Exercise of Algorithms

Problem 1.

In Olympics 2020, there were athletes from n countries participating in m games. Let $C = \{c_1, c_2, \cdots, c_n\}$ denote the n countries, and let $H = \{h_1, h_2, \cdots, h_m\}$ denote the m games. For simplicity, assume that every country sent at most one athlete to each game, and that every game has at least three countries participating in it. For $i = 1, 2, \cdots, m$, let $A_i \subseteq C$ denote the set of countries who sent athletes to participated in the game h_i. Every game produced three medals: the gold medal, the silver medal, and the bronze medal.

For $j = 1, 2, \cdots, n$, let x_j denote the total number of medals won by country c_j after all games ended. Clearly, the vector $(x_1, x_2, \cdots, x_n)$ (which is basically the Medal List of Countries) needs to satisfy certain constraints and therefore cannot be arbitrary. For example, the summation of $x_1, x_2, \cdots, x_n$ has to be $3m$.

The question we consider is this: *Given any vector $X = (x_1, x_2, \cdots, x_n)$, is X a possible outcome of Olympics 2020?* Design an efficient algorithm to determine if the answer if "yes" or "no", prove its correctness, and analyze its time complexity.