MATH300 Homework 11 (due 4/26)

1. (15 pts) Let $f : X \to Y$ and $A_1, A_2 \subseteq X$. Prove that $f[A_1 \cup A_2] = f[A_1] \cup f[A_2]$. You may use logical symbols in part of your final answer proof.

2. (8 pts) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 2024 - 2x$. Compute $f^{-1}([-3, 5])$. (No proof or justification needed.)

3. (8 pts) Let $D : \mathbb{R}[x] \to \mathbb{R}[x]$ be defined by $D(f) = f'$, where $\mathbb{R}[x]$ is the set of all polynomials over x with real coefficients. Simply put, D sends a polynomial to its derivative, e.g., $D(3x^2 + \pi x) = 6x + \pi$. Find $D^{-1}(\{4x^3\})$. (No proof or justification needed.)

4. (14 pts) Let $s : \mathbb{Z}^+ \times \mathbb{Z}^+ \to \mathbb{Z}^+$ be defined by $s(m, n) = m + n$. Simply put, s sends an ordered pair of positive integers to their sum, e.g., $s(3, 10) = 13$. Find the following preimages. (No proof or justification needed.)

   (a) $s^{-1}(\{4\})$

   (b) $s^{-1}(\{1\})$

5. (15 pts) Let $f : X \to Y$ and $B_1, B_2 \subseteq Y$. Prove that $f^{-1}[B_1 \cup B_2] = f^{-1}[B_1] \cup f^{-1}[B_2]$. You may use logical symbols in part of your final answer proof.

6. (24 pts) Let $f : X \to Y$ and $A \subseteq X$. Prove the following.

   (a) $A \subseteq f^{-1}[f[A]]$

   (b) If f is one-to-one, then $f^{-1}[f[A]] = A$ (make sure to use one-to-one in your proof).

7. (16 pts) Let $a = 1207$ and $b = 569$.

   (a) Use the Euclidean Algorithm to find $\gcd(a, b)$.

   (b) Find $x, y \in \mathbb{Z}$ such that $xa + yb = \gcd(a, b)$.