Question 1

(a) $\{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots\}$

(b) $\{\ldots, -13, -8, -3, 2, 7, 12, 17, \ldots\}$

Question 2

(a) False

(b) True

(c) True

(d) True

(e) True

(f) True

(g) False

Question 3

Proof. Let A and B be sets. We want to prove that $A = B$, which means that $A \subseteq B$ and $B \subseteq A$. First, assume that x is an element of A. Then, by the definition of A, we can write x as $x = 4k + 1$ for some integer k. We can then do the following manipulations:

$$\begin{aligned} x &= 4k + 1 \\ &= 4k + 1 - 8 + 8 \\ &= 4k + 8 - 7 \\ &= 4(k + 2) - 7. \end{aligned}$$

Since $k + 2$ is an integer, x is by definition an element of B. Therefore, $A \subseteq B$. Similarly, assume that x is an element of B. Then, by the definition of B, we can write x as $x = 4j - 7$ for some integer j. We can then do the following manipulations:

$$\begin{aligned} x &= 4j - 7 \\ &= 4j - 7 + 8 - 8 \\ &= 4j - 8 + 1 \\ &= 4(j - 2) + 1. \end{aligned}$$

Since $j - 2$ is an integer, x is by definition an element of A. Therefore, $B \subseteq A$. Since $A \subseteq B$ and $B \subseteq A$, we have proven that $A = B$. $\square$

Question 4

(a)

$$\begin{aligned}(A \cup \overline{B}) \cap C &= (\{a, b, \{2\}\} \cup \{1, \{2\}, a\}) \cap \{1, \{2\}, c\} \\ &= \{1, \{2\}, a, b\} \cap \{1, \{2\}, c\} \\ &= \{1, 2\}\end{aligned}$$

(b)

$$\begin{aligned}A \cup (\overline{B} \cap C) &= \{a, b, \{2\}\} \cup (\{1, \{2\}, a\} \cap \{1, \{2\}, c\}) \\ &= \{a, b, \{2\}\} \cup \{1, \{2\}\} \\ &= \{1, \{2\}, a, b\}\end{aligned}$$

Question 5

(a) *Disproof.* Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, and $C = \{2, 3, 5\}$. Then, $A \cap B = \{2, 3\} = A \cap C$. However, $B \neq C$, as $4 \in B$ but $4 \notin C$, and $5 \in C$ but $5 \notin B$. Therefore, the statement is false. □

(b) *Disproof.* Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, and $C = \{1, 2, 3, 4\}$. Then, $A \setminus B = \{1\} = A \setminus C$. However, $B \neq C$, as $4 \in B$ but $4 \notin C$. Therefore, the statement is false. □

Question 6

Proof. Let A and B be sets. We want to show that $A \subseteq B$ if and only if $\overline{B} \subseteq \overline{A}$. First, $A \subseteq B$ is equivalent to saying $x \in A \to x \in B$. This is logically equivalent to its contrapositive, that being $x \notin B \to x \notin A$, which is the definition of $\overline{B} \subseteq \overline{A}$. Since we have this sequence of logically equivalent statements, we have shown that $A \subseteq B \leftrightarrow \overline{B} \subseteq \overline{A}$. □

Question 7

(a) *Proof.* Let A and B be sets. We want to show that $\overline{A \cap B} = \overline{A} \cup \overline{B}$. Assuming that $x \in \overline{A \cap B}$, we have $x \notin A \cap B$. By the definition of the set intersection, this means that $x \notin A$ or $x \notin B$. By the definition of the set complement, this means that $x \in \overline{A}$ or $x \in \overline{B}$. This can be rewritten using the set union as $x \in \overline{A} \cup \overline{B}$. Since these are all bidirectional logical equivalences, we have shown that $\overline{A \cap B} = \overline{A} \cup \overline{B}$. □

(b) *Proof.* Let A be a set. We want to show that $A \cap \emptyset = \emptyset$. Seeking a contradiction, assume that $A \cap \emptyset \neq \emptyset$. Suppose that $x \in A \cap \emptyset$. By definition of the set intersection, we have $x \in A$ and $x \in \emptyset$. However, the empty set has no elements, so x cannot be in $\emptyset$. Thus, we have reached a contradiction, and our assumption that $A \cap \emptyset \neq \emptyset$ must be false. Therefore, $A \cap \emptyset = \emptyset$. $\square$