LAB 6: HARMONIC MOTION

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Abstract *This lab was performed to explore the principles of simple harmonic motion in a practical application. The goal of this experiment was to find the spring constant of three different springs that were used in an elastic pendulum. The experiment was conducted by hanging masses on each of the three springs then pulling each of the elastic pendulums past their equilibrium positions and letting them oscillate. A colored tracking point was placed on each of the masses such that a computer vision tracking system could record the position and time data for the mass. The computer generated data was then used to find the period of oscillation for each pendulum, which was used to find the spring constants.*

Keywords: *spring constant, pendulum, oscillation*

1. Introduction

The purpose of this lab was to calculate the spring constants for three different springs used in elastic pendulums. The angular frequency of a spring and mass system is related to the square root of the spring constant divided by the mass of the weight. This can be used in conjunction with the definition of angular frequency, which has the units of oscillations per second, to find the spring constant of each mass spring system if the mass of the weight and the period of oscillation are known. The spring constant is found using the following equation:

$$k = \frac{4\pi^2 m}{T^2}$$ ***Equation 1***

Where *m* is the mass of the hanging weight and *T* is the period of oscillation of the mass spring system. The uncertainty of the hanging mass is given, but the uncertainty of the period of oscillation must be calculated. This is done by taking the standard error of every period of oscillation in the data, found by the following equation:

$$SE = \frac{\sigma}{\sqrt{n}}$$ ***Equation 2***

Where σ is the standard deviation of the quantity and *n* is the number of samples taken. Since there are uncertainties in the quantities for m and T, these uncertainties must be propagated into the calculation for the spring constant. This propagated uncertainty is found using partial derivatives. If $Q = f(x, \dots, z)$, the propagated uncertainty in Q is found using the following formula:

$$\delta Q = \sqrt{\left(\frac{\partial Q}{\partial x}\delta x\right)^2 + \dots + \left(\frac{\partial Q}{\partial z}\delta z\right)^2}$$ ***Equation 3***

Where variables *x* to *z* are quantities in the original formula that have uncertainties related to them.

2. Experimental Procedure

For this experiment, data collection was done by individuals outside of the group. Data was collected by using the computer tracking system in the visualization studio. The camera was set up such that its line of sight was parallel to the ground, which allowed it to capture the height of the mass spring systems. The top of three springs colored red, white, and green were screwed to a fixed point. Then, for each spring, a hanging mass was attached to the bottom end of the spring. For each trial, a tracking sticker was then placed on the mass on the ends of the spring such that the computer can track the position of the mass. A python script was then executed on the lab computer to start tracking the mass, and the mass was stretched slightly past its equilibrium position so that it could start oscillating. The mass spring system was allowed to oscillate for around twenty seconds. After enough data was collected, the python script was terminated and the next spring was attached for data collection. This was repeated until there was oscillation data for all three springs. The recorded data was generated in comma-separated values files (.CSV) which included time, horizontal position, and vertical position data for the tracking sticker, which was attached to the end of the hanging mass. The results were then calculated using methods described in the results and analysis section.

3. Results and Analysis

The .CSV files were read using a python script and processed using the Pandas, NumPy, SciPy, and Uncertainties libraries. Since the mass spring systems were oscillating up and down, the horizontal position data included in the .CSV files were disregarded. Calculating the spring constant for each spring requires the period of oscillation for each mass spring system. To calculate the period of oscillation for each mass spring system, the vertical position data was first isolated from the first peak of oscillation to the last peak of oscillation. After the extraneous data from the beginning and end of data collection was discarded, the *find_peaks* function from the SciPy library was used to find the timestamp of the local maxima for the vertical position of each mass spring system. Then, the difference between each of these timestamps were calculated and averaged to find the average period of oscillation for each mass spring system. The standard error of each period of oscillation was calculated using the standard error formula (***Equation 2***).

Once the periods of oscillation were calculated for each spring, they were then used to calculate the spring constant for each mass spring system. The hanging mass on the green and red springs had masses of 0.4 kilograms and the hanging mass on the white spring had a mass of 0.3 kilograms. Each hanging mass had an uncertainty of 0.003 kilograms. The spring constants were calculated by using ***Equation 1***. **Table 1** shows the calculated spring constants as well as their respective uncertainties.

Table 1: Spring constant calculations with uncertainty

	k (N/m)	δk *(N/m)*
Green	4.49×10^{-5}	0.09×10^{-5}
Red	2.09×10^{-5}	0.02×10^{-5}
White	3.27×10^{-5}	0.04×10^{-5}

The associated uncertainties with the mass of the hanging weights and the periods of oscillation were propagated into the spring constant calculations by using partial derivatives, shown in ***Equation 3***. This was accomplished in Python by casting the mass and period quantities into *ufloat* objects from the *uncertainties* library. This made it so that the mass and period had their respective uncertainties associated with them, and any calculations involving them would return a new *ufloat* object with the new quantity as well as its propagated uncertainty.

In addition to calculating the spring constants, height versus time graphs were produced for each mass spring system to show the harmonic nature of the motion for the systems. **Figure 1**, **Figure 2**, and **Figure 3** show the height of the hanging mass plotted against the time elapsed after extraneous data was removed.

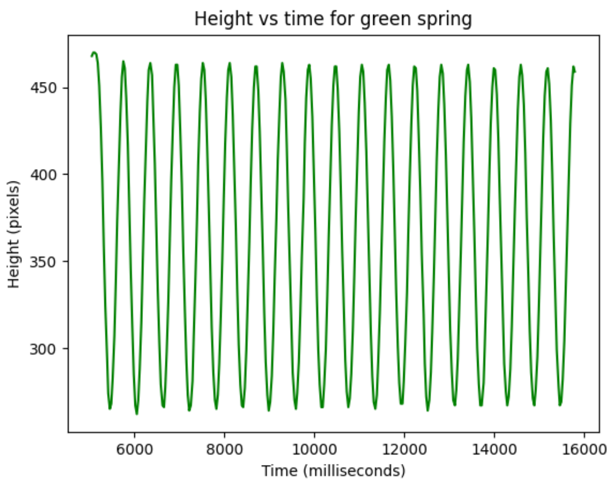

Figure 1: Height vs Time graph for the green spring

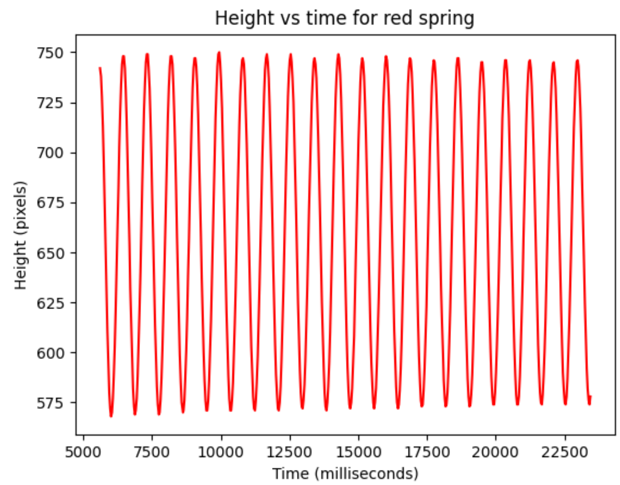

Figure 2: Height vs time graph for the red spring

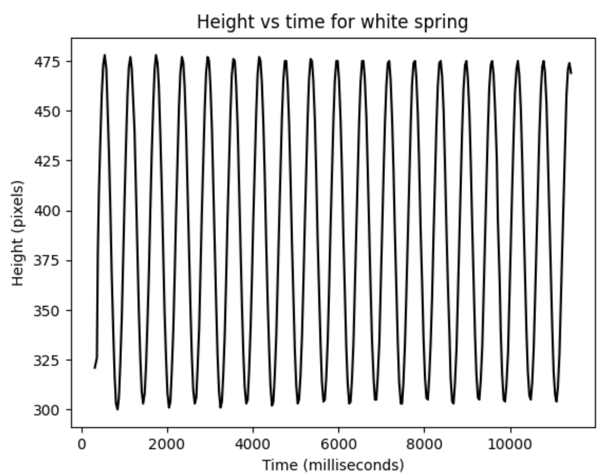

Figure 3: Height vs Time graph for the white spring

4. Conclusions

In conclusion, this experiment successfully calculated the spring constants of three mass spring systems by using the principles of simple harmonic motion. By utilizing a computer vision tracking system to record the position and time data of the oscillating mass-spring systems, precise and accurate measurements were obtained. This data was then processed and analyzed using various Python libraries to determine the periods of oscillation, which were subsequently used to calculate the spring constants. The results demonstrate the effectiveness of the experimental procedure and the data analysis approach in determining the spring constants and their respective uncertainties. Additionally, the height versus time graphs provide visual confirmation of the harmonic nature of the motion in the mass-spring systems. Overall, this experiment provided valuable insights into simple harmonic motion and emphasized the importance of precise and efficient data collection in enhancing our understanding of these systems.