CSCE 222 Discrete Structures for Computing – Fall 2023

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Homework 2 Solutions

Total 100 points.

Problem 1. ($5+5=10$ points) Section 2.6, Exercise 2.53 (a) and (c). Explain.

Solution. (a) There are infinitely many triples (a, b, c) of nonnegative integers that make the predicate true. For instance, $C(m, 0, m)$ is true for all positive integers m, as $m^3 + 0^3 = m^3$.

(c) Any triple (a, b, c) satisfying the predicate $S(a, b, c)$ is a Pythagorean triple. There are just two Pythagorean triples (a, b, c) with $a, b, c \in \{1, 2, 3, 4, 5\}$, namely $(3, 4, 5)$ and $(4, 3, 5)$ satisfy $3^2 + 4^2 = 5^2$ and $4^2 + 3^2 = 5^2$, respectively.

Problem 2. ($5+5=10$ points) Section 2.6, Exercise 2.54 (b) and (c)

Solution. (b) For each real number x there exists a larger real number y.

(c) For any pair of real numbers x and z such that x is strictly smaller than z, there exists a real number y that lies strictly between x and z.

Problem 3. $(5 + 5 = 10$ points) Section 2.7, Exercise 2.58 (a) and (e)

Solution. (a) $\exists x \forall y\, (P(x) \wedge \neg Q(y))$

(e) $\forall x \forall y\, (P(x) \vee Q(y))$

Problem 4. $(5 + 5 = 10$ points) Section 2.7, Exercise 2.59 (d) and (e)

Solution.

(d) For all integers a there exists an integer b such that $a + b \neq 1001$.

(e) There exists a positive integer a such that for all positive integers b, $b \geq a$.

Problem 5. (15 points) Section 2.9, Exercise 2.73 [Hint: Use the property of "consecutive integers" and the definition of an "odd integer".]

Solution. The definition of an odd integer is as follows: An integer i is odd if and only if there exists an integer k such that $i = 2k + 1$.

If m and n are consecutive integers and, say, $m < n$, then $n = m + 1$, so their sum $m + n = m + (m + 1) = 2m + 1$, which means that $m + n$ is an odd integer.

Problem 6. (15 points) Section 2.9, Exercise 2.80

Solution. We prove the contrapositive:

$$\text{If } \neg(m > 40 \vee n > 60) \text{ then } \neg(m + n > 100).$$

Suppose that $\neg(m > 40 \vee n > 60)$ holds. By de Morgan's law, it follows that $m \leq 40$ **and** $n \leq 60$. Adding these two inequalities yields $m + n \leq 100$, and this is exactly $\neg(m + n > 100)$. Therefore, we have proved the contrapositive statement, which implies the claim.

Problem 7. (15 points) Section 2.9, Exercise 2.84

Solution. Seeking a contradiction, let us suppose that there exist integers m_0 and n_0 such that $42m_0 + 70n_0 = 1000$. Since $42 = 7 \times 6$ and $70 = 7 \times 10$, we have $7(6m_0 + 10n_0) = 1000$. This implies that 7 is a prime factor of 1000, contradicting the fact that 2 and 5 are the only prime factors of 1000, since $1000 = 2^3 \times 5^3$. Therefore, we can conclude that $42m + 70n = 1000$ does not have an integer solution.

Problem 8. (15 points) Section 3.3, Exercise 3.20 [Hint: Use the definitions of $\subseteq$, $\cup$, and the power set.]

Solution. By the definition of the power set, $P(A)$ is a set that contains all subsets of A as elements, and $P(B)$ a set that contains all subsets of B as elements.

Suppose that $S \in P(A) \cup P(B)$. Therefore, $S \in P(A)$ or $S \in P(B)$, by the definition of $\cup$. It follows that $S \subseteq A$ or $S \subseteq B$ by the definition of the power set. Therefore, we can deduce that $S \subseteq A \cup B$ by the definition of $\cup$. In other words, we have $S \in P(A \cup B)$ by the definition of the power set.

We can conclude that $P(A) \cup P(B) \subseteq P(A \cup B)$ by the definition of $\subseteq$.