

MATH300 Homework 4 (Due Friday, 2/16)

1. (12 pts) Let $n \in \mathbb{Z}$. Prove that if $n^2 - 3n + 5$ is even, then n is odd.

2. (12 pts) Let $n \in \mathbb{Z}$. Prove that n is odd if and only if $n + 2$ is odd.

3. (14 pts) Let $m, n \in \mathbb{Z}$. Prove that mn is even if and only if m is even or n is even.

4. (16 pts) Prove that if a and b are odd integers, then $4 \nmid (a^2 + b^2)$.

5. (10 pts) Prove that there do not exist integers m and n such that $8m + 26n = 1$.

6. (20 pts) In this problem, you may use the following Lemma (which is proven in Chapter 6):

Lemma 1. An integer n is not divisible by 3 if and only if there exists an integer k such that $n = 3k + 1$ or $n = 3k + 2$.

Prove that for all integers n , if $3 \mid n^2$, then $3 \mid n$.

7. (16 pts) Prove that there do not exist integers m and n such that $m^2 = 4n + 3$. (Hint: use Corollary 2.2.5 of the notes.)