6th Homework — MATH 304 — Fall 2023
— Due October 26 —

1. Let
$$u_1 := (1,1,1)^T,\ u_2 := (1,2,2)^T,\ u_3 = (2,3,4)^T \text{ and}$$
$$v_1 := (4,6,7)^T,\ v_2 := (0,1,1)^T,\ v_3 = (0,1,2)^T.$$
   a. Find the transition matrix corresponding to the change of basis from $\{e_1, e_2, e_3\}$ to $\{u_1, u_2, u_3\}$.
   b. Find the transition matrix corresponding to the change of basis from $\{v_1, v_2, v_3\}$ to $\{e_1, e_2, e_3\}$.
   c. Find the transition matrix from $\{v_1, v_2, v_3\}$ to $\{u_1, u_2, u_3\}$
   d. Let $x = 2v_1 + 3v_2 - 4v_3$. Find the coordinates of x with respect to $\{u_1, u_2, u_3\}$.
   e. Verify your answer to previous one, by computing the coordinates in each case with respect to the standard basis.

2. Find a basis for the row space, column space and null space of the following matrices:
$$A = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 1 & 4 \\ 4 & 7 & 8 \end{pmatrix},\ B = \begin{pmatrix} -3 & 1 & 3 & 4 \\ 1 & 2 & -1 & -2 \\ -3 & 8 & 4 & 2 \end{pmatrix}\ C = \begin{pmatrix} 1 & 3 & -2 & 1 \\ 2 & 1 & 3 & 2 \\ 3 & 4 & 5 & 6 \end{pmatrix}$$

3. Let $E := [p_1(x) = 1, p_2(x) = x + 1, p_3(x) = x^2 - 1]$ and $F := [q_1(x) = 1, q_2(x) = x, q_3(x) = x^2]$. These are two basis of the vector space P_2 of all polynomials of degree at least 2. Find the transition matrix from E to F and the transition matrix from F to E. Express the polynomial
$$p(x) = 11x^2 - 2x + 5$$
   with respect to the basis E.

Show your work in each exercise.