

MATH 152 Lab 8

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```
In [1]: from sympy import *
from sympy.plotting import (plot, plot_parametric)
import matplotlib.pyplot as plt
import numpy as np
```

Question 1

1a

```
In [2]: n, ub = symbols('n ub')
expr = (n**50 * 50**n) / factorial(n)
s = Sum(expr, (n, 1, ub))

terms = [N(expr.subs(n, i)) for i in range(1, 6)]
print(f"The first 5 terms are: {terms}")
print(f"The terms appear to be increasing.")
```

The first 5 terms are: [50.0000000000000, 1.40737488355328e+18, 1.49562080769136e+28, 3.30117343809435e+35, 2.31296463463574e+41]
The terms appear to be increasing.

1b

```
In [3]: an1 = ((n + 1)**50 * 50**(n + 1)) / factorial(n + 1)
an = (n**50 * 50**n) / factorial(n)
L = Limit(an1 / an, n, oo)
L = simplify(L)
print(f"The limit is: {L}")
print(f"The limit is 0, so the series converges.")
```

The limit is: 0
The limit is 0, so the series converges.

1c

```
In [4]: print("The answer to part (b) tells that the terms of the series decrease fast enough
```

The answer to part (b) tells that the terms of the series decrease fast enough that the series converges.

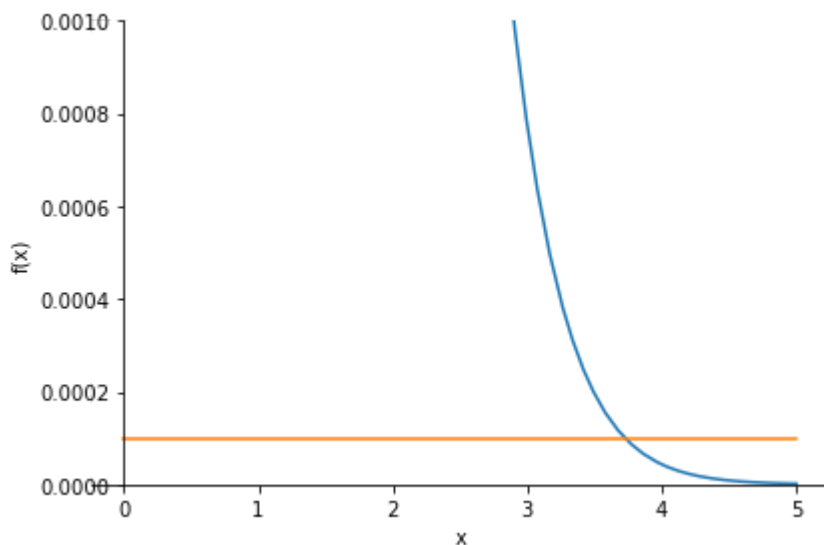
Question 2

2a

```
In [5]: an = (n**8) * E**(-5*n)

x = symbols('x')
```

```
plot((integrate(an, (n, x, oo))), 0.0001, (x, 0, 5), ylim = (0, 0.001))
print("It appears that at least four terms are needed to approximate the integral to 0.0001")
```



It appears that at least four terms are needed to approximate the integral to 0.0001 by looking at the graph.

2b

```
In [6]: terms = nsolve(integrate(an, (n, x, oo)) - 0.0001, 3.7)
print(f"The number of terms needed to approximate the series to 0.0001 is: {ceiling(terms)}")
```

The number of terms needed to approximate the series to 0.0001 is: 4

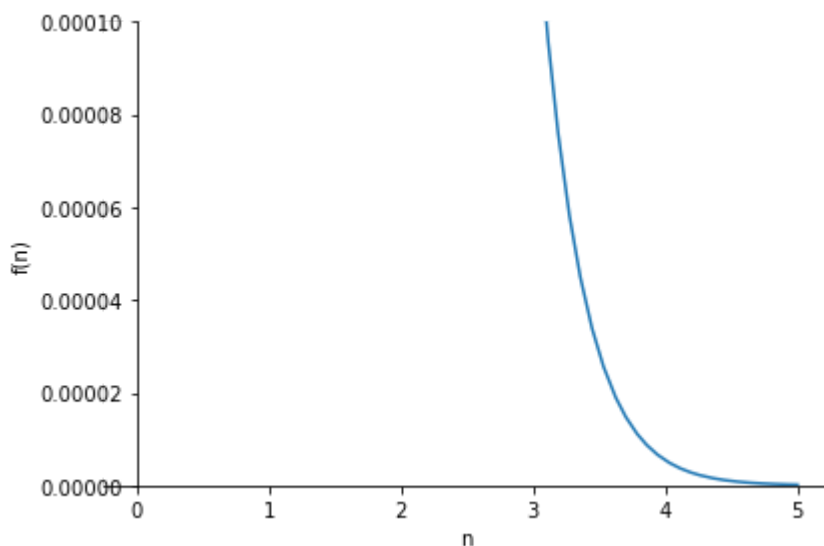
2c

```
In [7]: print(f"The sum of the series within 0.0001 using the integral is {N(integrate(an, (n, 0, 5)))}")
```

The sum of the series within 0.0001 using the integral is 0.0191950368579348

2d

```
In [8]: plot(an.subs(n, n+1), (n, 0, 5), ylim = (0, 0.0001))
print("It appears that at least three terms are needed to approximate the integral to 0.0001")
```



It appears that at least three terms are needed to approximate the integral to 0.0001 by looking at the graph.

2e

```
In [9]: terms = nsolve(an.subs(n, n+1) - 0.0001, 3)
print(f"The number of terms needed to approximate the series to 0.0001 is: {ceiling(terms)}
```

The number of terms needed to approximate the series to 0.0001 is: 4

2f

```
In [10]: print(f"The sum of the series within 0.0001 using the alternating series estimation theorem is: {sum(an.subs(n, n+1) for n in range(4))}")
```

The sum of the series within 0.0001 using the alternating series estimation theorem is 0.00301248965909831

Question 3

3a

```
In [11]: x, n = symbols('x n')
expr = factorial(n)**2 / factorial(2*n) * x**n
s = Sum(expr, (n, 0, oo))

print(f"The limit of the ratio test as n approaches infinity is: {limit(abs(simplify(expr.subs(n, n+1)/expr.subs(n, n)))}, oo) if it exists, otherwise None")
```

The limit of the ratio test as n approaches infinity is: Abs(x)/4

3b

```
In [12]: print(f"The radius of convergence is 4. The endpoints of the interval of convergence are -4 and 4.")
```

The radius of convergence is 4. The endpoints of the interval of convergence are -4 and 4.

3c

```
In [13]: try:
print(f"The sum of the series when x = 4 is: {N(s.subs(x, 4))}")
except ValueError:
print("The series diverges for x = 4.")
```

The series diverges for x = 4.

3d

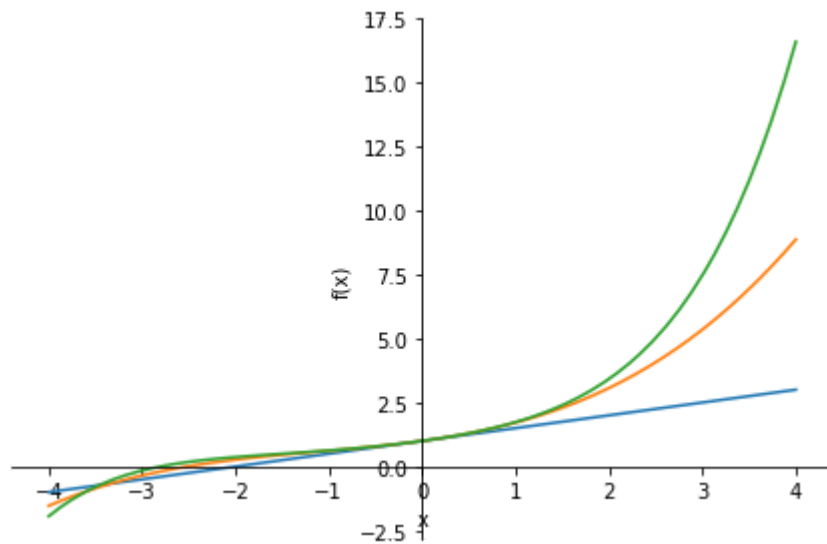
```
In [14]: nvals = [10, 100, 1000, 10000]
for i in nvals:
print(f"The value of |an| when n = {i} is: {N(abs(expr.subs(x, -4).subs(n, i)))}")

print(f"Based on the values of |an|, the series diverges for x = -4.")
```

The value of $|a_n|$ when $n = 10$ is: 5.67546385503042
The value of $|a_n|$ when $n = 100$ is: 17.7467079428307
The value of $|a_n|$ when $n = 1000$ is: 56.0569188406160
The value of $|a_n|$ when $n = 10000$ is: 177.247600671712
Based on the values of $|a_n|$, the series diverges for $x = -4$.

3e

```
In [15]: plot(Sum(expr, (n, 0, 1)), Sum(expr, (n, 0, 3)), Sum(expr, (n, 0, 5)), (x, -4, 4))
```



```
Out[15]: <sympy.plotting.plot.Plot at 0x26498d1f070>
```