

Practice for the Final exam — MATH 304 — Fall 2023
— No Due Date —

1. Find the general solution of each of the following systems:

a. $\begin{cases} y_1 + y_2 = y'_1 \\ -2y_1 + 4y_2 = y'_2 \end{cases}$

b. $\begin{cases} y_1 - y_2 = y'_1 \\ y_1 + y_2 = y'_2 \end{cases}$

c. $\begin{cases} y_1 + y_3 = y'_1 \\ 2y_2 + 6y_3 = y'_2 \\ y_2 + 3y_3 = y'_3 \end{cases}$

2. Solve the following initial value problems:

a. $\begin{cases} -y_1 + 2y_2 = y'_1 \\ 2y_1 - y_2 = y'_2 \end{cases}, y_1(0) = 3, y_2(0) = 1.$

b. $\begin{cases} y_1 - 2y_2 = y'_1 \\ 2y_1 + y_2 = y'_2 \end{cases}, y_1(0) = 1, y_2(0) = -2.$

3. In each of the following, "diagonalize" the matrix X and use it to compute A^{-1} , A^4 , e^A .

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, A = \begin{pmatrix} 2 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{pmatrix}, A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ 3 & 6 & -3 \end{pmatrix}$$

4. Let

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix}, b = \begin{pmatrix} 12 \\ 6 \\ 18 \end{pmatrix}.$$

- a. Use the Gram-Schmidt process to find an orthonormal basis for the column space of A .
- b. Factor A into QR .
- c. Use the above to solve the system $Ax = b$.
5. Let $\{x_1, x_2, x_3\} := \{(0, 1, 0), (2, 1, 2), (0, 0, 1)\}$, be a basis of \mathbb{R}^3 .
- a. Use the Gram-Schmidt process to obtain an orthonormal basis.

- b. Let $b := (1, 1, 1)$. Compute the projection of b onto $\text{span}\{x_1, x_2\}$ and to $\text{span}\{x_3, x_2\}$.
6. Consider the vector space $C[0, 1]$ with the inner product
- $$\langle f, g \rangle = \int_0^1 f(x)g(x)dx.$$
- a. Find an orthonormal basis of the subspace E spanned by $1, x, x^2$.
- c. Compute the length of $2x^2 + 3$.
- c. Compute the projection of e^x onto E
7. Find the orthogonal complement of the subspace of \mathbb{R}^4 spanned by $(1, 1, 1, 1), (1, -1, 1, -1)$.
8. For each of the following systems $Ax = b$ find all least squares solutions:

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}, \quad \text{and} \quad A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

This file covers the material after the second mid-term. Use the "practice" files for the first and second midterm exams to practice on problems on the previous chapters. Work on the Homework assignments

The final exam is

1. **Section 509:** December 11, 1:00 till 3:00 pm.
2. **Section 508:** December 12, 1:00 till 3:00 pm.
3. **Section 510:** December 12, 3:30 till 5:30 pm.