MATH300 Homework 9 (no submission; not for a grade)

1. (0 pts) Find $f \circ g$ and $g \circ f$ for the following.

   (a) $f, g : \mathbb{R} \to \mathbb{R}$ by $f(x) = x^2 - 3x$ and $g(x) = 5x - 2$

   (b) $f, g : \mathbb{Z} \to \mathbb{Z}$ by $f(n) = 2n + 3$ and $g(n) = \begin{cases} 2n - 1 & \text{if } n \text{ is even} \\ n + 1 & \text{if } n \text{ is odd} \end{cases}$

2. (0 pts) For each of the following functions f,

   (i) determine whether f is one-to-one;

   (ii) determine whether f is onto.

   Prove your answers.

   (a) $f : \mathbb{Z} \to \mathbb{Z}$, $f(n) = 7n + 3$

   (b) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \sqrt[5]{2x - 1}$

   (c) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ 2x & \text{if } x < 0 \end{cases}$

   (d) $f : \mathbb{Z} \to \mathbb{Z}$, $f(n) = \begin{cases} 2n + 1 & \text{if } n \text{ is even} \\ n + 3 & \text{if } n \text{ is odd} \end{cases}$

3. (0 pts) Determine whether the following functions are one-to-one. If one-to-one, provide a proof; otherwise, give a counterexample.

   (a) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = 10x^{10} + 4x^4 - 2x^2 + 5$

   (b) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = -x^3 - x$

   (c) $f : \mathbb{Z} \to \mathbb{Z}$, $f(n) = \begin{cases} n + 2 & \text{if } n \text{ is even} \\ 2 - n & \text{if } n \text{ is odd} \end{cases}$