Math 152 – Python Lab 7

Directions: Use Python to solve each problem. (Template link)

1. Euler found the sum of the p-series with $p = 4$:

$$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$$

   (a) Find the partial sum s_{10} of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$. Estimate the error in using s_{10} as an approximation to the sum of the series.

   (b) A variation of the Remainder Estimate tells us that:

$$s_n + \int_{n+1}^{\infty} f(x)\,dx \le s \le s_n + \int_{n}^{\infty} f(x)\,dx$$

   Use $n = 10$ to give an improved estimate of the sum.

   (c) Compare your estimates in part (b) with Euler's estimate.

   (d) Find a value of n so that s_n is within 10^{-6} of the sum.

2. Given the series $\displaystyle\sum_{n=2}^{\infty} n^2 e^{-n}$ and the function $f(x) = x^2 e^{-x}$:

   (a) Compute $\displaystyle\int f(x)\,dx$ and $\displaystyle\int_1^{\infty} f(x)\,dx$.

   (b) In a print statement, state your conclusion about the convergence or divergence of the series based on your answer to a).

   (c) Compute s_{10}, s_{50}, s_{100}, and s.

   (d) Use the Remainder Estimate for the Integral Test to estimate $s - s_{100}$. Compare the actual value of $s - s_{100}$. Which is larger?

   (e) According to the Remainder Estimate, how many terms are needed to sum the series to within 10^{-10}? Compute the sum to confirm $|s - s_N| < 10^{-10}$. (NOTE: To expedite the computation, convert the terms to floating point before summing)

(NOTE: #3 on the next page)

3. Given the series $\sum_{n=1}^{\infty} \frac{n \sin^2(n)}{1+n^3}$:

   (a) Let $a_n = \frac{n \sin^2(n)}{1+n^3}$. Define a series b_n with which to compare it.

   (b) Plot the first 50 terms of a_n and b_n on the same graph to determine which is larger. If the graph is not clear, use the logical test $a_n < b_n$ to test the logical value comparing each term.

   (c) Determine whether $\sum_{n=1}^{\infty} b_n$ converges or not, and state whether any conclusion can be made about the convergence of $\sum_{n=1}^{\infty} a_n$ as a result.

   (d) If (c) is conclusive, skip to (e). If (c) is inconclusive, determine whether $\frac{a_n}{b_n}$ converges or not, and state your conclusion about the convergence of $\sum_{n=1}^{\infty} a_n$. (NOTE: If you still cannot conclude anything, start over with a different b_n!)

   (e) Repeat parts (a) – (d) for $a_n = \frac{e^n + 1}{ne^n + 1}$

