LAB 5: ROTATIONAL MOTION

Kevin Lei, Kate Oliver, Zach Witte
Texas A&M University
College Station, TX 77843, US.

Abstract. *This experiment was performed to first confirm the principle of conservation of momentum, and then use the principle of conservation of momentum to calculate the moments of inertia of two spinning objects. The experiment was conducted by spinning three objects on an air table and then placing a mass on the spinning objects. Colored tracking points were placed on the centers of the spinning objects, a rotating portion of each spinning object, and the mass that was placed on the spinning objects. The velocity data of the tracking points were then processed and used to confirm the principle of conservation of angular momentum. Finally, the velocity data were used to determine the moments of inertia of two of the spinning objects.*

Keywords: *angular momentum, polar coordinates, propagation of error*

1. Introduction

The first portion of this experiment was to confirm the principle of conservation of momentum. The principle of conservation of angular momentum states that the total angular momentum of a closed system remains constant over time as long as there are no external torques applied. This principle is important and will be used in the latter portion of this experiment. Angular momentum is found using the following equation:

$$L = I\omega \qquad \textbf{\textit{Equation 1}}$$

Where L is the angular momentum of an object, I is the moment of inertia of an object, and ω is the angular velocity of an object. The second portion of this experiment was to calculate the moment of inertia of two objects using the velocity data, the moment of inertia of the dropped object, and the mass of the dropped object. The moments of inertia were calculated with the following formula:

$$I = \frac{(I_m + md^2)\omega_f}{\omega_i - \omega_f} \qquad \textbf{\textit{Equation 2}}$$

Where I_m is the moment of inertia of the dropped mass, m is the mass of the dropped mass, d is the distance between the dropped mass and the center of rotation, ω_f is the final angular velocity of the system, and ω_i is the final angular velocity of the system. There are uncertainties in the quantities I_m, d, ω_i, and ω_f, which means that the error in the final calculated moments of inertia will have a propagated uncertainty. The propagated uncertainty is found using partial derivatives. If $Q = f(x, \ldots, z)$, the propagated uncertainty in Q is found using the following formula:

$$\delta Q = \sqrt{\left(\frac{\partial Q}{\partial x}\delta x\right)^2 + \ldots + \left(\frac{\partial Q}{\partial z}\delta z\right)^2} \qquad \textbf{\textit{Equation 3}}$$

2. Experimental Procedure

For all three of the spinning objects, the center of mass was found and the objects were placed around a screw on the table such that the objects could spin freely and consistently. The objects had one tracking dot on the center of mass as well as another tracking dot an arbitrary distance away from the center of mass. The mass that would be eventually placed on the spinning objects also had a tracking dot on it. To run the experiment, a laptop was connected to the lab computer using the secure shell protocol. The lab computer contained a folder of Python scripts used to track colored dots visible on the table using the connected camera. One of these scripts was run for each of the spinning objects, which generated three .CSV files containing time and cartesian position data for all tracking points. To start the experiment, the fan under the table was activated to minimize friction. This is done because friction is a nonconservative force and will negatively affect the conservation of momentum. Each object was spun to an initial angular velocity, and the tracking script was executed. A mass was gently placed on the spinning object for each trial, and the script was terminated after a few seconds to end data collection. The results were then calculated using methods described in the results and analysis section.

3. Results and Analysis

The .CSV files were loaded into a Python script and processed using the Pandas and NumPy libraries. First, the cartesian positions were shifted such that the center of the spinning object was the origin, which was done by subtracting the positions of each point by the position of the center of mass. Once the positions were recentered to the center of mass, the cartesian position data was then converted into polar coordinates. The radius of each point was found by using the distance formula between the x and y coordinates, and the angle was found by taking the arctangent of the y coordinate divided by the x coordinate. This process was repeated for all three .CSV files.

The first objective of this experiment was to confirm the conservation of angular momentum. This was done by generating an angular momentum vs time graph for the spinning L-shaped object where the moment of inertia was already known. The graph plots angular momentum for the L-shaped object, the dropped mass, and the entire system. To find angular momentum, angular velocity first needs to be calculated. This is done by taking a finite difference between the angular position and time. This was done for both the spinning point and the dropped mass. Once angular velocity was calculated, the angular momentum of the object and mass was calculated using ***Equation 1***. Once angular momentum was calculated for the duration of the experiment, it was plotted against time to show the conservation of angular momentum as shown in **Figure 1**.

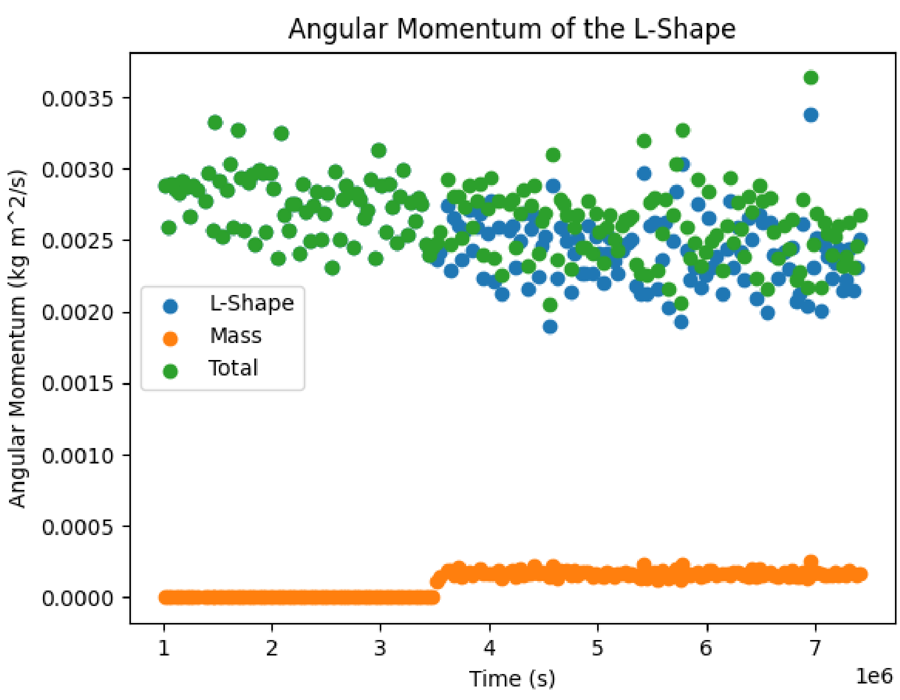

Figure 1: Angular momentum of the L-Shaped object

The graph shows that the angular momentum of the shape decreases when the mass is dropped, and the angular momentum of the mass remains approximately constant after it is dropped. The total angular momentum of the system remains approximately constant throughout the experiment, which means that the momentum of the system is conserved. It could be argued that there is a slight decrease in the angular momentum throughout the graph, which is entirely possible due to air resistance and friction.

The second objective of this experiment was to calculate the moments of inertia for the circular shape and the pentagon shape. Since it was proved earlier that the angular momentum of the system is conserved after the mass is dropped, the conservation of angular momentum can be combined with the parallel axis theorem to find the moments of inertia of the spinning objects. Before ***Equation 2*** can be used to calculate the moments of inertia for the objects, the distance between the mass and the center of rotation, average initial angular velocity, and average final angular velocity need to be calculated for the objects. The moment of inertia of the dropped mass and the mass of the dropped object are known. The distance between the dropped mass and the center of rotation was calculated as the average radial coordinate of the tracking point of the mass. The initial angular velocity of the system was calculated as the average angular velocity of the spinning tracking dot on the shape before the mass was dropped, and the final angular velocity of the system was calculated as the average angular velocity of the spinning tracking dot on the shape after the mass was dropped. After these quantities were determined, ***Equation 2*** was used to calculate the moments of inertia for the circular-shaped object and the pentagonal object. Values and uncertainties are shown in **Table 1**.

	I (kg m^2)	**Uncertainty**
Circle	0.00061	0.00005
Pentagon	0.00146	0.00022

Table 1: Moments of inertia and propagated uncertainty

Since there are multiple uncertainties involved with the calculation of the moment of inertia, these uncertainties must be propagated to find the final uncertainty of the moment of inertia. The uncertainty of the moment of inertia of the dropped mass is given, and the uncertainty in the mass of the dropped object is assumed to be zero. The uncertainties of the distance between the dropped mass and the center of rotation and the initial and final angular velocities are found as the standard error of all the points since the quantities themselves are averages. Once these uncertainties are found using standard error, ***Equation 3*** is used to propagate these uncertainties to find the uncertainty of the moments of inertia. **Table 1** also includes the propagated uncertainties.

4. Conclusions

The purpose of this lab was to confirm the principle of conservation of angular momentum and use the conservation of angular momentum to find the moments of inertia of two rotating objects. Conservation of angular momentum was confirmed by calculating the angular momentum of a spinning object whose moment of inertia is already known. Doing this required finding the angular velocity of the object, which was calculated by converting the cartesian position data into polar coordinates and then taking the finite difference of the angle of the moving tracking point with respect to time. Plotting the angular momentum of the spinning object and the dropped mass confirmed that the total angular momentum of the system remained constant after dropping the mass. Then, using the conservation of angular momentum, the moments of inertia of the two unknown objects were calculated after first finding the angular velocity of the system before and after dropping the mass. Uncertainties in the initial and final angular velocity, the moment of inertia of the dropped mass, and the radial distance of the mass were propagated using partial derivatives. Overall, this lab successfully demonstrated the conservation of angular momentum and its practical application in calculating the moments of inertia for two rotating objects.