Notes on pre-dispute/automatic appeal insurance

To fill in - points to make: Downsides - higher initial costs to parties, vault can run out of money, doesn't help attack victims that lost previous round (indeed if attacker is funded by this mechanism it makes the attack slightly easier), but can still be combined with crowdfunding. Could be integrated into Kleros (allowing a governance vote to veto funding a particular party in a given case in case of attack - downside that makes people voting in governance make that much more effort paying attention to individual cases). Alternatively could be done as an overlay that does not directly depend on Kleros - a contract that helps all parties using for example a given subcourt to mutually insure each other. Attack of trying to drain vault by appealing a bunch of times and hoping the result flips - using this to generate juror fees for yourself as juror is not particularly effective way to make money (say attacker also has a significant percentage of juror spots) as if could flip result like that generally would already make more money from pnk redistribution. However, this is a way to reinforce/set up a bank attack by hoping with some probability to preliminarily drain the vault in a previous case. Probably want to limit the number of rounds this insurance is paid for (potentially in function of funds in vault at time) so that not too large a risk on any one case. Want to add discussion here on how could set parameters for amount of deposit to pay in excess of arbitration fees - dependent on things like rate of appeals being overturned which are likely to mostly be empiracally observed.

We will denote f to be the per juror amount that must be paid in fees when paying into this insurance mechanism. Generally, we have $f \geq j$, where j is the required average arbitration fee per juror, so that the difference f - j, to the degree that it is not reimbursed to a winning party, is used to fund (future) pre-ruling appeal fee insurance. M_i is the number of jurors in the ith round, so $M_i j$ is the total arbitration fees required in the ith round.

Consider, the moment, the simplest situation where one has a binary case with outcomes a and b. Suppose we take the following model (which is relatively conservative with respect to the vault losing money): if a won the r-1st round, then in order to fund b, at least $\sum_{i=1}^r M_i f$ total must have been paid across all rounds on behalf of b, where M_i is the number of jurors in the ith. Particularly, if b had previously won a round and had its appeal fees covered in the next round by this mechanism, one must "make up" for this amount to fund b following a later round that a has won. Note that in this case, the only way that the vault can lose money is if the last ruling is different from the second to last ruling.

Otherwise all required fees would have been paid on behalf of the loser of the last round.

Then, if the case is very unclear so that the probability of any given juror ruling a or b is each roughly 1/2, and there are r appeal rounds, the probability of any given string of round-by-round outcomes (e.g $baa \dots ba$ or $aaa \dots ab$) is $\frac{1}{2r}$. So the expected change in the value of the vault is

$$= \frac{1}{2^{r}} \left(2^{r-1} (M_{1} + \dots + M_{r})(f-j) + 2^{r-2} [(M_{1} + \dots + M_{r-1})(f-j) - M_{r}j] + \dots + 2 (M_{1}(f-j) - M_{2}j - \dots - M_{r}j) \right) \\
\leq \frac{1}{2^{r}} \left(2^{r-1} (M_{1} + \dots + M_{r})(f-j) + 2^{r-1} (M_{1} + \dots + M_{r-1})(f-j) - 2^{r-1} M_{r}j \right) \\
\approx \frac{M_{0}}{2^{r}} \left(2^{2r} (f-j) + 2^{2r-1} (f-2j) \right) \\
\approx M_{r} \left(\frac{3}{2} f - 2j \right),$$

where in the last two lines we have used $M_i \approx M_0 \cdot 2^i$. Hence, in order for edge case like this to not have an average negative contribution to the value of the vault, one will need at least $f \geq \frac{4}{3}j$. (In fact, due to the approximation in the second step, even greater f may be required).

To get a sense of what fees would be sufficient to not result in the vault draining in cases such as the above, we similarly compute,

$$= \frac{1}{2^{r}} \left(2^{r-1} (M_{1} + \dots + M_{r})(f-j) + 2^{r-2} [(M_{1} + \dots + M_{r-1})(f-j) - M_{r}j] + \dots + 2 (M_{1}(f-j) - M_{2}j - \dots - M_{r}j) \right)$$

$$\geq \frac{1}{2^{r}} \left(2^{r-1} (M_{1} + \dots M_{r})(f-j) + 2^{r-1} M_{1}(f-j) - 2^{r-1} (M_{1} + \dots M_{r})j \right)$$

$$\approx \frac{M_{0}}{2^{r}} \left(2^{2r} (f-j) - 2^{2r}j \right)$$

$$\approx M_{r} (f-2j).$$

So according to this bound, taking $f \ge 2j$ would be sufficient to ensure that the vault does not lose money on average in such cases.

An less conservative estimates on the requirements we have on f relative to j would require a more sophisticated model for determining the probability that a or b (or generally a given option) wins a given appeal round. One could consider the probability of a given juror voting for an option and use properties of binomial distributions to determine the probability of the option winning a round, however this does not necessarily adequately take into account how the risk of attacks may change over the rounds. Moreover, as the pre-dispute appeal insurance cannot screen cases based on them being two uncertain to qualify for insurance, one would also need a model for how the probabilities of the outcomes

change from case to case to control the percentage of total cases that are have this level of uncertainty.

Verdict: Seems that such as mechanism would require substantially higher fees, on the order of 4/3 to 2 times what would be required without it. This factor is comparable to the added effect of requiring stake for crowdfunding. However,

- In our existing crowdfunding model, added stakes are preserved between the two parties to a given dispute. Hence, the requiring such stakes leading to higher risk for the parties, though a party that estimates that it has majority winning chances is no worse in expected value.
- For the pre-dispute/vault insurance, the flow of extra fees is across disputes, specifically from disputes that do not have overturned appeals to disputes that do. Then in some cases, parties with disputes that they do not expect to be appealed with have depressed expected value of using Kleros because of these extra fees.

Nonetheless, this mechanism is a possible structure that could be added on, and could be useful in some cases. Particularly, one could create this as a second layer system that parties opt in to at the time of creating an arbitrable contract, agreeing to pay higher fees for the service of being insuranced in such situations.