Exercise of Algorithms

Note: Problem 1 below is Exercise 34.5-1 in the textbook "Introduction to Algorithms" (4th edition), page 1098

Problem 1

Given two graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$, G is said to be isomorphic to H if there is a bijection (namely, one-to-one mapping) between the two sets of vertices V_G and V_H :

$$f: V_G \to V_H$$

such that for each pair of vertices $u,v\in V_G$, they are adjacent in G if and only if the two corresponding vertices $f(u),f(v)\in V_H$ are adjacent in H (namely, $(u,v)\in E_G$ if and only if $(f(u),f(v))\in E_H$). Clearly, if G is isomorphic to H, then H is isomorphic to G, and vice versa. In other words, if G and G are isomorphic, then they are essentially the same graph, except that they label their vertices differently.

The Subgraph-Isomorphism Problem takes two undirected graphs G_1 and G_2 , and asks whether G_1 is isomorphic to a subgraph of G_2 . Show that the subgraph-isomorphism problem is NP-complete.