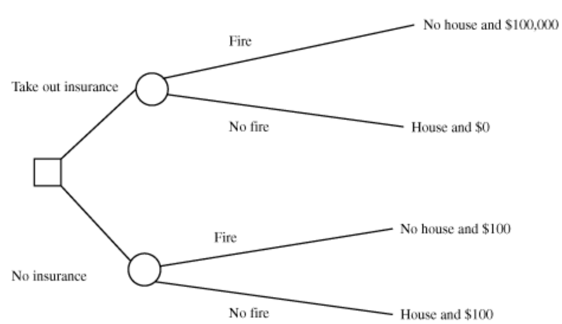
The Decision Matrix

- Before you make a decision you have somehow to determine what to decide about. Or, to put it differently, you have to specify what the relevant acts, states and outcomes are.
- Suppose, for instance, that you are thinking about taking out fire insurance on your apartment. Perhaps it costs $100 to take out insurance on an apartment worth $100,000, and you ask: Is it worth it?
- Before you decide, you have to get the formalization of the decision problem right. In this case, it seems that you face a decision problem with two acts, two states, and four outcomes. It is helpful to visualize this information in a decision matrix.

	Fire	No fire
Take out insurance	No house and $100,000	House and $0
No insurance	No house and $100	House and $100



Many decision theorists distinguish only between decision problems and a corresponding decision matrix or tree.

However, it is worth emphasizing that we are actually dealing with three levels of abstraction:

1 - The decision problem
2 - A formalization of the decision problem
3 - A visualization of the formalization

A decision problem is constituted by the entities of the world that prompt the decision maker to make a choice, or are otherwise relevant to that choice.

By definition, a formalization of a decision problem is made up of information about the decision to be made, irrespective of how that information is visualized. Formalizations thus comprise information about acts, states and outcomes, and sometimes also information about probabilities.

Of course, one and the same decision problem can be formalized in different ways, not all of which are likely to be equally good.

For example, some decision problems can be formalized either as decisions under risk or as decisions under ignorance, but if probabilities are known it is surely preferable to choose the former type of formalization (since one would otherwise overlook relevant information).

The basic building blocks of a decision problem are states, outcomes and acts.

States

Intuitively, a state is a part of the world that is not an outcome or an act (that can be performed by the agent in the present decision situation; acts performed by others can presumably be thought of as states).

Outcomes

Rational decision makers are not primarily concerned with states or acts. What ultimately matters is the outcome of the choice process. Acts are mere instruments for reaching good outcomes, and states are devices needed for applying these instruments.

However, in order to figure out which instrument to use (i.e. which act to choose given a set of states), outcomes must be ranked in one way or another, from the worst to the best. Exactly how this should be done is an important topic of debate in decision theory.

In order to measure the value of an outcome, as it is perceived by the decision maker, it is convenient to assign numbers to outcomes.

In decision theory, numbers referring to comparative evaluations of value are commonly called utilities (or values).

Values can be measured on two fundamentally different kinds of scales, viz. ordinal scales and cardinal scales.

Let us return to the issue of whether or not one should insure an apartment worth $100,000 at a rate of $100 per annum. Imagine that Jane has made a sincere effort to analyze her attitudes towards safety and money, and that she felt that the four possible outcomes should be ranked as follows, from the best to the worst.

1. House and $100 *is better than*
2. House and $0 *is better than*
3. No house and $100,000 *is better than*
4. No house and $100.

Consider the set of numbers assigned by Jane to the outcomes of her decision problem:

	Fire	No fire
Take out insurance	1	4
Do not	−100	10

If a higher number is assigned to one outcome than another, it means that the first outcome to be better than the second.

However, if the selected scale is an ordinal scale, nothing more than that follows. In particular, nothing can be concluded about how much better one outcome is in relation to another.

The numbers merely reflect the qualitative ranking of outcomes. No quantitative information about the 'distance' in value is reflected by the scale.

For example, the following scales could be used for representing exactly the same ordinal ranking.

	Original scale	Scale A	Scale B	Scale C
Best outcome	10	4	100	777
Second best	4	3	98	−378
Third best	1	2	97	−504
Worst outcome	−100	1	92	−777

1. Ordinal scale: Qualitative comparison of objects allowed; no information about differences or ratios. Example: The jury of a song contest award points to the participants. On this scale, 10 points is more than 5.

2. Cardinal scales

(a) Interval scale: Quantitative comparison of objects; accurately reflects differences between objects. Example: The Centigrade and Fahrenheit scales for temperature measurement are the most well-established examples. The difference between 10°C and 5°C equals that between 5°C and 0°C, but the difference between 10°C and 5°C does not equal that between 10°F and 5°F.

(b) Ratio scale: Quantitative comparison of objects; accurately reflects ratios between objects. Example: Height, mass, time, etc. 10 kg is twice as much as 5 kg, and 10 lb is also twice as much as 5 lb. But 10 kg is not twice as much as 5 lb.

In general, there are three types of utility functions:

- Risk Averse: concave

$$u(x) = \sqrt{x}.$$

- Risk Seeking: convex

$$u(x) = x^2$$

- Risk neutral: linear

$$u(x) = x$$

Acts

Imagine you want to make an omelette. You have already broken five eggs into the omelette, and plans to add a sixth. However, before breaking the last egg into the omelette, you suddenly start to worry that it might be rotten. After examining the egg carefully, you decide to take a chance and break the last egg into the omelette.

The act of adding the sixth egg can be conceived of as a function that takes either the first state (*The sixth egg is rotten*) or the second (*The sixth egg is not rotten*) as its argument.

If the first state happens to be the true state of the world, i.e. if it is inserted into the function, then it will return the outcome *No omelette*, and if the second state happens to be the true state, the value of the function will be *Six egg omelette*.

This definition of acts can be trivially generalized to cover cases with more than two states and outcomes: an act is a function from a set of states to a set of outcomes.

	The sixth egg is rotten	The sixth egg is not rotten
Add sixth egg	No omelette	Six egg omelette
Do not add sixth egg	Five egg omelette	Five egg omelette

Decision theory is primarily concerned with particular acts, rather than generic acts. A generic act, such as sailing, walking or swimming can be instantiated by different agents at different time intervals.

Hence, Columbus' first voyage to America and James Cook's trip to the southern hemisphere are both instantiations of the same generic act, viz. sailing.

Particular acts, on the other hand, are always carried out by specific agents at specific time intervals, and hence Columbus' and Cook's voyages were different particular acts.

Rival formalizations

The decision maker (or decision making system) may sometimes be confronted with rival formalizations of one and the same decision problem. Rival formalizations arise if two or more formalizations are equally reasonable and strictly better than all alternative formalizations.

Obviously, rival formalizations are troublesome if an act is judged to be rational in one optimal formalization of a decision problem, but non-rational in another optimal formalization of the same decision problem.

In such cases one may legitimately ask whether the act in question should be performed or not.

What should a rational decision maker do?

Imagine that you are a paparazzi photographer and that rumor has it that a famous actress will show up in either New York (NY), Los Angeles (LA) or Paris (P).

Nothing is known about the probability of these states of the world.

You have to decide if you should stay in America or catch a plane to Paris. If you stay and actress shows up in Paris you get $0; otherwise you get your photos, which you will be able to sell for $10,000. If you catch a plane to Paris and she shows up in Paris your net gain after having paid for the ticket is $5,000, and if she shows up in America you for some reason, never mind why, get $6,000. Your initial representation of the decision problem is visualized as:

	P	LA	NY
Stay	$0	$10k	$10k
Go to Paris	$5k	$6k	$6k

Since nothing is known about the probabilities of the states, you decide it makes sense to regard them as equally probable, i.e. you decide to assign probability 1/3 to each state. Consider the decision matrix in the following:

	P (1/3)	LA (1/3)	NY (1/3)
Stay	\$0	\$10k	\$10k
Go to Paris	\$5k	\$6k	\$6k

The two rightmost columns are exactly similar. Therefore, they can be merged into a single (disjunctive) column, by adding the probabilities of the two rightmost columns together :

	P (1/3)	LA or NY (2/3)
Stay	\$0	\$10k
Go to Paris	\$5k	\$6k

However, now suppose that you instead start with the original matrix and first merge the two repetitious states into a single state. You would then obtain the following decision matrix:

	P	LA or NY
Stay	\$0	\$10k
Go to Paris	\$5k	\$6k

Now, since you know nothing about the probabilities of the two states, you decide to regard them as equally probable, i.e. you assign a probability of 1/2 to each state. This yields the formal representation in the following table, which is clearly different from the one suggested in the above:

	P (1/2)	LA or NY (1/2)
Stay	\$0	\$10k
Go to Paris	\$5k	\$6k

Which formalization is best? It seems question begging to claim that one of them must be better than the other – so perhaps they are equally reasonable? If they are, we have an example of rival formalizations.