ENGR/PHYS 216 Spring 20223
HW Assignment 3: Finite Differences

Note: Graphs make great diagrams for some of these problems

1. A team of Aggie engineers are testing a solar car over long distances. The table below lists the time and odometer readings of a test run. Fill out the blank parts of the table (time interval, distance traveled during that time interval, and average speed) using finite difference equations discussed in lecture 3. Please show your work for **one** time interval, **one** distance, and **one** average speed. You do not have to show your work for the remaining calculations. What is the overall average speed of the car (the average of the averages)?

Clock time (hr:min:sec)	0:00:00	0:59:12	2:01:46	2:58:55	3:47:01	4:13:00	5:36:17
Odometer reading (mi)	102.0	157.8	217.6	264.1	315.2	341.7	420.3
Time interval (hr)							
Distance (mi)							
Average speed (mph)							

Note: Feel free to use Excel to check your work. Just remember that when entering times in Excel, it may be easier to use a special format – you can select formats for a cell using the format drop-down menu (the General format is the default).

The Clock times above were done using a format found under the Custom list. After entering the Clock time values, try changing the format of the cells to General. You will see that Excel stores times (and dates) as numbers. (In fact, the number 1 is equal to the start of Excel's time/date clock – midnight, January 1, 1900.) The fact that dates and times are numbers means you can do math with them!

2. Use the following data to calculate the velocity and acceleration of a moving ball at t = 2, 3, 4, 5, and 6 seconds using
   a. forward finite difference
   b. backward finite difference
   c. centered finite difference

Time t, seconds	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0
Position x, meters	0.00	4.10	7.53	10.92	12.31	12.35	11.83	10.49	7.95

Please show your work for **one** velocity and **one** acceleration for **all three** finite difference methods (6 in total). You do not have to show your work for the remaining calculations. Please write your final answers in a table format so they are easy to read.

3. The velocity, u (m/s), of air flowing across a flat surface is measured at several distances y (m) away from the surface. (Think about a fan blowing air over a table and measuring the air velocity at several heights above the table.) Use Newton's viscosity law

$$\tau = \mu \frac{du}{dy}$$

and the **second order centered first** finite difference method to determine the shear stress, τ (N/m^2), at distances $y = 0.006, 0.012$, and 0.018 m. Assume a constant value for the dynamic viscosity $\mu = 1.8 \times 10^{-5} Ns/m^2$. How do you think your answers would change if you use higher order finite differences?

y **(m)**	0.000	0.00200	0.00600	0.0120	0.0180	0.0240
u **(m/s)**	0.000	0.067	0.572	2.291	5.047	9.041

Please show your work for **one** distance. You do not have to show your work for the remaining calculations.

4. Consider the equation

$$f(x) = \frac{\ln(x)\sinh(x)}{e^x}$$

The true value of its derivative at $x = 1.5$ is $f'(1.5) = 0.336925$. Use **forward**, **backward**, and **centered** first finite differences to estimate the derivative numerically for the step size $\Delta x = 0.25$, and determine the percent error between the true value and each of the estimated values. Percent error is given by:

$$\varepsilon = \left|\frac{true\ value - estimated\ value}{true\ value}\right| \times 100\%$$

What value of Δx would you have to use for the backward and forward finite differences to get the same (or smaller) percent error as the centered finite difference using $\Delta x = 0.25$? *Hint: it should be less than 0.25!*

Note: $\sinh(x)$ is hyperbolic sine