4th Homework — MATH 304 — Fall 2023
— Due October 12th —

1. Determine whether the following subsets are subspaces:

   a. $S_1 := \{(x_1, x_2)^T \in \mathbb{R}^2 : x_1 = \sqrt{123}x_2\}$

   b. $S_2 := \{(x_1, x_2)^T \in \mathbb{R}^2 : x_1 x_2 = 1\}$

   c. $S_3 := \{\text{the set of singular } 2 \times 2 \text{ matrices}\}$

   d. Let A be a fixed (but arbitrary) 2×2 matrix.
   $$S_4 := \{B \in \mathbb{R}^{2\times 2} : BA = 0\}$$

   e. $S_5 := \{\text{the set of all polynomials ofdegree 2 or 4}\}$

   f. $S_6 := \{\text{the set of upper triangular } 2 \times 2 \text{ matrices}\}$

   g. $S_7 := \{p \in P_4 : p(0) = 0\}$. Here
   $$P_4 := \{\text{the set of all polynomials ofdegree} \leq 4\}$$

2. Find the null space of the following matrices:
$$A = \begin{pmatrix} 2 & 1 & 0 \\ 4 & -1 & 1 \end{pmatrix}, \ B = \begin{pmatrix} -1 & -2 & 2 & 1 \\ 2 & 4 & -4 & -2 \end{pmatrix} \ C = \begin{pmatrix} 0 & 1 & 4 \\ 1 & 0 & 3 \\ 4 & 3 & 0 \end{pmatrix}$$

3. Show that the following matrices is a spanning set for $\mathbb{R}^{2\times 2}$.
$$A_{11} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, A_{12} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, A_{21} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, A_{22} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$
   Show also that these matrices are linearly indepedent.

4. Let x_1, x_2, x_3 be linearly independent vectors in $\mathbb{R}^n$. Let
$$y_1 = x_1 + x_2, \ y_2 = x_2 + x_3, \ y_3 = x_3 + x_1.$$
   Decide if y_1, y_2, y_3 are linearly independent or not.

Show your work in each exercise.