

**Winter Is Coming: Maybe?  
An Investigation Into Volcanic Triggers of Global Glaciation**

EOSC 453 - Assignment 2

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## Introduction

Volcanic eruptions eject ash and aerosol particles into the atmosphere that can cause temporary cooling, in the most extreme cases of more than 10 degrees over several years. The climate system is nonlinear, a brief, high intensity event can precipitate a rapid transition to a different state. This different state could be a fully glaciated planet or ‘snowball earth’, with obvious consequences to life on the surface. In this assignment we aim to observe how the steady state earth would respond to various volcanic scenarios and if rapid state changes occur. First we build a simple radiative energy balance model for our planet, dividing it into 6 latitude zones each exchanging energy with their neighbours. We then test several scenarios, including volcanic forcing, volcanic forcing with an ice-albedo feedback, and the effect of a supervolcano in different latitude zones. First we present the model and the assumptions included in it. We then reveal the outputs of the model in a results section. Finally we discuss the implications of these results and what further work could be undertaken to take this experiment further.

We created a model of the earth’s atmosphere and active surface layer in this assignment. We are ultimately interested in the effects various inputs have on the climate—thus the output of our model will be temperature. Using two basic starting points, the Stefan-Boltzmann radiation law and the principle of conservation of energy, we develop an energy flux model often known as a ‘Budyko’ model, in recognition of its creator.

Temperature is strongly latitude dependant on Earth due to the equator receiving more solar flux, thus we divide our model into latitude bins to account for this. We create a total of six latitude zones: the North and South polar regions ( $60^{\circ}$  latitude+), the two mid latitude regions ( $30\text{--}60^{\circ}$  latitude) and the two tropical zones, with each hemisphere separated by the equator. Each zone acts as a separate box in the model, exchanging energy with its neighbours; due to conservation laws the flux into each zone is equal to the flux out. Due to the curvature of the planet, the areas of the zones are not all equal; the tropical zones are the largest and Polar regions the smallest. Table 1 sums up the transfer coefficients between zones: these are mostly set at a background value of  $1.00\text{E-}7 \text{ W/(m}^{\star}\text{K)}$ . The lower value of  $5.00\text{E-}8 \text{ W/(m}^{\star}\text{K)}$  at  $60^{\circ}\text{S}$  is here used to simulate reduced latitudinal heat transfer due to the southern ocean’s circulation. The higher value of  $2.00\text{E-}7 \text{ W/(m}^{\star}\text{K)}$  at  $30^{\circ}\text{N}$  is used to simulate the northerly transport of heat via the Gulf Stream.

Three main materials with very different thermal and radiative properties are present on the earth today: ice, liquid water and land. The thermal scale depth of each layer gives the relevant thickness of material that can be affected by surface radiative processes (see table 2). The amount of energy affected is given by the product of the material density multiplied by its specific heat capacity (again, see table 2). Finally the rough fractions of water, ice, and land in each area were worked out using Google Earth Pro via the area function (roughly 5%

uncertainty in the fraction of ice and 10% uncertainty for the fraction of water and land). Table 2 sums up all of these attributes and values that were defined for the model.

	Transfer Coefficients (W/(m*K))
Zone 1-2	5.00E-08
Zone 2-3	1.00E-07
Zone 3-4	1.00E-07
Zone 4-5	2.00E-07
Zone 5-6	1.00E-07

*Table 1: Transfer coefficients between the zones*

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We use an albedo of 0.6 for ice, 0.1 for water and 0.4 for land. These average values take into account the variability within each material. Using all the values summed up in table 2 we can calculate area averaged values for each of the zones using the following formulae:

$$pcZaverage(x) = fl(x) * pl * cl * Zl + fs(x) * ps * cs * Zs + fi(x) * pi * ci * Zi ;$$

and

$$ao(x) = fl(x) * al + fs(x) * as + fi(x) * ai ;$$

with x denoting the zone from 1 to 6; fl/fs/fi denoting the areas of land, sea and ice respectively; pl/fs/pi the densities; Zl/Zs/Zi the thermal scale depths and al/as/ai the albedos. These allow us to use a single value for each zone and ignore ice/sea/land inhomogeneities (at least for the initial steady state in which they don't change). We also added a greenhouse effect to our system of differential equations to approximate realistic temperatures. We did this by replacing a

term in the differential equations (outgoing radiative flux) with a function OLR which is dependent on atmospheric CO<sub>2</sub> concentration and temperature.

	Ice	Water	Land
Thermal scale depth (m)	1	70	1
Density (kg/m <sup>3</sup> )	900	1028	2500
Specific heat capacity (J*kg/K)	2060	4187	790
albedo	0.6	0.1	0.4
Fraction in zone 1 (60+°S)	0.45	0.5	0.05
Fraction in zone 2 (30-60°S)	0.01	0.74	0.25
Fraction in zone 3 (0-30°S)	0	0.7	0.3
Fraction in zone 4 (0-30°N)	0	0.55	0.45
Fraction in zone 5 (30-60°N)	0.01	0.54	0.45
Fraction in zone 6 (60+°N)	0.3	0.35	0.35

*Table 2: Properties used in this model and area fraction of water, land and ice in each zone*

Using these zonally averaged values we can build a heat (thermal energy) transfer model for the 6 box system with fluxes linking neighbouring boxes together. All the model inputs are known (shown in Table 1 or in the attached matlab code). We solved these equations using the matlab functions ODE15s and ODE45, depending on the timestep and resolution required for each experiment.

We defined several scenarios to test using this model, briefly described here:

### **1) Steady state system of the earth according to this model**

We examined the equilibrium state (temperature) of each zone with no additional inputs or forcings.

### **2) Steady state with volcanic forcing**

We created a new function ( $\Phi$ ) to simulate volcanic eruptions that we edit into the equations described above. We use Matlab's function 'randsample' to randomly sample

a population of integer magnitudes with a base 10 logarithmic distribution from magnitudes 4 to 8 (reflection of the VEI scale). Each magnitude is given a random year of occurrence, and randomly applied to a zone weighted by a zone's fractional area. This achieves simulating a naturally plausible situation where eruptions occur at random intervals and with random magnitudes according to a magnitude-frequency relationship. Magnitudes are scaled by a power law, so that a magnitude 4 is sixteen times smaller than a magnitude 8, but ten thousand times more likely. Thus large (VEI8 scale, for example Toba/Yellowstone) eruptions occur infrequently but have the largest climatic impact when they do occur. The climate cooling induced by volcanoes is accounted for by a pseudo-albedo term, where the amount of reflected radiation is increased immediately following an eruption by a factor linked to the magnitude, then decays linearly in a number of years also linked to this magnitude.

### **3) Volcanic forcing with ice albedo effect**

In this model we introduce a link between volcanic cooling and longer term climate change: the ice-albedo feedback. When volcanic eruptions temporarily cool the climate below a threshold, ice begins to form. This new ice then reflects more incoming radiation away from the surface and leads to slightly reduced steady state temperatures. If enough ice forms (the albedo increases enough), a positive feedback loop is initiated and the planet cools to the extent that ice covers its entire surface. This is known as a Snowball Earth episode. We run this case 10 times in order to investigate how often random volcanic activity is able to initiate Snowball Earth.

### **4) Super-eruption occurrence**

We run our volcanic  $\Phi$  function with the above volcanism rates, but also force an incredibly large eruption (VEI 8) if one had not been created randomly. This eruption not only injects more gasses into the atmosphere (higher reduction in incoming radiation), but also lasts for a longer duration. This strategy for injecting a super volcanic event is effectively the same as observing a 10,000 year period in which an 8.0 eruption randomly occurs, without having to rerun the model until such an eruption is generated. A case is run for a super volcano occurring in each zone, resulting in six total cases.

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## Results

### 1) Steady state system

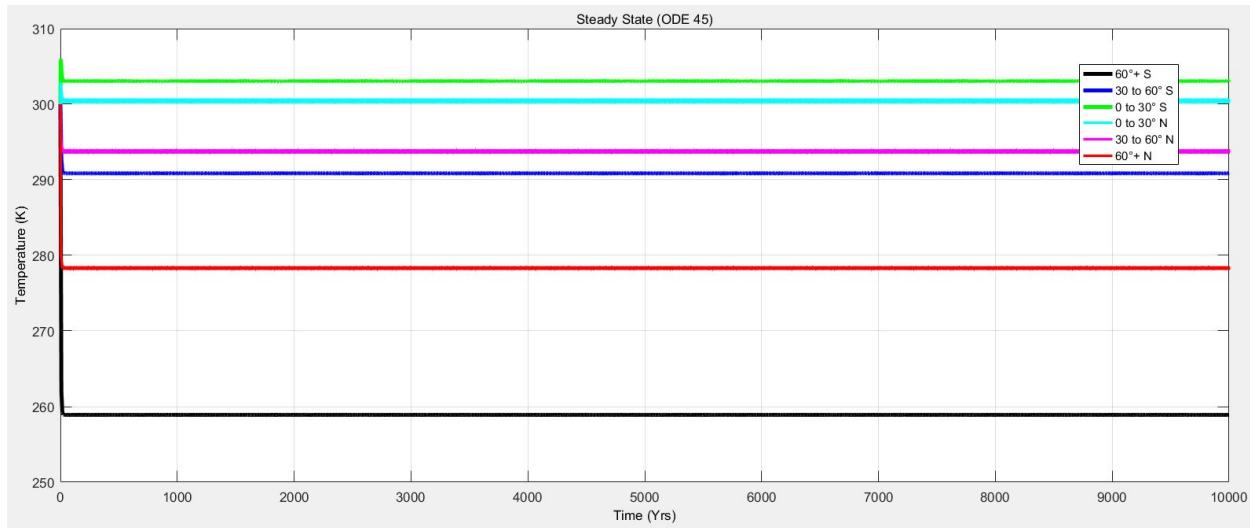


Figure 1: Results of running the model at steady state. The initial spike at  $t=0$  is an artefact of the model

At steady state, the temperature of each zone is given solely by its energy balance model, latitudinal heat transfers, and the greenhouse effect. Due to their area, the tropical zones receive more solar radiation and thus are warmer. The southern tropics are slightly warmer than the northern tropics due to a lower land percentage (oceans have a lower albedo than land). The Northern mid latitudes are slightly warmer than their southern counterparts due to the Gulf Stream's effects. The South pole is almost  $20^{\circ}$  colder than the North pole due to the isolating effects of the southern ocean and large ice fraction (Antarctica).

### 2) Volcanic forcing with no albedo feedback

Figure 2: The lower plot displays the time, magnitude, and zone of the randomly generated volcanic events. The vertical axis displays the zone in which the eruption occurred, and the horizontal axis displays the time in which the event occurred. Larger, brighter red dots indicate a larger scale eruption, as displayed in the colour bar (scale in VEI). The top plot displays the temperature of each zone with the occurrence of the volcanic events displayed in the lower figure. The temperatures all start at 290 degrees (K) and quickly reach steady state, after which volcanic activity begins to force temporary decreases in temperature. No temperature in any zone ever exceeds the original steady state and extended periods at the same temperature are not seen.

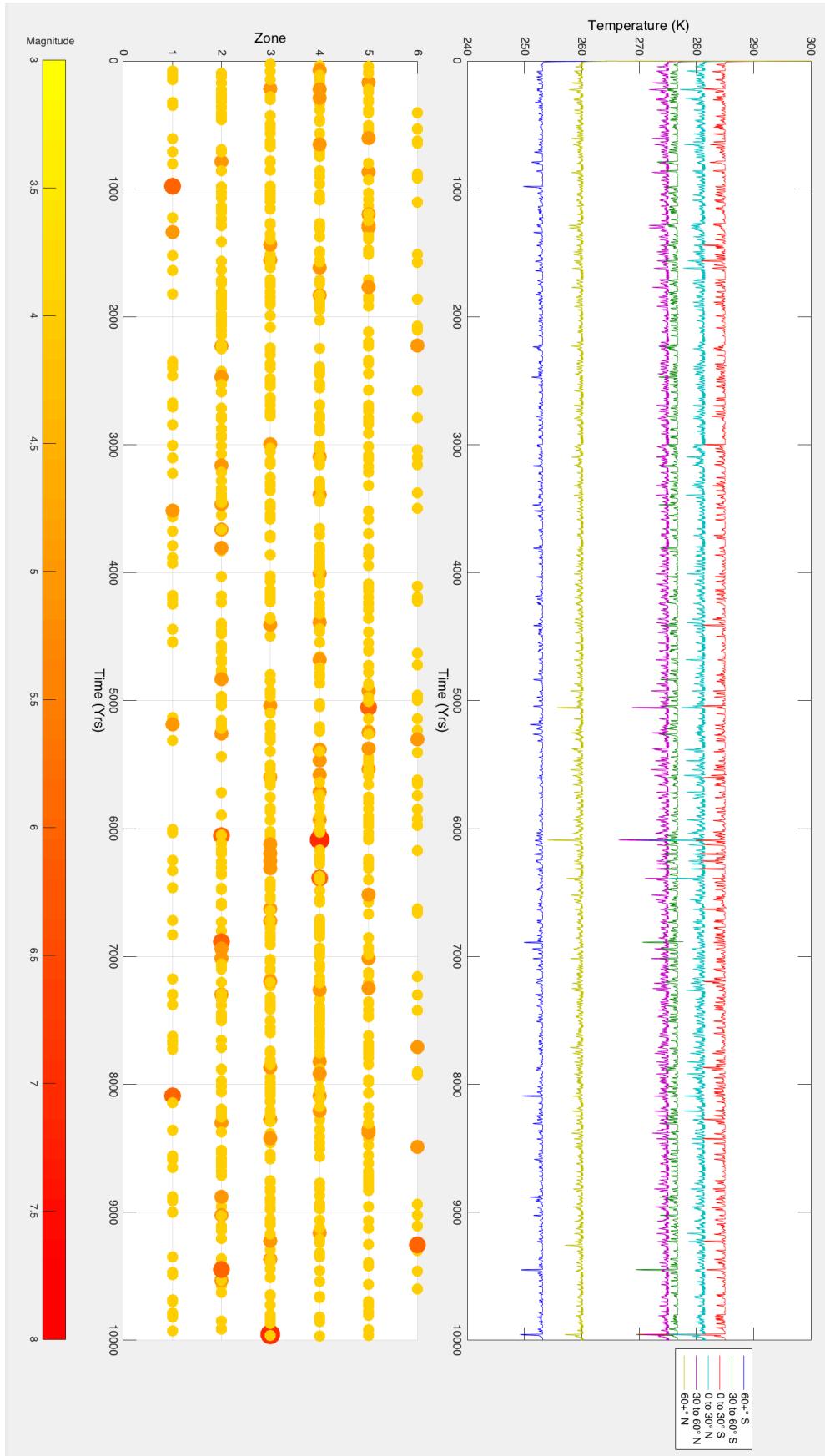


Fig 2. Results of running the model with volcanic events and no albedo feedback. The top plot displays the temperatures by zone, the bottom plot shows the distribution of volcanic events and the magnitude of these events.

### **3) Volcanic forcing with albedo feedback, figures 3 and 4**

Figure 3: The first of ten runs. The remainder have been included in the appendices. The top two plots display the same metrics as in figure two. In the bottom plot, the global solar irradiance coefficient changes with a high frequency over the observed time scale. There are many local highs and lows over the entirety of the observed time.

Figure 4: The fifth of ten runs. All of the plots display the same metrics as in figure 3. The top plot begins in a similar fashion as figure three but has a precipitous drop about 800 years into the simulation. The zone temperatures continue near these new lower temperatures and begin to undergo similar short-term temperature fluctuations.

### **4) Super-eruption occurrence**

Figure 5: This figure displays the same metrics as figure 3, however at 5000 years a magnitude 8 eruption was injected into zone 6. This simulates a massive eruption in the North Pole. A similar plot for the zone 1 (South Pole) has been included in appendix 9. The injection of the super volcano into zones 1 and 6 has a temporary effect on all zones temperatures.

Figure 6: This figure displays the same metrics as figure 3, however at 5000 years a magnitude 8 eruption was injected into zone 5. This simulates a massive eruption in the Northern mid-latitudes. A similar plots for the zones 2-4 have been included in appendices 10-12. The injection of a super volcano in zones 2-5 has an effect on all zone's temperatures that lasts the remainder of the simulation's time period.

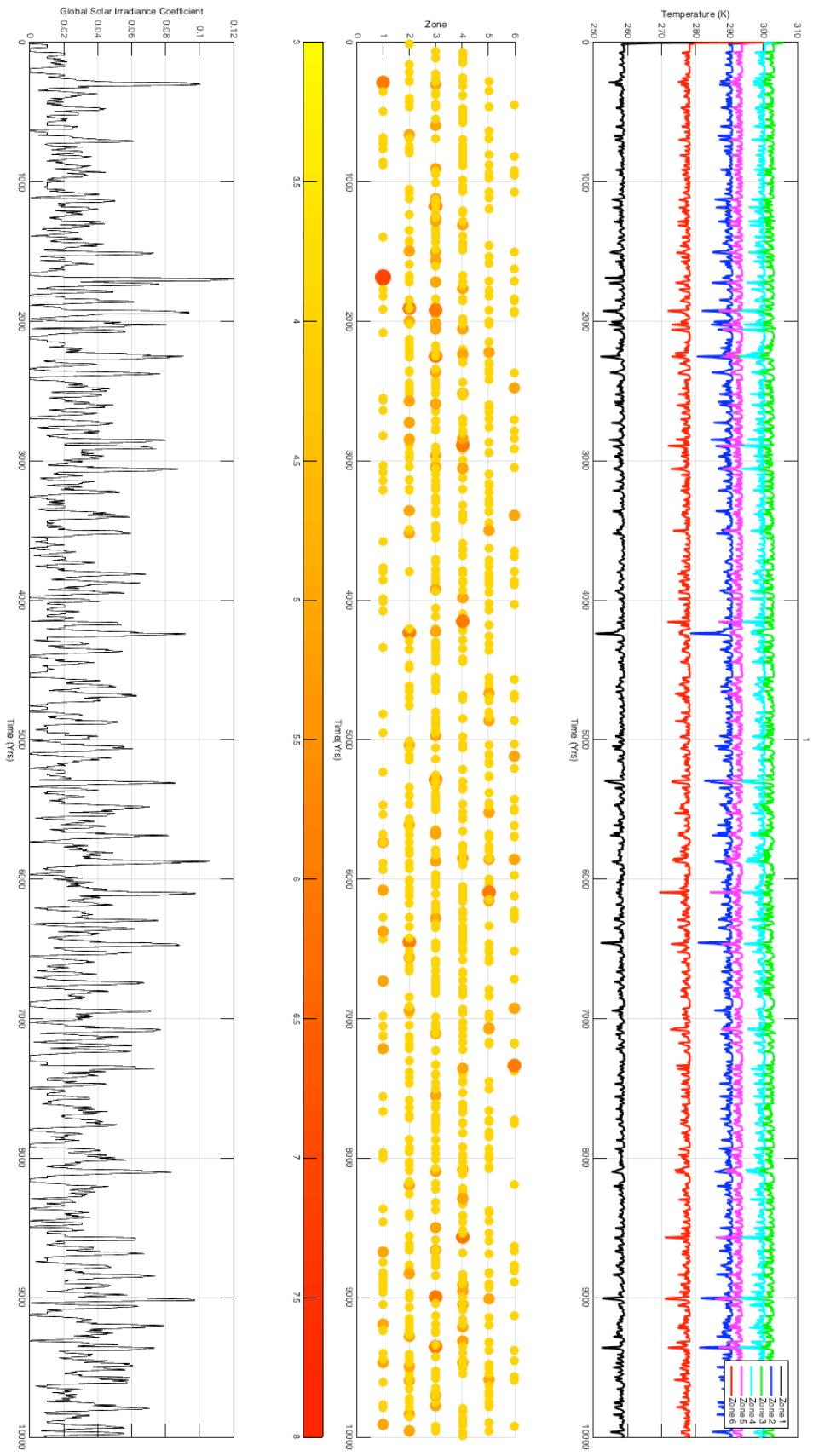
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## **Discussions**

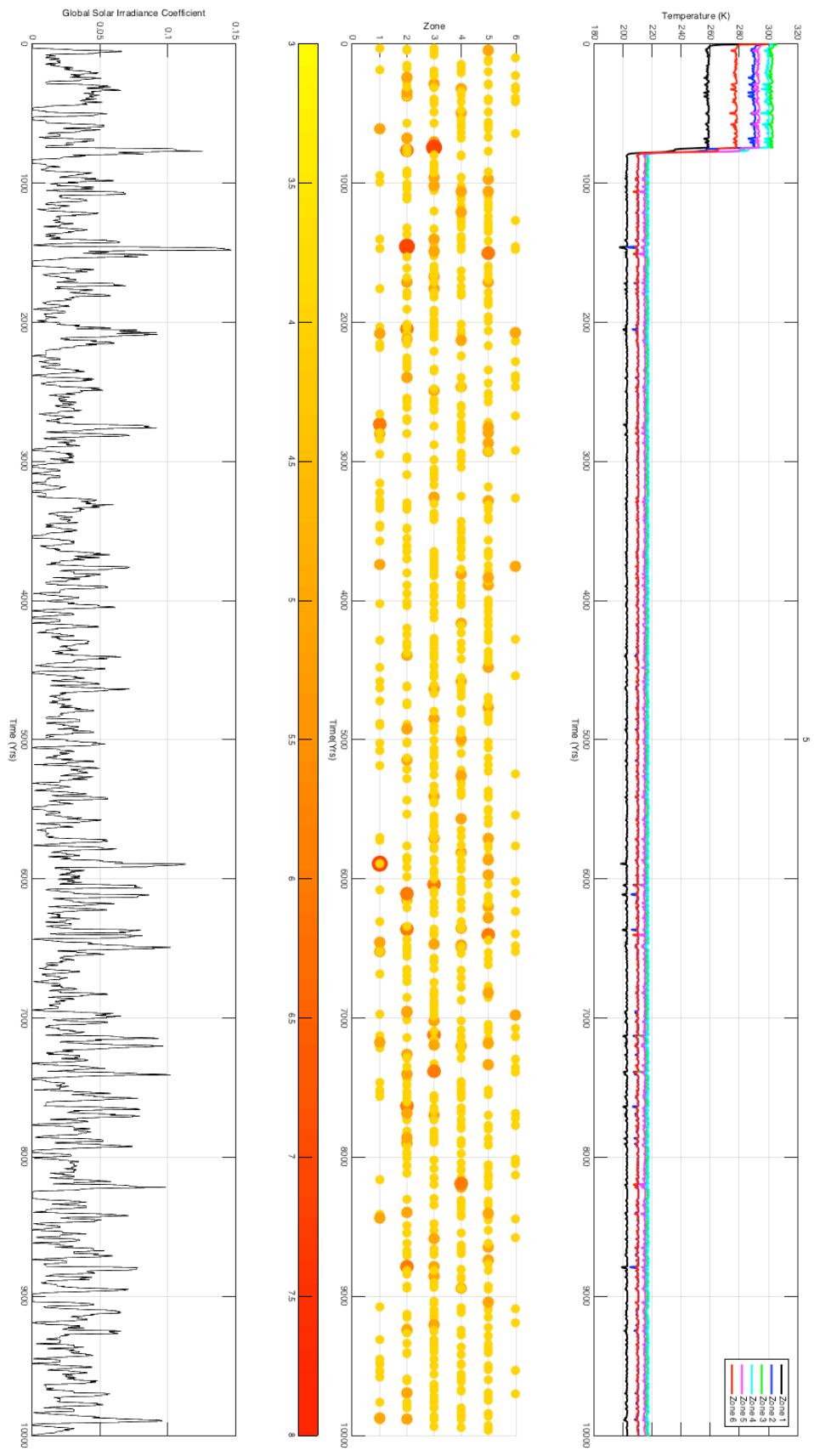
### **1) The steady state**

The steady state model has no forcings so it does not evolve over time, but the equilibrium temperatures of the zones do provide useful information about large scale patterns on the earth's surface today. For instance we observe that land distribution has a significant effect on surface temperatures. The northern tropical zone, with otherwise similar properties to the southern tropics, is 3-4° cooler due to the presence of a larger landmass.

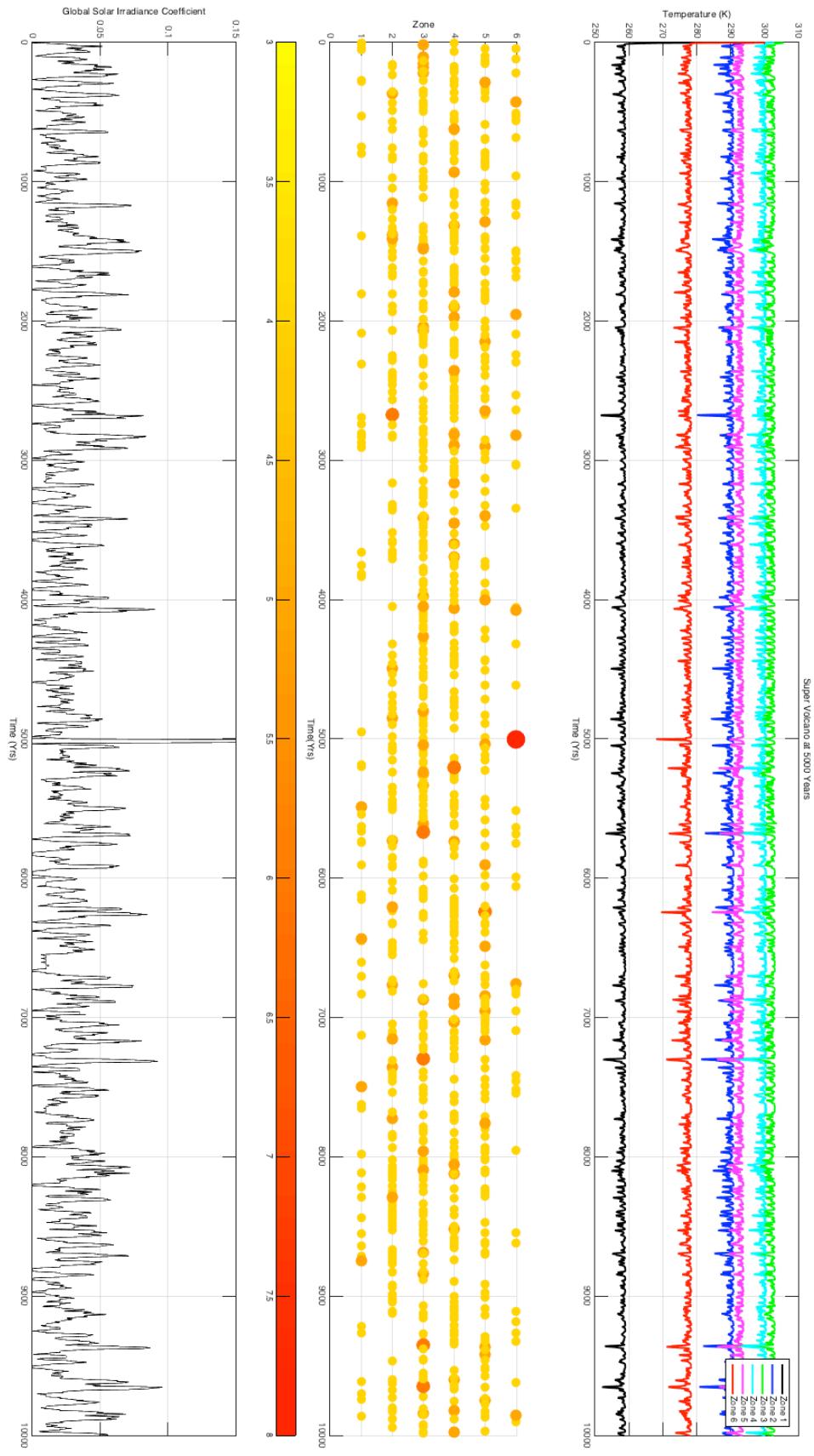
The modifications to the background circulations to account for the gulf stream and southern circumpolar current also had large effects, even after the gulf stream factor was reduced from the initial value of  $5.00E-7 \text{ W}/(\text{m}^*\text{K})$  to  $2.00E-7 \text{ W}/(\text{m}^*\text{K})$ . These effects keep the mean temperature of the north polar regions above freezing, but let the mean



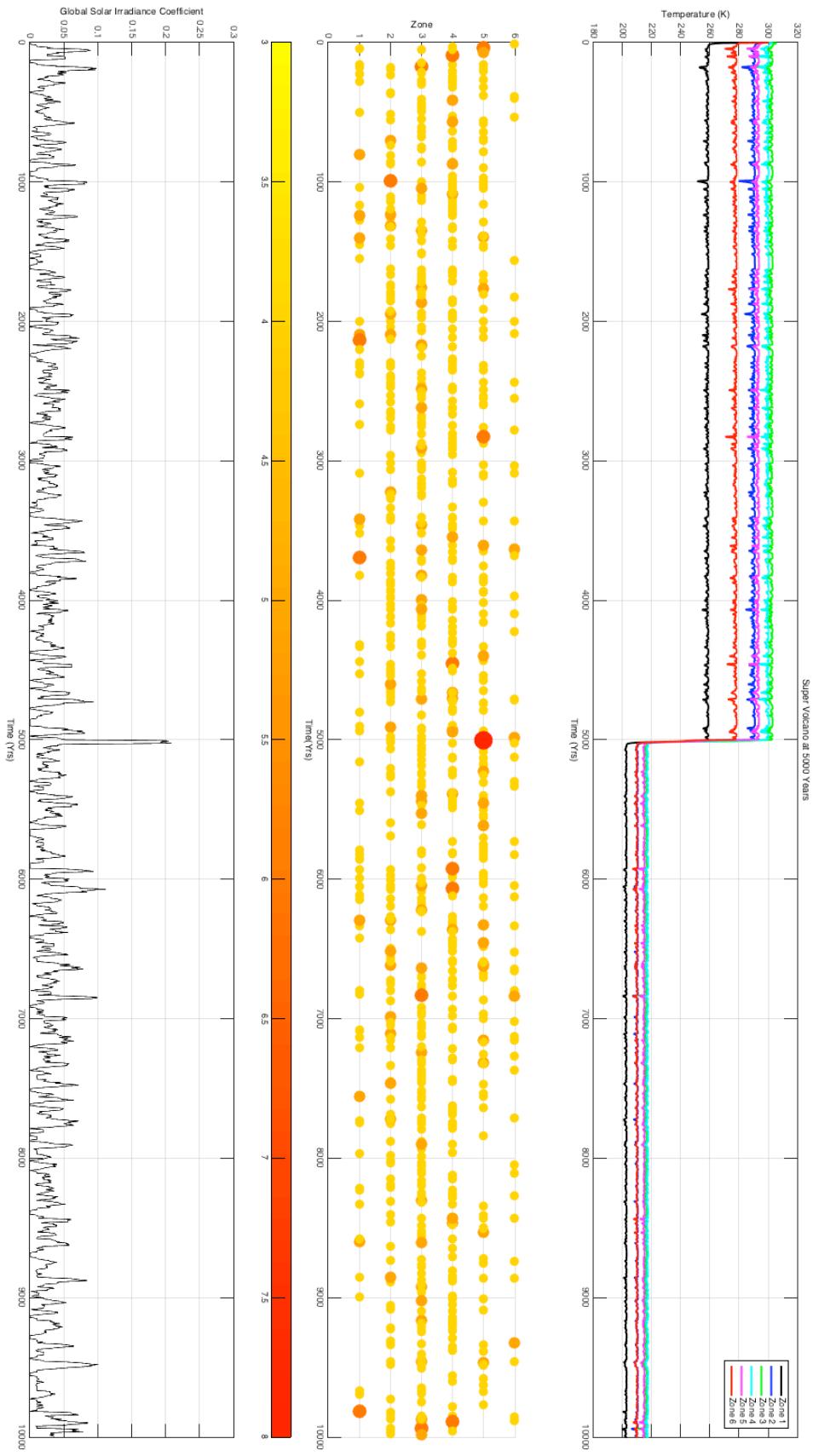
**Figure 3:** First of ten runs of the model over 10000 years with 1000 volcanic events. This model features volcanic forcing with an ice-albedo feedback. (Top): Temperatures in each of the 6 zones over the time span. (Middle): The zonal and temporal distribution of volcanic events generated in this run. Size and color both denote magnitude. (Bottom): Global Solar Irradiance Coefficient smoothed using a 25 year lagged moving average. The coefficient is sum of volcanic activity in each zone, where all of the zones are weighted equally.



**Figure 4:** Fifth of ten runs of the model over 1000 years with 1000 volcanic events. This model features volcanic forcing with an ice-albedo feedback. (Top): Temperatures in each of the 6 zones over the time span. (Middle): The zonal and temporal distribution of volcanic events generated in this run. Size and color both denote magnitude. (Bottom): Global Solar Irradiance Coefficient smoothed using a 25 year lagged moving average. The coefficient is sum of volcanic activity in each zone, where all of the zones are weighted equally.



**Figure 5:** This model features volcanic forcing with an ice-albedo feedback, as well as a Magnitude 8 Super Volcano injected at 5000 years into zone 6. (Top); Temperatures in each of the 6 zones over the time span. (Middle); The zonal and temporal distribution of volcanic events generated in this run. Size and color both denote magnitude. (Bottom); Global Solar Irradiance Coefficient smoothed using a 25 year lagged moving average. The coefficient is sum of volcanic activity in each zone, where all of the zones are weighted equally.



**Figure 6:** This model features volcanic forcing with an ice-albedo feedback, as well as a **Magnitude 8 Super Volcano** injected at 5000 years into zone 5. (Top): Temperatures in each of the 6 zones over the time span. (Middle): The zonal and temporal distribution of volcanic events generated in this run. Size and color both denote magnitude. (Bottom): Global Solar Irradiance Coefficient smoothed using a 25 year lagged moving average. The coefficient is sum of volcanic activity in each zone, where all of the zones are weighted equally.

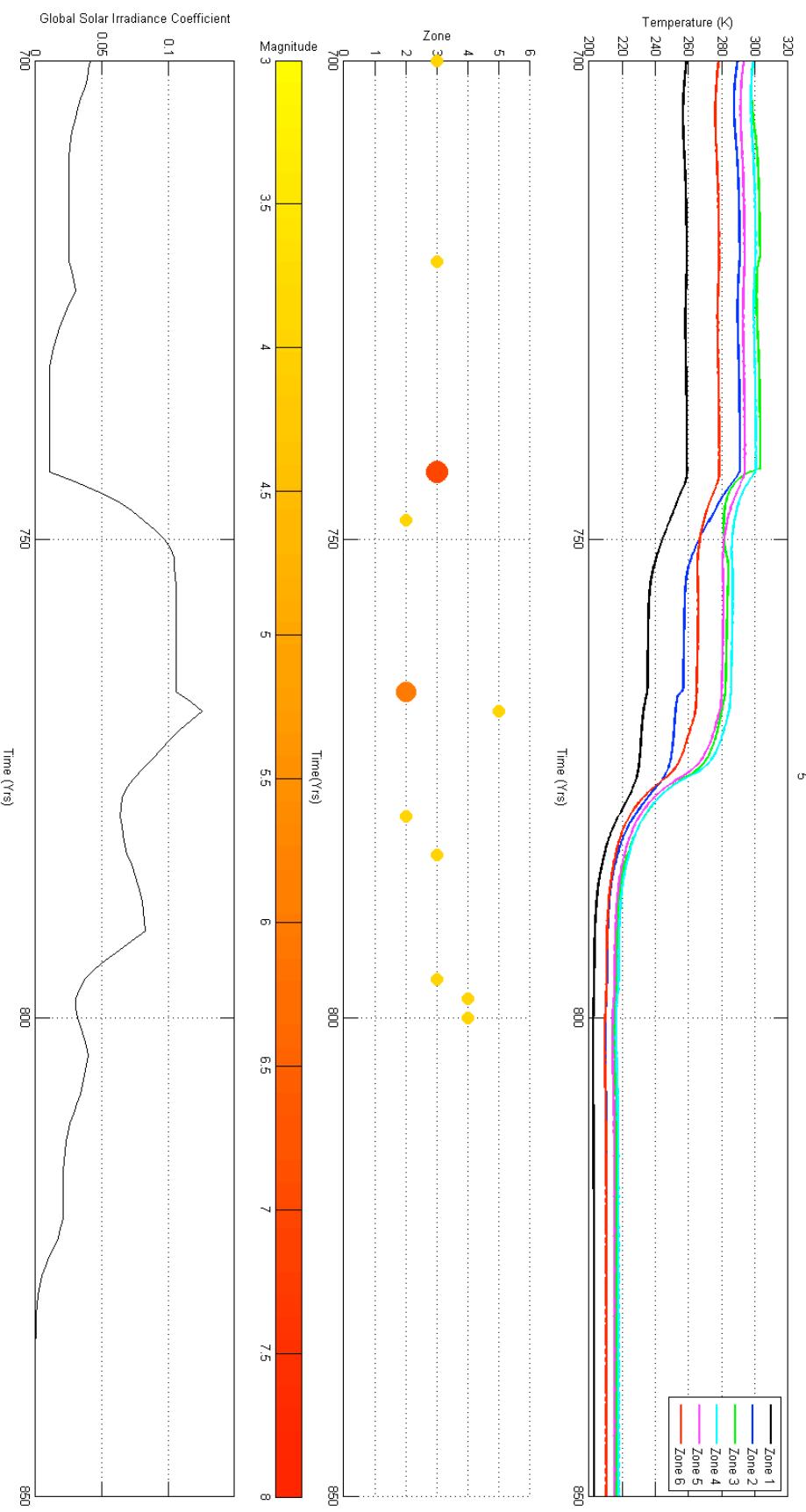
temperature in the Antarctic fall to roughly  $-15^{\circ}\text{C}$ . If these modifications are removed the Antarctic temperature rises again to near freezing point, suggesting that the circumpolar is a first order control on the stability of Antarctic Ice.

## 2) Volcanism

In this setup each volcanic eruption will cause a brief (up to a few years) dip in temperatures, however temperatures will rapidly return to steady state equilibrium following even the largest of events. We find that the zone in which the eruption occurs is very important; eruptions of identical magnitudes in the tropics and poles have very different effects. Eruptions in the tropics tend not only to cool the planet the most, but also to have the widest effects with significant cooling distributed in both hemispheres. Eruptions in the mid latitudes may have strong local cooling, but this tends to be more limited to one hemisphere, with reduced and delayed effects in other zones. Eruptions in the poles, in particular the South Pole, tend to have very limited effects: indeed eruptions in the associated tropics or mid latitudes have a stronger cooling spike on the pole than an eruption in the pole itself. This suggests that volcanoes in the high latitudes (Iceland, Alaska, Antarctica...) likely can only have limited effects on global climate (although local effects may be strong), however tropical volcanoes (Mexico, Indonesia...) can have stronger global cooling episodes.

## 3) Volcanism and ice-albedo

The addition of an ice albedo feedback provides a mechanism for volcanic cooling to be preserved over long timescales. As all zones within the climate system are deeply coupled within this model they either all turn to snowball conditions or all return to their original steady state. Due to modelling constraints the ice albedo effect was limited to a simple albedo-temperature relationship which causes an extremely rapid transition to a fully glaciated world. The random eruption rate is unlikely to cause a transition to a fully glaciated state unless the effects of several large magnitude events are superimposed. An example of this is shown in figure 7, which is figure 4 zoomed in to focus on the 150 years around the freezing event. We see that a large (7.0) event occurred in zone 3 in the year 743, which rapidly cooled the zone 20 degrees. Shortly afterwards zones 2 and 4 cool due to their transfer of heat with zone 3, and the event propagates through the remaining zones with diminishing effect and increasing lag. This single event would likely not have been enough to meet the ice-albedo threshold, but the superposition of the effects from a magnitude 6 in year 766, and a couple magnitude 4s, are enough to break the ice-albedo threshold and rapidly freeze all zones. We note that a peak in the global solar irradiance coefficient of 0.13 coincides with the freezing event. We have not thoroughly investigated the relationship between global solar irradiance coefficient values and freezing events, but in general we find that times where values exceed 0.12



**Figure 7:** Fifth of ten runs of the model focussing on the 700-850 year time period. This model features volcanic forcing with an ice-albedo feedback. (Top): Temperatures in each of the 6 zones over the time span. (Middle): The zonal and temporal distribution of volcanic events generated in this run. Size and color both denote magnitude. (Bottom): Global Solar Irradiance Coefficient smoothed using a 25 year lagged moving average. The coefficient is sum of volcanic activity in each zone, where all of the zones are weighted equally.

are likely freezing events. We feel that incorporating a mechanism to weight zones in this function will increase its applicability.

When we reduce the steady state temperatures, for example by reducing the amount of CO<sub>2</sub> in the atmosphere, we bring the steady state closer to the ice-albedo feedback threshold and this transition becomes more and more likely. For a low enough equilibrium temperature, the first large eruption causes an immediate and global transition into a snowball earth episode. Thus, volcanoes may not be the primary causes of global glaciation events, but may greatly accelerate their formation once background conditions are favourable.

#### 4) Volcanism, ice-albedo, and supervolcano eruption

Introducing supervolcanic eruptions to our model results in an Earth that freezes more often than the aforementioned volcanism model. Super eruptions in the polar regions have a lower ability to freeze the earth than eruptions in mid-latitudes. This is due to the decreased ability of polar regions to transfer heat to non-neighbouring zones. This effectively limits the global impact of supervolcanic eruptions in polar regions. This is best observed in figure 5. The eruption of a supervolcano at 5000 years in zone 6 (North Pole) has a large effect on the irradiance coefficient but a small effect on global temperatures outside of zone 6. Figure 6 has a supervolcanic eruption occur in zone 4. This event rapidly decreases the temperatures of all zones enough to enter a snowball earth event. This case is only run once for each zone, so the results are insufficient to say that a super volcano in equatorial zones will always cause global glaciation.

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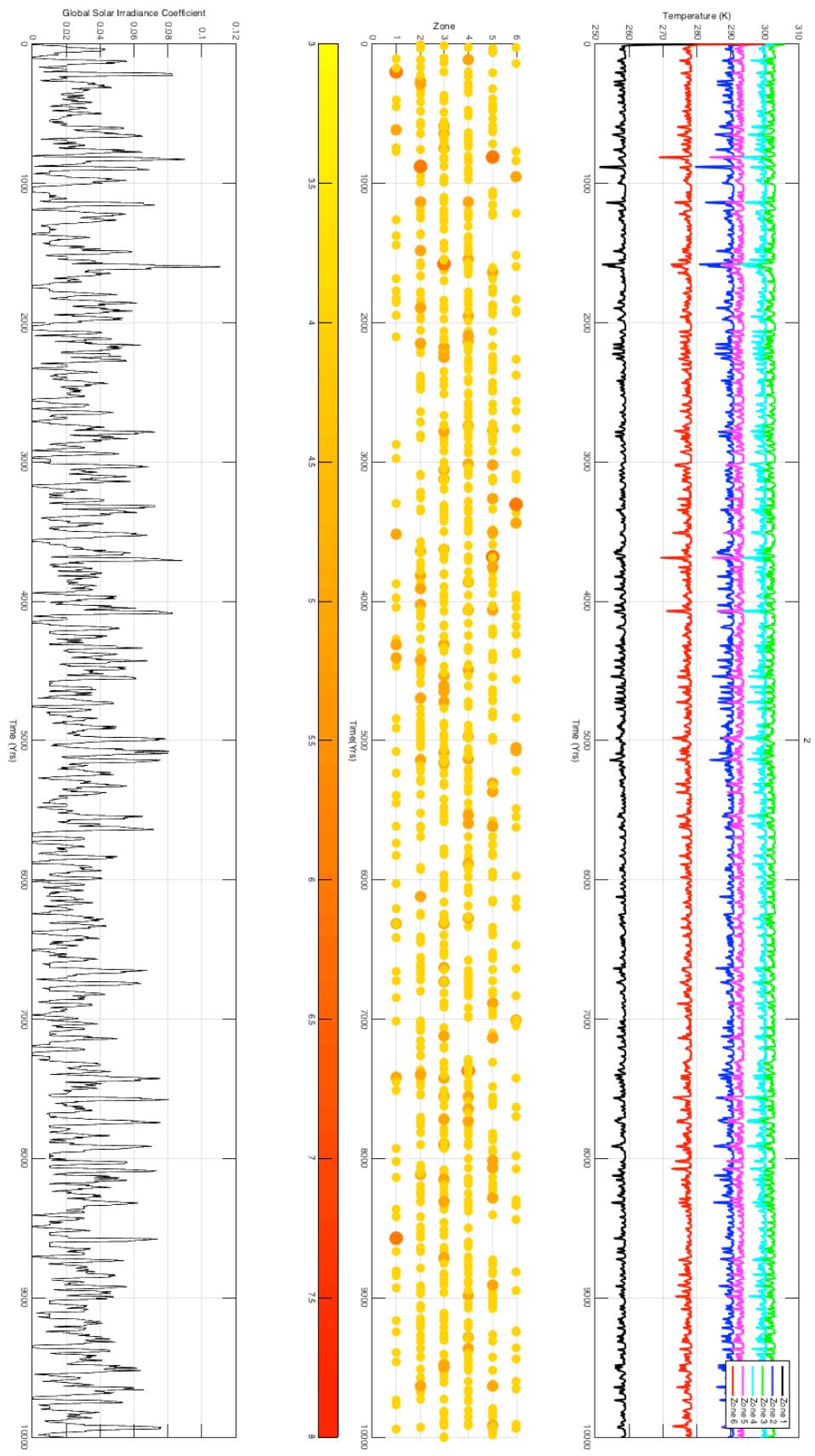
## Conclusions

In this study we built a simple radiative energy model of the climate system to gain insight into the effect of volcanic eruptions. We observe that the zone of an eruption is a strong control of its effects: polar eruptions barely affect other zones whereas tropical eruptions can cool the entire planet. One in ten of the tested 10,000 year cases result in random volcanic activity cooling the entire planet to a snowball earth state. It is shown that a super eruption (magnitude 8) occurring in either zones 2, 3, 4, or 5 (60°N to 60°S) froze the planet every time it was tested.

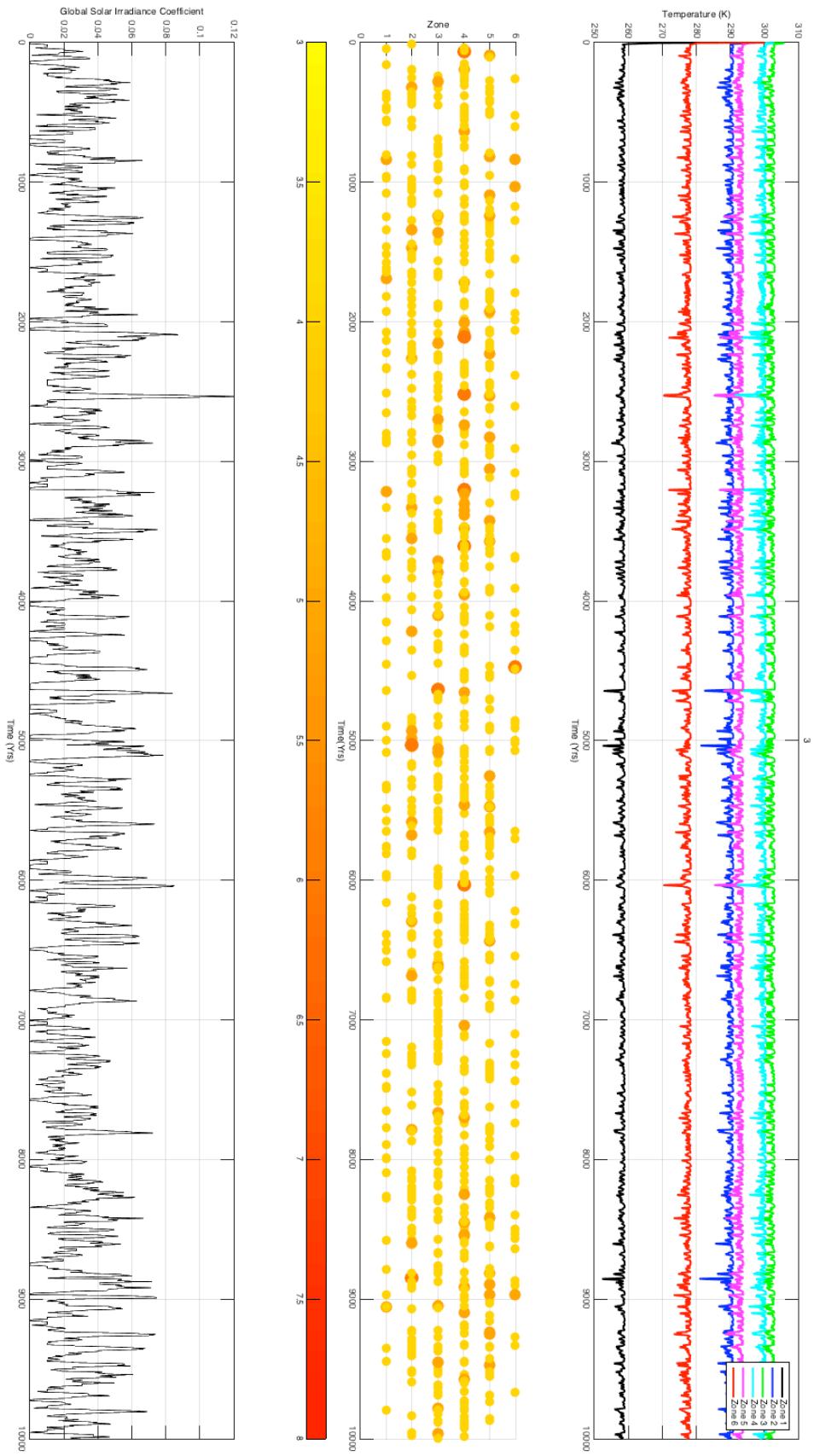
Our ice albedo effects could be improved by replacing the simple albedo relation in our model with a direct link to ice coverage of the zone—this should slow down the cooling but also allow smaller events to contribute to the overall effect. Our model also lacks a means to escape the snowball conditions: further modelling of this problem could include volcanic ash darkening of ice and ejection of volcanic CO<sub>2</sub> gradually increasing

the greenhouse effect (C-Si cycle). This would allow us to meaningfully run the model over longer timescales and observe whether glacial/interglacial cycles can be naturally generated without orbital forcing. The simplicity of our initial conditions, computational time step (1 year) and lack of longitudinal zoning mean that finer spatial and temporal differences cannot be resolved.

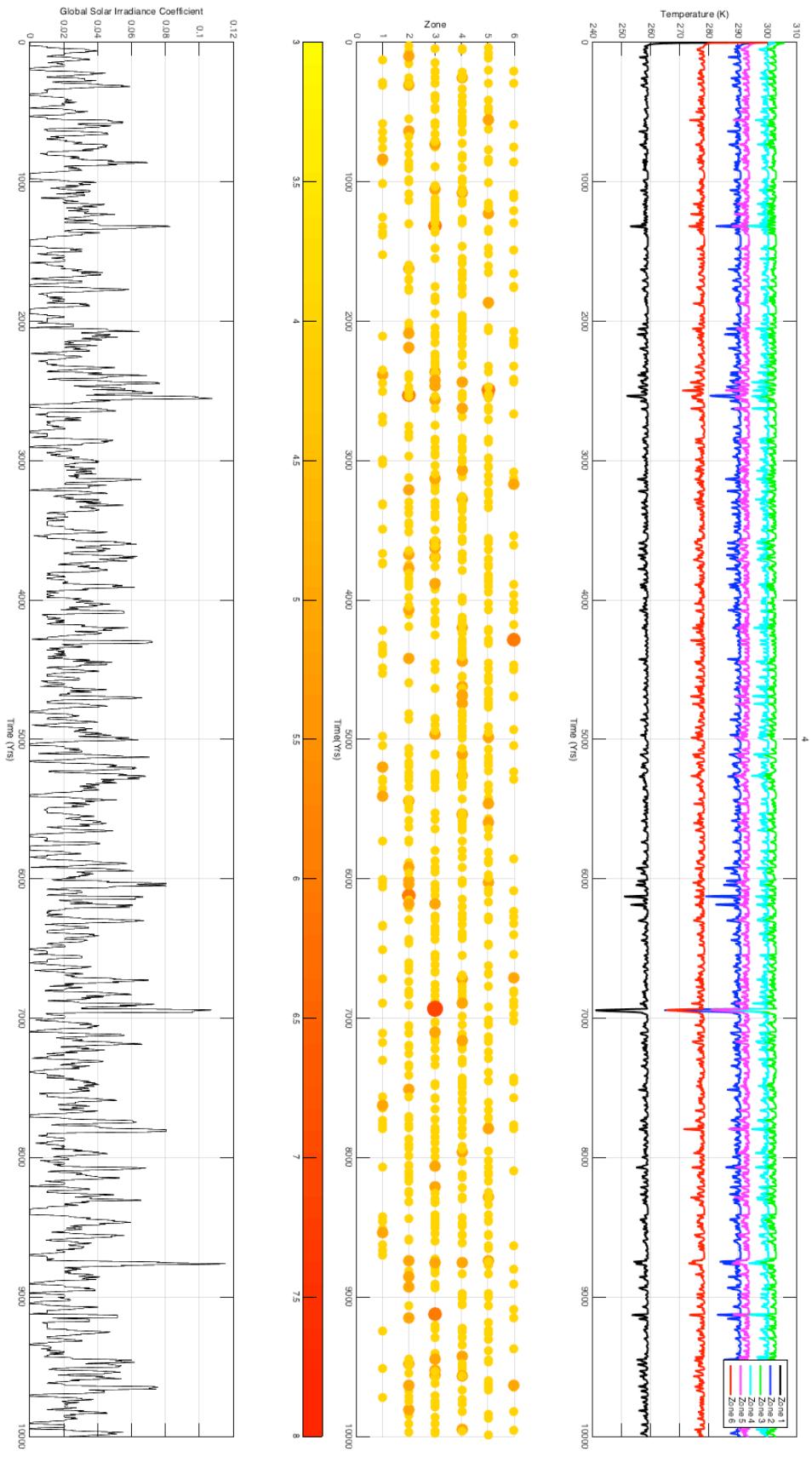
Our results show that snowball earth is a surprisingly possible (10%) outcome in a system with random volcanic activity. These results support the snowball earth theory, because if it was not possible to achieve this state with random volcanism, it would indicate that a specific combination of events is required. The likelihood of such volcanic activity actually having occurred in the past is slim.



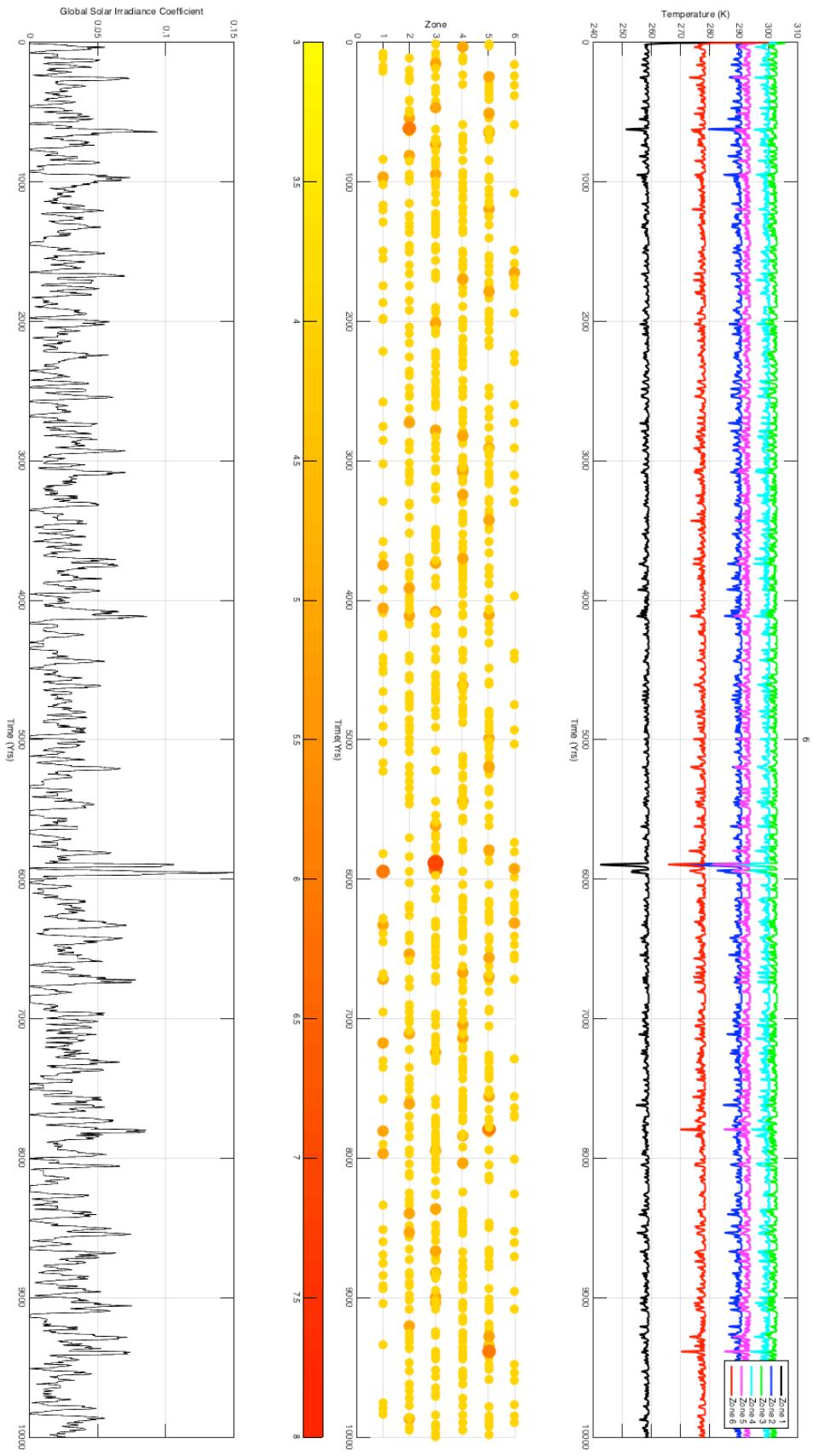
**Appendix I:** Second of ten runs of the model over 1000 years with 1000 volcanic events. This model features volcanic forcing with an ice-albedo feedback.  
 (Top): temperatures in each of the 6 zones over the time span. (Middle): The zonal and temporal distribution of volcanic events generated in this run.  
 Size and color both denote magnitude. (Bottom): Global Solar Irradiance Coefficient smoothed using a 25 year lagged moving average. The coefficient is sum of volcanic activity in each zone, where all of the zones are weighted equally.



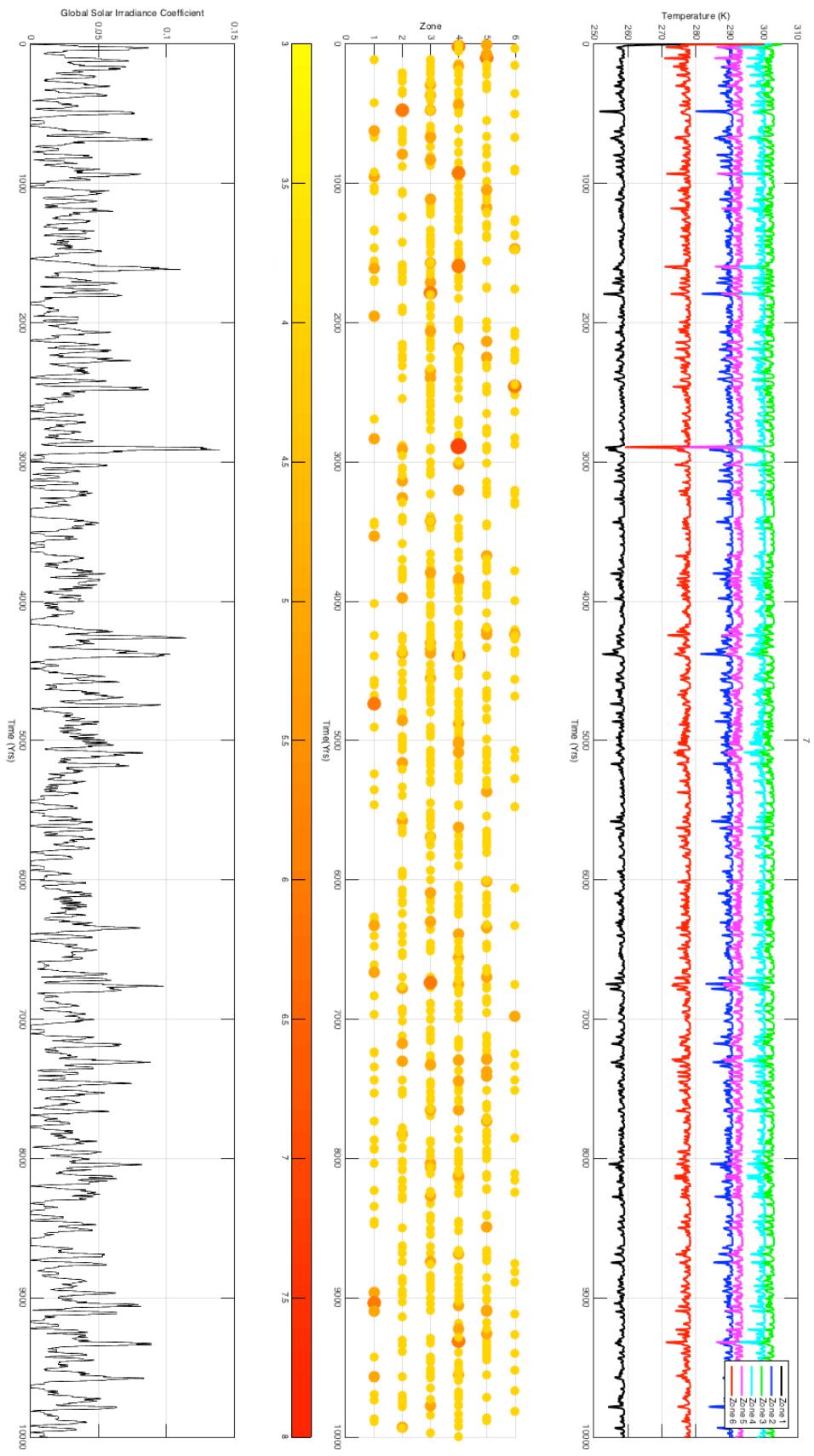
**Appendix 2:** Third of ten runs of the model over 10000 years with 1000 volcanic events. This model features volcanic forcing with an ice-albedo feedback.  
 (Top): Temperatures in each of the 6 zones over the time span. (Middle): The zonal and temporal distribution of volcanic events generated in this run. Site and color both denote magnitude. (Bottom): Global Solar Irradiance Coefficient smoothed using a 25 year lagged moving average. The coefficient is sum of volcanic activity in each zone, where all of the zones are weighted equally.



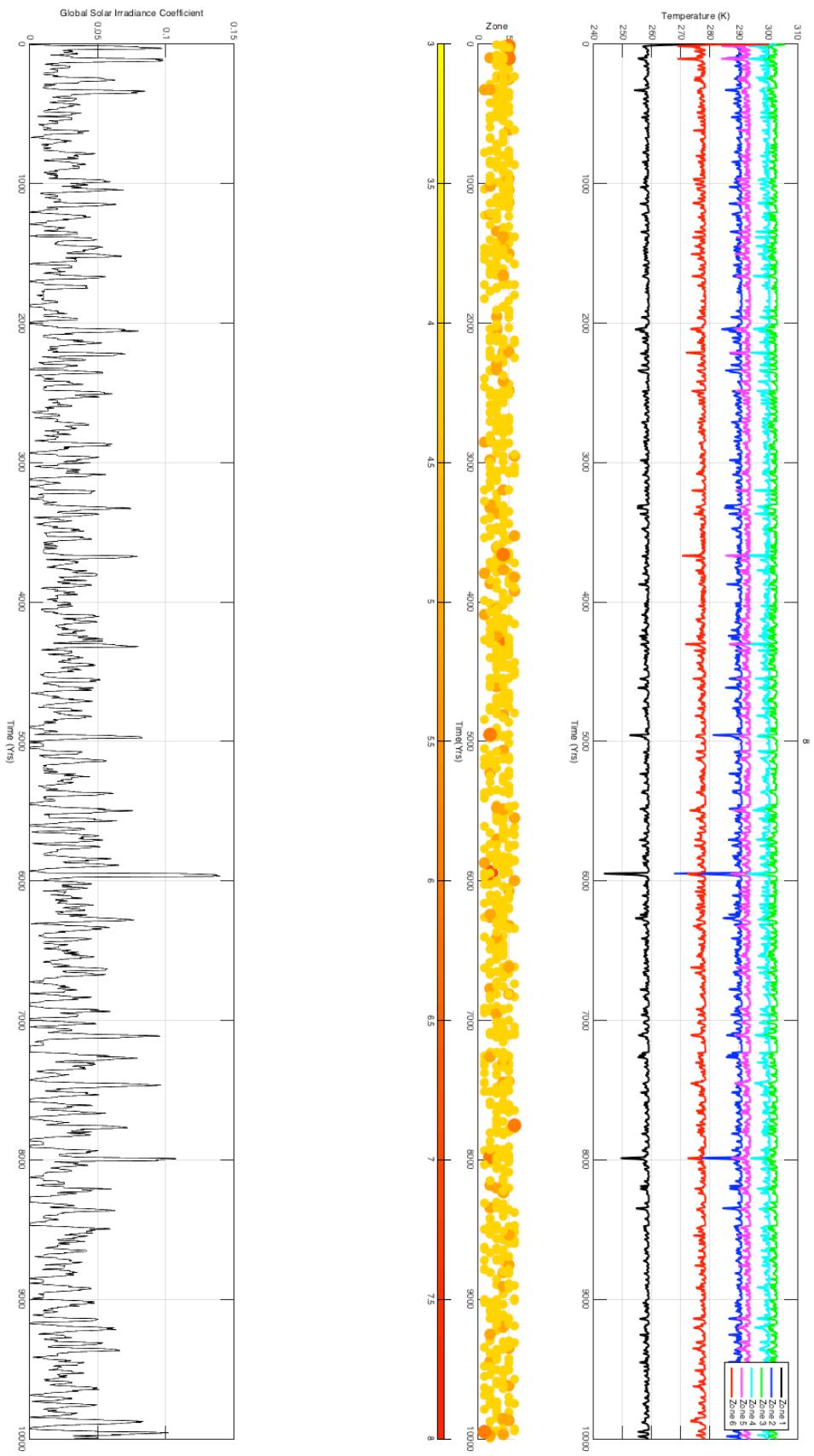
**Appendix 3:** Fourth of ten runs of the model over 10000 years with 1000 volcanic events. This model features volcanic forcing with an ice-albedo feedback. (Top): Temperatures in each of the 6 zones over the time span. (Middle): The zonal and temporal distribution of volcanic events generated in this run. Size and color both denote magnitude. (Bottom): Global Solar Irradiance Coefficient smoothed using a 25 year lagged moving average. The coefficient is sum of volcanic activity in each zone, where all of the zones are weighted equally.



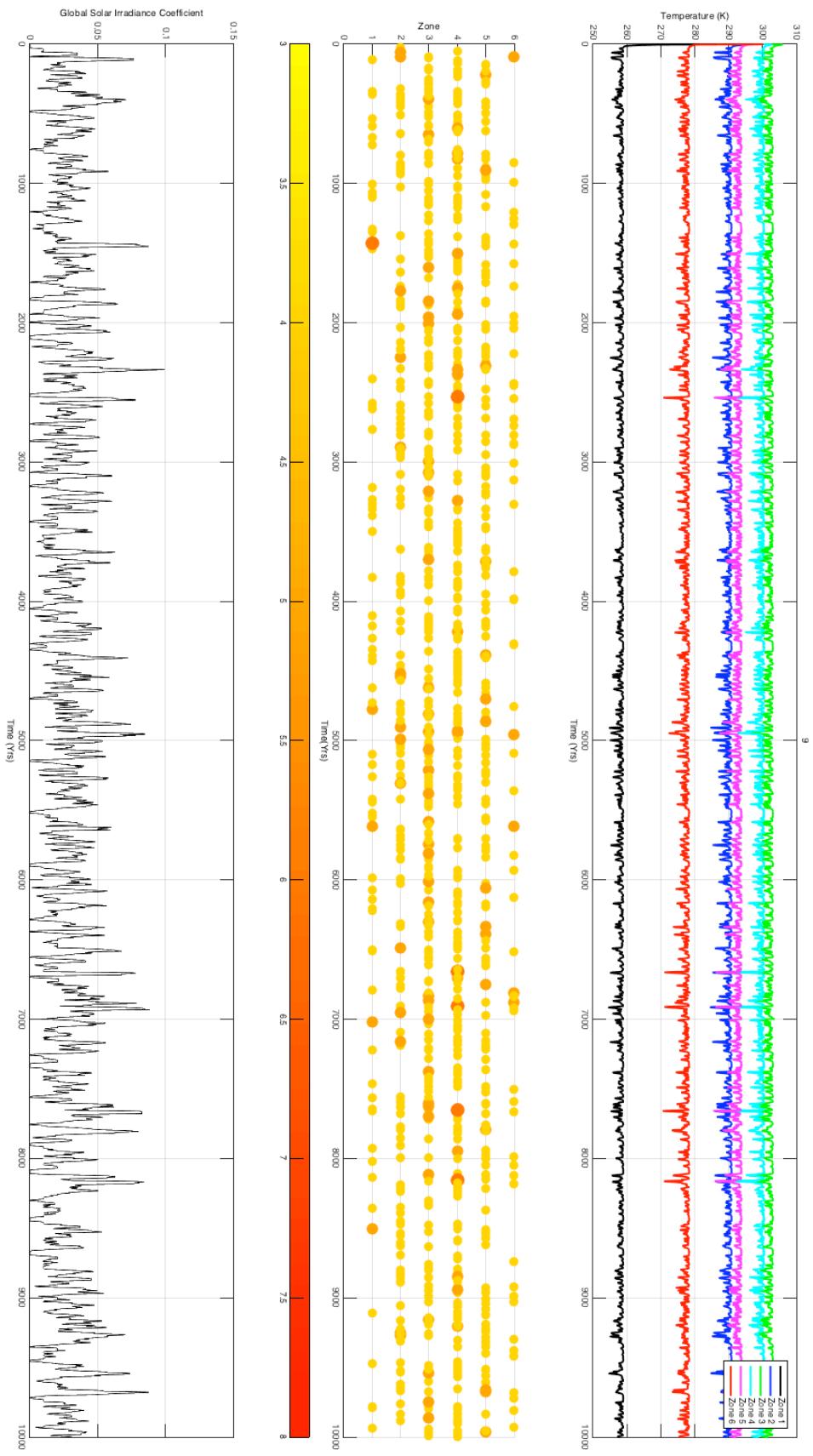
**Appendix 4:** Six of ten runs of the model over 10000 years with 1000 volcanic events. This model features volcanic forcing with an ice-albedo feedback.  
 (Top): Temperatures in each of the 6 zones over the time span. (Middle): The zonal and temporal distribution of volcanic events generated in this run. Site and color both denote magnitude. (Bottom): Global Solar Irradiance Coefficient smoothed using a 25 year lagged moving average. The coefficient is sum of volcanic activity in each zone, where all of the zones are weighted equally.



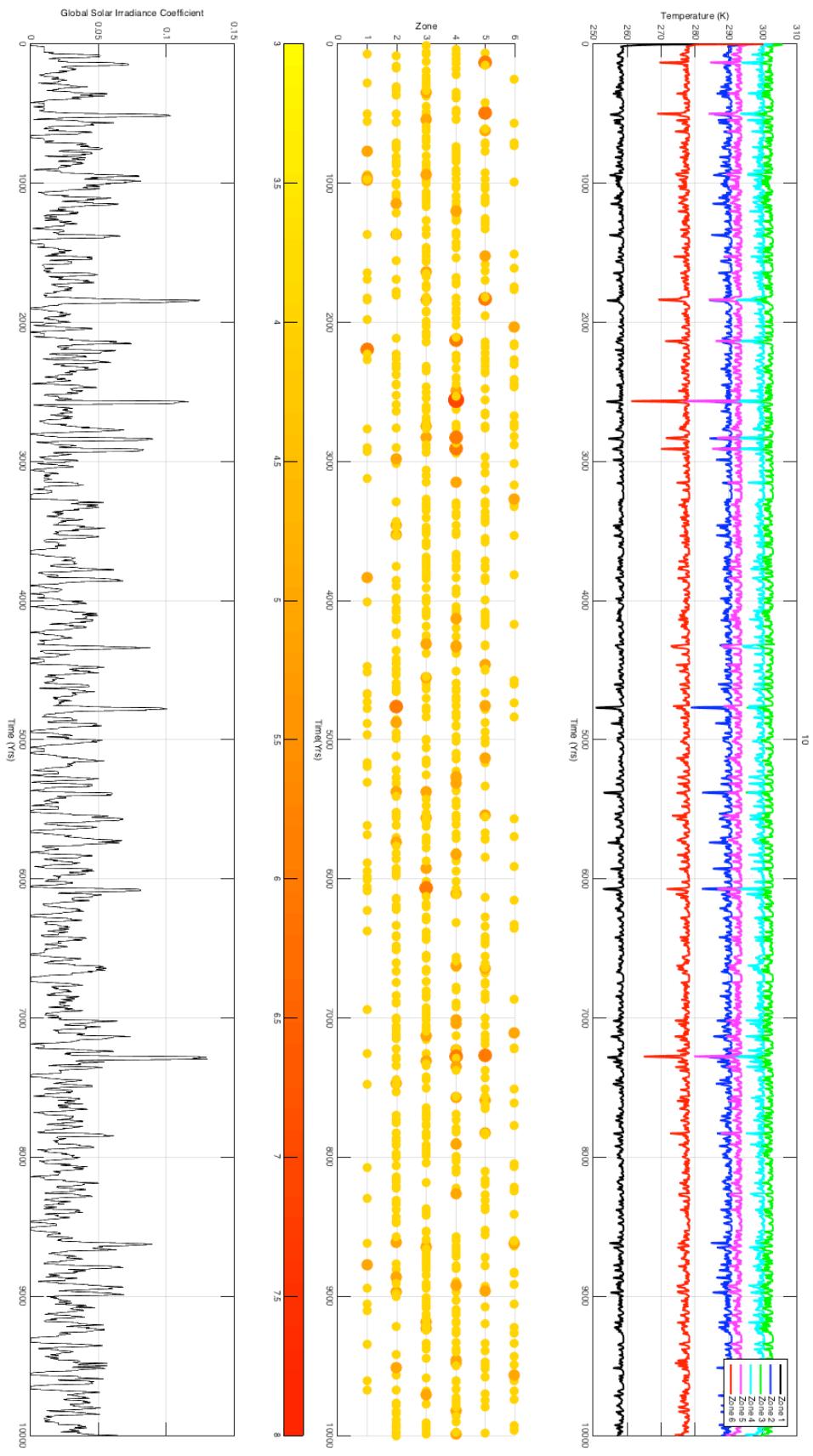
**Appendix 5:** Seventh of ten runs of the model over 10000 years with 1000 volcanic events. This model features volcanic forcing with an ice-albedo feedback. (Top): Temperatures in each of the 6 zones over the time span. (Middle): The zonal and temporal distribution of volcanic events generated in this run. Site and color both denote magnitude. (Bottom): Global Solar Irradiance Coefficient smoothed using a 25 year lagged moving average. The coefficient is sum of volcanic activity in each zone, where all of the zones are weighted equally.



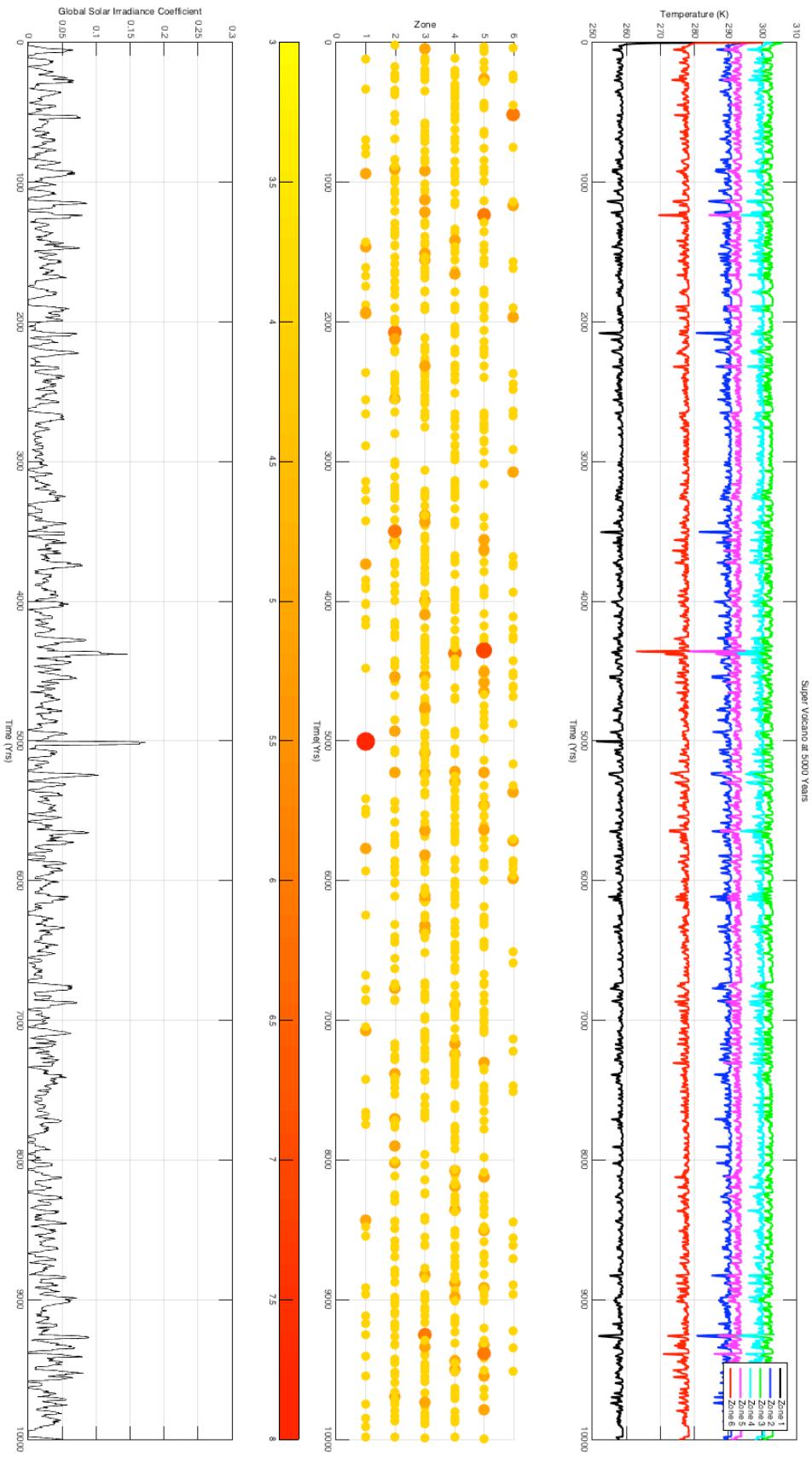
**Appendix 6:** Eighth of ten runs of the model over 10000 years with 1000 volcanic events. This model features volcanic forcing with an ice-albedo feedback.  
 (Top): Temperatures in each of the 6 zones over the time span. (Middle): The zonal and temporal distribution of volcanic events generated in this run. Site and color both denote magnitude. (Bottom): Global Solar Irradiance Coefficient smoothed using a 25 year lagged moving average. The coefficient is sum of volcanic activity in each zone, where all of the zones are weighted equally.



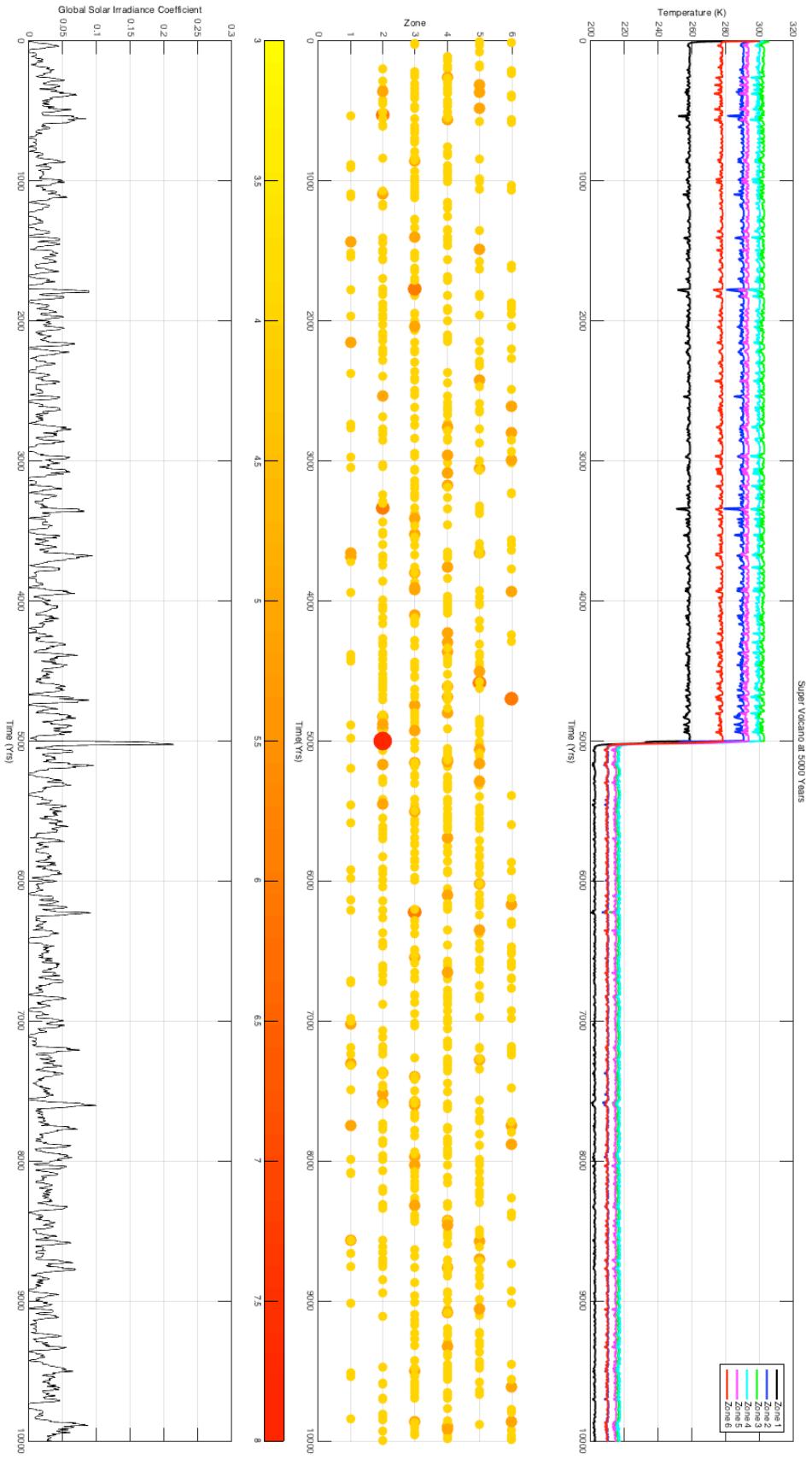
**Appendix 7:** Ninth of ten runs of the model over 10000 years with 1000 volcanic events. This model features volcanic forcing with an ice-albedo feedback.  
 (Top): Temperatures in each of the 6 zones over the time span. (Middle): The zonal and temporal distribution of volcanic events generated in this run. Site and color both denote magnitude. (Bottom): Global Solar Irradiance Coefficient smoothed using a 25 year lagged moving average. The coefficient is sum of volcanic activity in each zone, where all of the zones are weighted equally.



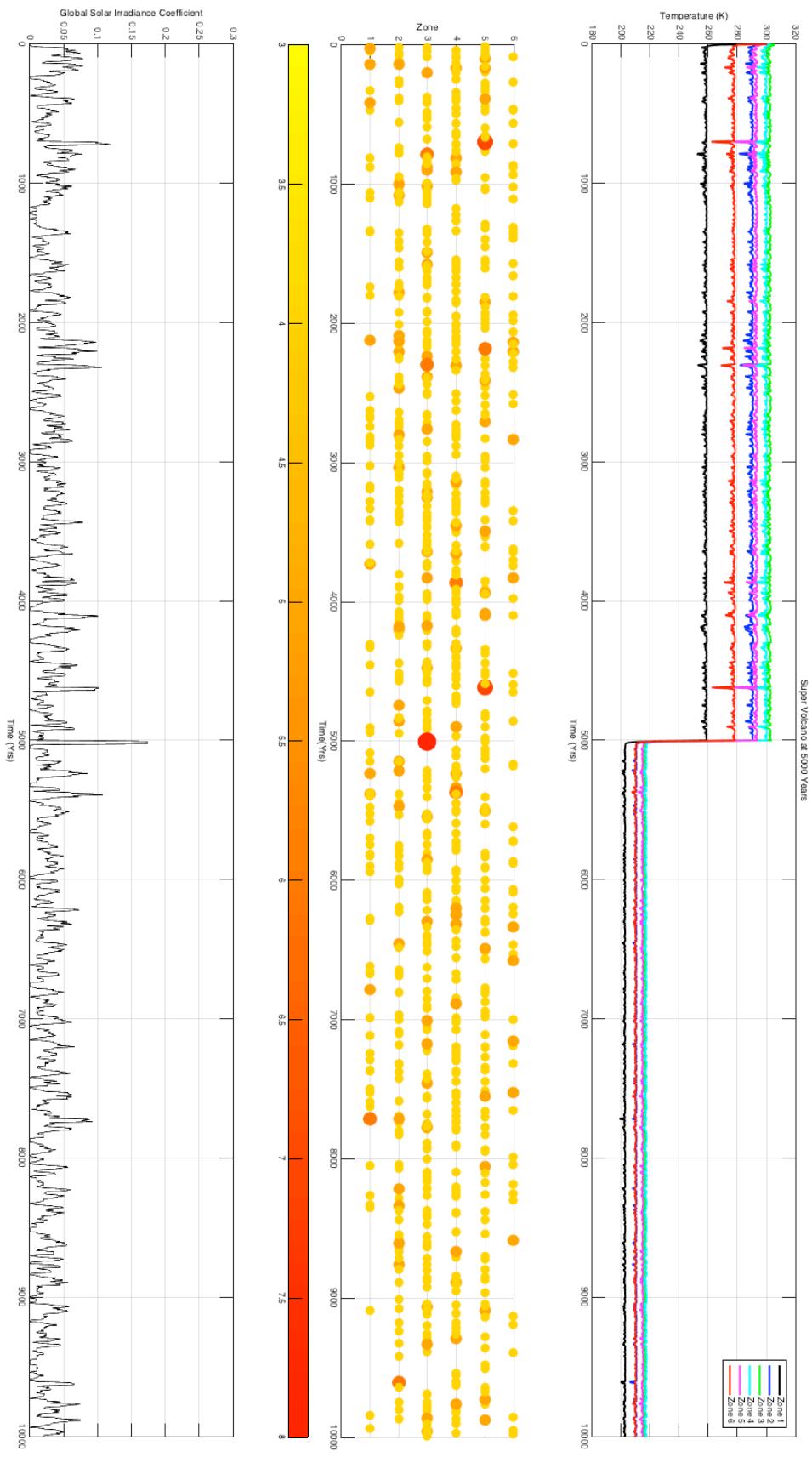
**Appendix 8:** Tenth of ten runs of the model over 10000 years with 1000 volcanic events. This model features volcanic forcing with an ice-albedo feedback.  
 (Top): Temperatures in each of the 6 zones over the time span. (Middle): The zonal and temporal distribution of volcanic events generated in this run. Site and color both denote magnitude. (Bottom): Global Solar Irradiance Coefficient smoothed using a 25 year lagged moving average. The coefficient is sum of volcanic activity in each zone, where all of the zones are weighted equally.



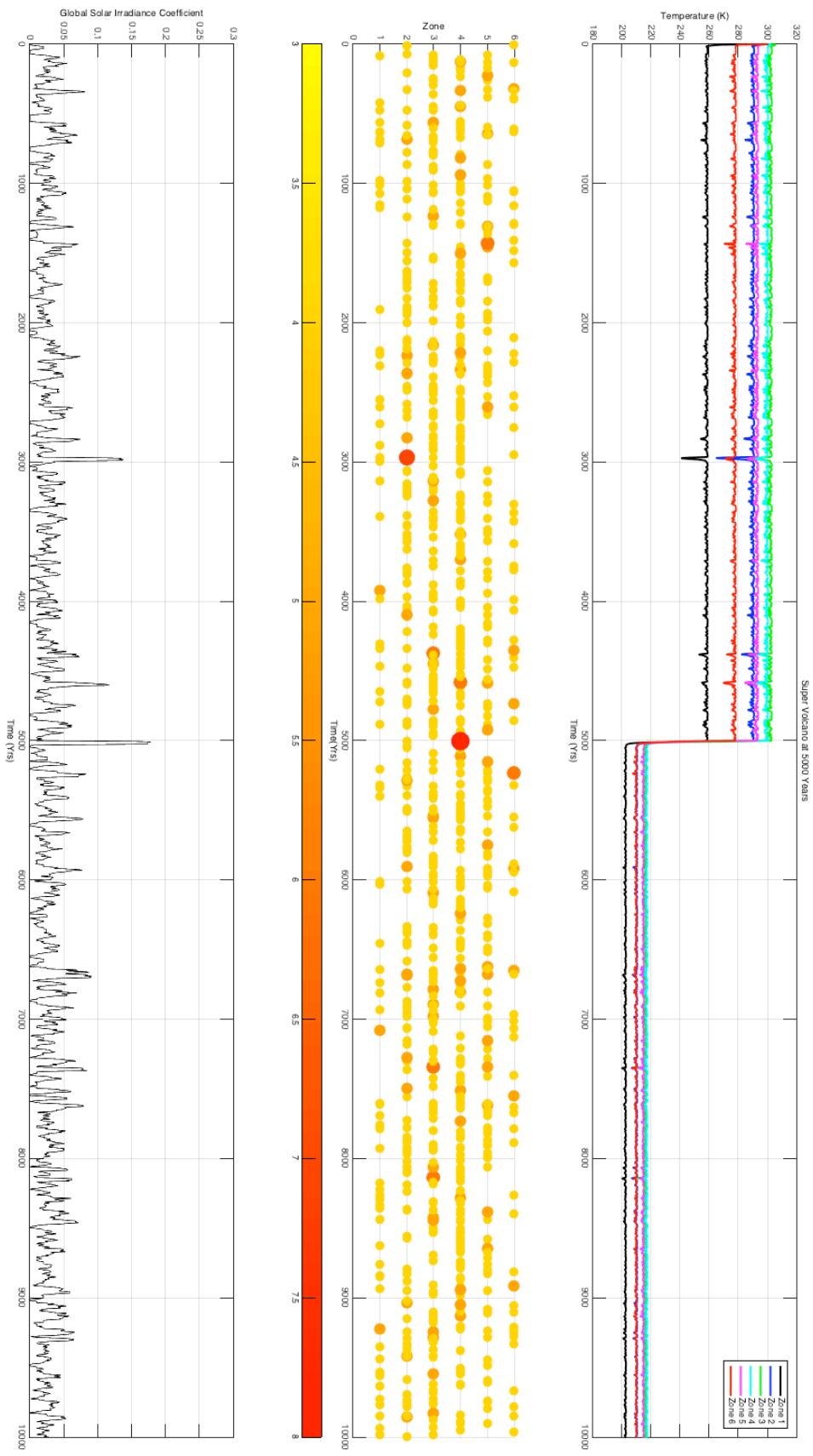
**Appendix 9:** This model features volcanic forcing with an ice-albedo feedback, as well as a Magnitude 8 Super Volcano injected at 5000 years. (Top): Temperatures in each of the 6 zones over the time span. (Middle): The zonal and temporal distribution of volcanic events generated in this run. Size and color both denote magnitude. (Bottom): Global Solar Irradiance Coefficient smoothed using a 25 year lagged moving average. The coefficient is sum of volcanic activity in each zone, where all of the zones are weighted equally.



**Appendix 10:** This model features volcanic forcing with an ice-albedo feedback, as well as a **Magnitude 8 SuperVolcano** injected at 5000 years. (Top): Temperatures in each of the 6 zones over the time span. (Middle): The zonal and temporal distribution of volcanic events generated in this run. Size and color both denote magnitude. (Bottom): Global Solar Irradiance Coefficient smoothed using a 25 year lagged moving average. The coefficient is sum of volcanic activity in each zone, where all of the zones are weighted equally.



**Appendix II:** This model features volcanic forcing with an ice-albedo feedback, as well as a **Magnitude 8 SuperVolcano** injected at 5000 years. (Top): Temperatures in each of the 6 zones over the time span. (Middle): The zonal and temporal distribution of volcanic events generated in this run. Size and color both denote magnitude. (Bottom): Global Solar Irradiance Coefficient smoothed using a 25 year lagged moving average. The coefficient is sum of volcanic activity in each zone, where all of the zones are weighted equally.



**Appendix I 2:** This model features volcanic forcing with an ice-albedo feedback, as well as a Magnitude 8 Super Volcano injected at 5000 years. (Top): Temperatures in each of the 6 zones over the time span. (Middle): The zonal and temporal distribution of volcanic events generated in this run. Size and color both denote magnitude. (Bottom): Global Solar Irradiance Coefficient smoothed using a 25 year lagged moving average. The coefficient is sum of volcanic activity in each zone, where all of the zones are weighted equally.

```
%% A2 EOSC 453
% Max, Joel and Keegan

%% Define Input Parameters
clear; close all; clear global
% Make variables that will be used in the ODE global
global k L ao a pcZaverage sigmaB S0 tau asky gamma A phi1 phi2 ...
    phi3 phi4 phi5 phi6 ai Tlow Thigh To count;

% % Load RNG Seed
% load('seed1.mat');
%
% % Reset RNG
% rng(s);

% Thermal Exchange Coefficients (W/mK)
k = zeros(6);
k(1,2) = 0.5*10^7; % Reduced Transfer from Southern Ocean
k(2,3) = 1*10^7;
k(3,4) = 1*10^7;
k(4,5) = 2*10^7; % Note: This was reduced in order to see temps we ↴
felt
k(5,6) = 1*10^7; % were more likely

% Boundary Length (m)
L = zeros(6);
L(1,2) = 2.0015*10^7;
L(2,3) = 3.4667*10^7;
L(3,4) = 4.0030*10^7;
L(4,5) = 3.4667*10^7;
L(5,6) = 2.0015*10^7;

% Area Fractions of Zones
areafrac = [0.0670 0.1830 0.2500 0.2500 0.1830 0.0670];

% Geometric Factor
gamma = [0.1076 0.2277 0.3045 0.3045 0.2277 0.1076];
```

## % Constants

```
sigmaB = 5.6696*10^-8; % W m?2 K?4 Stefan-Boltzmann constant
S0 = 1368; % W m?2 Solar constant
RE = 6371*10^3; %m Radius of Earth,
AE = 4*pi*RE^2; % Surface area of Earth
eta = 1; % Total emissivity of Earth
tau = 0.63; % Atmospheric transmissivity
asky = 0.2; % Atmospheric albedo
al = 0.4; % Albedo of land surface
as = 0.1; % Albedo of ocean surface
ai = 0.6; % Albedo of ice
pl = 2500; % kg m?3 Density of land surface
ps = 1028; % kg m?3 Density of ocean water
pi = 900; % kg m?3 Density of ice
Zl = 1.0; % m Thermal scale depth for land
Zs = 70.0; % m Thermal scale depth for ocean
Zi = 1.0; % m Thermal scale depth for ice
cl = 790; % J kg K?1 Specific heat capacity for land
cs = 4187; % J kg K?1 Specific heat capacity for water
ci = 2060; % J kg K?1 Specific heat capacity for ice
fl = [0.05 0.25 0.3 0.45 0.45 0.35]; % Land Fraction
fs = [0.50 0.74 0.7 0.55 0.54 0.35]; % Sea Fraction
fi = [0.45 0.01 0.0 0.00 0.01 0.30]; % Ice Fraction
sec_per_year = 365*24*3600; % Seconds per year
Tlow = 260;
Thigh = 290;
count = 1;
```

## % Area of Zones

```
A = AE*areafrac;
```

## % Initial Temperature Conditions

```
To = [290 290 290 290 290 290]+10;
```

## % Time Parameters

```
starttime_Ma = 0;
starttime_s = starttime_Ma*1000000*365*24*3600;

endtime_Ma = 0.01;
```

```

endtime_s = endtime_Ma*1000000*365*24*3600;

% Surface Average Values

pcZaverage(1)=f1(1)*pl*cl*Zl+fs(1)*ps*cs*zs+fi(1)*pi*ci*zi;
pcZaverage(2)=f1(2)*pl*cl*Zl+fs(2)*ps*cs*zs+fi(2)*pi*ci*zi;
pcZaverage(3)=f1(3)*pl*cl*Zl+fs(3)*ps*cs*zs+fi(3)*pi*ci*zi;
pcZaverage(4)=f1(4)*pl*cl*Zl+fs(4)*ps*cs*zs+fi(4)*pi*ci*zi;
pcZaverage(5)=f1(5)*pl*cl*Zl+fs(5)*ps*cs*zs+fi(5)*pi*ci*zi;
pcZaverage(6)=f1(6)*pl*cl*Zl+fs(6)*ps*cs*zs+fi(6)*pi*ci*zi;

% Avg Albedo
ao(1)= f1(1)*al+fs(1)*as+fi(1)*ai;
ao(2)= f1(2)*al+fs(2)*as+fi(2)*ai;
ao(3)= f1(3)*al+fs(3)*as+fi(3)*ai;
ao(4)= f1(4)*al+fs(4)*as+fi(4)*ai;
ao(5)= f1(5)*al+fs(5)*as+fi(5)*ai;
ao(6)= f1(6)*al+fs(6)*as+fi(6)*ai;
a = ao;

% %% Calc ODE for Steady State
% [time, Temps] = ode15s(@tempodes,[starttime_s endtime_s], To);
%
% figure(1)
% plot(time/sec_per_year,Temps)
% legend('1','2','3','4','5','6')
% title('Steady State (ODE15s)')
%
% t1 = Temps(end,:);

%% Random Events Effecting the Incoming Solar Radiation Coefficient ↵
(Phi)
% We want it to have random magnitudes, at random times, in random ↵
zones.
% This section will initially create a phi that is all ones, so ↵
that it
% will not effect steady state.

% Number of Volcanic Events
num_events = 1000;

```

```
% Create Incoming Solar Radiation Coefficient for each zone
phil(:,1) = starttime_s:sec_per_year:endtime_s; %s
phil(:,2) = ones(size(phi1(:,1)));
phi2=phi1;
phi3=phi1;
phi4=phi1;
phi5=phi1;
phi6=phi1;

% Find a Zone
zone_event = randi([1 100],1,num_events);

% Find percent of sphere each zone takes up
areaperc = round(areafrac*100);

% Distribute zones according to percent of sphere
for g = 1:length(areaperc);

    gg = zone_event <= sum(areaperc(1:g)) & zone_event > sum(areaperc(1:g-1));
    zone_event(gg) = g;

end

% Load a Magnitude Population that has a power rule distribution of
% magnitudes.
load('logdist.mat');
mag_power = 4;
mag_scale = 2;
% Randomly sample this data set to get the desired number of events
mag_event = randsample(mags, num_events)/10;

%Get a point in time
time_event = randi([starttime_s endtime_s]/sec_per_year,1,%
num_events); %s

% Force a 8.0 mag event in zone 4 at the center, if none happened
if nnz(mag_event == 8) == 0 & ~isempty(mag_event) > 0;
```

```
mag_event(end) = 0.8;
zone_event(end) = 1;
time_event(end) = (endtime_s - starttime_s)/(sec_per_year^2);
end

mag_event_effect = mag_scale*(mag_event.^mag_power);

% Set a linear rebound time scale for phi after an event. This value ↵
will be
% multiplied with the magnitude to determine how many years it takes ↵
to
% return to normal
rebound = 10;

% Loop over the number of volcanic events
for i = 1:num_events;

    %Find where it is in phi's time
    [dummy, closest_time] = min(abs(phi1(:,1) - time_event(i) ↵
*sec_per_year));

    %Define event
    event = linspace((1 - mag_event_effect(i)), 1, rebound*ceil(
(mag_event_effect(i))));

    % Index the rows in phi that will be replaced with the event
    ii = closest_time:closest_time+length(event)-1;

    % Replace values in phi with the event and distribute to zones
    switch zone_event(i)

        case 1
            phi1(ii,2) = event;
        case 2
            phi2(ii,2) = event;
        case 3
            phi3(ii,2) = event;
        case 4
            phi4(ii,2) = event;
```

```
case 5
    phi5(ii,2) = event;
case 6
    phi6(ii,2) = event;
end

end

%% Calc ODE for Phi Forcing
% [time_phi, Temps_phi] = ode45(@tempodes_phi, [starttime_s ↵
endtime_s], To);
%
% figure(2)
% subplot 211
% plot(time_phi/sec_per_year, Temps_phi)
% legend('1','2','3','4','5','6')
%
% subplot 212
% scatter(time_event, zone_event, (25*mag_event).^2, ↵
(10*mag_event), 'filled')
% map = [
%         1 0.8 0.8;
%         1 0.6 0.6;
%         1 0.4 0.4;
%         1 0.2 0.2
%         1 0 0];
% caxis([3 8])
% grid on
% colormap('autumn'); colormap(flipud(colormap)); colorbar ↵
('southoutside')
% ylim([0 6])
% xlim([0 endtime_s/sec_per_year])

%% Calc ODE for Phi Forcing w/ Ice - Albedo Feedback
[time_phi_AIF, Temps_phi_AIF] = ode45 ↵
(@tempodes_phi_feedbackGHEFFECT_noalb, [starttime_s endtime_s], To);
%%
figure(3)
```

```
subplot 311
plot(time_phi_AIF/sec_per_year,Temps_phi_AIF(:,1),'k-','LineWidth',2);
hold on
plot(time_phi_AIF/sec_per_year,Temps_phi_AIF(:,2),'b-','LineWidth',2);
plot(time_phi_AIF/sec_per_year,Temps_phi_AIF(:,3),'g-','LineWidth',2);
plot(time_phi_AIF/sec_per_year,Temps_phi_AIF(:,4),'c-','LineWidth',2);
plot(time_phi_AIF/sec_per_year,Temps_phi_AIF(:,5),'m-','LineWidth',2);
plot(time_phi_AIF/sec_per_year,Temps_phi_AIF(:,6),'r-','LineWidth',2);
legend('Zone 1','Zone 2','Zone 3','Zone 4','Zone 5','Zone 6')
grid on
ylabel('Temperature (K)')
xlabel('Time (Yrs)')
title('Super Volcano at 5000 Years')
%%
subplot 312
scatter(time_event, zone_event,(25*mag_event).^2,
(10*mag_event),'filled')
map = [
    1 0.8 0.8;
    1 0.6 0.6;
    1 0.4 0.4;
    1 0.2 0.2
    1 0 0];
caxis([3 8])
grid on
colormap('autumn'); colormap(flipud(colormap)); c = colorbar('southoutside');
ylim([0 6])
xlim([0 endtime_s/sec_per_year])
ylabel('Zone')
xlabel('Time (Yrs)')
ylabel(c,'Magnitude')

%% Volcanic Density
```

```
global_activity = (1-phi1(1:endtime_s/sec_per_year + 1,2)) + (1-phi2(1:endtime_s/sec_per_year + 1,2))...
+ (1-phi3(1:endtime_s/sec_per_year + 1,2)) + (1-phi4(1:endtime_s/sec_per_year + 1,2))...
+ (1-phi5(1:endtime_s/sec_per_year + 1,2)) + (1-phi6(1:endtime_s/sec_per_year + 1,2));

[short long] = movavg(global_activity,3,25);

figure(3)
subplot 313
%plot(1:length(global_activity),global_activity,'r-'); hold on;
plot(1:length(long),long,'k-','LineWidth',1);
ylim([0 0.3])
xlim([starttime_s endtime_s]/sec_per_year)
grid on
xlabel('Time (Yrs)')
ylabel('Global Solar Irradiance Coefficient')
```

```
function [ Tdot ] = tempodes_phi_feedbackGHEFFECT_noalb( t, T );
%Climate Model for ODES

%Bring in global vars from main script
global k L ao a pcZaverage sigmaB S0 tau asky gamma A phi1 phi2 ...
phi3 phi4 phi5 phi6 ai Tlow Thigh To count;

% Find Closest time in phi
[dummy, tphi] = min(abs(phi1(:,1) - t));

%% Ice Albedo Feedback

%Index current temps
i_below = T <= Tlow;
i_middle = T > Tlow & T < Thigh;
i_above = T >= Thigh;

% If below threshold, turn to ice
if nnz(i_below) > 0;

    a(count,i_below) = ai;

end

% If inbetween threshold, calc new albedo
if nnz(i_middle) > 0;

    a(count,i_middle) = ao(i_middle)' + (ai - ao(i_middle))' .* ((T -
(i_middle) - Thigh).^2)/((Tlow - Thigh)^2);

end

% If above threshold, turn to normal (ao)
if nnz(i_above) > 0;

    a(count,i_above) = ao(i_above);

end
```

```
%% System of ODES
```

```
CO2=.00004;
```

```
CO20=40;
```

```
Tdot = [(1/pcZaverage(1)) * (gamma(1) * (1-asky) * (1-a(count,1)) * S0 * phi1 ↵
(tphi,2)-tau*OLRfxn(T(1),To(1),CO2,CO20,ao(1))+(L(1,2)*k(1,2)/(A(1) ↵
*pcZaverage(1))) * (T(2)-T(1));
```

```
(1/pcZaverage(2)) * (gamma(2) * (1-asky) * (1-a(count,2)) * S0 * phi2 ↵
(tphi,2)-tau*OLRfxn(T(2),To(2),CO2,CO20,ao(2))+(1/(A(2)*pcZaverage(2))) * (-L(1,2)*k(1,2)*(T(2)-T(1))+L(2,3)*k(2,3)*(T(3)-T(2)));
```

```
(1/pcZaverage(3)) * (gamma(3) * (1-asky) * (1-a(count,3)) * S0 * phi3 ↵
(tphi,2)-tau*OLRfxn(T(3),To(3),CO2,CO20,ao(3))+(1/(A(3)*pcZaverage(3))) * (-L(2,3)*k(2,3)*(T(3)-T(2))+L(3,4)*k(3,4)*(T(4)-T(3)));
```

```
(1/pcZaverage(4)) * (gamma(4) * (1-asky) * (1-a(count,4)) * S0 * phi4 ↵
(tphi,2)-tau*OLRfxn(T(4),To(4),CO2,CO20,ao(4))+(1/(A(5)*pcZaverage(5))) * (-L(3,4)*k(3,4)*(T(4)-T(3))+L(4,5)*k(4,5)*(T(5)-T(4)));
```

```
(1/pcZaverage(5)) * (gamma(5) * (1-asky) * (1-a(count,5)) * S0 * phi5 ↵
(tphi,2)-tau*OLRfxn(T(5),To(5),CO2,CO20,ao(5))+(1/(A(5)*pcZaverage(5))) * (-L(4,5)*k(4,5)*(T(5)-T(4))+L(5,6)*k(5,6)*(T(6)-T(5)));
```

```
(1/pcZaverage(6)) * (gamma(6) * (1-asky) * (1-a(count,6)) * S0 * phi6 ↵
(tphi,2)-tau*OLRfxn(T(6),To(6),CO2,CO20,ao(6))-(L(5,6)*k(5,6)/(A(6) ↵
*pcZaverage(6))) * (T(6)-T(5))];
```

```
count = count + 1;
```

```
end
```