MATH 152 Lab 7

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In [1]:
```
from sympy import *
from sympy.plotting import (plot,plot_parametric)
import matplotlib.pyplot as plt
import numpy as np
```

Question 1

1a

In [2]:
```
n, ub = symbols("n ub")
expr = 1/n**4
sn = Sum(expr, (n, 1, ub))

s10 = sn.subs(ub, 10)
print(f"The partial sum of the series s10 is {N(s10)}")
print(f"The error in using s10 as an approximation of the sum of the series is {N(pi**
```

```
The partial sum of the series s10 is 1.08203658349376
The error in using s10 as an approximation of the sum of the series is 0.000286650217
381645
```

1b

In [3]:
```
lower_estimate = sn.subs(ub, 10) + integrate(expr, (n, 11, oo))
upper_estimate = sn.subs(ub, 10) + integrate(expr, (n, 10, oo))
print(f"The sum of the series is between {N(lower_estimate)} and {N(upper_estimate)}")
```

```
The sum of the series is between 1.08228702176072 and 1.08236991682709
```

1c

In [4]:
```
print(f"The error in the lower estimate is {N(pi**4/90 - lower_estimate)}")
print(f"The error in the upper estimate is {N(upper_estimate - pi**4/90)}")
```

```
The error in the lower estimate is 0.0000362119504144521
The error in the upper estimate is 0.0000466831159516886
```

1d

In [5]:
```
i = 10
while N(pi**4/90 - sn.subs(ub, i)) > 10**(-6):
    i += 1
print(f"The number of terms needed to get an error of 10^-6 is {i}")
```

```
The number of terms needed to get an error of 10^-6 is 69
```

Question 2

2a

In [6]:
```
x = symbols("x")
expr = n**2 * E**(-n)
series = Sum(expr, (n, 2, oo))
fx = x**2 * E**(-x)

print(f"The indefintie integral of f(x) is {integrate(fx, x)}")
print(F"The definite integral of f(x) from 1 to infinity is {N(integrate(fx, (x, 1, oo
```

```
The indefintie integral of f(x) is (-x**2 - 2*x - 2)*exp(-x)
The definite integral of f(x) from 1 to infinity is 1.83939720585721
```

2b

In [7]:
```
print("Based on the answer in part a, the series converges for all n >= 2 by the integ
```

```
Based on the answer in part a, the series converges for all n >= 2 by the integral te
st.
```

2c

In [8]:
```
sn = Sum(expr, (n, 2, ub))
s10 = sn.subs(ub, 10)
s50 = sn.subs(ub, 50)
s100 = sn.subs(ub, 100)
s = sn.subs(ub, oo)
print(f"The 10th partial sum of the series is {N(s10)}")
print(f"The 50th partial sum of the series is {N(s50)}")
print(f"The 100th partial sum of the series is {N(s100)}")
print(f"The sum of the series is {N(s)}")
```

```
The 10th partial sum of the series is 1.62084673541930
The 50th partial sum of the series is 1.62441532595354
The 100th partial sum of the series is 1.62441532595354
The sum of the series is 1.62441532595354
```

2d

In [9]:
```
r100 = integrate(expr, (n, 100, oo))
error = N(s - s100)
print(f"The remainder estimate for the integral test for the 100th partial sum is {N(r
print(f"The actual value of the error is {error}")
print(f"The larger error is the remainder estimate.")
```

```
The remainder estimate for the integral test for the 100th partial sum is 3.795221510
73646E-40
The actual value of the error is 2.23423822900972E-40
The larger error is the remainder estimate.
```

2e

In [10]:
```
i = 2
while integrate(expr, (n, i, oo)) > 10**(-10):
    i += 1
```

```
print(f"The number of terms needed to get an error of 10^-10 is {i}")
print(f"The value of s - s{i} is {N(s - sn.subs(ub, i))} which is less than 10^-10")
```

```
The number of terms needed to get an error of 10^-10 is 30
The value of s - s30 is 5.43688886516869E-11 which is less than 10^-10
```

Question 3

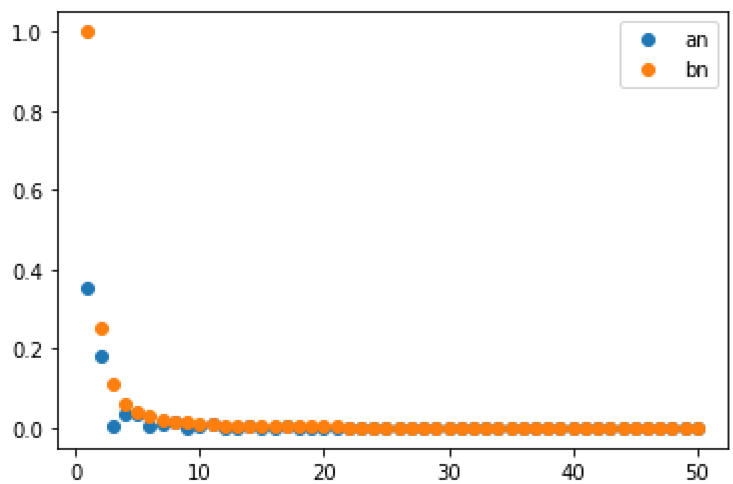
3a

In [11]:
```
an = (n * sin(n)**2) / (1 + n**3)
s = Sum(an, (n, 1, oo))

bn = 1 / n**2
print(f"The sequence an can be compared to the sequence bn which is {bn}")
```

```
The sequence an can be compared to the sequence bn which is n**(-2)
```

3b

In [12]:
```
anvals = [an.subs(n, i) for i in range(1, 51)]
bnvals = [bn.subs(n, i) for i in range(1, 51)]
plt.figure()
plt.plot(range(1, 51), anvals,"o",label="an")
plt.plot(range(1, 51), bnvals,"o",label="bn")
plt.legend()
plt.show()

print(f"The sequence bn is larger than an.")
```




```
The sequence bn is larger than an.
```

3c

In [13]:
```
print(f"The sum of bn is convergent by the integral test.")
print(f"It can be concluded that an is convergent by comparison to bn.")
```

```
The sum of bn is convergent by the integral test.
It can be concluded that an is convergent by comparison to bn.
```

3d

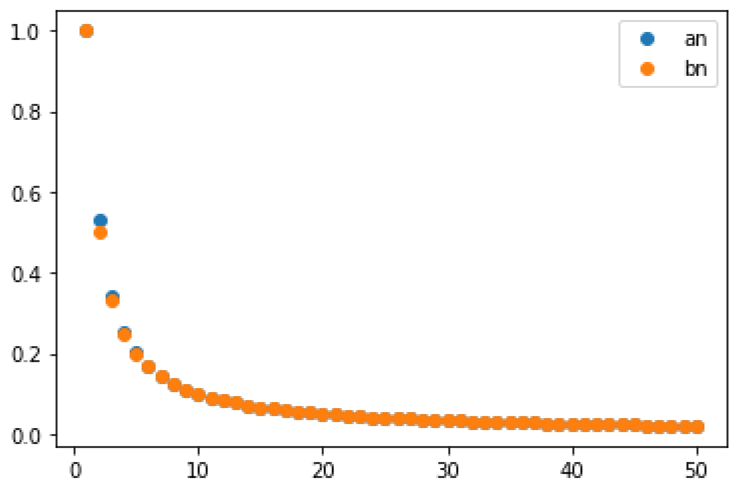
In [14]:
```
print(f"The conclusion in part c is conclusive")
```
```
The conclusion in part c is conclusive
```

Question 3 (part 2)

3a.2

In [15]:
```
an = (E**n + 1) / (n * E**n + 1)
bn = 1 / n
print(f"The new sequence an can be compared to the sequence bn which is {bn}")
```
```
The new sequence an can be compared to the sequence bn which is 1/n
```

3b.2

In [16]:
```
anvals = [an.subs(n, i) for i in range(1, 51)]
bnvals = [bn.subs(n, i) for i in range(1, 51)]
plt.figure()
plt.plot(range(1, 51), anvals,"o",label="an")
plt.plot(range(1, 51), bnvals,"o",label="bn")
plt.legend()
plt.show()

print(f"The sequence bn is smaller than an.")
```



```
The sequence bn is smaller than an.
```

3c.2

In [17]:
```
print(f"The sum of bn is divergent by the integral test.")
print(f"It can be concluded that an is divergent by comparison to bn.")
```
```
The sum of bn is divergent by the integral test.
It can be concluded that an is divergent by comparison to bn.
```

In [ ]: