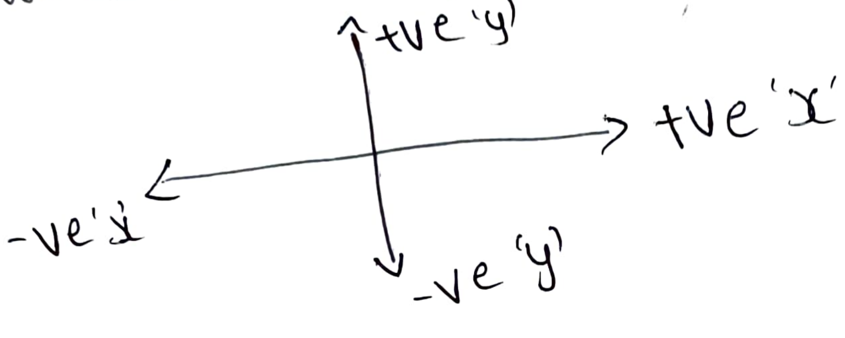
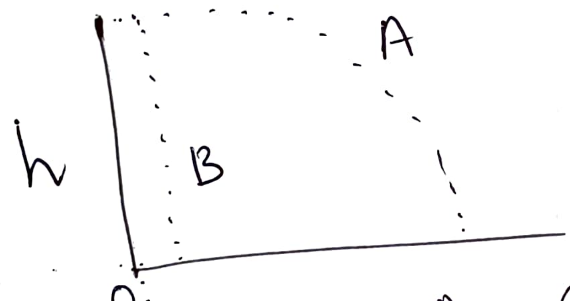
RECITATIONS

CHAPTER-3

NOTE:- Unless otherwise specified, these are the sign conventions I am going to use.



1)



A - $V_{0x} = V_0$, $V_{0y} = 0$, $a_x = 0$ $a_y = -g$

$0 = h + 0 - \frac{1}{2} g t_A^2$ $\quad [\because y = y_0 + V_{0y} t + \frac{1}{2} a_y t^2]$

$\Rightarrow t_A = \sqrt{\frac{2h}{g}}$

B - $V_{0x} = 0$, $V_{0y} = -V_0$, $a_x = 0$, $a_y = -g$

$0 = h - V_0 t_B - \frac{1}{2} g t_B^2$

Solve to get $t_B = \frac{-V_0 \pm \sqrt{V_0^2 + 2gh}}{g}$

$\Rightarrow t_B = \frac{-V_0 - \sqrt{V_0^2 + 2gh}}{g}$ [Other root is -ve; negative time makes no sense]

$$h = \frac{1}{2} g t_A^2$$

$$h = V_0 t_B + \frac{1}{2} g t_B^2$$

$$\Rightarrow \frac{1}{2} g t_A^2 = V_0 t_B + \frac{1}{2} g t_B^2$$

$$\frac{1}{2} g (t_A^2 - t_B^2) = V_0 t_B$$

$$\frac{1}{2} g (t_A - t_B)(t_A + t_B) = V_0 t_B$$

$$\boxed{t_A - t_B = \frac{2 V_0 t_B}{g(t_A + t_B)}}$$

$\downarrow$

V_0, t_B, g & $t_A + t_B$ are all positive numbers

$\Rightarrow t_A - t_B > 0$ or $t_A > t_B$

(a) Balloon B hits the ground first

(b) $t_A - t_B = \sqrt{\frac{2h}{g}} - \left[\frac{-V_0 + \sqrt{V_0^2 + 2gh}}{g}\right]$

$$\boxed{t_A - t_B = \sqrt{\frac{2h}{g}} + \frac{V_0 - \sqrt{V_0^2 + 2gh}}{g}}$$

(c) $\cancel{V_A - V_B =}$ $\cancel{V_x = V_{0x} + a_x t = , V_{0x} = V_x}$

$\cancel{V_y = V_{0y} + a_y t}$

(c) A - $V_x^2 = V_{0x}^2 + 2a_x(x - x_0)$

$V_x^2 = V_{0x}^2 = V_0^2$ $[a_x = 0]$

$V_y^2 = V_{0y}^2 + 2a_y(y - y_0)$

$= 0 + 2(-g)(0 - h)$

$V_y^2 = 2gh$

$(V_{final})_A = \sqrt{V_x^2 + V_y^2} = \sqrt{V_0^2 + 2gh}$

B - $V_x^2 = 0$

$V_y^2 = V_{0y}^2 + 2a_y(y - y_0)$

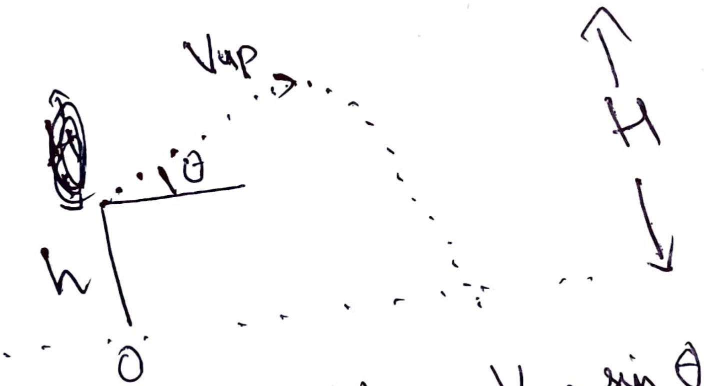
$= V_0^2 + 2(-g)(0 - h)$

$V_y^2 = V_0^2 + 2gh$

$(V_{final})_B = \sqrt{V_x^2 + V_y^2} = \sqrt{V_0^2 + 2gh}$

(c) $(V_{final})_A = (V_{final})_B = \sqrt{V_0^2 + 2gh}$

2)



$V_{0x} = V_{up}\cos\theta$, $V_{0y} = V_{up}\sin\theta$

$a_x = 0$, $a_y = -g$

If 't' is the time after which it hits the ground, then using $y = y_0 + V_{0y}t + \frac{1}{2}a_y t^2$.

$$0 = h + V_{up}\sin\theta\, t - \frac{1}{2}gt^2$$

[∵ displacement is h, it does not matter that it went up & came down]

Solve quadratic equation to get

$$t = \frac{V_{up}\sin\theta \pm \sqrt{V_{up}^2\sin^2\theta + 2gh}}{g}$$

Ignore the negative root because $V_{up}\sin\theta - \sqrt{V_{up}^2\sin^2\theta + 2gh} < 0$

(a) $\Rightarrow$ $\boxed{t = \frac{V_{up}\sin\theta + \sqrt{V_{up}^2\sin^2\theta + 2gh}}{g}}$

$y_{max} = h + H$

At its topmost point, $V_y = 0$

$$V_y^2 = V_{0y}^2 + 2a_y(y - y_0)$$

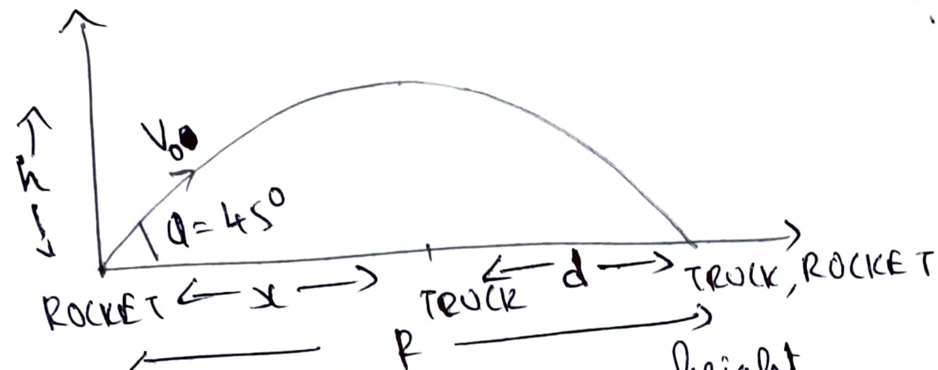
$$0 = V_{up}^2\sin^2\theta + 2(-g)(H - h)$$

(b) $\Rightarrow$ $2gH = V_{up}^2\sin^2\theta + 2gh$

$$\boxed{H = h + \frac{V_{up}^2\sin^2\theta}{2g}}$$

(c) $\boxed{x = V_{0x}t = V_{up}\cos\theta\, t}$ → Use t from problem (a)

3)



t - time taken to reach maximum height

~~R = V sin~~ $V_y = V_{oy} - gt$

$$t = \frac{V_{oy}}{g} = \frac{V_0 \sin 45}{g} \qquad [V_y = 0 \text{ at top}]$$

T - time of flight; in other words, time after which rocket drops onto the truck.

$$y = y_0 + V_{oy}T - \frac{1}{2}gT^2$$

$$\Rightarrow 0 = 0 + V_0 \sin 45\, T - \frac{1}{2}gT^2$$

$$T = \frac{2V_0 \sin 45}{g} = 2t$$

Now, $V_y^2 = V_{oy}^2 + 2(-g)(H)$

$$\Rightarrow 0 = V_0^2 \sin^2 45 - 2gH$$

$$V_0^2 = \frac{2gH}{\sin^2 45}$$

R - distance from launching point

$$R = 0 + V_{ox}T + 0$$

$$R = V_0 \cos 45 \cdot T = V_0 \cos 45 \cdot \frac{2V_0 \sin 45}{g}$$

$$R = \frac{2V_0^2 \sin 45 \cos 45}{g} = 2 \cdot \frac{2gH}{\sin^2 45} \cdot \frac{\cos 45 \cdot \sin 45}{g}$$

(a) Solve to get $\boxed{R = 4h}$

$$T = \frac{2V_0 \sin 45}{g} = 2 \cdot \frac{\sqrt{2gh}}{\sin 45} \frac{\sin 45}{g} = 2\sqrt{\frac{2h}{g}}$$

Since the truck has travelled 'T' seconds, the distance it has covered $d = V_T \cdot T = 2V_T\sqrt{\frac{2h}{g}}$

(b) $\Rightarrow \boxed{x = R - d = 4h - 2V_T\sqrt{\frac{2h}{g}}}$

At highest point, $\vec{V}_{\text{(rocket)}} = V_x\hat{i} + V_y\hat{j}$

$V_x = V_{0x}$ $= \cancel{V_0 \cos 45 \cdot \frac{V_0 \sin 45}{g}}$

$V_x = V_0 \cos 45$ $= \cancel{\frac{V_0^2 \cos 45 \sin 45}{g}}$

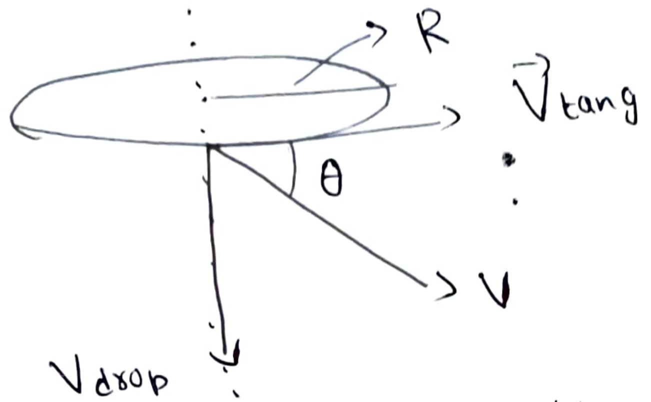
$= \frac{\sqrt{2gh} \cos 45}{\sin 45}$ $= \cancel{2gh}$

$V_x = \sqrt{2gh}$, $V_y = 0$, $a_x = 0$, $a_y = -g$

$\vec{V}(\text{truck}) = V_T\hat{i}$

$$\boxed{\begin{aligned} &\Rightarrow V_{R,T} = \vec{V}_R - \vec{V}_T = (\sqrt{2gh} - V_T)\hat{i} + 0\hat{j} \\ &\text{(c)}\ a_{R,T} = \vec{a}_R - \vec{a}_T = 0\hat{i} - g\hat{j} \end{aligned}}$$

4)



It has two velocities - $\vec{V}_{drop}$ & $\vec{V}_{tan}$ due to its rotation

$\cancel{\vec{V}_{drop}}\ \vec{V}_{tan} = \frac{d}{t} = \frac{2\pi R}{t_s}$

(a) ⟹ Total velocity $\boxed{|\vec{V}| = \sqrt{V_{tan}^2 + V_{drop}^2} = \sqrt{\left(\frac{2\pi R}{t_s}\right)^2 + V_{drop}^2}}$

(b) Centripetal acceleration $\vec{a} = \frac{v^2}{r}$. It is always directed towards centre of circle

$|\vec{a}_{rad}| = \cancel{\frac{|\vec{V}|^2}{R}}\ \frac{|V_{tan}|^2}{R} = \frac{(2\pi R)^2}{t_s^2 R}$

$$\boxed{a_{rad} = \frac{4\pi^2 R}{t_s^2}}$$

(c) From figure, $\tan\theta = \frac{V_{drop}}{V_{tan}}$

⟹ $\tan\theta = \frac{V_{drop}}{\frac{2\pi R}{t_s}}$

$$\boxed{\theta = \tan^{-1}\left(\frac{V_{drop}\, t_s}{2\pi R}\right)}$$

5)

Let the intersection be chosen as origin

A - $\vec{v} = v\hat{i}$, $\vec{a} = 0$

B - $v_0 = 0$, $\vec{a} = -2\hat{j}$

A - $x_A = v_0 t = 6v\hat{i}$

B - $\cancel{x_B = -\frac{1}{2}at^2\hat{j}}$ $x_B = 40\hat{j} - \frac{1}{2}at^2\hat{j}$

$x_B = (40 - 36)\hat{j} = 4\hat{j}$

$$\boxed{(a)\ \vec{x}_B - \vec{x}_A = 4\hat{j} - 6v\hat{i}}$$

A - $\vec{V}_A = v\hat{i}$

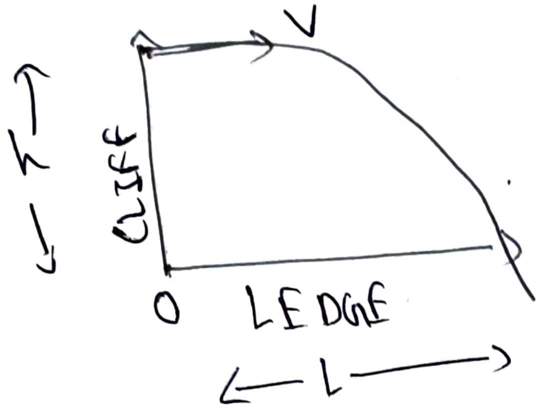
B - $\vec{V}_B = 0 - 2t\hat{j}$

$\vec{V}_B = -12\hat{j}$

$$(b)\ \boxed{\vec{V}_B - \vec{V}_A = -12\hat{j} - v\hat{i}}$$

$$(c)\ \boxed{\vec{a}_B - \vec{a}_A = -2\hat{j}}$$

6)



$V_{0x} = V$

$V_{0y} = 0$

$a_x = 0$

$a_y = -g$

$x = x_0 + V_{0x} t + \frac{1}{2} a_x t^2$

$L = Vt \Rightarrow t = \frac{L}{V}, \; V = \frac{L}{t}$

$y = y_0 + V_{0y} t + \frac{1}{2} a_y t^2$

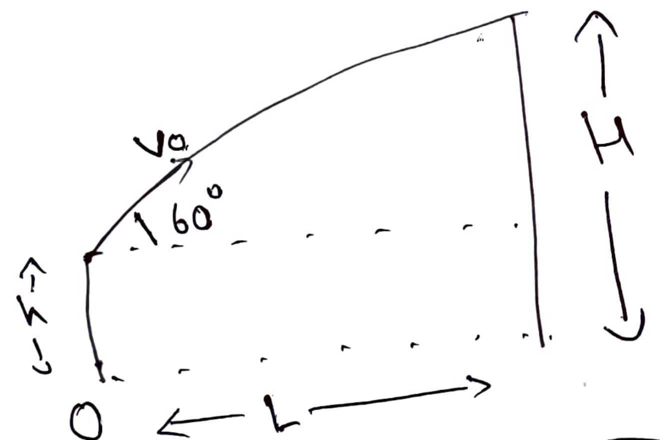
$0 = h + 0 - \frac{1}{2} g t^2$

$\Rightarrow t = \sqrt{\frac{2h}{g}}$

$V = \frac{L}{t} = \frac{L\sqrt{g}}{\sqrt{2h}}$

$$\boxed{V = \sqrt{\frac{gL^2}{2h}}}$$

7)



$V_{0x} = V\cos 60 = \frac{V_0}{2}, \; V_{0y} = V\sin 60 = \frac{\sqrt{3}V_0}{2}$

$a_x = 0, \; a_y = -g$

$L = 0 + V_{ox}t + 0$

$L = \frac{V_0 t}{2}$. $\Rightarrow t = \frac{2L}{V_0}$

$H = h + V_{oy}t - \frac{1}{2}gt^2 = h + \frac{\sqrt{3}}{2}V_0 t - \frac{1}{2}gt^2$

Use $t = \frac{2L}{V_0}$ in above equation

$H - h = \frac{\sqrt{3}}{2} V_0 \cdot \frac{2L}{V_0} - \frac{1}{2} g \frac{4L^2}{V_0^2}$

$H - h = \sqrt{3}L - \frac{2gL^2}{V_0^2}$

Solve to get

(a) $\boxed{V_0 = \sqrt{\frac{2gL^2}{\sqrt{3}L - (H-h)}}}$

$V_x = V_{ox} - a_x t = V_{ox} = V_0 \cos 60$

$V_x = \frac{V_0}{2} = \frac{1}{2}\sqrt{\frac{2gL^2}{\sqrt{3}L - (H-h)}}$

$V_y = V_{oy} + a_y t = V_0 \sin 60 - gt$

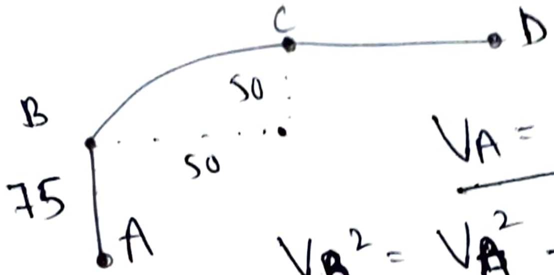
$= \frac{\sqrt{3}V_0}{2} - g \cdot \frac{2L}{V_0}$

Substitute value of V_0 to get

$V_y = \frac{\sqrt{3}}{2}\sqrt{\frac{2gL^2}{\sqrt{3}L - (H-h)}} - 2gL\sqrt{\frac{\sqrt{3}L - (H-h)}{2gL^2}}$

(b) $\boxed{V = \sqrt{V_x^2 + V_y^2}}$; Substitute V_x & V_y from above

9)



$\underline{V_A = 0}$, $\underline{V_D = 0}$

$V_B^2 = V_A^2 + 2 \cdot a \cdot 75$ $[a = 2]$

$0 \cancel{+ V_B^2}$ $V_B^2 = 2 \cdot 2 \cdot 75$

$V_B = \underline{10\sqrt{3}}$

Since its circular motion, $\underline{V_C = 10\sqrt{3}}$

Now, $V_D^2 = V_C^2 + 2a \cdot (x_D - x_C)$

$0 = 300 - 2(x_D - 50)$

$x_D = 200$

$\Rightarrow$ $\underline{A(0,0)}$, $\underline{B(0,75)}$, $\underline{C(50,125)}$ & $\underline{D(200,125)}$.

Now, $75 = 0 + 0 + \frac{1}{2} \cdot 2\, t_{AB}^2$

$\Rightarrow$ $\underline{t_{AB} = 5\sqrt{3}}$

Also, $t_{BC} = \frac{BC}{V_B} = \frac{2\pi R}{4 V_B}$ [$\because$ perimeter BC is $\frac{1}{4}$ th circumference]

$t_{BC} = \frac{2 \cdot \pi \cdot 50}{4 \cdot 10\sqrt{3}}$ $\Rightarrow$ $\underline{t_{BC} = \frac{5\pi\sqrt{3}}{6}}$

~~200 = 50 + Vct~~ $V_D = V_C + a t_{CD}$

$0 = 10\sqrt{3} - t_D$

$t_D = \underline{10\sqrt{3}}$

$$\vec{V}_{av} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j}$$

$\Delta x = 200$, $\Delta y = 125$, $\Delta t = 5\sqrt{3} + \frac{5}{6}\sqrt{3} + 10\sqrt{3}$

≈ 30.5

$\Rightarrow$ (a) $\vec{V}_{av} = \frac{200}{30.5}\hat{i} + \frac{125}{30.5}\hat{j}$

$$\boxed{\vec{V}_{av} = 6.55\hat{i} + 4.10\hat{j}}$$

$$\boxed{\vec{a}_{av} = \frac{\Delta v}{\Delta t} = \frac{V_D - V_A}{\Delta t} = 0}$$

(b) ~~A $x_c - x_A$~~ $\vec{r}_C - \vec{r}_A = (x_C - x_A)\hat{i} + (y_C - y_A)\hat{i}$

$$\boxed{\Delta\vec{r} = 50\hat{i} + 125\hat{j}}$$

(c) total distance from A to D = AB + BC + CD

~~A = (0,0), D =~~

$AB = 75$, $BC = \frac{2\pi R}{4} = 25\pi$, $CD = 150$

$d = 75 + 25\pi + 150$

$$\boxed{d = 225 + 25\pi}$$

8) SOLVED IN CLASS!!!