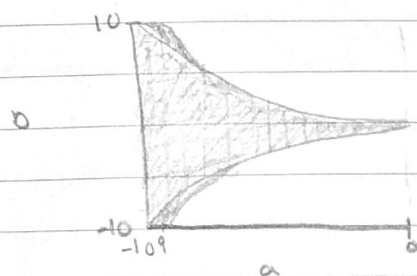


Desmos is a graphing calculator created (founded) by Eli Luberoff who was a math and physics student at Yale. The tool is a web and mobile application written in Javascript. The product was launched at a TechCrunch conference in 2011. After one year, they received \$1 million in funding. Desmos is capable of plotting equations and inequalities as well as lists, plots, regressions, interactive variables, graph restrictions, simultaneous graphing, piecewise functions, polar functions, etc. Users can save graphs by creating an account. Teachers may create a teacher account and activity modules can be used in class. Desmos also has accessibility features for those with disabilities.

One of the example graphs is an elliptic curve. The equation of an elliptic curve is given by:

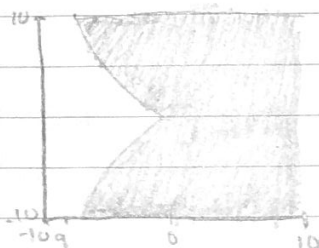
$$y^2 = x^3 + ax + b.$$

Desmos provides a slider for variable a and b . In order to produce 3 roots:



(a, b) must be in the shaded region.

In order to produce 1 root:



(a, b) must be in the shaded region.

$$\begin{aligned} \textcircled{a} \quad y^2 &= x^3 - 3x - 2 = (x+1)^2(x-2) \\ \textcircled{b} \quad y^2 &= x^3 - 3x + 2 = (x-1)^2(x+2) \\ \textcircled{c} \quad y^2 &= x^3 \end{aligned}$$

$$\text{discriminant: } \Delta = -16(4a^3 + 27b^2)$$

$$\textcircled{a} \quad \Delta = -16(4(27) + 27(4)) = 0$$

$$\textcircled{b} \quad \Delta = -16(4(-27) + 27(4)) = 0$$

$$\textcircled{c} \quad \Delta = -16(4(0) + 27(0)) = 0$$

} discriminant must
be non zero

By definition, an elliptic curve is required to be non-singular. Geometrically, this means that the graph can't have cusps, self-intersections or isolated points. Algebraically, the discriminant can't be zero. So, when $(a, b) = (-3, -2), (-3, 2)$ or $(0, 0)$, the discriminant is zero.