Question 1

(a) $\{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots\}$

(b) $\{\ldots, -13, -8, -3, 2, 7, 12, 17, \ldots\}$

Question 2

(a) False

(b) True

(c) True

(d) True

(e) True

(f) True

(g) False

Question 3

Proof. Let A and B be sets. We want to prove that $A = B$, which means that $A \subseteq B$ and $B \subseteq A$. First, assume that x is an element of A. Then, by the definition of A, we can write x as $x = 4k + 1$ for some integer k. We can then do the following manipulations:

$$\begin{aligned} x &= 4k + 1 \\ &= 4k + 1 - 8 + 8 \\ &= 4k + 8 - 7 \\ &= 4(k + 2) - 7. \end{aligned}$$

Since $k + 2$ is an integer, x is by definition an element of B. Therefore, $A \subseteq B$. Similarly, assume that x is an element of B. Then, by the definition of B, we can write x as $x = 4j - 7$ for some integer j. We can then do the following manipulations:

$$\begin{aligned} x &= 4j - 7 \\ &= 4j - 7 + 8 - 8 \\ &= 4j - 8 + 1 \\ &= 4(j - 2) + 1. \end{aligned}$$

Since $j - 2$ is an integer, x is by definition an element of A. Therefore, $B \subseteq A$. Since $A \subseteq B$ and $B \subseteq A$, we have proven that $A = B$. $\square$

Question 4

(a)

$$\begin{aligned}(A \cup \overline{B}) \cap C &= (\{a, b, \{2\}\} \cup \{1, \{2\}, a\}) \cap \{1, \{2\}, c\} \\ &= \{1, \{2\}, a, b\} \cap \{1, \{2\}, c\} \\ &= \{1, 2\}\end{aligned}$$

(b)

$$\begin{aligned}A \cup (\overline{B} \cap C) &= \{a, b, \{2\}\} \cup (\{1, \{2\}, a\} \cap \{1, \{2\}, c\}) \\ &= \{a, b, \{2\}\} \cup \{1, \{2\}\} \\ &= \{1, \{2\}, a, b\}\end{aligned}$$