Project of Algorithms on Condition Satisfiability

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1 Main Idea

In the "Condition Satisfiability Problem", we are given the following:

1. n boolean variables $x_1, x_2, \ldots, x_n$.
2. A set $L = \{T_1, T_2, \ldots, T_P\}$ of P "*lead-to*" conditions. A "*lead-to*" condition has the form $(x_{i_1} \wedge x_{i_2} \wedge \ldots \wedge x_{i_k}) \Rightarrow x_j$, meaning that if $x_{i_1} \wedge x_{i_2} \wedge \ldots \wedge x_{i_k}$ is true, then x_j is true. The degenerate case is when $k = 0$, in which case the condition is simply x_j.
3. A set $F = \{M_1, M_2, \ldots, M_Q\}$ of Q "*False-must-exist*" conditions. A "*False-must-exist*" condition has the form $(\neg x_{i_1} \vee \neg x_{i_2} \vee \ldots \vee \neg x_{i_k})$, meaning that if at least one of $x_{i_1}, x_{i_2}, \ldots, x_{i_k}$ is false, then the condition is true.

We want to find the truth values of the n boolean variables $x_1, x_2, \ldots, x_n$ that satisfy all of the P+Q conditions in L and F. If there is no such assignment, then we want to output "*No satisfying solution exists*".

The main idea behind this algorithm is to initialize a set of n boolean variables to be all false. Then, we will iteratively update the truth values based on the constraints in L and F.

2 Pseudocode

Algorithm 1: Conditoin Satisfiability

Input: n boolean variables $x_1, x_2, \ldots, x_n$, set $L = \{T_1, T_2, \ldots, T_P\}$ of P "*lead-to*" conditions, set $F = \{M_1, M_2, \ldots, M_Q\}$ of Q "*False-must-exist*" conditions

Output: Truth values of $x_1, x_2, \ldots, x_n$ that satisfy all of the $P + Q$ conditions in L and F, or "*No satisfying solution exists*"

x_i = false for $i = 1, 2, \ldots, n$;
cont = true;
while *cont* **do**
    cont = false;
    for $i = 1$ *to* P **do**
        $lhs = T_i[0] \wedge T_i[1] \wedge \ldots \wedge T_i[k]$;
    end
    for $i = 1$ *to* Q **do**
    end
end
for $i = 1$ *to* P **do**
end
for $i = 1$ *to* Q **do**
end
return $x_1, x_2, \ldots, x_n$

3 Proof of Correctness

4 Runtime Complexity Analysis