Question 1

Proof. Let $f : X \to Y$ and $A_1, A_2 \subseteq X$. Assume $y \in f[A_1 \cup A_2]$. Then,

$$\begin{aligned} y \in f[A_1 \cup A_2] &\iff y = f(x), x \in A_1 \cup A_2 \\ &\iff y = f(x), x \in A_1 \vee x \in A_2 \\ &\iff y \in f[A_1] \vee y \in f[A_2] \\ &\iff y \in f[A_1] \cup f[A_2]. \end{aligned}$$

Thus, $f[A_1 \cup A_2] = f[A_1] \cup f[A_2]$. $\square$

Question 2

$f^{-1}([-3, 5]) = [1009.5, 1013.5]$

Question 3

$D^{-1}(\{4x^3\}) = \{x^4\}$

Question 4

1. $s^{-1}(\{4\}) = \{(0, 4), (1, 3), (2, 2), (3, 1), (4, 0)\}$
2. $s^{-1}(\{1\}) = \{(0, 1), (1, 0)\}$

Question 5

Proof. Let $f : X \to Y$ and $B_1, B_2 \subseteq Y$. Assume $x \in f^{-1}[B_1 \cup B_2]$. Then,

$$\begin{aligned} x \in f^{-1}[B_1 \cup B_2] &\iff f(x) \in B_1 \cup B_2 \\ &\iff f(x) \in B_1 \vee f(x) \in B_2 \\ &\iff x \in f^{-1}[B_1] \vee x \in f^{-1}[B_2] \\ &\iff x \in f^{-1}[B_1] \cup f^{-1}[B_2]. \end{aligned}$$

Thus, $f^{-1}[B_1 \cup B_2] = f^{-1}[B_1] \cup f^{-1}[B_2]$. $\square$

Question 6

Part a

Proof. Let $f : X \to Y$ and $A \subseteq X$. Assume $x \in A$. Then $f(x) \in f[A]$. Since $x \in f^{-1}[f[A]]$, $A \subseteq f^{-1}[f[A]]$. $\square$

Part b

Proof. Let $f : X \to Y$ and $A \subseteq X$. Assume that f is injective. From part a, we know that $A \subseteq f^{-1}[f[A]]$. Now let $x \in f^{-1}[f[A]]$. Then, $f(x) \in f[A]$. Thus, there exists $a \in A$ such that $f(x) = f(a)$. Since f is injective, $x = a$. Since $x = a$, $x \in A$. Therefore, $f^{-1}[f[A]] \subseteq A$, so $f^{-1}[f[A]] = A$. $\square$

Question 7

Part a

Iteration 1: $a = 1207, b = 569$
$569 \nmid 1207$, so $q = \frac{1207}{569}$

Part b