Problem 1

Given:

Mass M = 2.50 +/- 0.020 kg

Radius R = 0.180 +/- 0.0030 m

Angular velocity ω = 17.5 +/- 0.250 rad/s

Find:

Angular momentum of a uniform disk with the given measurements

Diagram:

N/A

Theory:

Angular momentum L = $\frac{1}{2}MR^2\omega$

Assumptions:

The three original uncertainties are uncorrelated and random.

Solution:

1) $L=\frac{1}{2}MR^2\omega$ $\quad M = 2.50 \pm 0.020$ kg, $R = 0.180 \pm 0.0030$ m, $\omega = 17.5 \pm 0.250$ rad/s

$$L=\frac{1}{2}(2.50)(0.180)^2(17.5)$$
$$= 0.70875$$
$$= 0.709 \text{ kg}\cdot\text{m}/\text{s}^2$$

$$\frac{\delta L}{|L|}=\sqrt{\left(\frac{\delta M}{|M|}\right)^2+\left(\frac{\delta w}{|w|}\right)^2+\left(\frac{\delta R}{|R|}\right)^2+\left(\frac{\delta R}{|R|}\right)^2}$$

$$\delta L=\sqrt{\left(\frac{\delta M}{|M|}\right)^2+\left(\frac{\delta w}{|w|}\right)^2+\left(\frac{\delta R}{|R|}\right)^2+\left(\frac{\delta R}{|R|}\right)^2}\;|L|$$

$$=\sqrt{\left(\frac{0.020}{2.50}\right)^2+\left(\frac{0.250}{17.5}\right)^2+\left(\frac{0.0030}{0.180}\right)^2+\left(\frac{0.0030}{0.180}\right)^2}\;\left|\frac{1}{2}(2.50)(0.180)^2(17.5)\right|$$

$$= 0.020 \text{ kg}\cdot\text{m}/\text{s}^2$$

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$L = 0.709 \pm 0.020 \frac{kg * m}{s^2}$

Problem 2

Given:

Length L = 0.75 +/- 0.011 m

Find:

The period of the pendulum T

Diagram:

N/A

Theory:

$T = 2\pi\sqrt{\frac{L}{g}}$

$\delta T = \frac{1}{2}L^{-\frac{1}{2}} * \ \delta L * |T|$

Assumptions:

None

Solution:

2) $L = 0.75 \pm 0.011$ m $\quad T = 2\pi\sqrt{\frac{L}{g}}$

$T = 2\pi\sqrt{\frac{0.75}{9.8}} = 1.74$ s

$\delta T = \frac{1}{2}L^{-\frac{1}{2}}\delta L \ |T|$

$= \frac{1}{2\sqrt{0.75}}(0.011)(1.74)$

$= 0.011$ s

The predicted value of T is 1.74 ± 0.011 seconds. A measured value of 1.75 ± 0.010 seconds is consistent with the theoretical prediction because it is only very slightly different and within the margin of error.

Problem 3

Given:

$v_1 = 3.54 \pm 0.10$ m/s

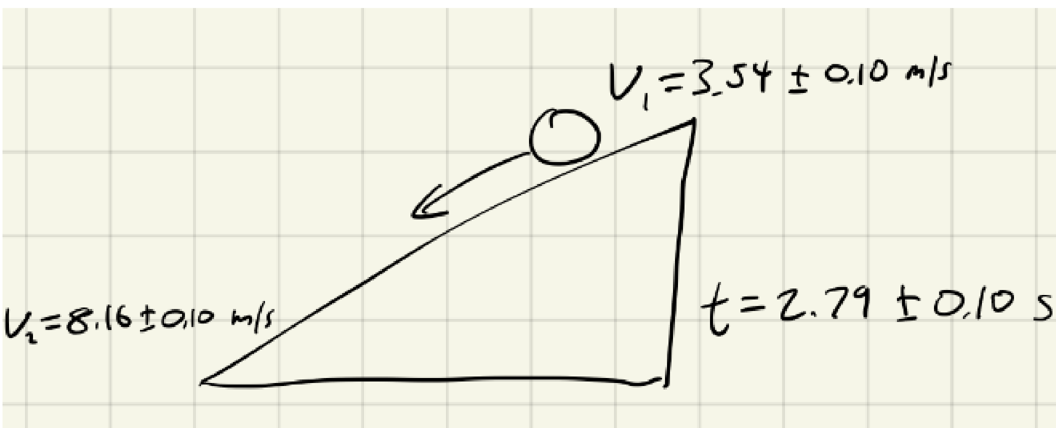
$v_2 = 8.16 \pm 0.10$ m/s

$t = 2.79 \pm 0.10$ s

Find:

The acceleration and uncertainty measurement of a ball rolling down a ramp with the given velocity and time data.

Diagram:



Theory:

$a = \frac{v_2 - v_1}{t}$

Assumptions:

All uncertainties are independent and random.

Solution:

$$a = \frac{8.16 - 3.54}{2.79} = 1.66 \text{ m/s}^2$$

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$$\Delta V = V_2 - V_1$$

$$\delta \Delta V = \sqrt{\delta V_2^2 + \delta V_1^2}$$

$$\delta a = \sqrt{\left(\frac{\delta \Delta V}{\Delta V}\right)^2 + \left(\frac{\delta t}{t}\right)^2}\ |a|$$

$$= \sqrt{\left(\frac{\sqrt{0.10^2 + 0.10^2}}{8.16 - 3.54}\right)^2 + \left(\frac{0.10}{2.79}\right)^2}\ \left|\frac{8.16 - 3.54}{2.79}\right|$$

$$= 0.078\ m/s^2$$

The acceleration is $a = 1.66 \pm 0.078 \frac{m}{s^2}$

The other calculated acceleration of $1.85 \pm 0.10 \frac{m}{s^2}$ does not agree with the prediction because it is not within the margin of error.

Problem 4

Given:

The textbook has 437 pages.

Thickness is 1.24 $\pm$ 0.0050 inches.

Find:

The thickness of 1 page including its uncertainty

Diagram:

N/A

Theory:

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$$T_p = \frac{T_B}{\Sigma P} \quad \delta T_p = \sqrt{\left(\frac{\delta T_B}{T_B}\right)^2} \, |T_p|$$

Assumptions:

All pages are equally thick.

Solution:

4)

$$T_p = \frac{T_B}{\Sigma P} = \frac{1.24}{437} = 2.84 \times 10^{-3} \text{ inches}$$

$$\delta T_p = \sqrt{\left(\frac{\delta T_B}{T_B}\right)^2} \, |T_p| = \sqrt{\left(\frac{0.0050}{1.24}\right)^2} \, \left|\frac{1.24}{437}\right| = 1.14 \times 10^{-5} \text{ inches}$$

$$5.0 \times 10^{-6} = \frac{0.0050}{437x} \Rightarrow x = 2.3 \quad \text{3 whole books}$$

Problem 5

Given:

$m_1 = 102 \pm 1.0$ grams

$m_2 = 86 \pm 0.9$ grams

Find:

- An equation for the uncertainty in the expected acceleration in terms of m1, m2, and their uncertainties.
- The expected acceleration and the propagated uncertainty

Diagram:

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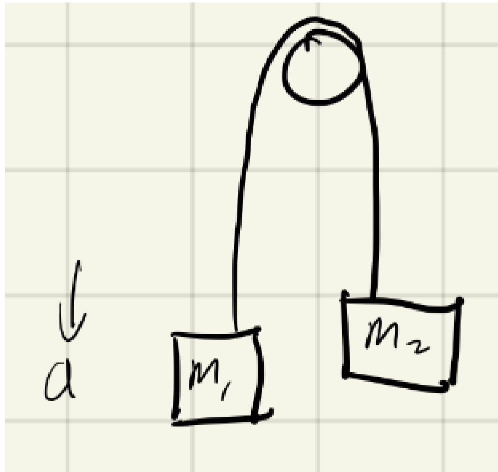

Theory:

$$\boldsymbol{\delta Q} = \sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2}$$ **for Q = ab**

Assumptions:

The pully is frictionless.

Solution:

$m_1 = 102 \pm 1.0$ grams

$m_2 = 86 \pm 0.90$ grams

$$a = g\frac{m_1 - m_2}{m_1 + m_2} = g\frac{1 - \frac{m_2}{m_1}}{1 + \frac{m_2}{m_1}} = g\left(\frac{1}{1+\frac{m_2}{m_1}} - \frac{\frac{m_2}{m_1}}{1+\frac{m_2}{m_1}}\right) = g\left(\overbrace{\frac{1}{1+\frac{m_2}{m_1}}}^{t_1} - \overbrace{\frac{1}{1+\frac{m_1}{m_2}}}^{t_2}\right)$$

$$\delta\frac{m_2}{m_1} = \sqrt{\left(\frac{\delta m_2}{|m_2|}\right)^2 + \left(\frac{\delta m_1}{|m_1|}\right)^2}\left|\frac{m_2}{m_1}\right|$$

$$\delta a = \sqrt{\left(\frac{\sqrt{\left(\frac{\delta m_2}{m_2}\right)^2 + \left(\frac{\delta m_1}{m_1}\right)^2}\left(\frac{m_2}{m_1}\right)}{\left|1+\frac{m_2}{m_1}\right|^2}\right)^2 + \left(\frac{\sqrt{\left(\frac{\delta m_2}{m_2}\right)^2 + \left(\frac{\delta m_1}{m_1}\right)^2}\left(\frac{m_1}{m_2}\right)}{\left|1+\frac{m_1}{m_2}\right|^2}\right)^2}$$

$$a = 9.8\left(\frac{102-86}{102+86}\right) = 0.83\ m/s^2$$

$$\delta a = 0.01\ m/s^2$$