MODEM

No.
Date

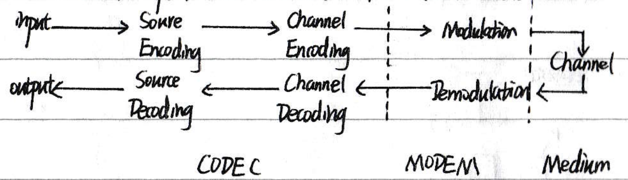
Digital Communication System

Purpose: communicate information at required rate between geographically separated locations reliably (quality)

- Important point : rate, quality ↔ spectral bandwidth, power requirements
- Information theory provides guilding principles for everything in communication

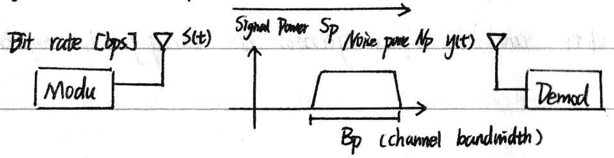
Major components : a pair of transmitter and receiver called transceiver

- "Horizontal" partition : transmitter, receiver and channel (transmission medium)
- "Vertical" partition : CODEC, MODEM and Channel



Recap of Channel Capacity

Information theory provides us basic theory for communication system design, including MODEM



- Assuming AWGN channel with Gaussian signal s(t), channel capacity

$$C = B_p \cdot \log_2 (1 + \frac{P_s}{N_0}) \text{ [bps]}$$

- Maximum rate could be achieved, i.e. upper limit

MODEM responsible: transfer the bit stream at required rate over the communication medium reliably

- Require rate [bps] with required quality ↔ spectral bandwidth and power requirement

Carrier communication: s(t) is radio frequency signal, because low frequency signal cannot travel far, also spectral resource (channels) are in RF

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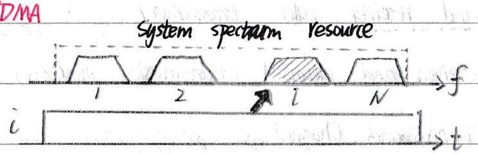
Channel Partition

FDMA

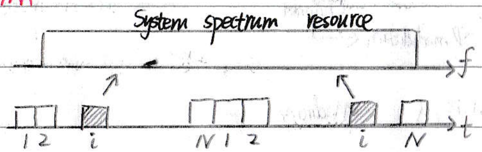
Frequency division multiple access: system spectral band is divided into frequency slots

- A user is assigned with a frequency slot (channel), who can transmit continuously in time, but its signal spectrum must be inside its allocated frequency slot.

FDMA



TDMA



Time division multiple access: transmission in time frames, and each frame divided into time slots

- A user is assigned with a time slot (channel), who can only transmit in time bursts i.e. in its allocated time slots, and its signal spectrum can occupy whole system spectral band.

Digital Modulation

In old day, communications were analogue, analogue modulation techniques include

- amplitude modulation (AM), frequency modulation (FM), and phase modulation (PM)

Communications today are all digital, and equivalent digital modulation forms

- amplitude keying (ASK), frequency shift keying (FSK), and phase shift keying (PSK)

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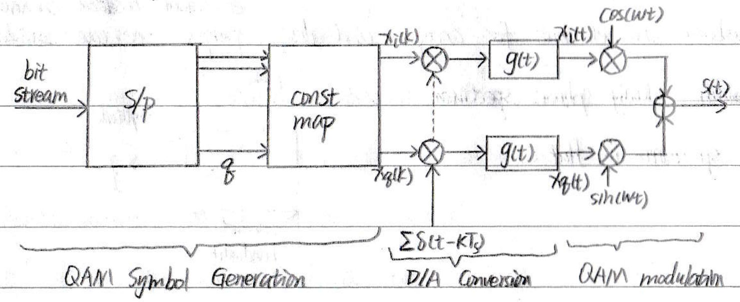
Carrier signal in digital communication is sin waveform $A \cdot \sin(2\pi f_c t + \theta)$, specified by amplitude A, frequency f_c, and phase θ

- Use amplitude, frequency, or phase of the carrier to "carry" information leads to ASK, FSK or PSK
- A large number of digital modulations are in use, and often combination of these three basic ways are employed

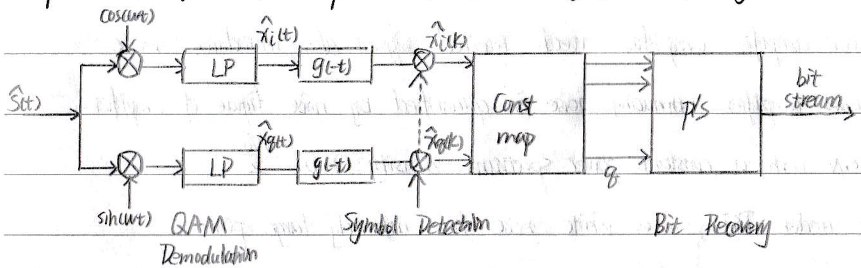
We will consider quadrature amplitude modulation (QAM), which is a combination of ASK and PSK

Quadrature Amplitude Modulation (Let us start our study from transmitter)



D/A conversion is not "correct name", should be "transmit filter", part of pulse shaping filter pair

Note: e.g. odd bits go to form $x_i(k)$ and even bits go to form $x_q(k)$; $x_i(k)$ and $x_q(k)$ are in-phase and quadrature components of the $x_i(k)+jx_q(k)$ QAM symbol; $x_i(k)$ and $x_q(k)$ are M-ary symbols



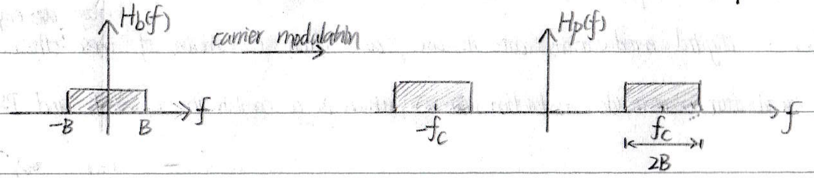
In-phase and quadrature braches are "identical"

- many issues, such as design of Tx/Rx filter $g(t)/g(-t)$, carrier recovery, synchronisation, can be studied using one branch

Channel Characteristics

Between modulator and demodulator is medium (channel)

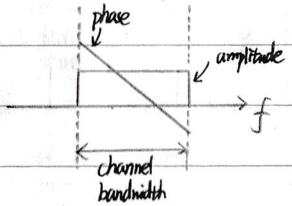
Passband channel and baseband (remove modulator/demodulator) equivalence:



— ~~Based~~ Baseband channel bandwidth $B \leftrightarrow$ passband channel bandwidth $B_p = 2B$

Communication is at passband channel but for analysis and design purpose one can consider equivalent ~~baseband~~ baseband channel

Channel has finite bandwidth, ideally phase spectrum is linear and amplitude spectrum is flat:



Channel Noise

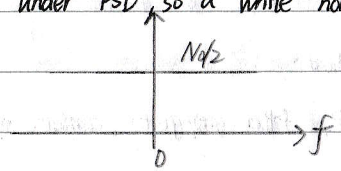
Bandwidth is a prime consideration, and another consideration is noise level

— At receiver, power amplifier amplifies weak received signal also introduces noise

— How serious power amplifier introducing noise is quantified by noise figure of amplifier

Channel noise: AWGN with a constant power spectrum density (PSD) $\frac{N_0}{2}$

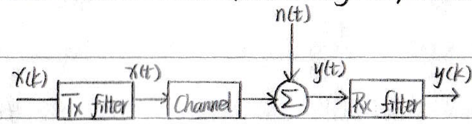
Power is the area under PSD, so a white noise has infinitely large power

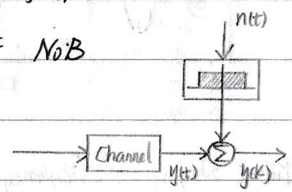


But communication channels are bandlimited, so noise is also bandlimited and has a finite power

— Noise $n(t)$ introduced by power amplifier passes through Rx filter who has a bandwidth of B

— Thus noise at the receiver signal, $n_B(t)$, has a power of $N_0 B$





Pulse Shaping — Starting Point

Unless transmission symbol rate f_s is very low, one cannot use impulse, narrow pulse or rectangular pulse to transmit data symbols

— Such pulses have large (infinite) bandwidth, but we only have finite baseband bandwidth B

Discrete samples have to be pulse shaped

— $\{x[k]\}$: transmitted symbols

— $\sum \delta(t-kT_s)$: pulse clock (every T_s s a symbol is transmitted)

— $r(t)$: combined impulse response of Tx/Rx filters, and channel

$r(t) = g(-t) * c(t) * g(t)$ or $R(f) = G_R(f) \cdot C(f) \cdot R_T(f)$

— Baseband (received) signal, assuming no noise

$$x(t) = r(t) * \left(\sum x[k]\,\delta(t-kT_s)\right) = \int \sum r(t-\tau)\cdot x[k]\cdot \delta(\tau-kT_s)\,d\tau = \sum_{k=-\infty}^{+\infty} x[k]\cdot r(t-kT_s)$$

Pulse shaping

⇓

In electronics and telecommunications, pulse shaping is the process of ~~the~~ changing the waveform of transmitted ~~waveform~~ pulses. It's purpose is to make the transmitted signal better suited to its purpose or the communication channel, typically by limiting the effective bandwidth of transmission.

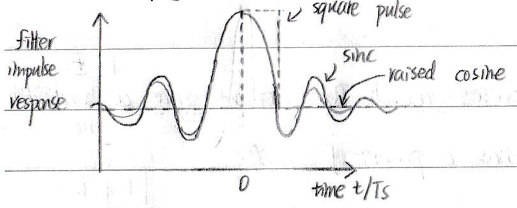
Intersymbol interface: a form of distortion of a signal in which one symbol interfaces with subsequent symbol

~~Transmitted~~

Transmitting a signal at high modulation rate (symbol rate) through a band-limited channel can create interface symbol

As modulation rate increases, the signal's bandwidth increases. When the signal's bandwidth becomes larger than the channel bandwidth, the channel starts to introduce distortion to the signal. This distortion usually manifests itself as intersymbol interference.

Pulse Shaping - Time Domain

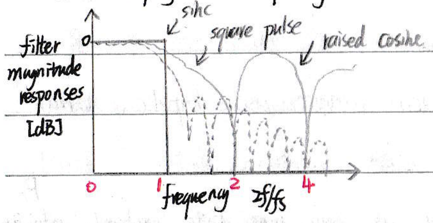


① Square : last one T_s

② Sinc : assume $t \to \pm\infty$

③ Raised cosine : truncate (截短成) to 8 T_s.S

Pulse Shaping - Frequency Domain



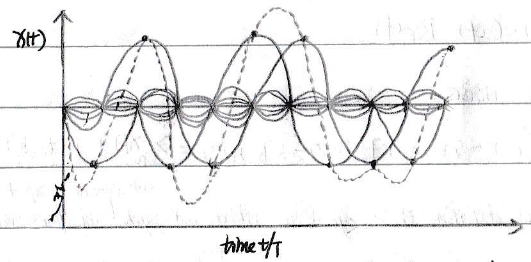
Channel bandwidth B is finite, and signal bandwidth B_T must fit in it.

- Square pulse produces considerable large excess bandwidth well beyond symbol rate f_s
- Sinc pulse has exactly finite bandwidth of $f_s/2$, but impractical to realize
- truncated raised ~~no~~ cosine has main bandwidth within f_s, and easy to realize

Right Pulse Shaping

Recall we are discussing how to choose r(t) so that we can recover {x[k]} from x(t)

Example - binary (±1) {x[k]}, each is transmitted as a sinc pulse : the peak of different shifted sinc functions (different x[k]) coincide (重合) with zero crossings of all other sincs (data symbols)



- At receiver, sampling at correct symbol rate enables recovery of transmitted x[k] !

Right pulse shaping seems : combined impulse response r(t) has regular zero-crossing at symbol-rate spacing except it peaks at t=0, a Nyquist filter

Transmit~~ted~~ and Receive Filters

Pulse shaping fulfils two purposes

- limit the transmission bandwidth B_T so it fits in channel bandwidth B
- enable to recover the correct sample values of transmitted symbols.

Such a pulse shaping $r(t) = g(-t) * c(t) * g(t)$ is called a Nyquist system

1. (Infinite) sinc has a (baseband) bandwidth $B_T = \frac{f_s}{2}$, (infinite) raised cosine has $\frac{f_s}{2} \le B_T \le f_s$ depending on roll-off factor
2. A Nyquist time pulse have regular zero-crossing at symbol-rate spacings to avoid interference with neighbouring pulses at correct sampling instances

Assuming ideal channel c(t), Nyquist system r(t) is separated into { transmit filter g(t) ; receive filter g(-t) }

1. The filter g(-t) in the receiver is also called a matched filter (to g(t)); g(t) and g(-t) are basically identical (square-root of r(t))
2. This division of r(t) enables suppression of out-of-band noise and results in the maximum received signal-to-noise ratio (SNR)

Summary

MODEM : responsible for transferring the bit stream at required rate over the communication medium ~~rea~~ reliably

- Required rate [bps] with required quality ⟺ spectral bandwidth and power requirements
- Transmission channel (medium) has finite bandwidth and introduces noise

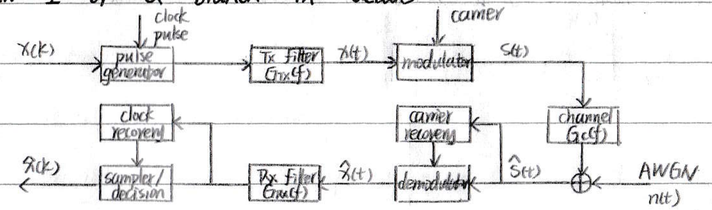
Purpose of pulse shaping, how to design transmit and receive filters.

- limit the transmission bandwidth so it can fit in channel bandwidth
- enable to recover the correct sample values of transmitted symbols.

MODEM - L2

Baseband System

Redraw I or Q branch in details



Assuming a perfect demodulation, we can consider the baseband system:

$G_c(f)$ is equivalent baseband channel

$$x(k)\cdot\delta(t-kT_s) \longrightarrow \boxed{R(f) = G_{Tx}(f)\cdot G_c(f)\cdot G_{Rx}(f)} \xrightarrow{\hat{y}(t)} \oplus \xleftarrow{} n_0(t) \longrightarrow y(t)$$

Pulse shaping is about combined response $r(t) = g(-t) * c(t) * g(t)$ or $R(f)$

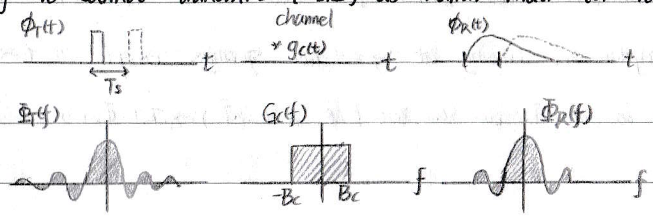
Pulse Shaping - Finite-Time Pulses

Narrow-width or rectangular pulses with time support $\le$ symbol period T_s

- At $t = kT_s$, we transmit $x[k]$, and $\{x[k]\}$ in $\sum x[k]\Phi_T(t-kT_s)$ will not overlap

  But $\{x[k]\}$ in $x(t) = \sum x[k]\Phi_R(t-kT_s)$ will overlap

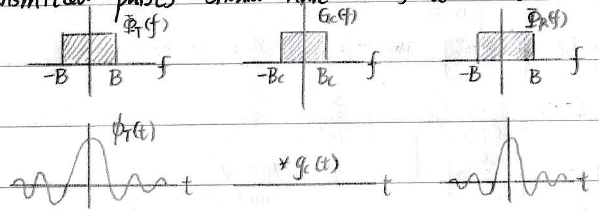
Why we cannot transmit $\{x[k]\}$ as narrow-width (or rectangular) pulses:



- Channel has finite bandwidth, and the bandwidth of a narrow impulse is not finite
- The pulses will spread out as their high-frequency components are suppressed (抑制), causing interference with nighbouring pulses (symbols) in time

Pulse - Shaping - Finite-Bandwidth Pulses

Transmitted pulses should have a finite bandwidth $\leq$ channel bandwidth



Then a pulse (symbol) will be passed through channel without distortion

But finite bandwidth means infinite time waveform, and this will cause interference in time domain with neighbouring pulses.

- $\{x[k]\}$ in $x(t) = \sum x[k]\,\Phi_R(t-kT_s)$ will overlap, as each $x[k]\cdot\Phi_R(t-kT_s)$ lasts infinite long in time.

Pulse Shaping - Right Pulses

Finite-bandwidth pulses are actually the right ones for transmitting $\{x[k]\}$

- But how you avoid infinite long pulses interference with each other in time? You cannot!

All we want is to recover transmitted symbols $\{x[k]\}$ from $x(t) = \sum x[k]\cdot\Phi_R(t-kT_s)$

- Pulse waveform $\Phi_R(t)$ has regular zero-crossing at symbol-rate spacings except at $t=0$
- Interference from other pulses at $t=kT_s$ are all zero! i.e. $x(t=kT_s) = x[k]\cdot\Phi_R(0)$

InterSymbol Interference (ISI)

Let a symbol $x[k]$ be transmitted as $x[k]\cdot\phi_T(t-kT_s)$ with T_s the symbol period.

Assume that the Tx pulse $\phi_T(t)$ peaks at $t=0$, with $\phi_T(0)=1$, and it has zero-crossing symbol-rate spacings, i.e. it is a Nyquist filter.

The ~~transmitted~~ transmitter output waveform is:

$$x(t) = \sum_{k=-\infty}^{+\infty} x[k]\cdot\phi_T(t-kT_s)$$

Assume that the Tx pulse $\phi_T(t)$ arrives at the receiver as $\phi_R(t-t_d)$, where the Rx pulse $\phi_R(t)$ peaks at $t=0$ with $\phi_R(0)=1$, and it is also a Nyquist filter.

The receiver output waveform is:

$$y(t) = \sum_{k=-\infty}^{+\infty} x[k]\cdot\phi_R(t-kT_s-t_d) + n_0(t)$$

- Clearly, all transmitted symbols $\{x[k]\}$ in $y(t)$ are mixed up - ISI

Achieving Zero ISI

The sampled receiver output at correct sampling instance $t_k = kT_s + t_d$ is:

$$y(t_k) = x[k] + \sum_{m\neq k} x[m]\,\phi_R((k-m)T_s) + n_0(t_k)$$

First term: correct symbol Second term: ISI Third term: due to channel noise

In order to eliminate ISI, the received pulse should satisfy:

$$\phi_R(kT_s) = \begin{cases} 0, & \text{for } k\neq 0 \\ 1, & \text{for } k=0 \end{cases}$$

That is, $\phi_R(t)$ has zero-crossing at symbol-rate spacings, leading to:

$$y(t_k) = x[k] + n_0(t_k)$$

Pulse shaping: achieve a desired finite transmission bandwidth and eliminate ISI

Nyquist Criterion for Zero ISI

Recall symbol rate f_s and symbol period T_s; if $\Phi_R(f)=\mathcal{F}[\phi_R(t)]$ satisfies:

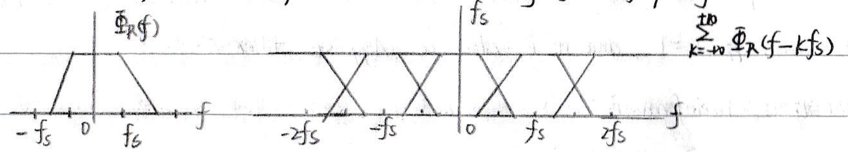
$$\sum_{k=-\infty}^{+\infty}\Phi_R(f-kf_s)=\text{constant} \quad \text{for } |f|\le \frac{f_s}{2}$$

then in time domain, the pulse waveform meets:

$$\phi_R(kT_s)=\begin{cases}1, & k=0\\ 0, & k\neq 0\end{cases}$$

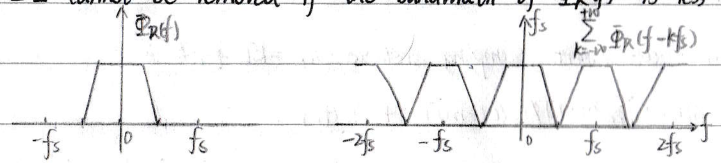
Filter (system) generates such a pulse is a Nyquist system.

Illustration of condition for zero ISI, seeing from frequency domain:



Minimum Transmission Bandwidth

ISI cannot be removed if the bandwidth of $\Phi_R(f)$ is less than $f_s/2$

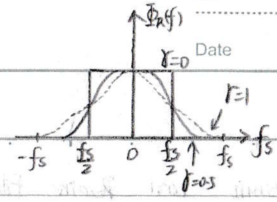


The minimum required bandwidth of $\Phi_R(f)$ required for zero ISI is $\frac{f_s}{2}$

This is in fact the sinc pulse: $\text{sinc}(f_s t)=\frac{\sin(\pi f_s t)}{\pi f_s t}$, the only Nyquist system with bandwidth $\frac{f_s}{2}$

From channel capacity: require rate, quality ↔ bandwidth, power requirements

- To transmit at symbol rate f_s, required transmission bandwidth must $B_T \ge \frac{f_s}{2}$



Raised ~~Cos~~ Cosine Nyquist System

The required baseband bandwidth: $\frac{f_s}{2}\le B\le f_s$, and the spectrum

$$\Phi_R(f)=\begin{cases}1 & |f|\le\frac{f_s}{2}-\beta\\ \cos^2\left(\frac{\pi}{4\beta}\left(|f|-\frac{f_s}{2}+\beta\right)\right) & \frac{f_s}{2}-\beta<|f|\le\frac{f_s}{2}+\beta\\ 0 & |f|>\frac{f_s}{2}+\beta\end{cases}$$

β: the extra bandwidth over the minimum $f_s/2$, and 滚降因子 roll-off factor γ:

$$\gamma=\frac{\beta}{f_s/2}=\frac{B-f_s/2}{f_s/2} \quad \text{or} \quad B=\frac{f_s}{2}(1+\gamma)$$

Raised cosine pulse is widely used pulse shaping, its time waveform of course satisfies

- Regular zero-crossing at symbol rate spacing, i.e. $\phi_R(kT_s)=0\ \forall k\neq 0$
- Its time waveform $\phi_R(t)$ decays from peak at $t=0$ much faster than sinc pulse

Effect of Sampling Error

It is clearly the ability of recovering transmitted symbols $\{x[k]\}$ all depends on sampling the received signal $y(t)$ at correct sampling instance.

- Sampling correctly at $t_k=kT_s+t_d$ leads to (忽略 omit noise and use $\phi_R(0)=1$)

$$y(t_k)=x[k]+\sum_{m\neq k}x[m]\phi_R((k-m)T_s)=x[k]$$

- In practice, small sampling error δt unavoidable, and sampling at $t_k=kT_s+t_d+\delta t$ leads to

$$y(t_k)=x[k]\phi_R(\delta t)+\sum_{m\neq k}x[m]\phi_R((k-m)T_s+\delta t)\neq x[k]\cdot\phi_R(\delta t)$$

- Two neighbouring symbols $x[k-1]$ and $x[k+1]$ cause the biggest ISI to $x[k]$

$$y(t_k)=\cdots+x[k-1]\phi_R(T_s+\delta t)+x[k]\phi_R(\delta t)+x[k+1]\phi_R(-T_s+\delta t)+\cdots$$

- sinc pulse decays from peak at $t=0$ at rate of $1/t$
  - Biggest ISI term due to small sampling error δt has magnitude of around $1/T_s$
- Raised cosine pulse with roll-off fact $\gamma=1$ decays from peak at $t=0$ at rate of $1/t^3$
  - Biggest ISI term due to small sampling error δt has magnitude of around $1/T_s^3$

Raised cosine pulse costs more bandwidth but is less sensitive to sampling error and more practical

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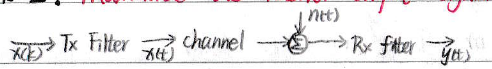
Optimal Transmit and Receive Filters

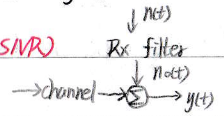
Task 1. Combined Tx / Rx filters provide desired spectrum shape:

$$R(f) = G_{Tx}(f) \cdot G_c(f) \cdot G_{Rx}(f) = \Phi_R(f)$$ the required spectrum shape

- Assume the (band~~base~~) channel bandwidth $B_c \geq B$, then $G_c(f) = 1$ and $R(f) = G_{Tx}(f) \cdot G_{Rx}(f) = \Phi_R(f)$
- Hence name 'pulse shaping' for transmit and receive filter pair $g(t)$ and $g(-t)$

Task 2. Maximise the receiver output signal to noise ratio (SNR)





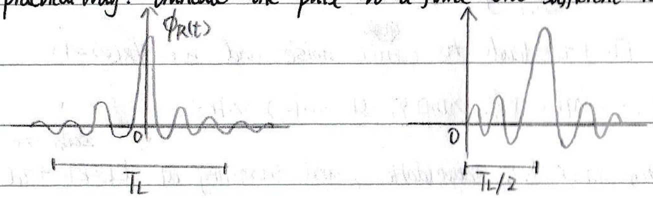
- This leads to $G_{Tx}(f) = G_{Rx}(f)$, that is, identical as a square-root of Nyquist system $\Phi_R(f)$
- Hence Rx filter $g(-t)$ matches Tx filter $g(t)$, this is, $g(-t)$ is 'mirror' of $g(t)$

Practical Implementation

A true Nyquist system (e.g. raised cosine) has absolute finite bandwidth but the corresponding time-waveform is non-causal and lasts infinite long

- Pulse shaping filters to realize such a true Nyquist system cannot be constructed physically.

A practical way: truncate the pulse to a finite but sufficient length and delay the truncated pulse:



- The bandwidth of the truncated pulse is no longer infinite
- But if the pulse is selected to decaying rapidly the resulting ISI can be made sufficiently small

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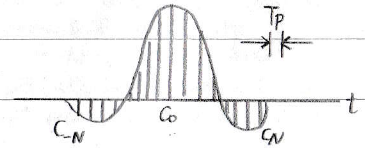
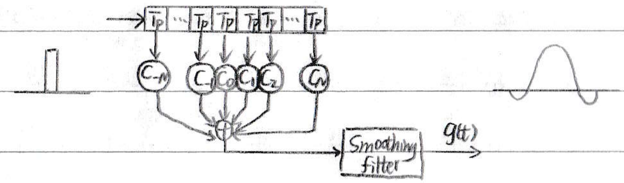
Tx / Rx filters Realization

Sampled values are obtained from the wave-form of g(t)



FIR or transversal filter is used to realize the required Tx / Rx filters:



Summary

Design of transmit and receiver filters (pulse shaping): to achieve zero ISI and to maximise the received signal to noise ratio

The combined Tx/Rx filters should form a Nyquist System (regular zero-crossings at symbol-rate spacing except at $t=0$), and Rx filter should be identical (matched) to Tx filter as a square-root Nyquist system.

Nyquist criterion for zero ISI, to transmit at symbol rate f_s requires at least a baseband B of $\frac{f_s}{2}$

The raised cosine pulse, roll-off factor, and the required baseband transmission bandwidth:

$$B = \frac{f_s}{2}(1+\gamma)$$ (passband bandwidth is doubled)

Practical considerations for implementing pulse shaping filters

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MODEM - L3 Carrier recovery and timing recovery

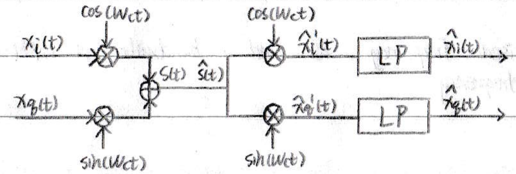
Revision

Pulse shaping Tx / Rx filter pair

- Design of Tx/Rx filters (pulse shaping) : to achieve zero ISI and to maximise received signal to noise ratio
- Combined Tx / Rx filters : Nyquist System (regular zero-crossings at symbol-rate spacings except at $t=0$), and Rx filter matched (identical) to Tx filter.
- Nyquist criterion for zero ISI ; to transmit at symbol rate f_s requires at least a baseband $B=\frac{f_s}{2}$
- Raised cosine pulse , roll-off factor , and required baseband transmission bandwidth $B=\frac{f_s}{2}(1+\gamma)$

QAM Modulator / Demodulator

Modulator and demodulator of the QAM scheme



Carrier Modulation at transmitter : low frequency or baseband analogue signals $x_i(t)$ and $x_q(t)$ at modulated by two carriers

Carrier Demodulation at receiver : two transmitted baseband signals are or obtained from received carrier signal

- Inphase and quadrature carriers are orthogonal (正交) , and they can be separated at receiver
- Inphase or quadrature rate is half of original transmission rate , meaning half of bandwidth

QAM - Modulation

Modulation of "in phase" and "quadrature" components to carrier frequency w_c:

$$\begin{cases} x_1(t) = x_i(t) \cdot \cos(w_c t) \\ x_2(t) = x_q(t) \cdot \sin(w_c t) \end{cases}$$

$S(t) = x_i(t) \cdot \cos(w_c t) + x_q(t) \cdot \sin(w_c t)$

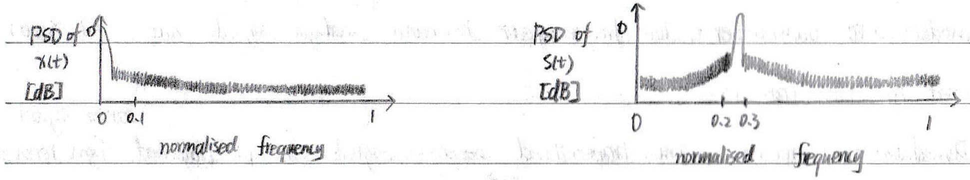
In phase and quadrature signals are mixed up and transmitted as $\boxed{s(t) = x_1(t) + x_2(t)}$

To explain demodulation, we assume perfect transmission $\hat{s}(t) = s(t)$

Real carriers are in hundreds of MHz or GHz

- In next slide we have a baseband transmission bandwidth 10kHz and carrier f_c=250kHz normalised by 1MHz in plots
- Thus, carrier f_c = 250kHz equals to normalised frequency 0.25, and a bandwidth of 10kHz is equal to a width of 0.01 in normalised frequency

I or Q Branch Modulation Example



QAM - Demodulation

Demodulation for the in-phase component:

$$\hat{x}_i'(t) = s(t) \cdot \cos(w_c t) = (x_i(t) \cdot \cos(w_c t) + x_q(t) \cdot \sin(w_c t)) \cdot \cos(w_c t)$$

$$= x_i(t) \cdot \cos^2(w_c t) + x_q(t) \cdot \sin(w_c t)\cos(w_c t)$$

$$= x_i(t) \cdot \tfrac{1}{2} \cdot (1 + \cos(2 w_c t)) + x_q(t) \cdot \tfrac{1}{2} \cdot \sin(2 w_c t)$$

- If the lowpass filter LP is selected appropriately (cut-off frequency $\le w_c$), the components modulated at frequency $2w_c$ can be filtered out, and hence

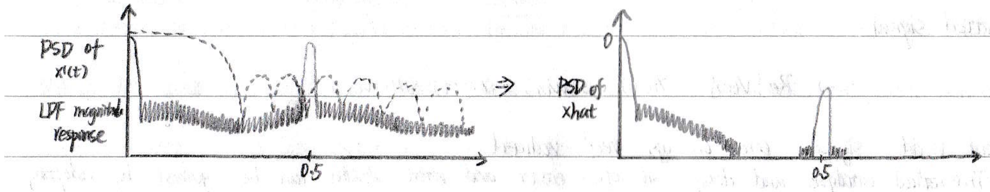
$$\hat{x}_i(t) = LP(\hat{x}_i'(t)) = \tfrac{1}{2} x_i(t)$$

A similar calculation can be performed for the demodulation of $\hat{x}_q(t)$:

$$\hat{x}_q'(t) = \cdots = x_i(t) \cdot \tfrac{1}{2} \cdot \sin(2 w_c t) + x_q(t) \cdot \tfrac{1}{2} \cdot (1 - \cos(2 w_c t))$$

- and hence $\hat{x}_q(t) = LP(\hat{x}_q'(t)) = \tfrac{1}{2} x_q(t)$

I or Q Branch Demodulation Example



Complex Notation Representation

The modulation/demodulation scheme is often expressed in complex notation

- in phase and quadrature components are considered to be real and imaginary parts of the complex signal, in which $j = \sqrt{-1}$ represents imaginary axis.

$$x(t) = x_i(t) + j \cdot x_q(t)$$

Carrier modulation is viewed as modulating a complex carrier $e^{-jw_c t}$ by $x(t)$, where angular frequency $w_c = 2\pi f_c$

- The transmitted signal is obtained by taking the real part of modulated carrier $x(t) \cdot e^{-jw_c t}$

$$s(t) = Re\{x(t) \cdot e^{-jw_c t}\}$$

Flow graph of modulator in complex notation

$$x(t) \longrightarrow \otimes \xrightarrow{v(t)} Re\{\cdot\} \longrightarrow s(t)$$

$\uparrow e^{-jw_c t}$ (input to $\otimes$)

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Modulation – Complex Notation

Modulation: $V(t) = e^{-j\omega_c t}\cdot x(t) = (\cos(\omega_c t) - j\cdot\sin(\omega_c t))\cdot(x_i(t)+j\cdot x_q(t))$

$$= \underbrace{x_i(t)\cdot\cos(\omega_c t)+x_q(t)\cdot\sin(\omega_c t)}_{\text{real}} \underset{\text{imaginary}}{- jx_i(t)\cdot\sin(\omega_c t)+jx_q(t)\cdot\cos(\omega_c t)}$$

Transmitted signal:

$$s(t)=Re\{V(t)\} = x_i(t)\cdot\cos(\omega_c t)+x_q(t)\cdot\sin(\omega_c t)$$

In real world, signals are always real-valued.

- Theoretical analysis and design are often easier and more insights can be gained by adopting complex representations

Demodulation – Complex Notation

Flow graph for the complex representation of the demodulation scheme:

$$\hat{s}(t) \longrightarrow \underset{\uparrow \; s(t)=e^{j\omega_c t}}{\otimes} \xrightarrow{\hat{x}'(t)} \boxed{LP} \longrightarrow \hat{x}(t)$$

The demodulated signal: $\hat{x}'(t) = e^{j\omega_c t}\cdot s(t)$, yielding

$$\hat{x}'(t) = (\cos(\omega_c t)+j\sin(\omega_c t))\cdot(x_i(t)\cdot\cos(\omega_c t)+x_q(t)\cdot\sin(\omega_c t))$$

$$= x_i(t)\cdot\frac{1}{2}(1+\cos(2\omega_c t)+j\sin(2\omega_c t)) + jx_q\cdot\frac{1}{2}(1-\cos(2\omega_c t)-j\sin(2\omega_c t))$$

Lowpass filter (LP) will again remove components modulated at $2\cdot\omega_c$

$$LP[\hat{x}'(t)] = \frac{1}{2}x_i(t) + j\cdot\frac{1}{2}x_q(t)$$

Local carrier: $\overset{\frown}{s}$

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Carrier Recovery – Phase Offset

Previously, we assume $\hat{s}(t) = x_i(t)\cdot\cos(\omega_c t)+x_q(t)\cdot\overset{\sin}{\cos}(\omega_c t)$, and we can use $e^{j\omega_c t} = \cos(\omega_c t) + j\sin(\omega_c t)$ to remove carrier – What we really assume:

1. Received carrier signal is $\hat{s}(t)=x_i(t)\cdot\cos(\omega_c t)+x_q(t)\cdot\sin(\omega_c t)$
2. At receiver we can generate a local carrier $\overset{\frown}{s}(t)=\cos(\omega_c t)+j\sin(\omega_c t)=e^{j\omega_c t}$
3. Hence we can carry out demodulation by $\hat{s}(t)\cdot\overset{\frown}{s}(t) \Rightarrow x_i(t)+jx_q(t)=x(t)$

Mostly likely, transmitted signal having travelled to receiver will accumulate a random and unknown phase φ

$$\hat{s}(t)=x_i(t)\cdot\cos(\omega_c t+\varphi)+x_q(t)\cdot\sin(\omega_c t+\varphi)$$

- At receiver we may generate a local carrier

$$\overset{\frown}{s}(t)=\cos(\omega_c t+\overset{\frown}{\varphi})+j\cdot\sin(\omega_c t+\overset{\frown}{\varphi})=e^{j(\omega_c t+\overset{\frown}{\varphi})}$$

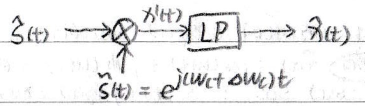
- Demodulation by $\hat{s}(t)\cdot\overset{\frown}{s}(t) \Rightarrow x(t)\cdot e^{j\Delta\varphi}$, where phase offset

$$\Delta\varphi=\varphi-\overset{\frown}{\varphi}$$

☆ Unless local carrier $\overset{\frown}{s}(t)$ happens to have same phase as incoming carrier signal $\hat{s}(t)$, i.e. phase offset $\Delta\varphi=0$, you cannot recover $x(t)$.

Carrier Recovery – Frequency Offset

Tx and Rx frequency generators are unlikely to match exactly, and consider demodulation with a Rx local "carrier" having a frequency offset $\Delta\omega_c$:



- Even assuming $\hat{s}(t) = s(t)$, demodulated signal prior to sampling is $\hat{x}(t) = x(t) \cdot e^{j\Delta\omega_c t}$, not $\hat{x}(t) = x(t)$
- The effect of carrier frequency mismatch $\Delta\omega_c t$, like the phase offset $\Delta\varphi$, has to be compensated at receiver to recover $x(t)$
- $\Delta\omega_c t + \varphi$ is called carrier offset between actual carrier and Rx local carrier

Thus, the receiver has to "recover" the actual carrier $e^{j(\omega_c t + \varphi)}$ (in fact the phase φ) in order to demodulate the signal correctly.

Synchronization

The process of selecting the correct sampling instances is called synchronization (timing or clock recovery)

- Tx and Rx clocks likely to mismatch, clock recovery synchronises receiver clock with transmitter clock to obtain samples at appropriate instances.
- Sampling demodulated signal $\hat{x}(t)$ at appropriate sampling instance is vital for recovering transmitted symbol $\{x[k]\}$, as this is condition for avoid ISI
- This is equivalent to replacing sampling impulse train $\sum\delta(t - kT_s)$ by $\sum\delta(t - kT_s - \tau)$, where $0 \le \tau \le T_s$, with correct τ value

$$\hat{x}(t) \xrightarrow{kT_s - \tau} \hat{x}[k]$$

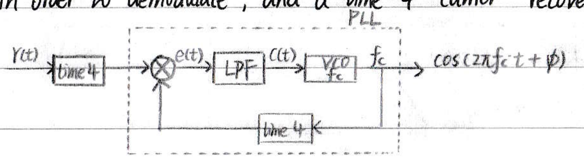
During link initialisation and between data frames, transmitter sends preamble (前同步码) which contains known training pseudo noise (PN) sequence

- Receiver generate local PN sequence, and by ~~overla~~ oversampling matches it with incoming PN sequence to obtain correct sampling information.

During data transmission, timing recovery has to rely on demodulated baseband signal $\hat{x}(t)$ only

Implementation

Carrier recovery ($\Delta\varphi$) matches the phase $\hat{\varphi}$ of local carrier to the unknown phase φ of incoming carrier, in order to demodulate, and a time-4 carrier recovery circuit:



- Electronic circuit for carrier recovery operates at very high carrier frequency, and is expensive.
- Receiver with carrier recovery is called coherent (相干接收) receiver and performs much better, as it can correctly demodulate the baseband signal $x(t)$, but is more complicated and expensive.

Receiver without carrier recovery is called non-coherent receiver, and its performance is poorer but it is less complicated and cheaper

- Using local carrier to demodulate without carrier recovery generates demodulated baseband signal $x(t) \cdot e^{j\varphi}$
- Other means must be implemented in order to remove unknown channel phase, e.g.: differential encoding at transmitter and differential detection at receiver.

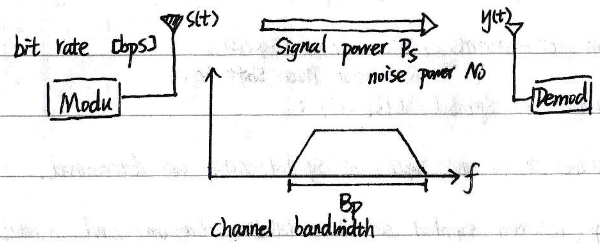
Timing recovery ($\Delta\omega_c t$) matches transmitter clock with receiver clock, in order to sample demodulated baseband signal at appropriate sampling instances

- Timing recovery operates at much lower frequency baseband signal, and it is required for any transceiver, coherent or non-coherent

MODEM - L4

Motivations

Recap MODEM aim and resource



To transmit at bit rate R_b would require baseband bandwidth $B = \frac{R_b}{2}(1+\gamma)$ with rolloff factor γ, and channel bandwidth $B_p = 2B$

{bits} → bits to symbols → {symbols} Tx filter → modulat → channel → demodu → Rx filter → {symbols} symbol to bits → {bits}

bit rate ... symbol rate $\underbrace{\qquad\qquad\qquad\qquad}_{\text{MODEM parts discussed so far}}$

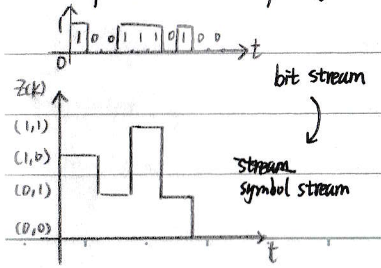
Image grouping every q bits into a symbol, thus convert binary source into a digital source with symbol set of size 2^q

- Transmitted symbol rate would be $f_s = R_b/q$, and required bandwidth is reduced by a factor of q
- No fee-lunch, you have to pay something (power) for this saving in bandwidth

Bits to Symbols

The bit stream to be transmitted is serial to parallel multiplexed onto a stream of symbols with q bits per symbol (discrete 2^q levels)

Example for $q=2$ bits per symbol (4-ary modulation): symbol period T_s is twice of bit period T_b



Symbol rate is half of bit rate;

Symbol stream is then pulse shaped, carrier modulated, ...

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Mapping to Constellation Pattern

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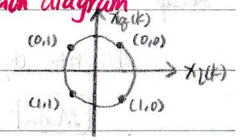
It is typically practice to describe a symbol $x(k)$ by a point in constellation diagram

i.e. its in-phase and quadrature components, $x_i(k)$ and $x_q(k)$

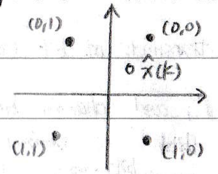


Quadrature Phase Shift Keying

Example for a case of $q=2$ bits per symbol (QPSK):

From the constellation pattern, the values $x_i(k)$ and $x_q(k)$ of symbol $x(k)$ are determined.

There is a one-to-one relationship between symbol set (constellation diagram) and modulation signal set (actually transmitted modulated signal)



In the receiver, the constellation point and therefore the transmitted symbol value is determined from the received signal sample $\hat{x}(k)$

Phase Shift Keying (PSK) 相移键控 / 相位偏移调制

In PSK, carrier phase used to carry symbol information, and modulation signal set:

$S_i(t) = A\cdot\cos(W_c t + \phi_i(t)),\ 0 \le t \le T_s,\ 1 \le i \le M = 2^q$

Constant carrier amplitude — Symbol Period — Number of symbol points in constellation diagram

"Phase" carries symbol information, namely to transmit i-th symbol value (point), signal $S(t)=S_i(t)$ is sent, note: $S(t) = A\cos(W_c t + \phi_i(t)) = \underbrace{A\cos(\phi_i(t))}_{\text{inphase symbol } x_i(t)}\cdot\cos(W_c t) + \underbrace{(-A\sin(\phi_i(t)))}_{\text{quadrature symbol } x_q(t)}\cdot\sin(W_c t)$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$S(t) = x_i(t)\cdot\cos(W_c t) + x_q(t)\cdot\sin(W_c t)$

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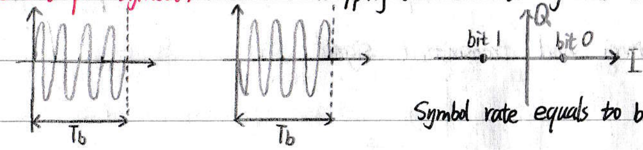
Binary Phase Shift Keying (BPSK)

※ quadrature branch is not used

One bit per symbol, note the mapping from bits to symbols in constellation diagram, where



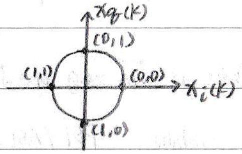
Symbol rate equals to bit rate and symbol period equals bit period

Modulation signal set $S_i(t) = A\cos(W_c t + \phi_i),\ i=1,2$ $\begin{cases} \text{bit 0 or symbol 1 } (+1): \phi_1 = 0 \\ \text{bit 1 or symbol 2 } (-1) = \phi_2 = \pi \end{cases}$

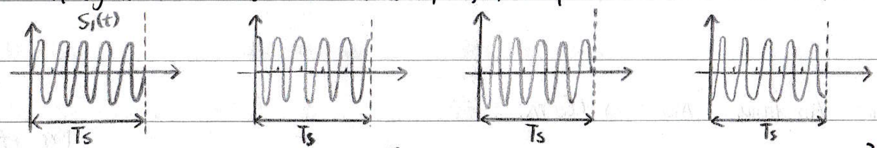
Phase separation: π

Quadrature Phase Shift Keying (QPSK)

Two bits per symbol with a minimum phase separation of $\frac{\pi}{2}$



Modulation signal set: $S_i(t) = A\cos(W_c t + \phi_i),\ 1 \le i \le 4$

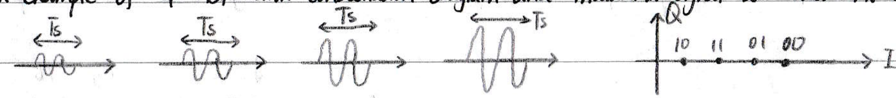


bit (0,0): $\phi_1 = 0$ bit (0,1): $\phi_2 = \frac{\pi}{2}$ bit (1,1): $\phi_3 = \pi$ bit (1,0): $\phi_4 = \frac{3}{2}\pi$

Amplitude Shift Keying (ASK) 振幅偏移键控

Pure ASK: carrier amplitude is used to carry symbol information

An example of 4-ASK with constellation diagram and modulation signal set $S(t) = A_i\cdot\cos(W_c t)$ $1 \le i \le 4$



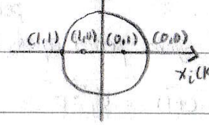
(1,0): A_1 (1,1): A_2 (0,1): A_3 (0,0): A_4

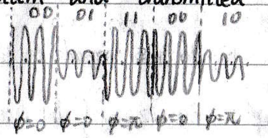
Note quadrature branch is not used, pure ASK rarely used itself as amplitude can easily be distorted by channel.

- Channel AWGN can seriously distort pure ASK.

Combined ASK/PSK

PSK and ASK can be combined. Here is an example of 4-ary or 4-PAM (pulse amplitude modulation) with constellation pattern and transmitted signal $s(t)$:





$s(t) = A\cos(\omega_c t + \phi(t))$

$\cos(\omega_c t)$:

2 amplitude level and phase shift of π are combined to represent 4-ary symbols

– 4-ary: 2 bits per symbol

Note in $\sqrt{M}$-ary or $\sqrt{M}$ PAM, quadrature component is not used, a more generic scheme of combining PSK/ASK is QAM, which uses both I and Q branches.

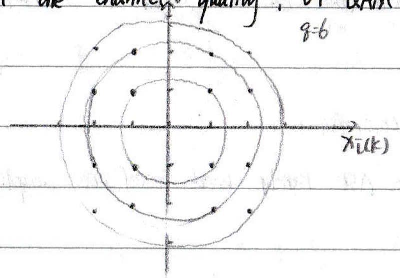
Quadrature Amplitude Modulation (QAM)

QAM: combines features of PSK and ASK, uses both I and Q components, and B is very efficient

An example of (squared) 16-QAM: – 4 bits per symbol

Note for squared M-QAM, I and Q branches are both $\sqrt{M}$-ary

Depending on the channel quality, 64-QAM ($q=6$) or 256-QAM ($q=8$) or higher order QAM are possible

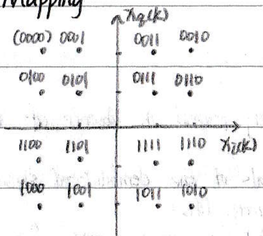


每个 symbol q [bits] — eg. 16: $q=4$

有 2^q M [symbols] — $M=16$

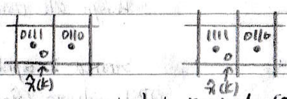
是 $\sqrt{M} \times \sqrt{M}$ ary — 4x4 ary

Gray Mapping



Gray coding: adjacent constellation points only differ in a single bit (minimum hamming distance)

Gray code



Non-gray code

Symbol (0110) was sent but received sample in neighbor region due to noise

If noise or distortions cause misclassification in the receiver, Gray coding can minimise the bit error rate

Frequency Shift Keying

恒包络调制

M-frequency shift keying: a constant envelope modulation with a set of frequencies $\{f_i, 1 \le i \le M = 2^q\}$ carrying symbol information

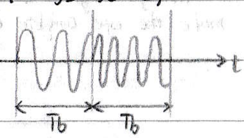
$S_i(t) = A\cos(2\pi f_i t + \theta),\ 1 \le i \le M = 2^q,\ 0 \le t \le T_s$

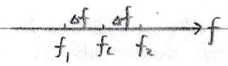
– BFSK, QFSK, etc. 1 bit per symbol, 2 bits per symbol, etc.

BFSK: $M=2$, bit 0: $f_1 = f_c - \Delta f$, bit 1: $f_2 = f_c + \Delta f$

$S_1(t) = A\cos(2\pi(f_c - \Delta f)t)$

$S_2(t) = A\cos(2\pi(f_c + \Delta f)t)$





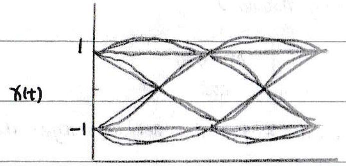
baseband signal bandwidth

With raised cosine pulse shaping, BFSK RF bandwidth is $B_p = 2\Delta f + 2B = 2\Delta f + (1+\gamma)B_0 R_b$

– Compared with BPSK's RF bandwidth $B_p = (1+\gamma)B_0$

Eye Diagram

[Perfect Channel]



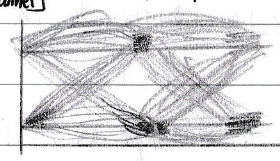
Design all components correctly, and in absence of noise

Stacked 2 symbol period intervals of the demodulated signal $\hat{x}_i(t)$ in QPSK scheme ($\hat{x}_i(k)$ is BPSK) looks like:

Ideal sampling of $\hat{x}_i(k)$ will sample the crossing points $\hat{x}_i(t) = \pm 1$

time/symbol period

[Noise Channel]

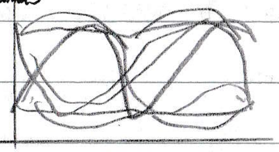


With channel noise at 10dB SNR

As long as the sampling points can be clearly determined and the eye is "open" $\hat{x}_i(k)$ will correctly resemble $x_i(k)$

At higher noise level, misclassification can occur if the eye is "closed"

[Distortion Channel]



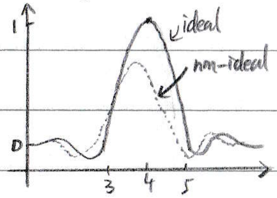
Although we design all components correctly for AWGN channel, channel may actually be Non-ideal.

Consider a very mild (温和) non-ideal channel with impulse response $c(t) = \delta(t) - \frac{1}{2}\cdot\delta(t - T_s/4)$, in absence of noise, the diagram ←

The eye diagram is closed, this ISI together with noise effect will make the eye complete closed, leading to misclassification

InterSymbol Interference (ISI)

Combined impulse response of an ideal pulse shaping filter of regular zero crossings with ideal channel $g_c(t) = \delta(t)$ and non-ideal channel $g_c(t) = \delta(t) - \frac{1}{2}\delta(t - t_s/4)$



For non-ideal channel, the combined Tx-filter-channel-Rx Filter has lost the property of a Nyquist system, no longer has regular $\boxed{\text{zero crossings}}$ (零交叉) at symbol spacing

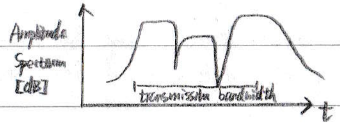
Dispersive (分散) Channel

Recall that zero ISI is achieved if combined Tx and Rx filter is a Nyquist System

But this is only true if the channel is ideal $\Rightarrow G_{Tx}(f)\cdot G_c(f)\cdot G_{Rx}(f) = G_{Tx}(f)\cdot R_{Rx}(f)$

If $G_c(f)$ is no-ideal, $G_{Tx}(f)\, G_c(f)\, G_{Rx}(f)$ will not be a Nyquist system;

E.g.: a distorting channel



Dispersive channel is caused by: (i) a restricted bandwidth (channel bandwidth is insufficient for the required transmission rate); or (ii) multipath distorting

Equalisation is needed for overcoming this channel distortion

MODEM Summary

Given bit rate R_b [bps] and resource of channel bandwidth B_p and power budget.

- Select a modulation scheme (bits to symbol map) so that symbol rate can fit into required baseband of $B = B_p/2$ and signal power can met power budget
- Pulse shaping ensures bandwidth constraint is met and maximizes receiver SNR
- At transmitter, baseband signal modulates carrier so transmitted signal is in required channel.
- At receiver, incoming carrier phase must be recovered to demodulate it, and timing must be recovered to correctly sampling demodulated signal

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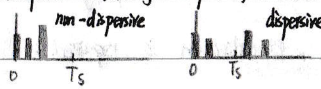
MODEM – L5 Equalisation 均衡

Dispersive Channel

Non-ideal channel has memory, i.e. is dispersive, which can be caused by

1. Restricted bandwidth, i.e. channel bandwidth is insufficient for the required transmission rate
2. Multipath distorting: copies of transmitted signal arrive at receiver with different excess delays



- If excess delay is small compared with symbol period Ts, channel is non-dispersive, i.e. ideal
- If exces delay is big compared with Ts, channel is dispersive, i.e. having memory

Discrete Channel Model

→ Tx filter → modulat → channel → demodu → Rx filter →

$\underbrace{\qquad\qquad\qquad\qquad\qquad}_{\text{Digital discrete channel}}$

$x[k] \rightarrow$ Symbol rate discrete channel $\xrightarrow{\hat{x}[k]} \oplus \rightarrow y[k]$, with $n[k]$ added at $\oplus$

If physical transmission channel is ideal, $y[k]$ is a noise corrupted delayed $x[k]$:

$$y[k] = x[k-k_d] + n[k]$$

If physical channel is dispersive (not ISI):

$$y[k] = \sum_{i=0}^{N_c} c_i \cdot x[k-i] + n[k]$$

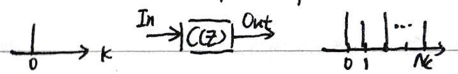
$\{c_i\}$ are the channel impulse response (CIR) taps, and N_c the length of CIR

Channel Impulse Response

Continuous-time signal/system → Fourier transform

Discrete-time signal/system → z-transform

Discrete channel with channel impulse response $\{c_0, c_1, c_2 \cdots, c_{N_c}\}$ $\quad C(z) = \sum_{i=0}^{N_c} c_i \cdot z^{-i}$



In practice, real signal/system are real-valued, ~~tou~~ but we can use equivalent baseband signal/system (as in QAM system) which are complex-valued

Equalisation - Solution

The system $C(z)$ is the z-transform of the discrete baseband channel model (including Tx and Rx filters, modulation, physical transmission channel, demodulation, and sampling.

If the channel has ~~several~~ severe amplitude and phase distortion, equalisation is required:

$$\xrightarrow{X(z)} C(z) \xrightarrow{Y(z)} W(z) \xrightarrow{\hat{X}(z)}$$

We want to find an equalisation filter $W(z)$ such that the recovered symbols $\hat{X}(z)$ are only delayed versions of the transmitted signal, $\hat{X}(z) = z^{-kd} \cdot X(z)$

The optimal solution for the noise-free case is (zero-forcing equalisation): ZF

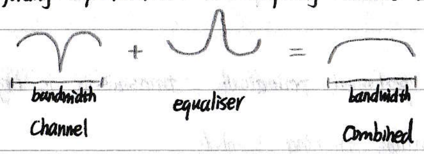
$$W(z) \cdot C(z) = z^{-kd} \quad \text{or} \quad W(z) = z^{-kd} \cdot C^{-1}(z)$$

- Since $C(z) = \sum_{i=0}^{N_c} c_i \cdot z^{-i}$ is a finite-duration impulse response (FIR) filter, $z^{-kd} \cdot C^{-1}(z)$ is an infinite-duration impulse (IIR) filter
- In practice we can only truncate $W(z)$ to a sufficiently long but finite-duration filter
$$W(z) = \sum_{i=0}^{N_e} w_i \cdot z^{-i} \approx z^{-kd} \cdot C^{-1}(z)$$

Another popular optimal equalisation is called minimum mean square error (MMSE) solution

Equalisation - Issues

Equaliser : aims to make the combined channel/equaliser a Nyquist system again
- Zero-forcing equalisation will completely remove ISI



- But the noise is amplified by the equaliser, and in high noise condition, ZF (zero-forcing) equalisation may enhance the noise to unacceptable level $(N(z) \cdot C^{-1}(z))$

Design of equaliser is a trade off between $\boxed{\text{eliminating ISI}}$ and $\boxed{\text{not enhancing noise too much}}$

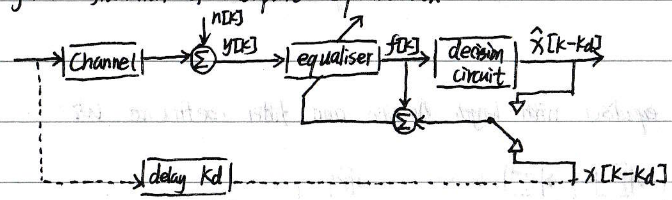
- MMSE equalisation provides better trade-off between eliminating ISI and enhancing noise

Also the channel can be time-varying, hence adaptive equalisation is needed

- Channel $\{c_i\}_{i=0}^{N_c}$ may change, and equaliser $\{w_i\}_{i=0}^{N_e}$ have to follow

Adaptive Equalisation - Architecture

The generic framework of adaptive equalisation



Equaliser sets its coefficients w_i to 'match' channel characteristics

- Training mode : Tx transmits a prefixed sequence known to Rx. The equaliser uses local generated symbols $x[k]$ as the desired response to adapt w_i
  - * As though, training data $\{x[k]\}$ were sent to receiver via a virtual pass
- Decision-directed mode : the equaliser assumes the decisions $\hat{x}[k-kd]$ are correct and uses them to substitute for $x[k-kd]$ as the desired response

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Adaptive Equalisation - Arrangement

For fixed (time-invariant) channel, equalisation is done once during link set up.
- During link set up, a prefixed training sequence is sent, and equaliser is trained based on locally generated this training sequence

For time-varying channel, equalisation must be performed periodically, transmission is organized in time frames, a small part of each frame contains training symbols.
- e.g. GSM mobile phone, middle of each Tx frame contains 26 training symbols

| data | training | data |

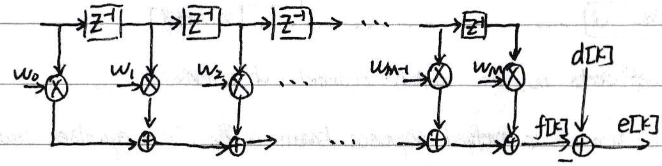
Frame structure

- Receiver uses locally generated training symbols for training equaliser, and the trained equaliser then detects the data in the frame

Blind equalisation: perform equalisation based on Rx signal $\{y[k]\}$ without access to training symbols $\{x[k]\}$, e.g. multipoint network, digital TV, etc
- Note training causes extra bandwidth, thus blind equalisation is attractive but is more difficult

Linear Equaliser

The setup of generic linear equaliser with length $Ne=M$ and filter coefficients w_i:



The aim of the equaliser is to set its co coefficients w_i to produce an output $f[k]$:

$$f[k] = \sum_{i=0}^{M} w_i^* \cdot y[k-i]$$

- that is as close as possible to the desired signal $d[k]$:

$$d[k] = \begin{cases} x[k-k_d], & \text{training} \\ \hat{x}[k-k_d], & \text{decision directed} \end{cases}$$

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- Conventionally, conjugate w_i^* of w_i is used in producing equaliser output
- Equaliser length M should be sufficiently long to cancel channel induced ISI, but not too long as to amplify noise too much
- Equaliser decision delay K_d depends the zero locations of the channel transfer function $C(z)$: for minimum phase $C(z)$, $K_d=0$; otherwise, $K_d>0$

后续请看PPT 中 Mean Square Error

$P(X_0)=P$ $\quad$ $P(Y_0|X_0)=P(Y_1|X_1)=1-P_e$

$P(X_1)=1-P$ $\quad$ $P(Y_0|X_1)=P(Y_1|X_0)=P_e$

$P-PP_e+P_e-PP_e$

$P(Y_0)=P(X_0)\cdot P(Y_0|X_0)+P(X_1)\cdot P(Y_0|X_1)=P\cdot(1-P_e)+(1-P)\cdot P_e = P+P_e-2PP_e = P+P_e(1-2P)$

$P(Y_1)=P(X_0)\cdot P(Y_1|X_0)+P(X_1)\cdot P(Y_1|X_1)=P\cdot P_e+(1-P)\cdot(1-P_e)=P\cdot P_e+1-P_e-P+PP_e = 1+2PP_e-P_e-P$

$=1-P+P_e(2P-1)$

$x=P_e(1-2P)$

$H(X)-H(X|Y)=H(Y)-H(Y|X)$ $\quad$ $P(Y_0)=P(X_0)+x$

$P(Y_1)=P(X_1)-x$

$H(X|Y)=$

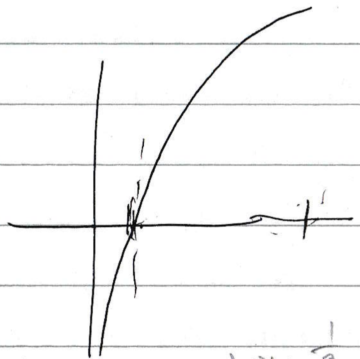
$H(X)=P(X_0)\cdot\log_2\frac{1}{P(X_0)}+P(X_1)\cdot\log_2\frac{1}{P(X_1)}$

$=P\cdot\log_2\frac{1}{P}+(1-P)\cdot\log_2\frac{1}{1-P}$

$H(Y)=(P+x)\cdot\log_2\frac{1}{P+x}+(1-P-x)\log_2\frac{1}{1-P-x}$

$P\cdot\log_2\frac{1}{P}+(1-P)\cdot\log_2\frac{1}{1-P}-P\cdot\log_2\frac{1}{P+x}-x\cdot\log_2\frac{1}{P+x}+\log_2\frac{1}{1-P-x}-P\cdot\log_2\frac{1}{1-P-x}-x\cdot\log_2\frac{1}{1-P-x}$

$=$



P

$t\log_2\frac{1}{t}+(1-t)\log_2\frac{1}{1-t}$

$P\log_2\frac{1}{P}+(1-P)\log_2\frac{1}{1-P}$

$\frac{1}{t}<\frac{1}{P}$

$t>P$ $\quad$ $1-t<1-P$

$\frac{1}{1-t}>\frac{1}{1-P}$

$\frac{d\,P\log_2\frac{1}{P}}{dP}$

$\log_2\frac{1}{P}+P.$

MODEM - L3

$$\cos 2w = \cos^2 w - \sin^2 w = 2\cos^2 w - 1 = 1 - 2\sin^2 w \Rightarrow \sin^2 w = \frac{1}{2}(1-\cos 2w)$$

$$\sin 2w = 2\sin w \cos w$$

$$\begin{aligned}\hat{x}'_i(t) = s(t)\cdot\cos(w_c t) &= (x_i(t)\cdot\cos(w_c t) + x_q(t)\cdot\sin(w_c t))\cdot\cos(w_c t)\\ &= x_i(t)\cdot\cos^2(w_c t) + x_q(t)\cdot\cos(w_c t)\cdot\sin(w_c t)\\ &= x_i(t)\cdot\frac{1}{2}\cdot(1+\cos(2w_c t)) + x_q(t)\cdot\frac{1}{2}\cdot\sin(2w_c t)\\ &= x_i(t)\cdot\frac{1}{2}\cdot(1+\underbrace{\cos(2w_c t)}_{\text{high } f}) + x_q(t)\cdot\underbrace{\frac{1}{2}\sin(2w_c t)}_{\text{high } f}\end{aligned}$$

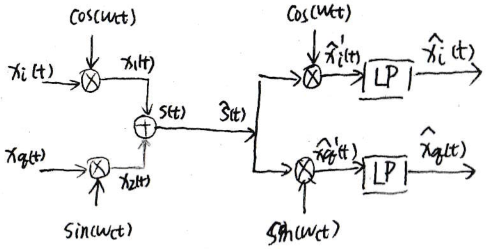
$$\xrightarrow{LP} \approx x_i(t)\cdot\frac{1}{2}$$

$$\begin{aligned}\hat{x}'_q(t) = s(t)\cdot\sin(w_c t) &= (x_i(t)\cdot\cos(w_c t) + x_q(t)\cdot\sin(w_c t))\cdot\sin(w_c t)\\ &= x_i(t)\cdot\cos(w_c t)\cdot\sin(w_c t) + x_q(t)\cdot\sin^2(w_c t)\\ &= \cancel{x_i(t)\cdot\frac{1}{2}\cdot\sin 2w_c t + x_q(t)\cdot\frac{1}{2}}\\ &= x_i(t)\cdot\frac{1}{2}\cdot\underbrace{\sin 2w_c t}_{X} + x_q(t)\cdot\frac{1}{2}(1-\underbrace{\cos^2 w}_{X})\end{aligned}$$

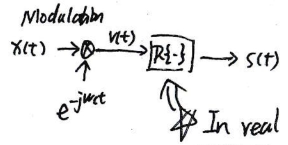
$$\xrightarrow{LP} \approx x_q(t)\cdot\frac{1}{2}$$

$$\begin{cases} x_1(t) = x_i(t)\cdot\cos(w_c t)\\ x_2(t) = x_q(t)\cdot\sin(w_c t)\end{cases}$$

$$s(t) = x_1(t) + x_2(t)$$



Complex notation : Modulation



In real world, signals are always real-valued.

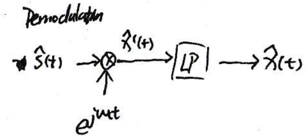
$$\Rightarrow x(t) = x_i(t) + j x_q(t)$$

$$e^{-jw_c t} = \cos w_c t - j\sin w_c t$$

$$\begin{aligned}s(t) = x(t)\cdot e^{-jw_c t} &= (x_i(t) + jx_q(t))\cdot(\cos w_c t - j\sin w_c t)\\ &= x_i(t)\cdot\cos w_c t + x_q(t)\cdot\sin w_c t + \cancel{j(\cdots)}\end{aligned}$$

$$\Rightarrow s(t) = x_i(t)\cdot\cos w_c t + x_q(t)\cdot\sin w_c t$$

Demodulation



$$\begin{aligned}\hat{x}'(t) &= (\cos(w_c t) + j\sin(w_c t))\cdot(x_i(t)\cdot\cos(w_c t) + x_q(t)\cdot\sin(w_c t))\\ &= x_i(t)\cdot\frac{1}{2}(1+\cos(2w_c t) + j\sin(2w_c t)) + j x_q(t)\cdot\frac{1}{2}(1-\cos(2w_c t) - j\sin(2w_c t))\end{aligned}$$

$$\xrightarrow{LP} \approx \frac{1}{2}x_i(t) + j\cdot\frac{1}{2}x_q(t)$$

Carrier recovery

$\hat{s}(t)$ vs $\check{s}(t)$ $\rightarrow \Delta\varphi = \varphi - \check{\varphi}$

$\hat{s}(t)\cdot\check{s}(t) \Rightarrow x(t)\cdot e^{j\Delta\varphi}$

Timing recovery

mismatch $\Delta w_c t$

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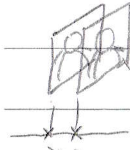
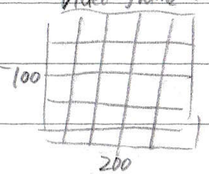
Video Compression

30 frames/second

identify the little movement Region

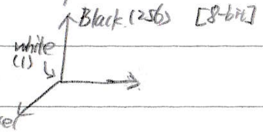


Video frame : $100 \times 200 = 2 \times 10^4$ pixel



Red Green Blue

Each pixel has three elements RGB



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① PCM video code at 8-bit per pixel

$2 \cdot 10^4 \cdot 8 = 1.6 \times 10^5 = 160000$ bits/frame

8-bits for per pixel

30 frames/per s → 4800000 4.8×10^6 bits/second ≈ 4MBit/s

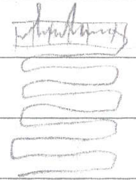
② DPCM

4 bit/symbol 16 QAM = B = 20MHz in 4G

80Mb/s

20 for PCM video

DPCM



in a single line 4bits/pixel

halving the bit rate of 8 bit PCM by using DPCM we can accommodate twice the # of width?

motion compensation

① PCM 4.8Mb/s

② DPCM 2.4Mb/s

③ VQ

a. Motion Compensation : 200 blocks

seach area: $\underbrace{\text{16 horizontal}}_{\text{4 bit/MVH}} + \underbrace{\text{16 vertical pos}}_{\text{4 bit/MVH}}$ → 8bits/frames

1600 bits/frame

b. VQ