

Follow the the given guideline to answer all questions.

1. Apply an appropriate test to determine convergence of the series  $\sum_{n=1}^{\infty} \left| \frac{(-1)^n \ln n}{n} \right|$

$$\sum_{n=1}^{\infty} \left| \frac{(-1)^n \ln(n)}{n} \right| = \sum_{n=1}^{\infty} \frac{\ln(n)}{n} > \sum_{n=1}^{\infty} \frac{1}{n} \quad \text{p-series } p=1 \rightarrow \text{diverges}$$

The series  $\sum_{n=1}^{\infty} \left| \frac{(-1)^n \ln(n)}{n} \right|$  diverges by comparison to the harmonic series

2. Apply an appropriate test to determine convergence of the series  $\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}$

$$\sum_{n=1}^{\infty} \frac{(-1)^n \ln(n)}{n} \quad \text{alternating} \quad b_n = \frac{\ln(n)}{n} \quad \lim_{n \rightarrow \infty} b_n = 0$$

The series  $\sum_{n=1}^{\infty} \frac{(-1)^n \ln(n)}{n}$  converges by the alternating series test

3. Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}$$

The series is conditionally convergent since it is not absolutely convergent but still converges when alternating.

4. Find  $s_6$ , the sum of the first six terms, of the series. Give your answer in exact form in the first line and also as a decimal rounded to 5 decimal places in the second line.

$$s_6 = \sum_{n=1}^6 \frac{(-1)^n \ln n}{n} = 0 + \frac{\ln 2}{2} - \frac{\ln 3}{3} + \frac{\ln 4}{4} - \frac{\ln 5}{5} + \frac{\ln 6}{6}$$

$$\approx 0.30368$$

The remainder of the series  $\sum_{n=1}^{\infty} a_n$  is defined by

$$R_n = s - s_n = a_{n+1} + a_{n+2} + a_{n+3} + \cdots$$

The Alternating Series Estimation Theorem provides the bound for the possible error  $R_n$  in the approximation of the sum of an alternating series.

5. Compute a bound for  $R_6$ . Give an exact value and also a decimal rounded to 5 decimal places. Write a paragraph to describe what this tells you about the approximation of the sum of the series by  $s_6$ .

$$|R_6| \leq b_{6+1} = \frac{(-1)^7 \ln 7}{7} = \frac{\ln 7}{7} \approx 0.27799$$

The value of  $R_6$  tells you that  $S_6$  can approximate the entire sum to an error of about 0.27799