MATH 152 Lab 6

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In [1]:
```
from sympy import *
import matplotlib.pyplot as plt
from numpy import cumsum
```

Question 1

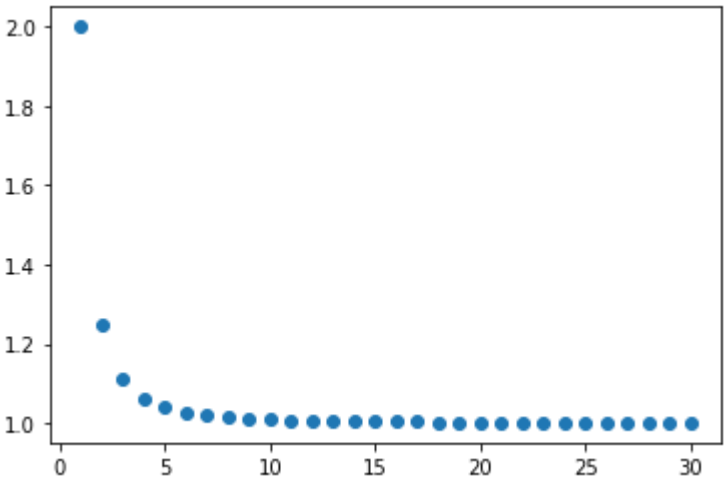
1a

In [2]:
```
n = symbols("n")
an = 1 + 1/n**2
print(f"The first ten terms of the sequence are: {[an.subs(n, i) for i in range(1, 11)
```

```
The first ten terms of the sequence are: [2, 5/4, 10/9, 17/16, 26/25, 37/36, 50/49, 6
5/64, 82/81, 101/100]
```

1b

In [3]:
```
nvals = [an.subs(n, i) for i in range(1, 31)]
plt.figure()
plt.plot(range(1, 31), nvals, "o")
plt.show()
```



1c

In [4]:
```
print(f"The limit of the sequence as n approaches infinity is {limit(an, n, oo)}")
```

```
The limit of the sequence as n approaches infinity is 1
```

Question 2

2a

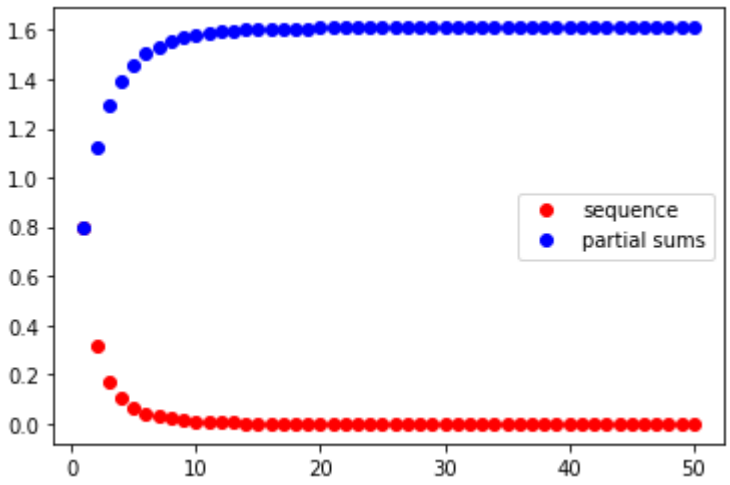
In [5]:
```
bn = (4/5)**n / n
assert limit(bn, n, oo) == 0
print(f"The limit of the sequence as n approaches infinity is: {limit(bn, n, oo)}")
```

```
The limit of the sequence as n approaches infinity is: 0
```

2b

In [6]:
```
b1_50 = [bn.subs(n, i) for i in range(1, 51)]
s1_50 = cumsum(b1_50)

plt.figure()
plt.plot(range(1, 51), b1_50, "ro", label="sequence")
plt.plot(range(1, 51), s1_50, "bo", label="partial sums")
plt.legend()
plt.show()
```



2c

In [7]:
```
print(f"The sum of the series is: {summation(bn, (n, 1, oo))}")
```

```
The sum of the series is: 1.60943791243410
```

Question 3

3a

In [8]:
```
an = 3 / (n**2 + 4*n)
print(f"The limit of the sequence as n approaches infinity is: {limit(an, n, oo)}")
```

```
The limit of the sequence as n approaches infinity is: 0
```

3b

In [9]:
```
print(f"First 12 terms: {[an.subs(n, i) for i in range(1, 13)]}")
print(f"First 12 partial sums: {cumsum([an.subs(n, i) for i in range(1, 13)])}")
```

```
First 12 terms: [3/5, 1/4, 1/7, 3/32, 1/15, 1/20, 3/77, 1/32, 1/39, 3/140, 1/55, 1/6
4]
First 12 partial sums: [3/5 17/20 139/140 1217/1120 775/672 4043/3360 6559/5280 1681/
1320
 7431/5720 10575/8008 4873/3640 39439/29120]
```

3c

In [10]:
```
print(f"The partial fraction decomposition of the series is: {apart(an)}")
```

```
The partial fraction decomposition of the series is: -3/(4*(n + 4)) + 3/(4*n)
```

3d

In [11]:
```
sn = 3/4 * ( 1 + 1/2 + 1/3 + 1/4 - 1/(n+1) - 1/(n+2) - 1/(n+3) - 1/(n+4) )
print(f"The sum of the series is: {limit(sn, n, oo)}")
```

```
The sum of the series is: 25/16
```

Question 4

4a

In [12]:
```
a = [1]
for n in range(1, 11):
    a.append(1/(3 * a[n-1]**2) + (2*a[n-1])/3)

print(f"The first ten terms of the sequence are: {a}")
print("The limit of the sequence as n approaches infinity is 1")
```

```
The first ten terms of the sequence are: [1, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0,
1.0, 1.0]
The limit of the sequence as n approaches infinity is 1
```

4b

In [13]:
```
a = [100]
for n in range(1, 11):
    a.append(1/(3 * a[n-1]**2) + (2*a[n-1])/3)

print(f"The first ten terms of the sequence with a0 = 100 are: {a}")
```

```
The first ten terms of the sequence with a0 = 100 are: [100, 66.6667, 44.444541666591
67, 29.629863193656167, 19.753621810618192, 13.169935457645675, 8.781878786203064, 5.
858908047611485, 3.915649283118309, 2.6321734377751667, 1.8028938852987562]
```

4c

In [14]:
```
print("Regardless of the starting term, the sequence always converges to 1 when n appr
```

```
Regardless of the starting term, the sequence always converges to 1 when n approaches
infinity.
```