Introduction to Decision Theory

Motivational background

- On September 6, 1492 Christopher Columbus sailed from Canary Islands, westbound to discover a new route between Europe and Far East.
- On October 12, 5 weeks later, he landed on middle America ('the West Indies') .
- After 29 years later, Magellan discovered the truth.

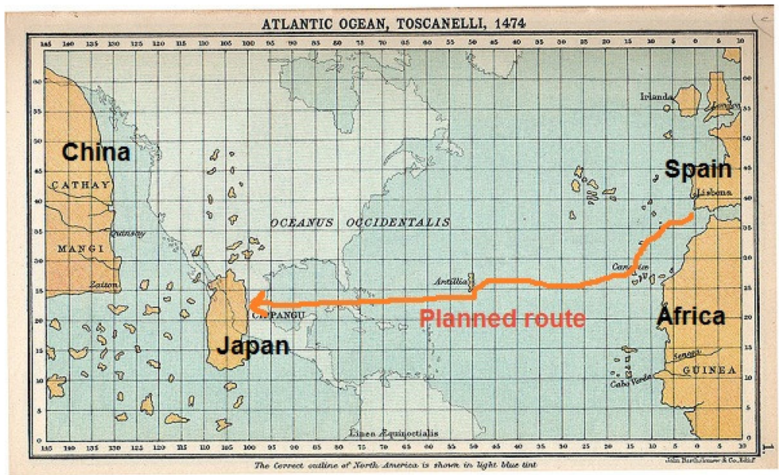
- Columbus's decision to sail west from the Canary Islands was arguably one of the bravest decisions ever made by an explorer.
- But, was it rational?

- Columbus believed in hypothesis that:

a) Earth is a small sphere.
b) Based on his geographical assumptions, he estimated the distance from Europe to East India to total of 2,330 miles. (The actual distance is about 12,200 miles, which is more than five times farther than what Columbus thought.)

- In the fifteenth century no ship would have been able to carry provisions for such a long journey.
- Had America not existed, or had the earth been flat, Columbus would certainly have faced a painful death.

Was it really worth risking everything for the sake of finding a new trade route?



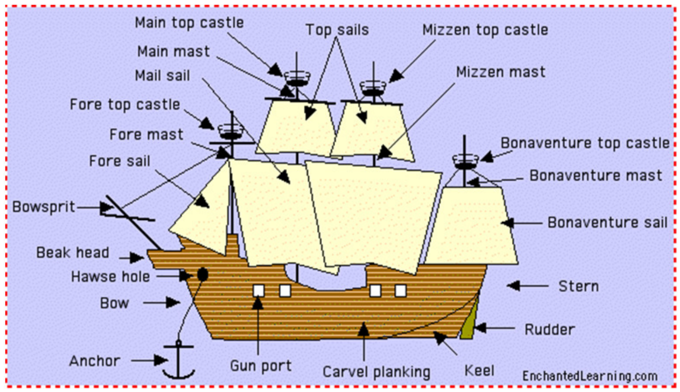
Introduction to Decision Theory

3

Introduction to Decision Theory

4

Fifteen's century ship industry



- A *decision maker*, in this case Columbus, chooses an act from a set of alternatives, such as sailing westwards or staying at home.

- The outcome depends on the true state of the world, which in many cases is only partially known to the decision maker.

- For example, had the Earth been a modest-sized sphere mostly covered by land and relatively small and navigable sea, Columbus' decision to sail westwards would have made him rich and famous, because the King and Queen of Spain had promised him ten percent of all revenue gained from a new trade route.

- However, Columbus' geographical hypothesis turned out to be false.

- Thus, in fifteen century the westward route was not a viable option for European wishing to trade with the Far east.

• All of these facts were unknown to Columbus.

•Despite this, the actual outcome of Columbus' decision was surprisingly good.

• When he returned to Spain he gained instant fame (though no financial reward).

•Another possible outcome would have been to never reach land again. Indeed a terrible way to die.

• The decision problem faced by Columbus on the Canary Islands in September 1492 can be summarized in the decision matrix shown in the following:

	Geographical hypothesis true	There is some other land westwards	There is no land westwards
Sail westwards	Rich and famous	Famous but not rich	Dead
Do not	Status quo	Status quo	Status quo

• Decision theory is an interdisciplinary study.

•There are distinction between descriptive and normative decision theory.

• Descriptive decision theory seek to explain and predict how people actually make decisions. This is an empirical discipline, stemming from experimental psychology.

• Normative theories seek to yield prescription about what decision makers are rationally – or ought – to do.

Risk, ignorance and uncertainty

In 1960s, Dr. Christian Barnard in Cape Town experimented on animals to develop a method for transplanting hearts.

In 1967 he offered 55-year-old Louis Washkansky the chance to become the first human to undergo a heart transplant.

Mr. Washkansky was dying of severe heart disease and in need of a new heart.

No one had ever before attempted to transplant a heart.

It would be meaningless to estimate the chance of success.

All Dr. Barnard can offer that his procedure worked successfully on animals.

Naturally, Mr. Washkansky knew he would not survive long without a new heart. He accepted the offer.

The operation was successful and Dr. Bernard's surgical method worked quite well.

Unfortunately, Mr. Washkansky died 18 days later.

The decision made by Mr. Washkansky was a decision under ignorance.

	Method works	Method fails
operation	Live on for some time	Death
No operation	Death	Death

- Since Mr. Washkansky surgery, thousands of patients had their lives prolonged by heart transplant.
- Interestingly enough, the decision to undergo a heart transplant is no longer a decision under ignorance and uncertainty.
- Increased medical knowledge has turned this kind of decision into decision under risk.
- For example, recent statistics show that 71.2% of all patients who undergoes a heart transplant survive on average of 14.8 years, 13.9% survive for 3.9 years, and 7.8% for 2.1 years. However, 7.1% die shortly after the operation.

•Let us assume that the patient's life expectancy after a heart transplant is determine entirely by her/his genes. For example,

Group I: people with this gene die on average 18 days after the operation (0.05 years).
Group II: people with this gene die on average 2.1 years after the operation.
Group III: people with this gene die on average 3.9 years after the operation.
Group IV: People with this gene die on average 14.8 years after the operation.

Nowadays such disease can be diagnosed at a very early stage with sophisticated drugs. The average life expectancy can be increased by 1.5 years.

	Group I: 7.1%	Group II: 7.8%	Group III: 13.9%	Group IV: 71.2%
operation	0.05 years	2.1 years	3.9 years	14.8 years
No operation	1.5 years	1.5 years	1.5 years	1.5 years

From principle of maximizing expected value which states the total value of an act equal to the sum of the values of its possible outcomes weighted by probability for each outcome, we can have:

Operation: (0.05*0.071)+(2.1*0.078)+(3.9*0.139)+(14.8*0.712)=11

No operation: (1.5*0.071)+(1.5*0.078)+(1.5*0.139)+(1.5*0.712)=1.5