Game Theory: Introduction and Overview

- Game theory deals with interactions among strategic agents.
- The term *game* in the phrase game theory corresponds to an interaction involving decision makers or players who are rational and intelligent.
- *Rationality* of a player implies that the player chooses his/her strategies so as to maximize a well define individualistic payoff while *intelligence* means that players are capable enough to compute their best strategies.
- Game theory is a tool for logical and mathematical analysis that models conflict as well as cooperation between the decision makers .
- It also provides a principle way of predicting the result of interactions among the players using equilibrium analysis.

Example 1: Student Coordination

- There are two students (1 and 2)
- The student derive utility by spending time together either studying at the SFU library or going to a Club.
- To spend time together they have two options (strategies): SFU or Club.
- If both of them are at SFU, each gets a payoff of 100.
- If both of them go to a Club, each gets a payoff of only 10.
- If one of them remains at SFU and the other goes to the Club, the payoff is 0 for each.

Payoffs for the students in different situations.

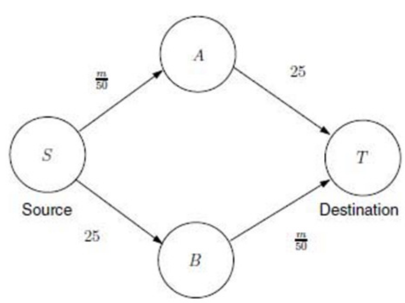
1	2	
	SFU	Club
SFU	100,100	0,0
Club	0,0	10,10

- Suppose the two friends have to choose their strategies simultaneously and independently of each other.
- Being rational and intelligent, each one would like to select the best possible strategy.
- They both select SFU as the best possible outcome and both opting for Club is also fine though clearly worse than both opting for SFU.
- The worst happens when they choose different options since each ends up with zero utility.
- Game theory helps us with a principle way of predicting the options that would be chosen by the students.
- In this case, the outcome of both opting for SFU and the outcome of both opting for Pub can be shown to be what are called Nash Equilibria which are strategy profiles in which no player is better off by unilaterally deviating from her/his equilibrium strategy.

- Game theory also provides one more prediction for this game which on the face of it is counter-intuitive but represents an equilibrium outcome that the students will not be averse to playing.
- This outcome which is technically called a mixed strategy Nash Equilibrium corresponds to the situation where each student chooses SFU with the probability 1/11 and Club with the probability 10/11.
- This perhaps explains why some students are found mostly in the Club and rarely at SFU.
- The above game which is often called the coordination game is an abstraction of many social, technical and emgineering situations in the real world.

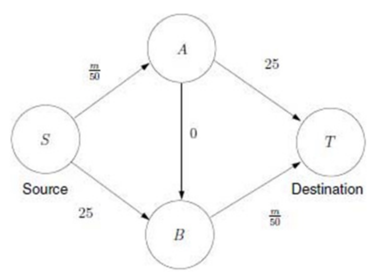
Example 2 – Braess' Paradox

- Figure shows a network that consists of a source S and a destination T, and two intermediate hubs A and B.



- All vehicles travelling from S can go via hub A or hub B.
- Suppose, regardless of the number of vehicles on the route, it takes 25 minutes to travel from S to B or from A to T.
- On the other hand, the travel time from S to A is m/50 minutes where m is the number of vehicles travelling on that link.
- Similarly, the travel time from B to T is m/50 minutes where m is the number of vehicles on that link.

- Suppose we now introduce an additional fast link A to B to ease the congestion in the network (let us assumed the degenerate case of having a travel time of zero minutes).
- Now the vehicles can go from S to T in three different ways: a) S to A to T; b) S to B to T; and 3) S to A to B to T.
- Intuition tells us that the second configuration where we have an additional link should make the users happier.
- However, game theoretic analysis proves, using equilibrium analysis, that the first configuration is in fact better for the users.



Example 3 – Divide the Dollar game

- Suppose there are three individuals who wish to divide a total wealth of 300 among themselves.
- Each player can propose an allocation such that no player's payoff is negative and the sum of all the payoffs does not exceed 300.
- Assume that if two or more players propose the same allocation, then that allocation will be implemented.
- For example, if players 1 and 2 proposes an allocation (150,150,0) and player 3 proposes (100,100,100), the allocation (150,150,0) will be implemented.

- However, player 3 may tempt player 2 with the allocation (0, 225,75) and if player 2 and 3 propose this, the original allocation (150,150,0) gets overturned. Note that this allocation is strictly better for both 2 and 3.
- Player 1 may now entice player 3 and jointly propose with player 3 an allocation (200,0,100) which is better for both 1 and 3.
- Bargaining of this kind can be never ending leading to the perpetual breaking and making of coalitions.

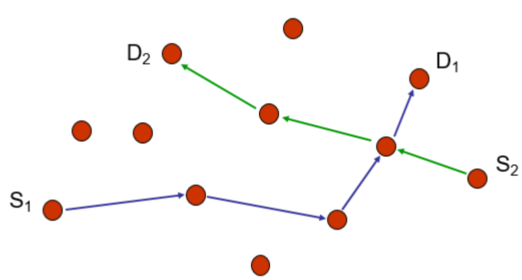
- Cooperative game theory helps us analyze such situations in a systematic and scientific way.
- For example, by modeling the above as a cooperative game, one can show that the *core of this game is empty implying that none of the allocations is stable and can always be derailed by a pair of players coming together.*
- *One can also show that the Shapley value of this game is (100,100,100) which provides a fair way of allocating the wealth among the three players in this case.*

Some Modern Applications

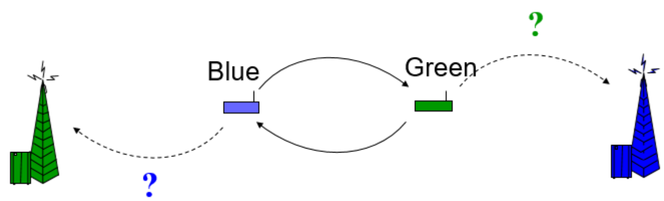
- Cooperative robotics, human/robot interaction, gaming.
- Matching Markets: Matching is the process of allocating one set of resources or individuals to another set of resources or individuals.
- Sponsored Search Auctions: is now a well known example of an extremely successful business model in Internet advertising. When a user searches a keyword, the search engine delivers a page with numerous results containing the links that are relevant to the keyword and also sponsored links that correspond to advertisements of selected advertisers.

- Crowdsourcing Mechanisms: it can be described as distribution of work to a possibly unknown group of human resources in the form of an open call.
- Social Network Analysis: Social network analysis is central to numerous Internet-based applications, for example, viral marketing, influence maximization, and influence limitation, that are based on social networks.

Cooperation in self-organized wireless networks



Usually, the devices are assumed to be cooperative. But what if they are not?



- users controlling the devices are ***rational*** = try to maximize their benefit
- game formulation: G = (P,S,U)
  - P: set of players
  - S: set of strategy functions
  - U: set of payoff functions

• Reward for packet reaching the destination: 1
• Cost of packet forwarding: c ($0 < c << 1$)

- ***strategic-form*** representation

Blue \ Green	Forward	Drop
Forward	(1-c, 1-c)	(-c, 1)
Drop	(1, -c)	(0, 0)

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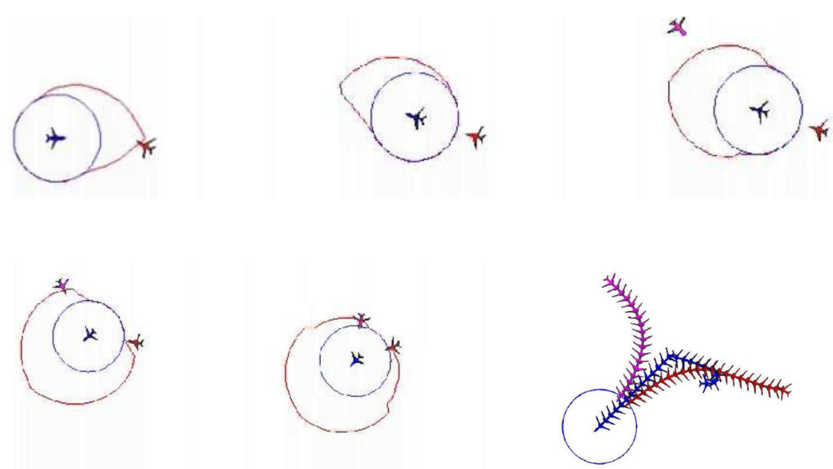
Evader and pursuer



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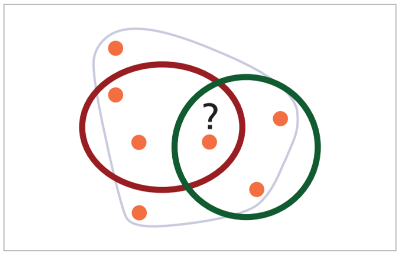
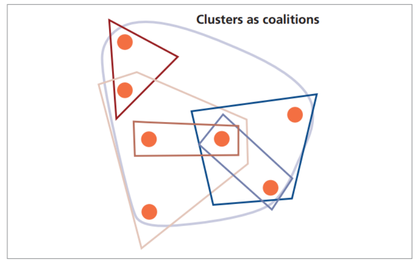
Clustering





An example of cooperative games strategies. In this example the player makes a choice between the "red" and the "green" coalitions – or stating with the ("blue") grand coalition.

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