Question 1

(a)

x	$P(x)$	$Q(x)$	$P(x) \iff Q(x)$
0	F	F	T
2	F	F	T
3	T	F	F
4	T	T	T
6	T	T	T

(b) The truth value for $\forall x \in \{0, 2, 3, 4, 6\} P(x) \iff Q(x)$ is false.

(c) The truth value for $\exists x \in \{0, 2, 3, 4, 6\} P(x) \iff Q(x)$ is true.

Question 2

(a) 20 is a member of the set of the integer multiples of 4.

(b) 3.14 is a member of the set of the rational numbers or π is a member of the set of the real numbers.

(c) There exists an integer n in the set of natural numbers such that the square root of n is not a member of the set of real numbers.

Question 3

(a) For all integers n, $n^2 - 4n + 3 \geq 0$.

(b) There exists a rational number x such that $x \geq 100$.

Question 4

(a) There exists an Aggie who does not follow the Aggie Honor Code.

(b) All students either do not live on campus or is a math major.

(c) There exists an integer m such that m^2 is even and $m^3 - 1$ is not divisible by 4.

Question 5

Converse: If f is continuous at 0, then f is a linear function.
Contrapositive: If f is not continuous at 0, then f is not a linear function.
Inverse: If f is not a linear function, then f is not continuous at 0.

Question 6

(a) For all real numbers x, there exists an integer n such that n is less than or equal to x and x is less than $n + 1$.

(b) $(\exists x \in \mathbb{R})(\forall n \in \mathbb{Z})(n \leq x \vee x < n + 1)$

Question 7

(a) If x is a multiple of 6, then x is even and is not a multiple of 4.
Negation: There exists a multiple of 6 such that it is not even or it is a multiple of 4.

(b) If x is an even integer, then x^2 is divisible by 4.
Negation: There exists an integer x such that x is even and x^2 is not divisible by 4.

Question 8

Proof. Let n be an odd integer. By definition of the odd integers, there exists an integer k such that $n = 2k + 1$. Then

$$\begin{aligned} n^2 + 1 &= (2k + 1)^2 + 1 \\ &= 4k^2 + 4k + 2 \\ &= 2(2k^2 + 2k + 1). \end{aligned}$$

Since the integers are closed under addition and multiplication, $2k^2 + 2k + 1$ is an integer. Therefore, $n^2 + 1 = 2(2k^2 + 2k + 1) = 2l$ for some integer l. The even integers are defined as integers that can be written as twice an arbitrary integer. Since $n^2 + 1$ is twice an arbitrary integer l, $n^2 + 1$ is an even integer. $\square$