

MATH300 Homework 2 (Due Friday, 2/2)

1. (14 pts) Consider the predicates $P(x) : x + 1 \geq 4$ and $Q(x) : 13 < 4x$, where $x \in \{0, 2, 3, 4, 6\}$.

(a) Determine the value(s) of x for which $P(x) \Leftrightarrow Q(x)$ is true by filling in the following truth table.

x	$P(x)$	$Q(x)$	$P(x) \Leftrightarrow Q(x)$
0			
2			
3			
4			
6			

(b) What is the truth value of $(\forall x \in \{0, 2, 3, 4, 6\})(P(x) \Leftrightarrow Q(x))$?

(c) What about $(\exists x \in \{0, 2, 3, 4, 6\})(P(x) \Leftrightarrow Q(x))$?

2. (12 pts) Write the following in words.

(a) $20 \in 4\mathbb{Z}$

(b) $3.14 \in \mathbb{Q} \wedge \pi \in \mathbb{R}$

(c) $(\exists n \in \mathbb{N})(\sqrt{n} \notin \mathbb{R})$

3. (8 pts) Negate the following statements as a **useful denial** (without using the word “not”).

(a) The inequality $n^2 - 4n + 3 < 0$ has an integer solution.

(b) Every rational number is less than 100.

4. (12 pts) Negate the following statements.

(a) All Aggies follow the Aggie Honor Code.

(b) There is a student who lives on campus and is not a math major.

(c) For every integer m , m^2 is odd or $m^3 - 1$ is divisible by 4.

5. (12 pts) Find the converse, contrapositive, and negation of

If f is a linear function, then f is continuous at 0.

6. (10 pts) Consider the quantified statement

$$(\forall x \in \mathbb{R})(\exists n \in \mathbb{Z})(n \leq x < n + 1).$$

(a) Express the statement in words.

(b) Express the negation of the statement in symbols.

(Side note: the order of quantifiers matters as $(\exists n \in \mathbb{Z})(\forall x \in \mathbb{R})(n \leq x < n + 1)$ has a completely different meaning from the original statement.)

7. (16 pts) Express the following statements in the form “*If ... , then ...*”, introducing variables as necessary. Then write their negations in words.

(a) Every multiple of 6 is even and is not a multiple of 4.

(b) The square of an even integer is divisible by 4.

8. (16 pts) Prove that if n is an odd integer, then $n^2 + 1$ is even.