8th Homework — MATH 304 — Fall 2023
— Due November 16th —

1. Let $\{u_1, u_2, u_3\}$ be an orthonormal set of vectors in some vector space with inner product. Let

$$u := u_1 + 2u_2 + 3u_3 \text{ and } v := u_1 - u_3.$$

   Compute $\langle u, v\rangle, \|u\|, \|v\|$.

2. Consider the vector space $C[-1, 1]$ equipped with the inner product:

$$\langle f, g\rangle := \int_{-1}^{1} f(x)g(x)dx.$$

   - Show that $1, x$ are orthogonal.
   - Compute the norms $\|1\|, \|x\|$.

3. Let

$$u_1 = (\frac{1}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, -\frac{4}{3, \sqrt{2}})^T, u_2 = \frac{1}{3}(2, 2, 1)^T, u_3 = \frac{1}{\sqrt{2}}(1, -1, 0)^T.$$

   - Show that u_1, u_2, u_3 is an orthonormal basis for $\mathbb{R}^3$.
   - Let $x = (1, 2, 2)^T$. Find the projection p of x onto $S := \text{span}\{u_2, u_3\}$.

4. Let $v_1 := (1, 2, 0, -1)^T, v_2 = (1, -1, 0, 0)^T, v_3 = (0, 1, 0, -1)^T$. Find the angle between v_1, v_2, v_2, v_3 and v_1, v_3. Find the norm of each o these vectors. Find the projection of v_1 onto v_2 and onto v_3.

5. Let A be an $m \times n$ matrix. Show that $A^T A$ and AA^T is a symmetric matrix. Assume that $m \geq n$ and $\text{rank}(A) = n$. Show that if $P = A(A^T A)^{-1}A^T$ then

$$P^2 = P.$$

Show your work in each exercise.