Question 1

Let

$$u_1 := (1,1,1)^T, u_2 := (1,2,2)^T, u_3 := (3,2,4)^T$$
$$v_1 := (4,6,7)^T, v_2 := (0,1,1)^T, v_3 := (0,1,2)^T$$

(a) Find the transition matrix corresponding to the change of basis from e_1, e_2, e_3 to u_1, u_2, u_3.

(b) Find the transition matrix corresponding to the change of basis from v_1, v_2, v_3 to e_1, e_2, e_3.

(c) Find the transition matrix from v_1, v_2, v_3 to u_1, u_2, u_3.

(d) Let $x = 2v_1 + 3v_2 - 4v_3$. Find the coordinates of x with respect to u_1, u_2, u_3.

(e) Verify your answer to previous one, by computing the coordinates in each case with respect to the standard basis.

Question 2

Find a basis for the row space, column space and null space of the following matrices.

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 4 \\ 4 & 7 & 8 \end{bmatrix} B = \begin{bmatrix} -3 & 1 & 3 & 4 \\ 1 & 2 & -1 & -2 \\ -3 & 8 & 4 & 2 \end{bmatrix} C = \begin{bmatrix} 1 & 3 & -2 & 1 \\ 2 & 1 & 3 & 2 \\ 3 & 4 & 5 & 6 \end{bmatrix}$$

Question 3

Let $E := [p_1(x) = 1, p_2(x) = x + 1, p_3(x) = x^2 - 1]$ and $F := q_1(x) = 1, q_2(x) = x, q_3(x) = x^2$. These are two basis of the vector space P_2 of all polynomials of degree at least 2. Find the transition matrix from E to F and the transition matrix from F to E. Express the polynomial

$$p(x) = 11x^2 - 2x + 5$$

with respect to the basis E.