LAB 2: Visual Odometry

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Abstract: *This lab involves using a tracking camera to collect position and time data of a puck released on an inclined plane in order to find the gravitational constant. First, the position data is differentiated using finite differences to derive velocity vs time and ultimately acceleration vs time data for the puck. Rolling averages are used to reduce noise within the data, and the average acceleration can be used with trigonometry to calculate the gravitational constant. Uncertainty in the pixel length of the meter stick used for calibration is propagated into the error of the conversion factor from pixels to meters. The standard error of acceleration and angle of the table are propagated into the uncertainty of the final gravitational constant.*

Keywords: *backwards finite difference, rolling average, uncertainty, unit conversion*

1. Introduction

The purpose of this lab was to calculate the earth's gravitational constant by tracking an object on an inclined plane with a camera, which requires a conversion factor between pixels meters. To find this conversion factor, the distance in pixels between the two tracking points on the ruler is needed. This is founding with the following equation:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Equation 1

Then the uncertainty for ruler distance in pixels was calculated using the standard error formula:

$$\delta f = \frac{\sigma_f}{\sqrt{N}}$$

Equation 2

where σ is the standard deviation for the average distance in pixels and N is the total number of distance values.
Then the uncertainty value for the pixel to meters conversion factor was calculated using the Division Error Formula:

$$\delta Q = |Q|\sqrt{(\frac{\delta f}{f})^2 + (\frac{\delta m}{m})^2}$$

Equation 3

Where f is the average distance of in pixels, δf is the uncertainty of pixel distance, m is the distance between the dots in meters and δm is the error of the meter distance. Q is the actual value of the pixel per meter conversion.
To find velocity and acceleration, the backward finite difference was used on the position vs time data:

$$V \cong \frac{r_1 - r_0}{t_1 - t_0}$$

Equation 4

$$a_y \cong \frac{v_1 - v_0}{t_1 - t_0}$$

Equation 5

r_1 and r_0 is the position, t_1 and t_0 is the time and v is velocity.
Then, gravity was calculated using the formula:

$$g = \frac{a}{sin\theta}$$

Equation 6

Finding the uncertainty for **g** requires the uncertainty of $sin(\Theta)$, which is found with the general uncertainty formula:

$$\delta Q = |\frac{dQ}{dx}|\delta x$$

Equation 7

A rolling average was calculated for position, velocity, and acceleration using the following formula:

$$\bar{x}_i = \frac{x_{i+1} + x_i + x_{i-1}}{3}$$

Equation 8

Where x_{i-1} is the preceding value, x_i is the current value and x_{i+1} is the succeeding value.

2. Experimental Procedure

This lab used the Jetson system which includes an inclined air table connected to a computer with a tracking camera. First, a ruler with two colored dots spaced 500 millimeters apart was placed on the air table without any incline. The Jetson's computer was connected using the secure shell protocol. The computer contained a directory of python scripts used to collect data. One of these scripts used to collect position and time data was copied to the home directory and executed to collect the position data of the two dots. Terminating the python script generated a comma-separated values file containing horizontal and vertical coordinates of each of the dots along with timestamps in milliseconds. This file was copied to the host machine using the secure shell protocol. Approximately 800 rows of data were collected for the ruler. After that, the ruler was taken off the table, and the table was inclined to 3.6 degrees below the horizontal. A puck with a colored tracking dot was then held on the higher end of the inclined table and released shortly after the same python script was executed. After the puck reached the bottom of the table, the python script was terminated, which generated another comma-separated values file containing position coordinates over the time the script was running. This file was also copied to the host machine using the secure shell protocol.

The data collected in this experiment were used to calculate a constant used to convert from pixels to meters, the acceleration of the puck, the earth's gravitational constant, as well as the uncertainties associated with the aforementioned quantities. To find these calculations, both comma-separated values files were loaded into respective DataFrame objects from the Pandas Python library. Extraneous data and labels for colors not concerned with the ruler and puck were removed from each DataFrame object before any calculations were made. Calculations are described in the results and analysis section.

3. Results and Analysis

Results are shown in **Table 1** below. The length of the ruler was calculated using the distance formula (**equation 1**), where the square root of the sum of the squares of the differences between the horizontal and vertical coordinates was taken for each row of data collected. This generated a new set of average lengths in pixels. To find the number of pixels per meter, the distance between the tracking dots was divided by the average length in pixels. Then, **equation 2** was used to find the standard error of the mean ruler length in pixels, using 0.5mm as the uncertainty for the length between the dots. This is the uncertainty for the length. The inverse of the number of pixels per meter yielded a quantity in meters per pixel, which was then used to convert the position data from pixels to meters.

To find the average acceleration of the puck, the puck's position data was isolated such that it only contained data after the initial release and before the puck bounced at the bottom of the table. Since the python script collected timestamp data for each row in milliseconds, the timestamp data was divided by 1000 to convert to seconds. Then, a rolling average of the position data was calculated with a period of 3 using the .rolling() and .mean() methods in Pandas, which are simply code implementations of **equation 8**. Velocities for each timestamp were calculated using a backward finite difference (**equation 4**) of the smoothed position data, which was accomplished using the .diff() method in Pandas. This value was then multiplied by the number of meters per pixel found earlier to convert from pixels per second to meters per second. Then, a rolling average (**equation 8**) with a period of 3 was calculated for the velocity data using the same method as before. Accelerations for each timestamp were found the same way velocity was calculated, which was by taking a backward finite difference of the smoothed velocity data (**equation 5**). Finally, a rolling average with a period of 3 was calculated using the same methods as before (**equation 8**). The average acceleration of the puck was calculated by taking the mean of each acceleration value, which was done using the .mean() method on the raw acceleration series in the DataFrame. The standard error of the mean acceleration was calculated using the .sem() method on the same raw acceleration series in the DataFrame (**equation 2**).

Finally, the earth's gravitational constant was calculated by dividing the puck's average acceleration by the sine of the table's angle (**equation 6**). The uncertainty for the gravitational constant was found using a combination of **equation 3** and **equation 7**. First, the uncertainty of sin(Θ) must be found in order to use **equation 3**, which is done using **equation 7** and plugging in 0.25 degrees for the uncertainty of theta. This yielded a result of $g = 9.85721\ \frac{m}{s^2} \pm 1.53395 \frac{m}{s^2}$, which is within the expected value of 9.8 meters per second squared, which confirms that the calculations were accurate. Plots of position vs time, velocity vs time, and acceleration vs time were made by plotting the respective DataFrame series using matplotlib. Both smoothed and raw data were plotted on each graph.

Table 1: Results of Calculations

Value	Measurement	Uncertainty
Ruler length from Camera	526.090 pixels	0.054428 pixels
Meters per pixel	$9.50407 * 10^{-4}$ m/pixel	$9.55480 * 10^{-7}$ m/pixel
Acceleration	0.618939 m/s^2	0.0899875 m/s^2
Gravitation Constant	9.85721 m/s^2	1.53395 m/s^2

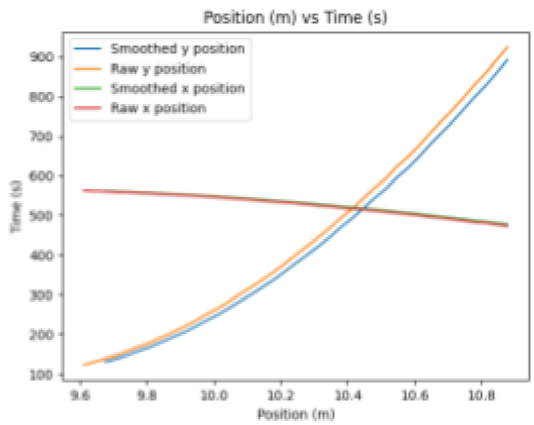

Figure 1: Graph of raw and smoothed position with respect to time in seconds

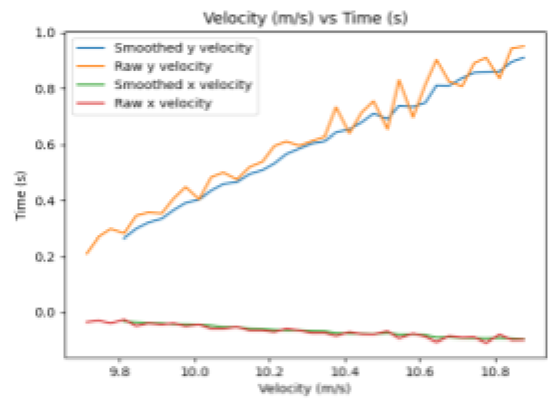

Figure 2: Graph of raw and smoothed velocity with respect to time in seconds

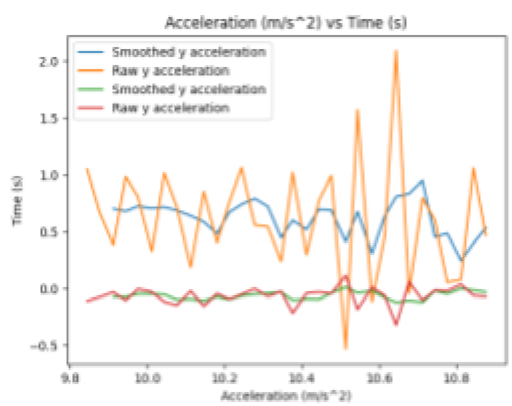

Figure 3: Graph of raw and smoothed acceleration with respect to time in seconds

4. Conclusions

Overall, the results of this lab experiment demonstrate the successful use of visual odometry to determine the gravitational constant of the Earth. By tracking the motion of a puck on an inclined plane with a camera, position and time data were collected and analyzed using various mathematical equations and techniques. The backward finite difference was used to derive velocity and acceleration data from the position data while rolling averages were used to reduce noise in the data. Uncertainty calculations were performed to propagate errors from the measurement devices into the final result. The calculated value for the gravitational constant of the earth was found to be within the range of accepted values, which validates the accuracy of the experiment.