Question 1

(a) Shrek is six feet tall - proposition

(b) Dr. Cantu and his son - neither

(c) Give an example of an integrable function - neither

(d) $20^2 + 23^2 > 2023^2$ - proposition

(e) $x^2 = 1$ - predicate

(f) Potatoes are awesome - neither

(g) n is a perfect square - predicate

(h) The product of every two prime numbers is odd - proposition

Question 2

(a) P: false

(b) Q: true

(c) $P \vee Q$: true

(d) $P \wedge Q$: false

(e) $P \to Q$: true

(f) $Q \to P$: false

Question 3

For the predicate $P(x) : (x^2 - 9)(x - 1) = 0$ where $x \in \mathbb{R}_{\geq 1}$, x must be either 3 or 1 for $P(x)$ to be true.

Question 4

(a) $\sqrt{3}$ is irrational

(b) 0 is a negative number

(c) The real number r is greater than π

Question 5

(a) **Hypothesis:** a is irrational
Conclusion: $2a$ is irrational

(b) **Hypothesis:** a is an even integer
Conclusion: a^3 is an even integer

(c) **Hypothesis:** $\lim_{x\to 0^+} f(x) = 3$
Conclusion: $\lim_{x\to 0} f(x) = 3$

Question 6

(a)

P	Q	R	$P \vee Q$	$(P \vee Q) \wedge R$	$P \wedge R$	$Q \wedge R$	$(P \wedge R) \vee (Q \wedge R)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	F	T
T	F	F	T	F	F	F	F
F	T	T	T	T	F	T	T
F	T	F	T	F	F	F	F
F	F	T	F	F	F	F	F
F	F	F	F	F	F	F	F

(b)

P	Q	$P \to Q$	$\neg(P \to Q)$	$\neg Q$	$P \wedge (\neg Q)$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	F

Question 7

(a)
$P \wedge \neg P$ is a contradiction.

(b)

$$
\begin{aligned}
& P \to (Q \to P) \\
\equiv & P \to (\neg Q \vee P) \\
\equiv & \neg P \vee (\neg Q \vee P) \\
\equiv & (P \vee \neg P) \vee \neg Q \\
\equiv & T \vee \neg Q \\
\equiv & T
\end{aligned}
$$

This is a tautology.

Question 8

(a)
(b)