MATH300 Homework 8 (Due Monday, 4/1)

1. (8 pts) For $i \in \mathbb{Z}^+$, let $A_i = (-i, i)$. Find the following (you do not need to prove your answers.)

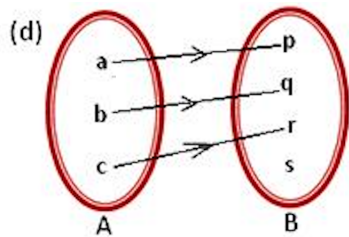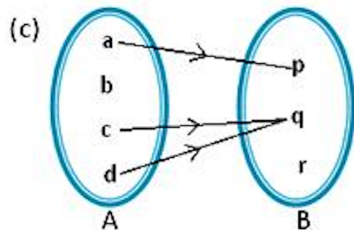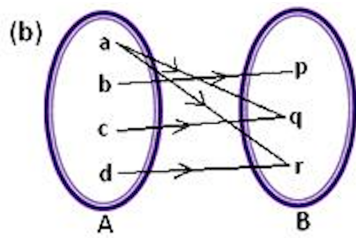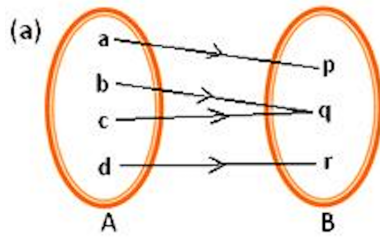   (a) $\displaystyle\bigcup_{i=1}^{\infty} A_i$

   (b) $\displaystyle\bigcap_{i=1}^{\infty} A_i$

2. (8 pts) For $i \in \mathbb{Z}^+$, let $A_i = \left[1 - \frac{1}{i}, 3 - \frac{1}{i}\right)$. Find the following (you do not need to prove your answers.)

   (a) $\displaystyle\bigcup_{i \in \mathbb{Z}^+} A_i$

   (b) $\displaystyle\bigcap_{i \in \mathbb{Z}^+} A_i$

3. (16 pts) Determine whether the following diagrams define functions from A to B. If not, give a reason. If yes, find its range.

(a)

(b)

(c)

(d)

4. (14 pts) Let $f : [-2, \infty) \to \mathbb{R}$ be defined by $f(x) = \sqrt[4]{x + 2}$. Prove that $[0, \infty) \subseteq \text{Ran}(f)$.

5. (18 pts) Let $f : \mathbb{R} - \{3\} \to \mathbb{R}$ be defined by $f(x) = \dfrac{x}{x-3}$. Prove that $\mathrm{Ran}(f) = \mathbb{R} - \{1\}$.

6. (18 pts) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = -x^2$. Prove that $\mathrm{Ran}(f) = (-\infty, 0]$.

7. (18 pts) Let $A = \{a, b, c, d, e\}$, $B = \{w, x, y, z\}$, and $C = \{0, 1, 3, 6, 10\}$.

    (a) Using a graph (not diagram), define a function $f : A \rightarrow C$ whose range coincides with its codomain.

    (b) Using a graph, define a function $g : B \rightarrow C$ so that w is mapped to 10 under g and z is a preimage of 0 under g.