CS 109A/STAT 121A/AC 209A/CSCI E-109A: Homework 3

Multiple Linear Regression, Subset Selection, Cross Validation

Harvard University
Fall 2017
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INSTRUCTIONS

- To submit your assignment follow the instructions given in canvas.
- Restart the kernel and run the whole notebook again before you submit.
- Do not include your name(s) in the notebook if you are submitting as a group.
- If you submit individually and you have worked with someone, please include the name of your [one] partner below.

```
Your partner's name (if you submit separately): Walter Thornton and Dwayne
Kennemore

Enrollment Status (109A, 121A, 209A, or E109A): E109A
```

Import libraries:

In [1]:
```
import numpy as np
import pandas as pd
import matplotlib
import matplotlib.pyplot as plt
from sklearn.metrics import r2_score
import statsmodels.api as sm
from statsmodels.api import OLS
from sklearn.preprocessing import PolynomialFeatures
from sklearn.linear_model import Ridge
from sklearn.linear_model import LinearRegression
from sklearn.linear_model import Lasso
from sklearn import linear_model as lm
from sklearn.linear_model import RidgeCV
from sklearn.cross_validation import cross_val_score, cross_val_predict
from sklearn import metrics
from sklearn.linear_model import LassoCV
import statsmodels.formula.api as formulas
%matplotlib inline
```

```
C:\Users\wlt42\Anaconda3\lib\site-packages\statsmodels\compat\pandas.py:56: Fut
ureWarning: The pandas.core.datetools module is deprecated and will be removed
 in a future version. Please use the pandas.tseries module instead.
  from pandas.core import datetools
C:\Users\wlt42\Anaconda3\lib\site-packages\sklearn\cross_validation.py:44: Depr
ecationWarning: This module was deprecated in version 0.18 in favor of the mode
l_selection module into which all the refactored classes and functions are move
d. Also note that the interface of the new CV iterators are different from that
of this module. This module will be removed in 0.20.
  "This module will be removed in 0.20.", DeprecationWarning)
```

Forecasting Bike Sharing Usage

In this homework, we will focus on multiple linear regression and will explore techniques for subset selection. The specific task is to build a regression model for a bike share system that can predict the total number of bike rentals in a given day, based on attributes about the day. Such a demand forecasting model would be useful in planning the number of bikes that need to be available in the system on any given day, and also in monitoring traffic in the city. The data for this problem was collected from the Capital Bikeshare program in Washington D.C. over two years.

The data set is provided in the files `Bikeshare_train.csv` and `Bikeshare_test.csv`, as separate training and test sets. Each row in these files contains 10 attributes describing a day and its weather:

- season (1 = spring, 2 = summer, 3 = fall, 4 = winter)
- month (1 through 12, with 1 denoting Jan)
- holiday (1 = the day is a holiday, 0 = otherwise)
- day_of_week (0 through 6, with 0 denoting Sunday)
- workingday (1 = the day is neither a holiday or weekend, 0 = otherwise)
- weather
  - 1: Clear, Few clouds, Partly cloudy, Partly cloudy
  - 2: Mist + Cloudy, Mist + Broken clouds, Mist + Few clouds, Mist

- 3: Light Snow, Light Rain + Thunderstorm + Scattered clouds, Light Rain + Scattered clouds
- 4: Heavy Rain + Ice Pallets + Thunderstorm + Mist, Snow + Fog

- temp (temperature in Celsius)
- atemp (apparent temperature, or relative outdoor temperature, in Celsius)
- humidity (relative humidity)
- windspeed (wind speed)

and the last column 'count' contains the response variable, i.e. total number of bike rentals on the day.

Part (a): Data Exploration & Preprocessing

As a first step, identify important characteristics of the data using suitable visualizations when necessary. Some of the questions you may ask include (but are not limited to):

- How does the number of bike rentals vary between weekdays and weekends?
- How about bike rentals on holidays?
- What effect does the season have on the bike rentals on a given day?
- Is the number of bike rentals lower than average when there is rain or snow?
- How does temperature effect bike rentals?
- Do any of the numeric attributes have a clear non-linear dependence with number of the bike rentals?

In [2]:
```
#open the files
train_data = pd.read_csv('Bikeshare_train.csv', sep=",", header=0)
test_data = pd.read_csv('Bikeshare_test.csv', sep=",", header=0)

#For a while, we thought a malfunction in our code was being caused by overloading
#This turned out not to be the case, but after it was already changed, we just le
train_data.rename(index=str, columns={"count": "rentals"}, inplace=True)
test_data.rename(index=str, columns={"count": "rentals"}, inplace=True)
train_data
test_data
```

Out[2]:

	Unnamed: 0	season	month	holiday	day_of_week	workingday	weather	temp	atemp	humidi
0	0	1.0	2.0	0.0	4.0	1.0	1.0	2.0	6.0	56.208
1	1	1.0	12.0	0.0	4.0	1.0	2.0	1.0	3.0	65.291
2	2	2.0	6.0	0.0	4.0	1.0	2.0	36.0	37.0	56.833
3	3	1.0	2.0	0.0	1.0	1.0	1.0	8.0	12.0	49.083
4	4	2.0	5.0	0.0	0.0	0.0	1.0	28.0	29.0	69.708
5	5	3.0	8.0	0.0	2.0	1.0	1.0	25.0	26.0	54.833
6	6	4.0	11.0	0.0	4.0	1.0	1.0	3.0	8.0	55.565
7	7	3.0	9.0	1.0	1.0	0.0	2.0	27.0	28.0	79.041
8	8	1.0	12.0	0.0	4.0	1.0	2.0	12.0	16.0	75.750
9	9	2.0	6.0	0.0	2.0	1.0	2.0	26.0	27.0	83.333
10	10	2.0	5.0	0.0	0.0	0.0	2.0	13.0	17.0	76.208
11	11	1.0	3.0	0.0	5.0	1.0	2.0	5.0	8.0	64.956
12	12	1.0	1.0	0.0	0.0	0.0	1.0	-6.0	-4.0	43.416
13	13	1.0	12.0	0.0	6.0	0.0	1.0	11.0	15.0	61.583
14	14	2.0	6.0	0.0	5.0	1.0	1.0	26.0	27.0	46.791
15	15	1.0	1.0	0.0	3.0	1.0	2.0	-8.0	-3.0	41.458
16	16	3.0	6.0	0.0	4.0	1.0	1.0	29.0	29.0	43.416
17	17	2.0	5.0	0.0	4.0	1.0	1.0	17.0	20.0	55.208
18	18	3.0	9.0	0.0	6.0	0.0	2.0	16.0	19.0	71.833
19	19	2.0	5.0	0.0	1.0	1.0	2.0	23.0	25.0	81.125
20	20	1.0	3.0	0.0	2.0	1.0	1.0	2.0	6.0	53.500
21	21	3.0	6.0	0.0	3.0	1.0	1.0	29.0	29.0	36.000
22	22	3.0	8.0	0.0	1.0	1.0	1.0	29.0	29.0	47.000
23	23	1.0	2.0	0.0	0.0	0.0	2.0	3.0	6.0	51.583
24	24	2.0	6.0	0.0	6.0	0.0	1.0	31.0	32.0	65.458
25	25	1.0	3.0	0.0	6.0	0.0	1.0	3.0	6.0	35.041
26	26	3.0	8.0	0.0	6.0	0.0	1.0	35.0	36.0	61.333

	Unnamed: 0	season	month	holiday	day_of_week	workingday	weather	temp	atemp	humidit
27	27	1.0	2.0	0.0	2.0	1.0	1.0	7.0	12.0	49.625
28	28	1.0	2.0	0.0	0.0	0.0	1.0	3.0	6.0	41.000
29	29	1.0	2.0	0.0	4.0	1.0	1.0	12.0	16.0	50.500
...	...	...	...	...	...	...	...	...	...	
370	370	3.0	8.0	0.0	2.0	1.0	1.0	29.0	30.0	57.833
371	371	3.0	9.0	0.0	0.0	0.0	2.0	29.0	30.0	81.500
372	372	3.0	7.0	0.0	2.0	1.0	1.0	34.0	33.0	49.208
373	373	4.0	11.0	0.0	5.0	1.0	1.0	9.0	13.0	64.375
374	374	4.0	10.0	0.0	2.0	1.0	2.0	21.0	23.0	80.875
375	375	4.0	11.0	0.0	5.0	1.0	1.0	8.0	11.0	54.083
376	376	4.0	12.0	0.0	2.0	1.0	2.0	7.0	10.0	59.666
377	377	3.0	7.0	0.0	2.0	1.0	2.0	30.0	31.0	66.750
378	378	2.0	4.0	0.0	5.0	1.0	1.0	14.0	18.0	40.083
379	379	2.0	4.0	0.0	1.0	1.0	1.0	21.0	23.0	42.625
380	380	3.0	8.0	0.0	2.0	1.0	1.0	34.0	34.0	49.125
381	381	3.0	9.0	0.0	5.0	1.0	2.0	15.0	18.0	59.041
382	382	4.0	12.0	0.0	0.0	0.0	1.0	6.0	11.0	77.583
383	383	4.0	10.0	0.0	3.0	1.0	3.0	19.0	21.0	89.521
384	384	3.0	9.0	0.0	5.0	1.0	1.0	29.0	30.0	73.625
385	385	2.0	4.0	0.0	0.0	0.0	1.0	14.0	17.0	47.958
386	386	2.0	4.0	0.0	2.0	1.0	1.0	25.0	26.0	72.916
387	387	2.0	6.0	0.0	3.0	1.0	1.0	34.0	35.0	62.208
388	388	1.0	12.0	0.0	3.0	1.0	3.0	0.0	3.0	82.333
389	389	3.0	8.0	0.0	2.0	1.0	1.0	31.0	32.0	68.666
390	390	1.0	2.0	0.0	0.0	0.0	1.0	3.0	7.0	56.833
391	391	2.0	5.0	0.0	5.0	1.0	1.0	19.0	22.0	36.041
392	392	1.0	12.0	0.0	4.0	1.0	1.0	1.0	6.0	57.416
393	393	1.0	3.0	0.0	0.0	0.0	1.0	6.0	8.0	40.333
394	394	3.0	7.0	0.0	0.0	0.0	1.0	37.0	39.0	55.083
395	395	3.0	9.0	0.0	5.0	1.0	2.0	26.0	27.0	72.708
396	396	3.0	8.0	0.0	3.0	1.0	1.0	33.0	32.0	42.416
397	397	3.0	9.0	0.0	0.0	0.0	1.0	30.0	31.0	74.208
398	398	1.0	1.0	0.0	0.0	0.0	1.0	8.0	13.0	69.250
399	399	2.0	4.0	0.0	4.0	1.0	1.0	16.0	20.0	61.250

400 rows × 12 columns

your answers here

In [3]:
```
print(train_data.dtypes)
pd.set_option('display.max_rows', 500)
test_data
```

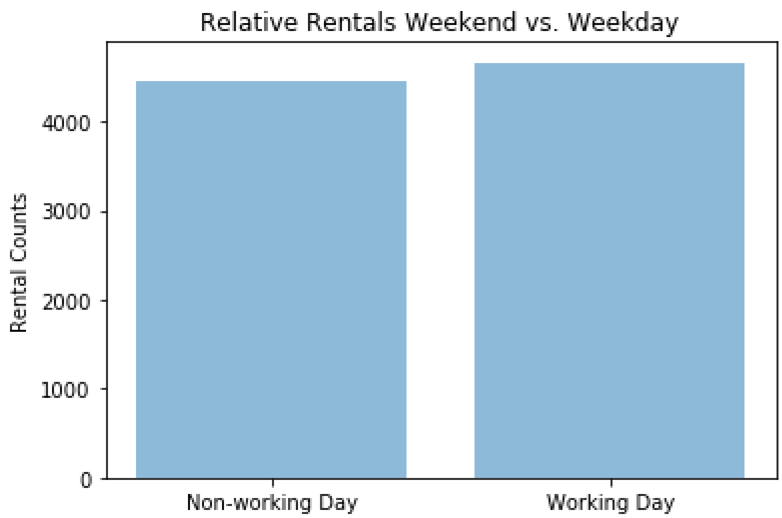
```
Unnamed: 0       int64
season         float64
month          float64
holiday        float64
day_of_week    float64
workingday     float64
weather        float64
temp           float64
atemp          float64
humidity       float64
windspeed      float64
rentals        float64
dtype: object
```

In [4]:
```
avg = [0, 0]

#Q1. How do the average rentals compare on weekdays vs weekend (training data set,
#A. The average rentals on a non-working day were 4,461, while average rentals on
#   4,666. This was unexpected; you'd think that bike rentals would be hiked duri

objects = ('0', '1')
y_pos = np.arange(len(objects))

day = [train_data.loc[train_data['workingday'] == 0, 'rentals'].sum()]
day.append(train_data.loc[train_data['workingday'] == 1, 'rentals'].sum())
#print("Day = ", day)
number_of_entries = list(pd.value_counts(train_data['workingday'].values, sort=Fa
number_of_entries = list(reversed(number_of_entries))
#print("Number of Entries = ", number_of_entries)
avg[0] = day[0] / number_of_entries[0]
avg[1] = day[1] / number_of_entries[1]
print("Avg rentals on a non-working day: ", avg[0])
print("Avg rentals on a working day: ", avg[1])
plt.bar(y_pos, avg, align='center', alpha=0.5)
plt.xticks(y_pos, ['Non-working Day', 'Working Day'])
plt.ylabel('Rental Counts')
plt.title('Relative Rentals Weekend vs. Weekday')
plt.show()

#Q2. How do bike rentals vary on holidays vs. non-holidays?
#A. Avg rentals on holiday were 4,612 on average, but 4,199 on average for a non-
objects = ('0', '1')
y_pos = np.arange(len(objects))
day = [train_data.loc[train_data['holiday'] == 0, 'rentals'].sum()]
day.append(train_data.loc[train_data['holiday'] == 1, 'rentals'].sum())
number_of_entries = list(pd.value_counts(train_data['holiday'].values, sort=False
avg[0] = day[0] / number_of_entries[0]
avg[1] = day[1] / number_of_entries[1]
print("Avg rentals on holiday: ", avg[0])
print("Avg rentals on non-holiday: ", avg[1])
plt.bar(y_pos, avg, align='center', alpha=0.5)
plt.xticks(y_pos, ['Holiday', 'Non-holiday'])
plt.ylabel('Rental Counts')
plt.title('Relative Rentals Holidy vs. Non-holiday')
plt.show()

#Q3: How are rentals distributed by season?
#A: Spring has the lowest number, just over 2,000 per day. The other seasons see
#5,000 per day (rounded).
avg_2 = [0, 0, 0, 0]
objects = ('1', '2', '3', '4')
y_pos = np.arange(len(objects))
day = [train_data.loc[train_data['season'] == 1, 'rentals'].sum()]
day.append(train_data.loc[train_data['season'] == 2, 'rentals'].sum())
day.append(train_data.loc[train_data['season'] == 3, 'rentals'].sum())
day.append(train_data.loc[train_data['season'] == 4, 'rentals'].sum())
number_of_entries = list(pd.value_counts(train_data['season'].values, sort=False)
#number_of_entries = list(reversed(number_of_entries))
#print("Number of Entries = ", number_of_entries)
avg_2[0] = day[0] / number_of_entries[0]
```

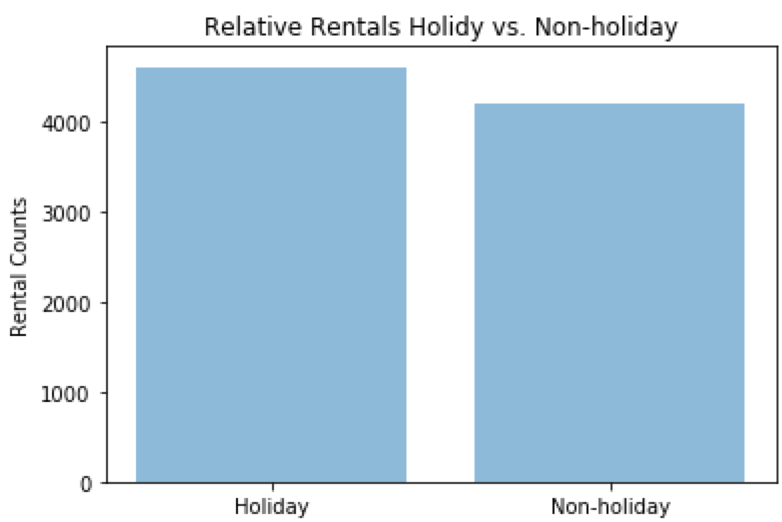
```
avg_2[1] = day[1] / number_of_entries[1]
avg_2[2] = day[2] / number_of_entries[2]
avg_2[3] = day[3] / number_of_entries[3]
print("Avg rentals for each season: ", avg_2)
plt.bar(y_pos, avg_2, align='center', alpha=0.5)
plt.xticks(y_pos, ['Spring', 'Summer', 'Fall', 'Winter'])
plt.ylabel('Rental Counts')
plt.title('Relative Rentals by Season')
plt.show()

#Q4: How are rentals impacted by weather conditions?
#A: Unsurprisingly, rentals are highest when the weather is clear or at worst par
avg_2 = [0, 0, 0, 0]
objects = ('1', '2', '3', '4')
y_pos = np.arange(len(objects))
day = [train_data.loc[train_data['weather'] == 1, 'rentals'].sum()]
day.append(train_data.loc[train_data['weather'] == 2, 'rentals'].sum())
day.append(train_data.loc[train_data['weather'] == 3, 'rentals'].sum())
day.append(train_data.loc[train_data['weather'] == 4, 'rentals'].sum())
number_of_entries = list(pd.value_counts(train_data['weather'].values, sort=False
mean = np.mean(train_data['rentals'])
#Note: the try..excepts below are due to the fact that some weather parameters ar
#which was creating a division by zero.
try:
    avg_2[0] = day[0] / number_of_entries[0]
except:
    avg_2[0] = 0
try:
    avg_2[1] = day[1] / number_of_entries[1]
except:
    avg_2[1] = 0
try:
    avg_2[2] = day[2] / number_of_entries[2]
except:
    avg_2[2] = 0
try:
    avg_2[3] = day[3] / number_of_entries[3]
except:
    avg_2[3] = 0
print("Avg rentals for each type of weather: ", avg_2)
plt.bar(y_pos, avg_2, align='center', alpha=0.5)
plt.axhline(y=mean, color='r', linestyle='-')
plt.xticks(y_pos, objects)
plt.ylabel('Rental Counts')
plt.title('Relative Rentals by Weather')
plt.legend( ('1: Clear, Few clouds, Partly cloudy, Partly cloudy',
             '2: Mist + Cloudy, Mist + Broken clouds, Mist + Few clouds, Mist',
             '3: Light Snow, Light Rain + Thunderstorm + Scattered clouds, Light
             '4: Heavy Rain + Ice Pallets + Thunderstorm + Mist, Snow + Fog') )
plt.show()

#Q5: How are rentals impacted by temperature?
#A: We prepared a histogram to demonstrate this impact. The bulk of rentals occur
#from 20-40. We thought the histogram would be improved if each bar had a low-hig
#not get this feature to work.
mean = np.mean(train_data['temp'])
plt.hist(train_data['temp'], weights=train_data['rentals'])
```

```
plt.axvline(x=mean, color = "black")
plt.ylabel('rentals')
plt.xlabel('Temp')
plt.show()
```

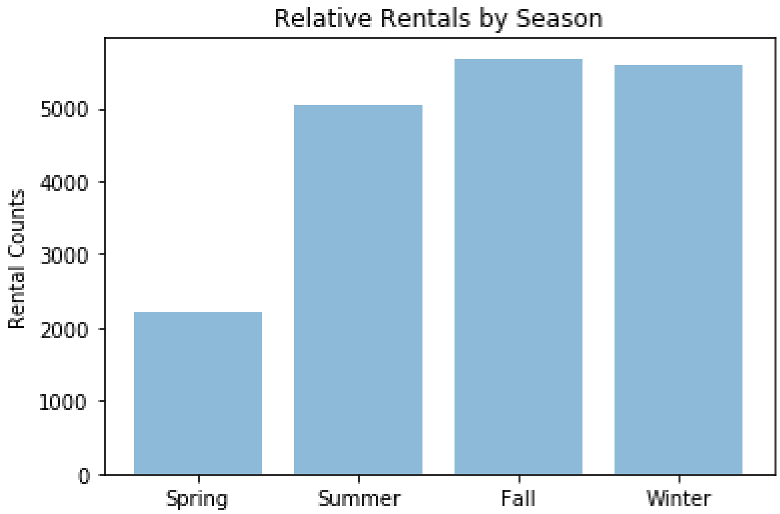
```
Avg rentals on a non-working day:  4461.0733945
Avg rentals on a working day:  4665.8963964
```



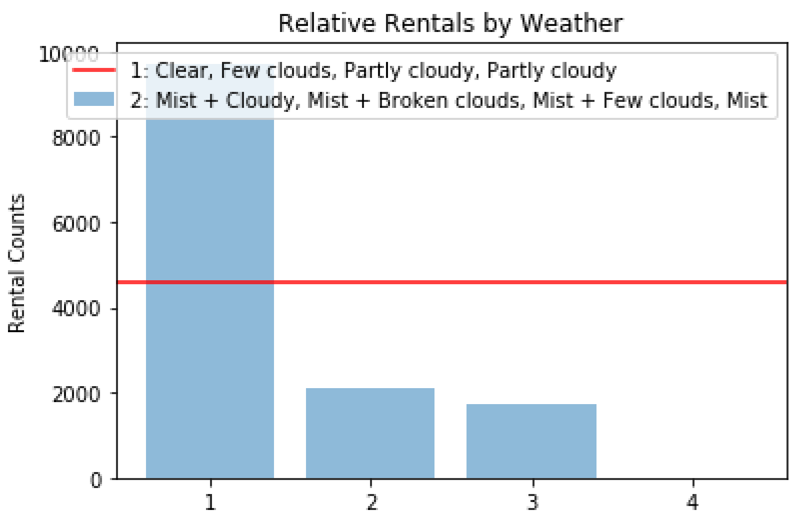

```
Avg rentals on holiday:  4612.171875
Avg rentals on non-holiday:  4199.18181818
```

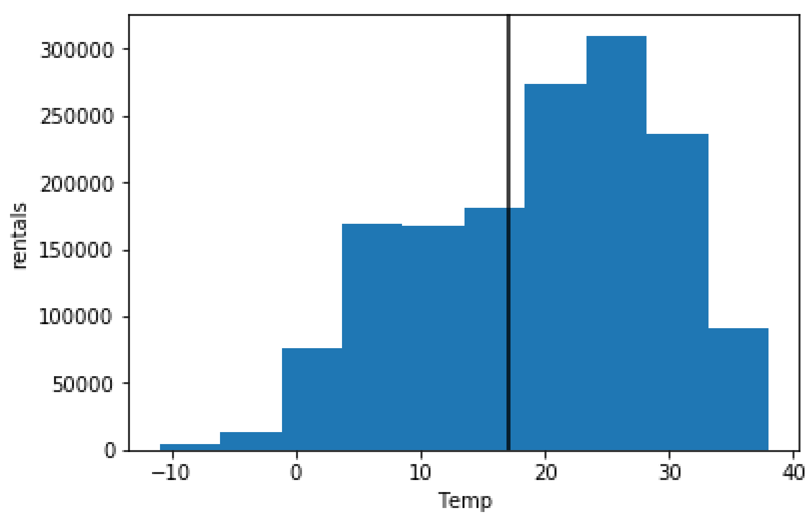



```
Avg rentals for each season:  [2210.0, 5044.3999999999996, 5680.738636363636, 5
602.041666666667]
```




```
Avg rentals for each type of weather:  [9727.6422018348621, 2096.2783018867926, 1736.2, 0]
```

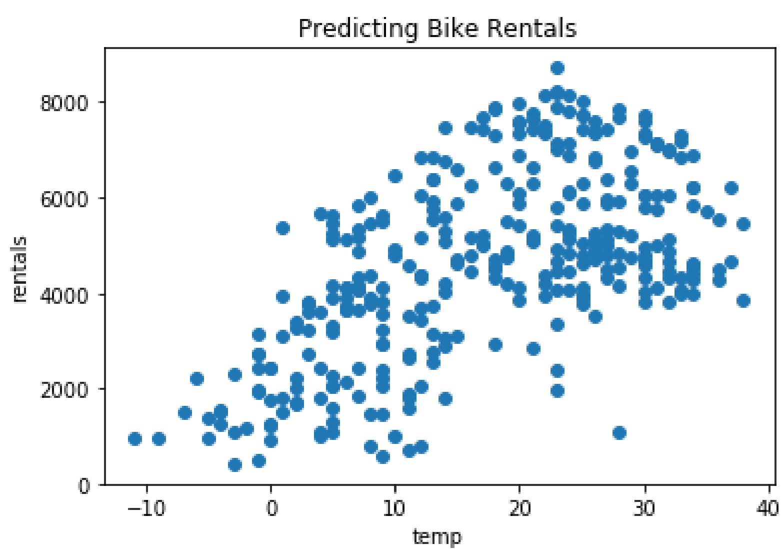


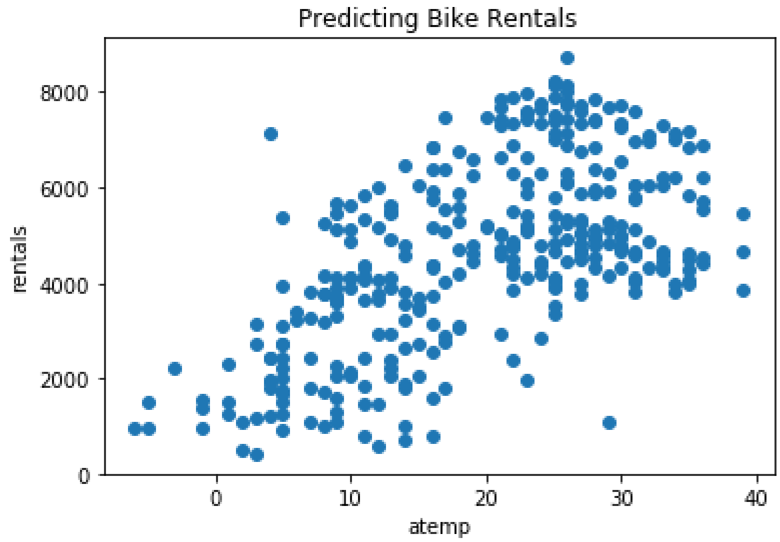


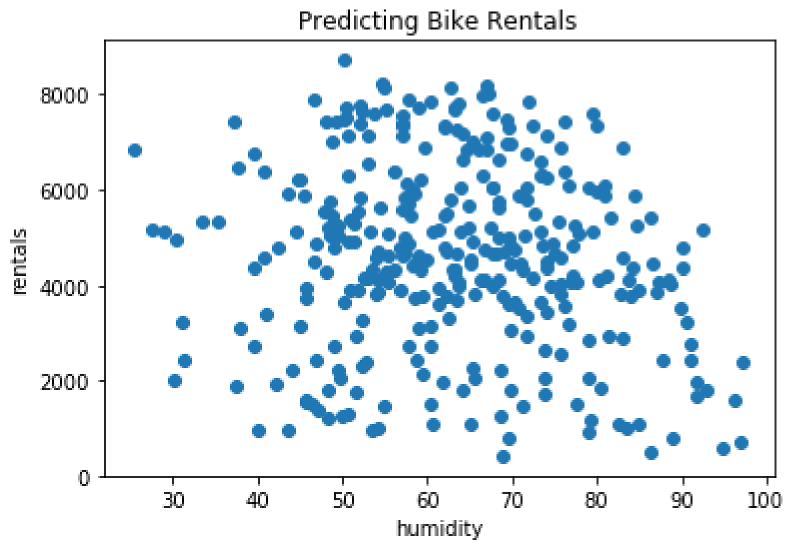
In [5]:

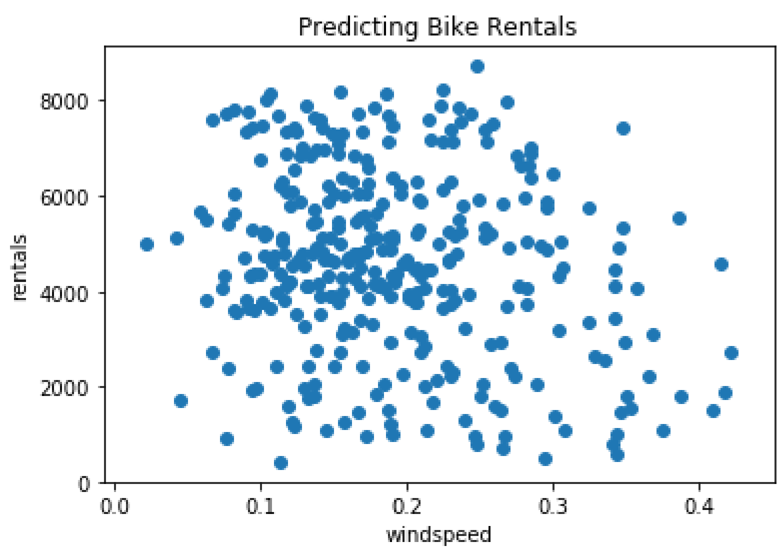
```
factors = ['temp', 'atemp', 'humidity', 'windspeed']
i = 0
k_list = []
rtrain_list = []
rtest_list = []
for i, k in enumerate(factors):

    x = train_data[k]
    x = x.reshape(-1, 1)
    y = train_data['rentals']
    y = y.reshape(-1, 1)

    # Build graph
    plt.scatter(x, y)
    plt.xlabel(k)
    plt.ylabel('rentals')
    plt.title('Predicting Bike Rentals')
    plt.show()
```

```
C:\Users\wlt42\Anaconda3\lib\site-packages\ipykernel_launcher.py:10: FutureWarn
ing: reshape is deprecated and will raise in a subsequent release. Please use .
values.reshape(...) instead
  # Remove the CWD from sys.path while we load stuff.
C:\Users\wlt42\Anaconda3\lib\site-packages\ipykernel_launcher.py:12: FutureWarn
ing: reshape is deprecated and will raise in a subsequent release. Please use .
values.reshape(...) instead
  if sys.path[0] == '':
```









In [6]:
```
#Q6: Do any variables have a clear non linear relationship with rentals.
#A: We tested temp, atemp, humidity and windspeed. Temp and Atemp had a linear re
#noisy, but humidity and wind speed looked almost *random* in their dispersions. 
#in the scatter plots above.
```

In [7]:
```
# one hot encoding of test and training set
train_data = pd.get_dummies(train_data, prefix = ['season', 'month', 'day_of_week
test_data = pd.get_dummies(test_data, prefix = ['season', 'month', 'day_of_week',
```

In [8]:
```
#train_data.columns
#Note: You might well wonder the purpose for the code below. When we put together
#formulas to run our regressions later here, the code was tripping over the ".0"
#ends these data items. Neither us nor the TFs in Office Hours could figure out wh
#this posed a problem, so we (manually, painstakingly) renamed them all.

train_data.rename(index=str, columns={'season_2.0' : 'season_2'}, inplace=True)
train_data.rename(index=str, columns={'season_3.0' : 'season_3'}, inplace=True)
train_data.rename(index=str, columns={'season_4.0' : 'season_4'}, inplace=True)
train_data.rename(index=str, columns={'month_2.0' : 'month_2'}, inplace=True)
train_data.rename(index=str, columns={'month_3.0' : 'month_3'}, inplace=True)
train_data.rename(index=str, columns={'month_4.0' : 'month_4'}, inplace=True)
train_data.rename(index=str, columns={'month_5.0' : 'month_5'}, inplace=True)
train_data.rename(index=str, columns={'month_6.0' : 'month_6'}, inplace=True)
train_data.rename(index=str, columns={'month_7.0' : 'month_7'}, inplace=True)
train_data.rename(index=str, columns={'month_8.0' : 'month_8'}, inplace=True)
train_data.rename(index=str, columns={'month_9.0' : 'month_9'}, inplace=True)
train_data.rename(index=str, columns={'month_10.0' : 'month_10'}, inplace=True)
train_data.rename(index=str, columns={'month_11.0' : 'month_11'}, inplace=True)
train_data.rename(index=str, columns={'month_12.0' : 'month_12'}, inplace=True)
train_data.rename(index=str, columns={'day_of_week_1.0' : 'day_of_week_1'}, inpla
train_data.rename(index=str, columns={'day_of_week_2.0' : 'day_of_week_2'}, inpla
train_data.rename(index=str, columns={'day_of_week_3.0' : 'day_of_week_3'}, inpla
train_data.rename(index=str, columns={'day_of_week_4.0' : 'day_of_week_4'}, inpla
train_data.rename(index=str, columns={'day_of_week_5.0' : 'day_of_week_5'}, inpla
train_data.rename(index=str, columns={'day_of_week_6.0' : 'day_of_week_6'}, inpla
train_data.rename(index=str, columns={'weather_2.0' : 'weather_2'}, inplace=True)
train_data.rename(index=str, columns={'weather_3.0' : 'weather_3'}, inplace=True)
train_data.columns

test_data.rename(index=str, columns={'season_2.0' : 'season_2'}, inplace=True)
test_data.rename(index=str, columns={'season_3.0' : 'season_3'}, inplace=True)
test_data.rename(index=str, columns={'season_4.0' : 'season_4'}, inplace=True)
test_data.rename(index=str, columns={'month_2.0' : 'month_2'}, inplace=True)
test_data.rename(index=str, columns={'month_3.0' : 'month_3'}, inplace=True)
test_data.rename(index=str, columns={'month_4.0' : 'month_4'}, inplace=True)
test_data.rename(index=str, columns={'month_5.0' : 'month_5'}, inplace=True)
test_data.rename(index=str, columns={'month_6.0' : 'month_6'}, inplace=True)
test_data.rename(index=str, columns={'month_7.0' : 'month_7'}, inplace=True)
test_data.rename(index=str, columns={'month_8.0' : 'month_8'}, inplace=True)
test_data.rename(index=str, columns={'month_9.0' : 'month_9'}, inplace=True)
test_data.rename(index=str, columns={'month_10.0' : 'month_10'}, inplace=True)
test_data.rename(index=str, columns={'month_11.0' : 'month_11'}, inplace=True)
test_data.rename(index=str, columns={'month_12.0' : 'month_12'}, inplace=True)
test_data.rename(index=str, columns={'day_of_week_1.0' : 'day_of_week_1'}, inplac
test_data.rename(index=str, columns={'day_of_week_2.0' : 'day_of_week_2'}, inplac
test_data.rename(index=str, columns={'day_of_week_3.0' : 'day_of_week_3'}, inplac
test_data.rename(index=str, columns={'day_of_week_4.0' : 'day_of_week_4'}, inplac
test_data.rename(index=str, columns={'day_of_week_5.0' : 'day_of_week_5'}, inplac
test_data.rename(index=str, columns={'day_of_week_6.0' : 'day_of_week_6'}, inplac
test_data.rename(index=str, columns={'weather_2.0' : 'weather_2'}, inplace=True)
test_data.rename(index=str, columns={'weather_3.0' : 'weather_3'}, inplace=True)
```

In [9]:
```
train_data
```

Out[9]:

	Unnamed: 0	holiday	workingday	temp	atemp	humidity	windspeed	rentals	season_2	se
0	0	0.0	1.0	24.0	26.0	76.5833	0.118167	6073.0	1	
1	1	0.0	1.0	15.0	19.0	73.3750	0.174129	6606.0	0	
2	2	0.0	1.0	26.0	28.0	56.9583	0.253733	7363.0	1	
3	3	0.0	0.0	0.0	4.0	58.6250	0.169779	2431.0	0	
4	4	0.0	1.0	23.0	23.0	91.7083	0.097021	1996.0	0	
5	5	0.0	1.0	24.0	26.0	69.7083	0.342667	4451.0	1	
6	6	0.0	1.0	7.0	11.0	33.3478	0.347835	5315.0	0	
7	7	0.0	1.0	23.0	26.0	50.7083	0.269283	4891.0	1	
8	8	0.0	0.0	23.0	26.0	50.1667	0.247521	8714.0	0	
9	9	0.0	1.0	30.0	30.0	61.9583	0.169771	7347.0	0	

We next require you to pre-process the categorical and numerical attributes in the data set:

- Notice that this data set contains categorical attributes with two or more categories. **Why can't they be directly used as predictors?** Convert these categorical attributes into multiple binary attributes using one-hot encoding: in the place of every categorical attribute x_j that has categories $1, \ldots, K_j$, introduce $K_j - 1$ binary predictors $x_{j1}, \ldots, x_{j,K_j-1}$ where x_{jk} is 1 whenever $x_j = k$ and 0 otherwise. **Why is it okay to not have a binary column for the K_j-th category?**
- Since the attributes are in different scales, it is a good practice to standardize the continuous predictors, i.e. to scale each continuous predictor to have zero mean and a standard deviation of 1. This can be done by applying the following transform to each continuous-valued predictor j : $\hat{x}_{ij} = (x_{ij} - \bar{x}_j)/s_j$, where $\bar{x}_j$ and s_j are the sample mean and sample standard deviation (SD) of predictor j in the training set. We emphasize that the mean and SD values used for standardization must be estimated using only the training set observations, while the transform is applied to both the training and test sets. **Why shouldn't we include the test set observations in computing the mean and SD?**
- Provide a table of the summary statistics of the new attributes (`pd.describe()' function will help).

Hint: You may use the `pd.get_dummies` function to convert a categorical attribute in a data frame to one-hot encoding. This function creates K binary columns for an attribute with K categories. We suggest that you delete the last (or first) binary column generated by this function.

Note: We shall use the term "attribute" to refer to a categorical column in the data set, and the term "predictor" to refer to the individual binary columns resulting out of one-hot encoding.

In [10]:
```
# When using one-hot encoding, for example, for season or month, we omit one montl
# 0, it is implicitly season one, and that is already baked into the numbers (in
# explicitly declared.
#
# We do not include the test set observations in computing mean and SD because we
# pure out-of-sample test to ensure our regressions are robust.
```

In [11]:
```
train_data.describe()
```

Out[11]:

	Unnamed: 0	holiday	workingday	temp	atemp	humidity	windspeed	r
count	331.000000	331.000000	331.000000	331.000000	331.000000	331.000000	331.000000	331.C
mean	165.000000	0.033233	0.670695	17.018127	19.543807	63.385776	0.190833	4598.4
std	95.695698	0.179515	0.470672	11.192515	9.930991	14.334789	0.078240	1935.3
min	0.000000	0.000000	0.000000	-11.000000	-6.000000	25.416700	0.022392	431.C
25%	82.500000	0.000000	0.000000	7.500000	11.000000	52.702900	0.133083	3370.C
50%	165.000000	0.000000	1.000000	18.000000	21.000000	63.291700	0.178479	4648.C
75%	247.500000	0.000000	1.000000	26.000000	27.000000	73.500000	0.235380	5981.C
max	330.000000	1.000000	1.000000	38.000000	39.000000	97.250000	0.421642	8714.C

8 rows × 30 columns

In [12]:
```
# normalize training data
mean_temp = train_data.temp.dropna().mean()
sd_temp = train_data.temp.dropna().std()
mean_atemp = train_data.atemp.dropna().mean()
sd_atemp = train_data.atemp.dropna().std()
mean_humidity = train_data.humidity.dropna().mean()
sd_humidity = train_data.humidity.dropna().std()
mean_windspeed = train_data.windspeed.dropna().mean()
sd_windspeed = train_data.windspeed.dropna().std()

train_data['temp'] = train_data['temp'].apply(lambda x: (x - mean_temp ) / (sd_te
train_data['atemp'] = train_data['atemp'].apply(lambda x: (x - mean_atemp ) / (sd
train_data['humidity'] = train_data['humidity'].apply(lambda x: (x - mean_humidit
train_data['windspeed'] = train_data['windspeed'].apply(lambda x: (x - mean_winds

train_data
```

Out[12]:

	Unnamed: 0	holiday	workingday	temp	atemp	humidity	windspeed	rentals	seas
0	0	0.0	1.0	0.623798	0.650106	0.920664	-0.928758	6073.0	
1	1	0.0	1.0	-0.180310	-0.054759	0.696852	-0.213502	6606.0	
2	2	0.0	1.0	0.802489	0.851495	-0.448383	0.803926	7363.0	
3	3	0.0	0.0	-1.520492	-1.565182	-0.332113	-0.269099	2431.0	
4	4	0.0	1.0	0.534453	0.348021	1.975789	-1.199027	1996.0	
5	5	0.0	1.0	0.623798	0.650106	0.441062	1.940601	4451.0	
6	6	0.0	1.0	-0.895074	-0.860318	-2.095460	2.006654	5315.0	
7	7	0.0	1.0	0.534453	0.650106	-0.884385	1.002672	4891.0	
8	8	0.0	0.0	0.534453	0.650106	-0.922167	0.724530	8714.0	
9	9	0.0	1.0	1.159871	1.052885	-0.099581	-0.269202	7347.0	

In [13]:
```
train_data.columns
test_data.columns
```

Out[13]:
```
Index(['Unnamed: 0', 'holiday', 'workingday', 'temp', 'atemp', 'humidity',
       'windspeed', 'rentals', 'season_2', 'season_3', 'season_4', 'month_2',
       'month_3', 'month_4', 'month_5', 'month_6', 'month_7', 'month_8',
       'month_9', 'month_10', 'month_11', 'month_12', 'day_of_week_1',
       'day_of_week_2', 'day_of_week_3', 'day_of_week_4', 'day_of_week_5',
       'day_of_week_6', 'weather_2', 'weather_3'],
      dtype='object')
```

In [14]:
```
# normalize test data
test_data['temp'] = test_data['temp'].apply(lambda x: (x - mean_temp ) / (sd_temp
test_data['atemp'] = test_data['atemp'].apply(lambda x: (x - mean_atemp ) / (sd_a
test_data['humidity'] = test_data['humidity'].apply(lambda x: (x - mean_humidity
test_data['windspeed'] = test_data['windspeed'].apply(lambda x: (x - mean_windspe
```

In [15]:
```
y_train = train_data['rentals'].values
X_train = train_data[['holiday', 'workingday', 'temp', 'atemp', 'humidity', 'wind
                  'season_3', 'season_4', 'month_2', 'month_3', 'month_4', 'month_
                  'month_7', 'month_8', 'month_9', 'month_10', 'month_11', 'month_
                  'day_of_week_2', 'day_of_week_3', 'day_of_week_4', 'day_of_week_
                  'weather_3']].values

y_test = test_data['rentals'].values
X_test = test_data[['holiday', 'workingday', 'temp', 'atemp', 'humidity', 'windsp
                  'season_3', 'season_4', 'month_2', 'month_3', 'month_4', 'month_
                  'month_7', 'month_8', 'month_9', 'month_10', 'month_11', 'month_
                  'day_of_week_2', 'day_of_week_3', 'day_of_week_4', 'day_of_week_
                  'weather_3']].values
```

your answers here

Part (b): Multiple Linear Regression

We are now ready to fit a linear regression model and analyze its coefficients and residuals.

- Fit a multiple linear regression model to the training set, and report its R^2 score on the test set.
- *Statistical significance*: Using a t-test, find out which of estimated coefficients are statistically significant at a significance level of 5% (p-value<0.05). Based on the results of the test, answer the following questions:
  - Which among the predictors have a positive correlation with the number of bike rentals?
  - Does the day of a week have a relationship with bike rentals?
  - Does the month influence the bike rentals?
  - What effect does a holiday have on bike rentals?
  - Is there a difference in the coefficients assigned to `temp` and `atemp`? Give an explanation for your observation.
- *Residual plot:* Make a plot of residuals of the fitted model $e = y - \hat{y}$ as a function of the predicted value $\hat{y}$. Note that this is different from the residual plot for simple linear regression. Draw a horizontal line denoting the zero residual value on the Y-axis. Does the plot reveal a non-linear relationship between the predictors and response? What does the plot convey about the variance of the error terms?

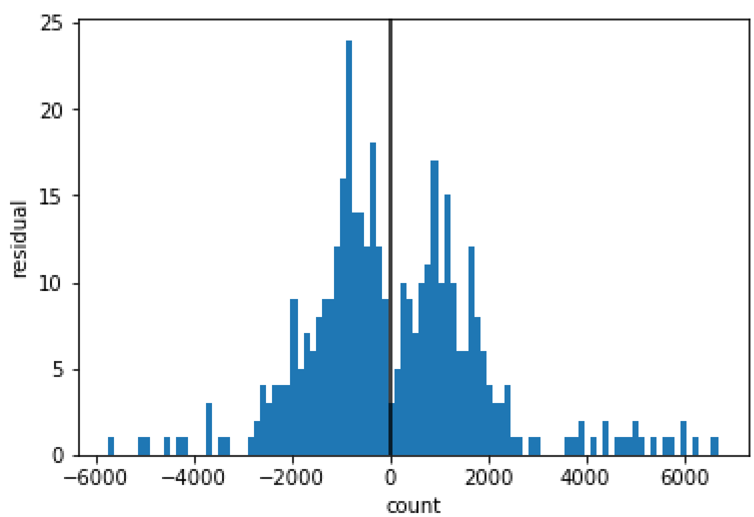
In [16]:
```
y_train = y_train.reshape(len(y_train), 1)
y_test = y_test.reshape(len(y_test), 1)
print(X_train.shape, y_train.shape, X_test.shape , y_test.shape)
```

```
(331, 28) (331, 1) (400, 28) (400, 1)
```

In [17]:
```
X_train_lm = X_train
y_train_lm = y_train


lm_1 = LinearRegression(fit_intercept=True)
lm_1.fit(X_train, y_train)

y_pred = lm_1.predict(X_test)

print('The equation of the regression plane is: {} + {}^T . x'.format(lm_1.interc
```

```
The equation of the regression plane is: [ 3681.77653152] + [[ -284.35627461
 308.15314174   925.73384986   312.43407189
   -548.49294906  -255.12258899   898.82903949  1032.88157484
   1226.18654265    88.94009267   239.18089841   333.34908642
    -65.81249999  -792.2628985  -1279.98700626  -776.4754899    405.1445661
    486.25090367   112.68164522  -118.83581872  -123.75147639
   -195.28593262   170.51134687    61.25602987   111.06689939
    465.14500996   -16.56658402 -1581.97828361]]^T . x
```

In [20]:
```
# Residual plot
plt.hist(y_test - y_pred, bins=100)
plt.axvline(x=0, color = "black")
plt.ylabel('residual')
plt.xlabel('count')
plt.show()
```



In [ ]:
```
# The only factor that seems to have a positive corrleation with rentals (which do
# statistically insignificant), is factor 9, which is the season 4 (winter) flag.

# Day of the week, month, and holiday seem to have no relationship to rentals.

# CAUTION: The values here were computed with a number of factors that need to be
# consider the current relationships, as observed, reliable.

# The coefficients for temp and atemp are different nominally, as shown below:
# x3           925.7338    474.536      1.951      0.052      -8.070    1859.538
# x4           312.4341    429.987      0.727      0.468    -533.705    1158.573
# One is 926 and the other is 312, as can be seen, but in a statistical sense they
# we cannot say they are reliably different from zero (i.e., the variation observe
```

In [ ]:
```
train_MSE= np.mean((y_train - lm_1.predict(X_train))**2)
test_MSE= np.mean((y_test - lm_1.predict(X_test))**2)
print('The train MSE is {}, the test MSE is {}'.format(train_MSE, test_MSE))

train_R_sq = lm_1.score(X_train, y_train)
test_R_sq = lm_1.score(X_test, y_test)
print('The train R^2 is {}, the test R^2 is {}'.format(train_R_sq, test_R_sq))
```

In [ ]:
```
# statsmodel regression
# create the X matrix by appending a column of ones to x_train
X = sm.add_constant(X_train)
X_test_sm = sm.add_constant(X_test)
# build the OLS model from the training data
smm = sm.OLS(y_train, X)

#save regression info in results_sm
results_sm = smm.fit()

print(results_sm)
```

In [ ]:
```
print(results_sm.summary())
print('Parameters: ', results_sm.params)
```

your answers here

Part (c): Checking Collinearity

Does the data suffer from multi-collinearity? To answer this question, let us first analyze the correlation matrix for the data. Compute the (Pearson product-moment) correlation matrix for the predictor variables in the training set, and visualize the matrix using a heatmap. For categorical attributes, you should use each binary predictor resulting from one-hot encoding to compute their correlations. Are there predictors that fall into natural groups based on the correlation values?

Hint: You may use the `np.corrcoef` function to compute the correlation matrix for a data set (do not forget to transpose the data matrix). You may use `plt.pcolor` function to visualize the correlation matrix.

In [ ]:
```
print(train_data.head())
dimensions = ['holiday', 'workingday', 'temp', 'atemp', 'humidity', 'windspeed',
    'season_3', 'season_4', 'month_2', 'month_3', 'month_4', 'month_5', 'month_6'
    'month_7', 'month_8', 'month_9', 'month_10', 'month_11', 'month_12', 'day_of_
    'day_of_week_2', 'day_of_week_3', 'day_of_week_4', 'day_of_week_5', 'day_of_w
    'weather_3']
corr_matrix = np.corrcoef(train_data[dimensions].T)
for y in range(0, len(corr_matrix)):
    for x in range(0, len(corr_matrix)):
        print ("%5.2f " % corr_matrix[y][x], end="")
    print()
heatmap = plt.pcolor(corr_matrix, cmap=matplotlib.cm.Blues)
plt.show()

dimensions_2 = ['holiday', 'workingday', 'temp', 'humidity', 'windspeed', 'month_
    'month_6', 'month_7', 'month_8', 'month_9', 'month_10', 'month_11', 'month_12
    'day_of_week_2', 'day_of_week_3', 'day_of_week_4', 'day_of_week_5', 'day_of_w
    'weather_3']
corr_matrix_2 = np.corrcoef(train_data[dimensions_2].T)
for y in range(0, len(corr_matrix_2)):
    for x in range(0, len(corr_matrix_2)):
        print ("%5.2f " % corr_matrix_2[y][x], end="")
    print()
heatmap_2 = plt.pcolor(corr_matrix_2, cmap=matplotlib.cm.Blues)
plt.show()
```

''' The first heatmap provided above does reflect correlation concentrations that suggest colinearity. They have been removed in the second.

Atemp and the seasons have been removed. Atemp should correlate highly with temp and the months all neatly bucket within the seasons; there's no good reason to have both. '''

Part (d): Subset Selection

Apply either one of the following subset selection methods discussed in class to choose a minimal subset of predictors that are related to the response variable:

- Step-wise forward selection
- Step-wise backward selection

We require you to implement both these methods *from scratch*. You may use the Bayesian Information Criterion (BIC) to choose the subset size in each method. Do these methods eliminate one or more of the redundant predictors (if any) identified in Part (c)? In each case, fit linear regression models using the identified subset of predictors to the training set. How do the test R^2 scores for the fitted models compare with the model fitted in Part (b) using all predictors?

In [ ]:
```
dimensions_3 = ['holiday', 'workingday', 'temp', 'humidity', 'windspeed', 'season_
y = train_data.rentals
X = train_data[dimensions_3]

# We'll use a slightly narrower feature set going into the stepwise algorithm,
# having pruned the months and atemp due to covariance
```

In [ ]:
```
# step forward model selection
# gets the bic of a given model, creates dict entry of model with its bic
def get_bic(predictors):
    model = sm.OLS(y, X[list(predictors)]).fit()
    return {"model": model, "bic" : model.bic}

# determine the best of a given set of models
def best_of(predictors):
    remaining_predictors = [p for p in X.columns if p not in predictors]
    results = []
    for p in remaining_predictors:
        results.append(get_bic(predictors+[p]))
    models = pd.DataFrame(results)
    best_model = models.loc[models['bic'].argmin()]
    return best_model

models = pd.DataFrame(columns=["bic", "model"])
predictors = []

# go through predictors stepwise until adding more predictors raises bic
for i in range(1, len(X.columns)+1):
    models.loc[i] = best_of(predictors)
    predictors = models.loc[i]["model"].model.exog_names
    if i == 1:
        best_score = models.loc[i]["bic"]
    else:
        if models.loc[i]["bic"] < best_score:
            best_score = models.loc[i]["bic"]
        if models.loc[i]["bic"] > best_score:
            best_model = models.loc[i-1]
            print(best_model)
            break
```

In [ ]:
```
print(best_model[1].summary())

# save model for further use in cross-validation
forward_step_model = best_model[1]
```

In [ ]:
```
dimensions_4 = ['holiday', 'workingday', 'temp', 'humidity', 'season_2', 'season_
```

In [ ]:
```
X_train = train_data[['holiday', 'workingday', 'temp', 'humidity', 'season_2', 's

X_test = test_data[['holiday', 'workingday', 'temp', 'humidity', 'season_2', 'sea


lm = LinearRegression(fit_intercept=True)
lm.fit(X_train, y_train)

y_pred = lm.predict(X_test)
```

In [ ]:
```
train_MSE= np.mean((y_train - lm.predict(X_train))**2)
test_MSE= np.mean((y_test - lm.predict(X_test))**2)
print('The train MSE is {}, the test MSE is {}'.format(train_MSE, test_MSE))

train_R_sq = lm.score(X_train, y_train)
test_R_sq = lm.score(X_test, y_test)
print('The train R^2 is {}, the test R^2 is {}'.format(train_R_sq, test_R_sq))
```

This process dramatically reduced the number of predictors to around 10 with similar performance. Ising the correlation matrix and stewise selection, we were able to eliminate much of the colinearity.The performance on the training set is about the same as the earlier models with more predictores. Although the r2 score was lower for the training data with the simpler model. Despite all this improvement in the complexity of the model, the result hint that maybe a linear model is not the best choice.

Part (e): Cross Validation

- Perform a 10-fold cross-validation procedure to select between the 3 competing models you have so far: the model with the best BIC from Step-wise forward selection, the model with the best BIC from Step-wise backward selection (if it is different), and the model with all possible predictors. Report the average R^2 across all 10 validation sets for each model and compare the results. Why do you think this is the case?
- Evaluate each of the 3 models on the provided left out test set by calculating R^2. Do the results agree with the cross-validation? Why or why not?

In [ ]:
```
# our models - lm_1 all predictors
#              lm   pruned predictors

# first model
scores = cross_val_score(lm, X_train_lm, y_train_lm, cv=10)
lm_avg = np.mean(scores)
print('Cross-validated scores', scores, 'Average of folds', lm_avg)
print
```

your answers here

In [ ]:
```
# second model
scores = cross_val_score(lm_1, X_train, y_train, cv=10)
lm_1_avg = np.mean(scores)
print('Cross-validated scores', scores, 'Average of folds', lm_1_avg)
```

In the second model we have a higher average score, inline with its earlier outperforming of the first model.