SOCIETY OF ACTUARIES

EXAM FAM   FUNDAMENTALS OF ACTUARIAL MATHEMATICS

EXAM FAM SAMPLE QUESTIONS

The weight of topics in these sample questions is not representative of the weight of topics on the exam. The syllabus indicates the exam weights by topic.

September 2022 changes: Edits made to Questions/Solutions 1, 20, 38; Question 5 deleted; Questions 77 and 78 added.

December 2022 changes: Questions 79-101 added

May 2024 change: Question 102 added

1. You are given:

(i) The number of claims has a Poisson distribution.

(ii) Claim sizes have a Pareto distribution with parameters $\theta = 0.5$ and $\alpha = 6$

(iii) The number of claims and claim sizes are independent.

(iv) The observed average total payment should be within 2% of the expected average total payment 90% of the time.

Calculate the expected number of claims needed for full credibility.

(A) Less than 7,000

(B) At least 7,000, but less than 10,000

(C) At least 10,000, but less than 13,000

(D) At least 13,000, but less than 16,000

(E) At least 16,000

2. You are given:

(i) Losses follow a single-parameter Pareto distribution with density function:

$$f(x) = \frac{\alpha}{x^{\alpha+1}}, \quad x > 1, \quad 0 < \alpha < \infty$$

(ii) A random sample of size five produced three losses with values 3, 6 and 14, and two losses exceeding 25.

Calculate the maximum likelihood estimate of α .

(A) 0.25

(B) 0.30

(C) 0.34

(D) 0.38

(E) 0.42

3. The distribution of accidents for 84 randomly selected policies is as follows:

Number of Accidents	Number of Policies
0	32
1	26
2	12
3	7
4	4
5	2
6	1
Total	84

Determine which of the following models best represents these data.

(A) Negative binomial

(B) Discrete uniform

(C) Poisson

(D) Binomial

(E) Zero-modified Poisson

4. You are given:

(i) Low-hazard risks have an exponential claim size distribution with mean θ.

(ii) Medium-hazard risks have an exponential claim size distribution with mean 2θ.

(iii) High-hazard risks have an exponential claim size distribution with mean 3θ.

(iv) No claims from low-hazard risks are observed.

(v) Three claims from medium-hazard risks are observed, of sizes 1, 2 and 3.

(vi) One claim from a high-hazard risk is observed, of size 15.

Calculate the maximum likelihood estimate of θ.

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

5. Deleted

6. You are given:

Claim Size (X)	Number of Claims
(0, 25]	25
(25, 50]	28
(50, 100]	15
(100, 200]	6

Assume a uniform distribution of claim sizes within each interval.

Calculate $E(X^2) - E[(X \wedge 150)^2]$.

(A) Less than 200

(B) At least 200, but less than 300

(C) At least 300, but less than 400

(D) At least 400, but less than 500

(E) At least 500

7. The number of claims follows a negative binomial distribution with parameters β and r, where β is unknown and r is known. You wish to estimate β based on n observations, where $\bar{x}$ is the mean of these observations.

Determine the maximum likelihood estimate of β.

(A) $\bar{x} / r^2$

(B) $\bar{x} / r$

(C) $\bar{x}$

(D) $r\bar{x}$

(E) $r^2\bar{x}$

8. You are given the following information about a commercial auto liability book of business:

(i) Each insured's claim count has a Poisson distribution with mean λ, where λ has a gamma distribution with $\alpha = 1.5$ and $\theta = 0.2$.

(ii) Individual claim size amounts are independent and exponentially distributed with mean 5000.

(iii) The full credibility standard is for aggregate losses to be within 5% of the expected with probability 0.90.

Calculate the expected number of claims required for full credibility using limited fluctuated credibility.

(A) 2165

(B) 2381

(C) 3514

(D) 7216

(E) 7938

9. You are given:

(i) Losses follow an exponential distribution with mean θ.

(ii) A random sample of 20 losses is distributed as follows:

Loss Range	Frequency
[0, 1000]	7
(1000, 2000]	6
(2000, ∞)	7

Calculate the maximum likelihood estimate of θ.

(A) Less than 1950

(B) At least 1950, but less than 2100

(C) At least 2100, but less than 2250

(D) At least 2250, but less than 2400

(E) At least 2400

10. You are given:

Number of Claims	Probability	Claim Size	Probability
0	1/5		
1	3/5	25	1/3
		150	2/3
2	1/5	50	2/3
		200	1/3

Claim sizes are independent.

Calculate the variance of the aggregate loss.

(A) 4,050

(B) 8,100

(C) 10,500

(D) 12,510

(E) 15,612

11. You observe the following five ground-up claims from a data set that is truncated from below at 100:

125 150 165 175 250

You fit a ground-up exponential distribution using maximum likelihood estimation.

Calculate the mean of the fitted distribution.

(A) 73

(B) 100

(C) 125

(D) 156

(E) 173

12. You are given the following information about a general liability book of business comprised of 2500 insureds:

(i) $X_i = \sum_{j=1}^{N_i} Y_{ij}$ is a random variable representing the annual loss of the *i*th insured.

(ii) $N_1, N_2, \ldots, N_{2500}$ are independent and identically distributed random variables following a negative binomial distribution with parameters $r = 2$ and $\beta = 0.2$.

(iii) $Y_{i1}, Y_{i2}, \ldots, Y_{iN_i}$ are independent and identically distributed random variables following a Pareto distribution with $\alpha = 3$ and $\theta = 1000$.

(iv) The full credibility standard is to be within 5% of the expected aggregate losses 90% of the time.

Calculate the partial credibility of the annual loss experience for this book of business using limited fluctuation credibility theory.

(A) 0.34

(B) 0.42

(C) 0.47

(D) 0.50

(E) 0.53

13. Losses come from a mixture of an exponential distribution with mean 100 with probability p and an exponential distribution with mean 10,000 with probability $1 - p$.

Losses of 100 and 2000 are observed.

Determine the likelihood function of p.

(A) $$\left(\frac{pe^{-1}}{100} \bullet \frac{(1-p)e^{-0.01}}{10,000}\right) \bullet \left(\frac{pe^{-20}}{100} \bullet \frac{(1-p)e^{-0.2}}{10,000}\right)$$

(B) $$\left(\frac{pe^{-1}}{100} \bullet \frac{(1-p)e^{-0.01}}{10,000}\right) + \left(\frac{pe^{-20}}{100} \bullet \frac{(1-p)e^{-0.2}}{10,000}\right)$$

(C) $$\left(\frac{pe^{-1}}{100} + \frac{(1-p)e^{-0.01}}{10,000}\right) \bullet \left(\frac{pe^{-20}}{100} + \frac{(1-p)e^{-0.2}}{10,000}\right)$$

(D) $$\left(\frac{pe^{-1}}{100} + \frac{(1-p)e^{-0.01}}{10,000}\right) + \left(\frac{pe^{-20}}{100} + \frac{(1-p)e^{-0.2}}{10,000}\right)$$

(E) $$p\left(\frac{e^{-1}}{100} + \frac{e^{-0.01}}{10,000}\right) + (1-p)\left(\frac{e^{-20}}{100} + \frac{e^{-0.2}}{10,000}\right)$$

14. A health plan implements an incentive to physicians to control hospitalization under which the physicians will be paid a bonus B equal to c times the amount by which total hospital claims are under 400 $(0 \le c \le 1)$.

The effect the incentive plan will have on underlying hospital claims is modeled by assuming that the new total hospital claims will follow a Pareto distribution with $\alpha = 2$ and $\theta = 300$.

$E(B) = 100$

Calculate c.

(A) 0.44

(B) 0.48

(C) 0.52

(D) 0.56

(E) 0.60

15. Computer maintenance costs for a department are modeled as follows:

(i) The distribution of the number of maintenance calls each machine will need in a year is Poisson with mean 3.

(ii) The cost for a maintenance call has mean 80 and standard deviation 200.

(iii) The number of maintenance calls and the costs of the maintenance calls are all mutually independent.

The department must buy a maintenance contract to cover repairs if there is at least a 10% probability that aggregate maintenance costs in a given year will exceed 120% of the expected costs.

Calculate the minimum number of computers needed to avoid purchasing a maintenance contract using the normal approximation for the distribution of the aggregate maintenance costs.

(A) 80

(B) 90

(C) 100

(D) 110

(E) 120

16. Aggregate losses for a portfolio of policies are modeled as follows:

(i) The number of losses before any coverage modifications follows a Poisson distribution with mean λ.

(ii) The severity of each loss before any coverage modifications is uniformly distributed between 0 and b.

The insurer would like to model the effect of imposing an ordinary deductible, d $(0 < d < b)$, on each loss and reimbursing only a percentage, c $(0 < c \leq 1)$, of each loss in excess of the deductible.

It is assumed that the coverage modifications will not affect the loss distribution.

The insurer models its claims with modified frequency and severity distributions. The modified claim amount is uniformly distributed on the interval $[0, c(b-d)]$.

Determine the mean of the modified frequency distribution.

(A) λ

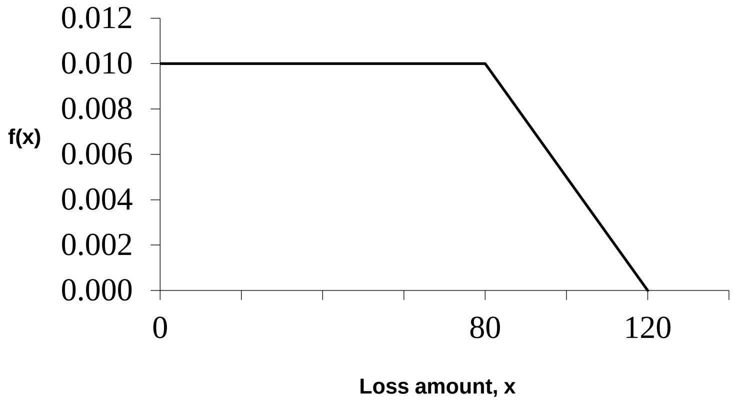
(B) λc

(C) $\lambda \dfrac{d}{b}$

(D) $\lambda \dfrac{b-d}{b}$

(E) $\lambda c \dfrac{b-d}{b}$

17. The graph of the density function for losses is:



Calculate the loss elimination ratio for an ordinary deductible of 20.

(A) 0.20

(B) 0.24

(C) 0.28

(D) 0.32

(E) 0.36

18. A towing company provides all towing services to members of the City Automobile Club. You are given:

Towing Distance	Towing Cost	Frequency
0-9.99 miles	80	50%
10-29.99 miles	100	40%
30+ miles	160	10%

(i) The automobile owner must pay 10% of the cost and the remainder is paid by the City Automobile Club.

(ii) The number of towings has a Poisson distribution with mean of 1000 per year.

(iii) The number of towings and the costs of individual towings are all mutually independent.

Calculate the probability that the City Automobile Club pays more than 90,000 in any given year using the normal approximation for the distribution of aggregate towing costs.

(A) 3%

(B) 10%

(C) 50%

(D) 90%

(E) 97%

19. You are given:

(i) Losses follow an exponential distribution with the same mean in all years.

(ii) The loss elimination ratio this year is 70%.

(iii) The ordinary deductible for the coming year is 4/3 of the current deductible.

Calculate the loss elimination ratio for the coming year.

(A) 70%

(B) 75%

(C) 80%

(D) 85%

(E) 90%

20. Prescription drug losses, *S*, are modeled assuming the number of claims has a geometric distribution with mean 4, and the amount of each prescription is 40.

Calculate $E[(S-100)_+]$.

(A) 60

(B) 82

(C) 92

(D) 114

(E) 146

21. At the beginning of each round of a game of chance the player pays 12.5. The player then rolls one die with outcome N. The player then rolls N dice and wins an amount equal to the total of the numbers showing on the N dice. All dice have 6 sides and are fair.

Calculate the probability that a player starting with 15,000 will have at least 15,000 after 1000 rounds using the normal approximation.

(A) 0.01

(B) 0.04

(C) 0.06

(D) 0.09

(E) 0.12

22. X is a discrete random variable with a probability function that is a member of the (a,b,0) class of distributions.

You are given:

(i) $\mathrm{P}(X = 0) = \mathrm{P}(X = 1) = 0.25$

(ii) $\mathrm{P}(X = 2) = 0.1875$

Calculate $\mathrm{P}(X = 3)$.

(A) 0.120

(B) 0.125

(C) 0.130

(D) 0.135

(E) 0.140

23. A group dental policy has a negative binomial claim count distribution with mean 300 and variance 800.

Ground-up severity is given by the following table:

Severity	Probability
40	0.25
80	0.25
120	0.25
200	0.25

You expect severity to increase 50% with no change in frequency. You decide to impose a per claim deductible of 100.

Calculate the expected total claim payment after these changes.

(A) Less than 18,000

(B) At least 18,000, but less than 20,000

(C) At least 20,000, but less than 22,000

(D) At least 22,000, but less than 24,000

(E) At least 24,000

24. You own a light bulb factory. Your workforce is a bit clumsy – they keep dropping boxes of light bulbs. The boxes have varying numbers of light bulbs in them, and when dropped, the entire box is destroyed.

You are given:

- Expected number of boxes dropped per month: 50
- Variance of the number of boxes dropped per month: 100
- Expected value per box: 200
- Variance of the value per box: 400

You pay your employees a bonus if the value of light bulbs destroyed in a month is less than 8000.

Assuming independence and using the normal approximation, calculate the probability that you will pay your employees a bonus next month.

(A) 0.16

(B) 0.19

(C) 0.23

(D) 0.27

(E) 0.31

25. For a certain company, losses follow a Poisson frequency distribution with mean 2 per year, and the amount of a loss is 1, 2, or 3, each with probability 1/3. Loss amounts are independent of the number of losses, and of each other.

An insurance policy covers all losses in a year, subject to an annual aggregate deductible of 2.

Calculate the expected claim payments for this insurance policy.

(A) 2.00

(B) 2.36

(C) 2.45

(D) 2.81

(E) 2.96

26. The unlimited severity distribution for claim amounts under an auto liability insurance policy is given by the cumulative distribution:

$$F(x) = 1 - 0.8e^{-0.02x} - 0.2e^{-0.001x}, \quad x \geq 0$$

The insurance policy pays amounts up to a limit of 1000 per claim.

Calculate the expected payment under this policy for one claim.

(A) 57

(B) 108

(C) 166

(D) 205

(E) 240

27. The random variable for a loss, X, has the following characteristics:

x	$F(x)$	$E(X \wedge x)$
0	0.0	0
100	0.2	91
200	0.6	153
1000	1.0	331

Calculate the mean excess loss for a deductible of 100.

(A) 250

(B) 300

(C) 350

(D) 400

(E) 450

28. A dam is proposed for a river that is currently used for salmon breeding. You have modeled:

(i) For each hour the dam is opened the number of salmon that will pass through and reach the breeding grounds has a distribution with mean 100 and variance 900.

(ii) The number of eggs released by each salmon has a distribution with mean 5 and variance 5.

(iii) The number of salmon going through the dam each hour it is open and the numbers of eggs released by the salmon are independent.

Calculate the least number of whole hours the dam should be left open so the probability that 10,000 eggs will be released is greater than 95% using the normal approximation for the aggregate number of eggs released.

(A) 20

(B) 23

(C) 26

(D) 29

(E) 32

29. For a stop-loss insurance on a three person group:

(i) Loss amounts are independent.

(ii) The distribution of loss amount for each person is:

Loss Amount	Probability
0	0.4
1	0.3
2	0.2
3	0.1

(iii) The stop-loss insurance has a deductible of 1 for the group.

Calculate the net stop-loss premium.

(A) 2.00

(B) 2.03

(C) 2.06

(D) 2.09

(E) 2.12

30. For a discrete probability distribution, you are given the recursion relation

$$p(k) = \frac{2}{k} p(k-1), \quad k = 1, 2, \ldots$$

Calculate $p(4)$.

(A) 0.07

(B) 0.08

(C) 0.09

(D) 0.10

(E) 0.11

31. A company insures a fleet of vehicles. Aggregate losses have a compound Poisson distribution. The expected number of losses is 20. Loss amounts, regardless of vehicle type, have exponential distribution with $\theta = 200$.

To reduce the cost of the insurance, two modifications are to be made:

(i) A certain type of vehicle will not be insured. It is estimated that this will reduce loss frequency by 20%.

(ii) A deductible of 100 per loss will be imposed.

Calculate the expected aggregate amount paid by the insurer after the modifications.

(A) 1600

(B) 1940

(C) 2520

(D) 3200

(E) 3880

32. The number of claims, N, made on an insurance portfolio follows the following distribution:

n	$\Pr(N = n)$
0	0.7
2	0.2
3	0.1

If a claim occurs, the benefit is 0 or 10 with probability 0.8 and 0.2, respectively.

The number of claims and the benefit for each claim are independent.

Calculate the probability that aggregate benefits will exceed expected benefits by more than 2 standard deviations.

(A) 0.02

(B) 0.05

(C) 0.07

(D) 0.09

(E) 0.12

33. Annual prescription drug costs are modeled by a Pareto distribution with $\theta = 2000$ and $\alpha = 2$.

A prescription drug plan pays annual drug costs for an insured member subject to the following provisions:

(i) The insured pays 100% of costs up to the ordinary annual deductible of 250.

(ii) The insured then pays 25% of the costs between 250 and 2250.

(iii) The insured pays 100% of the costs above 2250 until the insured has paid 3600 in total.

(iv) The insured then pays 5% of the remaining costs.

Calculate the expected annual plan payment.

(A) 1120

(B) 1140

(C) 1160

(D) 1180

(E) 1200

34. Two types of insurance claims are made to an insurance company. For each type, the number of claims follows a Poisson distribution and the amount of each claim is uniformly distributed as follows:

Type of Claim	Poisson Parameter λ for Number of Claims in one year	Range of Each Claim Amount
I	12	(0, 1)
II	4	(0, 5)

The numbers of claims of the two types are independent and the claim amounts and claim numbers are independent.

Calculate the normal approximation to the probability that the total of claim amounts in one year exceeds 18.

(A) 0.37

(B) 0.39

(C) 0.41

(D) 0.43

(E) 0.45

35. The number of annual losses has a Poisson distribution with a mean of 5. The size of each loss has a Pareto distribution with $\theta = 10$ and $\alpha = 2.5$. An insurance policy for the losses has an ordinary deductible of 5 per loss.

Calculate the expected value of the aggregate annual payments for this policy.

(A) 8

(B) 13

(C) 18

(D) 23

(E) 28

36. Losses in Year 1 follow a Pareto distribution with $\alpha = 2$ and $\theta = 5$. Losses in Year 2 are uniformly 20% higher than in Year 1. An insurance covers each loss subject to an ordinary deductible of 10.

Calculate the Loss Elimination Ratio in Year 2.

(A) 0.567

(B) 0.625

(C) 0.667

(D) 0.750

(E) 0.800

37. You are given the following three observations:

0.74 0.81 0.95

You fit a distribution with the following density function to the data:

$$f(x) = (p+1)x^p, \quad 0 < x < 1,\ p > -1$$

Calculate the maximum likelihood estimate of p.

(A) 4.0

(B) 4.1

(C) 4.2

(D) 4.3

(E) 4.4

38. You are given:

(i) The number of claims has probability function:

$$p(x) = \binom{m}{x} q^x (1-q)^{m-x}, \quad x = 0, 1, \ldots, m$$

(ii) The actual number of claims must be within 1% of the expected number of claims with probability 0.95.

(iii) The expected number of claims for full credibility is 34,574.

Calculate q.

(A) 0.05

(B) 0.10

(C) 0.20

(D) 0.40

(E) 0.80

39. You are given:

(i) A sample of losses is:

600 700 900

(ii) No information is available about losses of 500 or less.

(iii) Losses are assumed to follow an exponential distribution with mean θ.

Calculate the maximum likelihood estimate of θ.

(A) 233

(B) 400

(C) 500

(D) 733

(E) 1233

40. You are given:

(i) The number of claims follows a Poisson distribution with mean λ.

(ii) Observations other than 0 and 1 have been deleted from the data.

(iii) The data contain an equal number of observations of 0 and 1.

Calculate the maximum likelihood estimate of λ.

(A) 0.50

(B) 0.75

(C) 1.00

(D) 1.25

(E) 1.50

41. For a collective risk model the number of losses, N, has a Poisson distribution with $\lambda = 20$.

The common distribution of the individual losses has the following characteristics:

(i) $E[X] = 70$

(ii) $E[X \wedge 30] = 25$

(iii) $\Pr(X > 30) = 0.75$

(iv) $E[X^2 \mid X > 30] = 9000$

An insurance covers aggregate losses subject to an ordinary deductible of 30 per loss.

Calculate the variance of the aggregate payments of the insurance.

(A) 54,000

(B) 67,500

(C) 81,000

(D) 94,500

(E) 108,000

42. For a collective risk model:

(i) The number of losses has a Poisson distribution with $\lambda = 2$.

(ii) The common distribution of the individual losses is:

x	$f_X(x)$
1	0.6
2	0.4

An insurance covers aggregate losses subject to a deductible of 3.

Calculate the expected aggregate payments of the insurance.

(A) 0.74

(B) 0.79

(C) 0.84

(D) 0.89

(E) 0.94

43. A discrete probability distribution has the following properties:

(i) $p_k = c\left(1+\frac{1}{k}\right)p_{k-1}$ for $k = 1, 2, \ldots$

(ii) $p_0 = 0.5$

Calculate c.

(A) 0.06

(B) 0.13

(C) 0.29

(D) 0.35

(E) 0.40

44. The repair costs for boats in a marina have the following characteristics:

Boat type	Number of boats	Probability that repair is needed	Mean of repair cost given a repair	Variance of repair cost given a repair
Power boats	100	0.3	300	10,000
Sailboats	300	0.1	1000	400,000
Luxury yachts	50	0.6	5000	2,000,000

At most one repair is required per boat each year. Repair incidence and cost are mutually independent.

The marina budgets an amount, Y, equal to the aggregate mean repair costs plus the standard deviation of the aggregate repair costs.

Calculate Y.

(A) 200,000

(B) 210,000

(C) 220,000

(D) 230,000

(E) 240,000

45. For an insurance policy:

(i) Losses can be 100, 200 or 300 with respective probabilities 0.2, 0.2, and 0.6.

(ii) The policy has an ordinary deductible of 150 per loss.

(iii) Y^P is the claim payment per payment random variable.

Calculate $\text{Var}(Y^P)$.

(A) 1500

(B) 1875

(C) 2250

(D) 2625

(E) 3000

46. The distribution of a loss, X, is a two-point mixture:

(i) With probability 0.8, X has a Pareto distribution with $\alpha = 2$ and $\theta = 100$.

(ii) With probability 0.2, X has a Pareto distribution with with $\alpha = 4$ and $\theta = 3000$.

Calculate $\text{P}(X \leq 200)$.

(A) 0.76

(B) 0.79

(C) 0.82

(D) 0.85

(E) 0.88

47. You are given:

(i) The number of claims follows a negative binomial distribution with parameters r and $\beta = 3$.

(ii) Claim severity has the following distribution:

Claim Size	Probability
1	0.4
10	0.4
100	0.2

(iii) The number of claims is independent of the severity of claims.

Calculate the expected number of claims needed for aggregate losses to be within 10% of expected aggregate losses with 95% probability.

(A) Less than 1200

(B) At least 1200, but less than 1600

(C) At least 1600, but less than 2000

(D) At least 2000, but less than 2400

(E) At least 2400

48. You are given the following 20 bodily injury losses before the deductible is applied:

Loss	Number of Losses	Deductible	Policy Limit
750	3	200	∞
200	3	0	10,000
300	4	0	20,000
>10,000	6	0	10,000
400	4	300	∞

Past experience indicates that these losses follow a Pareto distribution with parameters α and $\theta = 10,000$.

Calculate the maximum likelihood estimate of α.

(A) Less than 2.0

(B) At least 2.0, but less than 3.0

(C) At least 3.0, but less than 4.0

(D) At least 4.0, but less than 5.0

(E) At least 5.0

49. Personal auto property damage claims in a certain region are known to follow the Weibull distribution:

$$F(x) = 1 - \exp\left[-\left(\frac{x}{\theta}\right)^{0.2}\right], \quad x > 0$$

A sample of four claims is:

130 240 300 540

The values of two additional claims are known to exceed 1000.

Calculate the maximum likelihood estimate of θ.

(A) Less than 300

(B) At least 300, but less than 1200

(C) At least 1200, but less than 2100

(D) At least 2100, but less than 3000

(E) At least 3000

50. In Year 1 a risk has a Pareto distribution with $\alpha = 2$ and $\theta = 3000$. In Year 2 losses inflate by 20%.

An insurance on the risk has a deductible of 600 in each year. P_i, the premium in year *i*, equals 1.2 times the expected claims.

The risk is reinsured with a deductible that stays the same in each year. R_i, the reinsurance premium in year *i*, equals 1.1 times the expected reinsured claims.

$$\frac{R_1}{P_1} = 0.55$$

Calculate $\dfrac{R_2}{P_2}$.

(A) 0.46

(B) 0.52

(C) 0.55

(D) 0.58

(E) 0.66

51. For an insurance:

(i) The number of losses per year has a Poisson distribution with $\lambda = 10$.

(ii) Loss amounts are uniformly distributed on (0, 10).

(iii) Loss amounts and the number of losses are mutually independent.

(iv) There is an ordinary deductible of 4 per loss.

Calculate the variance of aggregate payments in a year.

(A) 36

(B) 48

(C) 72

(D) 96

(E) 120

52. The random variable X has survival function:

$$S_X(x) = \frac{\theta^4}{(\theta^2 + x^2)^2}$$

Two values of X are observed to be 2 and 4. One other value exceeds 4.

Calculate the maximum likelihood estimate of θ.

(A) Less than 4.0

(B) At least 4.0, but less than 4.5

(C) At least 4.5, but less than 5.0

(D) At least 5.0, but less than 5.5

(E) At least 5.5

53. You are given:

(i) The distribution of the number of claims per policy during a one-year period for 10,000 insurance policies is:

Number of Claims per Policy	Number of Policies
0	5000
1	5000
2 or more	0

(ii) You fit a binomial model with parameters m and q using the method of maximum likelihood.

Calculate the maximum value of the loglikelihood function when $m = 2$.

(A) –10,397

(B) –7,781

(C) –7,750

(D) –6,931

(E) –6,730

54. You are given:

(i) At time 4 hours, there are 5 working light bulbs.

(ii) The 5 bulbs are observed for p more hours.

(iii) Three light bulbs burn out at times 5, 9, and 13 hours, while the remaining light bulbs are still working at time $4 + p$ hours.

(iv) The distribution of failure times is uniform on $(0, \omega)$.

(v) The maximum likelihood estimate of ω is 29.

Calculate p.

(A) Less than 10

(B) At least 10, but less than 12

(C) At least 12, but less than 14

(D) At least 14, but less than 16

(E) At least 16

55. A company has determined that the limited fluctuation full credibility standard is 2000 claims if:

(i) The total number of claims is to be within 3% of the true value with probability *p*.

(ii) The number of claims follows a Poisson distribution.

The standard is changed so that the total cost of claims is to be within 5% of the true value with probability *p*, where claim severity has probability density function:

$$f(x) = \frac{1}{10{,}000}, \quad 0 \le x \le 10{,}000$$

Calculate the expected number of claims necessary to obtain full credibility under the new standard using limited fluctuation credibility.

(A) 720

(B) 960

(C) 2160

(D) 2667

(E) 2880

56. For a group of policies, you are given:

(i) Losses follow the distribution function

$$F(x) = 1 - \theta / x, \quad x > \theta.$$

(ii) A sample of 20 losses resulted in the following:

Interval	Number of Losses
(0,10]	9
(10, 25]	6
$(25, \infty)$	5

Calculate the maximum likelihood estimate of θ.

(A) 5.00

(B) 5.50

(C) 5.75

(D) 6.00

(E) 6.25

57. Loss amounts have the distribution function

$$F(x) = \begin{cases} (x/100)^2, & 0 \le x \le 100 \\ 1, & x > 100 \end{cases}$$

An insurance policy pays 80% of the amount of the loss in excess of an ordinary deductible of 20, subject to a maximum payment of 60 per loss.

Calculate the conditional expected claim payment, given that a payment has been made.

(A) 37

(B) 39

(C) 43

(D) 47

(E) 49

58. Aggregate losses are modeled as follows:

(i) The number of losses has a Poisson distribution with $\lambda = 3$.

(ii) The amount of each loss has a Burr distribution with $\alpha = 3, \theta = 2, \gamma = 1$.

(iii) The number of losses and the amounts of the losses are mutually independent.

Calculate the variance of aggregate losses.

(A) 12

(B) 14

(C) 16

(D) 18

(E) 20

59. A risk has a loss amount that has a Poisson distribution with mean 3.

An insurance policy covers the risk with an ordinary deductible of 2. An alternative insurance policy replaces the deductible with coinsurance α, which is the proportion of the loss paid by the policy, so that the expected cost remains the same.

Calculate α.

(A) 0.22

(B) 0.27

(C) 0.32

(D) 0.37

(E) 0.42

60. An individual performs dangerous motorcycle jumps at extreme sports events around the world.

The annual cost of repairs to their motorcycle is modeled by a Pareto distribution with $\theta = 5000$ and $\alpha = 2$.

An insurance policy reimburses motorcycle repair costs subject to the following provisions:

(i) The annual ordinary deductible is 1000.

(ii) The policyholder pays 20% of repair costs between 1000 and 6000 each year.

(iii) The policyholder pays 100% of the annual repair costs above 6000 until they have paid 10,000 in out-of-pocket repair costs each year.

(iv) The policyholder pays 10% of the remaining repair costs each year.

Calculate the expected annual insurance reimbursement.

(A) 2300

(B) 2500

(C) 2700

(D) 2900

(E) 3100

61. For an aggregate loss distribution S:

(i) The number of claims has a negative binomial distribution with $r = 16$ and $\beta = 6$.

(ii) The claim amounts are uniformly distributed on the interval (0, 8).

(iii) The number of claims and claim amounts are mutually independent.

Calculate the premium such that the probability that aggregate losses will exceed the premium is 5% using the normal approximation for aggregate losses.

(A) 500

(B) 520

(C) 540

(D) 560

(E) 580

62. An insurance company sells a policy with a linearly disappearing deductible such that no payment is made on a claim of 250 or less and full payment is made on a claim of 1000 or more.

Calculate the payment made by the insurance company for a loss of 700.

(A) 450

(B) 500

(C) 550

(D) 600

(E) 700

63. The random variable X represents the random loss, before any deductible is applied, covered by an insurance policy. The probability density function of X is

$$f(x) = 2x, \quad 0 < x < 1.$$

Payments are made subject to a deductible, d, where $0 < d < 1$.

The probability that a claim payment is less than 0.5 is equal to 0.64.

Calculate the value of d.

(A) 0.1

(B) 0.2

(C) 0.3

(D) 0.4

(E) 0.5

64. You are given the following loss data:

Size of Loss	Number of Claims	Ground-Up Total Losses
0 – 99	1100	58,500
100 – 249	400	70,000
250 – 499	300	120,000
500 – 999	200	150,000
> 999	100	200,000
Total	**2100**	**598,500**

Calculate the percentage reduction in loss costs by moving from a 100 deductible to a 250 deductible.

(A) 25%

(B) 27%

(C) 29%

(D) 31%

(E) 33%

65. An individual purchases a homeowners policy with an 80% coinsurance clause. The home is insured for 150,000. The home was worth 180,000 on the day the policy was purchased. Lightning causes 20,000 worth of damage. On the day of the storm the home is worth 250,000.

Calculate the benefit payment the individual receives from the policy.

(A) 15,000

(B) 16,000

(C) 17,500

(D) 18,000

(E) 20,000

66. A company purchases a commercial insurance policy with a property policy limit of 70,000. The actual value of the property at the time of a loss is 100,000. The insurance policy has a coinsurance provision of 80% and a 200 deductible, which is applied to the loss before the limit or coinsurance are applied. A storm causes damage in the amount of 20,000.

Calculate the insurance company's payment.

(A) 15,840

(B) 16,000

(C) 17,300

(D) 17,325

(E) 19,800

67. Individual 1 has an automobile insurance policy with the ABC Insurance Company featuring 200,000 of third-party liability coverage (bodily injury/property damage) and a 1,000 deductible for collision coverage.

Individual 1 is at fault for an accident that injures Individual 2 who is insured by XYZ Insurance Company. Individual 1 is sued by Individual 2 for these injuries. The court orders Individual 1 to pay 175,000 to Individual 2.

Other expenses incurred are:

i) Legal fees to ABC on behalf of Individual 1: 45,000

ii) Collision costs to repair Individual 1's car: 20,000

Calculate the total amount ABC pays for this occurrence.

(A) 175,000

(B) 195,000

(C) 200,000

(D) 219,000

(E) 239,000

68. You are given the following earned premiums for three calendar years:

Calendar Year	Earned Premium
CY5	7,706
CY6	9,200
CY7	10,250

All policies have a one-year term and policy issues are uniformly distributed through each year.

The following rate changes have occurred:

Date	Rate Change
July 1, CY3	+ 7%
Nov. 15, CY5	– 4%
October 1, CY6	+ 5%

Rates are currently at the level set on October 1, CY6.

Calculate the earned premium at the current rate level for CY6.

(A) 9300

(B) 9400

(C) 9500

(D) 9600

(E) 9700

69. You use the following information to determine a rate change using the loss ratio method.

(i)

Accident Year	Earned Premium at Current Rates	Incurred Losses	Weight Given to Accident Year
AY8	4252	2260	40%
AY9	5765	2610	60%

(ii) Trend Factor: 7% per annum effective

(iii) Loss Development Factor (to Ultimate):

AY8: 1.08
AY9: 1.18

(iv) Permissible Loss Ratio: 0.657

(v) All policies are one-year policies, issued uniformly through the year, and rates will be in effect for one year.

(vi) Proposed Effective Date: July 1, CY10

Calculate the required portfolio-wide rate change.

(A) –26%

(B) –16%

(C) –8%

(D) –1%

(E) 7%

70. You are given:

i) Policies are written uniformly throughout the year.

ii) Policies have a term of 6 months.

iii) The following rate changes have occurred:

Date	**Amount**
October 1, CY1	+7%
July 1, CY2	+10%
September 1, CY3	–6%

Rates are currently at the September 1, CY3 level.

Calculate the on-level factor needed to adjust CY2 earned premiums to the current rate level.

(A) 0.97

(B) 0.98

(C) 0.99

(D) 1.00

(E) 1.01

71. You are given the following information:

			Cumulative Loss Payments through Development Month			
Accident Year	**Earned Premium**	**Expected Loss Ratio**	**12**	**24**	**36**	**48**
AY5	19,000	0.90	4,850	9,700	14,100	16,200
AY6	20,000	0.85	5,150	10,300	14,900	
AY7	21,000	0.91	5,400	10,800		
AY8	22,000	0.88	7,200			

There is no development past 48 months.

Calculate the indicated loss reserve using the Bornhuetter-Ferguson method and volume-weighted average loss development factors.

(A) 22,600

(B) 23,400

(C) 24,200

(D) 25,300

(E) 26,200

72. You are given:

i)

Accident Year	Cumulative Paid Losses through Development Year						Earned premium
	0	**1**	**2**	**3**	**4**	**5**	
AY4	1,400	5,200	7,300	8,800	9,800	9,800	18,000
AY5	2,200	6,400	8,800	10,200	11,500		20,000
AY6	2,500	7,500	10,700	12,600			25,000
AY7	2,800	8,700	12,900				26,000
AY8	2,500	7,900					27,000
AY9	2,600						28,000

ii) The expected loss ratio for each Accident Year is 0.550.

Calculate the total loss reserve using the Bornhuetter-Ferguson method and three-year arithmetic average paid loss development factors.

(A) 21,800

(B) 22,500

(C) 23,600

(D) 24,700

(E) 25,400

73. You are given:

i) An insurance company was formed to write workers compensation business in CY1.

ii) Earned premium in CY1 was 1,000,000.

iii) Earned premium growth through CY3 has been constant at 20% per year (compounded).

iv) The expected loss ratio for AY1 is 60%.

v) As of December 31, CY3, the company's reserving actuary believes the expected loss ratio has increased two percentage points each accident year since the company's inception.

vi) Selected incurred loss development factors are as follows:

12 to 24 months	1.500
24 to 36 months	1.336
36 to 48 months	1.126
48 to 60 months	1.057
60 to 72 months	1.050
72 to ultimate	1.000

Calculate the total IBNR reserve as of December 31, CY3 using the Bornhuetter-Ferguson method.

(A) 964,000

(B) 966,000

(C) 968,000

(D) 970,000

(E) 972,000

74. A primary insurance company has a 100,000 retention limit. The company purchases a catastrophe reinsurance treaty, which provides the following coverage

Layer 1:	85% of 100,000 excess of 100,000
Layer 2:	90% of 100,000 excess of 200,000
Layer 3:	95% of 300,000 excess of 300,000

The primary insurance company experiences a catastrophe loss of 450,000.

Calculate the total loss retained by the primary insurance company.

(A) 100,000

(B) 112,500

(C) 125,000

(D) 132,500

(E) 150,000

75. A primary liability insurer has a book of business with the following characteristics:

- All policies have a policy limit of 500,000
- The expected loss ratio is 60% on premiums of 4,000,000

A reinsurer provides an excess of loss treaty for the layer 300,000 in excess of 100,000.

The following table of increased limits factors is available:

Limit	**ILF**
100,000	1.00
200,000	1.25
300,000	1.45
400,000	1.60
500,000	1.70

Calculate the reinsurer's expected losses for this coverage (answer to the nearest 000s).

(A) 840,000

(B) 847,000

(C) 850,000

(D) 862,000

(E) 871,000

76. Company XYZ sells homeowners insurance policies. You are given:

i) The loss costs by accident year are:

Accident Year	Loss Cost
AY1	1300
AY2	1150
AY3	1550
AY4	1800

ii) The slope of the straight line fitted to the natural log of the loss costs is 0.1275.

iii) AY4 experience is weighted 80% and AY3 experience is weighted 20% for rate development.

New rates take effect November 1, CY5 for one-year policies and will be in effect for one year.

Calculate the expected loss cost for these new rates.

(A) 2124

(B) 2217

(C) 2264

(D) 2381

(E) 2413

77. You are given:

i) The current price to buy one share of XYZ stock is 500.

ii) The stock does not pay dividends.

iii) The continuously compounded risk-free interest rate is 6%.

iv) A European call option on one share of XYZ stock with a strike price of K that expires in one year costs 66.59.

v) A European put option on one share of XYZ stock with a strike price of K that expires in one year costs 18.64.

Using put-call parity, calculate the strike price, K.

(A) 449

(B) 452

(C) 480

(D) 559

(E) 582

78. The current price of a non-dividend paying stock is 40 and the continuously compounded risk-free interest rate is 8%. You are given that the price of a 35-strike call option is 3.35 higher than the price of a 40-strike call option, where both options expire in 3 months.

Calculate the amount by which the price of an otherwise equivalent 40-strike put option exceeds the price of an otherwise equivalent 35-strike put option.

(A) 1.55

(B) 1.65

(C) 1.75

(D) 3.25

(E) 3.35

79. You are given the following information about a homeowners insurance policy:

i) The deductible is 400 for claims up to 500.

ii) For claims of 500 or more, the deductible disappears linearly, until completely disappearing for claims of 2100 and beyond. There is no policy limit.

The policyholder submits a claim of X and receives a payment of $0.6X$.

Calculate X.

(A) 618

(B) 808

(C) 1000

(D) 1300

(E) 1420

80. An individual health insurance policy issued by Company X has a 2,000 deductible, 70% coinsurance, and a 5,000 out of pocket expense maximum that includes the deductible.

While the policy is effective, a policyholder incurs 18,500 of billed charges from a single provider for covered services. This provider is contracted with Company X to provide a 40% discount on all covered services.

No other charges are incurred by the policyholder.

Calculate Company X's payment to the provider.

(A) 3,780

(B) 4,100

(C) 4,620

(D) 6,100

(E) 6,370

81. Annual losses in Year 1 follow an exponential distribution with mean θ. An inflation factor of 20% applies to all Year 2 losses.

The ordinary deductible for Year 1 is 0.25θ. The deductible is doubled in Year 2.

Calculate the percentage increase in the loss elimination ratio from Year 1 to Year 2.

(A) 19%

(B) 28%

(C) 37%

(D) 54%

(E) 78%

82. For a medical insurance company, you are given:

i) Losses for a new product are assumed to follow a lognormal distribution with parameters $\mu = 6$ and $\sigma = 1.5$.

ii) The new product has a per-loss deductible that results in a loss elimination ratio of 0.33.

In a review of the business after five years of experience, it is determined that:

i) Losses for this product actually followed an exponential distribution.

ii) The initial mean for the exponential distribution is the same as the initial mean under the lognormal assumption.

iii) Since it was introduced, the expected value of a loss for this product increased at an annual compound rate of 4%.

iv) The per-loss deductible required to target the same loss elimination ratio is d.

Calculate d.

(A) 605

(B) 659

(C) 722

(D) 775

(E) 852

83. The distribution of the number of claims, N, is a member of the $(a, b, 0)$ class. You are given:

i) $P(N = k) = p_k$

ii) $\frac{p_6}{p_4} = 0.5$ and $\frac{p_5}{p_4} = 0.8$

A zero-modified distribution, N^M, associated with N has $P(N^M = 0) = 0.1$.

Calculate $E(N^M)$.

(A) 3.64

(B) 3.73

(C) 3.85

(D) 4.00

(E) 4.05

84. You are given the following properties of the distribution of the annual number of claims, N:

i) $P(N = k) = p_k$, $k = 0, 1, 2, \ldots$

ii) $p_0 = 0.45$

iii) $\frac{p_n}{p_m} = \frac{m!}{n!}$ for $m \geq 1$ and $n \geq 1$

Calculate the probability that at least two claims occur during a year.

(A) 0.16

(B) 0.18

(C) 0.21

(D) 0.23

(E) 0.26

85. For the workers' compensation claims of a construction company you are given:

i) The annual number of claims follows the Poisson distribution with mean 20.

ii) Claim sizes X follow the lognormal distribution with $\mu = 4.2$ and $\sigma = 1.1$.

iii) The company retains the first 500 of each claim.

iv) Annual aggregate retained claims approximately follow the normal distribution.

v) $E\left[\left(X \wedge 500\right)^2\right] = 26{,}189$

Determine the 90th percentile of the aggregate distribution of retained claims.

(A) Less than 2900

(B) At least 2900, but less than 3100

(C) At least 3100, but less than 3300

(D) At least 3300, but less than 3500

(E) At least 3500

86. For a product liability policy, you are given:

i) The following statistics for products X and Y

		Product X	Product Y
Number of Claims	Mean	10	2
	Standard Deviation	3	1
Loss Amount	Mean	20	50
	Standard Deviation	5	10

ii) Loss amounts and claim numbers are independent within and between products.

iii) Aggregate losses, for both products combined, approximately follow the normal distribution.

Determine the probability that aggregate losses for both products combined exceed 400.

(A) Less than 0.10

(B) At least 0.10, but less than 0.15

(C) At least 0.15, but less than 0.20

(D) At least 0.20, but less than 0.25

(E) At least 0.25

87. You are asked to consider whether the risk measure $\rho(X) = E(X)$ is coherent.

Determine which of the following statements is correct.

(A) $\rho(X)$ does NOT possess subadditivity.

(B) $\rho(X)$ does NOT possess monotonicity.

(C) $\rho(X)$ does NOT possess positive homogeneity.

(D) $\rho(X)$ does NOT possess translation invariance.

(E) $\rho(X)$ is a coherent risk measure.

88. You are given:

i) The random variable X has probability density function

$$f(x) = \alpha(1500)^{\alpha}(1500 + x)^{-(\alpha+1)}, \qquad \alpha > 0 \text{ and } x > 0$$

ii) Five sample observations are:

50	250	450	650	850

Calculate the maximum likelihood estimate of α .

(A) 0.16

(B) 0.79

(C) 1.85

(D) 2.91

(E) 3.97

89. The following random sample of five claims is drawn from a Pareto distribution with parameters α (unknown) and $\theta = 1$:

1.2 1.6 1.4 and two other values that exceed 2.0

Calculate the maximum likelihood estimate of α.

(A) 0.62

(B) 0.72

(C) 0.94

(D) 1.04

(E) 1.15

90. You are given the following observations on 185 small business policies:

Number of Claims	Number of Policies
0	80
1 or more	105

The number of claims per policy follows a Poisson distribution with parameter λ.

Using the maximum likelihood estimate of λ, determine the estimated probability of a policy having fewer than two claims.

(A) 0.79

(B) 0.84

(C) 0.89

(D) 0.95

(E) 0.98

91. You are given:

i) The number of claims follows a Poisson distribution.

ii) The number of claims and claim severity are independent.

iii) The severity distribution is:

Claim Size	Probability
5	0.60
40	0.35
60	0.05

Calculate the expected number of claims needed so the total cost of claims is within 5% of the expected with probability 0.90.

(A) 511

(B) 726

(C) 1083

(D) 2044

(E) 3126

92. You are given:

i) Claim frequency follows a Poisson distribution.

ii) Claim frequency and claim severity are independent.

iii) The probability density function of claim severity is

$$f(x) = \frac{3(100)^3}{x^4}, \quad x > 100$$

Calculate the expected number of claims needed so the observed total cost of claims is within 5% of the true value with probability 0.95.

(A) 1444

(B) 1537

(C) 2049

(D) 3586

(E) 8196

93. You are given the following information on losses paid for each of AY1 through AY4:

			Incremental Loss Payments through Development Year			
Accident Year	Earned Premium	Expected Loss Ratio	0	1	2	3
AY1	35,500	0.71	10,500	7,500	4,800	1,340
AY2	31,200	0.73	13,050	5,025	1,400	
AY3	X	0.75	12,500	7,250		
AY4	X + 5,000	0.75	18,400			

The estimated loss reserve using the expected loss ratio method is 43,412.

Calculate X.

(A) 48,964

(B) 49,674

(C) 51,875

(D) 52,174

(E) 54,785

94. An actuary is establishing reserves for a group of policies as of December 31, CY3. You are given the following table of reserve estimates for AY1 and AY2:

	Reserve estimates as of December 31, CY3		
	R_{BF}	R_{LR}	R_{CL}
AY1	400,000	250,000	437,500
AY2	1,120,000	1,200,000	1,050,000

where R_{BF} is the loss reserve under the Bornhuetter-Ferguson method, R_{LR} is the loss reserve under the Expected Loss Ratio method, and R_{CL} is the loss reserve under the Chain Ladder method.

Calculate f_2, the loss development factor from the paid-loss-development triangle at duration 2.

(A) 1.250

(B) 1.500

(C) 1.875

(D) 2.150

(E) 2.500

95. You use the following information to calculate projected loss costs:

i) Based on historical experience, the slope of the straight line fitted to the natural log of the loss cost is 0.182.

ii) The loss cost per unit exposure for the two most recent experience periods is:

Accident Year	Loss Cost per Unit Exposure
AY3	400
AY4	450

iii) The current experience period is weighted 90% and the prior experience period is weighted 10%.

New rates for one-year policies take effect May 1, CY5 and are in effect for one year.

Calculate the projected loss cost for these new rates.

(A) 527

(B) 577

(C) 615

(D) 632

(E) 666

96. An insurance company uses the following information to set new rates:

i) The projected loss cost grows at an annual effective rate of 7% per year.

ii) The table below shows loss cost data for two policy years, with corresponding weights:

Policy Year	Loss Cost	Weight
PY3	600	$1 - p$
PY4	670	p

iii) New rates will be effective on April 1, CY9. Policies are one-year policies and rates will be in effect for one year.

The projected expected loss cost is 951.

Calculate p.

(A) 0.089

(B) 0.119

(C) 0.567

(D) 0.881

(E) 0.911

97. You are given the following information:

i) Case reserves = 187,047

ii) Losses paid-to-date = 243,005

iii) Age-to-ultimate incurred loss development factor = 1.08

iv) IBNR using Bornhuetter-Ferguson method = 47,387

Calculate the absolute difference in the IBNR based on the chain-ladder method and the expected loss ratio method.

(A) 12,983

(B) 43,787

(C) 98,101

(D) 146,754

(E) 175,268

98. An insurance company sells one-year policies that have uniformly distributed effective dates.

The following rate changes have occurred:

Date	Rate Change
June 1, CY1	+10%
August 1, CY2	r

Rates are currently at the level set on August 1, CY2.

The earned premium at current rates for CY2 is 1.03 times the CY2 earned premium.

Calculate r.

(A) 1.7%

(B) 2.4%

(C) 3.3%

(D) 5.3%

(E) 7.5%

99. Company XYZ sells dental insurance policies. You are given:

i) All policies have a one-year term.

ii) Policy effective dates are uniformly distributed.

iii) The earned premium for Calendar Year 3 is 600.

The following rate changes have occurred:

Date	Rate change
September 1, CY1	3.0%
May 15, CY3	2.0%
August 1, CY4	5.0%

Rates are currently at the level set on August 1, CY4.

Calculate the earned premium at current rates for CY3.

(A) 610

(B) 617

(C) 632

(D) 640

(E) 659

100. A non-dividend paying stock has a current price of S. The continuously compounded risk-free interest rate is 2.75%.

The price of the stock over a six-month period follows a binomial model with u = 1.2903 and d = 0.7966. A six-month European put option on the stock with a strike price of $(S-4.50)$ has a price of 2.482.

Calculate S.

(A) 44.22

(B) 45.72

(C) 46.97

(D) 49.11

(E) 50.24

101. You are given:

(i) The Black-Scholes-Merton framework applies.

(ii) The prices of some 6-month European options on non-dividend paying Stock Y are:

Strike Price	Price of Call Option	Price of Put Option
525	55.92	*x*
550	45.46	64.57

The continuously compounded risk-free rate is 3.25%.

Calculate *x*.

(A) 50.03

(B) 50.43

(C) 50.83

(D) 51.03

(E) 51.23

102. You are given:

(i) For any random variable *X*, the risk measure is $\rho(X) = \mu_X + k\sigma_X$.

(ii) The variables *Y* and *Z* are defined as follows:

y	$P(Y = y)$
0	0.2
1	0.8

and *Z* is constant with $Z = 1$.

Calculate the largest value of *k* such that ρ has the monotonicity property with respect to *Y* and *Z*.

(A) 0.50

(B) 0.75

(C) 1.00

(D) 1.25

(E) 1.50