MATH 152 Lab 9

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In [1]:
```
from sympy import *
from sympy.plotting import (plot,plot_parametric)
import matplotlib.pyplot as plt
import numpy as np
```

Question 1

1a

In [2]:
```
x, n = symbols('x n')
expr = ((-1)**(n+1) * (-2 + 4*x**2)**n) / (4**n * n)
s = Sum(expr, (n, 1, oo))

ratio = abs(simplify(expr.subs(n, n+1) / expr))
print(f"The limit of the ratio of the nth and (n+1)th terms is {limit(ratio, n, oo)}")
```

```
The limit of the ratio of the nth and (n+1)th terms is Abs(2*x**2 - 1)/2
```

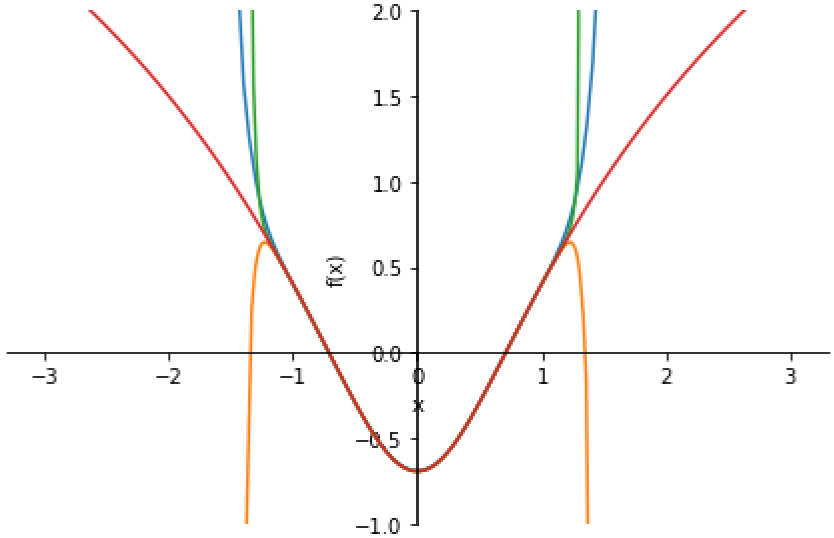
1b

In [3]:
```
print(f"The radius of convergence is 2*sqrt(3/2) and the endpoints are -sqrt(3/2) and 

if Sum(((-1)**(n+1) * (-2 + 4*(sqrt(3/2))**2)**n) / (4**n * n), (n, 1, oo)).is_converg
    print(f"The series converges when x = sqrt(3/2), so sqrt(3/2) is in the interval c
else:
    print(f"The series does not converge when x = sqrt(3/2), so sqrt(3/2) is not in th

if Sum(((-1)**(n+1) * (-2 + 4*(-sqrt(3/2))**2)**n) / (4**n * n), (n, 1, oo)).is_conver
    print(f"The series converges when x = -sqrt(3/2), so -sqrt(3/2) is in the interval
else:
    print(f"The series does not converge when x = -sqrt(3/2), so -sqrt(3/2) is not in 
```

```
The radius of convergence is 2*sqrt(3/2) and the endpoints are -sqrt(3/2) and sqrt(3/
2).
The series converges when x = sqrt(3/2), so sqrt(3/2) is in the interval of convergen
ce.
The series converges when x = -sqrt(3/2), so -sqrt(3/2) is in the interval of converg
ence.
```

1c

In [4]:
```
plot(Sum(expr, (n, 1, 5)), Sum(expr, (n, 1, 10)), Sum(expr, (n, 1, 15)), log((2*x**2 -
```



Out[4]: <sympy.plotting.plot.Plot at 0x1f185507130>

Question 2

2a

In [5]:

```
expr = ((-1)**n * sqrt(pi) * x**(2*n + 1)) / ((2*n + 1) * factorial(n))
s = Sum(expr, (n, 0, oo))

print(f"|a_n+1 / a_n| = {abs(simplify(expr.subs(n, n+1) / expr))}")
print(f"The limit of the ratio test as n approaches infinity is {limit(abs(simplify(ex
```

```
|a_n+1 / a_n| = Abs(x**2*(2*n + 1)/((n + 1)*(2*n + 3)))
The limit of the ratio test as n approaches infinity is 0
```

2b

In [6]:

```
print(f"The radius of convergence is infinite and the endpoints are -oo and oo exclusi

if Sum(((-1)**n * sqrt(pi) * oo**(2*n + 1)) / ((2*n + 1) * factorial(n)), (n, 0, oo)).
    print(f"The series converges when x = oo, so oo is in the interval of convergence.
else:
    print(f"The series does not converge when x = oo, so oo is not in the interval of

if Sum(((-1)**n * sqrt(pi) * (-oo)**(2*n + 1)) / ((2*n + 1) * factorial(n)), (n, 0, oo
    print(f"The series converges when x = -oo, so -oo is in the interval of convergenc
else:
    print(f"The series does not converge when x = -oo, so -oo is not in the interval o
```

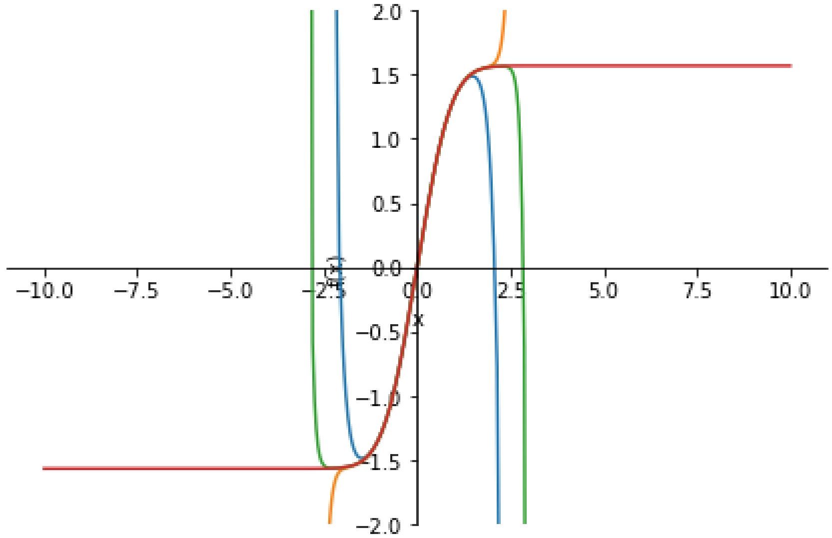
```
The radius of convergence is infinite and the endpoints are -oo and oo exclusive.
The series does not converge when x = oo, so oo is not in the interval of convergenc
e.
The series does not converge when x = -oo, so -oo is not in the interval of convergen
ce.
```

2c

In [7]:

```
t = symbols('t')
fx = sqrt(pi) * integrate(E**(-1 * t**2), (t, 0, x))
```

```python
plot(Sum(expr, (n, 0, 5)), Sum(expr, (n, 0, 10)), Sum(expr, (n, 0, 15)), fx, (x, -10,
```



Out[7]: <sympy.plotting.plot.Plot at 0x1f1854f80d0>

2d

In [8]:
```python
s100 = Sum(expr, (n, 0, 100))
print(f"f(5) is approximatly {N(s100.subs(x, 5))} using s100 as an approximation.")
print(f"The decimal approximation of pi/2 is {N(pi/2)}.")
print(f"The two approximations are about equal.")
```

```
f(5) is approximatly 1.57079632679248 using s100 as an approximation.
The decimal approximation of pi/2 is 1.57079632679490.
The two approximations are about equal.
```

Question 3

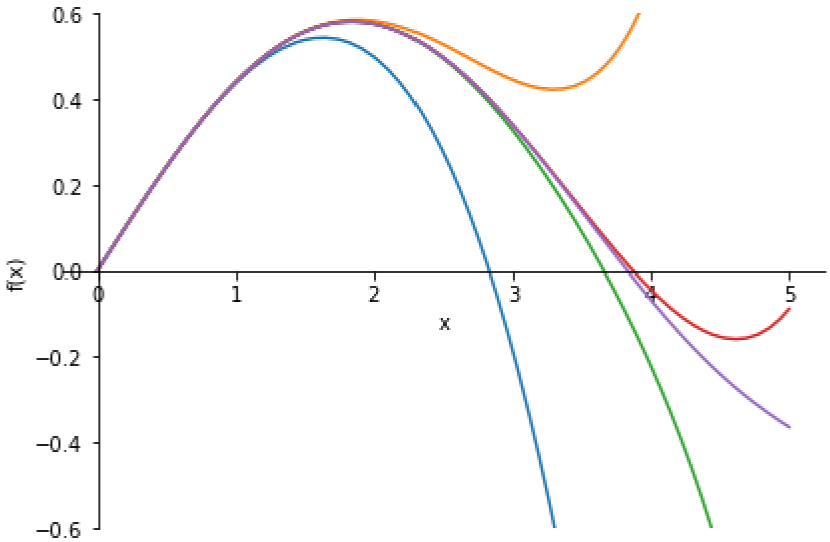
3a

In [9]:
```python
expr = ((-1)**n * x**(2*n + 1)) / (factorial(n) * factorial((n + 1)) * 2**(2*n + 1))
J1x = Sum(expr, (n, 0, oo))

ratio = abs(simplify(expr.subs(n, n+1) / expr))
print(f"The radius of convergence is infinite since the limit of the ratio test is 0.'
```

```
The radius of convergence is infinite since the limit of the ratio test is 0.
```

3b

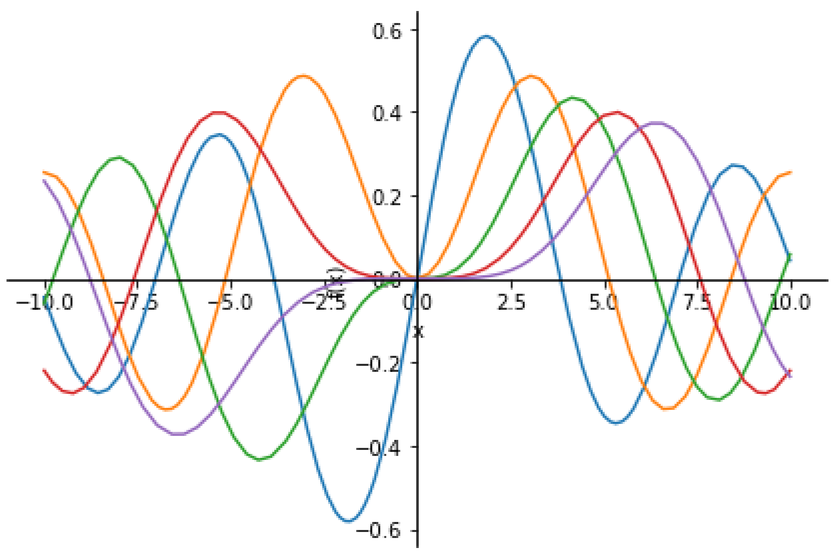
In [10]:
```python
plot(Sum(expr, (n, 0, 1)), Sum(expr, (n, 0, 2)), Sum(expr, (n, 0, 3)), Sum(expr, (n, 0
```



Out[10]: <sympy.plotting.plot.Plot at 0x1f185ec2910>

3c

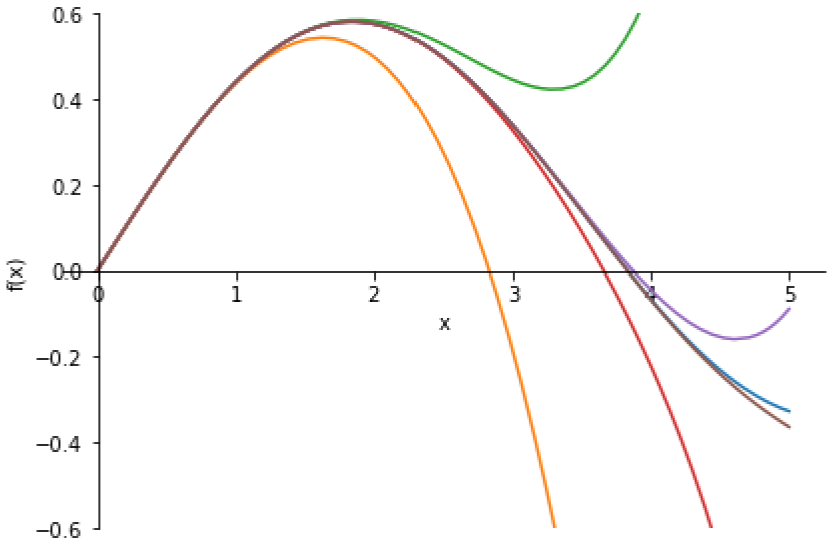
In [11]:
```
plot(besselj(1, x), besselj(2, x), besselj(3, x), besselj(4, x), besselj(5, x))
```



Out[11]: <sympy.plotting.plot.Plot at 0x1f185cfbbe0>

3d

In [12]:
```
plot(besselj(1, x), Sum(expr, (n, 0, 1)), Sum(expr, (n, 0, 2)), Sum(expr, (n, 0, 3)),
```



Out[12]: <sympy.plotting.plot.Plot at 0x1f1860fb700>