Question 1

Let $\{u_1, u_2, u_3\}$ be an orthonormal set of vectors in some vector space with inner product. Let

$$u := u_1 + 2u_2 + 3u_3 \text{ and } v := u_1 - u_3$$

Computer $\langle u, v\rangle$, $\|u\|$, and $\|v\|$.

Question 2

Consider the vector space $mathbbC[-1, 1]$ equipped with the inner product:

$$\langle f, g\rangle := \int_{-1}^{1} f(x)g(x)dx$$

1. Show that $1, x$ are orthogonal.
2. Compute the norms $\|1\|$, $\|x\|$.

Question 3

Let

$$u_1 = \left(\frac{1}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, -\frac{4}{3\sqrt{2}}\right)^T,\ u_2 = \frac{1}{3}(2, 2, 1)^T,\ u_3 = \frac{1}{\sqrt{2}}(1, -1, 0)^T$$

1. Show that u_1, u_2, u_3 is an orthonormal basis for $\mathbb{R}^3$.
2. Let $x = (1, 2, 2)^T$. Find the projection of p of x onto $S := \operatorname{span}\{u_2, u_3\}$.

Question 4

Let $v_1 := (1, 2, 0, -1)^T$ $v_2 := (1, -1, 0, 0)^T$ $v_3 := (0, 1, 0, -1)^T$. Find the angle between v_1, v_2, v_2, v_3, and v_1, v_3. Find the norm of each of these vectors. Find the projection of v_1 onto v_2 and onto v_3.

Question 5

Let A be an $m \times n$ matrix. Show that A^TA and AA^T is a symmetric matrix. Assume that $m \geq n$ and $\operatorname{rank}(A) = n$. Show that if $P = A(A^TA)^{-1}A^T$ then

$$P^2 = P$$