CSCE 222 Discrete Structures for Computing – Fall 2023
Hyunyoung Lee

Problem Set 2

Due dates: Electronic submission of *yourLastName-yourFirstName-hw2.tex* and *yourLastName-yourFirstName-hw2.pdf* files of this homework is due on **Monday, 9/18/2023 11:59 p.m.** on `https://canvas.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two files are missing, you will receive zero points for this homework.**

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Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Electronic signature: **Kevin Lei**

Total 100 points.

The intended formatting is that this first page is a cover page and each problem solved on a new page. You only need to fill in your solution between the `\begin{solution}` and `\end{solution}` environment. Please do not change this overall formatting.

Checklist:
- ☐ Did you type in your name and UIN?
- ☐ Did you disclose all resources that you have used? (This includes all people, books, websites, etc. that you have consulted.)
- ☐ Did you sign that you followed the Aggie Honor Code?
- ☐ Did you solve all problems?
- ☐ Did you submit both the .tex and .pdf files of your homework to each correct link on Canvas?

Problem 1. ($5+5 = 10$ points) Section 2.6, Exercise 2.53 (a) and (c). Explain.

Solution. Part (a). For the universe U of nonnegative integers, the only values of a, b, c that make predicate C(a, b, c) true are (0, 0, 0). This is because Fermat's Last Theorem states that there are no positive integer solutions to $a^n + b^n = c^n$ for values of n greater than 2. With positive integer solutions ruled out, the only other possible solution is zero, which is the only solution.

Part (c). For the universe U of {1, 2, 3, 4, 5}, there are two values of (a, b, c) that satisfies the predicate S(a, b, c), which is (3, 4, 5) and (4, 3, 5). Since $a^2 + b^2 = c^2$ is just the pythagorean theorem, (3, 4, 5) is the only pythagorean triple in this universe, and this can be written in two ways since the tuple is ordered.

Problem 2. $(5 + 5 = 10$ points) Section 2.6, Exercise 2.54 (b) and (c)

Solution. Given that the universe is the set of real numbers:

Part (b). $\forall x \exists y (x < y)$ can be translated as for all real numbers x, there exists a real number y such that x is less than y.

Part (c). $\forall x \forall z \exists y (x < z) \rightarrow ((x < y) \wedge (y < z))$ can be translated as for all real numbers x and z where x is less than z, there exists a real number y such that y is greater than x and less than z.

Problem 3. ($5 + 5 = 10$ points) Section 2.7, Exercise 2.58 (a) and (e)

Solution. Part (a). $\neg\forall x \exists y (P(x) \rightarrow Q(y))$
$\Leftrightarrow \exists x \forall y \neg(\neg P(x) \vee Q(x))$
$\Leftrightarrow \exists x \forall y (P(x) \wedge \neg Q(x))$
Part(e). $\neg\exists x \exists y (\neg P(x) \wedge \neg Q(y)) \Leftrightarrow \forall x \forall y (P(x) \vee Q(y))$

Problem 4. ($5 + 5 = 10$ points) Section 2.7, Exercise 2.59 (d) and (e)

Solution. Part (d). Let $P(a, b)$ be the predicate such that $a + b = 1001$ and assume the universe is integers. The statement can be formalized as $\exists a \forall b P(a, b)$. Negating this statement, we end up with $\forall a \exists b \neg P(a, b)$. In english, this would mean that for all integers a there exists an integer b such that a + b is not equal to 1001.

Part (e). Let $P(a, b)$ be the predicate such that $b < a$ where b and a are positive integers. The statement can be formalized as $\forall a \exists b P(a, b)$. The negation of this statement would be $\exists a \forall b \neg Q(a, b)$. In english, this would mean there exists a positive integer a such that for all positive integers b, the statement a is less than or equal to b is true.

Problem 5. (15 points) Section 2.9, Exercise 2.73 [Hint: Use the property of "consecutive integers" and the definition of an "odd integer".]

Solution. Suppose m and n are consecutive integers. Numbers m and n being consecutive means that one of them must be even and the other must be odd. By definition, an integer is even if it can be written as $2k$, and an integer is odd if it can be written as $2k + 1$. If we say that m is equal to $2k$ and n is equal to $2k + 1$, then $m + n$ would be equal to $2k + 2k + 1$. This simplifies to $4k + 1$, and since it has the $+1$ at the end, following the definition of an odd integer, the sum of m and n must be odd.

Problem 6. (15 points) Section 2.9, Exercise 2.80

Solution. The statement can be written as $(m+n > 100) \rightarrow (m > 40 \vee n > 60)$ where m and n are integers. The contrapositive of this would be $(m \leq 40 \wedge n \leq 60) \rightarrow (m + n \leq 100)$ If we assume that $m \leq 40$ and $n \leq 60$, the the maximum values of m and n are 40 and 60 respectively. This would mean that the maximum value of $m + n$ is 100, and the implication holds. Since the contrapositive is true, the original statement $(m+n > 100) \rightarrow (m > 40 \vee n > 60)$ is also true.

Problem 7. (15 points) Section 2.9, Exercise 2.84

Solution. Seeking a contradiction, assume the equation $42m + 70n = 1000$ has integer solutions. The fundamental theorem of arithmetic states that every integer greater than 1 has a unique prime factorization. Factoring both sides of the equation, we have $(2 \cdot 3 \cdot 7)m + (5 \cdot 7)n = 2^3 \cdot 5^3$. Since the m and n terms both have a factor of 7, the equation can be rewritten as $7(2 \cdot 3 \cdot m + 5 \cdot n) = 2^3 \cdot 5^3$ Furthermore, dividing both sides by 7, the equation is now $2 \cdot 3 \cdot m + 5 \cdot n = \frac{2^3 \cdot 5^3}{7}$. Since m and n are integers, $2 \cdot 3 \cdot m + 5 \cdot n$ must also be an integer. However, this brings us to a contradiction, since $\frac{2^3 \cdot 5^3}{7}$ is not an integer, but a rational number. Therefore, we have proved by contradiction that the equation $42m + 70n = 1000$ has no integer solutions.

Problem 8. (15 points) Section 3.3, Exercise 3.20 [Hint: Use the definitions of $\subseteq$, $\cup$, and the power set.]

Solution. Suppose that arbitrary element $X \in P(A) \cup P(B)$. This means that either $X \subseteq A$ or $X \subseteq B$. If $X \subseteq A$, then $X \subseteq A \cup B$ since $A \subseteq A \cup B$. Since $X \subseteq A \cup B$, $X \in P(A \cup B)$. The same reasoning holds if $X \subseteq B$. If $X \subseteq B$, then $X \subseteq A \cup B$ since $B \subseteq A \cup B$. Since $X \subseteq A \cup B$, $X \in P(A \cup B)$. For both cases, $X \in P(A \cup B)$, and we have shown that any element in $P(A) \cup P(B)$ is also in $P(A \cup B)$. By definition of the subset, $P(A) \cup P(B) \subseteq P(A \cup B)$. Therefore, the statement is proved.