ENGR HW 11

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Question 1:

Given: $y(t) = 26\sin\left(\frac{\pi}{2}t + \frac{\pi}{3}\right)$, mass = 18 kg

Find: Amplitude, period, angular frequency, phase difference, KE at equilibrium

Diagram:

Theory: $\omega = \frac{2\pi}{T} = 2\pi f$ $\quad \theta = \omega t = 2\pi f t$

$y(t) = A\sin(\omega t + \phi)$ $\quad KE = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$

Assumptions: Motion is simple and harmonic

Solution: Amplitude = 26 cm $\quad$ Period = $\frac{2\pi}{\frac{\pi}{2}} = 4$ seconds

Angular frequency = $\frac{\pi}{2}$ s $\quad KE = \frac{1}{2}(18)\left(\frac{\pi}{2}\right)^2(0.26)^2\sin^2\left(\frac{\pi}{2}\left(\frac{1}{3}\right) + \frac{\pi}{3}\right)$

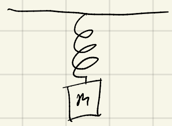
Phase difference = $\frac{\pi}{3}$ s $\quad = 1.5$ J

Question 2:

Given: $m = 0.25$ kg $\quad T = 0.50$ s

Find: Spring constant

Diagram:



Theory: $k = m\omega^2$ $\quad T = 2\pi\sqrt{\frac{M}{k}}$

Assumptions: there are no external forces

Solution: $T = 2\pi\sqrt{\frac{M}{k}}$

$0.5 = 2\pi\sqrt{\frac{0.25}{k}}$

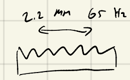
$k = 40$ N/m

Question 3:

Given: $d = 2.2$ mm $\quad f = 65$ Hz

Find: amplitude, max speed of blade, magnitude of maximum acceleration

Diagram:



Theory: $\omega = \frac{2\pi}{T} = 2\pi f$ $\quad \theta = \omega t = 2\pi f t$

Assumptions: no nonconservative forces

Solution: $A = \frac{2.2}{2} = \underline{1.1 \text{ mm}}$

$V_{max} = \omega x_{max} = (408.407)\,1.1/1000 = \underline{0.45 \text{ m/s}}$

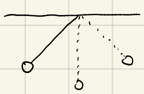
$a_{max} = \omega^2 x_{max} = (408.407)^2\,1.1/1000 = \underline{183 \text{ m/s}^2}$

Question 4:

Given: $T = 3.8$ s $\quad V_{max} = 0.65$ m/s

Find: length of pendulum, max displacement in degrees

Diagram:



Theory: $\omega = \frac{2\pi}{T} = 2\pi f$ $\quad \theta = \omega t = 2\pi f t$

Assumptions: No external forces

Solution: $T = 2\pi\sqrt{\frac{L}{g}}$

$L = \frac{gT^2}{4\pi^2} = \underline{3.6 \text{ m}}$

$\frac{1}{2}\cancel{m}v^2 = \cancel{m}gL(1-\cos\theta)$

$\frac{1}{2}(0.65)^2 = 9.8(3.6)(1-\cos\theta)$

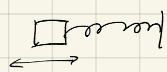
$\underline{\theta = 6.3^\circ}$

Question 5:

Given: $m = 28$ g $\quad k = 1.4$ N/m $\quad$ damping = 252 g/s

Find: is system under or over damped? Damping coefficient for critical damp?

Diagram:



Theory: Critically damped when $\beta^2 - \omega^2 = 0$

underdamped: $\beta^2 - \omega^2 < 0$

overdamped: $\beta^2 - \omega^2 > 0$

Assumptions: Simple harmonic motion

Solution: $c = 2\sqrt{mk} = 2\sqrt{0.028\,(1.4)} = 0.3959 > 0.252 \rightarrow$ underdamped

damping coefficient = 396 g/s