Natural Language Processing

Lecture 6: Parsing with Context Free Grammars I.
CKY algorithm

10/1/2021

COMS W4705
Yassine Benajiba

Formal Grammar and Parsing

- Formal Grammars are used in linguistics, NLP, programming languages.
- We want to build a compact model that describes a complete language.
- Need efficient algorithms to determine if a sentence is in the language or not **(recognition problem)**.
- We also want to recover the structure imposed by the grammar (**parsing problem**).

Syntactic Parsing

- Formalisms like CFGs and Finite State Automata define the (possibly infinite) set of legal strings of a language.

- **Parsing algorithms** determine if an input string is part of this language or not. For CFGs, they assign each string one or more syntactic analyses.

Two Approaches to Parsing

- Bottom-up: Start at the words (terminal symbols) and see which subtrees you can build. Then combine these subtrees into larger trees. (Driven by the input sentence.)
  CKY algorithm - requires Grammars in Chomsky Normal Form.

- Top-down: Start at the start symbol (S), try to apply production rules that are compatible with the input. (Driven by the grammar - next week)
  Earley algorithm

- Both approaches can be seen as a kind of search problem (next week).

Chomsky Normal Form

- A CFG $G=(N, \Sigma, R, S)$ is in Chomsky Normal Form (CNF) if the rules take one of the following forms:
  - $A \rightarrow B\ C$, where $A \in N$, $B \in N$, $C \in N$.
  - $A \rightarrow b$, where $A \in N$, $b \in \Sigma$.

S	→ NP VP	V	→ saw
VP	→ V NP	P	→ with
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ tail
NP	→ NP PP	N	→ student

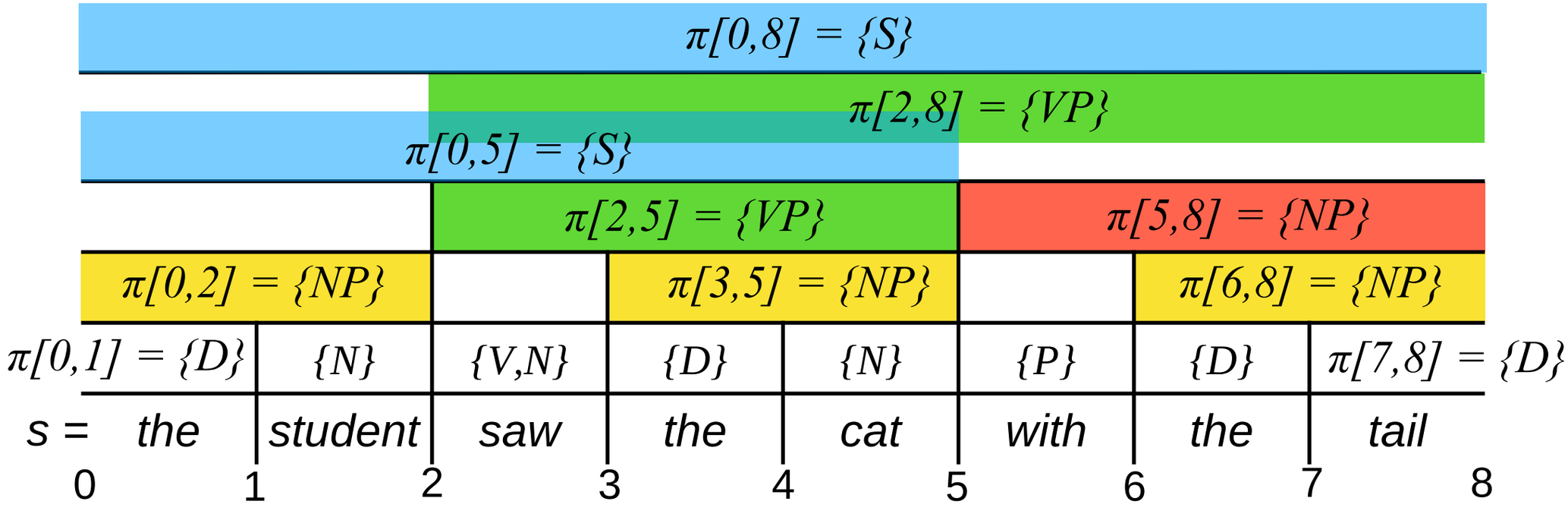
Any CFG can be converted to an equivalent grammar in CNF that expresses the same language.

Cocke-Kasami-Younger (CKY) Algorithm - Motivation

- A nonterminal A *covers* a sub-span $[i,j]$ of the input string s if the rules in the grammar can derive $s[i,j]$ from A.
  Let π[i,j] be the set of nonterminals that cover $[i,j]$.

- The string is recognized by the grammar if $S \in \pi[i,j]$.

- Approach: Compute π[i,j] for all sub-spans bottom-up, using dynamic-programming.



CKY Data Structure

- Use a 2-dimensional “parse table” to represent $\pi[i,j]$.

	0 she *1*	saw *2*	the *3*	cat *4*	with *5*	glasses
0	0,1	0,2	0,3	0,4	0,5	0,6
1	1,2	1,3	1,4	1,5	1,6	
2	2,3	2,4	2,5	2,6		
3	3,4	3,5	3,6			
4	4,5	4,6				
5	5,6					
6						

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

CKY Initialization

- For i=0…length(s-1):

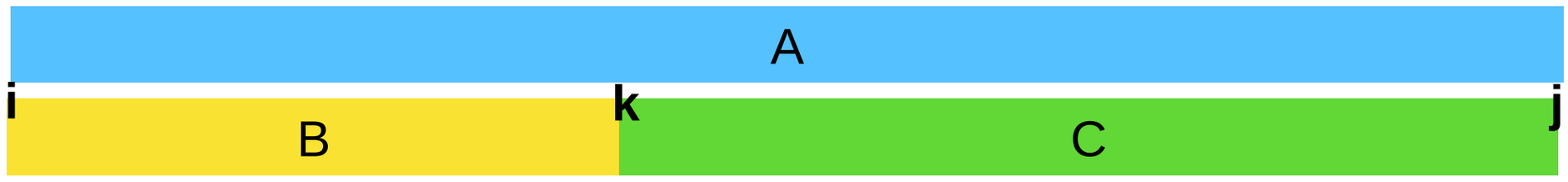
$$\pi[i, i+1] = \{A \mid A \rightarrow s[i:i+1] \in R\}$$

S → NP VP		**NP → she**				
VP → V NP		**NP → glasses**				
VP → VP PP		**D → the**				
PP → P NP		**N → cat**				
NP → D N		**N → glasses**				
NP → NP PP		**V → saw**				
		P → with				

	0 she	*1* saw	*2* the	*3* cat	*4* with	*5* glasses
0	NP	0,2	0,3	0,4	0,5	0,6
1	V	1,3	1,4	1,5	1,6	
2	D	2,4	2,5	2,6		
3	N	3,5	3,6			
4	P	4,6				
5	NP,N					
6						

CKY - finding the split

- CKY requires grammar to be in CNF.
- Assume subspan [i,j] is covered by nonterminal A.
  - Then this nonterminal was recognized by some production of the form $A \rightarrow B\ C$, where $A \in N$, $B \in N$, $C \in N$ (grammar is in CNF).
- Span [i,j] can be split into two parts:
  [i,k], which is covered by B, and
  [k,j] which is covered by C.



CKY - Recursive Definition

- To compute $\pi[i, j]$, try all possible split points k, such that $i < k < j$.
  - For each k, check if the nonterminals in $\pi[i,k]$ and $\pi[k,j]$ match any of the rules in the grammar.
- Recursive definition for $\pi[i, j]$:

$$\pi[i,j] = \bigcup_{k=i+1\ldots j-1} \{A | A \rightarrow B\,C \in R \text{ and } B \in \pi[i,k] \text{ and } C \in \pi[k,j]\}$$

CKY Full Algorithm

- **Input:** Grammar $G=(N, \Sigma, R, S)$, input string s of length n.

- for i=0…n-1: initialization

  $\pi[i, i+1] = \{A \mid A \rightarrow s[i]\}$

- for *length=2…n*: main loop

  for *i=0…(n-length):*

  $j = i+length$

  for $k=i+1 \ldots j-1$:

  $$M = \{A | A \rightarrow B\,C \in R \text{ and } B \in \pi[i,k] \text{ and } C \in \pi[k,j]\}$$
  $$\pi[i,j] = \pi[i,j] \cup M$$

- if $S \in \pi[0, i+1]$ return True, otherwise False

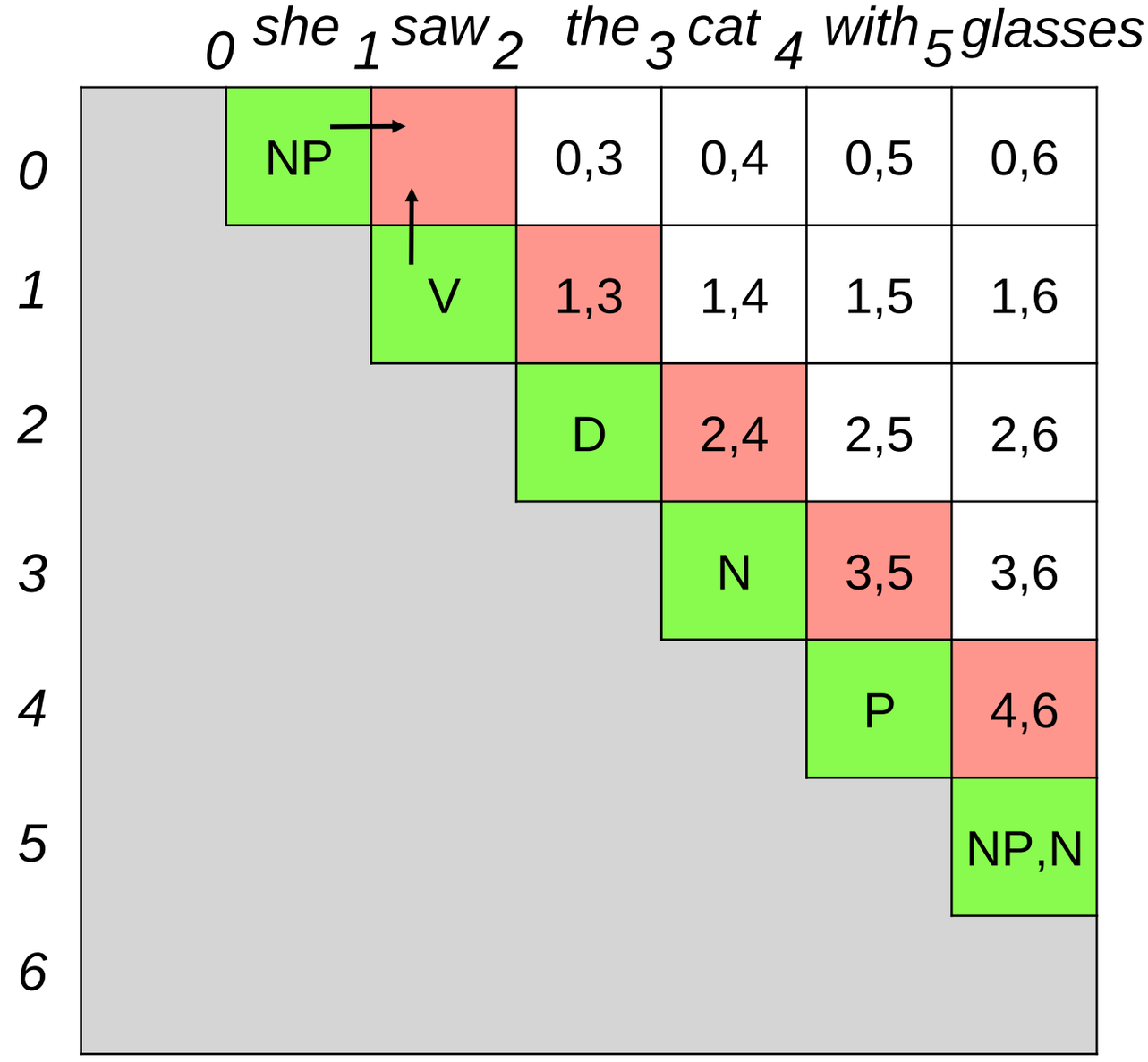
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=2

i=0,k=1,j=2



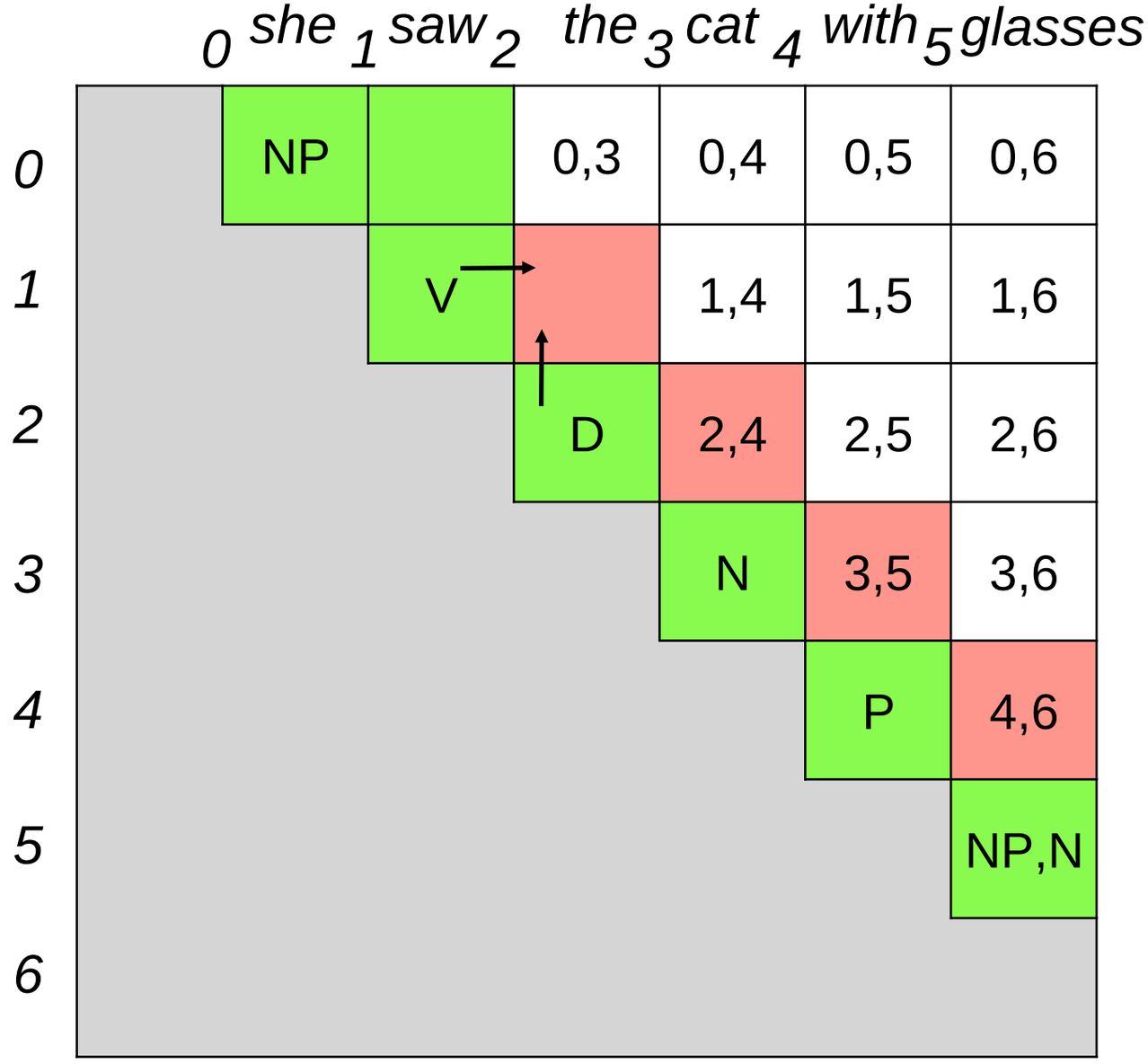
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=2

i=1,k=2,j=3



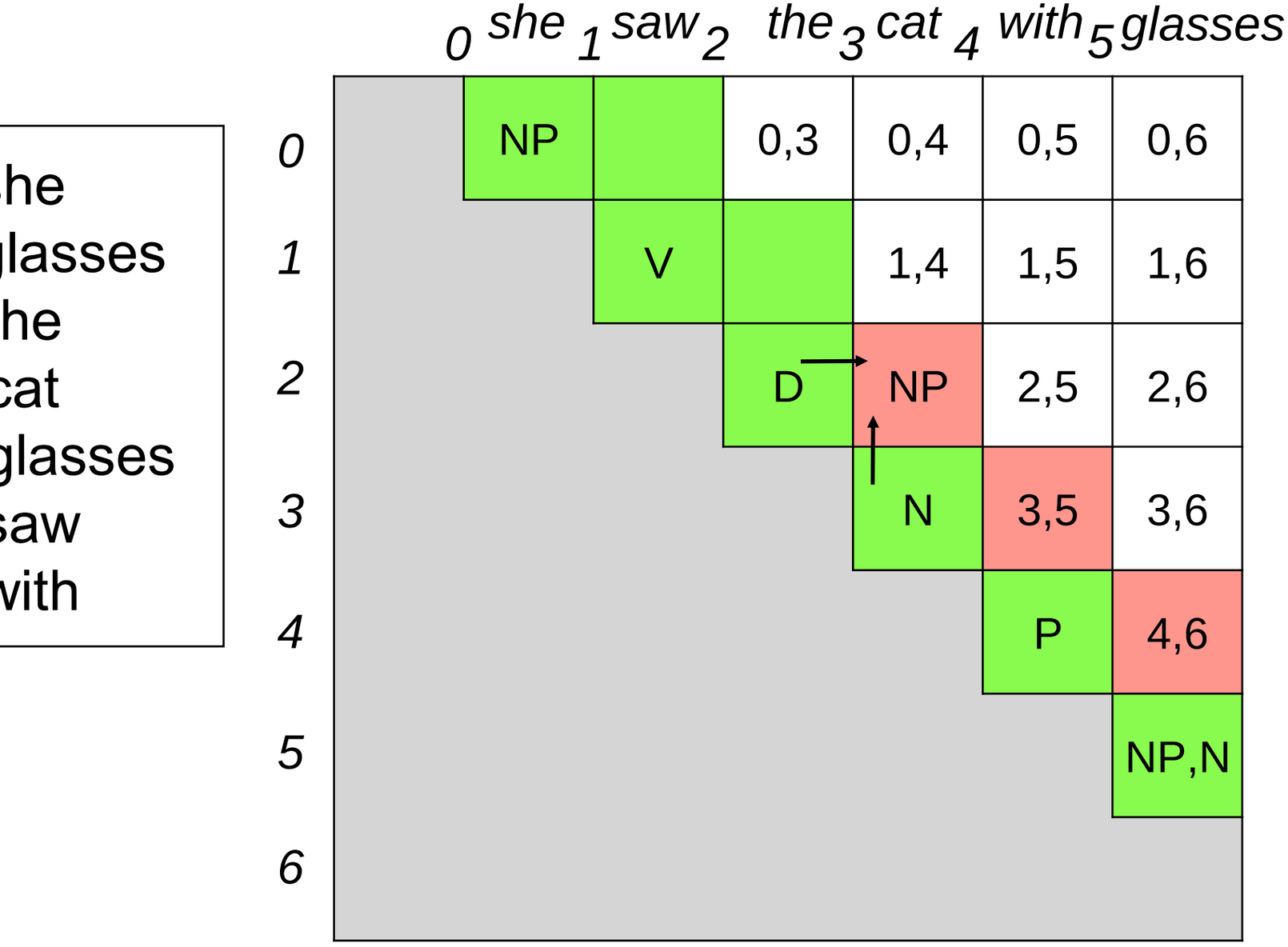
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ **D N**	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=2

i=2,k=3,j=4



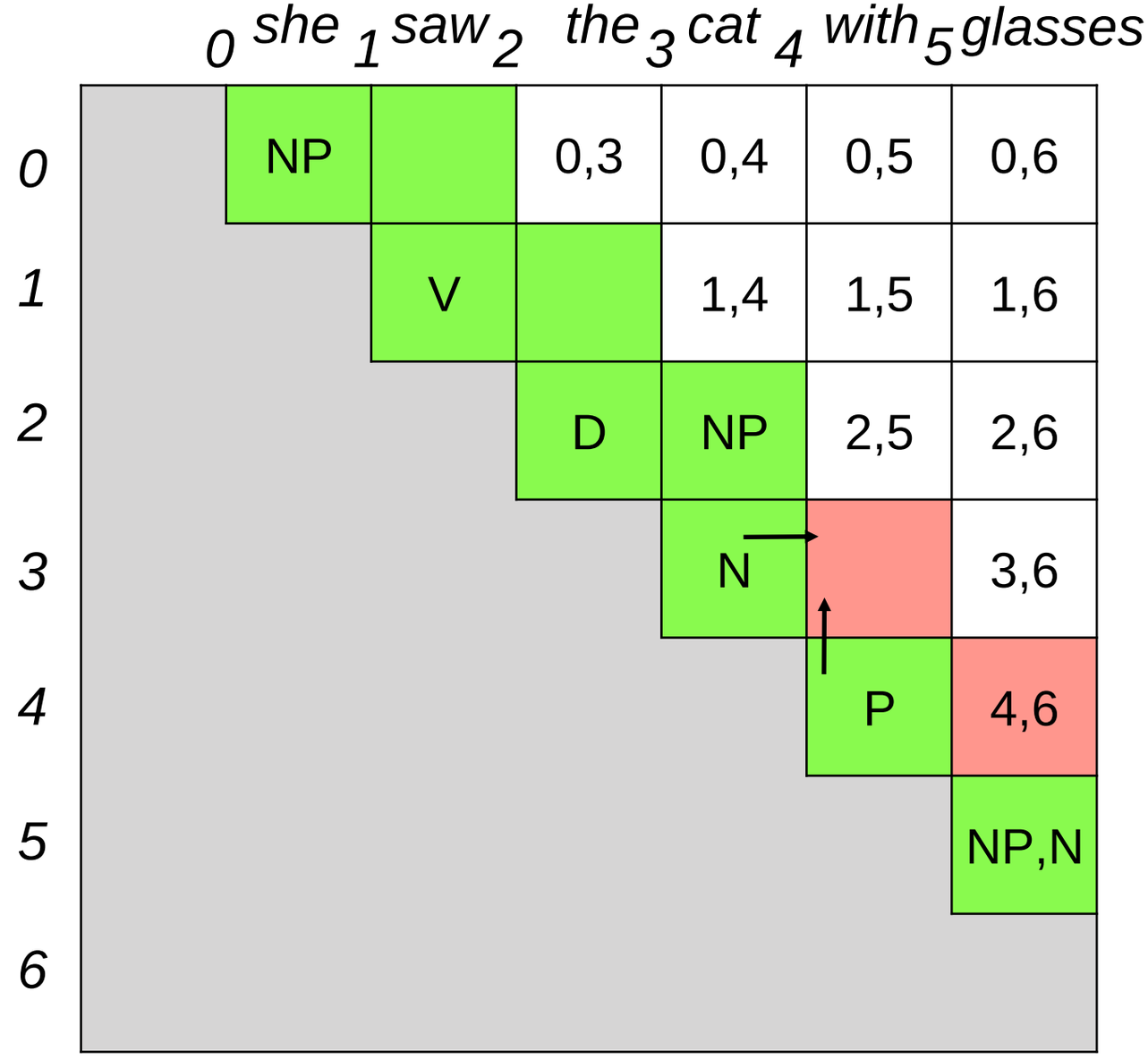
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=2

i=3,k=4,j=5



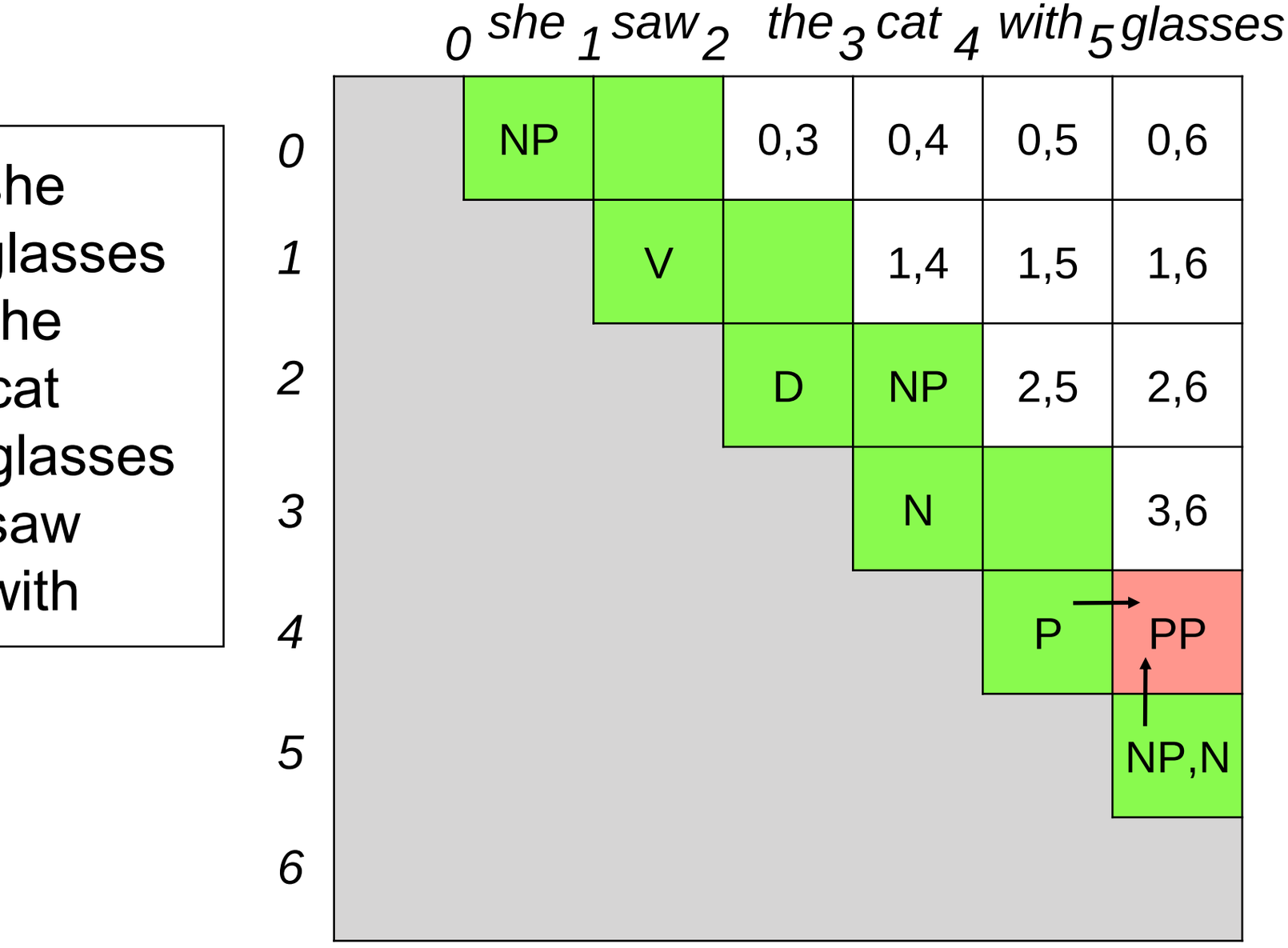
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ **P NP**	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=2

i=4,k=5,j=6



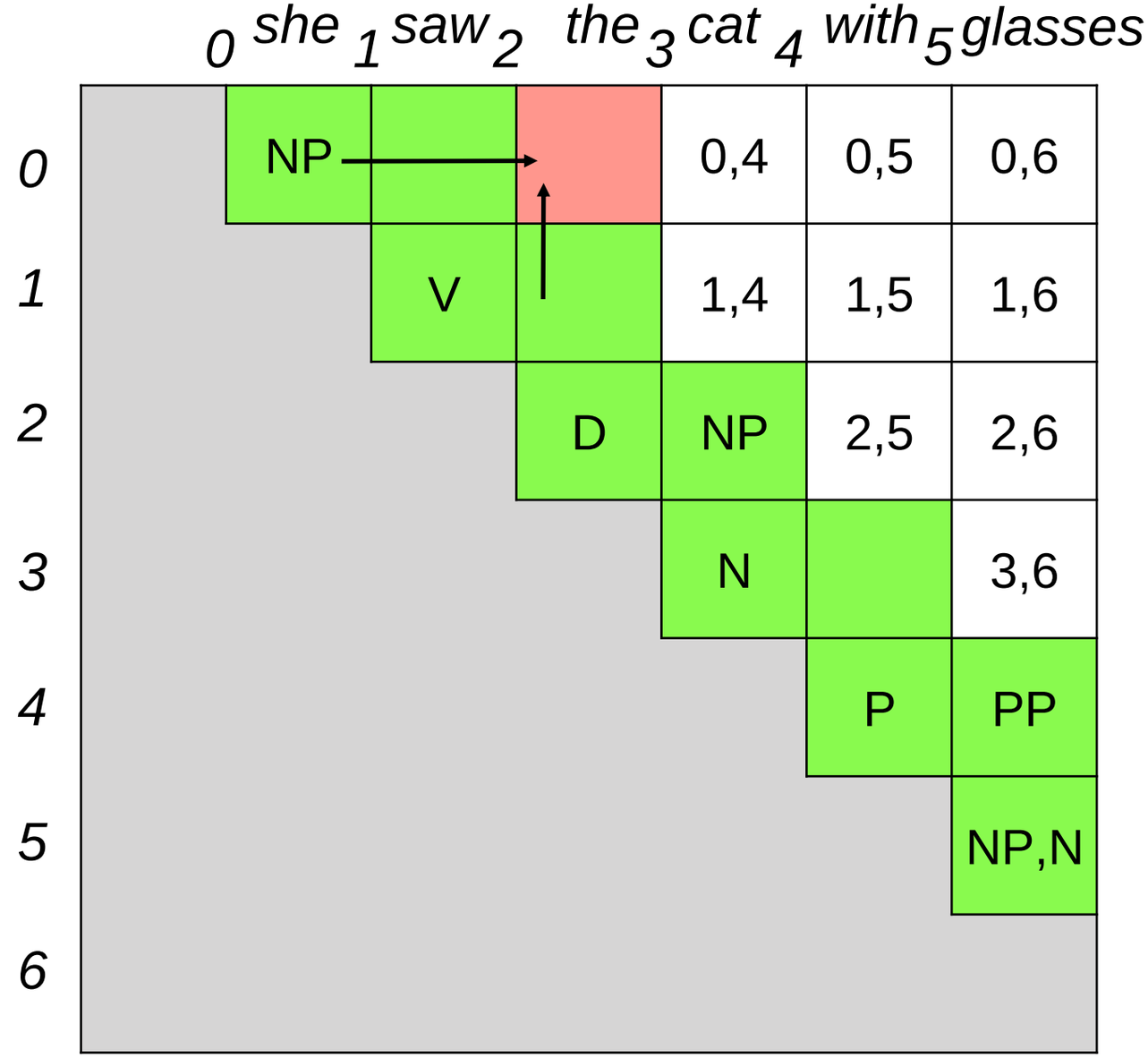
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=3

i=0,k=1,j=3



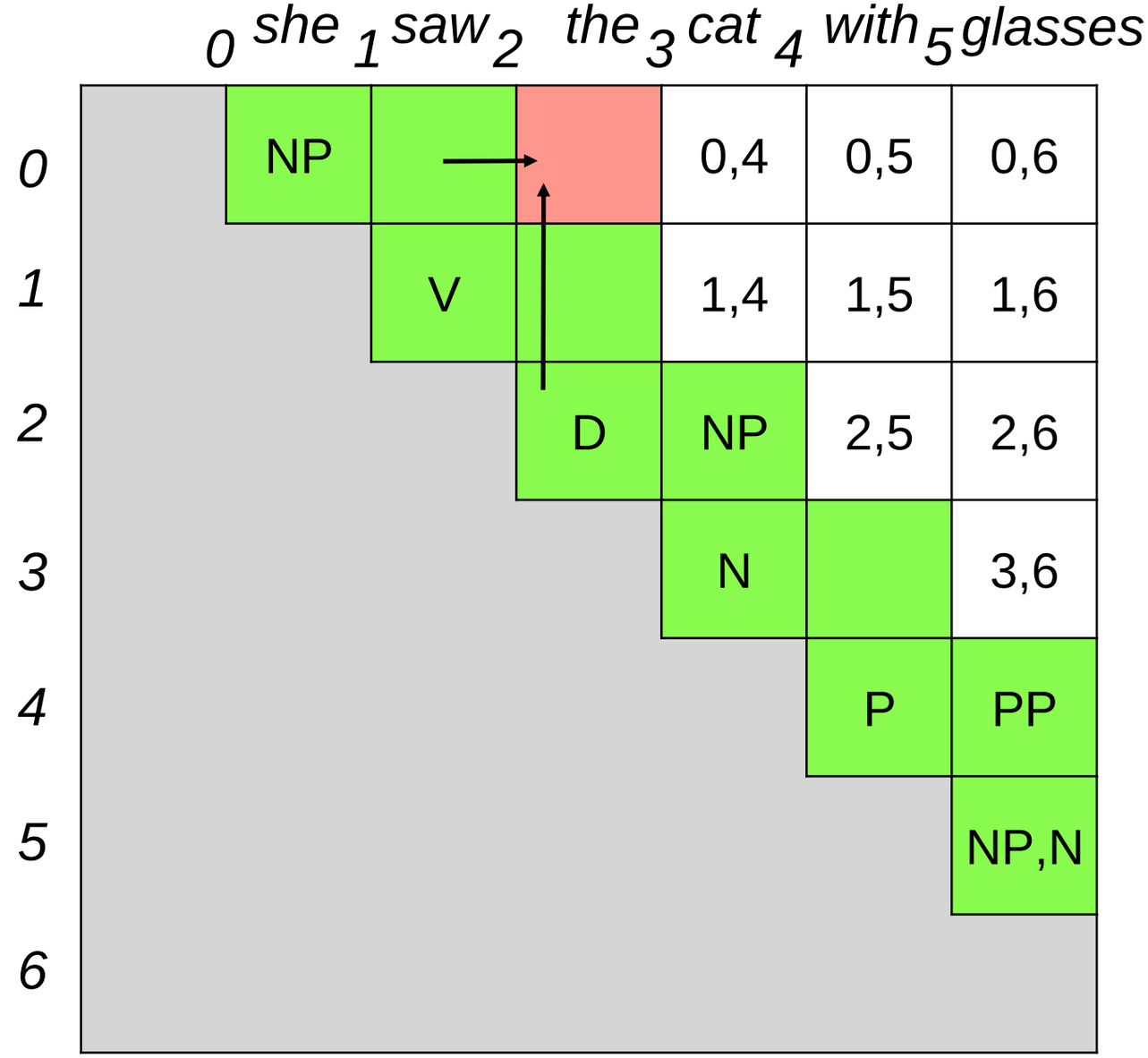
CKY Algorithm

for *i=0…(n-length):*
j = i+length
for *k=i+1…j-1:*
….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=3

i=0,k=2,j=3



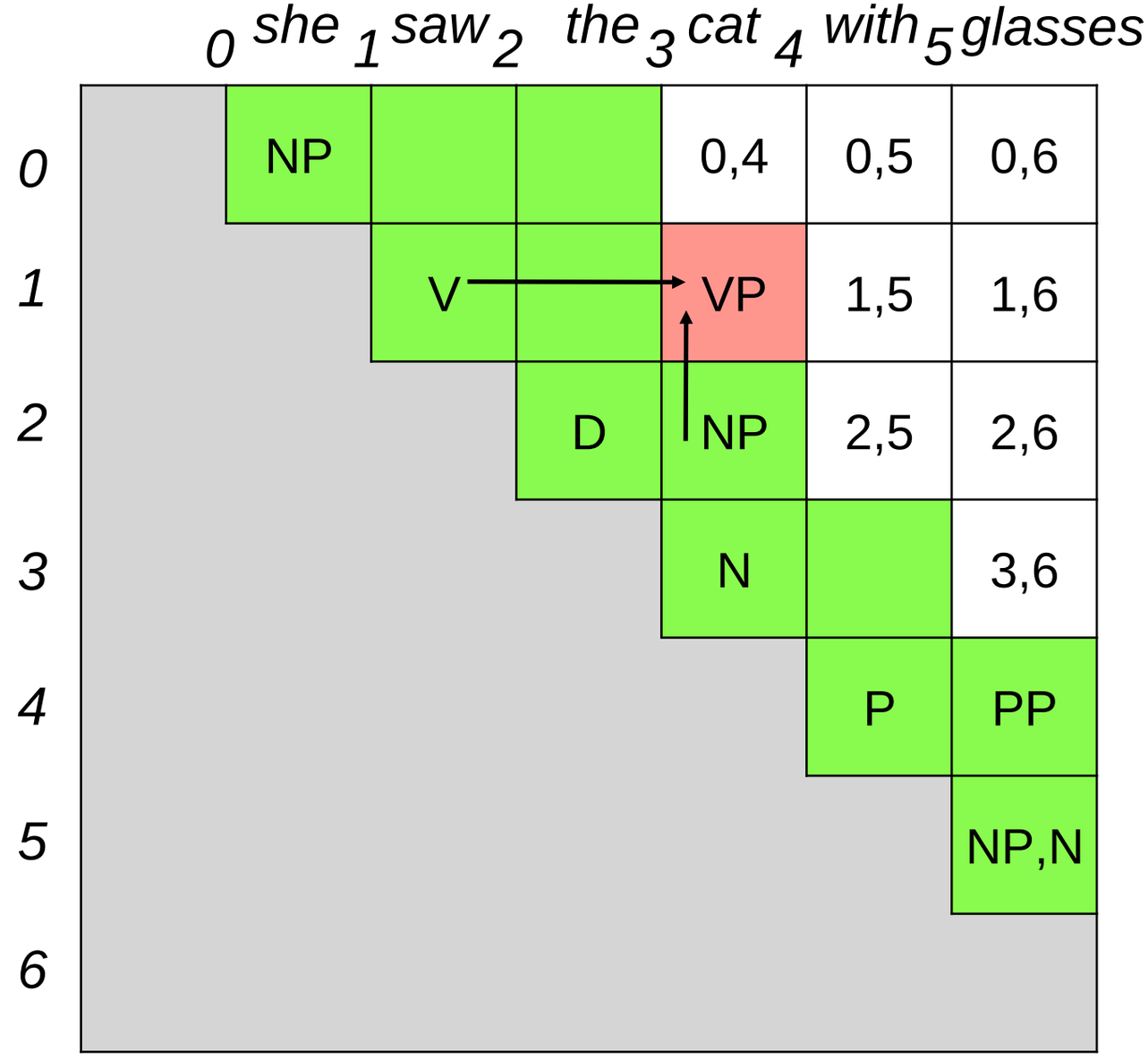
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

length=3

i=1,k=2,j=4

S → NP VP	NP → she
VP → V NP	NP → glasses
VP → VP PP	D → the
PP → P NP	N → cat
NP → D N	N → glasses
NP → NP PP	V → saw
	P → with



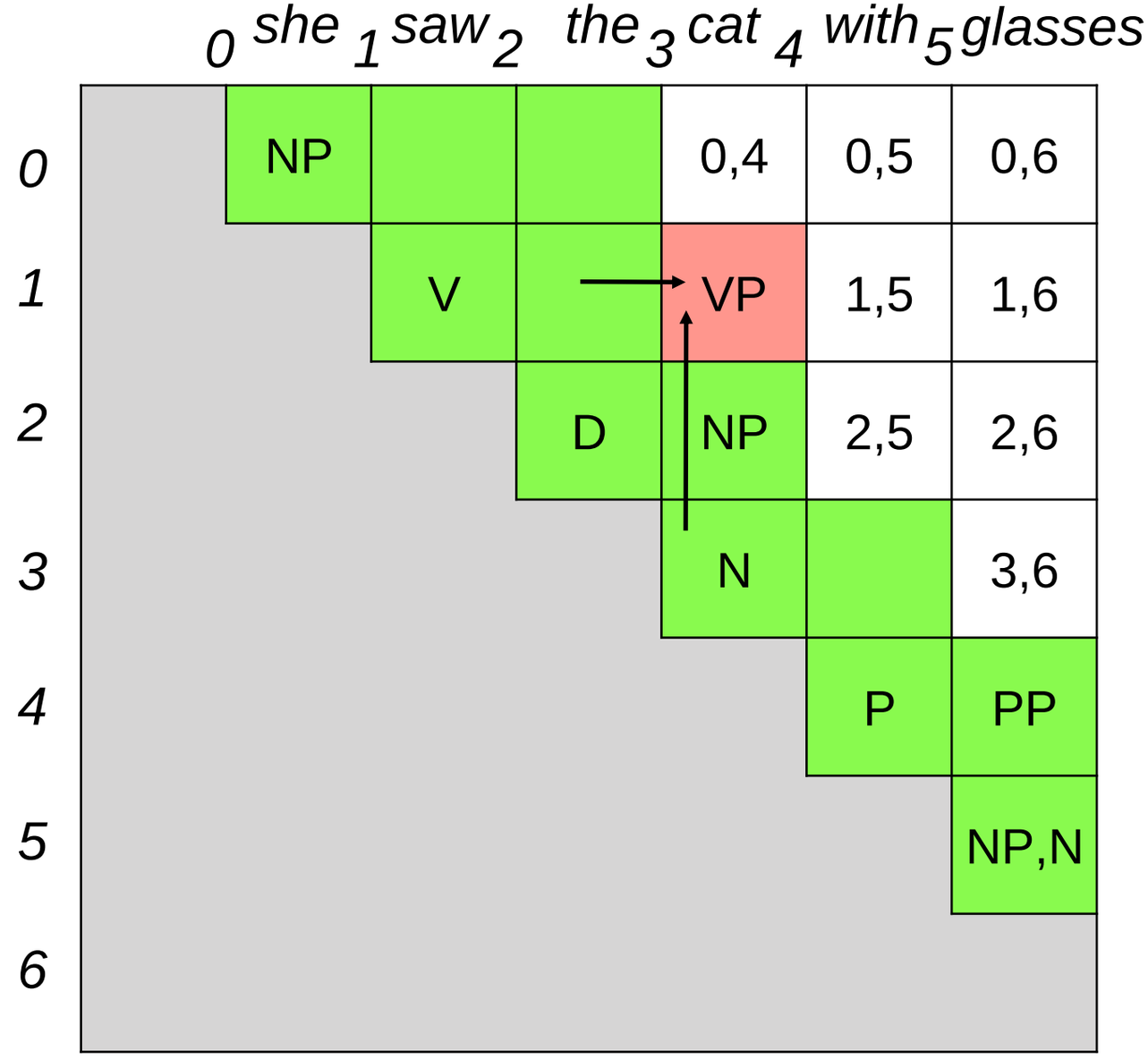
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=3

i=1,k=3,j=4



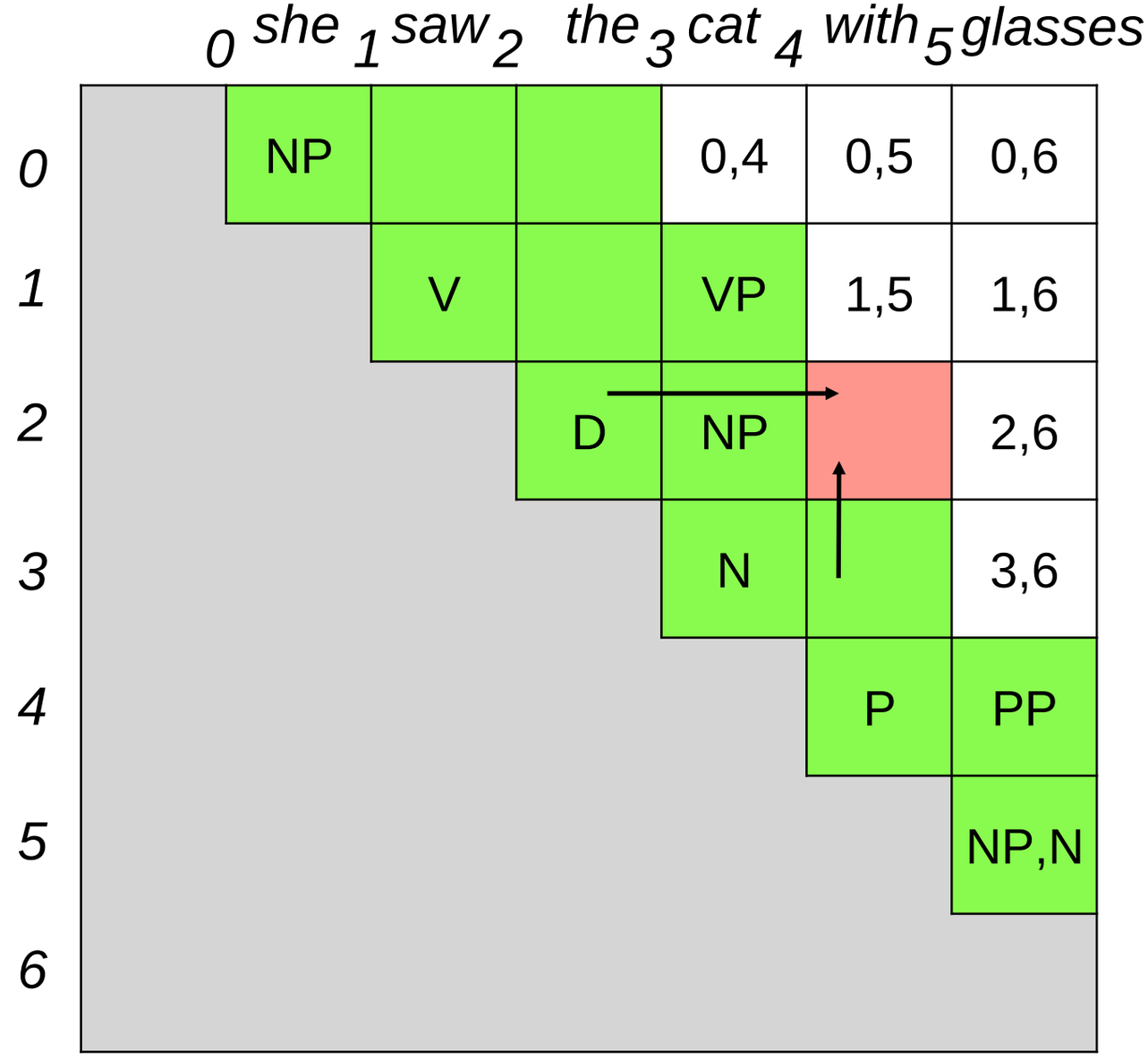
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=3

i=2,k=3,j=5



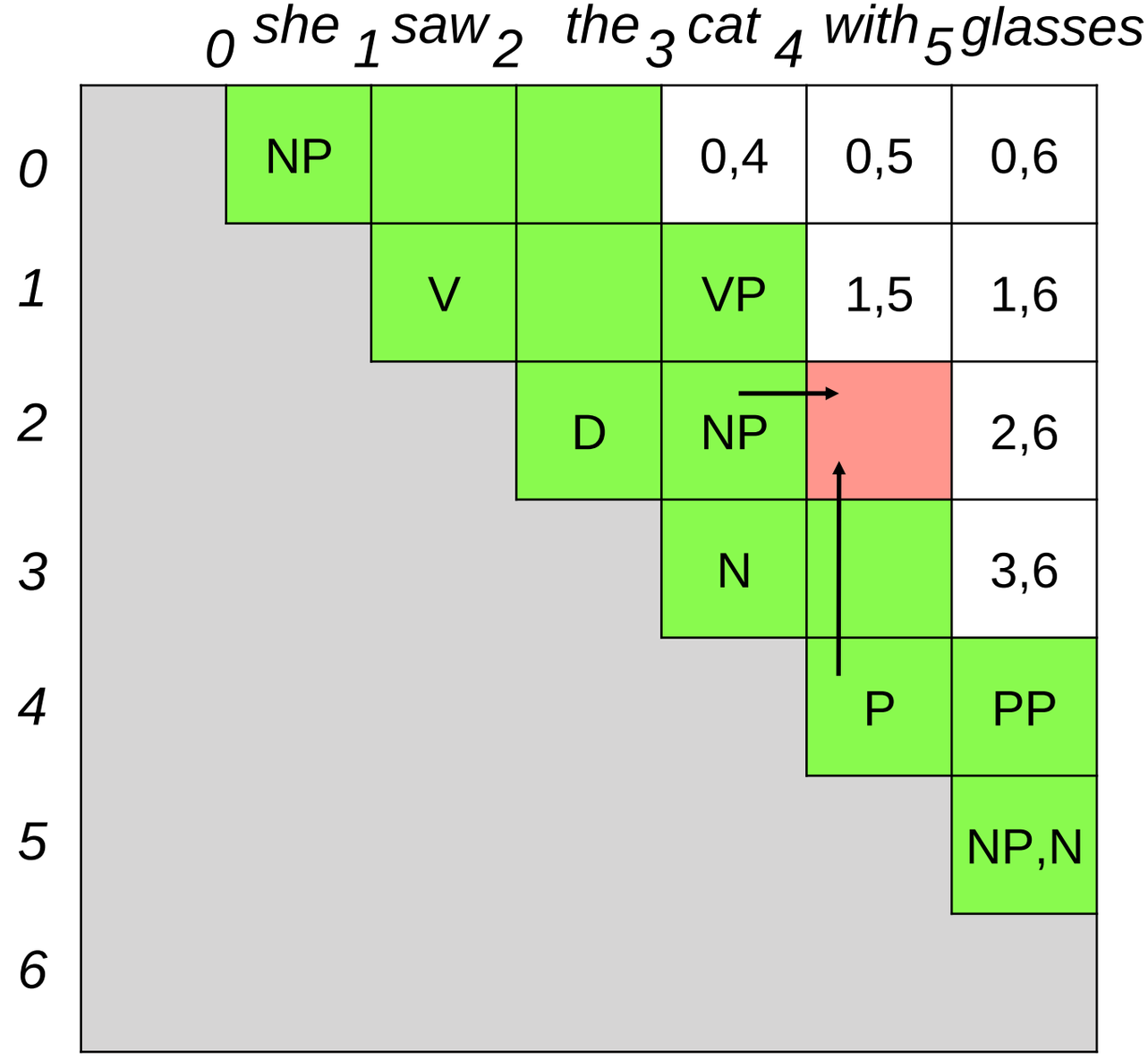
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=3

i=2,k=4,j=5



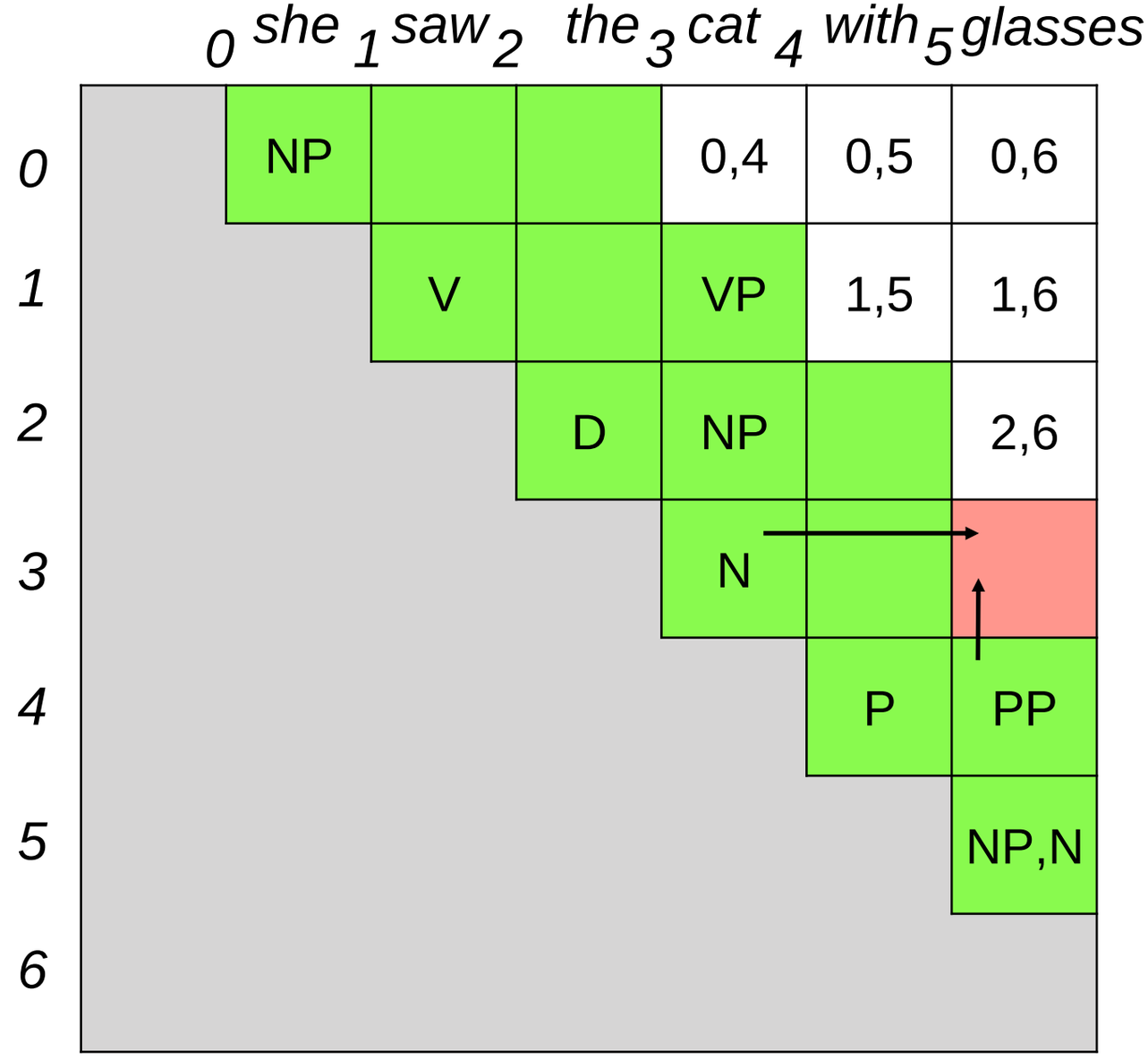
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=3

i=3,k=4,j=6



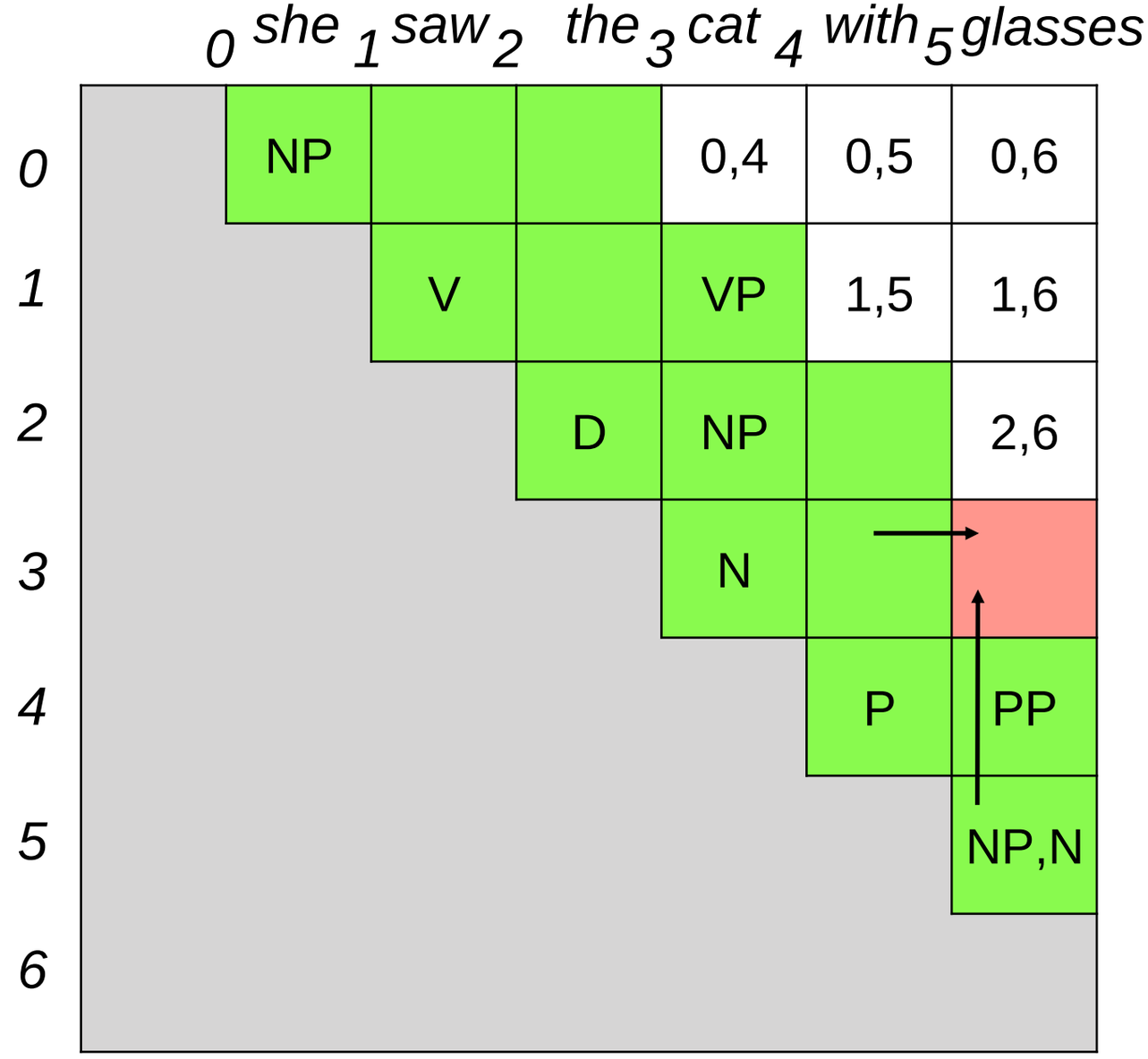
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=3

i=3,k=5,j=6



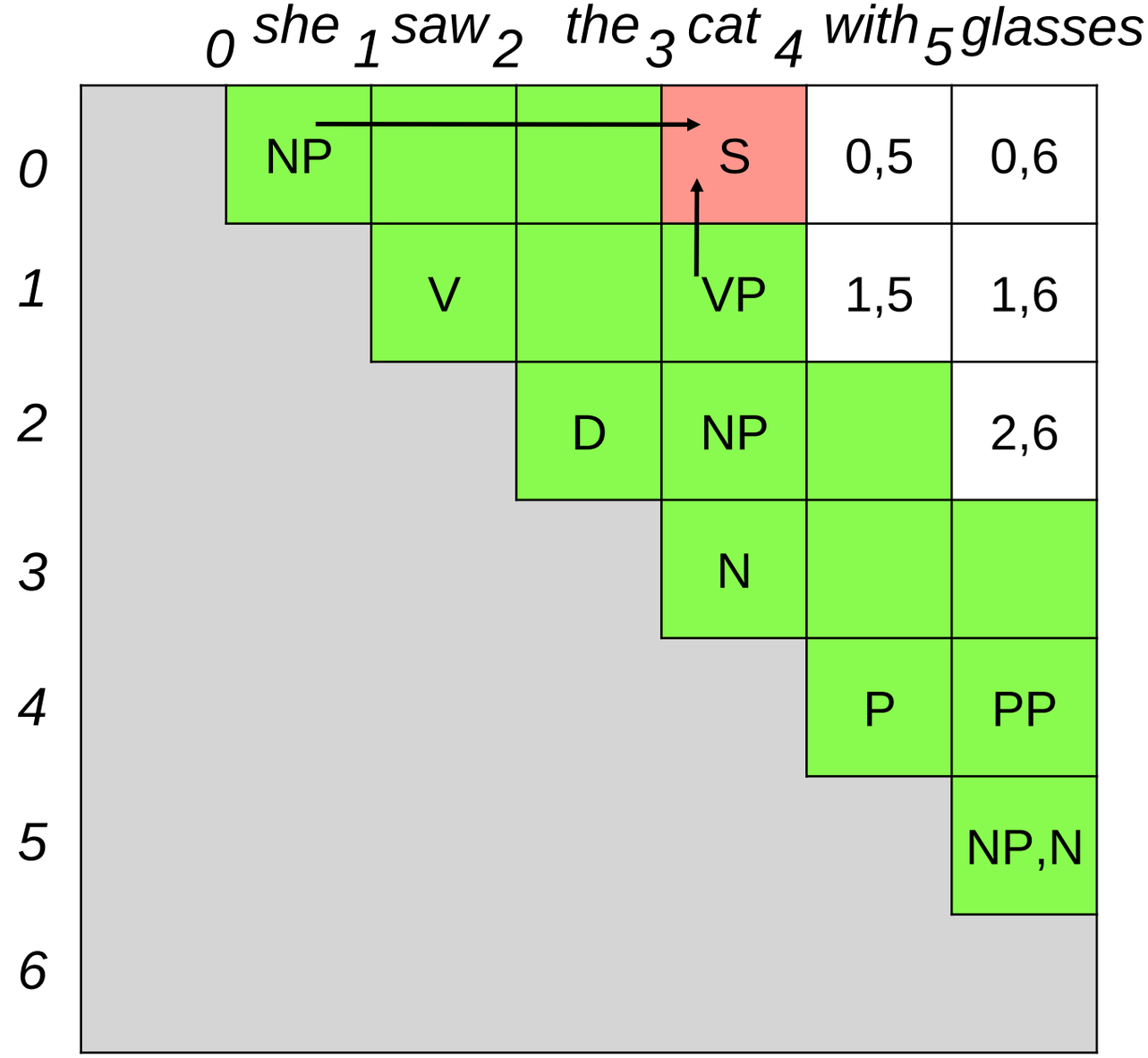
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ **NP VP**	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=4

i=0,k=1,j=4



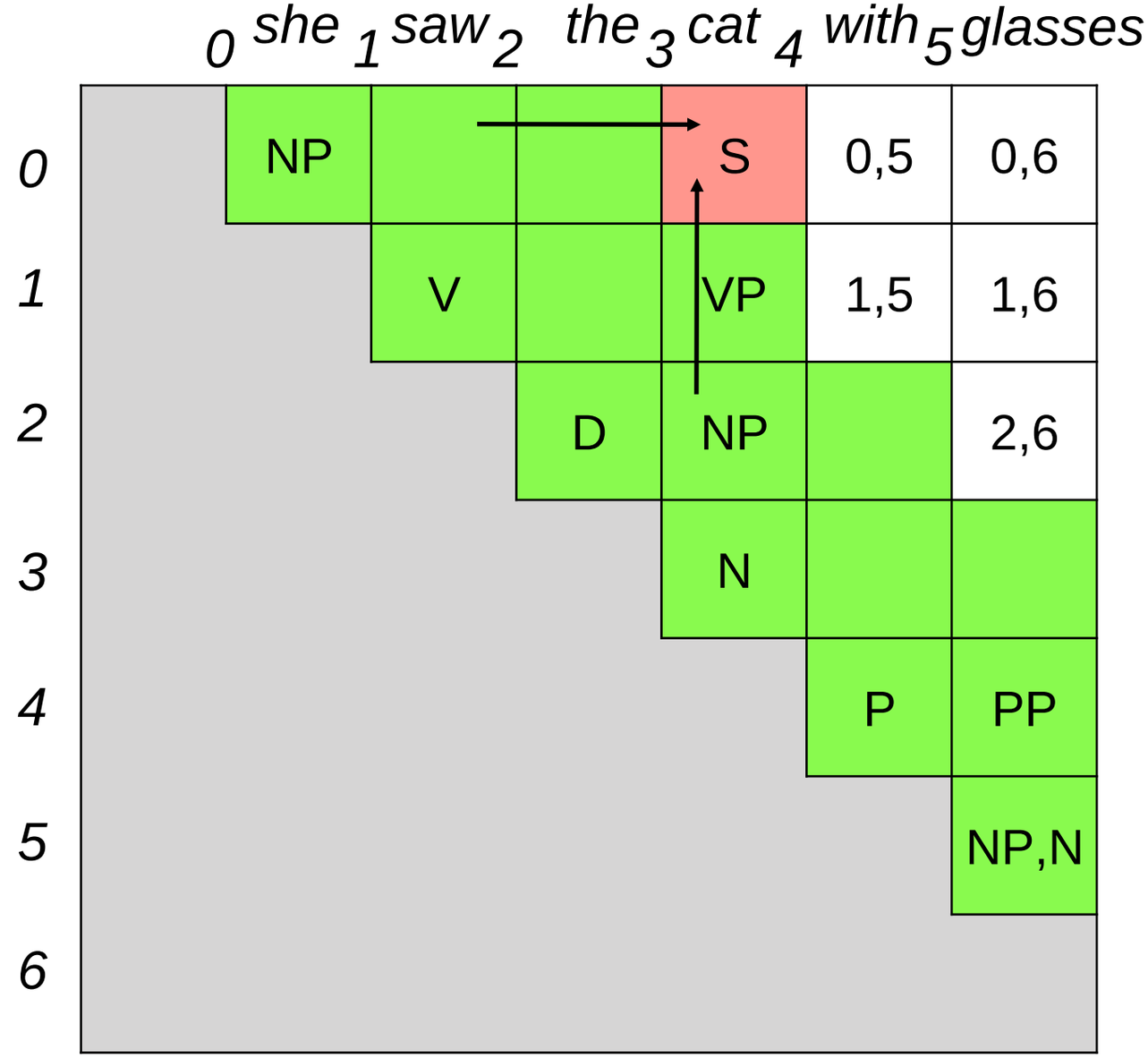
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=4

i=0,k=2,j=4



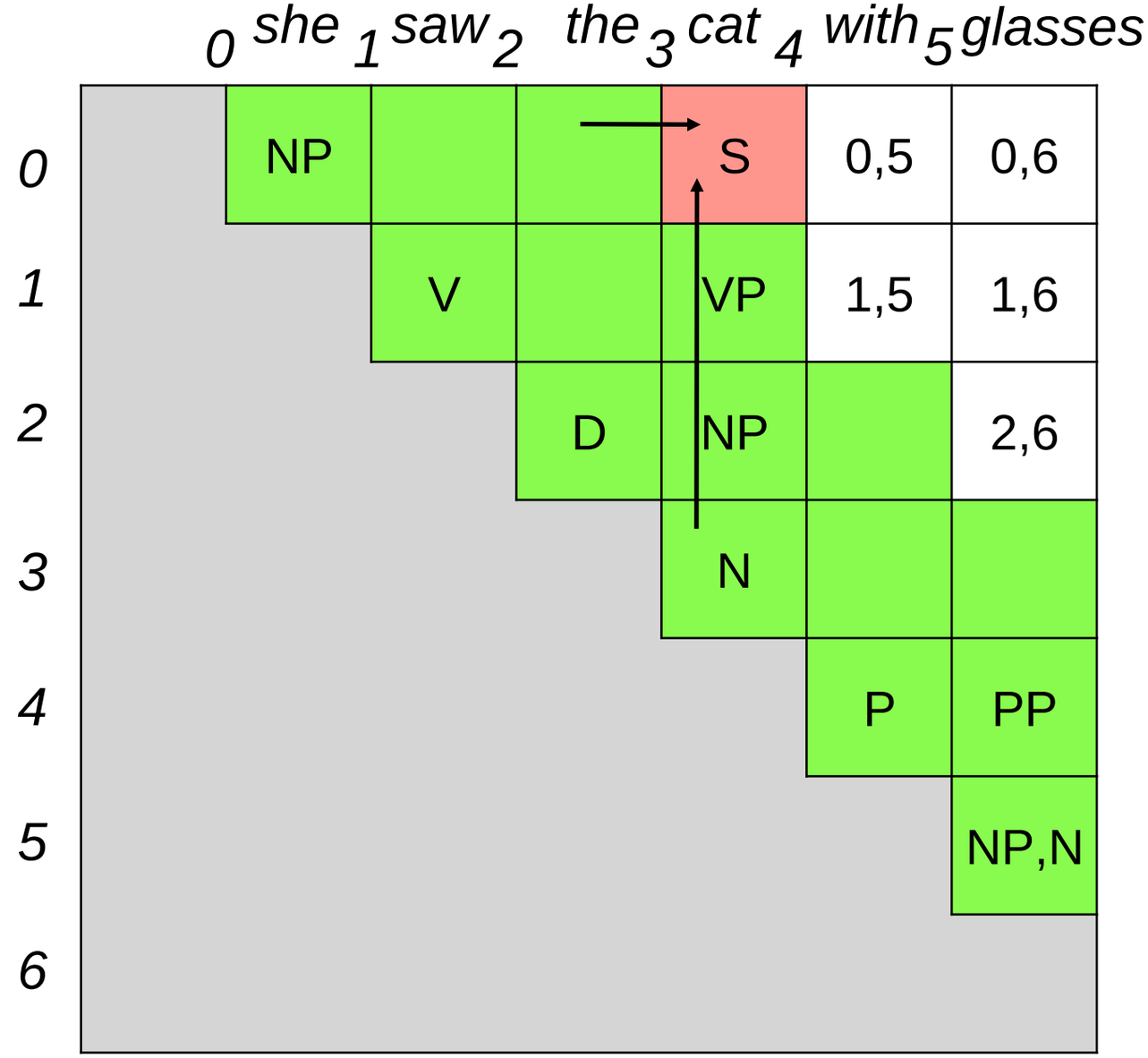
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=4

i=0,k=3,j=4



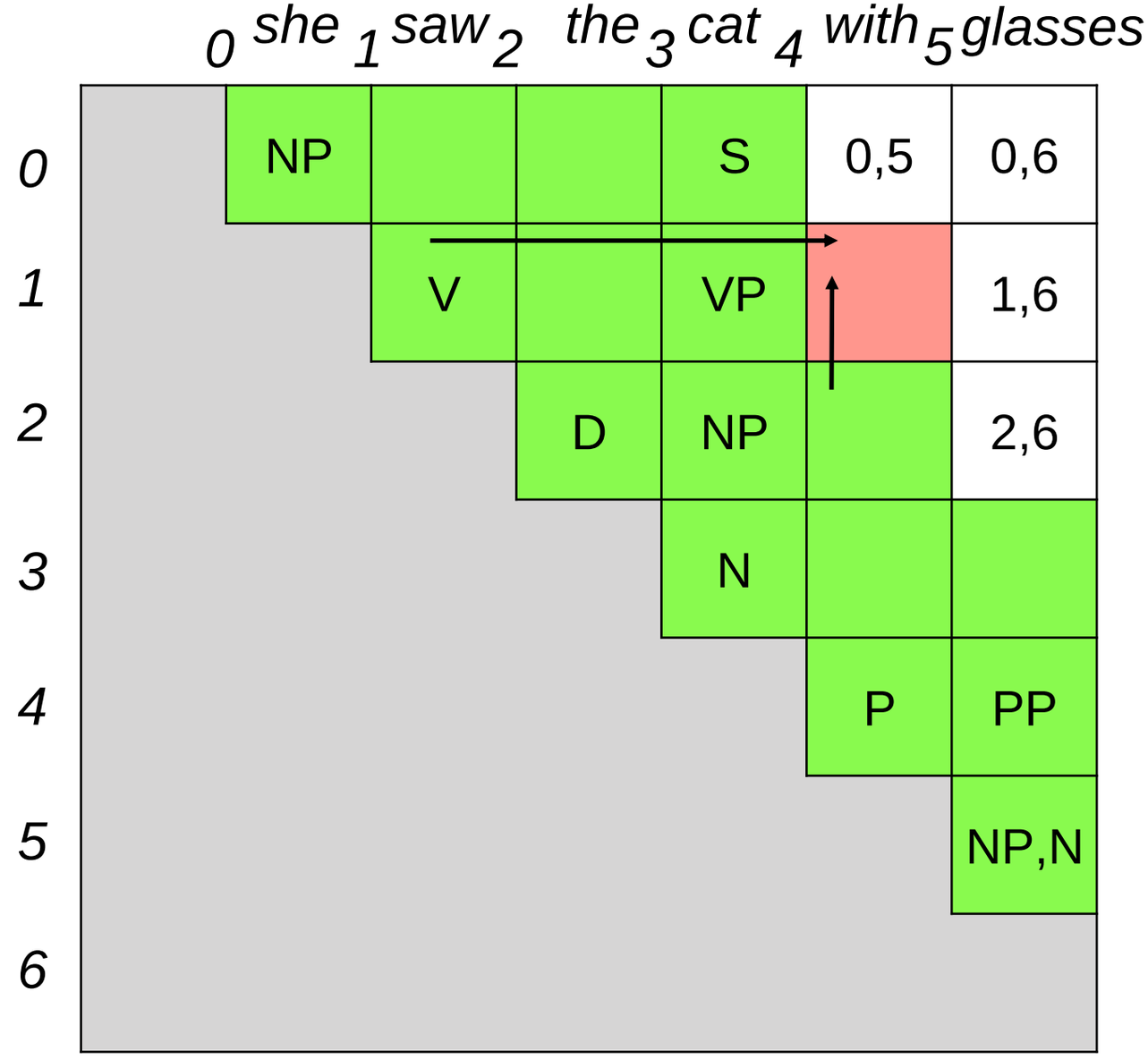
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=4

i=1,k=2,j=5



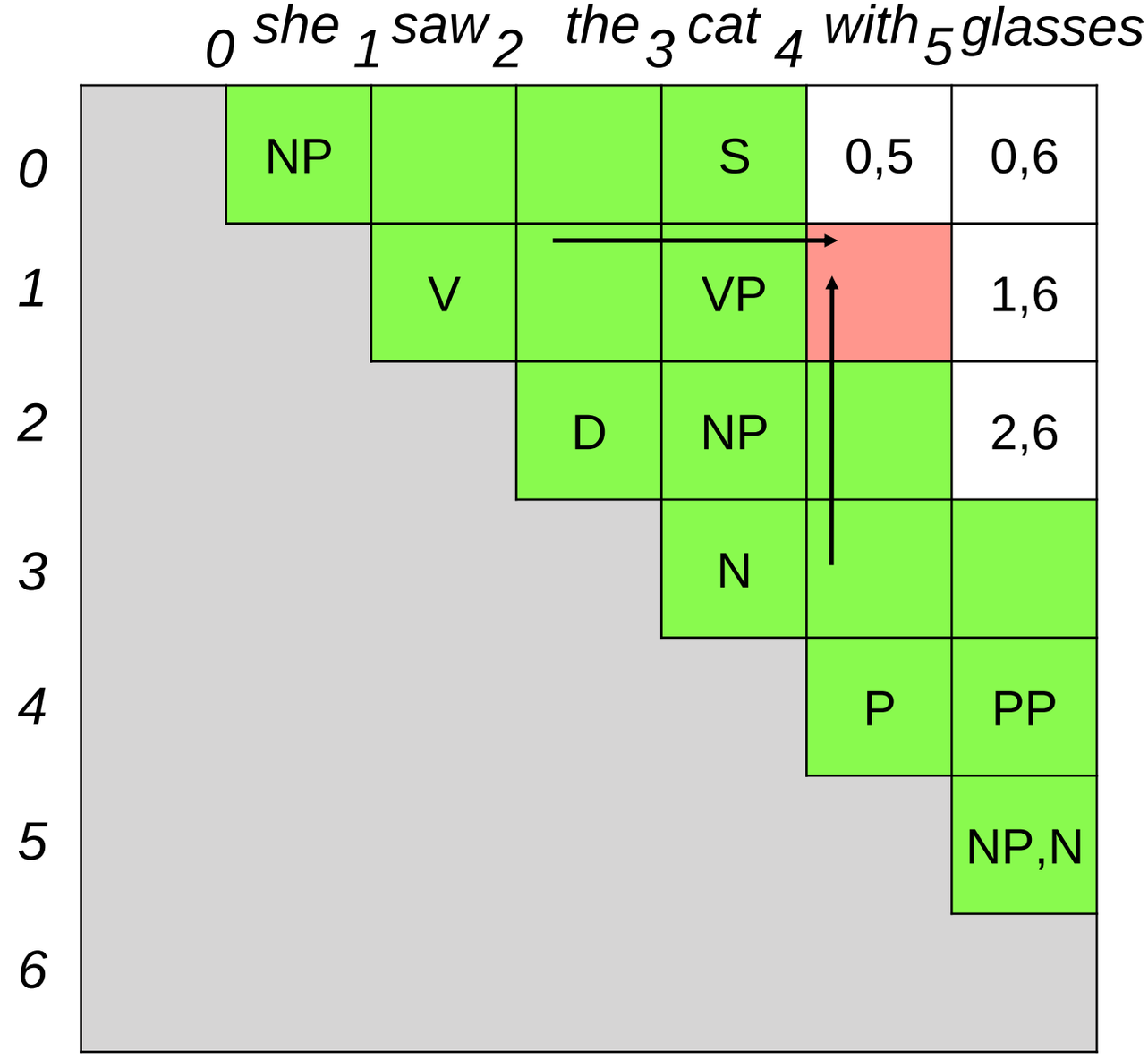
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=4

i=1,k=3,j=5



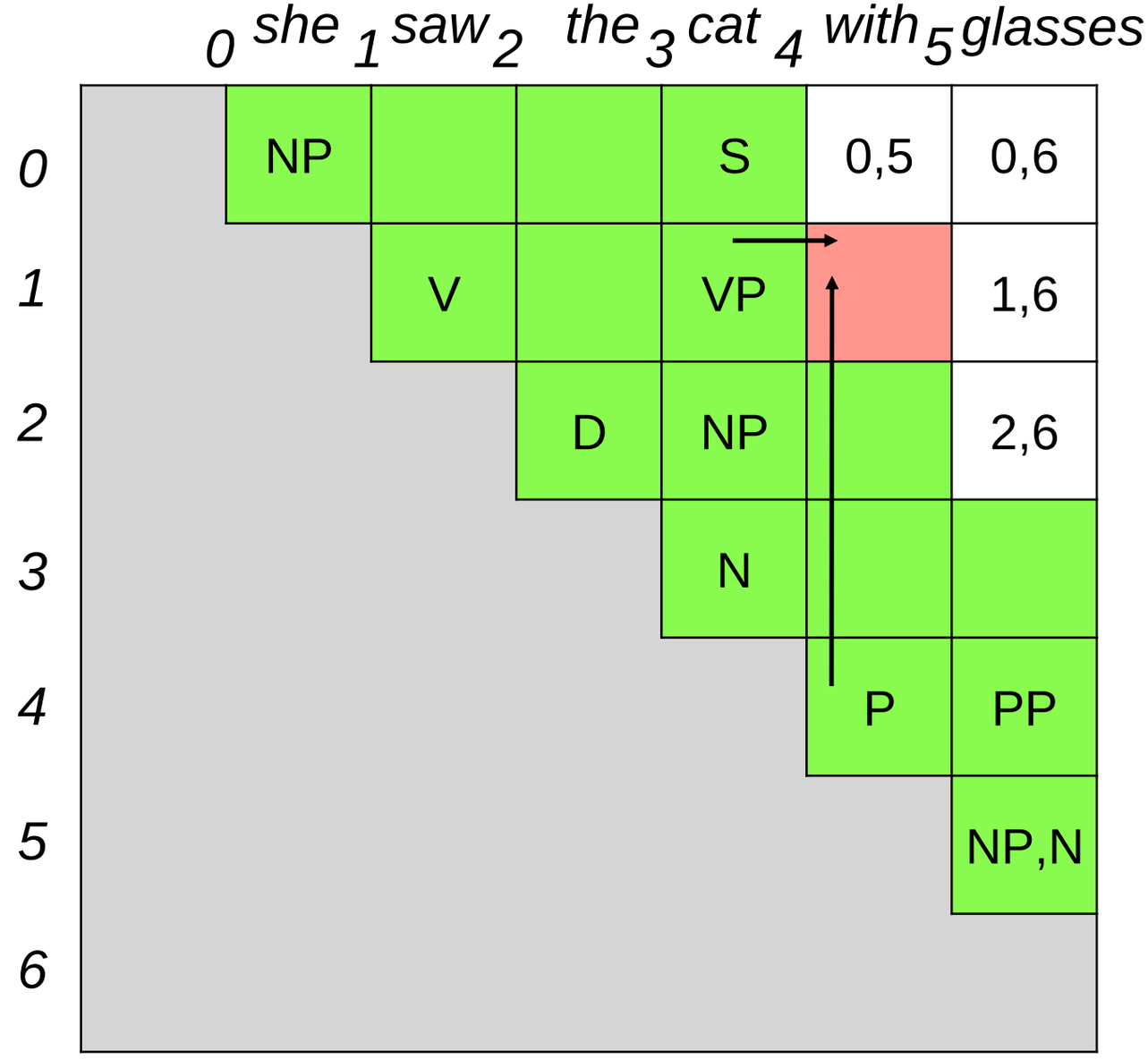
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=4

i=1,k=4,j=5



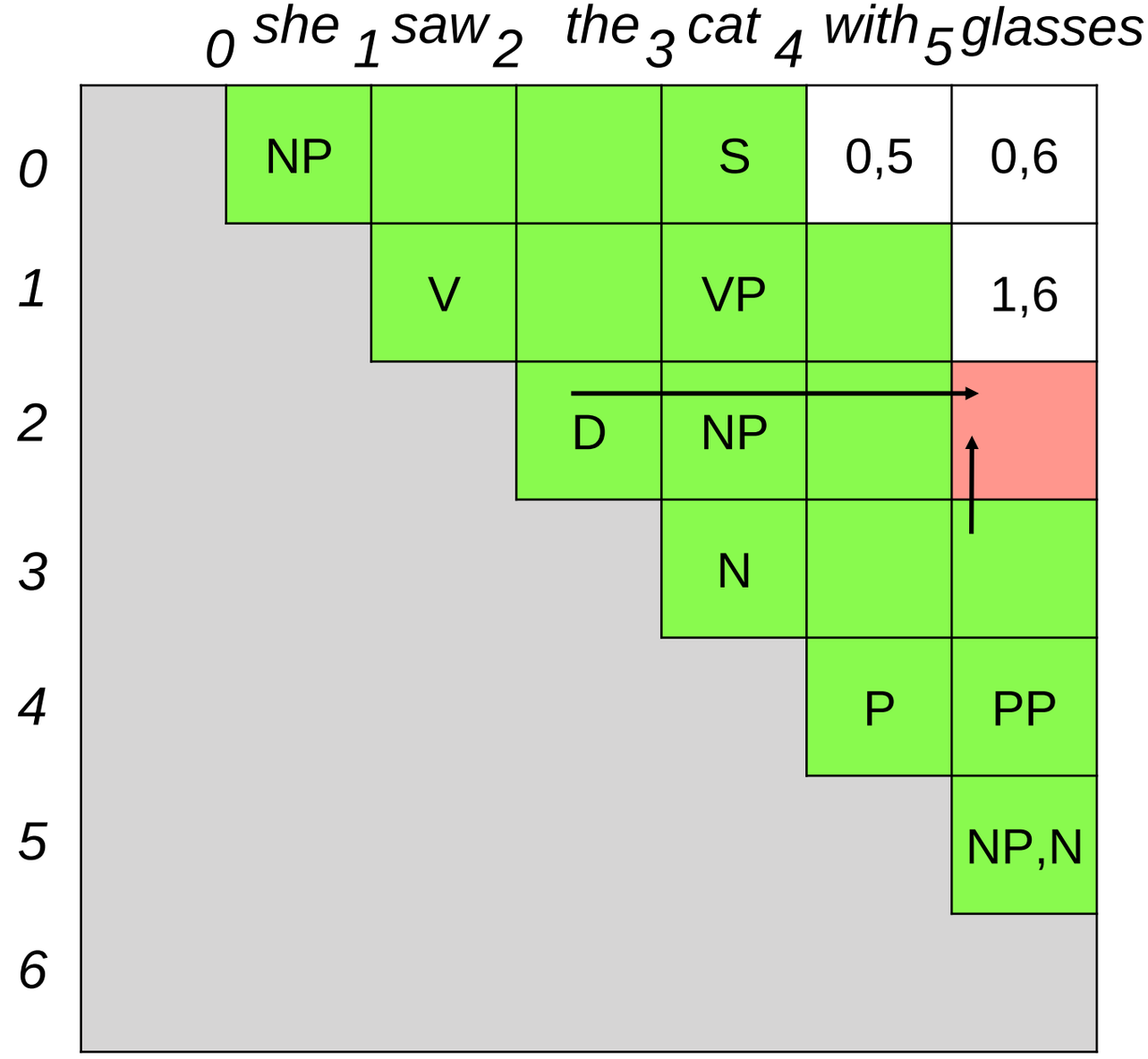
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=4

i=2,k=3,j=6



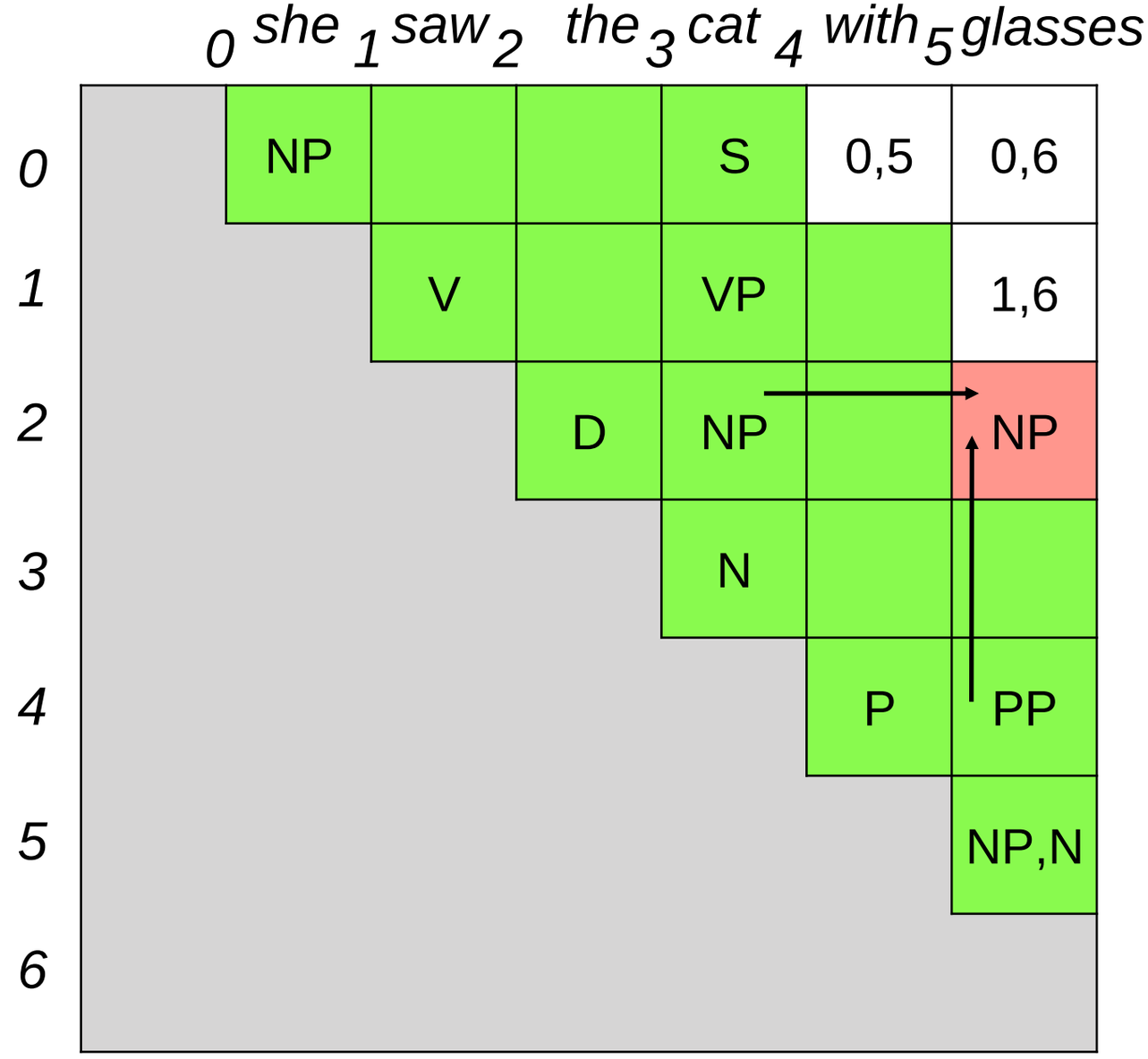
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ **NP PP**	V	→ saw
		P	→ with

length=4

i=2,k=4,j=6



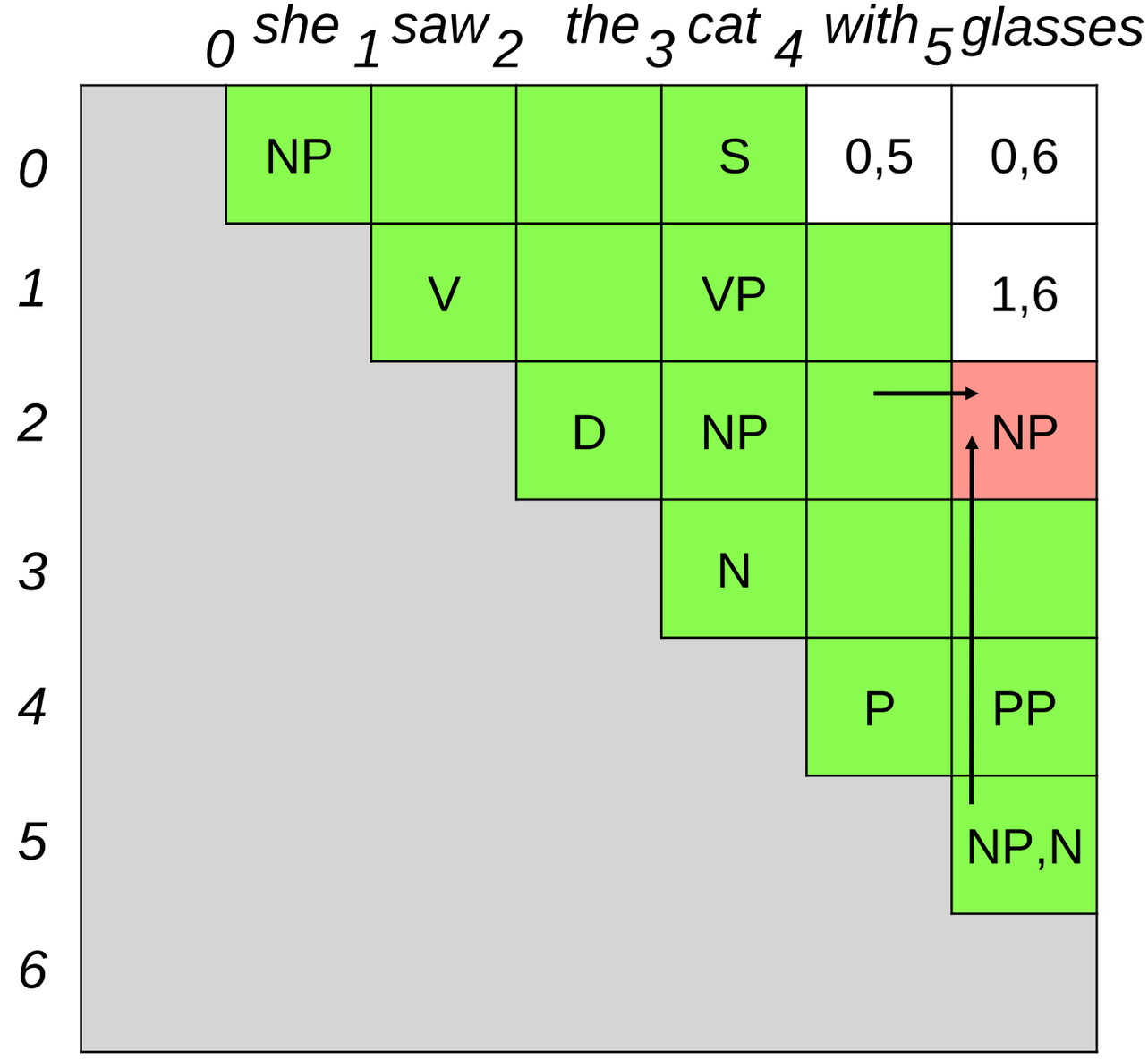
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=4

i=2,k=5,j=6



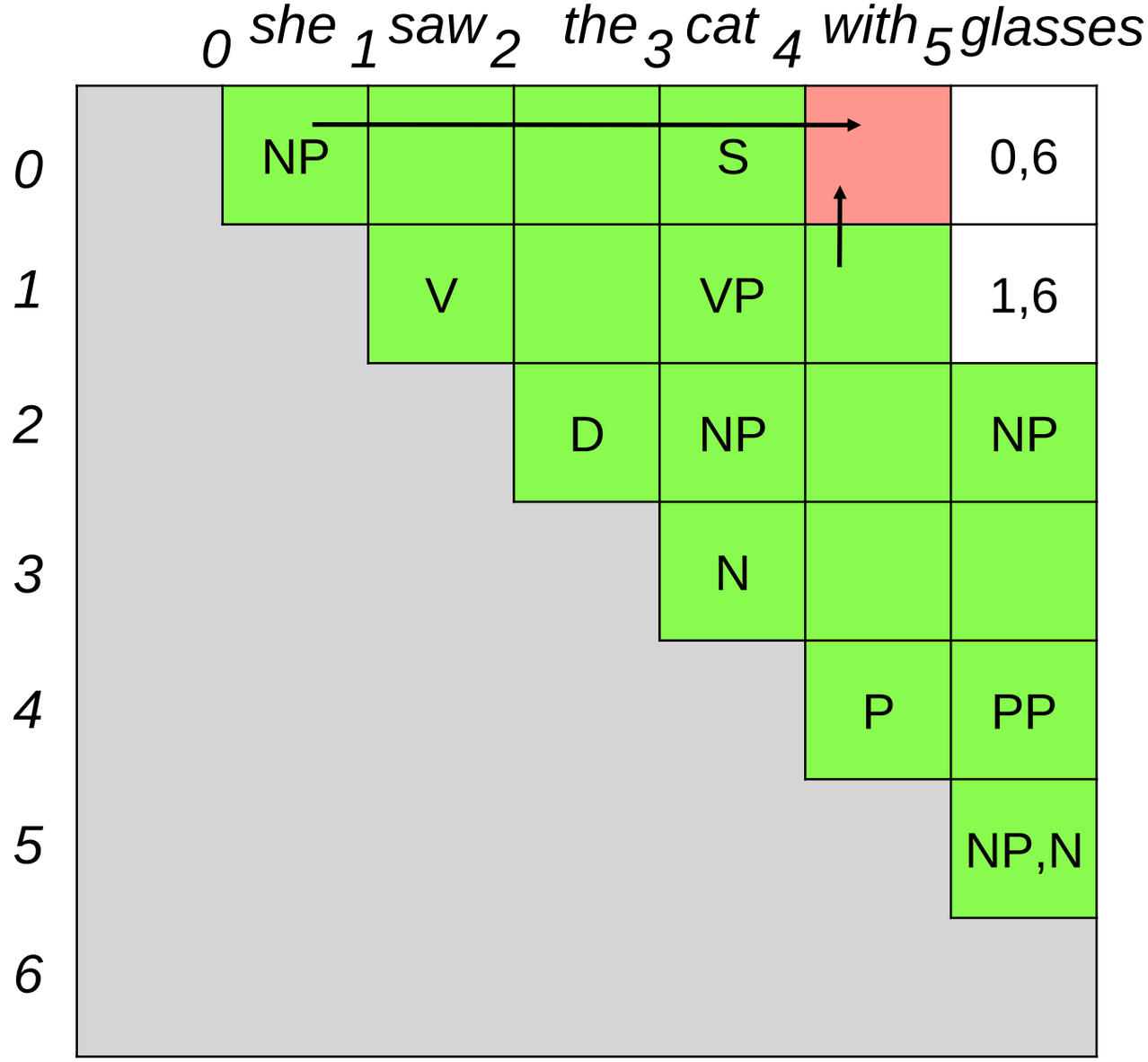
CKY Algorithm

for *i=0…(n-length):*
j = i+length
for *k=i+1…j-1:*
….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=5

i=0,k=1,j=5



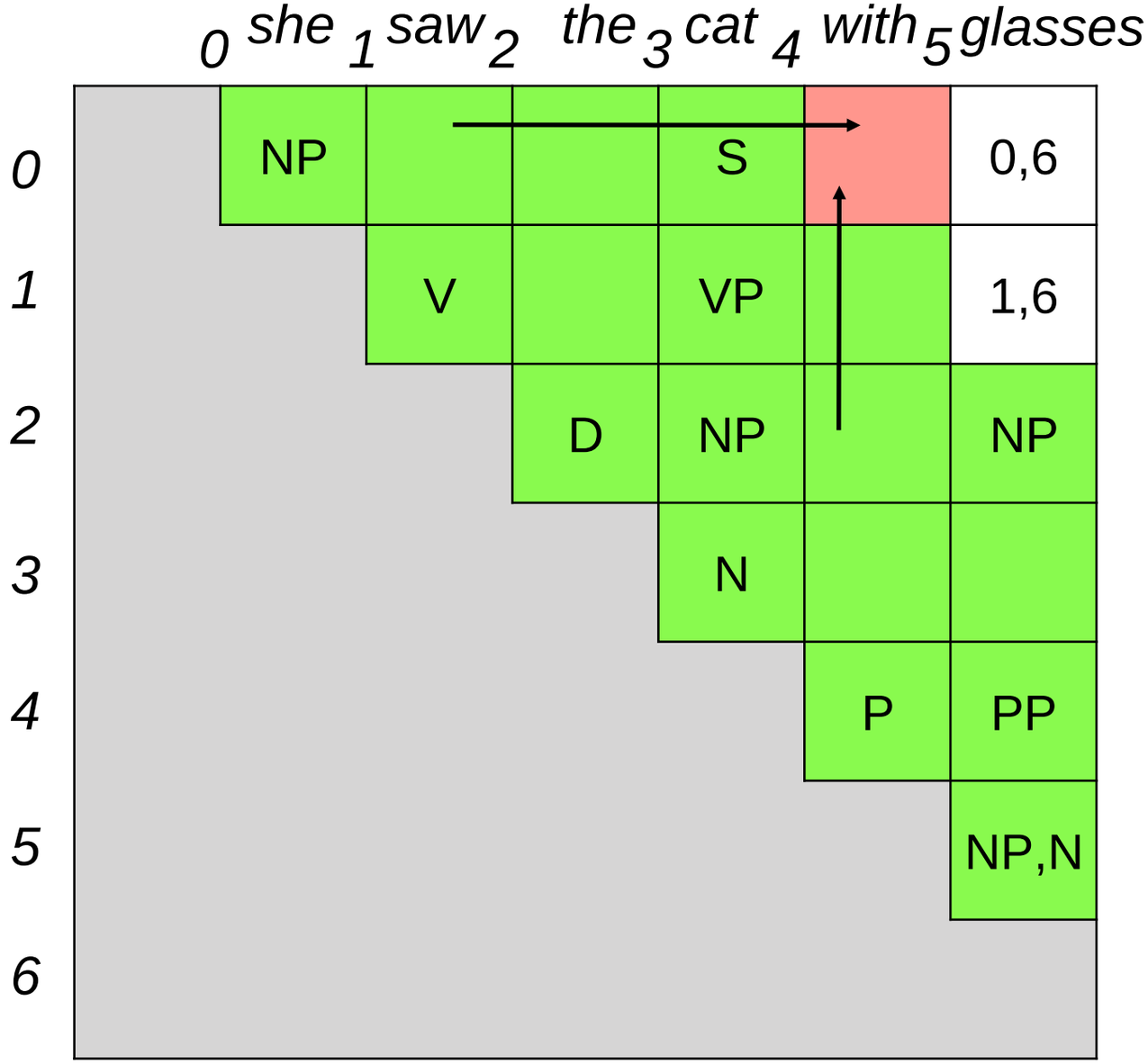
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=5

i=0,k=2,j=5



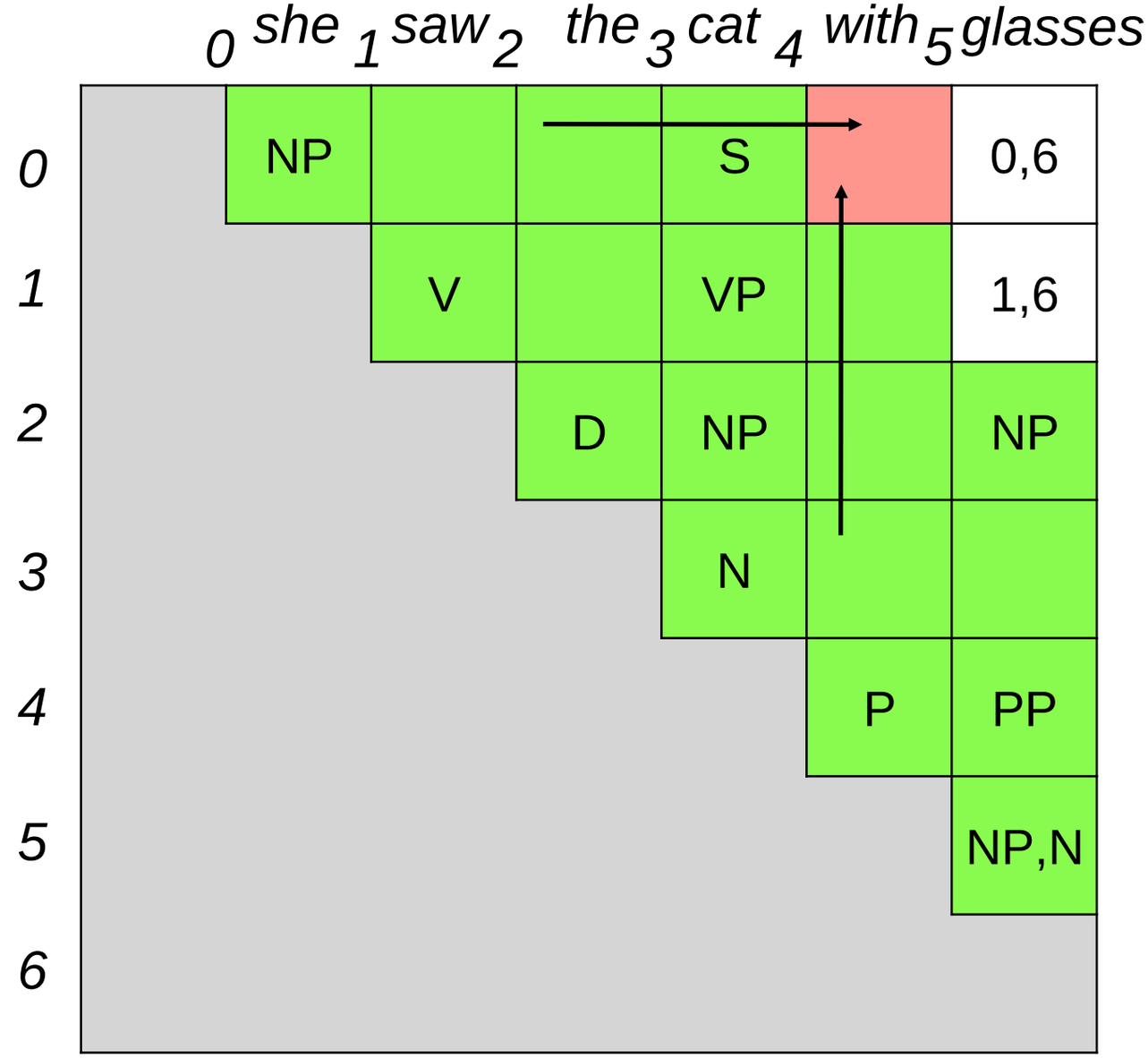
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=5

i=0,k=3,j=5



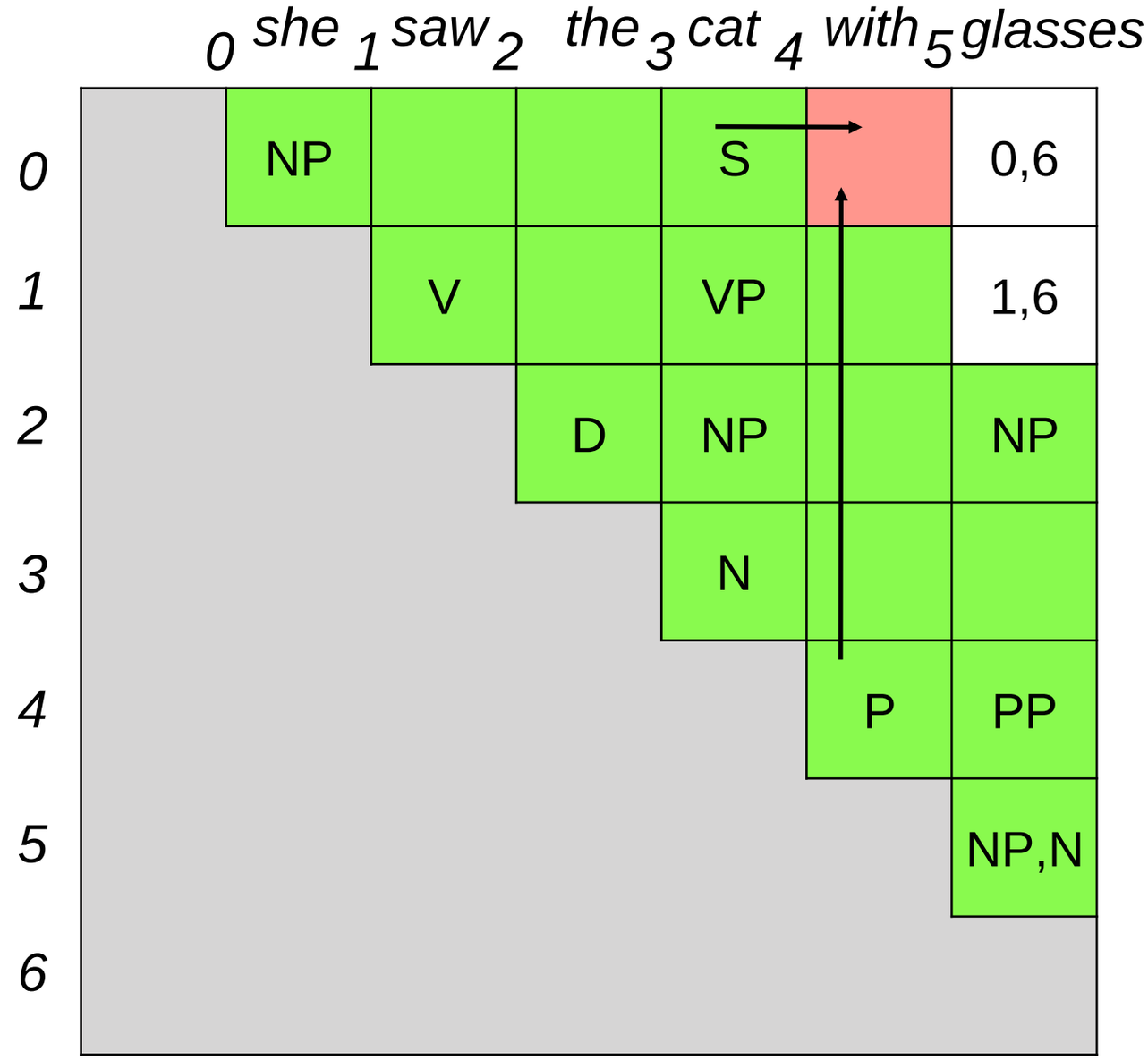
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=5

i=0,k=4,j=5



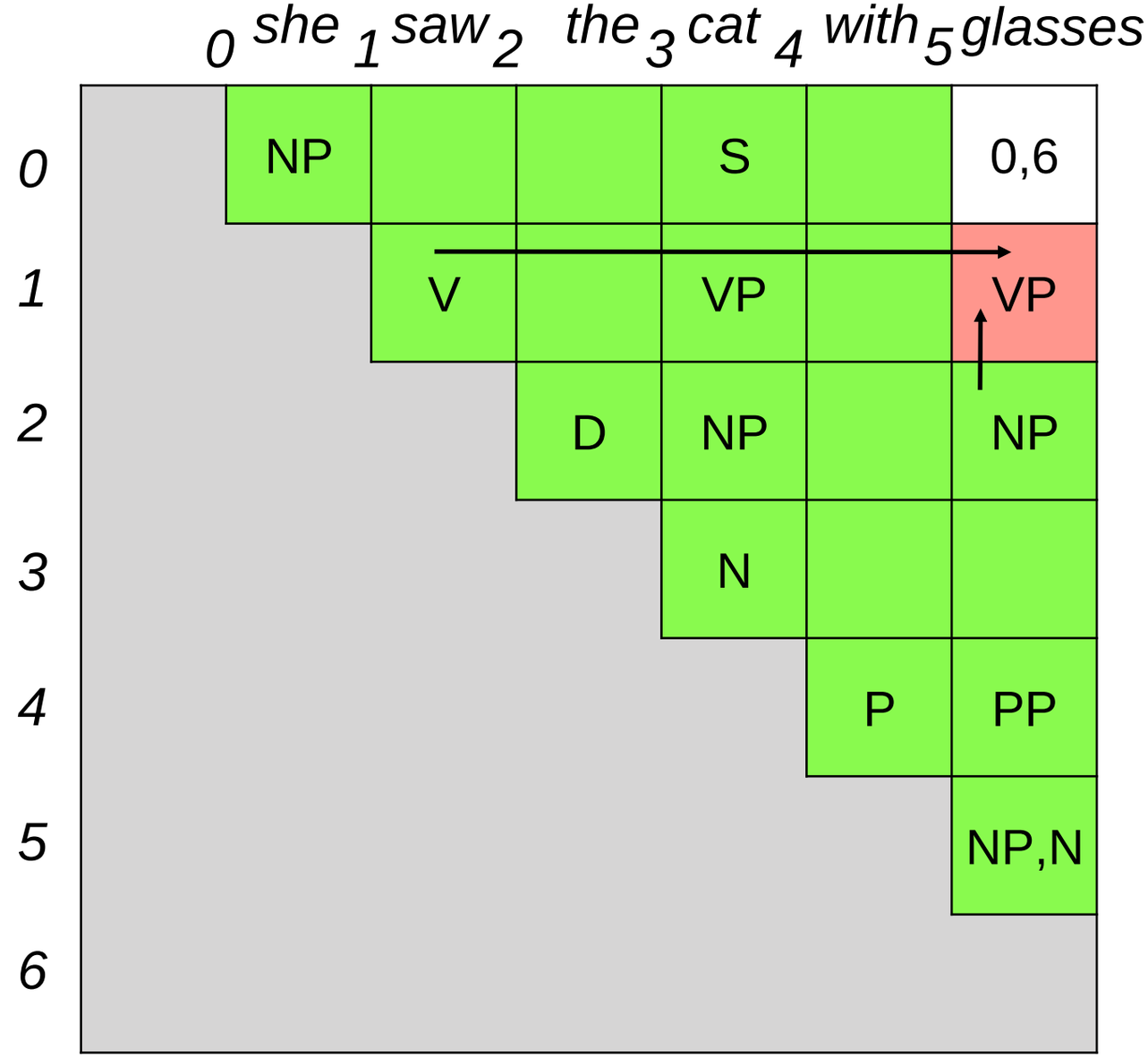
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ **V NP**	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=5

i=1,k=2,j=6



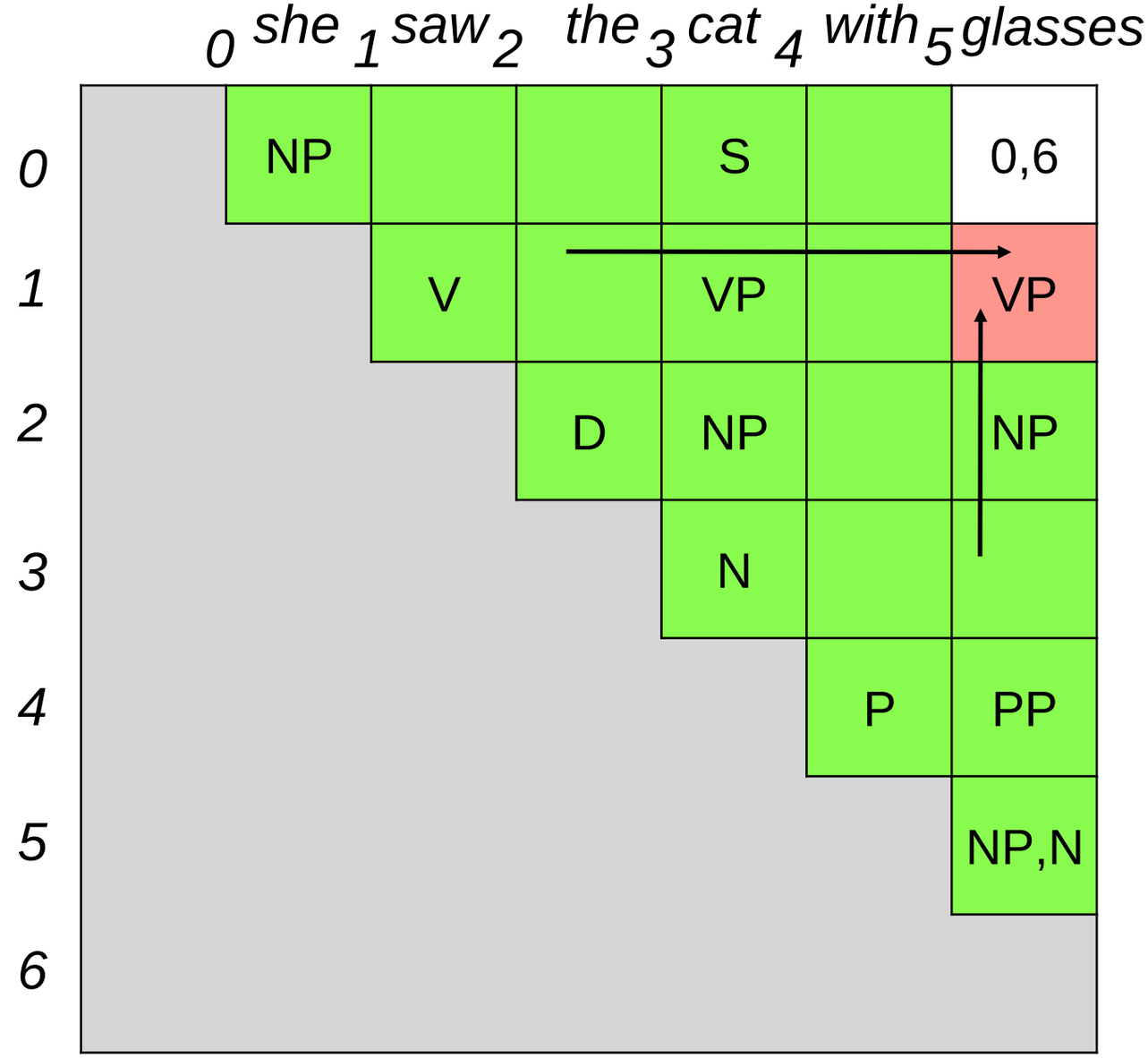
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ **V NP**	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=5

i=1,k=3,j=6



CKY Algorithm

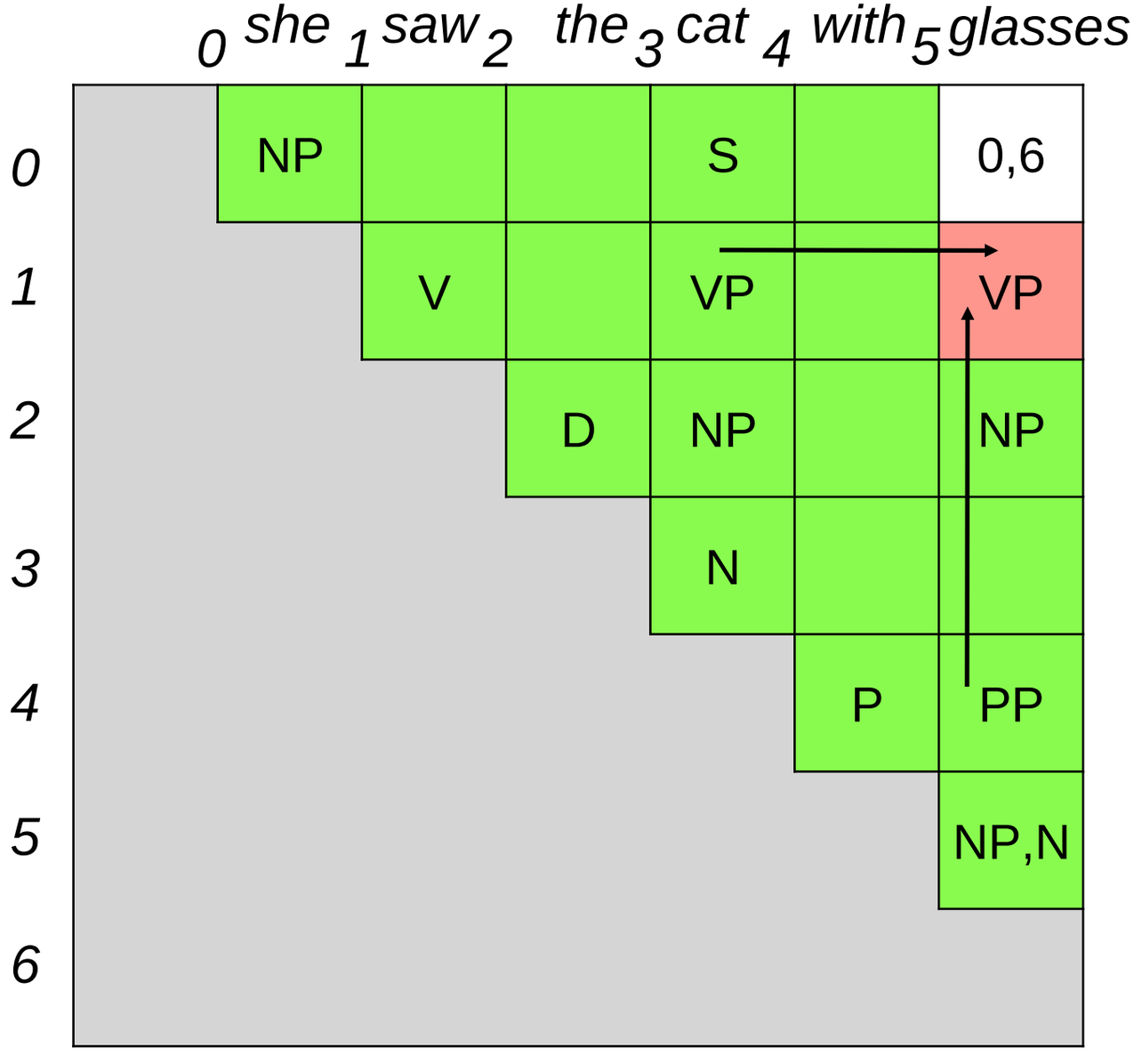
for *i=0…(n-length):*
j = i+length
for *k=i+1…j-1:*
….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ **VP PP**	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

! We can build VP over [1,6] in two ways!

length=5

i=1,k=4,j=6



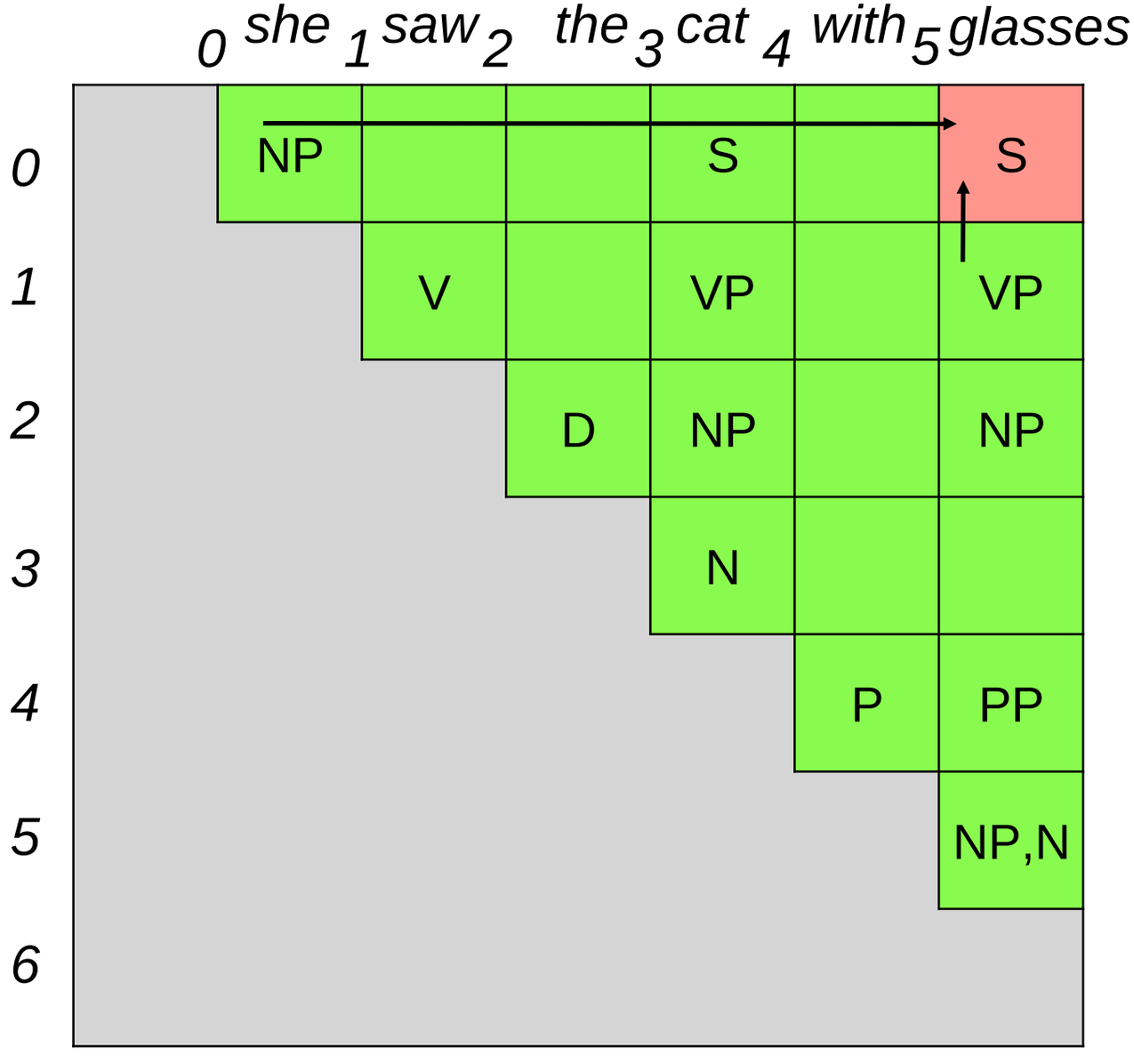
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=5

i=0,k=1,j=6



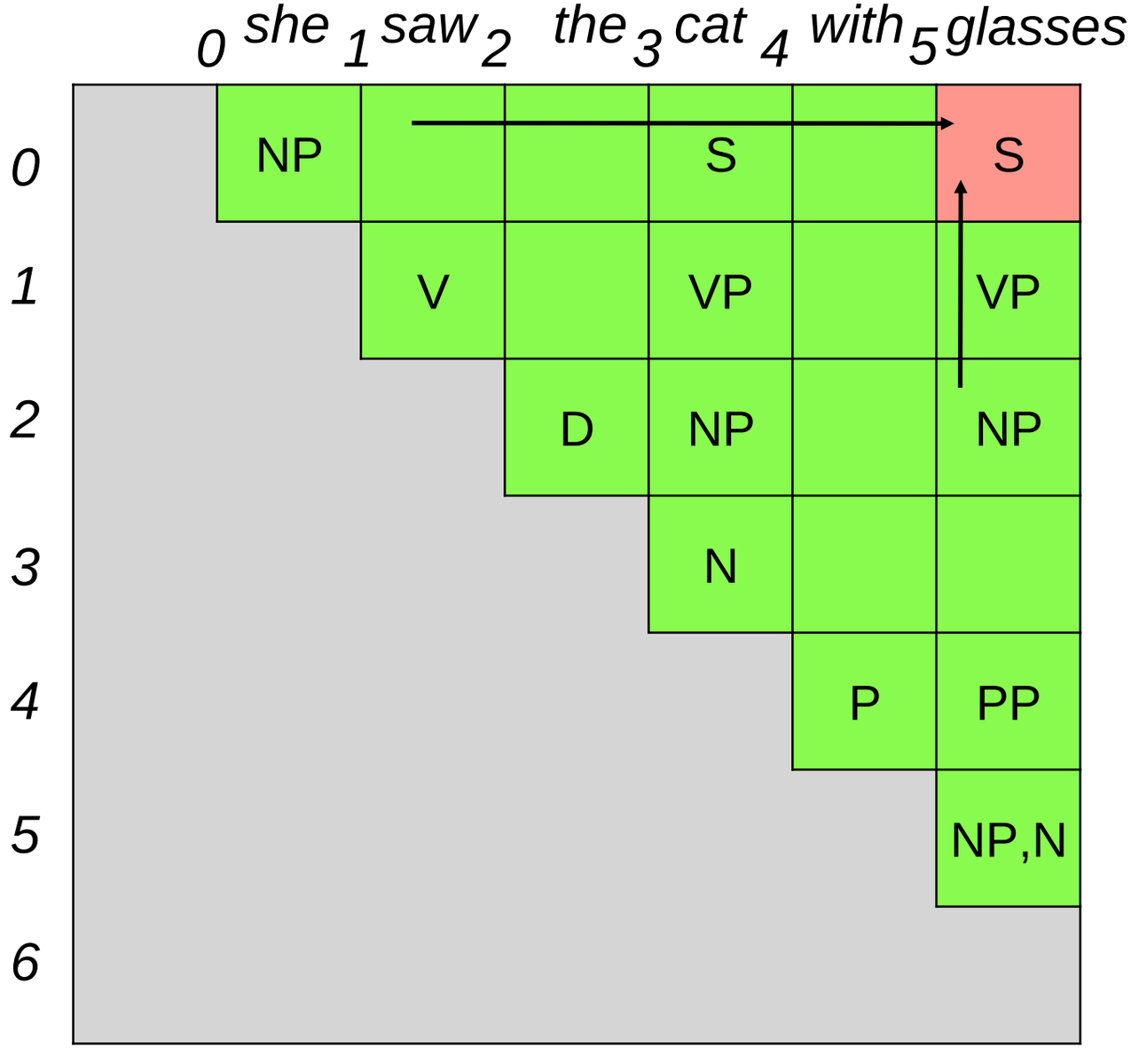
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=5

i=0,k=2,j=6



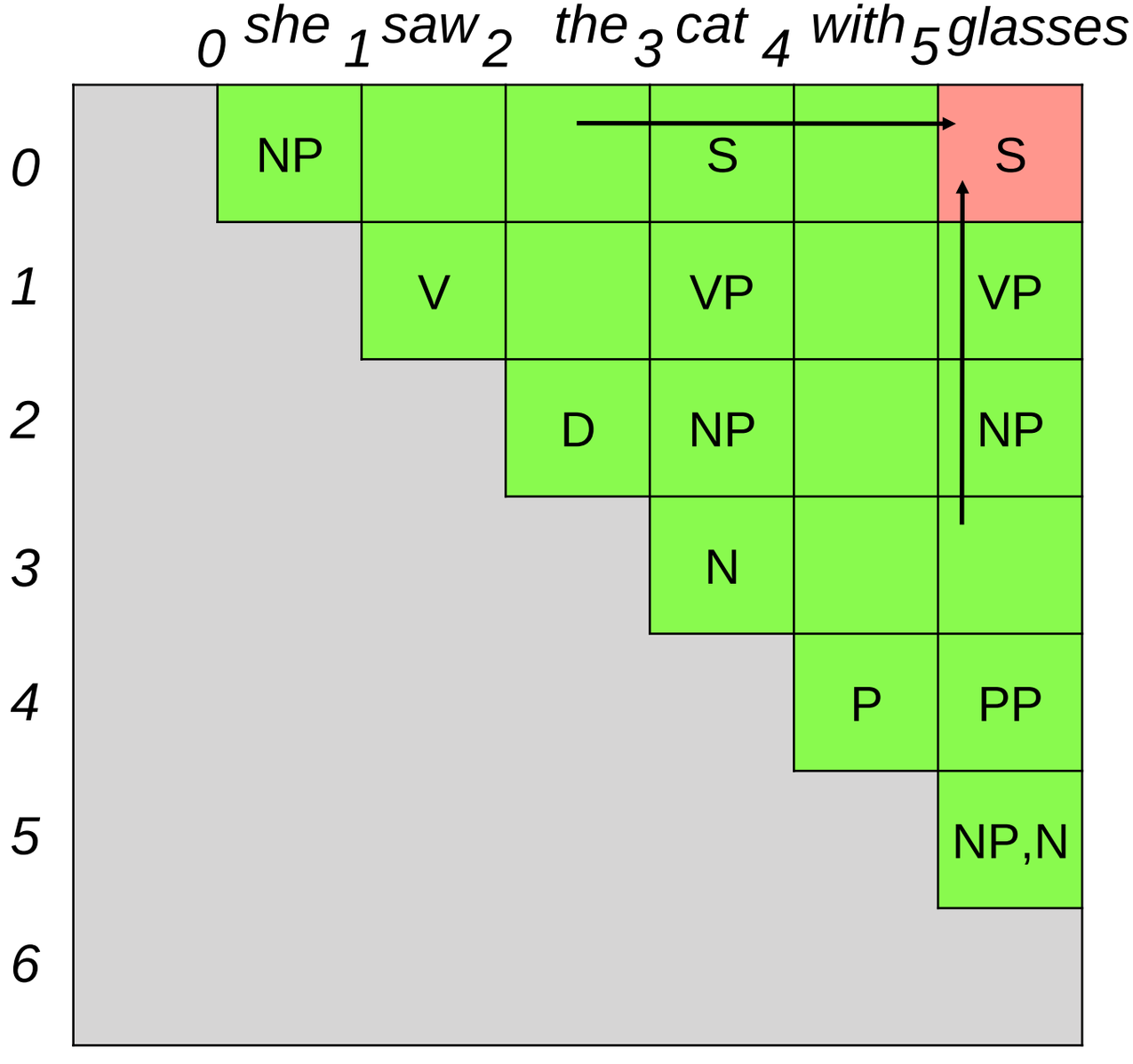
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=5

i=0,k=3,j=6



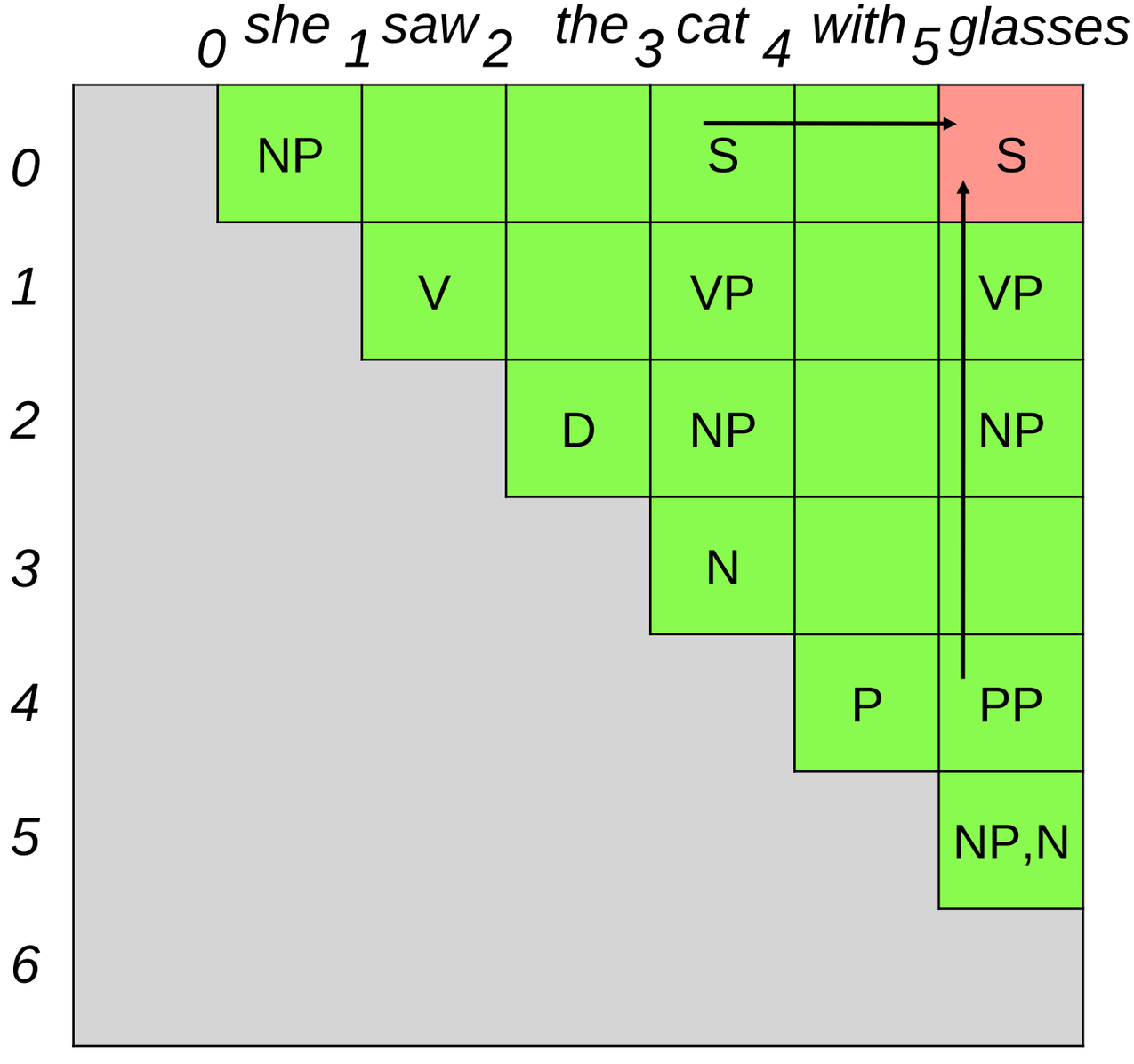
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=5

i=0,k=4,j=6



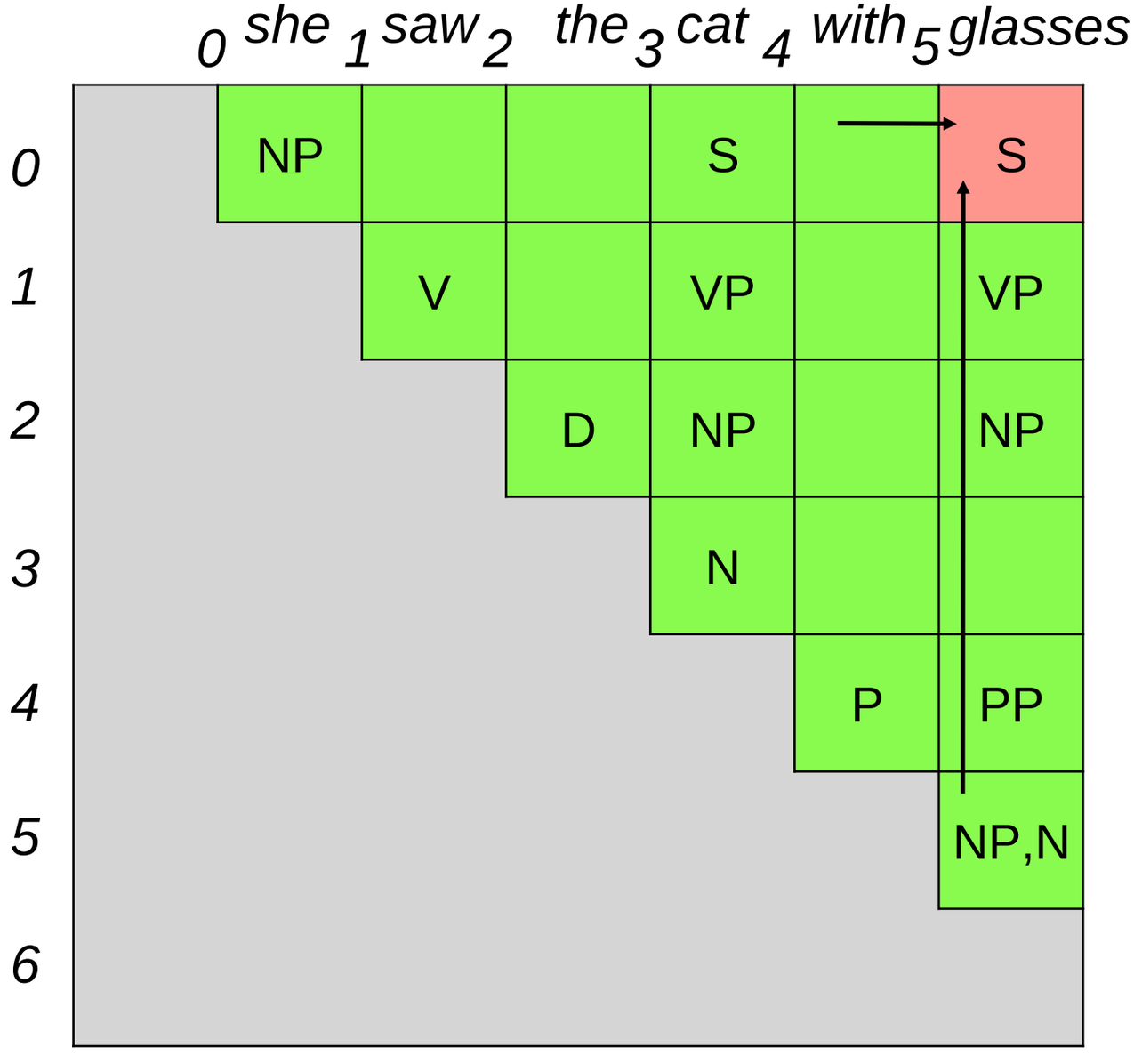
CKY Algorithm

for *i=0…(n-length):*
  j = i+length
  for *k=i+1…j-1:*
    ….

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with

length=5

i=0,k=5,j=6



CKY Runtime

- **Input:** Grammar $G=(N, \Sigma, R, S)$, input string s of length n.

- for i=0…n-1: O(N x |R|)
  $\pi[i, i+1] = \{A \mid A \rightarrow s[i]\}$

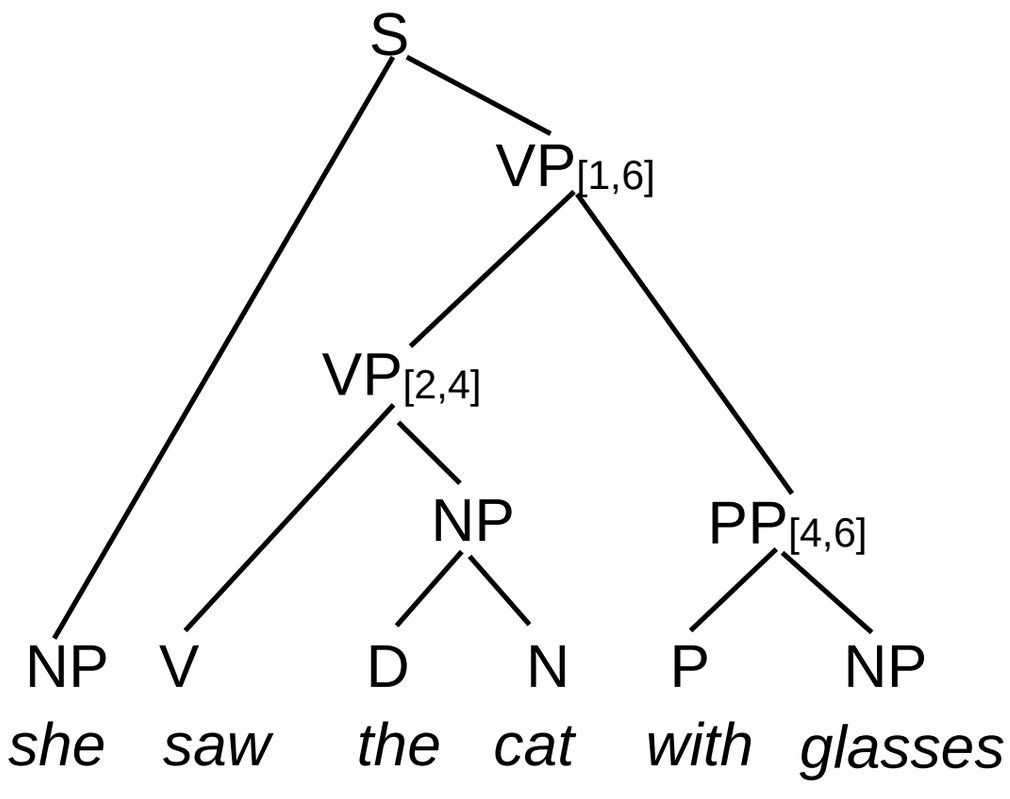
- for *length=2…n*: O(N)
  for *i=0…(n-length)*: O(N) Total : $O(N^3 \times |R|)$
  j = i+length O(N)
  for *k=i+1…j-1:*

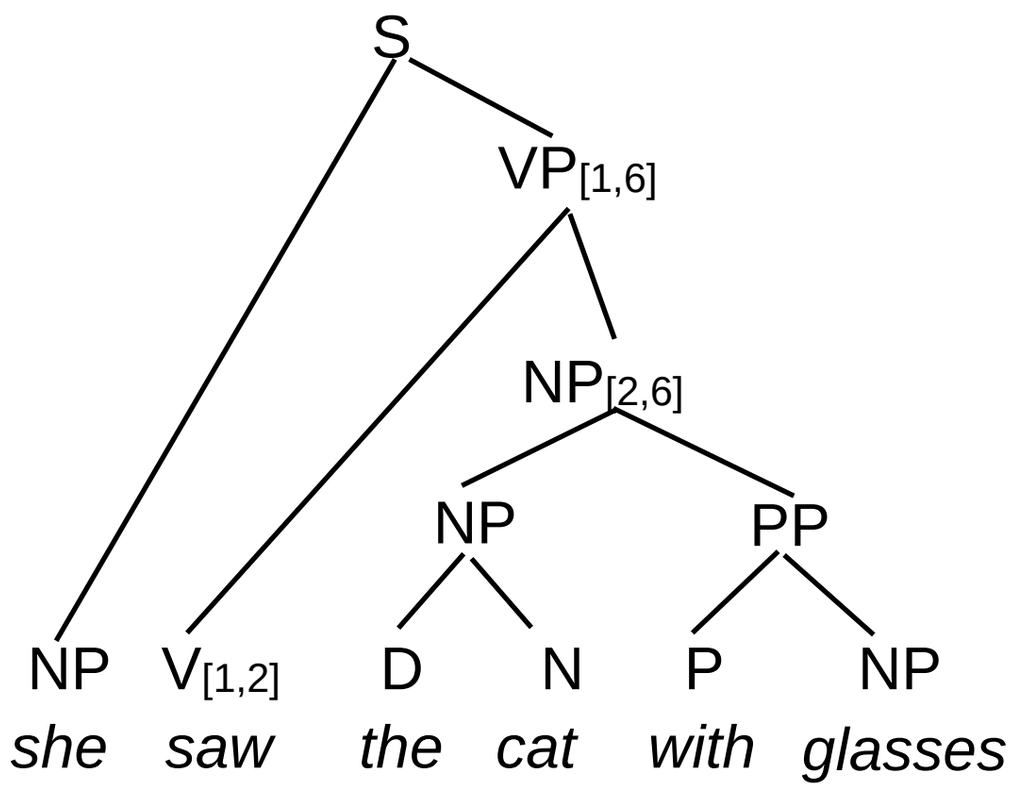
$$M = \{A | A \rightarrow B\,C \in R \text{ and } B \in \pi[i,k] \text{ and } C \in \pi[k,j]\}$$
$$\pi[i,j] = \pi[i,j] \cup M$$

- if $S \in \pi[0, i+1]$ return True, otherwise False

Syntactic Ambiguity

S	→ NP VP	NP	→ she
VP	→ V NP	NP	→ glasses
VP	→ VP PP	D	→ the
PP	→ P NP	N	→ cat
NP	→ D N	N	→ glasses
NP	→ NP PP	V	→ saw
		P	→ with





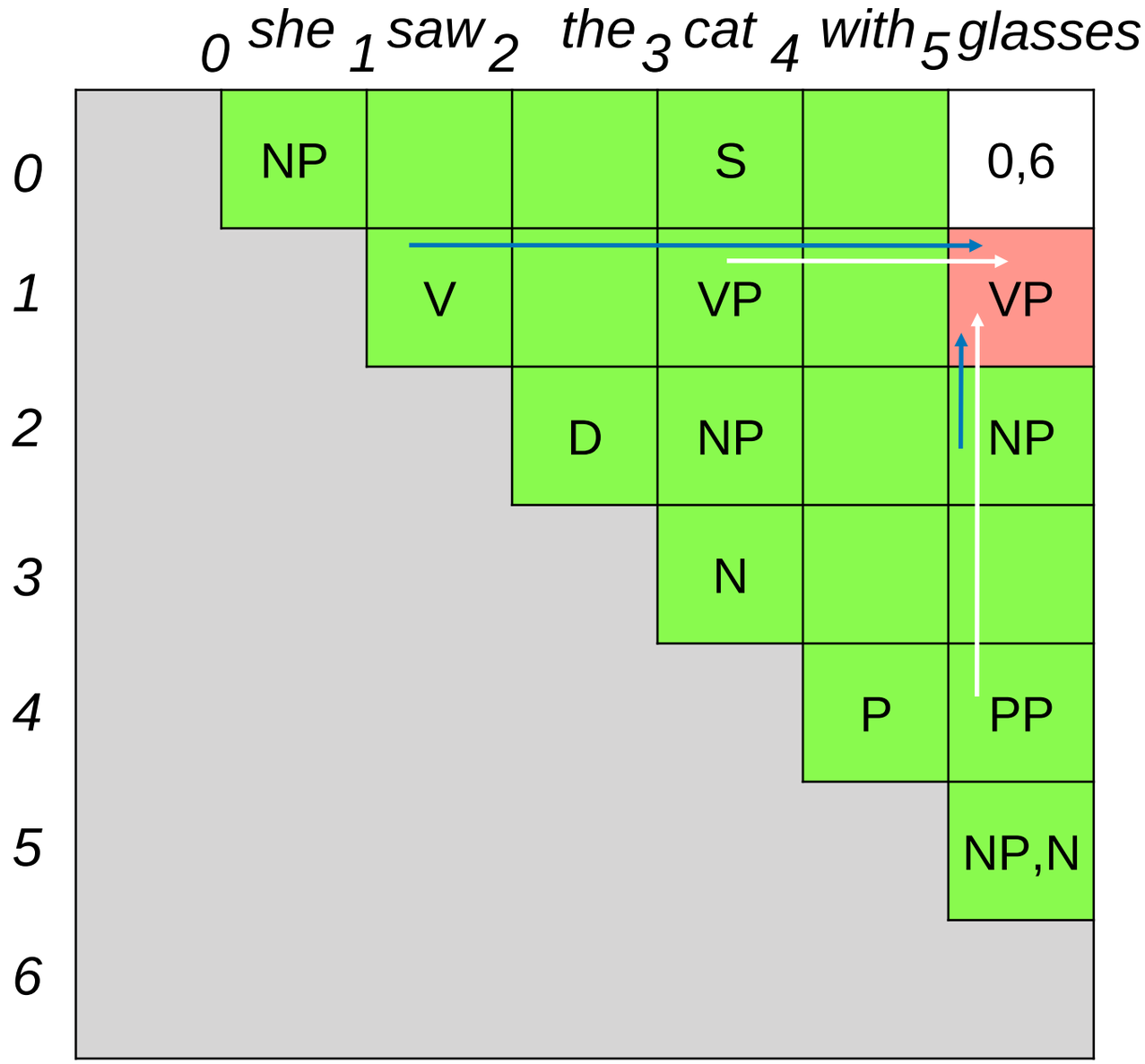
Backpointers

- The CKY algorithm presented so far determines if a sentence is recognized by a grammar.

- Also want to retrieve the parse trees!

- Instead of a set of nonterminals, store a list of instantiated rules and backpointers.

$$\left\{\begin{array}{l} VP_{[1,6]} \rightarrow V_{[1,2]} \quad NP_{[2,6]} \\ VP_{[1,6]} \rightarrow VP_{[1,4]}\, PP_{[4,6]} \end{array}\right\}$$



Retrieving Parse-Trees

- Start at the [0,n] entry and recursively follow the backpointers. Return a set of of subtrees from the recursion.