Dominant Strategy Equilibria

•<u>Strongly Dominated Strategy</u> –
•*Given a strategic form game* $\Gamma = \langle N, (S_i), (u_i) \rangle$, a strategy $s_i \in S_i$ of player i is said to be strongly dominated by another strategy $s_i' \in S_i$ if:

$$u_i(s_i', s_{-i}) > u_i(s_i, s_{-i}) \quad \forall s_{-i} \in S_{-i}$$

Example: The prisoner's Dilemma (revisited) and Strict Dominance

- Two thieves plan to rob an electronics store. As they approach the backdoor, the police arrest them for trespassing.
- The cops suspect that the pair planned to break in but lack the evidence to support such accusation.
- They therefore require a confession to charge the suspects with the greater crime.
- The police individually sequesters both robbers and tells each of them the following:

•We are currently charging you with trespassing, which implies a one month jail sentence.

•I know you were planning on robbing the store, but right now I cannot prove it – I need your testimony. In exchange for your cooperation, I will dismiss your trespassing charge, and your partner will be charged to the fullest extend of the law: a twelve month jail sentence.

•I am offering your partner the same deal. If both of you confess, your individual testimony is no longer as valuable, and your jail sentence will be eight months each.

•If both criminals are self-interested and only care about minimizing their jail time, should they take the interrogator's deal?

Player 1	**Player 2**	
	Quiet	Confess
Quiet	-1,-1	-12,0
Confess	0,-12	-8,-8

Which strategy should each player choose?

- Consider the game from the player 1's perspective: suppose player 2 keep quiet. How should she response.
- Since player 1 only cares about her time in jail, we block payoff of player 2 payoffs with ?
- Player 1 should confess since if she does not, she will spend one month in the jail. Note that player 2's payoff is completely irrelevant to player 1's decision.

Player 1	**Player 2**
	Quiet
Quiet	-1,?
Confess	0,?

Suppose player 1 knew that player 2 will confess. What should she do?

Confession wins a second time: confession leads to eight months of jail time, whereas silence buys twelve.

So player 1 would want to confess if player 2 confesses.

Considering both cases, player 1 is better off confession regardless of player 2's strategy.

Player 1	Player 2
	confess
Quiet	-12,?
Confess	-8,?

Suppose player 2 knew that player 1 will keep quiet, even we realize she should not. Player 2 should confess as he get zero jail time.

Player 1	Player 2	
	quiet	confess
Quiet	?,-1	?,0

Suppose player 2 knew that player 1 will confess. How should he response?

He should confess and spend four fewer months in jail.

Player 1	Player 2	
	quiet	confess
Confess	?,-12	?,-8

Player 2 prefers confession regardless of what player 1 does.

We reached a solution: both players confess, and both players spend eight months in jail.

- Looking at the game matrix, it can be seen that the <quiet, quiet> outcome leaves both players better off than the <confess, confess> outcome.
- Then why the players cannot coordinate on keeping quiet.
- But as we just saw, promises to remain silent are unsustainable.
- Player 1 wants player 2 to keep quiet so when she confesses she walks away free.
- The same goes for player 2.
- As a result, the <quiet, quiet> outcome is inherently unstable.
- Ultimately, the players finish in the inferior (but sustainable) <confess, confess> outcome.

Player 1	**Player 2**	
	Quiet	Confess
Quiet	-1,-1	-12,0
Confess	0,-12	-8,-8

•Recall the original example prisoner's dilemma where N={1,2} and the payoff matrix is given by:

1	**2**	
	NC	C
NC	-2,-2	-10,-1
C	-1,-10	-5,-5

•Note that the strategy C is strongly dominated over strategy NC for player 1 since

$$u_1(C,NC) > u_1(NC,NC)$$
$$u_1(C,C) > u_1(NC,C)$$

- Similarly, the strategy *C* is strongly dominated over strategy NC for player 2 since:

$$u_2(NC,C) > u_2(NC,NC)$$
$$u_2(C,C) > u_2(C,NC)$$

- Thus *C* is a strongly dominant strategy for player 1 and also for player 2. Therefore (*C*,*C*) is a strongly dominant strategy equilibrium for this game.

1	2	
	NC	C
NC	-2,-2	-10,-1
C	-1,-10	-5,-5

- Suppose we change the payoff matrix to this:

	Quiet	Confess
Quiet	3,3	1,4
Confess	4,1	2,2

- Here we have replaced the month of jail time with an ordering of most to least preferred outcomes with 4 representing a player's most preferred outcome and 1 representing a player's least preferred outcome.

Even with these changes, confess is still always better than keep quiet. To see this, suppose player 2 kept quiet:

	Quiet
Quiet	3,?
Confess	4,?

Player 1 should confess, since 4 beats 3.

Likewise, suppose player 2 confessed:

	Confess
Quiet	1,?
Confess	2,?

Then player 1 should still confess, as 2 beats 1.

The same is true for player 2.

First, suppose player 1 kept quiet:

	Quiet	Confess
Quiet	?,3	?,4

Player 2 ought to confess, since 4 beats 3.
Alternatively, if player 1 confessed:

	Quiet	Confess
Confess	?,1	?,2

Player 2 should confess as well , as 2 greater than 1.

Thus, regardless of what the other player does, each player's best strategy is to confess.

Application of the Prisoner's Dilemma

- Consider two states considering whether to go to war.
- The military technology available to these countries gives the side that strikes first a large advantage in the fighting.
- In fact, the first-strike benefit is so great that each country would prefer attaching the other state even if its rival plays a peaceful strategy.
- However, because war destroys property and kills people, both prefer remaining at peace to simultaneously declaring war.

Using the above, here we define a game matrix

	Defend	Attack
Defend	3,3	1,4
Attack	4,1	2,2

It can be seen that states most prefer attacking while the other one playas defensively (this is due to the first-strike advantage).

Their next best outcome is to maintain the peace through mutual defensive strategies.

After that, they prefer declaring war simultaneously.

Each state's worst outcome is to choose defense while the other side acts as the aggressor.

- A similar problem exists with arm races.
- Imagine two nations must simultaneously choose whether to develop a new military technology.
- Considering weapons is expensive but provides greater security against rival nations. The matrix for this scenario can be defined as:

	Pass	Build
Pass	3,3	1,4
Build	4,1	2,2

- Here, a nation most prefer building while other nation passes.
- Following that, they prefer the <pass, pass> outcome to <build, build> outcome; the states maintain the same relative military strength in both of these outcomes, but they do not waste money on weaponry if they both pass.
- The worst possible outcome is for the other side to build while the original side passes.
- Again, we already know the solution to this game. Both sides engage in the arms race and build.

Strict Dominance in Asymmetric Games

- Consider the arms race from earlier. Suppose that player 2 maintains her same payoffs. That is, she most prefers arming while her opponent passes and least prefers the opposite outcome.
- Meanwhile, she prefers neither side arming to both arming, as the balance of power remains the same but she saves on the costs of weapons.

	Pass	Build
Pass	3,3	1,4
Build	4,1	2,2

- On the other hand, suppose that player 1 is a pacifist. He simply receives -1 for either nation that builds weapons.

	pass	build
pass	0,3	-1,4
build	-1,1	-2,2

- Unlike before, each player has a distinct set of payoffs. But, if we run through the same process as before, we will see that <pass, build> is the only reasonable solution.

- Lets begin with player 1's choices. Suppose player 2 moved passes. How should player 1 respond?

	Pass
Pass	0,?
Build	-1,?

- Recall that player 1 wants to minimize the total number of weapons. If he passes while player 2 passes, he achieves his best possible outcome. If he build, he received a -1. As such, he would want to pass in this situation.

- Now suppose player 2 chose builds. Again, we need to find how player 1 should optimally respond:

	Build
Pass	-1,?
Build	-2,?

- This time, player 1 cannot reach his best possible outcome. He can, however, minimize his losses by passing instead of building. Consequently, he would pass in this situation as well.

- Combining the last two inferences together, we know player 1's optimal strategy: he will pass regardless of how player 2 behaves.

- That leaves us to solve for player 2's strategy. Let's start on how she should respond to pass:

	Pass	Build
Pass	?,3	?,4

- Player 2 can achieve her best possible outcome here by building, since she can exploit the shift in power. Since 4 beats 3, she will build in this situation.

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- Now suppose player 1 builds instead:

	Pass	Build
Build	?,1	?,2

- Although player 2 can no longer reach her favorite outcome, she can at least keep peace with player 1's power by building here. As such, she should build if she knew that player 1 would build.

- Despite the game's asymmetry, the game still has a solution in dominant strategies: <pass, build>. Player 2 achieves her best outcome, while player 1 must settle for a moderate result since he cannot stop her from arming.

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