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ENGR 216-445

Problem 1

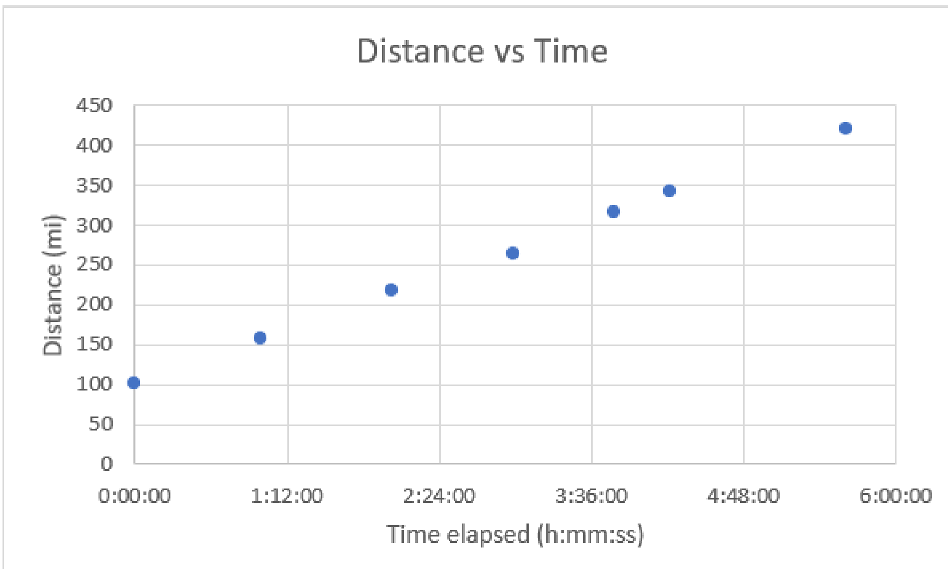
Given:

A table of time and odometer readings.

Find:

Time intervals, distance, and speed for each time reading.

Diagram:



Theory:

$$\frac{dx}{dt} = \frac{x_2 - x_1}{t_2 - t_1}$$

Assumptions:

The car works for the entirety of the experiment.

Solution:

first time interval: $\left(\frac{59}{60} + \frac{12}{3600}\right) - 0 = 0.9867\ hours$

first distance 1: $157.8 - 102.0 = 55.80\ miles$

first average speed 1: $\frac{55.8}{0.9867} = 56.55\ miles\ per\ hour$

clock time (hr:min:sec)	0:00:00	0:59:12	2:01:46	2:58:55	3:47:01	4:13:00	5:36:17
odometer reading (mi)	102	157.8	217.6	264.1	315.2	341.7	420.3
time interval (hr)	0.04111	0.04345	0.03969	0.0334	0.01804	0.05784	[redacted]
distance (mi)	55.8	59.8	46.5	51.1	26.5	78.6	[redacted]
average speed (mph)	56.55	57.35	48.82	63.74	61.19	56.63	[redacted]

Problem 2

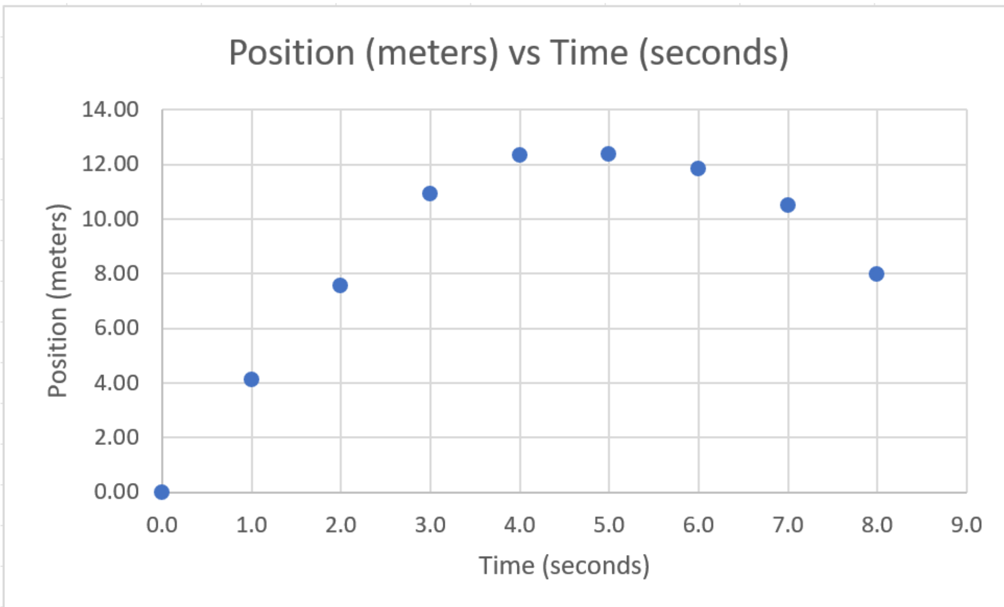
Given:

Position and time data of a moving ball

Find:

The velocity and acceleration of the ball at t=2, 3, 4, 5 and 6 seconds

Diagram:



Theory:

Forward finite difference: $f'(x) = \frac{f(x-\Delta x) - f(x)}{\Delta x}$

Backward finite difference: $f'(x) = \frac{f(x) - f(x-\Delta x)}{\Delta x}$

Center finite difference: $f'(x) = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$

Assumptions:

There is a ball and the ball is moving

Solution:

time t, seconds	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0
position x, meters	0.00	4.10	7.53	10.92	12.31	12.35	11.83	10.49	7.95
velocity vf, m/s (forward finite difference)	4.10	3.43	3.39	1.39	0.04	-0.52	-1.34	-2.54	
velocity vb, m/s (backward finite difference)		4.1	3.43	3.39	1.39	0.04	-0.52	-1.34	-2.54
velocity vc, m/s (center finite difference)		3.765	3.41	2.39	0.715	-0.24	-0.93	-1.94	
acceleration af, m/s^2 (forward finite difference)	-0.67	-0.04	-2	-1.35	-0.56	-0.82	-1.2		
acceleration ab, m/s^2 (backward finite difference)			-0.67	-0.04	-2	-1.35	-0.56	-0.82	-1.2
acceleration ac, m/s^2 (center finite difference)			-0.6875	-1.3475	-1.315	-0.8225	-0.85		

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2) At $t = 2$ seconds:

Velocity

forward: $\frac{10.92-7.53}{3-2} = 3.39$ m/s

backward: $\frac{7.53-4.10}{2-1} = 3.43$ m/s

center: $\frac{10.92-4.10}{3-1} = 3.41$ m/s

Acceleration

forward: $\frac{1.39-3.39}{3-2} = -2.00$ m/s^2

backward: $\frac{3.43-4.1}{2-1} = -0.67$ m/s^2

center: $\frac{2.39-3.765}{3-1} = -0.688$ m/s^2

Problem 3

Given:

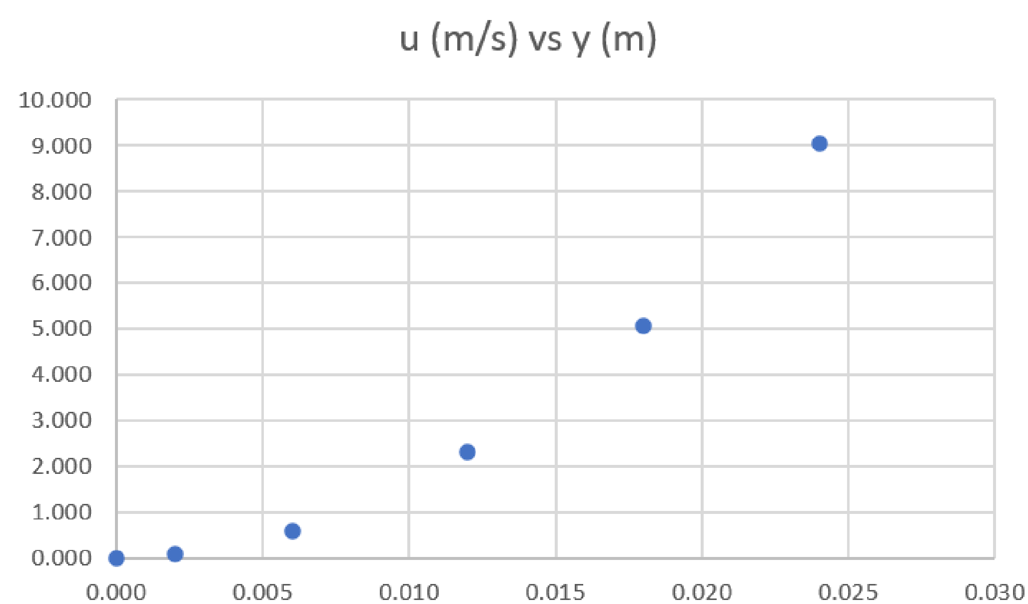
A table of distance and air velocity measurements.

$\mu = 1.8 * 10^{-5}\ Ns/m^2$

Find:

The shear stress of a surface at distances y = 0.006, 0.012, and 0.018 meters.

Diagram:



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Theory:

Newton's viscosity law: $\tau = \mu \frac{du}{dy}$

Second order center first finite difference: $f'(x) = \frac{f(x+\Delta x)-f(x-\Delta x)}{2\Delta x}$

Assumptions:

Dynamic velocity μ is constant.

Solution:

y (m)	0.000	0.00200	0.00600	0.0120	0.0180	0.0240
u (m/s)	0.000	0.067	0.572	2.291	5.047	9.041
τ (N/m^2)	[redacted]		0.004	0.00671	0.01013	[redacted]

Shear stress τ at y=0.006 m: $\tau = \frac{2.291-0.067}{0.0120-0.002} * 1.8 * 10^{-5} = 0.004$

Higher order finite differences find higher order derivatives of a function, so the change would be the slope of the function.

Problem 4

Given:

The function $f(x) = \frac{\ln(x)\sinh(x)}{e^x}$

$f'(x) = 0.336925$

Find:

1. Numeric estimates of the derivative of the function f(x) using forward, backward, and centered finite differences and step size $\Delta x = 0.25$
2. The percent error between the estimated values and the true value
3. The value of Δx needed for the forward and backward finite differences to reach the same percent error as the centered finite difference using the original Δx

Diagram:

N/A

Theory:

Forward finite difference: $f'(x) = \frac{f(x-\Delta x) - f(x)}{\Delta x}$

Backward finite difference: $f'(x) = \frac{f(x)-f(x-\Delta x)}{\Delta x}$

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Center finite difference: $f'(x) = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$

Percent error: $\varepsilon = \left|\frac{true\ value - estimated\ value}{true\ value}\right| * 100\%$

Assumptions:

The true value of the derivative at x=1.5 is accurate

Solution:

4)

error:

forward: $\frac{0.27135 - 0.19263}{1.75 - 1.5} = 0.31488$ $\quad \left|\frac{0.336925 - 0.31488}{0.336925}\right| \cdot 100\% = 6.54\%$

backward: $\frac{0.19263 - 0.10241}{1.5 - 1.25} = 0.36088$ $\quad \left|\frac{0.336925 - 0.36088}{0.336925}\right| \cdot 100\% = 7.11\%$

center: $\frac{0.27135 - 0.10241}{1.75 - 1.25} = 0.33788$ $\quad \left|\frac{0.336925 - 0.33788}{0.336925}\right| \cdot 100\% = 0.28\%$

$\Delta x = 0.0104$ $\quad \frac{f(1.5) - f(1.5 - 0.0104)}{0.0104} = 0.3375$

$\left|\frac{0.336925 - 0.3375}{0.336925}\right| \cdot 100\% = 0.17\% < 0.28\%$