Sections 5.3-5.5 Review

INSTRUCTIONS:

1. You will upload completed pages 2-4 of this assignment as a single PDF in Canvas under "Sections 5.3-5.5 Review Part A". To upload your pdf:
   (a) Click Start Assignment
   (b) Make sure you are on the File Upload tab
   (c) Click Upload File and choose your pdf file
   (d) Once your file is uploaded, click the Submit Assignment button

2. You will upload a video (problem 2b) in Canvas under "Sections 5.3-5.5 Review Part B". To upload your video:
   (a) Click Start Assignment
   (b) Click Record / Upload Media Assignment
   (c) Click the Upload Media tab and Select Video File
   (d) Attach your video file and wait for the file to upload
   (e) Once your file is uploaded, click the Submit Assignment button

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MATH151
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Sections 5.3-5.5 Review

1. Find $g'(x)$ if $g(x) = \int_{\sin(2x)}^{1} (t^3 + t)^8 \, dt$. $= -\int_{1}^{\sin(2x)} (t^3+t)^8 \, dt$

$g'(x) = -\left((\sin(2x))^3 + \sin(2x)\right)^8 \cdot 2\cos(2x)$

2. (a) Evaluate $\int_1^4 \frac{32 + \sqrt{x^5}}{x^3} \, dx$.

$= \int_1^4 32 \cdot x^{-3} + x^{\frac{5}{2}} \cdot x^{-3} \, dx = \int_1^4 32x^{-3} + x^{\frac{-1}{2}} \, dx$

$= 32 \cdot \frac{-1}{2} x^{-2} + 2x^{\frac{1}{2}} \Big]_1^4 = \frac{-16}{x^2} + 2\sqrt{x} \Big]_1^4$

$= \left[\left(\frac{-16}{4^2} + 2\sqrt{4}\right) - \left(\frac{-16}{1^2} + 2\sqrt{1}\right)\right] = \left[\left(-1+4\right) - \left(-16+2\right)\right] = \left[3 - (-14)\right]$

$= \boxed{17}$

(b) Record a short video explaining the steps in your solution from part (a). Note you do not need to appear in the video. Some options include:

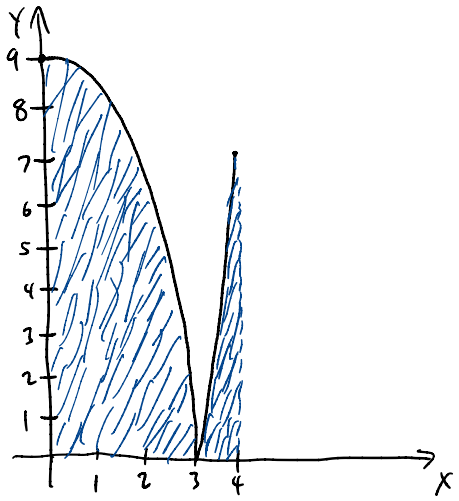
i. Use screen record (with audio) on your device with your solution displayed on-screen and explain audibly.

ii. Use your phone camera to record your solution (written or on-screen) and explain audibly.

iii. Record yourself working the problem live (e.g., on device, paper, whiteboard) while explaining audibly.

3. Let $f(t) = |t^2 - 9|$.

(a) Create a sketch of the area under f from $t = 0$ to $t = 4$.



(b) Calculate the exact area from part (a).

$$\int_0^4 f(t)\,dt = \int_0^4 |t^2-9|\,dt = -\int_0^3 (t^2-9)\,dt + \int_3^4 (t^2-9)\,dt$$

$$= -\left[\tfrac{1}{3}t^3 - 9t\right]_0^3 + \left[\tfrac{1}{3}t^3 - 9t\right]_3^4 = -\left[\left(\tfrac{1}{3}\cdot 3^3 - 9\cdot 3\right) - \left(\tfrac{1}{3}\cdot 0^3 - 9\cdot 0\right)\right] + \left[\left(\tfrac{1}{3}\cdot 4^3 - 9\cdot 4\right) - \left(\tfrac{1}{3}\cdot 3^3 - 9\cdot 3\right)\right]$$

$$= -\left[(9-27) - (0)\right] + \left[\left(\tfrac{64}{3} - 36\right) - (9-27)\right] = 18 + \tfrac{10}{3}$$

$$= \boxed{\tfrac{64}{3}}$$

(c) If $v(t) = t^2 - 9$ is the velocity of an object at time t, what does the value from part (b) represent? **Explain your answer in two or more sentences.**

The value from part B would represent the object's displacement from time $t=0$ to $t=4$ if the velocity function were $v(t) = |t^2-9|$, however, since the function is missing the absolute value bars, the value from part B is not the displacement for $v(t)$.

4. Find $\int \frac{e^{2x}}{3e^{2x}-1}\,dx$.

$u = 3e^{2x} - 1 \quad du = 6e^{2x}\,dx \quad dx = \frac{1}{6e^{2x}}\,du$

$$\int \frac{e^{2x}}{3e^{2x}-1}\,dx = \frac{1}{6}\int \frac{1}{u}\,du = \frac{1}{6}\ln|u|\,du = \boxed{\frac{\ln|3e^{2x}-1|}{6} + C}$$

5. Evaluate $\int_0^2 x(1-x)^6\,dx$.

$u = 1 - x \quad x = 1 - u$

$du = -dx \quad dx = -du$

$$\int_0^2 x(1-x)^6\,dx = -\int_1^{-1}(1-u)u^6\,du = \int_{-1}^{1} u^6 - u^7\,du$$

$$= \left[\frac{u^7}{7} - \frac{u^8}{8}\right]_{-1}^{1} = \left[\left(\frac{1^7}{7} - \frac{1^8}{8}\right) - \left(\frac{(-1)^7}{7} - \frac{(-1)^8}{8}\right)\right] = \left[\frac{1}{56} - \left(\frac{-15}{56}\right)\right]$$

$$= \frac{16}{56} = \boxed{\frac{2}{7}}$$