LAB 4: COLLISIONS

Anthony Capone, Kevin Lei, Irene Otterbach

Texas A&M University
College Station, TX 77843, US.

Abstract *The purpose of this experiment was to investigate the collision of two air hockey pucks at different angles. The experiment was conducted by recording the positions and velocities of the pucks using a camera and analyzing the data using the Python programming language. The velocity data from the camera was used to calculate the total momentum and kinetic energy of the system. There were eight angles of collision between zero and one hundred eighty degrees. The results tend to confirm that the total momentum and kinetic energy of the system is conserved after collision. The results also show that the angle of collision does not have an effect on the conservation of momentum or kinetic energy.*

Keywords: *Conservation, Momentum, Kinetic Energy, Elasticity*

1. Introduction

The goal of this experiment was to compare the momentum and kinetic energy of two pucks before and after colliding with each other. The entire experiment involved eight different angles of collision. The team operated the tracking camera to record the movement of the pucks before, during, and after they had collided with each other. The experiment tested different angles to determine whether or not the momentum was conserved based on the change in the kinetic energy from the different collisions. Since the exact angle of collision was unknown at the time of data collection, the angle of collision had to be calculated using the velocity vector data.

This was done by taking the inverse cosine of the dot product of the two vectors divided by the product of the magnitudes of each vector:

$$\theta = \arccos\left(\frac{v_1 \cdot v_2}{|v_1||v_2|} \right) \qquad \textit{Equation 1}$$

Where $\mathbf{v_1}$ and $\mathbf{v_2}$ are the velocity vectors for the first and second pucks respectively. After determining the angle of collision, the momentum of the pucks were calculated using the following equation:

$$p = mv \qquad \textit{Equation 2}$$

Where **p** is the momentum, **m** is the mass of the puck, and **v** is the velocity of the puck. The experiment also involved calculating the kinetic energy of the system before and after the collision, which was done with the following equation:

$$\mathrm{K}E = \frac{1}{2}mv^2 \qquad \textit{Equation 3}$$

Where **E** is the kinetic energy of one puck, **m** is the mass of the puck, and **v** is the velocity of the puck. Variation in the velocity data means that error would propagate into the momentum and kinetic energy calculations. To find the propagated error of the momentum and kinetic energy calculations, the following equation was used:

$$\frac{\delta Q}{|Q|} = \sqrt{\left(\frac{\delta a}{|a|}\right)^2 + \left(\frac{\delta b}{|b|}\right)^2 + \ \ldots \ + \left(\frac{\delta z}{|z|}\right)^2} \qquad \textit{Equation 4}$$

Where δQ is the error in the resulting quantity, $|Q|$ is the magnitude of the resulting quantity, δa is the error in one factor, $|a|$ is the magnitude in one factor, and so on for each remaining factor in the original equation.

2. Experimental Procedure

The team began the experiment by making sure the camera was in the correct position to gather all of the data for the experiment. After ensuring each puck had a marker sticker placed in the center, the team was able to turn on the frictionless table. The pucks were placed on the table and the team began to slide the pucks into each other at a controllable speed that would allow for good, near frictionless data points. The team conducted the first trial by running the program and having the pucks collide from about a 180° angle from each other. The team then did 2 sets of 3 trials each at different angles between 180°. The first of the 2 sets was done by doing 3 different angles between 0° and 90°, while the second set was done with angles between 90° and 180°. Once the team had finished collecting the data, the mass was given to determine the conservation of momentum, the possible loss of kinetic energy, and any possible uncertainties that may have also occurred.

3. Results and Analysis

Three types of graphs were generated using the data collected through the experiment. One graph compares the total momentum of the system (both pucks combined) before the collision and after the collision for all eight angles of collision. This is done for both horizontal and vertical components of the momentum. The second graph plots the ratio of the total momentum of the system for all eight angles of collision. Once again, this is done for both horizontal and vertical components. The last graph plots the kinetic energy of the system before and after the collision for all eight angles of collision. This graph is not split into horizontal and vertical components since kinetic energy is a scalar quantity with no direction.

Data from the experiment was generated as a comma-separated values (CSV) file with timestamp, position, and velocity data. CSV files for each trial were loaded into a python script as dataframes for calculation with the help of the Pandas library. The point of collision was found by locating the frame where the distance between the pucks was smallest. The data was then isolated such that it only contained frames where the velocity was approximately constant for times before and after the collision. Once the data was cleaned and prepared, the momentum of the pucks before and after the collision were calculated and compared using **Equation 2** for both horizontal and vertical components as shown by **Figure 1** and **Figure 2** below:

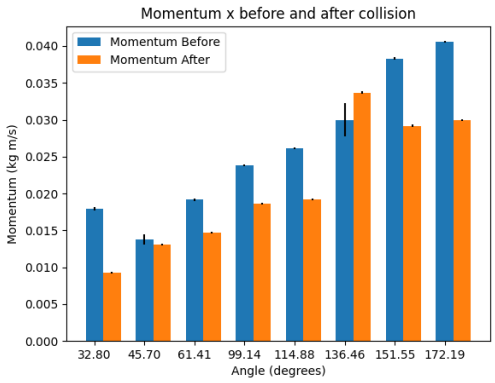

Figure 1: Horizontal Momentum

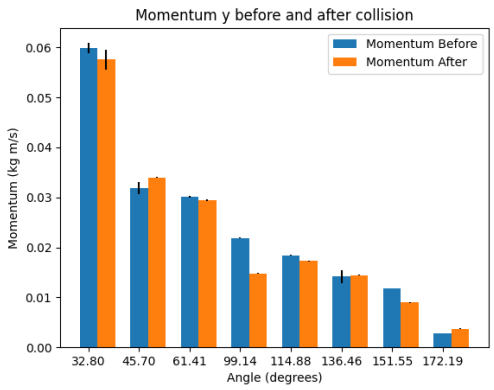

Figure 2: Vertical Momentum

After the momentum of the pucks before and after collision were calculated, the values were then used to find the ratio of the momentum of the pucks before the collision to the momentum of the pucks after the collision, which was done simply by dividing the momentum before by the momentum after. The ratio of momentum for all eight angles of collision are shown in **Figure 3** and **Figure 4** below:

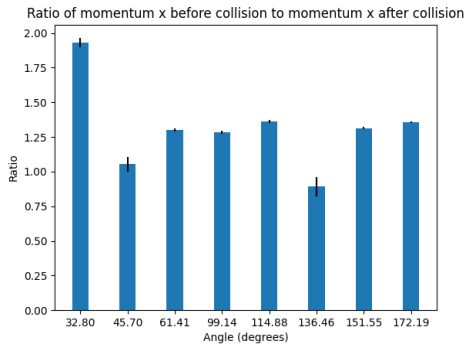

Figure 3: Horizontal Momentum Ratio

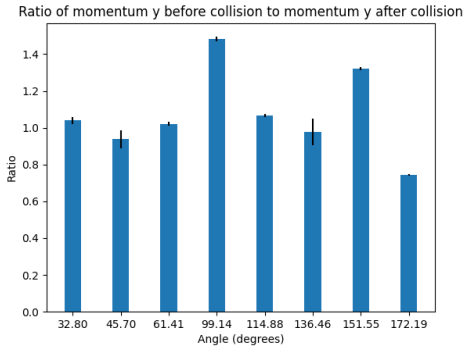

Figure 4: Vertical Momentum Ratio

Finally, the kinetic energy for the pucks before and after collision were calculated using **Equation 3**, which is shown by **Figure 5**.

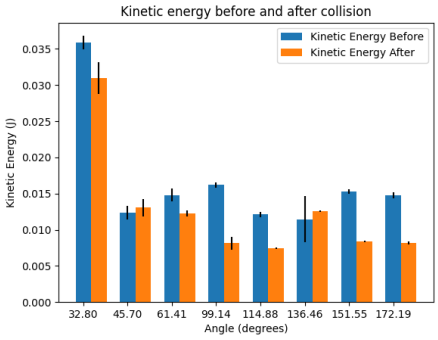

Figure 5: Kinetic Energy

The calculations above (momentum, ratio of momentum, and kinetic energy) all require the use of the velocity of the pucks. Since the velocity of the pucks was not perfectly linear, the error in the velocity would propagate out into the errors for the calculations above. The standard error for the velocity was found by dividing the standard deviation by the square root of the number of samples in the duration of constant velocity. **Equation 4** was then used to propagate this error into the momentum calculation and the kinetic energy calculation. The momentum error was then used with **Equation 4** again to find the error in the ratio of momentums before and after the collision.

The concept of conservation of momentum and kinetic energy is demonstrated in some of the trials in certain graphs, but not in every trial in every graph. In theory, the confidence intervals for the momentum bars before and after collision should be overlapping with each other for all trials for both horizontal and vertical components. However, some of the confidence intervals do not overlap with each other. This pattern is repeated in the momentum ratio graphs and the kinetic energy graphs, where the confidence interval for the ratio should be close to one, and the kinetic energy confidence intervals for before and after the collision should be overlapping with each other in order to show conservation of energy. This deviation from theory can be explained by a number of possible experimental errors such as throwing the puck too fast leading to a small sample size and more varied data, suboptimal timing of collisions, and accelerating the puck for too long before release. Although there were a few trials where the data reflected a violation of the law of conservation of momentum and energy, consideration for potential experimental errors would point that the experiment still mostly confirms that momentum and energy are conserved throughout collision.

4. Conclusions

This experiment investigated the collision of two pucks hovering on an air table where multiple trials were performed at different angles of collision. The results mostly confirmed that momentum and kinetic energy were conserved after collision regardless of the angle of collision, and the slight deviations from theoretical results can be explained in part by human error during experimentation and imperfect data collection. In addition, these errors could have resulted from air resistance on the puck.