MATH300 Homework 6 (Due Friday, 3/8)

1. (6 pts) Describe the following sets in roster notation by listing enough elements to indicate a pattern.

   (a) The set of all positive real numbers x for which $\cos x = 0$.

   (b) $\{n \in \mathbb{Z} \mid (\exists k \in \mathbb{Z})(n = 5k + 2)\}$

2. (14 pts) Determine whether the following are true or false.

   (a) True / False. $\{1, 3\} \in \{\{1, 2, 3\}, \{1, 2\}, 1, 3\}$

   (b) True / False. $\{1, 2\} \subseteq \{\{1, 2, 3\}, \{1, 2\}, 1, 3\}$

   (c) True / False. $\{0\} \in \{0, \{0\}\}$

   (d) True / False. $\{0\} \subseteq \{0, \{0\}\}$

   (e) True / False. $\{\{0\}\} \subseteq \{0, \{0\}\}$

   (f) True / False. $[5, 6) \subseteq (4, 6]$

   (g) True / False. $(7, 9] \subseteq [6, 9)$

3. (20 pts) Consider the sets

$$A = \{n \in \mathbb{Z} \mid (\exists k \in \mathbb{Z})(n = 4k + 1)\},$$
$$B = \{n \in \mathbb{Z} \mid (\exists j \in \mathbb{Z})(n = 4j - 7)\}.$$

Prove that $A = B$.

4. (10 pts) Let $\mathcal{U} = \{1, \{2\}, a, b, c\}$ be the universal set for $A = \{a, b, \{2\}\}$, $B = \{b, c\}$, and $C = \{1, \{2\}, c\}$. Determine the following.

(a) $(A \cup \overline{B}) \cap C$

(b) $A \cup (\overline{B} \cap C)$

5. (16 pts) Let A, B, and C be nonempty sets. Disprove the following statements.

    (a) If $A \cap B = A \cap C$, then $B = C$.

    (b) If $A - B = C - B$, then $A = C$.

6. (16 pts) Let A and B be sets. Prove that $A \subseteq B$ if and only if $\overline{B} \subseteq \overline{A}$.

7. (18 pts) Let A and B be sets. Prove the following statements.

    (a) (10 pts) $\overline{A \cap B} = \overline{A} \cup \overline{B}$

    (b) (8 pts) $A \cap \emptyset = \emptyset$.