SOCIETY OF ACTUARIES

EXAM FAM FUNDAMENTALS OF ACTUARIAL MATHEMATICS

EXAM FAM SAMPLE SOLUTIONS

The weight of topics in these sample questions is not representative of the weight of topics on the exam. The syllabus indicates the exam weights by topic.

September 2022 changes: Edits made to Questions/Solutions 1, 20, 38; Question 5 deleted; Questions 77 and 78 added.

December 2022 changes: Questions 79-101 added

May 2024 change: Question 102 added

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Question #1
Key: E

The standard for full credibility is $\left(\frac{1.645}{0.02}\right)^2\left(1+\frac{Var(X)}{E(X)^2}\right)$ where X is the claim size variable.

For the Pareto variable, $E(X)=0.5/5=0.1$ and $Var(X)=\frac{2(0.5)^2}{5(4)}-(0.1)^2=0.015$. Then the standard is $\left(\frac{1.645}{0.02}\right)^2\left(1+\frac{0.015}{0.1^2}\right)=16,913$ claims.

Question #2
Key: A

The distribution function is $F(x)=\int_1^x \alpha t^{-\alpha-1}dt=-t^{-\alpha}\Big|_1^x=1-x^{-\alpha}$. The likelihood function is

$$
\begin{aligned}
L &= f(3)f(6)f(14)[1-F(25)]^2 \\
&= \alpha 3^{-\alpha-1}\alpha 6^{-\alpha-1}\alpha 14^{-\alpha-1}(25^{-\alpha})^2 \\
&\propto \alpha^3[3(6)(14)(625)]^{-\alpha}.
\end{aligned}
$$

Taking logs, differentiating, setting equal to zero, and solving:

$\ln L = 3\ln\alpha - \alpha\ln 157,500$ plus a constant

$d\ln L/d\alpha = 3\alpha^{-1}-\ln 157,500 = 0$

$\hat{\alpha}=3/\ln 157,500 = 0.2507.$

Question #3
Key: A

k	kn_k / n_{k-1}
0	
1	0.81
2	0.92
3	1.75
4	2.29
5	2.50
6	3.00

Positive slope implies that the negative binomial distribution is a good choice. Alternatively, the sample mean and variance are 1.2262 and 1.9131 respectively. With the variance substantially exceeding the mean, the negative binomial model is again supported.

Question #4
Key: B

The likelihood function is $\frac{e^{-1/(2\theta)}}{2\theta}\frac{e^{-2/(2\theta)}}{2\theta}\frac{e^{-3/(2\theta)}}{2\theta}\frac{e^{-15/(3\theta)}}{3\theta}=\frac{e^{-8/\theta}}{24\theta^4}$. The loglikelihood function is $-\ln(24)-4\ln(\theta)-8/\theta$. Differentiating with respect to θ and setting the result equal to 0 yields $-\frac{4}{\theta}+\frac{8}{\theta^2}=0$ which produces $\hat{\theta}=2$.

Question #5
Deleted

Question #6
Key: C

In general,

$$E(X^2)-E[(X\wedge 150)^2]=\int_0^{200}x^2 f(x)dx-\int_0^{150}x^2 f(x)dx-150^2\int_{150}^{200}f(x)dx$$

$$=\int_{150}^{200}(x^2-150^2)f(x)dx.$$

Assuming a uniform distribution, the density function over the interval from 100 to 200 is 6/7400 (the probability of 6/74 assigned to the interval divided by the width of the interval). The answer is

$$\int_{150}^{200}(x^2-150^2)\frac{6}{7400}dx=\left(\frac{x^3}{3}-150^2x\right)\frac{6}{7400}\Bigg|_{150}^{200}=337.84.$$

Question #7
Key: B

The likelihood is:

$$L=\prod_{j=1}^{n}\frac{r(r+1)\cdots(r+x_j-1)\beta^{x_j}}{x_j!(1+\beta)^{r+x_j}}\propto\prod_{j=1}^{n}\beta^{x_j}(1+\beta)^{-r-x_j}.$$

The loglikelihood is:

$$l=\sum_{j=1}^{n}[x_j\ln\beta-(r+x_j)\ln(1+\beta)]$$

$$l'=\sum_{j=1}^{n}\left[\frac{x_j}{\beta}-\frac{r+x_j}{1+\beta}\right]=0$$

$$0=\sum_{j=1}^{n}[x_j(1+\beta)-(r+x_j)\beta]=\sum_{j=1}^{n}x_j-rn\beta=n\bar{x}-rn\beta$$

$$\hat{\beta}=\bar{x}/r.$$

Question #8
Key: B

X is the random sum $Y_1+Y_2+\cdots+Y_N$.
N has a negative binomial distribution with $r = a = 1.5$ and $\beta=\theta=0.2$.

$$E(N)=r\beta=0.3, Var(N)=r\beta(1+\beta)=0.36$$
$$E(Y)=5{,}000, Var(Y)=25{,}000{,}000$$
$$E(X)=0.3(5{,}000)=1{,}500$$
$$Var(X)=0.3(25{,}000{,}000)+0.36(25{,}000{,}000)=16{,}500{,}000$$

Number of exposures (insureds) required for full credibility
$n_{FULL}=(1.645/0.05)^2(16{,}500{,}000/1{,}500^2)=7{,}937.67.$

Number of expected claims required for full credibility
$E(N)n_{FULL}=0.3(7{,}937.67)=2{,}381.$

Question #9
Key: B

$$L = F(1000)^7[F(2000) - F(1000)]^6[1 - F(2000)]^7$$
$$= (1 - e^{-1000/\theta})^7(e^{-1000/\theta} - e^{-2000/\theta})^6(e^{-2000/\theta})^7$$
$$= (1 - p)^7(p - p^2)^6(p^2)^7 = p^{20}(1 - p)^{13}$$

where $p = e^{-1000/\theta}$ The maximum occurs at p = 20/33 and so $\hat{\theta} = -1000/\ln(20/33) = 1996.90.$

Question #10
Key: B

First obtain the distribution of aggregate losses:

Value	Probability
0	1/5
25	(3/5)(1/3) = 1/5
100	(1/5)(2/3)(2/3) = 4/45
150	(3/5)(2/3) = 2/5
250	(1/5)(2)(2/3)(1/3) = 4/45
400	(1/5)(1/3)(1/3) = 1/45

$$\mu = (1/5)(0) + (1/5)(25) + (4/45)(100) + (2/5)(150) + (4/45)(250) + (1/45)(400) = 105$$
$$\sigma^2 = (1/5)(0^2) + (1/5)(25) + (4/45)(100^2) + (2/5)(150^2) + (4/45)(250^2)$$
$$+(1/45)(400^2) - 105^2 = 8100$$

Question #11
Key: A

Because the exponential distribution is memoryless, the excess over the deductible is also exponential with the same parameter. So subtracting 100 from each observation yields data from an exponential distribution and noting that the maximum likelihood estimate is the sample mean gives the answer of 73.

Working from first principles,

$$L(\theta) = \frac{f(x_1)f(x_2)f(x_3)f(x_4)f(x_5)}{[1-F(100)]^5} = \frac{\theta^{-1}e^{-125/\theta}\theta^{-1}e^{-150/\theta}\theta^{-1}e^{-165/\theta}\theta^{-1}e^{-175/\theta}\theta^{-1}e^{-250/\theta}}{(e^{-100/\theta})^5}$$

$$= \theta^{-5}e^{-365/\theta}.$$

Taking logarithms and then a derivative gives

$$l(\theta) = -5\ln(\theta) - 365/\theta,\ l'(\theta) = -5/\theta + 365/\theta^2 = 0$$

$$\hat{\theta} = 365/5 = 73.$$

Question #12
Key: C

$$E(N) = r\beta = 0.4$$

$$Var(N) = r\beta(1+\beta) = 0.48$$

$$E(Y) = \theta/(\alpha-1) = 500$$

$$Var(Y) = \frac{\theta^2\alpha}{(\alpha-1)^2(\alpha-2)} = 750,000$$

Therefore,

$$E(X) = 0.4(500) = 200$$

$$Var(X) = 0.4(750,000) + 0.48(500)^2 = 420,000.$$

The full credibility standard is $n = \left(\frac{1.645}{0.05}\right)^2 \frac{420,000}{200^2} = 11,365,\ Z = \sqrt{2,500/11,365} = 0.47.$

Question #13
Key: C

$$f(x) = p\frac{1}{100}e^{-x/100} + (1-p)\frac{1}{10,000}e^{-x/10,000}$$

$$\begin{aligned} L(100,200) &= f(100)f(2000) \\ &= \left(\frac{pe^{-1}}{100} + \frac{(1-p)e^{-0.01}}{10,000}\right)\left(\frac{pe^{-20}}{100} + \frac{(1-p)e^{-0.2}}{10,000}\right) \end{aligned}$$

Question #14
Key: A

$$B = \begin{cases} c(400-x) & x < 400 \\ 0 & x \geq 400 \end{cases}$$

$$\begin{aligned} 100 = E(B) &= c400 - cE(X \wedge 400) \\ &= c400 - c300\left(1 - \frac{300}{300+400}\right) \\ &= c\left(400 - 300\frac{4}{7}\right) \end{aligned}$$

$$c = \frac{100}{228.6} = 0.44$$

Question #15
Key: C

Let N = number of computers in department
Let X = cost of a maintenance call
Let S = aggregate cost

$$\text{Var}(X) = [\text{Standard Deviation}(X)]^2 = 200^2 = 40{,}000$$
$$E(X^2) = \text{Var}(X) + [E(X)]^2$$
$$= 40{,}000 + 80^2 = 46{,}400$$
$$E(S) = N\lambda E(X) = N(3)(80) = 240N$$
$$\text{Var}(S) = N\lambda \times E(X^2) = N(3)(46{,}400) = 139{,}200N$$

We want

$$0.1 \ge \Pr\left(S > 1.2E(S)\right)$$
$$\ge \Pr\left(\frac{S-E(S)}{\sqrt{139{,}200N}} > \frac{0.2E(S)}{\sqrt{139{,}200N}}\right) \Rightarrow \frac{0.2(240)N}{373.1\sqrt{N}} \ge \Phi^{-1}(0.9) = 1.282$$
$$N \ge \left(\frac{1.282(373.1)}{48}\right)^2 = 99.3$$

Question #16
Key: D

The modified severity, X^*, represents the conditional payment amount given that a payment occurs. Given that a payment is required ($X > d$), the payment must be uniformly distributed between 0 and $c(b - d)$.

The modified frequency, N^*, represents the number of losses that result in a payment. The deductible eliminates payments for losses below d, so only $1 - F_X(d) = \frac{b-d}{b}$ of losses will require payments. Therefore, the Poisson parameter for the modified frequency distribution is $\lambda\frac{b-d}{b}$. (Reimbursing c% after the deductible affects only the payment amount and not the frequency of payments).

Question #17
Key: E

$$f(x) = 0.01, \quad 0 \le x \le 80$$
$$= 0.01 - 0.00025(x-80) = 0.03 - 0.00025x, \quad 80 < x \le 120$$

$$E(x) = \int_0^{80} 0.01x\,dx + \int_{80}^{120} (0.03x - 0.00025x^2)dx$$
$$= \frac{0.01x^2}{2}\Big|_0^{80} + \frac{0.03x^2}{2}\Big|_{80}^{120} - \frac{0.00025x^3}{3}\Big|_{80}^{120}$$
$$= 32 + 120 - 101.33 = 50.66667$$

$$E(X-20)_+ = E(X) - \int_0^{20} x\,f(x)dx - 20\left[1 - \int_0^{20} f(x)dx\right]$$
$$= 50.6667 - \frac{0.01x^2}{2}\Big|_0^{20} - 20\left(1 - 0.01x\Big|_0^{20}\right)$$
$$= 50.6667 - 2 - 20(0.8) = 32.6667$$

Loss Elimination Ratio = $1 - \dfrac{32.6667}{50.6667} = 0.3553$

Question #18
Key: B

First restate the table to be CAC's cost, after the 10% payment by the auto owner:

Towing Cost, x	$p(x)$
72	50%
90	40%
144	10%

Then $E(X) = 0.5(72) + 0.4(90) + 0.1(144) = 86.4$.

$E(X^2) = 0.5(72^2) + 0.4(90^2) + 0.1(144^2) = 7905.6$

$\text{Var}(X) = 7905.6 - 86.4^2 = 440.64$

Because Poisson,

$E(N) = Var(N) = 1000$

$E(S) = E(X)E(N) = 86.4(1000) = 86,400$

$\text{Var}(S) = E(N)\text{Var}(X) + E(X)^2\text{Var}(N) = 1000(440.64) + 86.4^2(1000) = 7,905,600$

$$\Pr(S > 90,000) + \Pr\left(\frac{S - E(S)}{\sqrt{\text{Var}(S)}} > \frac{90,000 - 86,400}{\sqrt{7,905,600}}\right) = \Pr(Z > 1.28) = 1 - \Phi(1.28) = 0.10$$

Since the frequency is Poisson, you could also have used

$\text{Var}(S) = \lambda E(X^2) = 1000(7905.6) = 7,905,600$.

That way, you would not need to have calculated Var(X).

Question #19
Key: C

$$\text{LER} = \frac{E(X \wedge d)}{E(X)} = \frac{\theta(1 - e^{-d/\theta})}{\theta} = 1 - e^{-d/\theta}$$

Last year $\quad 0.70 = 1 - e^{-d/\theta} \Rightarrow -d = \theta \log(0.30)$

Next year: $\quad -d_{new} = \theta \log(1 - \text{LER}_{new})$

Hence $\theta \log(1 - \text{LER}_{\text{new}}) = -d_{\text{new}} = \frac{4}{3}\theta \log(0.30)$

$\log\left(1 - \text{LER}_{new}\right) = -1.6053$

$\left(1 - \text{LER}_{new}\right) = e^{-1.6053} = 0.20$

$\text{LER}_{new} = 0.80$

Question #20
Key: C

Let N = number of prescriptions then

n	$f_N(n)$
0	0.2000
1	0.1600
2	0.1280
3	0.1024

$$E[(S-100)_+] = E[S] - E[S \wedge 100]$$

$$E[S] = 40(4) = 160$$

$$E[S \wedge 100] = 0(0.2) + 40(0.16) + 80(0.128) + 100(1 - 0.2 - 0.16 - 0.128) = 67.84$$

$$E[(S-100)_+] = 160 - 67.84 = 92.16$$

Question #21
Key: E

In each round,
N = result of first roll, to see how many dice you will roll
X = result of for one of the N dice you roll
S = sum of X for the N dice

$$E(X) = E(N) = 3.5$$

$$Var(X) = Var(N) = 2.9167$$

$$E(S) = E(N)E(X) = 12.25$$

$$Var(S) = E(N)Var(X) + Var(N)E(X)^2 = 3.5(2.9167) + 2.9167(3.5)^2 = 45.938$$

Let S_{1000} the sum of the winnings after 1000 rounds

$$E(S_{1000}) = 1000(12.25) = 12,250$$

$$SD(S_{1000}) = \sqrt{1000(45.938)} = 214.33$$

After 1000 rounds, you have your initial 15,000, less payments of 12,500, plus winnings for a total of $2,500 + S_{1000}$ Since actual possible outcomes are discrete, the solution tests for continuous outcomes greater than 15000-0.5. In this problem, that continuity correction has negligible impact.

$$\Pr(2,500 + S_{1000} > 14,999.5) = \Pr(S_{1000} > 12,499.5) \approx \Pr\left(Z > \frac{12,499.5 - 12,250}{214.33} = 1.17\right) = 0.12.$$

Question #22
Key: B

$$p_k = \left(a + \frac{b}{k}\right) p_{k-1}$$
$$0.25 = (a+b)0.25 \Rightarrow a = 1-b$$
$$0.1875 = \left(a + \frac{b}{2}\right)(0.25) \Rightarrow 0.1875 = (1-0.5b)(0.25) \Rightarrow b = 0.5,\ a = 0.5$$
$$p_3 = \left(0.5 + \frac{0.5}{3}\right)(0.1875) = 0.125$$

Question #23
Key: D

Severity after increase	Severity after increase and deductible
60	0
120	20
180	80
300	200

Expected payment per loss = 0.25(0) +0.25(20) + 0.25(80) + 0.25(200) = 75

Expected payments = Expected number of losses x Expected payment per loss
= 300(75) = 22,500

Question #24
Key: A

E(S) = E(N)E(X) = 50(200) = 10,000
$$Var(S) = E(N)Var(X) + E(X)^2 Var(N) = 50(400) + 200^2(100) = 4,020,000$$
$$\Pr(S < 8,000) \approx \Pr\left(Z < \frac{8,000 - 10,000}{\sqrt{4,020,000}} = -0.998\right) = 0.16$$

Question #25
Key: B

Let S denote aggregate loss before deductible.

E(S) = 2(2) = 4, since mean severity is 2.

$f_S(0) = \frac{e^{-2}2^0}{0!} = 0.1353$, since must have 0 losses to get aggregate loss = 0.

$f_S(1) = \frac{e^{-2}2^1}{1!}\frac{1}{3} = 0.0902$, since must have 1 loss whose size is 1 to get aggregate loss = 1.

$$E(S \wedge 2) = 0 f_S(0) + 1 f_S(1) + 2[1 - f_S(0) - f_S(1)]$$
$$= 0(0.1353) + 1(0.0902) + 2(1 - 0.01353 - 0.0902) = 1.6392$$
$$E[(S-2)_+] = E(S) - E(S \wedge 2) = 4 - 1.6392 = 2.3608$$

Question #26
Key: C

Limited expected value =

$$\int_0^{1000}[1-F(x)]dx = \int_0^{1000} 0.8e^{-0.02x} + 0.2e^{-0.001x}dx = -40e^{-0.02x} - 200e^{-0.001x}\Big|_0^{1000}$$
$$= -0 - 73.576 + 40 + 200 = 166.424$$

Question #27
Key: B

Mean excess loss = $\frac{E(X) - E(X \wedge 100)}{1 - F(100)} = \frac{331 - 91}{0.8} = 300$

Note that $E(X) = E(X \wedge 1000)$ because F(1000) = 1.

Question: #28
Key: B

N = number of salmon in t hours
X = eggs from one salmon
S = total eggs.
$E(N) = 100t$
$Var(N) = 900t$

$$E(S) = E(N)\,\mathrm{E(X)} = (100\,\mathrm{t})(5) = 500\,\mathrm{t}$$

$$\mathrm{Var(S)} = \mathrm{E(N)\,Var(X)} + \mathrm{E(X)}^2 Var(N) = (100t)(5) + (5^2)(900t) = 23,000t$$

$$0.95 < \Pr(S > 10,000) = \Pr\left(Z > \frac{10,000 - 500t}{\sqrt{23,000t}}\right) \Rightarrow \frac{10,000 - 500t}{\sqrt{23,000t}} = -1.645$$

$$10,000 - 500t = -1.645(151.66)\sqrt{t} = -249.48\sqrt{t}$$

$$500t - 249.48\sqrt{t} - 10,000 = 0$$

$$\sqrt{t} = \frac{249.48 \pm \sqrt{(-249.48)^2 - 4(500)(-10,000)}}{2(500)} = 4.73$$

$t = 22.26$
Round up to 23

Question: #29
Key: C

X = losses on one life
$E(X)$ = 0.3(1) +0.2(2) + 0.1(3) = 1
S = total losses
$E(S) = 3E(X) = 3(1) = 3$

$$E[(S-1)_+] = E(S) - 1[1 - F_S(0)] = 3 - 1[1 - f_S(0)] = 3 - (1 - 0.4^3) = 2.064$$

Alternatively, the expected retained payment is $0 f_S(0) + 1[1 - f_S(0)] = 0.936$ and the stop-loss premium is 3 – 0.936 = 2.064.

Question: #30
Key: C

$$p(k)=\frac{2}{k}p(k-1)=\left(0+\frac{2}{k}\right)p(k-1)$$

Thus an (*a*, *b*, 0) distribution with *a* = 0, *b* = 2.

Thus Poisson with $\lambda = 2$.

$$p(4)=\frac{e^{-2}2^4}{4!}=0.09$$

Question: #31
Key: B

By the memoryless property, the distribution of amounts paid in excess of 100 is still exponential with mean 200.

With the deductible, the probability that the amount paid is 0 is $F(100)=1-e^{-100/200}=0.393$.

Thus the average amount paid per loss is (0.393)(0) + (0.607)(200) = 121.4
The expected number of losses is (20)(0.8) = 16.
The expected amount paid is (16)(121.4) = 1942.

Question: #32
Key: E

$$E(N)=0.7(0)+0.2(2)+0.1(3)=0.7$$
$$Var(N)=0.7(0)+0.2(4)+0.1(9)-0.7^2=1.21$$
$$E(X)=0.8(0)+0.2(10)=2$$
$$Var(X)=0.8(0)+0.2(100)-2^2=16$$
$$E(S)=E(N)E(X)=0.7(2)=1.4$$
$$Var(S)=E(N)Var(X)+E(X)^2Var(N)=0.7(16)+4(1.21)=16.04$$
$$SD(S)=\sqrt{16.04}=4$$
$$\Pr(S>1.4+2(4)=9.4)=1-\Pr(S=0)=1-0.7-0.2(0.8)^2-0.1(0.8)^3=0.12$$

The last line follows because there are no possible values for *S* between 0 and 10. A value of 0 can be obtained three ways: no claims, two claims both for 0, three claims all for 0.

Question #33
Key: C

$$E\left(X \wedge x\right) = \frac{\theta}{\alpha - 1}\left[1 - \left(\frac{\theta}{x+\theta}\right)^{\alpha-1}\right] = \frac{2000}{1}\left[1 - \frac{2000}{x+2000}\right] = \frac{2000x}{x+2000}$$

x	$E(X \wedge x)$
∞	2000
250	222
2250	1059
5100	1437

$$0.75[E(X \wedge 2250) - E(X \wedge 250)] + 0.95\left[E(X) - E(X \wedge 5100)\right]$$
$$0.75(1059 - 222) + 0.95(2000 - 1437) = 1162.6$$

The 5100 breakpoint was determined by when the insured's share reaches 3600:

3600 = 250 + 0.25 (2250 – 250) + (5100 – 2250)

Question #34
Key: A

N_I, N_{II} denote the random variables for # of claims for Types I and II in one year
X_I, X_{II} denote the claim amount random variables for Types I and II
S_I, S_{II} denote the total claim amount random variables for Types I and II
$S = S_I + S_{II}$
$E(N_I) = Var(N_I) = 12,\ E(N_{II}) = Var(N_{II}) = 4$
$E(X_I) = (0+1)/2 = 1/2, Var(X_I) = (1-0)^2/12 = 1/12$
$E(X_{II}) = (0+5)/2 = 5/2, Var(X_{II}) = (5-0)^2/12 = 25/12$
$E(S) = E(N_I)E(X_I) + E(N_{II})E(X_{II}) = 12(1/2) + 4(5/2) = 16$
$$Var(S) = E(N_I)Var(X_I) + E(X_I)^2 Var(N_I) + E(N_{II})Var(X_{II}) + E(X_{II})^2 Var(N_{II})$$
$$= 12(1/12) + (1/2)^2(12) + 4(25/12) + (5/2)^2(4) = 37.33$$
$$\Pr(S > 18) = \Pr\left(Z > \frac{18-16}{\sqrt{37.33}} = 0.327\right) = 1 - \Phi(0.327) = 0.37$$

Question #35
Key: C

Let X be the loss random variable. Then, $(X-5)_+$ is the claim random variable.

$$E(X)=\frac{10}{2.5-1}=6.667$$

$$E(X\wedge 5)=\left(\frac{10}{2.5-1}\right)\left[1-\left(\frac{10}{5+10}\right)^{2.5-1}\right]=3.038$$

$$E[(X-5)_+]=E(X)-E(X\wedge 5)=6.667-3.038=3.629$$

Expected aggregate claims = $E(N)E[(X-5)_+]=5(3.629)=18.15.$

Question #36
Key: B

A Pareto $(\alpha=2,\theta=5)$ distribution with 20% inflation becomes Pareto with $\alpha=2,\theta=5(1.2)=6$. In Year 2

$$E(X)=\frac{6}{2-1}=6$$

$$E(X\wedge 10)=\frac{6}{2-1}\left[1-\left(\frac{6}{10+6}\right)^{2-1}\right]=3.75$$

$$E[(X-10)_+]_+=E(X)-E(X\wedge 10)=6-3.75-2.25$$

$$\text{LER}=1-\frac{E[(X-10)_+]}{E(X)}=1-\frac{2.25}{6}=0.625$$

Question #37
Key: D

$$L(p)=f(0.74)f(0.81)f(0.95)=(p+1)0.74^p(p+1)0.81^p(p+1)0.95^p$$
$$=(p+1)^3(0.56943)^p$$

$$l(p)=\ln L(p)=3\ln(p+1)+p\ln(0.56943)$$

$$l'(p)=\frac{3}{p+1}-0.563119=0$$

$$p+1=\frac{3}{0.563119}=5.32747,\ p=4.32747.$$

Question #38
Key: B

Let n be the number of observations. For full credibility,

$$n = \left(\frac{1.96}{0.01}\right)^2 \frac{mq(1-q)}{(mq)^2} = 38,416\frac{1-q}{mq}.$$

The required expected number of claims is

$$nmq = 34,574 = 38,416\frac{1-q}{mq}mq = 38,416(1-q).$$

Then q = 1 – 34,574/38,416 = 0.1.

Question #39
Key: A

These observations are truncated at 500. The contribution to the likelihood function is

$\dfrac{f(x)}{1-F(500)} = \dfrac{\theta^{-1}e^{-x/\theta}}{e^{-500/\theta}}$. Then the likelihood function is

$$L(\theta) = \frac{\theta^{-1}e^{-600/\theta}\theta^{-1}e^{-700/\theta}\theta^{-1}e^{-900/\theta}}{\left(e^{-500/\theta}\right)^3} = \theta^{-3}e^{-700/\theta}$$

$$l(\theta) = \ln L(\theta) = -3\ln\theta - 700\theta^{-1}$$

$$l'(\theta) = -3\theta^{-1} + 700\theta^{-2} = 0$$

$$\theta = 700/3 = 233.33.$$

Question #40
Key: C

There are n/2 observations of N = 0 (given N = 0 or 1) and n/2 observations of N = 1 (given N = 0 or 1). The likelihood function is

$L = \left(\dfrac{e^{-\lambda}}{e^{-\lambda}+\lambda e^{-\lambda}}\right)^{n/2}\left(\dfrac{\lambda e^{-\lambda}}{e^{-\lambda}+\lambda e^{-\lambda}}\right)^{n/2} = \dfrac{\lambda^{n/2}e^{-n\lambda}}{(e^{-\lambda}+\lambda e^{-\lambda})^n} = \dfrac{\lambda^{n/2}}{(1+\lambda)^n}$. Taking logs, differentiating and solving provides the answer.

$$l = \ln L = (n/2)\ln\lambda - n\ln(1+\lambda)$$

$$l' = \frac{n}{2\lambda} - \frac{n}{1+\lambda} = 0$$

$$n(1+\lambda) - n2\lambda = 0$$

$$1-\lambda = 0, \quad \lambda = 1.$$

Question #41
Key: B

Losses in excess of the deductible occur at a Poisson rate of $\lambda^* = [1 - F(30)]\lambda = 0.75(20) = 15$
The expected payment squared per payment is

$$E[(X-30)^2 \mid X>30) = E(X^2 - 60X + 900 \mid X > 30)$$

$$E[X^2 - 60(X-30) - 900 \mid X > 30]$$

$$E[X^2 \mid X > 30] - 60\frac{E(X) - E(X \wedge 30)}{1 - F(30)} - 900$$

$$= 9000 - 60\frac{70-25}{0.75} - 900 = 4500$$

The variance of *S* is the expected number of payments times the second moment, 15(4500) = 67,500.

Question #42
Key: A

$$E[(S-3)_+] = E(S) - 3 + 3f_S(0) + 2f_S(1) + 1f_S(2)$$

$$E(S) = 2[0.6 + 2(0.4)] = 2.8$$

$$f_S(0) = e^{-2},\ f_S(1) = e^{-2}(2)(0.6) = 1.2e^{-2}$$

$$f_S(2) = e^{-2}(2)(0.4) + \frac{e^{-2}2^2}{2!}(0.6)^2 = 1.52e^{-2}$$

$$E[(S-3)_+] = 2.8 - 3 + 3e^{-2} + 2(1.2e^{-2}) + 1(1.52e^{-2}) = -0.2 + 6.92e^{-2} = 0.7365$$

Question #43
Key: C

Write (i) as $\frac{p_k}{p_{k-1}} = c + \frac{c}{k}$

This is an (a, b, 0) distribution with $a = b = c$.

Which?

1. If Poisson, $a = 0$, so $b = 0$ and thus $p_0 = 0.5$ and $p_1 = p_2 = \cdots = 0$. The probabilities do not sum to 1 and so not Poisson.
2. If Geometric, $b = 0$, so $a = 0$. By same reasoning as #1, not Geometric.
3. If binomial, a and b have opposite signs. But here $a = b$, so not binomial.
4. Thus negative binomial.

$$1 = \frac{a}{b} = \frac{\beta/(1+\beta)}{(r-1)\beta/(1-\beta)} = \frac{1}{r-1} \text{ so } r = 2.$$

$$p_0 = 0.5 = (1+\beta)^{-r} = (1+\beta)^{-2} \Rightarrow \beta = \sqrt{2} - 1 = 0.414$$

$$c = a = \beta/(1+\beta) = 0.29$$

Question #44
Key: B

The number of repairs for each boat type has a binomial distribution.

For power boats:

$E(S) = 100(0.3)(300) = 9{,}000,$

$Var(S) = 100(0.3)10{,}000) + 100(0.3)(0.7)(300^2) = 2{,}190{,}000$

For sail boats:

$E(S) = 300(0.1)(1{,}000) = 30{,}000,$

$Var(S) = 300(0.1)(400{,}000) + 300(0.1)(0.9)(1{,}000^2) = 39{,}000{,}000$

For luxury yachts:

$E(S) = 50(0.6)(5{,}000) = 150{,}000,$

$Var(S) = 50(0.6)(0.4)(2{,}000{,}000) + 50(0.6)(0.4)(5{,}000^2) = 360{,}000{,}000$

The sums are 189,000 expected and a variance of 401,190,000 for a standard deviation of 20,030. The mean plus standard deviation is 209,030.

Question #45
Key: B

$$S_X(150) = 1 - 0.2 = 0.8$$

$$f_{Y^P}(y) = \frac{f_X(y+150)}{S_X(150)},\ f_{Y^P}(50) = \frac{0.2}{0.8} = 0.25,\ f_{Y^P}(150) = \frac{0.6}{0.8} = 0.75$$

$$E(Y^P) = 0.25(50) + 0.75(150) = 125$$

$$E\left[(Y^P)^2\right] = 0.25(50^2) + 0.75(150^2) = 17,500$$

$$Var(Y^P) = 17,500 - 125^2 = 1,875$$

Question #46
Key: A

$$F(200) = 0.8\left[1 - \left(\frac{100}{200+100}\right)^2\right] + 0.2\left[1 - \left(\frac{3000}{3000+200}\right)^4\right] = 0.7566$$

Question #47
Key: E

For claim severity,
$\mu_S = 1(0.4) + 10(0.4) + 100(0.2) = 24.4,$
$\sigma_S^2 = 1^2(0.4) + 10^2(0.4) + 100^2(0.2) - 24.4^2 = 1,445.04.$
For claim frequency,
$\mu_F = r\beta = 3r, \quad \sigma_F^2 = r\beta(1+\beta) = 12r,$
For aggregate losses,
$\mu = \mu_S\mu_F = 24.4(3r) = 73.2r,$
$\sigma^2 = \mu_S^2\sigma_F^2 + \sigma_S^2\mu_F = 24.4^2(12r) + 1,445.04(3r) = 11,479.44r.$
For the given probability and tolerance, $\lambda_0 = (1.96/0.1)^2 = 384.16.$
The number of observations needed is
$\lambda_0\sigma^2/\mu^2 = 384.16(11,479.44r)/(73.2r)^2 = 823.02/r.$
The average observation produces 3*r* claims and so the required number of claims is $(823.02/r)(3r) = 2,469.$

Question #48
Key: C

$$L = \left[\frac{f(750)}{1-F(200)}\right]^3 f(200)^3 f(300)^4 [1-F(10,000)]^6 \left[\frac{f(400)}{1-F(300)}\right]^4$$

$$= \left[\frac{\alpha 10,200^\alpha}{10,750^{\alpha+1}}\right]^3 \left[\frac{\alpha 10,000^\alpha}{10,200^{\alpha+1}}\right]^3 \left[\frac{\alpha 10,000^\alpha}{10,300^{\alpha+1}}\right]^4 \left[\frac{10,000^\alpha}{20,000^\alpha}\right]^6 \left[\frac{\alpha 10,300^\alpha}{10,400^{\alpha+1}}\right]^4$$

$$= \alpha^{14} 10,200^{-3} 10,000^{13\alpha} 10,300^{-4} 10,750^{-3\alpha-3} 20,000^{-6\alpha} 10,400^{-4\alpha-4}$$

$$\propto \alpha^{14} 10,000^{13\alpha} 10,750^{-3\alpha} 20,000^{-6\alpha} 10,400^{-4\alpha},$$

$$\ln L = 14\ln\alpha + 13\alpha\ln(10,000) - 3\alpha\ln(10,750) - 6\alpha\ln(20,000) - 4\alpha\ln(10,400)$$

$$= 14\ln\alpha - 4.5327\alpha.$$

The derivative is $14/\alpha - 4.5327$ and setting it equal to zero gives $\hat{\alpha} = 3.089$.

Question #49
Key: E

The density function is $f(x) = \dfrac{0.2x^{-0.8}}{\theta^{0.2}} e^{-(x/\theta)^{0.2}}$. The likelihood function is

$$L(\theta) = f(130)f(240)f(300)f(540)[1-F(1000)]^2$$

$$= \frac{0.2(130)^{-0.8}}{\theta^{0.2}} e^{-(130/\theta)^{0.2}} \frac{0.2(240)^{-0.8}}{\theta^{0.2}} e^{-(240/\theta)^{0.2}} \frac{0.2(300)^{-0.8}}{\theta^{0.2}} e^{-(300/\theta)^{0.2}} \frac{0.2(540)^{-0.8}}{\theta^{0.2}} e^{-(540/\theta)^{0.2}} e^{-(1000/\theta)^{0.2}} e^{-(1000/\theta)^{0.2}}$$

$$\propto \theta^{-0.8} e^{-\theta^{-0.2}(130^{0.2}+240^{0.2}+300^{0.2}+540^{0.2}+1000^{0.2}+1000^{0.2})},$$

$$l(\theta) = -0.8\ln(\theta) - \theta^{-0.2}(130^{0.2}+240^{0.2}+300^{0.2}+540^{0.2}+1000^{0.2}+1000^{0.2})$$

$$= -0.8\ln(\theta) - 20.2505\theta^{-0.2},$$

$$l'(\theta) = -0.8\theta^{-1} + 0.2(20.2505)\theta^{-1.2} = 0,$$

$$\theta^{-0.2} = 0.197526, \quad \hat{\theta} = 3,325.67.$$

Question #50
Key: D

For any deductible *d* and the given severity distribution

$$E[(X-d)_+] = E(X) - E(X \wedge d) = 3000 - 3000\left(1 - \frac{3000}{3000+d}\right) = 3000\left(\frac{3000}{3000+d}\right) = \frac{9,000,000}{3000+d}$$

So $P_1 = 1.2\dfrac{9,000,000}{3000+600} = 3000$

Let *r* denote the reinsurer's deductible relative to insured losses. Thus, the reinsurer's deductible is 600 + *r* relative to losses. Thus

$$R_1 = 1.1\left(\frac{9,000,000}{3000+600+r}\right) = 0.55P_1 = 0.55(3000) = 1650 \Rightarrow r = 2400$$

In Year 2, after 20% inflation, losses will have a Pareto distribution with $\alpha = 2$ and $\theta = 1.2(3000) = 3600$. The general formula for expected claims with a deductible of *d* is

$$E[(X-d)_+] = 3600\left(\frac{3600}{3600+d}\right) = \frac{12,960,000}{3600+d}$$

$$P_2 = 1.2\frac{12,960,000}{3000+600} = 3703,\ R_2 = 1.1\frac{12,960,000}{3000+600+2400} = 2160,\ \frac{R_2}{P_2} = \frac{2160}{3703} = 0.583$$

Question #51
Key: C

Since loss amounts are uniform on (0, 10), 40% of losses are below the deductible (4), and 60% are above. Thus, claims occur at a Poisson rate $\lambda^* = 0.6(10) = 6$.

Since loss amounts were uniform on (0, 10), claims are uniform on (0, 6).

Let *N* = number of claims; *X* = claim amount; *S* = aggregate claims.

$$E(N) = Var(N) = \lambda^* = 6$$

$$E(X) = (6-0)/2 = 3$$

$$Var(X) = (6-0)^2/12 = 3$$

$$Var(S) = E(N)Var(X) + E(X)^2 Var(N) = 6(3) + 3^2(6) = 72$$

Question #52
Key: E

$$f(x) = -S'(x) = \frac{4x\theta^4}{(\theta^2 + x^2)^3}$$

$$L(\theta) = f(2)f(4)S(4) = \frac{4(2)\theta^4}{(\theta^2 + 2^2)^3} \frac{4(4)\theta^4}{(\theta^2 + 4^2)^3} \frac{\theta^4}{(\theta^2 + 4^2)^2} = \frac{128\theta^{12}}{(\theta^2 + 4)^3(\theta^2 + 16)^5}$$

$$l(\theta) = \ln 128 + 12\ln\theta - 3\ln(\theta^2 + 4) - 5\ln(\theta^2 + 16)$$

$$l'(\theta) = \frac{12}{\theta} - \frac{6\theta}{\theta^2 + 4} - \frac{10\theta}{\theta^2 + 16} = 0; 12(\theta^4 + 20\theta^2 + 64) - 6(\theta^4 + 16\theta^2) - 10(\theta^4 + 4\theta^2) = 0$$

$$0 = -4\theta^4 + 104\theta^2 + 768 = \theta^4 - 26\theta^2 - 192$$

$$\theta^2 = \frac{26 \pm \sqrt{26^2 + 4(192)}}{2} = 32; \theta = 5.657$$

Question #53
Key: B

$$L(q) = \left[\binom{2}{0}(1-q)^2\right]^{5000} \left[\binom{2}{1}q(1-q)\right]^{5000} = 2^{5000} q^{5000} (1-q)^{15000}$$

$$l(q) = 5000\ln(2) + 5000\ln(q) + 15000\ln(1-q)$$

$$l'(q) = 5000q^{-1} - 15000(1-q)^{-1} = 0$$

$$\hat{q} = 0.25$$

$$l(0.25) = 5000\ln(2) + 5000\ln(0.25) + 15000\ln(0.75) = -7780.97.$$

Question #54
Key: D

$$L(\omega) = \frac{\frac{1}{\omega}\frac{1}{\omega}\frac{1}{\omega}\left(\frac{\omega - 4 - p}{\omega}\right)^2}{\left(\frac{\omega - 4}{\omega}\right)^5} = \frac{(\omega - 4 - p)^2}{(\omega - 4)^5}$$

$$l(\omega) = 2\ln(\omega - 4 - p) - 5\ln(\omega - 4), \ l'(\omega) = \frac{2}{\omega - 4 - p} - \frac{5}{\omega - 4} = 0$$

$$0 = l'(29) = \frac{2}{25 - p} - \frac{5}{25} \Rightarrow p = 15.$$

The denominator in the likelihood function is $S(4)$ to the power of five to reflect the fact that it is known that each observation is greater than 4.

Question #55
Key: B

For the severity distribution the mean is 5,000 and the variance is $10,000^2/12$. For credibility based on accuracy with regard to the number of claims,

$$2000 = \left(\frac{z}{0.03}\right)^2, \quad z^2 = 1.8$$

Where z is the appropriate value from the standard normal distribution. For credibility based on accuracy with regard to the total cost of claims, the number of claims needed is

$$\frac{z^2}{0.05^2}\left(1+\frac{10000^2/12}{5000^2}\right) = 960.$$

Question #56
Key: B

$$L(\theta) = \left(1-\frac{\theta}{10}\right)^9 \left(\frac{\theta}{10}-\frac{\theta}{25}\right)^6 \left(\frac{\theta}{25}\right)^5 \propto (10-\theta)^9\theta^{11}$$

$$l(\theta) = 9\ln(10-\theta) + 11\ln(\theta)$$

$$l'(\theta) = -\frac{9}{10-\theta} + \frac{11}{\theta} = 0$$

$$11(10-\theta) = 9\theta$$

$$110 = 20\theta$$

$$\theta = 110/20 = 5.5.$$

Question #57
Key: B

Pays 80% of loss over 20, with cap of payment at 60, hence u = 60/0.8 + 20 = 95.

$$E(Y \text{ per loss}) = \alpha[E(X\wedge 95) - E(X\wedge 20)] = 0.8\left[\int_0^{95} S(x)dx - \int_0^{20} S(x)dx\right]$$

$$= 0.8\int_{20}^{95} S(x)dx = 0.8\int_{20}^{95}\left(1-\frac{x^2}{10,000}\right)dx = 0.8\left(x-\frac{x^3}{30,000}\right)\Bigg|_{20}^{95} = 37.35$$

$$E(Y \text{ per payment}) = \frac{E(Y \text{ per loss})}{1-F(20)} = \frac{37.35}{0.96} = 38.91$$

Question #58
Key: A

Let S = aggregate losses, X = severity
Since the frequency is Poisson,

$$Var(S) = \lambda E\left(X^2\right)$$

$$E(X^2) = \frac{2^2\Gamma(3)\Gamma(1)}{\Gamma(3)} = 4 \quad \text{(table lookup)}$$

$$Var(S) = 3(4) = 12$$

You would get the same result if you used

$$Var(S) = E(N)Var(X) + Var(N)E(X)^2$$

Question #59
Key: E

$$E(X \wedge 2) = 1f(1) + 2[1 - F(1)] = 1f(1) + 2[1 - f(0) - f(1)]$$
$$= 1(3e^{-3}) + 2(1 - e^{-3} - 3e^{-3}) = 2 - 5e^{-3} = 1.75$$

Cost per loss with deductible is

$$E(X) - E(X \wedge 2) = 3 - 1.75 = 1.25$$

Cost per loss with coinsurance is $\alpha E(X) = 3\alpha$

Equating cost: $3\alpha = 1.25 \Rightarrow \alpha = 0.42$

Question #60
Key: C

Insurance pays 80% of the portion of annual claim between 6,000 and 1,000, and 90% of the portion of annual claims over 14,000.

The 14,000 breakpoint is where the policyholder has paid 10,000:

1000 = deductible
1000 = 20% of costs between 1000 and 6000
8000 = 100% of costs between 14,000 and 6,000

$$E(X \wedge x) = \theta\left(1 - \frac{\theta}{x+\theta}\right) = \frac{5000x}{x+5000}$$

x	$E(X \wedge x)$
1000	833.33
6000	2727.27
14000	3684.21
∞	5000

$$\begin{aligned}&0.80[E(X \wedge 6000) - E(X \wedge 1000)] + 0.90[E(X) - \mathrm{E}(X \wedge 14000)]\\&= 0.80[2727,27 - 833.33] + 0.90[5000 - 3684.21]\\&= 1515.15 + 1184.21 = 2699.36\end{aligned}$$

Question #61
Key: D

We have the following table:

Item	Dist	$E(\ )$	$Var(\ )$
Number claims	$NB(16,6)$	16(6) = 96	16(6)(7) = 672
Claims amounts	$U(0,8)$	(8 – 0)/2 = 4	$(8-0)^2/12 = 5.33$
Aggregate		4(96) = 384	$96(5.33) + 672(4^2) = 11,264$

Premium $= E(S) + 1.645\sqrt{Var(S)} = 384 + 1.645\sqrt{11,264} = 559$

Question #62
Key: D

At 250 the payment is 0. At 1000 the payment is 1000. Interpolating:
$\frac{700-250}{1000-250}=\frac{x-0}{1000-0} \Rightarrow x=450(1000)/750=600.$

Question #63
Key: C

$F(x)=\int_0^x 2y\,dy=x^2$. Let C be a random claim payment. Then $C = 0$ if $X < d$ and $C = X - d$ if $X \geq d$. Then,

$$P(C<0.5)=0.64$$
$$P(C\geq 0.5)=0.36$$
$$P(X-d\geq 0.5)=0.36$$
$$P(X\geq 0.5+d)=0.36$$
$$F(0.5+d)=0.64$$
$$(0.5+d)^2=0.64$$
$$0.5+d=0.8$$
$$d=0.3$$

Question #64
Key: B

With no deductible, the loss cost is proportional to 598,500.
With a 100 deductible it is proportional (in the same proportion) to 598,500 – 58,500 – 1000(100) = 440,000. With a 250 deductible it is 598,500 – 58,500 – 70,000 – 600(250) = 320,000. The reduction is 120,000/440,000 = 0.273 = 27%.

Question #65
Key: A

At the time of the loss the coverage is 150,000/250,000 = 60% < 80%. Then the benefit payment is $\min\left\{150,000, \frac{150,000}{0.8(250,000)}20,000\right\}=15,000$.

Question #66
Key: D

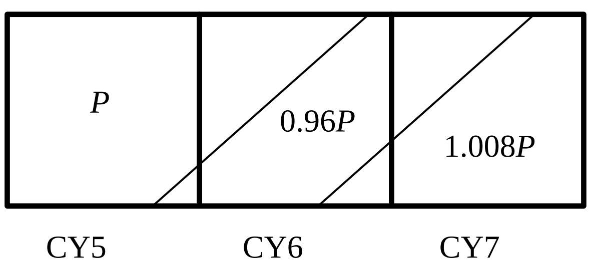
$$\min\left\{70{,}000, \frac{70{,}000}{0.8(100{,}000)}(20{,}000-200)\right\} = 17{,}325$$

Question #67
Key: E

The payment is 175,000 + 45,000 + (20,000 – 1,000) = 239,000. Note that the legal fees do not count against the liability limit.

Question #68
Key: C

Let P be the premium after the July 1, CY3 rate change. On November 15, CY5 the premium is $0.96P$ and on October 1, CY6 it becomes $1.05(0.96)P = 1.008P$. The relevant parallelogram is:



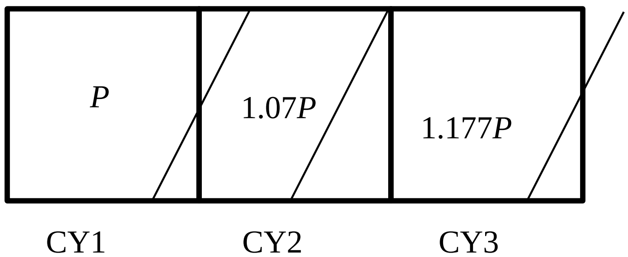
The upper left triangle for CY6 has area $(1/2)(7/8)^2 = 49/128$ and the lower right triangle has area $(1/2)(1/4)^2 = 4/128$. The weighted average is $[49 + 4(1.008) + 75(0.960)]P/128 = 0.9768P$. The current premium is 9200(1.008)/(0.9768) = 9494.

Question #69
Key: D

Trend periods for losses: Average accident date in experience period is July 1, AY8 and July 1, AY9, respectively. New rates will be in effect from July 1, CY10 through June 30, CY12, with an average accident date of July 1, CY11. Trend period for losses are 3 years for AY8 and 2 years for AY9. Ultimate losses trended and developed are, AY8: $2260(1.07^3)(1.08) = 2990$, and AY9: $2610(1.07^2)(1.18) = 3526$. Weighted average loss ratio = 0.4(2990/4252) + 0.6(3526/5765) = 0.648. Required portfolio-wide rate change = 0.648/0.657 – 1 = –1.4%.

Question #70
Key: E

Let P be the premium prior to the October 1, CY1 rate change. After the change, the premium is $1.07P$ and on July 1, CY2 it becomes $1.10(1.07)P = 1.177P$. The relevant parallelogram is:



The upper left triangle for CY2 has area (1/2)(1/4)(1/2) = 1/16 and the lower right triangle has area (1/2)(1/2)(1) = 4/16. The weighted average is $[1 + 4(1.177) + 11(1.07)]P/16 = 1.092375P$. The factor is (1.177×.94)/(1.092375) = 1.0128.

Question #71
Key: B

The year-to-year development factors are 12-24: 30,800/15,400 = 2; 24-36: 29,000/20,000 = 1.45; and 36-48: 16,200/14,100 = 1.149. Then the factor for 24-48 is 1.45(1.149) = 1.666 and for 12-48 is 2(1.666) = 3.332. The expected ultimate losses are AY6: 20,000(0.85) = 17,000; AY7: 21,000(0.91) = 19,110; and AY8: 22,000(0.88) = 19,360. The B-F reserves are 17,000(1 – 1/1.149) = 2205, 19,110(1 – 1/1.666) = 7639, and 19,360(1 – 1/3.332) = 13,550. The total is 23,394.

Question #72
Key: D

The development factors are:

$$0-1:\frac{1}{3}\left(\frac{7,900}{2,500}+\frac{8,700}{2,800}+\frac{7,500}{2,500}\right)=3.089 \quad 1-2:\frac{1}{3}\left(\frac{12,900}{8,700}+\frac{10,700}{7,500}+\frac{8,800}{6,400}\right)=1.428$$

$$2-3:\frac{1}{3}\left(\frac{12,600}{10,700}+\frac{10,200}{8,800}+\frac{8,800}{7,300}\right)=1.181 \quad 3-4:\frac{1}{2}\left(\frac{11,500}{10,200}+\frac{9,800}{8,800}\right)=1.1205 \quad 4-5:1$$

The cumulative factors are 0-5: 5.8372, 1-5: 1.8897, 2-5: 1.3233, 3-5: 1.1205; 4-5: 1.
The B-F reserve is [25,000(1 – 1/1.1205) + 26,000(1 – 1/1.3233) + 27,000(1 – 1/1.8897) + 28,000(1 – 1/5.8372)](0.55) = 24,726.

Question #73
Key: E

CY1: ELR = 60%, earned premium = 1,000,000, expected losses = 0.6(1,000,000) = 600,000. Cumulative development factor (CDF) = (1.05)(1.057)(1.126) = 1.250. Reserve using BF method = 600,000(1 – 1/1.250) = 120,000.
CY2: ELR = 62%, earned premium = 1,200,000, expected losses = 0.62(1,200,000) = 744,000. CDF = (1.05)(1.057)(1.126)(1.336) = 1.670. Reserve using BF method = 744,000(1 – 1/1.670) = 298,491.
CY3: ELR = 64%, earned premium = 1,440,000, expected losses = 0.64(1,440,000) = 921,600. CDF = (1.05)(1.057)(1.126)(1.336)(1.5) = 2.504. Reserve using BF method = 921,600(1 – 1/2.504) = 553,549. Total reserve = 120,000 + 298,491 + 553,549 = 972,040.

Question #74
Key: D

The retained loss is 100,000 + 0.15(100,000) + 0.10(100,000) + 0.05(150,000) = 132,500.

Question #75
Key: B

The expected losses for the primary insurer are 0.6(4,000,000) = 2,400,000. The expected proportion of losses in the treaty layer is (1.6/1.7 – 1/1.7 = 0.352941). The expected cost is 0.352941(2,400,000) = 847,058.
The relative cost of the layer can be derived using formulas from *Loss Models* as follows:

$$\frac{E(X\wedge 400,000)-E(X\wedge 100,000)}{E(X\wedge 500,000)}$$

$$=\frac{E(X\wedge 400,000)/E(X\wedge 100,000)-E(X\wedge 100,000)/E(X\wedge 100,000)}{E(X\wedge 500,000)/E(X\wedge 100,000)}$$

$$=\frac{ILF(400,000)-ILF(100,000)}{ILF(500,000)}=\frac{1.60-1.00}{1.70}=0.352941$$

Question #76
Key: E

Policies sold from November 1, CY5 to November 1, CY6 will be in effect through November 1, CY 7 and thus have an average accident date of November 1, CY6. For losses in AY4 the projection is 2.333 years and the projected cost is $1800e^{0.1275(2.333)}=2423.58$. For losses in AY3 the projection is 3.333 years and the projected cost is $1550e^{0.1275(3.333)}=2370.76$. The projected loss cost is the weighted average, (0.8)(2423.58) + (0.2)(2370.76) = 2413.02 = 2413.

Question #77
Key: C

66.59 – 18.64 = 500 – Kexp(–0.06) and so K = (500 – 66.59 + 18.64)/exp(–0.06) = 480.

Question #78
Key: A

Let C be the price for the 40-strike call option. Then, C + 3.35 is the price for the 35-strike call option. Similarly, let P be the price for the 40-strike put option. Then, $P - x$ is the price for the 35-strike put option, where x is the desired quantity. Using put-call parity, the equations for the 35-strike and 40-strike options are, respectively,

$$(C+3.35)+35e^{-0.02}-40=P-x$$
$$C+40e^{-0.02}-40=P.$$

Subtracting the first equation from the second, $5e^{-0.02}-3.35=x, \quad x=1.55.$

Question #79
Key: B

We need to find the value of the claim X for which the deductible is 40% of the claim.
Clearly the deductible is at least 80% of the claim for $X < 500$ and is 0% of the claim for $X > 2100$. So we consider only the interval $500 < X < 2100$.
Since the deductible is linear on this interval, 400 for a claim of 500, and 0 for a claim of 2100, the deductible is given by the linear expression

$$400+(X-500)\left(\frac{0-400}{2100-500}\right)=400-0.25(X-500)=525-0.25X.$$

Therefore, we have $525-0.25X=0.4X$, which gives X = 807.69.

Question #80
Key: E

18,500 * 60% = 11,100 of allowed charges

11,100 – 2,000 = 9,100 of allowed charges in excess of the deductible

9,100 * 70% = 6,370

9,100 * 30% = 2,730

2,000 + 2,730 = 4,730, which is below the 5,000 out of pocket maximum

Company X's payment to the provider should be 6,370.

Question #81
Key: D

For Year 1, we have: $E(X) = \theta$, $E\left(X \wedge 0.25\theta\right) = \theta\left(1 - e^{-0.25\theta/\theta}\right) = \theta\left(1 - e^{-0.25}\right)$

Therefore, $LER_1 = \dfrac{E\left(X \wedge 0.25\theta\right)}{E(X)} = 1 - e^{-0.25} = 0.2212$

For Year 2, $E(Y) = 1.20\theta$, $E\left(Y \wedge 0.5\theta\right) = 1.2E\left(X \wedge \dfrac{0.5\theta}{1.20}\right) = 1.2\theta\left(1 - e^{-5/12}\right)$

and $LER_2 = \dfrac{E\left(Y \wedge 0.5\theta\right)}{E(Y)} = 1 - e^{-5/12} = 0.34076$

$\dfrac{LER_2}{LER_1} = \dfrac{0.34076}{0.22120} = 1.54 \Rightarrow 54\%$ increase

Question #82
Key: A

Under the initial assumptions, losses, X, have $E(X) = \exp\left(6 + \dfrac{1.5^2}{2}\right) = 1242.65$

After 5 years: exponential distribution with mean $1242.65(1.04^5) = 1511.87$ and LER = 0.33,

so: $0.33 = \dfrac{E(Y \wedge d)}{E(Y)} = \dfrac{\theta\left(1 - e^{-d/\theta}\right)}{\theta} = 1 - e^{-d/1511.87} \Rightarrow d = -1511.87\ln(1 - 0.33) = 605.47$

Question #83
Key: A

$$\frac{p_k}{p_{k-1}} = a + \frac{b}{k} \Rightarrow 0.8 = a + \frac{b}{5} \text{ and } \frac{0.5}{0.8} = 0.625 = a + \frac{b}{6}$$

$$\Rightarrow 0.8 - 0.625 = b\left(\frac{1}{5} - \frac{1}{6}\right) \Rightarrow b = 5.25 \text{ and } a = -0.25$$

$$\Rightarrow N \text{ is binomial with } \frac{-q}{1-q} = -0.25 \text{ and } (m+1)\frac{q}{1-q} = 5.25 \Rightarrow m = 20 \text{ and } q = 0.2$$

$$\Rightarrow E(N^M) = (1 - p_0^M)\frac{mq}{1-(1-q)^m} = 3.64$$

Question #84
Key: D

Members of the $(a,b,1)$ class have $\frac{p_k}{p_{k-1}} = a + \frac{b}{k}$ for $k = 2,3,4,\ldots$, so point iii) implies that N is an $(a,b,1)$ distribution with $a = 0$ and $b = \lambda = 1$. This, together with point ii), implies the distribution is a zero-modified Poisson distribution with $p_0^M = 0.45$.

$$P(N \geq 2) = 1 - P(N \leq 1) = 1 - (P(N=0) + P(N=1)) = 1 - \left(0.45 + \frac{1-p_0^M}{1-P(L=0)}P(L=1)\right),$$

where L has a Poisson distribution with λ=1 and $P(L=0) = P(L=1) = e^{-1}$,

so $P(N \geq 2) = 1 - \left(0.45 + \frac{0.55}{1-e^{-1}}e^{-1}\right) = 0.2299$

Question #85
Key: C

$$E(X \wedge 500) = \exp\left(4.2 + \frac{1.1^2}{2}\right)\Phi(0.73) + 500\left(1 - \Phi(1.83)\right) = 110.5$$

Let S denote the aggregate distribution of retained claims.

$$E(S) = E(N)E(X \wedge 500) = 20(110.5) = 2210$$
$$Var(S) = E(N)Var(X \wedge 500) + E\left((X \wedge 500)^2\right)Var(N) = 20(26{,}189) = 523{,}780$$

The 90th percentile of S is $2210 + 1.282\sqrt{523{,}780} = 3137.82$

Question #86
Key: B

For Product X: aggregate losses have mean 10*20 = 200 and variance 10*25 + 400*9 = 3850.
For Product Y: aggregate losses have mean 2*50 = 100 and variance 2*100 + 2500 = 2700.

Because Product X and Product Y are independent, total aggregate losses, *S*, have mean 300 and variance 6550.
Using the normal approximation, we have:

$$P(S > 400) = P\left(\frac{S-300}{\sqrt{6550}} > \frac{400-300}{\sqrt{6550}}\right) = P(Z > 1.24) = 0.11$$

Question #87
Key: E

Subadditivity holds: $\rho(X+Y) = E(X+Y) = E(X) + E(Y) = \rho(X) + \rho(Y)$

Monotonicity holds: If $X \le Y$, then $\rho(X) = E(X) \le \rho(Y) = E(Y)$

Positive homogeneity holds: $\rho(cX) = E(cX) = cE(X) = c\rho(X)$

Translation invariance holds: $\rho(X+c) = E(X+c) = E(X) + c = \rho(X) + c$

Since $\rho(X) = E(X)$ satisfies all four properties, it is coherent.

Question #88
Key: E

The likelihood function is $L(\alpha) = \alpha^5 1500^{5\alpha} \prod \left(1500 + x_i\right)^{-(\alpha+1)}$
The loglikelihood function is $l(\alpha) = 5\ln\alpha + 5\alpha \ln 1500 - \left(\alpha+1\right)\sum \ln\left(1500 + x_i\right)$

$$l'(\alpha) = \frac{5}{\alpha} + 5\ln 1500 - \sum \ln\left(1500 + x_i\right)$$

Setting $l'(\alpha) = 0$, we find: $\frac{5}{\alpha} = -5\ln 1500 + \sum \ln\left(1500 + x_i\right) \Rightarrow \hat{\alpha} = 3.974$

Question #89
Key: A

The likelihood function is:
$L(\alpha) = f(1.2)\, f(1.6)\, f(1.4)\left[S(2.0)\right]^2 = \alpha^3 (1/3)^{2\alpha} \prod (x_i + 1)^{-(\alpha+1)}$
The loglikelihood function is $l(\alpha) = 3\ln\alpha + 2\alpha\ln(1/3) - (\alpha+1)\sum \ln(x_i + 1)$
$l'(\alpha) = \dfrac{3}{\alpha} + 2\ln(1/3) - \sum \ln(x_i + 1) = \dfrac{3}{\alpha} - 4.81666$
Setting $l'(\alpha) = 0$, we find $\hat{\alpha} = 0.623$

Question #90
Key: A

The likelihood function is $L(\lambda) = \left(e^{-\lambda}\right)^{80}\left(1 - e^{-\lambda}\right)^{105}$

The loglikelihood function is $l(\lambda) = -80\lambda + 105\ln(1 - e^{-\lambda})$

Setting $l'(\lambda) = -80 + \dfrac{105e^{-\lambda}}{1 - e^{-\lambda}}$ equal to 0, we find $e^{-\lambda} = \dfrac{80}{185} \Rightarrow \hat{\lambda} = -\ln\dfrac{80}{185} = 0.838329$

The probability the number of claims, *N*, is less than 2 is $P(N < 2) = e^{-\hat{\lambda}} + \hat{\lambda}e^{-\hat{\lambda}} = 0.79495$

Question #91
Key: D

For the severity distribution, we have $\mu_X = 20$ and $\sigma_X^2 = 355$. The standard for full credibility is $\left(\dfrac{1.645}{0.05}\right)^2\left(1 + CV_X^2\right) = \left(\dfrac{1.645}{0.05}\right)^2\left(1 + \dfrac{355}{20^2}\right) = 2043.05$, round up to 2044

Question #92
Key: C

For the severity distribution, we have
$E(X) = \int_{100}^{\infty} 3(100)^3 x^{-3}dx = 150,\ E(X^2) = \int_{100}^{\infty} 3(100)^3 x^{-2}dx = 30,000$ and
$\sigma_X^2 = 30,000 - 150^2 = 7500$. The standard for full credibility is
$\left(\dfrac{1.96}{0.05}\right)^2\left(1 + CV_X^2\right) = \left(\dfrac{1.96}{0.05}\right)^2\left(1 + \dfrac{7500}{150^2}\right) = 2048.85$, round up to 2049

Question #93
Key: A

Accident Year	Expected Ultimate Loss	Cumulative Paid-to-date	Reserve
AY1	$35,500*0.71=25,205$	24,140	1065
AY2	$31,200*0.73=22,776$	19,475	3301
AY3	$0.75X$	19,750	$0.75X-19,750$
AY4	$0.75(X+5000)$	18,400	$0.75(X+5000)-18,400$

$$43,412=1065+3301+0.75X-19,750+0.75(X+5000)-18,400 \Rightarrow X=48,964$$

Question #94
Key: B

$$R_{BF}=\left(1-\frac{1}{f}\right)R_{LR}+\frac{1}{f}R_{CL}$$

$$400,000=\left(1-\frac{1}{f}\right)250,000+\frac{1}{f}437,500 \Rightarrow f=1.25=\prod_{j=3}^{\infty} f_j$$

$$1,120,000=\left(1-\frac{1}{f}\right)1,200,000+\frac{1}{f}1,050,000 \Rightarrow f=1.875=\prod_{j=2}^{\infty} f_j$$

$$f_2=\frac{1.875}{1.25}=1.5$$

Question #95
Key: D

The midpoint of AY3 is July 1, AY3, the midpoint of AY4 is July 1, AY4, and May 1, AY6 is the midpoint of the effective period.

AY3 lost cost projected to May 1, AY6 is $400\exp(0.182*2.8333)=669.90$
AY4 lost cost projected to May 1, AY6 is $450\exp(0.182*1.8333)=628.23$
Projected loss cost = (0.9)(628.23) + (0.1)(669.90) = 632.40

Question #96
Key: D

For any given policy year, the average accident date is a full year after the beginning of that year. Similarly for new rates effective on April 1, CY9, the midpoint of the effective period is April 1, CY10.

Therefore, the PY3 and PY4 loss costs will be projected 6.25 and 5.25 years, respectively.

Using the projected loss cost, we find:
$600(1.07)^{6.25}(1-p)+670(1.07)^{5.25}(p)=951 \Rightarrow p=0.8813$

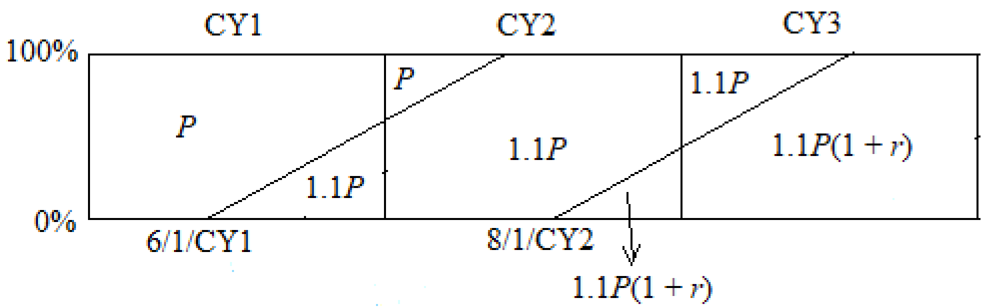
Question #97
Key: E

First, use the Borhuetter-Ferguson method to calculate the estimated ultimate loss as $\frac{47,387}{1-1/1.08}=639,724.5$. Next, calculate IBNR under the expected loss ratio method as $639,724.5-(187,047+243,005)=209,672.5$. Then, IBNR under the chain-ladder method is $(187,047+243,005)*0.08=34,404.16$. The absolute difference is $209,672.5-34,404.16=175,268.34$.

Question #98
Key: B

Let P represent the rate level before the rate change on June 1, CY1. The rate level $1.1P$ takes effect on June 1, CY1. The rate level $(1.1P)(1+r)$ takes effect on August 1, CY2, so this level is the current rate level.

The parallelogram method is shown in the diagram below.



For CY2, the average rate level for the earned exposure is

$$\frac{1}{2}\left(\frac{5}{12}\right)^2 P+\left[1-\frac{1}{2}\left(\frac{5}{12}\right)^2-\frac{1}{2}\left(\frac{5}{12}\right)^2\right](1.1P)+\frac{1}{2}\left(\frac{5}{12}\right)^2(1.1P)(1+r)\qquad .$$

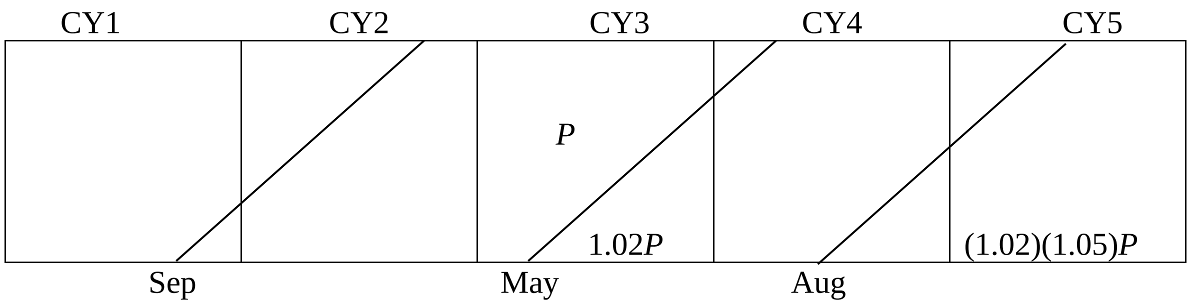
$$=0.086806P+0.909028P+0.0954861P(1+r)=0.99583\overline{3}P+0.095486\overline{1}P(1+r)$$

The ratio of the earned premium at current rates for CY2 to the CY2 earned premium, which is the on-level factor for CY2, is

$$\frac{(1.1P)(1+r)}{0.99583\overline{3}P+0.095486\overline{1}P(1+r)}=\frac{1.1}{0.99583\overline{3}/(1+r)+0.095486\overline{1}}=1.03$$

$$=>1+r=1.02402=>r=2.402\%$$

Question #99
Key: D



Let P denote the rate following the Sept 1, CY1 rate change.

For CY3, the average rate level for the earned exposure is:

$$\left[1-\frac{1}{2}\left(\frac{7.5}{12}\right)^2\right]P+\frac{1}{2}\left(\frac{7.5}{12}\right)^2(1.02P)=1.00390625P$$

Current rates are $1.02*1.05P=1.071P$

CY3 earned premium at current rates = $600*\dfrac{1.071P}{1.00390625P}=640.10$

Question #100
Key: A

The risk-neutral probability of an increase is:

$$1-q=\frac{e^{0.0275*0.5}-d}{u-d}=0.44003$$

Then:

$$2.482=e^{-0.0275*0.5}((1-q)*0+q*(S-4.5-0.7966S)) \Rightarrow S=44.22$$

Question #101
Key: B

Using put-call parity and the 550-strike options, we have:

$$45.46-64.57=S_0-550e^{-0.0325*0.5} \Rightarrow S_0=522.0247$$

Using the current stock price and put-call parity with the 525-strike options:

$$55.92-x=522.0247-525e^{-0.0325*0.5} \Rightarrow x=50.43$$

Question #102
Key: A

The monotonicity property states that if $Y \le Z$ for all possible outcomes, then $\rho(Y) \le \rho(Z)$. From the variable definitions, we have $\mu_Y = 0.8$, $\sigma_Y^2 = 0.8 - 0.8^2 = 0.16$, $\mu_Z = 1$, and $\sigma_Z^2 = 0$. Monotonicity requires $0.8 + 0.4k \le 1$. Therefore, $k \le 0.5$.