

### Decision under ignorance

- Jane needs to select from the main course in this newly opened French Bistro.
- There are only two options in the menu, Hamburger and Lotte de mer.
- Jane recalled that Lotte de mer means monkfish, and she feels that this would be a nice option as long as it is cooked by a first-class chef.
- However, she has some vague suspicions that this may not be the case in this particular restaurant.
- The starter was rather poor and cooking monkfish is difficult.
- Virtually any restaurant can serve edible hamburgers, however.

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- Jane feels that she can not assign any probability to the prospect of getting good monkfish.
- She simply knows too little about the newly operated restaurant.
- She is facing a decision under ignorance.
- In Decision Theory, Ignorance refers to cases in which the decision maker (i) knows what alternatives are and what outcomes they may results in, but (ii) unable to assign any probabilities to the states corresponding to the outcomes.
- Sometimes the term 'decision under uncertainty' is used synonymously.

	Good Chef	Bad Chef
Lotte de mer	Good monkfish	Terrible monkfish
Hamburger	Edible hamburger	Edible hamburger
No main course	hungry	hungry

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- The ranking can be represented on an ordinal scale. Since ordinal scales only preserve the order between objects, any set of numbers will do as long as better outcomes are represented by higher numbers. For example,

	s1	s2
a1	4	1
a2	3	3
a3	2	2

Which alternative should Jane chooses?

To answer this question the decision maker has to apply some decision rule, or provide some other kind of structured reasoning for choosing one alternative over another.

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### Dominance

- When Jane is about to choose a main course, rationality forbids her to choose a3.
- This is because a2 will lead to a better outcome than a3 no matter which state happens to be the true state of the world.
- Decision theorist say that alternative a2 dominates alternative a3.
- One alternative is better than another if (but not only if) the agent knows for sure that she will be at least as well off if the first alternative is chosen, no matter what state of the world turns out to be the true state.

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- Unfortunately, the dominance principle cannot tell the whole truth about decision making under ignorance.
- It merely provides a partial answer. This is because it sometimes fails to single out a set of optimal acts.
- For example, in Jane's decision at the restaurant, act  $a_1$  is not dominated by  $a_2$ , nor is  $a_2$  dominated by  $a_1$ . All that can be concluded is that it would not be rational to do  $a_3$ .
- This means that some additional criterion is needed to reach a decision.

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This *dominance principle* comes in two forms:

- Weak dominance*: One act is *more* rational than another if (1) all its possible outcomes are at least as good as those of the other, and if (2) there is at least one possible outcome that is better than that of the other act.
- Strong dominance*: One act is *more* rational than another if all of its possible outcome are better than that of the other act.









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**Strong Dominance**

	Good Chef	Bad Chef
Hamburger	 	 
No main	 	 

**Weak Dominance**

	Good Chef	Bad Chef
Hamburger (if good chef) No Main (otherwise)	 	 
No main	 	 

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**Maximin and Leximin**

- The maximin principle focuses on the worst possible outcome of each alternative.
- According to this principle, one should maximize the minimal value obtainable with each act.
- If the worst possible outcome of one alternative is better than that of another, then the former should be chosen.

	s1	s2	s3	s4
a1	6	9	3	0
a2	-5	7	4	12
a3	6	4	5	2
a4	14	-8	5	7

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Returning to Jane's example, we can write:

	Good chef	Bad chef
Monkfish	Best outcome	Worst outcome
Hamburger	2 <sup>nd</sup> best outcome	2 <sup>nd</sup> best outcome
No main course	3 <sup>rd</sup> best outcome	3 <sup>rd</sup> best outcome

What is the action related to maximin value?

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- If the worst outcomes of two or more acts are equally good the maximin rule tells you to be indifferent.
- The standard remedy to this objection is to invoke the lexical maximin rule (leximin rule).
- It holds that if the worst outcomes are equal, one should choose an alternative such that *the second worst outcome* is certain to be as good as possible.
- If this does not single out a unique act, then the third worst outcome should be considered, and so on.

	s1	s2	s3	s4
a1	6	14	8	5
a2	5	6	100	100
a3	7	7	7	5

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### Maximax and the optimism-pessimism rule

- The maximin rule singles out the worst possible outcome of a decision.
- However, one could equally well single out the best possible outcome.
- When J.K. Rowling is just about to release her seventh Potter book, we all knew that it will sell millions of copies.
- Clearly, as Ms. Rowling wrote her first story about an unknown boy called Harry Potter and submitted it to a publisher, she did not think much about the worst-case scenario, i.e. to have the manuscript rejected.
- What triggered her to actually submit the book to the publisher was no doubt her desire to achieve *the best possible outcome* of this decision.

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- Luckily for her, the best possible outcome also turned out to be the *actual* outcome.
- If all the authors considering submitting a manuscript always decided to focus on the worst-case, i.e. rejection and disappointment, no books would ever be published.
- The above example illustrates the maximax rule, i.e. one should maximize the maximal value obtainable with an act.
- The maximax rule has surprisingly few adherents among decision theorists.

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However, maximax also serves as a helpful introduction to another rule, which has attracted considerably more attention by decision theorists.

This is the optimism-pessimism rule, originally proposed by Hurwicz. (A synonymous name for this rule is the alpha-index rule.)

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- Optimism-pessimism rule asks the decision maker to consider both the best and the worst possible outcome of each alternative, and then choose an alternative according to her degree of optimism and pessimism.

- Suppose the decision maker's degree of optimism can be represented by a real number  $\alpha$  between 0 and 1, such that  $\alpha = 1$  corresponds to maximal optimism and  $\alpha = 0$  to maximal pessimism.

- Then if  $\max(a_i)$  is the best possible outcome of alternative  $a_i$  and  $\min(a_i)$  its worst possible outcome, its value is:

$$\alpha \cdot \max(a_i) + (1 - \alpha) \cdot \min(a_i).$$

- Then  $a_i \succ a_j$  iff

$$\alpha \max(a_i) + (1 - \alpha) \min(a_i) > \alpha \cdot \max(a_j) + (1 - \alpha) \cdot \min(a_j).$$

Naturally,  $\alpha$  is assumed to be fixed throughout the evaluation of all alternatives  $a_i$ .

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Consider the following table:

a1	55	18	28	10	36	100
a2	50	87	55	90	75	70

Let degree of optimism be defined by  $\alpha = 0.70$ .

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### Minimax regret

- Imagine that you are invited to invest in a newly launched engineering company.
- Some of your friends, who invested in similar companies a few years ago, made a fortune.
- Of course, you may lose the money, but the investment is rather small, so the fact that the company may go bankrupt is not a genuine issue of concern.
- However, you know that if you refrain from making the investment and the company turns out to be yet another success story, you will feel a lot of **regret**.
- The minimax regret rule maintain that the best alternative is one that minimizes the maximum amount of regret.

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For example,

a1	12	8	20	20
a2	10	15	16	8
a3	30	6	25	14
a4	20	4	30	10

Find the minmax of the decision matrix and the associated regret matrix.

### The principle of insufficient reason

- A number of prominent thinkers, including Bernoulli (1654-1705) and Laplace (1749-1827), have defended a rule known as the principle of insufficient reason.
- Imagine that you have booked a weekend trip to a foreign country. Unfortunately, you know nothing about the weather at your destination (e.g. before the age of Internet).
- What kind of clothes should you take?
- You have three types of wardrobe for three types of weather, viz. hot, medium and cold weather.

### The decision Matrix

	Actual temp is 38 C	Actual Temp is 10 C	Actual Temp is -17 C
Bring cloths suitable for 38 C	15	0	-30
Bring cloths suitable for 10 C	0	15	0
Bring cloths suitable for -17 C	-15	0	15

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The principle of insufficient reason prescribes that if one has *no* reasons to think that one state of the world is more probable than another, then all states should be assigned *equal* probability.

	s1(1/3)	s2(1/3)	s3(1/3)
a1	15	0	-30
a2	0	15	0
a3	-15	0	15

- By applying the principle of insufficient reason, an initial problem under ignorance is transformed into a decision problem under risk, that is, a decision problem with known probabilities and values.
- One can now maximize expected value in decision under risk, e.g. the expected value of bringing clothes suitable for 38 C).
- What are the expected values?

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