CSCE 222 Discrete Structures for Computing – Fall 2023

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Homework 1 Solutions

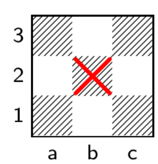
Total 100 points.

Problem 1. ($10 + 10 = 20$ points) Section 1.1, Exercise 1.3. For (b), give the knight's graph in a text format by giving all edges in the graph such that the knight's move from vertex v_i to vertex v_{i+1} is given as (v_i, v_{i+1}). Once you have all of the edges written, you can also give the path in the form of $v_i - v_{i+1} - v_{i+2} - \ldots$

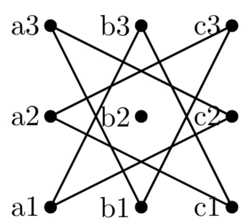
Use the common convention of expressing the columns and rows of a chessboard as a, b, and c, and 1, 2, and 3, respectively.

The graphical, visual representation of the graph is not required.

Solution. (a) The middle square b2 of the chess board cannot be reached from any other square by a knight's move. Therefore, it is impossible to have a knight's tour visiting every square of the board precisely once.



(b) The graph of the 3×3 chess board is given by



In a text format (which is a supposed/recommended way for the students to answer), the graph is given by $G = (V, E)$ where $V = \{$a1, a2, a3, b1, b2, b3, c1, c2, c3$\}$ and $E = \{\{$a1, b3$\}$, $\{$a1, c2$\}$, $\{$a2, c1$\}$, $\{$a2, c3$\}$, $\{$a3, b1$\}$, $\{$a3, c2$\}$, $\{$b1, c3$\}$, $\{$b3, c1$\}\}$. (There are total eight edges, connecting the eight vertices in a circle. The vertices in V and edges in E can be given in different orders since both V and E are sets.)

Notice that the vertex b2 is isolated, that is, there is no edge connecting b2 to any other vertices. Notice also that the remaining vertices are all connected in one cycle, namely a1 – c2 – a3 – b1 – c3 – a2 – c1 – b3 – a1.

This is essentially unique, as the only other choice would be to travel the same path in reverse. So if the square b2 would have been omitted from the chess board, then this would indeed be a Hamiltonian path on the 3×3 board with the middle square omitted.

Problem 2. (2 points $\times$ 5 subproblems $=$ 10 points) Section 2.1, Exercise 2.1

Solution. (a) Yes, this is a mathematical statement, which happens to be false.

(b) Yes, this is a true statement.

(c) Yes, this is a true statement, since the only solution to the equation are $2+\sqrt{2}$ and $2-\sqrt{2}$.

(d) Yes, this is a mathematical statement, but it is false, since the left-hand side also equals 23.

(e) No, this is not a mathematical statement, since x still needs to be specified.

Problem 3. (3 points $\times$ 5 subproblems $=$ 15 points) Section 2.1, Exercise 2.3

Solution. (a) This is true. Indeed, let us write $r = 0.1111\cdots$. Then $10r = 1.1111\cdots$. So $10r - r = 1$. In other words, $(10-1)r = 9r = 1$, so dividing both sides by 9 yields $r = 1/9$.

(b) This is false, since $0.121212\cdots = 4/33$. Similarly to the previous question, let $x = 0.121212\cdots$, then $100x = 12.121212\cdots$, and so $100x - x = 99x = 12$, yielding $x = 12/99 = 4/33$.

(c) This is false, as $11111111 = 1111 \cdot 10001$. Therefore, the greatest common divisor of 1111 and 11111111 is 1111.

(d) This is true. Indeed, $1 + (-1) = 0$. Multiplying an integer k by 0 yields $k \cdot 0 = 0$. Therefore, $0 = (-1)0 = (-1)(1 + (-1))$. By the distributive law, $0 = (-1)1 + (-1)(-1) = -1 + (-1)(-1)$. Adding 1 to both sides yields $1 = (-1)(-1)$.

(e) This is true. Seeking a contradiction, let us suppose that the set P of positive integers is finite. As P is finite, it must contain a maximal number m. Since $m+1$ is also an integer, it must be contained in P, contradicting the fact that m was maximal. Therefore, P must be infinite.

Problem 4. (2 points $\times$ 2 subproblems = 4 points) Section 2.2, Exercise 2.7 (a) and (b)

Solution. (a) Albert cooks pasta and Emmy is not happy.

(b) If Albert cooks pasta, then Albert and Emmy are happy.

Problem 5. (3 points $\times$ 2 subproblems = 6 points) Section 2.2, Exercise 2.8 (a) and (d)

Solution. (a) $C \to \neg S$

(d) $S \leftrightarrow \neg C$

Problem 6. (15 points) Section 2.2, Exercise 2.18. Use a truth table to show your reasoning.

Example LaTeX source for how to draw a truth table is shown in the truth-table.tex and truth-table.pdf files.

Solution. Let A be the statement "$\mathcal{A}$ is a knight" and B the statement "$\mathcal{B}$ is a knight". Then $\mathcal{A}$ asserts $\neg A \vee B$. Here the idea is that if A is true (i.e., $\mathcal{A}$ is a knight), then what $\mathcal{A}$ says is true, and if A is false (i.e., $\mathcal{A}$ is not a knight), then what $\mathcal{A}$ says must be false. Thus, $A \leftrightarrow (\neg A \vee B)$ is true.

A	B	$\neg A \vee B$	$A \leftrightarrow (\neg A \vee B)$
F	F	T	F
F	T	T	F
T	F	F	F
T	T	T	T

Therefore, $A \leftrightarrow (\neg A \vee B)$ is true if and only if both A and B are true. Therefore, $\mathcal{A}$ and $\mathcal{B}$ must both be knights.

Problem 7. (10 points) Section 2.3, Exercise 2.25. Use a truth table.

Solution. Consider the truth table of both expressions.

A	B	$A \to B$	$B \to A$	$(A \to B) \land (B \to A)$	$A \leftrightarrow B$
F	F	T	T	T	T
F	T	T	F	F	F
T	F	F	T	F	F
T	T	T	T	T	T

It is apparent that $(A \to B) \land (B \to A)$ takes on the same truth values as $A \leftrightarrow B$ for all assignments of truth values, so the two expressions are logically equivalent.

Problem 8. (20 points) Section 2.3, Exercise 2.26. Your answer should consist of a series of logical equivalences you learned in the text, and the final step must resolve to T. Do not use a truth table. Study the proofs of Proposition 2.8 (b) and (c) for the expected style of your answer. Watching the video "Problem Solving Exercise 1" in Module 2.1 will also be helpful.

Example LaTeX source for how to align the steps nicely is shown in the truth-table.tex and truth-table.pdf files.

Solution.

$$
\begin{aligned}
(A \wedge (A \to B)) \to B &\equiv (A \wedge (\neg A \vee B)) \to B \quad \text{by previous result: } p \to q \equiv \neg p \vee q \\
&\equiv ((A \wedge \neg A) \vee (A \wedge B)) \to B \quad \text{by distributive law: } \wedge \text{ over } \vee \\
&\equiv (F \vee (A \wedge B)) \to B \quad \text{since } A \wedge \neg A \equiv F \\
&\equiv (A \wedge B) \to B \quad \text{since } F \vee (A \wedge B) \equiv (A \wedge B) \text{ (identity law)} \\
&\equiv \neg(A \wedge B) \vee B \quad \text{by previous result: } p \to q \equiv \neg p \vee q \\
&\equiv \neg A \vee \neg B \vee B \quad \text{by de Morgan's law} \\
&\equiv \neg A \vee (\neg B \vee B) \quad \text{by associative law of } \vee \\
&\equiv \neg A \vee T \quad \text{since } \neg B \vee B = T \\
&\equiv T \quad \text{since } T \text{ dominates in } \vee
\end{aligned}
$$