MATH 151 Lab 8

Put team members' names and section number here.

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Section number 576

In [1]:
```
from sympy import *
from sympy.plotting import (plot,plot_parametric)
```

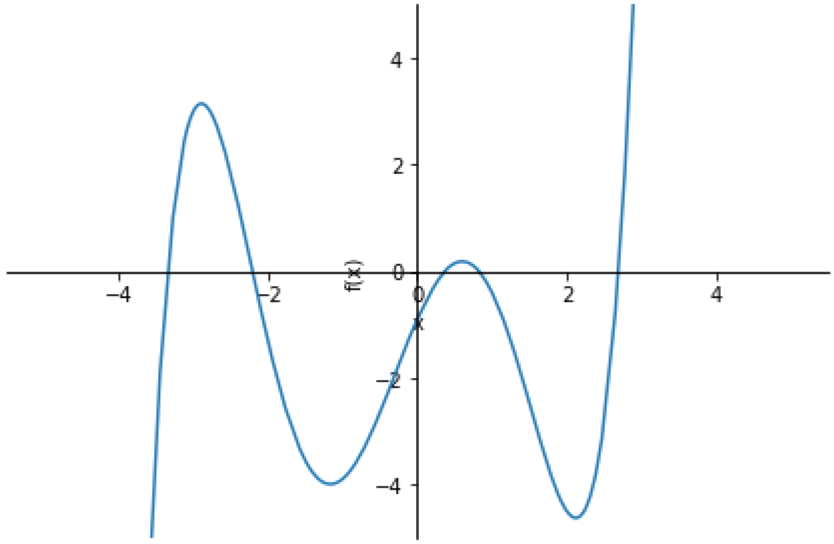
Question 1

1a

In [2]:
```
x = symbols("x")
fx = (1/40) * (x**6 + 2*x**5 - 16*x**4 - 20*x**3 + 64*x**2 - 36*x + 72)

print(f"f'(x) = {diff(fx, x, 1)}")
print(f"Approximate critical numbers: x = {solve(diff(fx, x, 1))}")
print(f"f'(-4) = {diff(fx, x, 1).subs(x, -4)} is negative, so f(x) is decreasing on th
print(f"f'(-3) = {diff(fx, x, 1).subs(x, -3)} is positive, so f(x) is increasing on th
print(f"f'(0) = {diff(fx, x, 1).subs(x, 0)} is negative, so f(x) is decreasing on the
print(f"f'(0.5) = {diff(fx, x, 1).subs(x, 0.5)} is positive, so f(x) is increasing on
print(f"f'(2) = {diff(fx, x, 1).subs(x, 2)} is negative, so f(x) is decreasing on the
print(f"f'(4) = {diff(fx, x, 1).subs(x, 4)} is positive, so f(x) is increasing on the
graph1 = plot(diff(fx, x, 1), (x, -5 ,5), ylim = (-5 ,5))
```

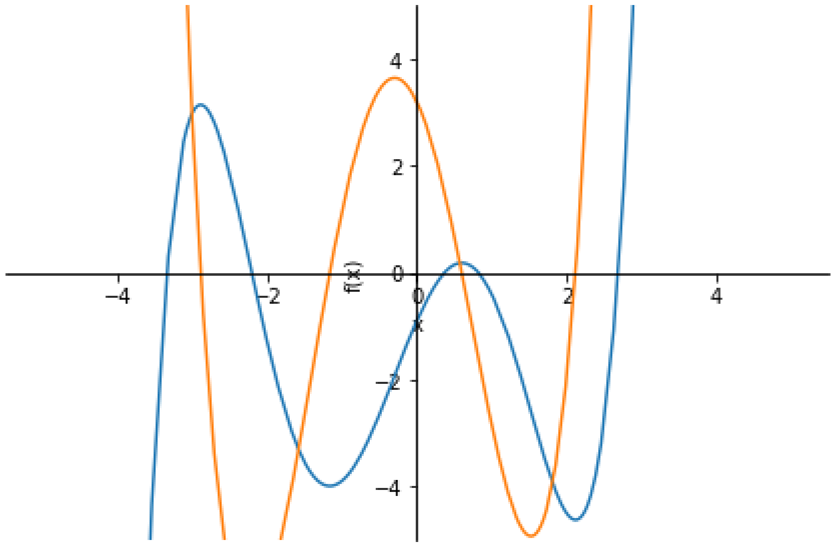
```
f'(x) = 0.15*x**5 + 0.25*x**4 - 1.6*x**3 - 1.5*x**2 + 3.2*x - 0.9
Approximate critical numbers: x = [-3.34365086397455, -2.20571930723638, 0.3677857145
82751, 0.821156998770767, 2.69376079119075]
f'(-4) = -24.9000000000000 is negative, so f(x) is decreasing on the interval (-oo, -
3.34365086397455)
f'(-3) = 3.00000000000000 is positive, so f(x) is increasing on the interval (-3.3436
5086397455, -2.20571930723638)
f'(0) = -0.900000000000000 is negative, so f(x) is decreasing on the interval (-2.205
71930723638, 0.367785714582751)
f'(0.5) = 0.145312500000000 is positive, so f(x) is increasing on the interval (0.367
785714582751, 0.821156998770767)
f'(2) = -4.50000000000000 is negative, so f(x) is decreasing on the interval (0.82115
6998770767, 2.69376079119075)
f'(4) = 103.100000000000 is positive, so f(x) is increasing on the interval (2.693760
79119075, oo)
```



1b

In [3]:
```
print(f"f''(X) = {diff(fx, x, 2)}")
inflectionPoints = [N(i) for i in real_roots(diff(fx, x, 2), x)]
print(f"Approximate inflection values: x = {inflectionPoints}")
print(f"f''(-3) = {diff(fx, x, 2).subs(x, -3)} is , so f(x) is concave up on the inter
print(f"f''(-2) = {diff(fx, x, 2).subs(x, -2)} is , so f(x) is concave down on the int
print(f"f''(0) = {diff(fx, x, 2).subs(x, 0)} is , so f(x) is concave up on the interva
print(f"f''(1) = {diff(fx, x, 2).subs(x, 1)} is , so f(x) is concave down on the inter
print(f"f''(3) = {diff(fx, x, 2).subs(x, 3)} is , so f(x) is concave up on the interva
graph1 = plot((diff(fx, x, 1), (x, -5 ,5)), (diff(fx, x, 2), (x, -5 ,5)), ylim = (-5 ,
```

```
f''(X) = 0.75*x**4 + 1.0*x**3 - 4.8*x**2 - 3.0*x + 3.2
Approximate inflection values: x = [-2.89174218338126, -1.16242859299527, 0.597894461
879547, 2.12294298116364]
f''(-3) = 2.75000000000000 is , so f(x) is concave up on the interval (-oo, -2.891742
18338126)
f''(-2) = -6.00000000000000 is , so f(x) is concave down on the interval (-2.89174218
338126, -1.16242859299527)
f''(0) = 3.20000000000000 is , so f(x) is concave up on the interval (-1.162428592995
27, 0.597894461879547)
f''(1) = -2.85000000000000 is , so f(x) is concave down on the interval (0.5978944618
79547, 2.12294298116364)
f''(3) = 38.7500000000000 is , so f(x) is concave up on the interval (2.1229429811636
4, oo)
```



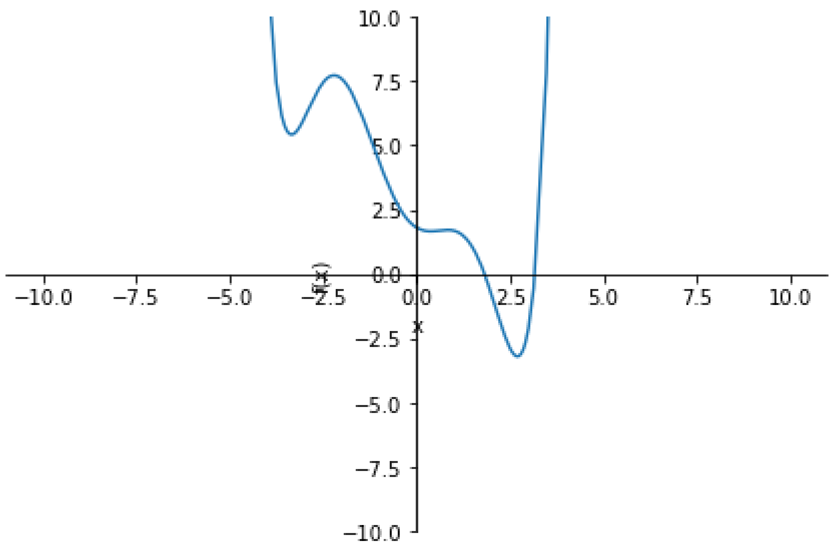
1c

In [4]:
```
print(f"f(x) has {len(solve(diff(fx, x, 1)))} local extrema and {len(inflectionPoints)
```

```
f(x) has 5 local extrema and 4 inflection points.
```

1d

In [5]:
```
plot(fx, (x, -10, 10), ylim = (-10, 10))
```



Out[5]: `<sympy.plotting.plot.Plot at 0x17f76cb57f0>`

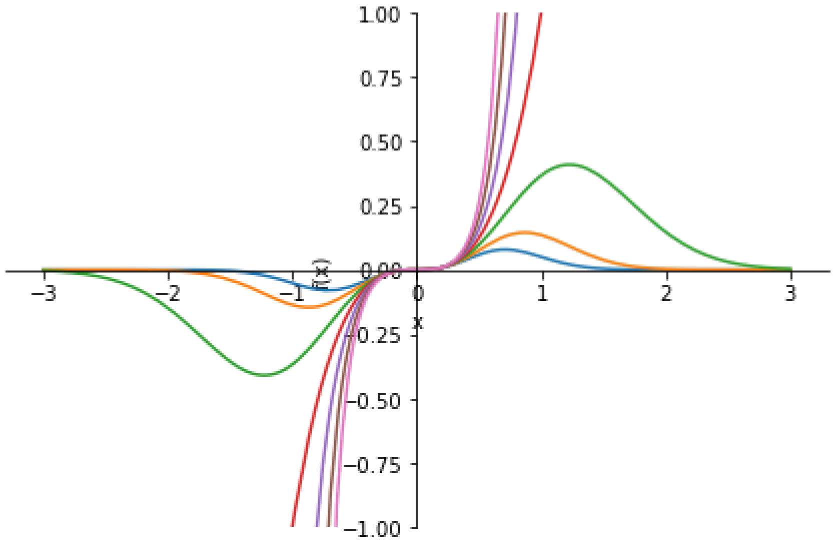
Question 2

2a

In [6]:
```
b = symbols("b")
gx = x**3 * E**(b*x**2)

graph2 = plot((gx.subs(b, -3), (x, -3, 3)), ylim = (-1, 1), show = False)
for i in range(-2, 4):
```

```
    graph2.append(plot(gx.subs(b, i), (x, -3, 3), show = False)[0])
graph2.show()
```



2b

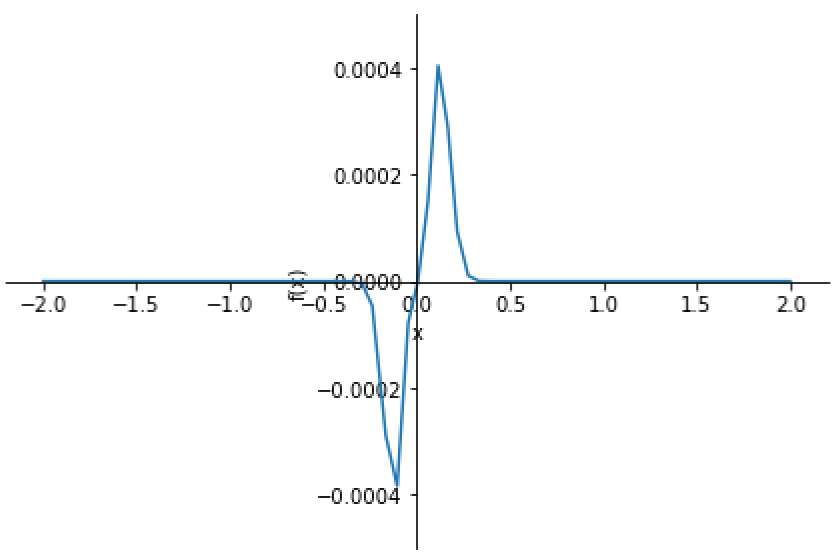
In [7]:
```
print(f"Critical values of g: b = {solve(diff(gx, x), x)}")
print("In order for the critical values -sqrt(6)*sqrt(-1/b)/2 and sqrt(6)*sqrt(-1/b)/2
```

```
Critical values of g: b = [0, -sqrt(6)*sqrt(-1/b)/2, sqrt(6)*sqrt(-1/b)/2]
In order for the critical values -sqrt(6)*sqrt(-1/b)/2 and sqrt(6)*sqrt(-1/b)/2 to be
real, b must be negative
```

2c

In [8]:
```
print("As b approaches negative infinity, the critical values approach 0.")
plot(gx.subs(b, -100), (x, -2, 2), ylim = (-0.0005, 0.0005), depth = 10000)
```

```
As b approaches negative infinity, the critical values approach 0.
```



Out[8]:
```
<sympy.plotting.plot.Plot at 0x17f79bd63d0>
```

2d

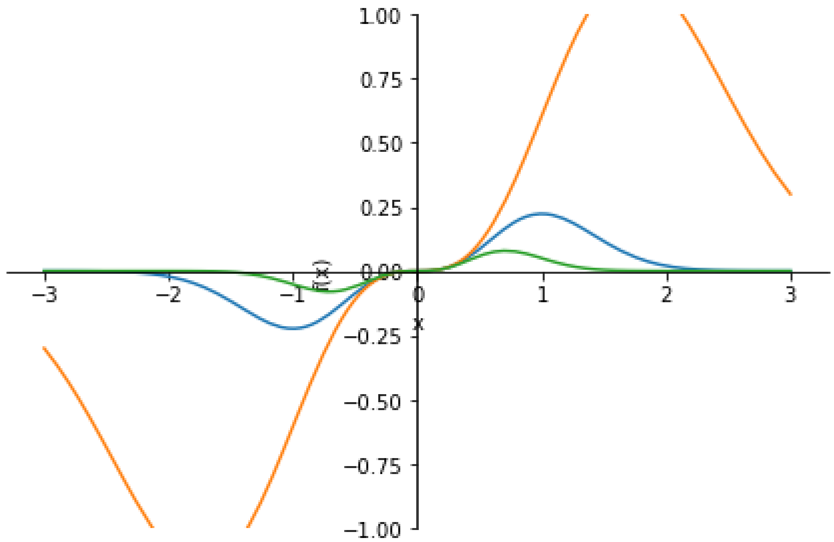
In [9]:
```
print(f"Inflection points of g: b = {solve(diff(gx, x, 2), x)}")
print("All inflection points are real when b is negative.")
```

```
Inflection points of g: b = [0, -sqrt(2)*sqrt(-1/b)/2, sqrt(2)*sqrt(-1/b)/2, -sqrt(3)
*sqrt(-1/b), sqrt(3)*sqrt(-1/b)]
All inflection points are real when b is negative.
```

2e

In [10]:
```
print(f"The critical values include -1 and 1 when b = {solve(solve(diff(gx, x, 1), x)[
print(f"The inflection points include -1 and 1 when b = {solve(solve(diff(gx, x, 2), x
plot((gx.subs(b, -3/2), (x, -3, 3)), (gx.subs(b, -1/2), (x, -3, 3)), (gx.subs(b, -3),
```

```
The critical values include -1 and 1 when b = -3/2
The inflection points include -1 and 1 when b = -1/2 and -3
```



Out[10]:
```
<sympy.plotting.plot.Plot at 0x17f76e284f0>
```

Question 3

3a

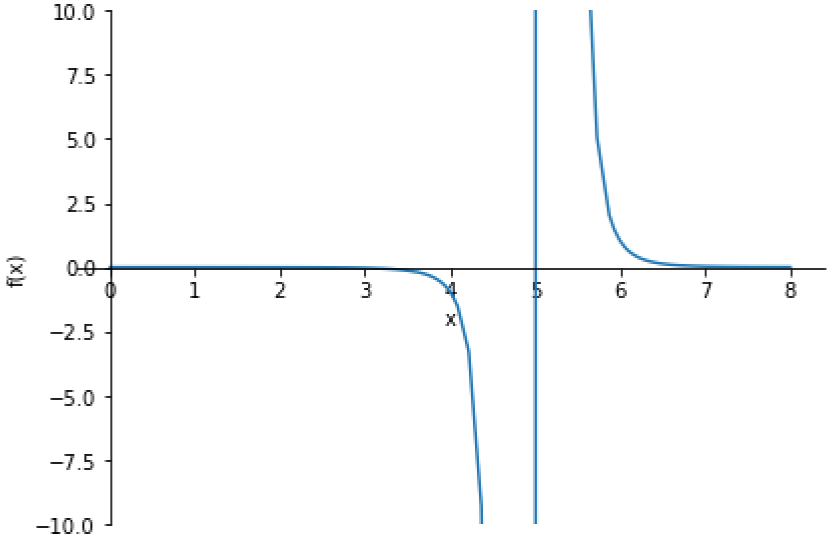
In [11]:
```
fx = ln(5 - x)
a, b = 1, 4
secant = (fx.subs(x, b) - fx.subs(x, a)) / (b - a)
print(f"c = {solve(diff(fx, x, 1) - secant, x)[0].evalf()} or {solve(diff(fx, x, 1) -
```

```
c = 2.83595743866655 or (-3 + log(1024))/log(4)
```

3b

In [12]:
```
gx = (x - 5)**(-1 * 5)
print("The Mean Value Theorem cannot be applied here since g(x) is not continuous on t
plot(gx, (x, 0, 8), ylim = (-10, 10))
```

```
The Mean Value Theorem cannot be applied here since g(x) is not continuous on the ope
n interval (0, 8).
```



Out[12]: <sympy.plotting.plot.Plot at 0x17f76cb56d0>

3c

In [13]:

```
hx = 8*x**2 * cos(4*x)
a, b = pi/4, 3*pi/4
secant = (hx.subs(x, b) - hx.subs(x, a)) / (b - a)
print(f"c = {nsolve(diff(hx, x, 1) - secant, x, 1.7)}")
```

```
c = 1.70739757583842
```