MATH300 Homework 1 (Due Friday, 1/26)

You can print and write answers, fill in digitally (e.g., with tablet), or write/type up your answers separately. Either way, submit a pdf through Gradescope. *Note that problems 6 and 7 contain material for our 1/23 lecture, but you may be able to figure them out before then.*

1. (24 pts) Determine whether each of the following is a proposition, predicate, or neither (circle one).

   (a) proposition / predicate / neither. Shrek is six feet tall.

   (b) proposition / predicate / neither. Dr. Cantu and his son.

   (c) proposition / predicate / neither. Give an example of an integrable function.

   (d) proposition / predicate / neither. $20^2 + 23^2 > 2023^2$.

   (e) proposition / predicate / neither. $x^2 = 1$.

   (f) proposition / predicate / neither. Potatoes are awesome.

   (g) proposition / predicate / neither. n is a perfect square.

   (h) proposition / predicate / neither. The product of every two prime numbers is odd.

2. (12 pts) Consider the following propositions

$$P : 2024 \text{ is odd}, \qquad Q : 42 \text{ is a multiple of } 3.$$

   Determine whether each statement is true or false (circle one).

   (a) True / False. P

   (b) True / False. Q

   (c) True / False. $P \vee Q$

   (d) True / False. $P \wedge Q$

   (e) True / False. $P \Rightarrow Q$

   (f) True / False. $Q \Rightarrow P$

3. (4 pts) For the predicate $P(x) : (x^2 - 9)(x - 1) = 0$, where x is a positive real number, determine the value(s) of x for which $P(x)$ is a true statement.

4. (9 pts) State the negation for each of the following. If possible, state the negation "positively".

   (a) $\sqrt{3}$ is a rational number.

   (b) 0 is not a negative number.

   (c) The real number r is at most π.

5. (12 pts) In each of the following statements identify the hypothesis and conclusion.

   (a) If a is irrational, then $2a$ is irrational.

   Hypothesis:

   Conclusion:

   (b) a^3 is an even integer whenever a is an even integer.

   Hypothesis:

   Conclusion:

   (c) $\lim_{x \to 0^+} f(x) = 3$ is necessary for $\lim_{x \to 0} f(x) = 3$.

   Hypothesis:

   Conclusion:

6. (12 pts) Let P, Q, and R be statements. Show that the two expressions in each pair are **logically equivalent** (same truth values) by completing the truth tables.

   (a) $(P \vee Q) \wedge R$ and $(P \wedge R) \vee (Q \wedge R)$. (this is a *distributive law*)

P	Q	R	$P \vee Q$	$(P \vee Q) \wedge R$	$P \wedge R$	$Q \wedge R$	$(P \wedge R) \vee (Q \wedge R)$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

   (b) $\neg(P \Rightarrow Q)$ and $P \wedge (\neg Q)$

P	Q	$P \Rightarrow Q$	$\neg(P \Rightarrow Q)$	$\neg Q$	$P \wedge (\neg Q)$
T	T				
T	F				
F	T				
F	F				

7. (6 pts) Let P and Q be statements. A **tautology** is a proposition that is true for every possible assignment of truth values to the statement letters that occur in it. A **contradiction** is a proposition that is false for every possible assignment of truth values to the statement letters that occur in it. Determine whether each of the following statements is a tautology, a contradiction, or neither.

   (a) $P \wedge \neg P$

   (b) $P \Rightarrow (Q \Rightarrow P)$

8. Watch at least two Numberphile or 3Blue1Brown videos on YouTube.

   (a) (1 pt) List the ones you watched.

   (b) (20 pts) Write a paragraph or two on your favorite video. Please indicate what it was about and what you learned.