CSCE 222 Discrete Structures for Computing – Fall 2023
Hyunyoung Lee

Problem Set 7

Due dates: Electronic submission of *yourLastName-yourFirstName-hw7.tex* and *yourLastName-yourFirstName-hw7.pdf* files of this homework is due on **Monday, 11/20/2023 before 11:59 p.m.** on `https://canvas.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two files are missing, you will receive zero points for this homework.** Your files must contain your first and last names and UIN in the given spaces and the electronic signature (your full name) correctly.

Name: Kevin Lei   **UIN: 432009232**

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to answer this homework.

Electronic signature: Kevin Lei

Total $100 + 10$ (bonus) points. *Explanation will be about 90% of the grade for each problem.*

The intended formatting is that this first page is a cover page and each problem solved on a new page. You only need to fill in your solution between the `\begin{solution}` and `\end{solution}` environment. Please do not change this overall formatting.

Checklist:
☐ Did you type in your name and UIN?
☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted)
☐ Did you sign that you followed the Aggie Honor Code?
☐ Did you solve all problems?
☐ Did you submit both the .tex and .pdf files of your homework to each correct link on Canvas?

Problem 1. (20 points) Section 13.1, Exercise 13.4. Explain your reasoning carefully, including (but not limited to) why you set up your generating function in the way you do.

Solution. We know that albert must receive an even amount of cards that is at least 8 and at most 14. To represent this with a generating function, we can use the following:

$$A(z) = z^8 + z^{10} + z^{12} + z^{14}$$

Here, the coefficient of z^k represents the number of ways to give Albert k cards. The same applies for Bella and Clara, but we have different restraints on the number of cards they can receive. Bella and Clara can both receive an odd number of cards that is at least 3 and at most 9. Therefore, we have the following:

$$B(z) = z^3 + z^5 + z^7 + z^9, \quad C(z) = z^3 + z^5 + z^7 + z^9$$

To find all combinations that satisfy the given conditions, we can multiply the generating functions together, and then find the coefficient of z^{20}, since Grandpa Dell has 20 cards to give out.

$$A(z)B(z)C(z) = (z^8 + z^{10} + z^{12} + z^{14})(z^3 + z^5 + z^7 + z^9)(z^3 + z^5 + z^7 + z^9)$$
$$= x^{32} + 3x^{30} + 6x^{28} + 10x^{26} + 12x^{24} + 12x^{22} + 10x^{20} + 6x^{18} + 3x^{16} + x^{14}$$

$[z^k]A(z)B(z)C(z) = 10$

Thus, we have 10 different ways to give out the cards.

Problem 2. (15 points) Section 13.2, Exercise 13.7. Explain.

Solution. The sequence $(1, 0, 1, 0, ...)$ can be represented by the following generating function:

$$A(z) = 1z^0 + 0z^1 + 1z^2 + 0z^3 + ...$$

since the first term is 1 which corresponds with z^0, and the second term is 0, and so on. This can be simplified as:

$$A(z) = 1 + z^2 + z^4 + z^6 + ...$$

In summation notation, this can be represented as:

$$A(z) = \sum_{k=0}^{\infty} z^{2k}$$

Since this is a geometric series with common ratio z^2 and first term 1, the closed form is:

$$A(z) = \frac{1}{1 - z^2}$$

The multiplicative inverse of a generating function must multiply with the original generating function to be identically 1. This means the multiplicative inverse of $A(z)$ is simply just:

$$B(z) = 1 - z^2$$

Problem 3. (15 points) Section 13.3, Exercise 13.12. Explain.

Solution.

Problem 4. $(15 + 15 = 30$ points) Section 14.2, Exercise 14.10. *Explain* your steps carefully.

For (a), study carefully how the example in Section 14.2 is solved using generating functions, and solve it in a similar way. (Be reminded that a "closed form" means a rational function that does not use a power series.)

For (b), do the partial fraction decomposition of $H(z)$ and expand it into a sum of two power series and then combine them into a power series to find the coefficient for the z^k power term (like we did for the Fibonacci recurrence in the problem solving video and in the lecture notes).

Solution.
(a)

(b)

Problem 5. (15 points) Section 14.7, Exercise 14.30. Study Example 14.14 in Section 14.7 and solve this exercise problem in a very similar way. Also, *explain* in a similar way as in Example 14.14.

Solution.

Problem 6. (15 points) Section 14.7, Exercise 14.32. Study Example 14.15 in Section 14.7 and solve this exercise problem in a very similar way. Also, *explain* in a similar way as in Example 14.15.

Solution.