

9th Homework — MATH 304 — Fall 2023
— Due November 28th —

1. Consider the vectors $\{x_1, x_2, x_3\}$ of \mathbb{R}^4 where

$$x_1 = (4, 2, 2, 1)^T, \quad x_2 = (2, 0, 0, 2)^T, \quad x_3 = (1, 1, -1, 1)^T.$$

Let $S := \text{span}\{x_1, x_2, x_3\}$ Use the Gram-Schmidt process to obtain an orthonormal basis for S .

2. Find the orthogonal complement of the subspace of \mathbb{R}^3 spanned by $(1, 2, 1)^T, (1, -1, 2)^T$.
3. Let A be an $m \times n$ matrix. Show that A and $A^T A$ have the same rank. Show that

$$N(A^T A) = N(A).$$

4. Let A be an $m \times n$ matrix and $\text{rank}(A) = r$. What are the dimensions of $N(A)$ and $N(A^T)$?
5. For each of the following systems $Ax = b$ find all least squares solutions:

$$A = \begin{pmatrix} 1 & 1 \\ 3 & 4 \\ -1 & 0 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \quad \text{and} \quad A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

6. Consider the basis $\{x_1, x_2, x_3\}$ of \mathbb{R}^3 where

$$x_1 = (1, 2, -2)^T, \quad x_2 = (4, 3, 2)^T, \quad x_3 = (1, 2, 1)^T.$$

Use the Gram-Schmidt process to obtain an orthonormal basis.

Show your work in each exercise.