Question 1

Consider the vectors $\{x_1, x_2, x_3\}$ of $\mathbb{R}^4$ where

$$x_1 = (4, 2, 2, 1)^T, \ x_2 = (2, 0, 0, 2)^T, \ x_3 = (1, 1, -1, 1)^T$$

Let $S := \text{span}\{x_1, x_2, x_3\}$. Use the Gram-Schmidt process to obtain an orthonormal basis for S.

Solution:

Question 2

Find the orthogonal complement of the subspace of $\mathbb{R}^3$ spanned by $(1, 2, 1)^T$, $(1, -1, 2)^T$.

Solution:

Question 3

Let A be an $m \times n$ matrix. Show that A and $A^T A$ have the same rank. Show that

$$N(A^T A) = N(A)$$

Solution:

Question 4

Let A be an $m \times n$ matrix and $\text{rank}(A) = r$. What are the dimensions of $N(A)$ and $N(A^T)$?

Solution:

Question 5

For each of the following systems $Ax = b$ find all least squares solutions.

$$A = \begin{bmatrix} 1 & 1 \\ 3 & 4 \\ -1 & 0 \end{bmatrix}, \ b = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \ \text{and} \ A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}, \ b = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Solution:

Question 6

Consider the basis $\{x_1, x_2, x_3\}$ of $\mathbb{R}^3$ where

$$x_1 = (1, 2, -2)^T, \ x_2 = (4, 3, 2)^T, \ x_3 = (1, 2, 1)^T$$

Use the Gram-Schmidt process to obtain an orthonormal basis.

Solution: