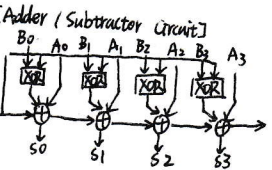


Adder
[Binary Adder]
 $S = A \oplus B \oplus C_{in}$
 $C = AB + AC_{in} + BC_{in}$
 $\rightarrow = (A \oplus B)C_{in} + AB$
Can share the term $A \oplus B$

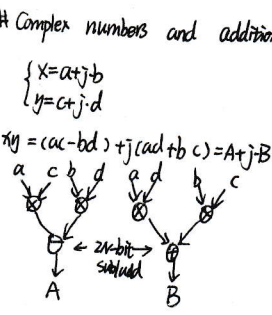
[Look from a different angle]
Generate signal: $G = A \cdot B$
Propagate function: $P = A \oplus B$
We use $P = A \oplus B$ since it shows the same behaviour and also can be used to generate the sum term
 $S = P \oplus C_{in}$
 $C = G + P \cdot C_{in}$

[Multi-bit adder]
Ripple carry/carry propagate
 $T_d (worst) = (N-1)T_{cp} + T_{sum}$

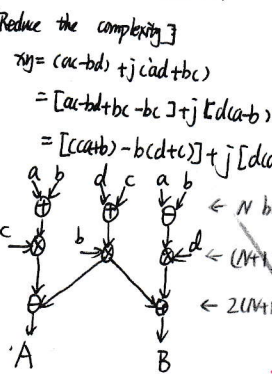
[Carry chain Optimisation]
 $S = ABC_{in} + (A+B)C_{in}$
For each of the full adders the C signal is coming earlier than the sum signal.



Multiplier
[Array Multiplier]
 $N \text{ bit} \rightarrow \Rightarrow (N(N-2)/2) \text{ FAs}$
 $N \text{ HAS } \approx \frac{N}{2} \text{ FA}$
 $N^2 \text{ ANDs}$



Complexity: $4[(N(N-2)/2) \text{ FA} + \frac{N}{2} \text{ FA} + N^2 \text{ AND}] + 2(2N-1) \text{ FA}$



Carry-Lookahead Adder (CLA)
To predict C_k earlier than $T_{C^*}K$

A	B	C_{in}	C	
0	0	x	0	Kill
0	1	C	C	Propagate
1	0	C	C	Propagate
1	1	x	1	Generate

$P = A \oplus B$ or $(A \oplus B)$
 $G = A \cdot B$
 $\rightarrow C_4 = 1$
 G_3
 G_2P_3
 $G_1P_2P_3$
 $G_0P_1P_2P_3$
 $C_{in}P_0P_1P_2P_3$
 $C_4 = G_3 + G_2P_3 + G_1P_2P_3 + G_0P_1P_2P_3 + C_{in}P_0P_1P_2P_3$

Booth's algorithm

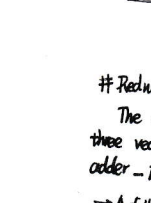
V_0	V_1	V_2	V_3	Operation
0	0	0	0	Add Zero
1	0	-1		Subtract
0	1	1		Add
1	1	0		Add Zero



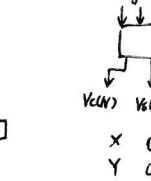
Modified Booth Algorithm

V_{m-1}	V_i	V_{i-1}	Operation
0	0	0	Do nothing
0	0	1	Add 1^*
0	1	0	Add 2^*
0	1	1	Sub 2^*
1	0	0	Sub 1^*
1	0	1	Sub 2^*
1	1	1	Do nothing

One bit shifter



Multiplication using shifter



Redundant Arithmetic

The basic idea is to perform an addition of three vectors (or inputs) using an array of 1-bit adder - BUT without propagating the carries.
 \rightarrow A full adder takes 3 inputs (a, b, carry-ins) and converts them to 2 bits (sum and carry-out)
 $(X_{N-1}Y_{N-1})Z_{N-1}$
 $(X_{N-2}Y_{N-2})Z_{N-2}$
 $(X_{N-3}Y_{N-3})Z_{N-3}$
 $(X_{N-4}Y_{N-4})Z_{N-4}$
 $(X_{N-5}Y_{N-5})Z_{N-5}$
 $(X_{N-6}Y_{N-6})Z_{N-6}$
 $(X_{N-7}Y_{N-7})Z_{N-7}$
 $(X_{N-8}Y_{N-8})Z_{N-8}$
 $(X_{N-9}Y_{N-9})Z_{N-9}$
 $(X_{N-10}Y_{N-10})Z_{N-10}$
 $(X_{N-11}Y_{N-11})Z_{N-11}$
 $(X_{N-12}Y_{N-12})Z_{N-12}$
 $(X_{N-13}Y_{N-13})Z_{N-13}$
 $(X_{N-14}Y_{N-14})Z_{N-14}$
 $(X_{N-15}Y_{N-15})Z_{N-15}$
 $(X_{N-16}Y_{N-16})Z_{N-16}$
 $(X_{N-17}Y_{N-17})Z_{N-17}$
 $(X_{N-18}Y_{N-18})Z_{N-18}$
 $(X_{N-19}Y_{N-19})Z_{N-19}$
 $(X_{N-20}Y_{N-20})Z_{N-20}$
 $(X_{N-21}Y_{N-21})Z_{N-21}$
 $(X_{N-22}Y_{N-22})Z_{N-22}$
 $(X_{N-23}Y_{N-23})Z_{N-23}$
 $(X_{N-24}Y_{N-24})Z_{N-24}$
 $(X_{N-25}Y_{N-25})Z_{N-25}$
 $(X_{N-26}Y_{N-26})Z_{N-26}$
 $(X_{N-27}Y_{N-27})Z_{N-27}$
 $(X_{N-28}Y_{N-28})Z_{N-28}$
 $(X_{N-29}Y_{N-29})Z_{N-29}$
 $(X_{N-30}Y_{N-30})Z_{N-30}$
 $(X_{N-31}Y_{N-31})Z_{N-31}$
 $(X_{N-32}Y_{N-32})Z_{N-32}$
 $(X_{N-33}Y_{N-33})Z_{N-33}$
 $(X_{N-34}Y_{N-34})Z_{N-34}$
 $(X_{N-35}Y_{N-35})Z_{N-35}$
 $(X_{N-36}Y_{N-36})Z_{N-36}$
 $(X_{N-37}Y_{N-37})Z_{N-37}$
 $(X_{N-38}Y_{N-38})Z_{N-38}$
 $(X_{N-39}Y_{N-39})Z_{N-39}$
 $(X_{N-40}Y_{N-40})Z_{N-40}$
 $(X_{N-41}Y_{N-41})Z_{N-41}$
 $(X_{N-42}Y_{N-42})Z_{N-42}$
 $(X_{N-43}Y_{N-43})Z_{N-43}$
 $(X_{N-44}Y_{N-44})Z_{N-44}$
 $(X_{N-45}Y_{N-45})Z_{N-45}$
 $(X_{N-46}Y_{N-46})Z_{N-46}$
 $(X_{N-47}Y_{N-47})Z_{N-47}$
 $(X_{N-48}Y_{N-48})Z_{N-48}$
 $(X_{N-49}Y_{N-49})Z_{N-49}$
 $(X_{N-50}Y_{N-50})Z_{N-50}$
 $(X_{N-51}Y_{N-51})Z_{N-51}$
 $(X_{N-52}Y_{N-52})Z_{N-52}$
 $(X_{N-53}Y_{N-53})Z_{N-53}$
 $(X_{N-54}Y_{N-54})Z_{N-54}$
 $(X_{N-55}Y_{N-55})Z_{N-55}$
 $(X_{N-56}Y_{N-56})Z_{N-56}$
 $(X_{N-57}Y_{N-57})Z_{N-57}$
 $(X_{N-58}Y_{N-58})Z_{N-58}$
 $(X_{N-59}Y_{N-59})Z_{N-59}$
 $(X_{N-60}Y_{N-60})Z_{N-60}$
 $(X_{N-61}Y_{N-61})Z_{N-61}$
 $(X_{N-62}Y_{N-62})Z_{N-62}$
 $(X_{N-63}Y_{N-63})Z_{N-63}$
 $(X_{N-64}Y_{N-64})Z_{N-64}$
 $(X_{N-65}Y_{N-65})Z_{N-65}$
 $(X_{N-66}Y_{N-66})Z_{N-66}$
 $(X_{N-67}Y_{N-67})Z_{N-67}$
 $(X_{N-68}Y_{N-68})Z_{N-68}$
 $(X_{N-69}Y_{N-69})Z_{N-69}$
 $(X_{N-70}Y_{N-70})Z_{N-70}$
 $(X_{N-71}Y_{N-71})Z_{N-71}$
 $(X_{N-72}Y_{N-72})Z_{N-72}$
 $(X_{N-73}Y_{N-73})Z_{N-73}$
 $(X_{N-74}Y_{N-74})Z_{N-74}$
 $(X_{N-75}Y_{N-75})Z_{N-75}$
 $(X_{N-76}Y_{N-76})Z_{N-76}$
 $(X_{N-77}Y_{N-77})Z_{N-77}$
 $(X_{N-78}Y_{N-78})Z_{N-78}$
 $(X_{N-79}Y_{N-79})Z_{N-79}$
 $(X_{N-80}Y_{N-80})Z_{N-80}$
 $(X_{N-81}Y_{N-81})Z_{N-81}$
 $(X_{N-82}Y_{N-82})Z_{N-82}$
 $(X_{N-83}Y_{N-83})Z_{N-83}$
 $(X_{N-84}Y_{N-84})Z_{N-84}$
 $(X_{N-85}Y_{N-85})Z_{N-85}$
 $(X_{N-86}Y_{N-86})Z_{N-86}$
 $(X_{N-87}Y_{N-87})Z_{N-87}$
 $(X_{N-88}Y_{N-88})Z_{N-88}$
 $(X_{N-89}Y_{N-89})Z_{N-89}$
 $(X_{N-90}Y_{N-90})Z_{N-90}$
 $(X_{N-91}Y_{N-91})Z_{N-91}$
 $(X_{N-92}Y_{N-92})Z_{N-92}$
 $(X_{N-93}Y_{N-93})Z_{N-93}$
 $(X_{N-94}Y_{N-94})Z_{N-94}$
 $(X_{N-95}Y_{N-95})Z_{N-95}$
 $(X_{N-96}Y_{N-96})Z_{N-96}$
 $(X_{N-97}Y_{N-97})Z_{N-97}$
 $(X_{N-98}Y_{N-98})Z_{N-98}$
 $(X_{N-99}Y_{N-99})Z_{N-99}$
 $(X_{N-100}Y_{N-100})Z_{N-100}$
 $(X_{N-101}Y_{N-101})Z_{N-101}$
 $(X_{N-102}Y_{N-102})Z_{N-102}$
 $(X_{N-103}Y_{N-103})Z_{N-103}$
 $(X_{N-104}Y_{N-104})Z_{N-104}$
 $(X_{N-105}Y_{N-105})Z_{N-105}$
 $(X_{N-106}Y_{N-106})Z_{N-106}$
 $(X_{N-107}Y_{N-107})Z_{N-107}$
 $(X_{N-108}Y_{N-108})Z_{N-108}$
 $(X_{N-109}Y_{N-109})Z_{N-109}$
 $(X_{N-110}Y_{N-110})Z_{N-110}$
 $(X_{N-111}Y_{N-111})Z_{N-111}$
 $(X_{N-112}Y_{N-112})Z_{N-112}$
 $(X_{N-113}Y_{N-113})Z_{N-113}$
 $(X_{N-114}Y_{N-114})Z_{N-114}$
 $(X_{N-115}Y_{N-115})Z_{N-115}$
 $(X_{N-116}Y_{N-116})Z_{N-116}$
 $(X_{N-117}Y_{N-117})Z_{N-117}$
 $(X_{N-118}Y_{N-118})Z_{N-118}$
 $(X_{N-119}Y_{N-119})Z_{N-119}$
 $(X_{N-120}Y_{N-120})Z_{N-120}$
 $(X_{N-121}Y_{N-121})Z_{N-121}$
 $(X_{N-122}Y_{N-122})Z_{N-122}$
 $(X_{N-123}Y_{N-123})Z_{N-123}$
 $(X_{N-124}Y_{N-124})Z_{N-124}$
 $(X_{N-125}Y_{N-125})Z_{N-125}$
 $(X_{N-126}Y_{N-126})Z_{N-126}$
 $(X_{N-127}Y_{N-127})Z_{N-127}$
 $(X_{N-128}Y_{N-128})Z_{N-128}$
 $(X_{N-129}Y_{N-129})Z_{N-129}$
 $(X_{N-130}Y_{N-130})Z_{N-130}$
 $(X_{N-131}Y_{N-131})Z_{N-131}$
 $(X_{N-132}Y_{N-132})Z_{N-132}$
 $(X_{N-133}Y_{N-133})Z_{N-133}$
 $(X_{N-134}Y_{N-134})Z_{N-134}$
 $(X_{N-135}Y_{N-135})Z_{N-135}$
 $(X_{N-136}Y_{N-136})Z_{N-136}$
 $(X_{N-137}Y_{N-137})Z_{N-137}$
 $(X_{N-138}Y_{N-138})Z_{N-138}$
 $(X_{N-139}Y_{N-139})Z_{N-139}$
 $(X_{N-140}Y_{N-140})Z_{N-140}$
 $(X_{N-141}Y_{N-141})Z_{N-141}$
 $(X_{N-142}Y_{N-142})Z_{N-142}$
 $(X_{N-143}Y_{N-143})Z_{N-143}$
 $(X_{N-144}Y_{N-144})Z_{N-144}$
 $(X_{N-145}Y_{N-145})Z_{N-145}$
 $(X_{N-146}Y_{N-146})Z_{N-146}$
 $(X_{N-147}Y_{N-147})Z_{N-147}$
 $(X_{N-148}Y_{N-148})Z_{N-148}$
 $(X_{N-149}Y_{N-149})Z_{N-149}$
 $(X_{N-150}Y_{N-150})Z_{N-150}$
 $(X_{N-151}Y_{N-151})Z_{N-151}$
 $(X_{N-152}Y_{N-152})Z_{N-152}$
 $(X_{N-153}Y_{N-153})Z_{N-153}$
 $(X_{N-154}Y_{N-154})Z_{N-154}$
 $(X_{N-155}Y_{N-155})Z_{N-155}$
 $(X_{N-156}Y_{N-156})Z_{N-156}$
 $(X_{N-157}Y_{N-157})Z_{N-157}$
 $(X_{N-158}Y_{N-158})Z_{N-158}$
 $(X_{N-159}Y_{N-159})Z_{N-159}$
 $(X_{N-160}Y_{N-160})Z_{N-160}$
 $(X_{N-161}Y_{N-161})Z_{N-161}$
 $(X_{N-162}Y_{N-162})Z_{N-162}$
 $(X_{N-163}Y_{N-163})Z_{N-163}$
 $(X_{N-164}Y_{N-164})Z_{N-164}$
 $(X_{N-165}Y_{N-165})Z_{N-165}$
 $(X_{N-166}Y_{N-166})Z_{N-166}$
 $(X_{N-167}Y_{N-167})Z_{N-167}$
 $(X_{N-168}Y_{N-168})Z_{N-168}$
 $(X_{N-169}Y_{N-169})Z_{N-169}$
 $(X_{N-170}Y_{N-170})Z_{N-170}$
 $(X_{N-171}Y_{N-171})Z_{N-171}$
 $(X_{N-172}Y_{N-172})Z_{N-172}$
 $(X_{N-173}Y_{N-173})Z_{N-173}$
 $(X_{N-174}Y_{N-174})Z_{N-174}$
 $(X_{N-175}Y_{N-175})Z_{N-175}$
 $(X_{N-176}Y_{N-176})Z_{N-176}$
 $(X_{N-177}Y_{N-177})Z_{N-177}$
 $(X_{N-178}Y_{N-178})Z_{N-178}$
 $(X_{N-179}Y_{N-179})Z_{N-179}$
 $(X_{N-180}Y_{N-180})Z_{N-180}$
 $(X_{N-181}Y_{N-181})Z_{N-181}$
 $(X_{N-182}Y_{N-182})Z_{N-182}$
 $(X_{N-183}Y_{N-183})Z_{N-183}$
 $(X_{N-184}Y_{N-184})Z_{N-184}$
 $(X_{N-185}Y_{N-185})Z_{N-185}$
 $(X_{N-186}Y_{N-186})Z_{N-186}$
 $(X_{N-187}Y_{N-187})Z_{N-187}$
 $(X_{N-188}Y_{N-188})Z_{N-188}$
 $(X_{N-189}Y_{N-189})Z_{N-189}$
 $(X_{N-190}Y_{N-190})Z_{N-190}$
 $(X_{N-191}Y_{N-191})Z_{N-191}$
 $(X_{N-192}Y_{N-192})Z_{N-192}$
 $(X_{N-193}Y_{N-193})Z_{N-193}$
 $(X_{N-194}Y_{N-194})Z_{N-194}$
 $(X_{N-195}Y_{N-195})Z_{N-195}$
 $(X_{N-196}Y_{N-196})Z_{N-196}$
 $(X_{N-197}Y_{N-197})Z_{N-197}$
 $(X_{N-198}Y_{N-198})Z_{N-198}$
 $(X_{N-199}Y_{N-199})Z_{N-199}$
 $(X_{N-200}Y_{N-200})Z_{N-200}$
 $(X_{N-201}Y_{N-201})Z_{N-201}$
 $(X_{N-202}Y_{N-202})Z_{N-202}$
 $(X_{N-203}Y_{N-203})Z_{N-203}$
 $(X_{N-204}Y_{N-204})Z_{N-204}$
 $(X_{N-205}Y_{N-205})Z_{N-205}$
 $(X_{N-206}Y_{N-206})Z_{N-206}$
 $(X_{N-207}Y_{N-207})Z_{N-207}$
 $(X_{N-208}Y_{N-208})Z_{N-208}$
 $(X_{N-209}Y_{N-209})Z_{N-209}$
 $(X_{N-210}Y_{N-210})Z_{N-210}$
 $(X_{N-211}Y_{N-211})Z_{N-211}$
 $(X_{N-212}Y_{N-212})Z_{N-212}$
 $(X_{N-213}Y_{N-213})Z_{N-213}$
 $(X_{N-214}Y_{N-214})Z_{N-214}$
 $(X_{N-215}Y_{N-215})Z_{N-215}$
 $(X_{N-216}Y_{N-216})Z_{N-216}$
 $(X_{N-217}Y_{N-217})Z_{N-217}$
 $(X_{N-218}Y_{N-218})Z_{N-218}$
 $(X_{N-219}Y_{N-219})Z_{N-219}$
 $(X_{N-220}Y_{N-220})Z_{N-220}$
 $(X_{N-221}Y_{N-221})Z_{N-221}$
 $(X_{N-222}Y_{N-222})Z_{N-222}$
 $(X_{N-223}Y_{N-223})Z_{N-223}$
 $(X_{N-224}Y_{N-224})Z_{N-224}$
 $(X_{N-225}Y_{N-225})Z_{N-225}$
 $(X_{N-226}Y_{N-226})Z_{N-226}$
 $(X_{N-227}Y_{N-227})Z_{N-227}$
 $(X_{N-228}Y_{N-228})Z_{N-228}$
 $(X_{N-229}Y_{N-229})Z_{N-229}$
 $(X_{N-230}Y_{N-230})Z_{N-230}$
 $(X_{N-231}Y_{N-231})Z_{N-231}$
 $(X_{N-232}Y_{N-232})Z_{N-232}$
 $(X_{N-233}Y_{N-233})Z_{N-233}$
 $(X_{N-234}Y_{N-234})Z_{N-234}$
 $(X_{N-235}Y_{N-235})Z_{N-235}$
 $(X_{N-236}Y_{N-236})Z_{N-236}$
 $(X_{N-237}Y_{N-237})Z_{N-237}$
 $(X_{N-238}Y_{N-238})Z_{N-238}$
 $(X_{N-239}Y_{N-239})Z_{N-239}$
 $(X_{N-240}Y_{N-240})Z_{N-240}$
 $(X_{N-241}Y_{N-241})Z_{N-241}$
 $(X_{N-242}Y_{N-242})Z_{N-242}$
 $(X_{N-243}Y_{N-243})Z_{N-243}$
 $(X_{N-244}Y_{N-244})Z_{N-244}$
 $(X_{N-245}Y_{N-245})Z_{N-245}$
 $(X_{N-246}Y_{N-246})Z_{N-246}$
 $(X_{N-247}Y_{N-247})Z_{N-247}$
 $(X_{N-248}Y_{N-248})Z_{N-248}$
 $(X_{N-249}Y_{N-249})Z_{N-249}$
 $(X_{N-250}Y_{N-250})Z_{N-250}$
 $(X_{N-251}Y_{N-251})Z_{N-251}$
 $(X_{N-252}Y_{N-252})Z_{N-252}$
 $(X_{N-253}Y_{N-253})Z_{N-253}$
 $(X_{N-254}Y_{N-254})Z_{N-254}$
 $(X_{N-255}Y_{N-255})Z_{N-255}$
 $(X_{N-256}Y_{N-256})Z_{N-256}$
 $(X_{N-257}Y_{N-257})Z_{N-257}$
 $(X_{N-258}Y_{N-258})Z_{N-258}$
 $(X_{N-259}Y_{N-259})Z_{N-259}$
 $(X_{N-260}Y_{N-260})Z_{N-260}$
 $(X_{N-261}Y_{N-261})Z_{N-261}$
 $(X_{N-262}Y_{N-262})Z_{N-262}$
 $(X_{N-263}Y_{N-263})Z_{N-263}$
 $(X_{N-264}Y_{N-264})Z_{N-264}$
 $(X_{N-265}Y_{N-265})Z_{N-265}$
 $(X_{N-266}Y_{N-266})Z_{N-266}$
 $(X_{N-267}Y_{N-267})Z_{N-267}$
 $(X_{N-268}Y_{N-268})Z_{N-268}$
 $(X_{N-269}Y_{N-269})Z_{N-269}$
 $(X_{N-270}Y_{N-270})Z_{N-270}$
 $(X_{N-271}Y_{N-271})Z_{N-271}$
 $(X_{N-272}Y_{N-272})Z_{N-272}$
 $(X_{N-273}Y_{N-273})Z_{N-273}$
 $(X_{N-274}Y_{N-274})Z_{N-274}$
 $(X_{N-275}Y_{N-275})Z_{N-275}$
 $(X_{N-276}Y_{N-276})Z_{N-276}$
 $(X_{N-277}Y_{N-277})Z_{N-277}$
 $(X_{N-278}Y_{N-278})Z_{N-278}$
 $(X_{N-279}Y_{N-279})Z_{N-279}$
 $(X_{N-280}Y_{N-280})Z_{N-280}$
 $(X_{N-281}Y_{N-281})Z_{N-281}$
 $(X_{N-282}Y_{N-282})Z_{N-282}$
 $(X_{N-283}Y_{N-283})Z_{N-283}$
 $(X_{N-284}Y_{N-284})Z_{N-284}$
 $(X_{N-285}Y_{N-285})Z_{N-285}$
 $(X_{N-286}Y_{N-286})Z_{N-286}$
 $(X_{N-287}Y_{N-287})Z_{N-287}$
 $(X_{N-288}Y_{N-288})Z_{N-288}$
 $(X_{N-289}Y_{N-289})Z_{N-289}$
 $(X_{N-290}Y_{N-290})Z_{N-290}$
 $(X_{N-291}Y_{N-291})Z_{N-291}$
 $(X_{N-292}Y_{N-292})Z_{N-292}$
 $(X_{N-293}Y_{N-293})Z_{N-293}$
 $(X_{N-294}Y_{N-294})Z_{N-294}$
 $(X_{N-295}Y_{N-295})Z_{N-295}$
 $(X_{N-296}Y_{N-296})Z_{N-296}$
 $(X_{N-297}Y_{N-297})Z_{N-297}$
 $(X_{N-298}Y_{N-298})Z_{N-298}$
 $(X_{N-299}Y_{N-299})Z_{N-299}$
 $(X_{N-300}Y_{N-300})Z_{N-300}$
 $(X_{N-301}Y_{N-301})Z_{N-301}$
 $(X_{N-302}Y_{N-302})Z_{N-302}$
 $(X_{N-303}Y_{N-303})Z_{N-303}$
 $(X_{N-304}Y_{N-304})Z_{N-304}$
 $(X_{N-305}Y_{N-305})Z_{N-305}$
 $(X_{N-306}Y_{N-306})Z_{N-306}$
 $(X_{N-307}Y_{N-307})Z_{N-307}$
 $(X_{N-308}Y_{N-308})Z_{N-308}$
 $(X_{N-309}Y_{N-309})Z_{N-309}$
 $(X_{N-310}Y_{N-310})Z_{N-310}$
 $(X_{N-311}Y_{N-311})Z_{N-311}$
 $(X_{N-312}Y_{N-312})Z_{N-312}$
 $(X_{N-313}Y_{N-313})Z_{N-313}$
 $(X_{N-314}Y_{N-314})Z_{N-314}$
 $(X_{N-315}Y_{N-315})Z_{N-315}$
 $(X_{N-316}Y_{N-316})Z_{N-316}$
 $(X_{N-317}Y_{N-317})Z_{N-317}$
 $(X_{N-318}Y_{N-318})Z_{N-318}$
 $(X_{N-319}Y_{N-319})Z_{N-319}$
 $(X_{N-320}Y_{N-320})Z_{N-320}$
 $(X_{N-321}Y_{N-321})Z_{N-321}$
 $(X_{N-322}Y_{N-322})Z_{N-322}$
 $(X_{N-323}Y_{N-323})Z_{N-323}$
 $(X_{N-324}Y_{N-324})Z_{N-324}$
 $(X_{N-325}Y_{N-325})Z_{N-325}$
 $(X_{N-326}Y_{N-326})Z_{N-326}$
 $(X_{N-327}Y_{N-327})Z_{N-327}$
 $(X_{N-328}Y_{N-328})Z_{N-328}$
 $(X_{N-329}Y_{N-329})Z_{N-329}$
 $(X_{N-330}Y_{N-330})Z_{N-330}$
 $(X_{N-331}Y_{N-331})Z_{N-331}$
 $(X_{N-332}Y_{N-332})Z_{N-332}$
 $(X_{N-333}Y_{N-333})Z_{N-333}$
 $(X_{N-334}Y_{N-334})Z_{N-334}$
 $(X_{N-335}Y_{N-335})Z_{N-335}$
 $(X_{N-336}Y_{N-336})Z_{N-336}$
 $(X_{N-337}Y_{N-337})Z_{N-337}$
 $(X_{N-338}Y_{N-338})Z$

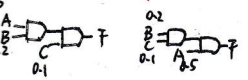
Low Power Design

$$P_d = C_L V_{DD}^2 \cdot f$$

Internal nodes are making power consuming transitions but output node is not showing the effect

$$P_{avg} = \left(\sum_{i=1}^{N} \alpha_i C_i \cdot V_i \right) \cdot V_{DD} \cdot f_{clk}$$

In general $V_i = V_{DD}$



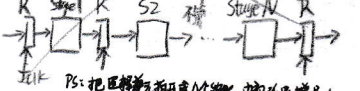
The higher probability signals should be introduced at a later stage in the logic pipeline to reduce toggling activity!

Low-Power Design Using Pipelining Approach

Reference = $C_{total} \cdot V_{DD}^2 \cdot f_{clk}$

Pipelining = $[C_{total} + (N-1) \cdot C_{reg}] \cdot V_{DD, new}^2 \cdot f_{clk}$

~~$$\frac{P_{pipelined}}{P_{reference}} = \left[1 + \frac{C_{reg}}{C_{total}} (N-1) \right] \frac{V_{DD, new}^2}{V_{DD}^2}$$~~

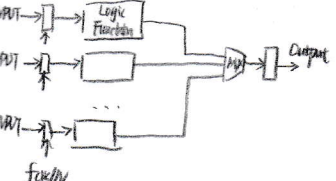


Logic function $F(\text{input})$ has to be partitioned into N successive stage each having approximately equal delay.

A total of $(N-1)$ registers arrays have to be introduced in addition to the original input and output registers

All registers are clocked at f_{clk} though it is possible to increase the clock speed to $N \cdot f_{clk}$ and thus, delay for each logic block can be increased by a factor of N without sacrificing throughput (if f_{clk})

Parallelism



Assume that the consecutive input data arrive at the same rate as in the single-stage case.

The input data are routed to all registers in N processing blocks.

- Global clock signals with clock period of $N T_{clk}$ are used to load each register every N clk
- The individual clock signals are skewed by T_{clk} , such that each one of the N consecutive data is loaded into a different input register.
- Each register is thus clocked at a lower frequency f_{clk}/N , the time allowed to compute the function for each input data is increased by a factor of N

The supply voltage can be reduced until the path delay equals the new clock period of $N T_{clk}$

$$P_{parallel} = \left[1 + \frac{C_{reg}}{C_{total}} \right] \cdot C_{total} \cdot V_{DD, new}^2 \cdot f_{clk}$$

$$\frac{P_{parallel}}{P_{reference}} = \frac{V_{DD, new}^2}{V_{DD}^2} \left[1 + \frac{C_{reg}}{C_{total}} \right]$$

Pipelining vs Parallelism

- Increase Area
- Increase of latency from 1 cycle to N cycles
- Increase area and latency
- Silicon area will grow faster than the number of processor because of signal routing and the overhead circuitry.

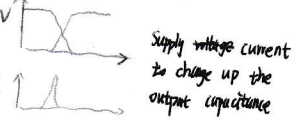
Short Circuit Power Dissipation

Power dissipation due to non-ideal switching characteristic of CMOS logic gates

The input of a real life logic gate (e.g. an inverter) is actually driven by input voltage waveforms that has finite rise and fall time.

Both the NMOS and PMOS transistors may conduct simultaneously for a very short amount of time during switching, forming a direct current path between the supply and the ground.

May be reduced by making the output voltage transition longer and/or making the input voltage transition time smaller. Should be balanced carefully.



Leakage Power Dissipation

NMOS and PMOS transistors generally have nonzero reverse leakage and subthreshold currents

So even when the gate is idle, it still draws a very small amount of current. For a chip having millions of transistors, this leakage power dissipation may contribute significantly.

- Two main components: biggest come in deep submicron
 - Reverse gate leakage current
 - Subthreshold current

[Leakage Reduction]

- Process level
 - Channel engineering
 - Well engineering
- Circuit level: Control voltages at different device terminals
 - standby leakage control using transistor stack
 - Multiple threshold design
 - sleep transistor insertion

Two main components of Leakage Power

- ① Reverse gate leakage current
 - Tunneling of electrons through gate oxide.
- ② Subthreshold current.

A weak current that flows due to carrier diffusion between the source and drain of a transistor in weak inversion region (V_{DS} is less than the threshold voltage but not exactly zero).

If V_{DS} is smaller than but very close to the threshold voltage then the subthreshold power dissipation may become comparable in magnitude to the switching power dissipation.

Timing Constraint: The data applied to the D-flip-flop should be stable for at least T_{setup} and should stay stable for at least T_{hold} .

Setup time (T_{setup}): is the minimum amount of time the data should be stable before the application of the clock signal.

Hold time (T_{hold}): is the minimum amount of time the data should be stable after the application of the clock signal.

[Setup Timing Constraint]

$$T_{clk-min} \geq T_D + T_{ic-max} + T_{setup}$$

[Holding Timing Constraint]

$$T_{hold} \leq T_D + T_{ic-min}$$

The role of Interconnect

Buses are implemented as long metal lines on a silicon wafer transmitting data using electromagnetic waves (finite speed limit)

In addition to data transfer Metal lines (interconnect) are also used power distribution networks and for clock distribution network in synchronous design.

Interconnect networks are have become increasingly complicated with greater integration

The Impact of Interconnection in Modern Technologies

① Limit Performance

Signals cannot travel across the entire die within a global clock cycle.

For example 6-to 20% clock cycles are needed to transfer data on a bus at 50nm technology

Unpredictability in signal propagation time has serious consequences for performance and correct functioning of synchronous digital circuits

② Consume up to 50% of chip power

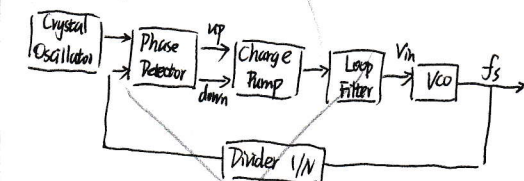
③ Affect system reliability (e.g. crosstalk errors)

$$D = 0.4 * R * C$$

Phase-Locked Loop

PLL is typically used to generate the high frequency required by SoC. A PLL takes an external low-frequency reference crystal frequency signal and multiplies its frequency by a rational number N

The reference clock is typically generated off-chip from an accurate crystal reference. The reference clock is compared to a divided version of the system clock (i.e. the divided clock) using a phase detector that compares the phase difference between the signals and produces an Up or Down signal when the local clock lags or leads the reference signal. It detects which of the two inputs signals arrives earlier and produces an appropriate output signal.



Phase detector: Determine the relative phase and frequency difference between two incoming signals and output a signal that is proportional to this difference.

Charge Pump: It converts the Up/Down pulses into an Analog voltage that controls the VCO.

Loop Filter: Low-pass filter that removes the high-frequency components from the VCO control voltage and smoothens out its response, which results in a reduction of the jitter

Clock Skew

Static point-to-point variation (of clock arrival time to FFs)

① Designed Variations - mismatch in buffer load sizes, interconnect lengths

② Process variations - process spread across die yielding different T_{prop}

③ Temperature gradients

④ IR voltage drop in power supply

Clock Jitter

is the variation in clock edge timing between clock cycle. (cycle to cycle variation)

① Variation in PLL oscillation frequency (PLL is highly sensitive to intrinsic device)

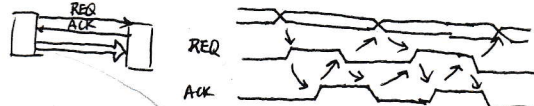
② High frequency power supply variations (caused by instantaneous IR drops along the power grid)

③ Variation in capacitive load (Due to coupling between the clock lines and adjacent signal wires)

$$T_{clk-min} \geq T_D + T_{ic-max} + T_{setup} + 2 \cdot \text{Jitter} - \text{Skew}$$

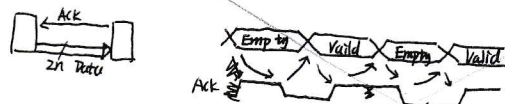
$$T_{hold} \leq T_D + T_{ic-min} - 2 \cdot \text{Jitter} - \text{Skew}$$

Bundled data Protocol (single rail)



data should be valid precedes REQ

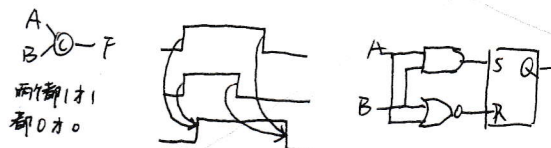
4-phase dual rail protocol



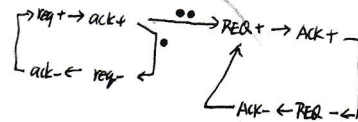
Delay Insensitive

Each change is acknowledged

Problems: Glitches, hazards



If left cycle goes fast and right cycle lags, then one $ack \rightarrow req$ + accumulates tokens



Boundedness

STG is bounded (safe) if no place or arc can ever contain more than one token

Consistent

$A^+ \dots A^+$ Two subsequent up-transitions without a down-transition in between
 $A^- \dots A^-$ Two subsequent down-transitions without an up-transition in between

Boundedness + Consistent State Assignment \Rightarrow Finite Binary State Graph

State Coding

{ Concurrent reduction
 Signal insertion

Output Persistence

Output disables output (metastability or glitch)

Metastability is a condition where the voltage level of a signal is at an intermediate level (neither 0 or 1) and which may persist for an indeterminate amount of time

$$MR = W * F_c * F_D$$

$$MTBF = \frac{e^{\frac{S}{\sigma}}}{W * F_c * F_D}$$

1FF, No $e^{\frac{S}{\sigma}}$
 2FF, $S = F_c$
 3FF, $S = 2F_c$