MATH300 Homework 7 (Due Friday, 3/22)

1. (24 pts) Let A, B, and C be sets.

   (a) Prove or disprove: if $A \subseteq B \cup C$, then $A \subseteq B$ or $A \subseteq C$.

   (b) Prove or disprove: if $A \subseteq B \cap C$, then $A \subseteq B$ and $A \subseteq C$.

2. (12 pts) Let A and B be subsets of a universal set $\mathcal{U}$. Prove $(A \cap B) \cup (A - B) = A$ using set algebra (similar to "Proof 2" of Example 4.2.14 of our notes).

3. (9 pts) Let A be a set. Prove that $A - A = \emptyset$.

4. (5 pts) Let $R = \{2, 8, J, Q, A\}$ and $S = \{\heartsuit, \diamondsuit\}$. Find $R \times S$.

5. (12 pts) Let A, B, and C be sets. Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

6. (6 pts) Find the following cardinalities.

   (a) $|\{2, 4, 6, \ldots, 20\} \times \{a, b, c, d, e, f\}|$.

   (b) $|\mathcal{P}(\mathcal{P}(A))|$, where $|A| = 3$.

7. (12 pts) Let $A = \{1, \{2, \{3\}\}\}$.

   (a) What are the elements of A?

   (b) Find $\mathcal{P}(A)$.

   (c) True / False. $A \subseteq \mathcal{P}(A)$

8. (12 pts) Let A and B be sets. Prove that if $A \subseteq B$, then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

9. (8 pts) For $i \in \mathbb{Z}^+$, let $A_i = [i-4, i]$. Find the following.

   (a) $\bigcup_{i=4}^{7} A_i$

   (b) $\bigcap_{i=4}^{7} A_i$