Natural Language Processing

Lecture 7: Parsing with Context Free Grammars II.
CKY for PCFGs. Earley Parser.

10/8/2020

COMS W4705
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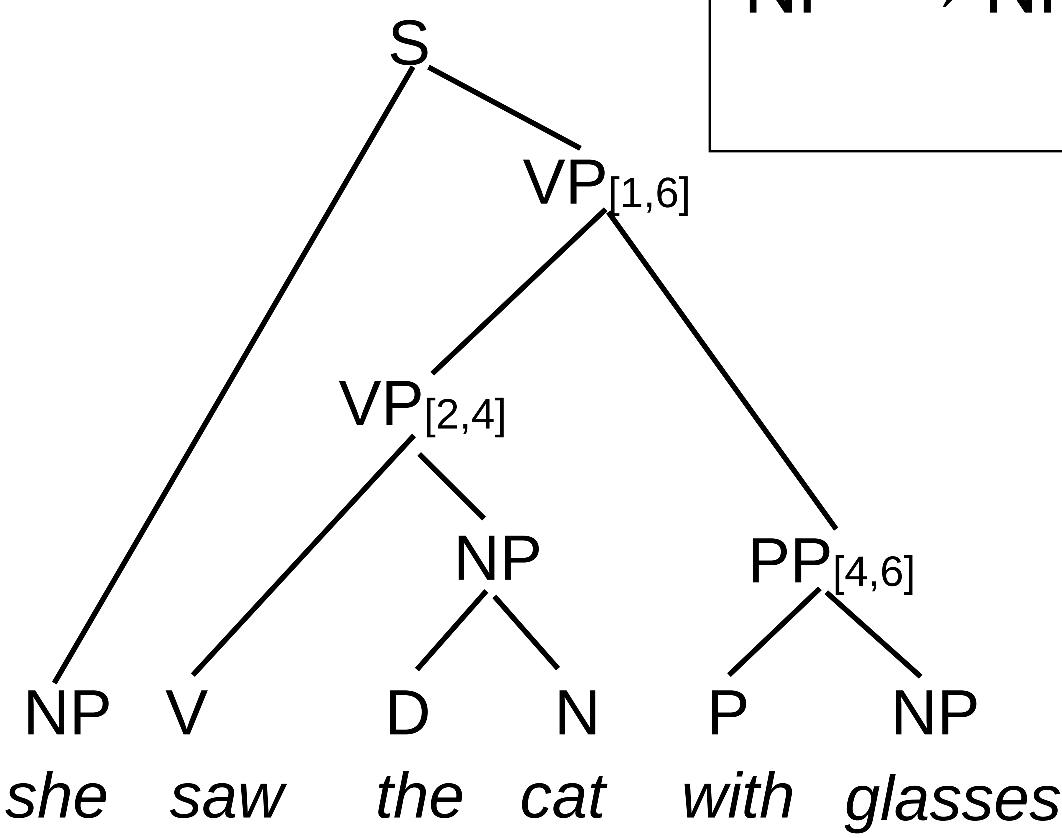
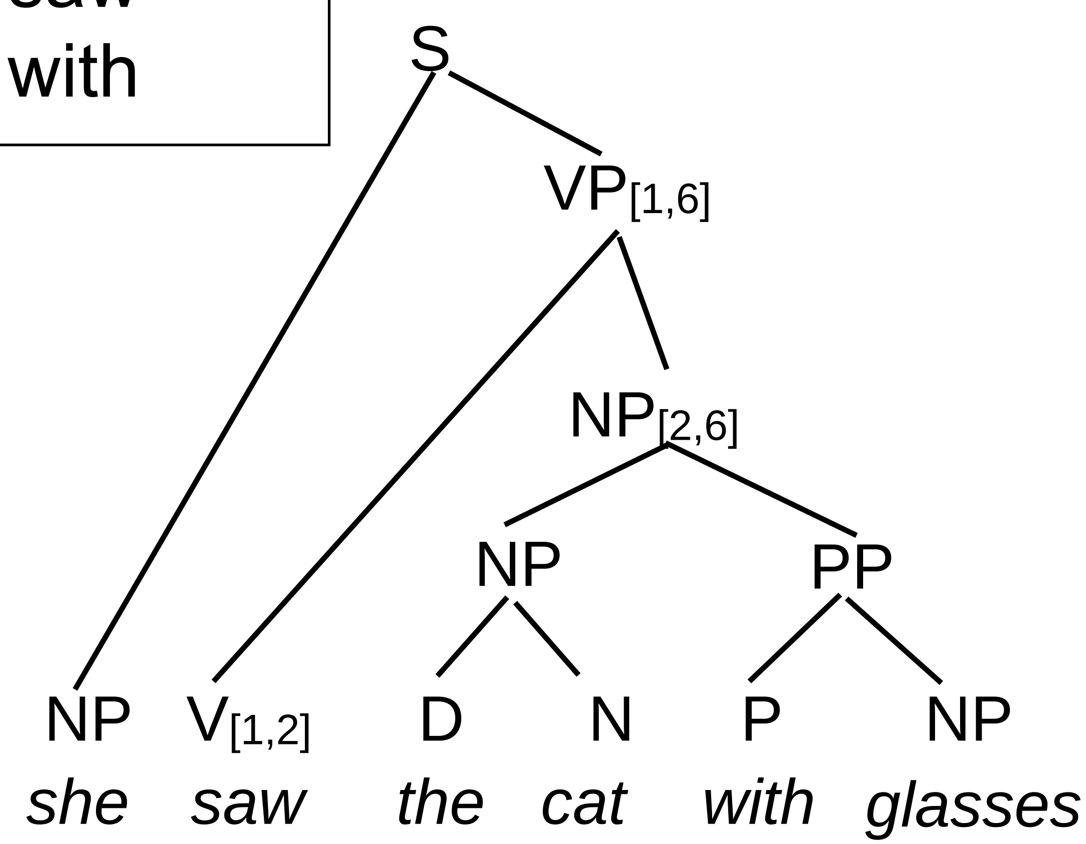
Recall: Syntactic Ambiguity

S → NP VP	NP → she
VP → V NP	NP → glasses
VP → VP PP	D → the
PP → P NP	N → cat
NP → D N	N → glasses
NP → NP PP	V → saw
	P → with





Which parse tree is "better"? More probable?

Probabilities for Parse Trees

- Let $\mathcal{T}_G$ be the set of all parse trees generated by grammar G.
- We want a model that assigns a probability to each parse tree, such that $\sum_{t \in \mathcal{T}_\mathcal{G}} P(t) = 1$.
- We can use this model to select the most probable parse tree compatible with an input sentence.
  - This is another example of a generative model!

Selecting Parse Trees

- Let $\mathcal{T}_G(s)$ be the set of trees generated by grammar G whose *yield* (sequence of leafs) is string s.
- The most likely parse tree produced by G for string s is

$$\arg\max_{t \in \mathcal{T}_G(s)} P(t)$$

  - How do we define $P(t)$?
  - How do we learn such a model from training data (annotated or un-annotated).
  - How do we find the highest probability tree for a given sentence? (*parsing/decoding*)

Probabilistic Context Free Grammars (PCFG)

- A PCFG consists of a Context Free Grammar $G=(N, \Sigma, R, S)$ and a probability $P(A \rightarrow \beta)$ for each production $A \rightarrow \beta \in R$.

  - The probabilities for all rules with the same left-hand-side sum up to 1:

$$\sum_{A \rightarrow \beta : A = X} P(A \rightarrow \beta) = 1 \text{ for all } X \in N$$

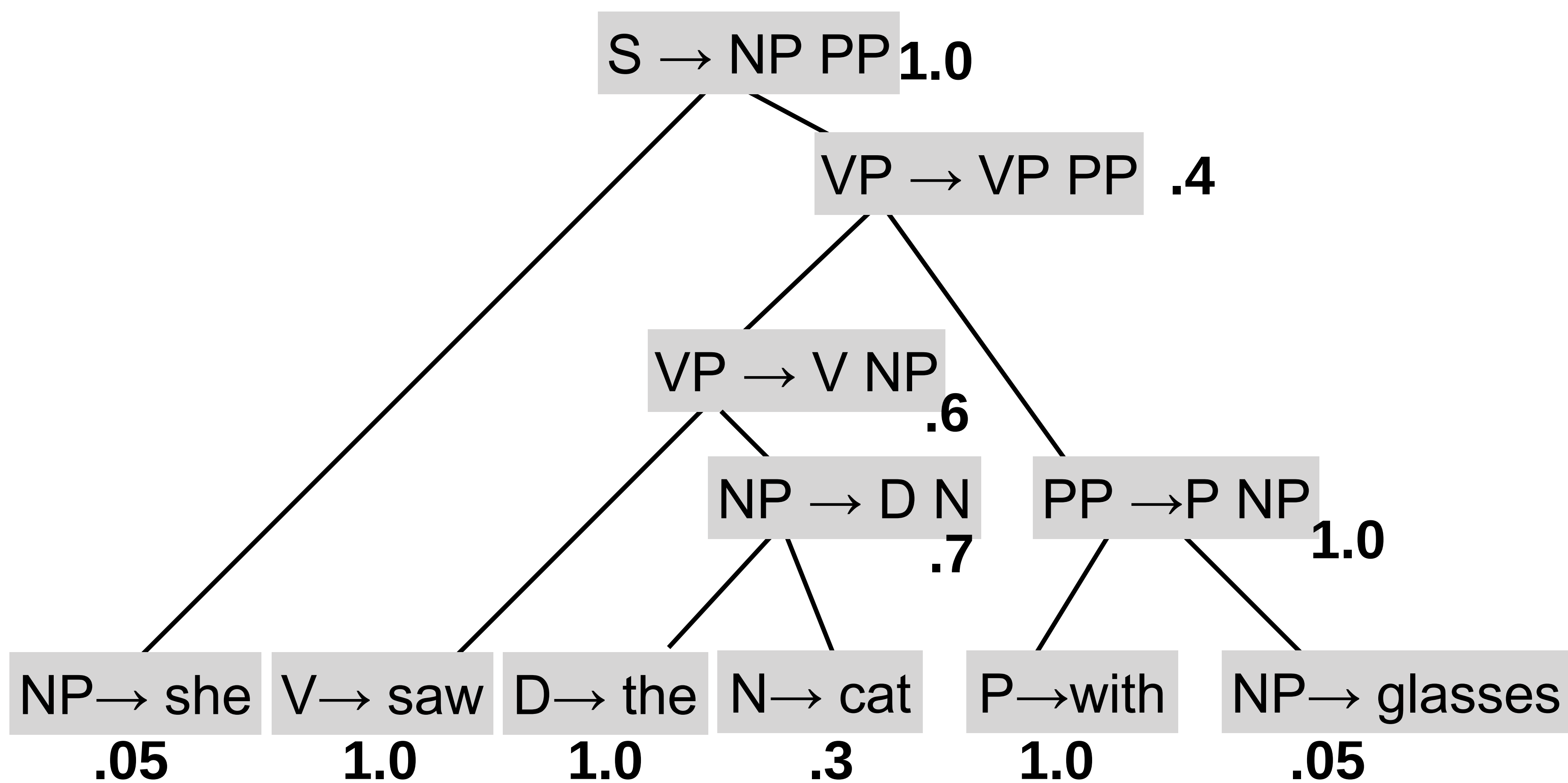
  - Think of this as the conditional probability for $A \rightarrow \beta$, given the left-hand-side nonterminal A.

PCFG Example

S → NP VP	[1.0]		NP → she	[0.05]
VP → V NP	[0.6]		NP → glasses	[0.05]
VP → VP PP	[0.4]		D → the	[1.0]
PP → P NP	[1.0]		N → cat	[0.3]
NP → D N	[0.7]		N → glasses	[0.7]
NP → NP PP	[0.2]		V → saw	[1.0]
			P → with	[1.0]

Parse Tree Probability

- Given a parse tree $t \in \mathcal{T}_G$, containing rules $A_1 \to \beta_1, \ldots, A_n \to \beta_n$ the probability of t is $$P(t) = \prod_{i=1}^{n} P(A_i \to \beta_i)$$



1 x .05 x .4 x .6 x 1 x 0.7 x 1 x 0.3 x 1 x 1 x .05 = .000126

Parse Tree Probability

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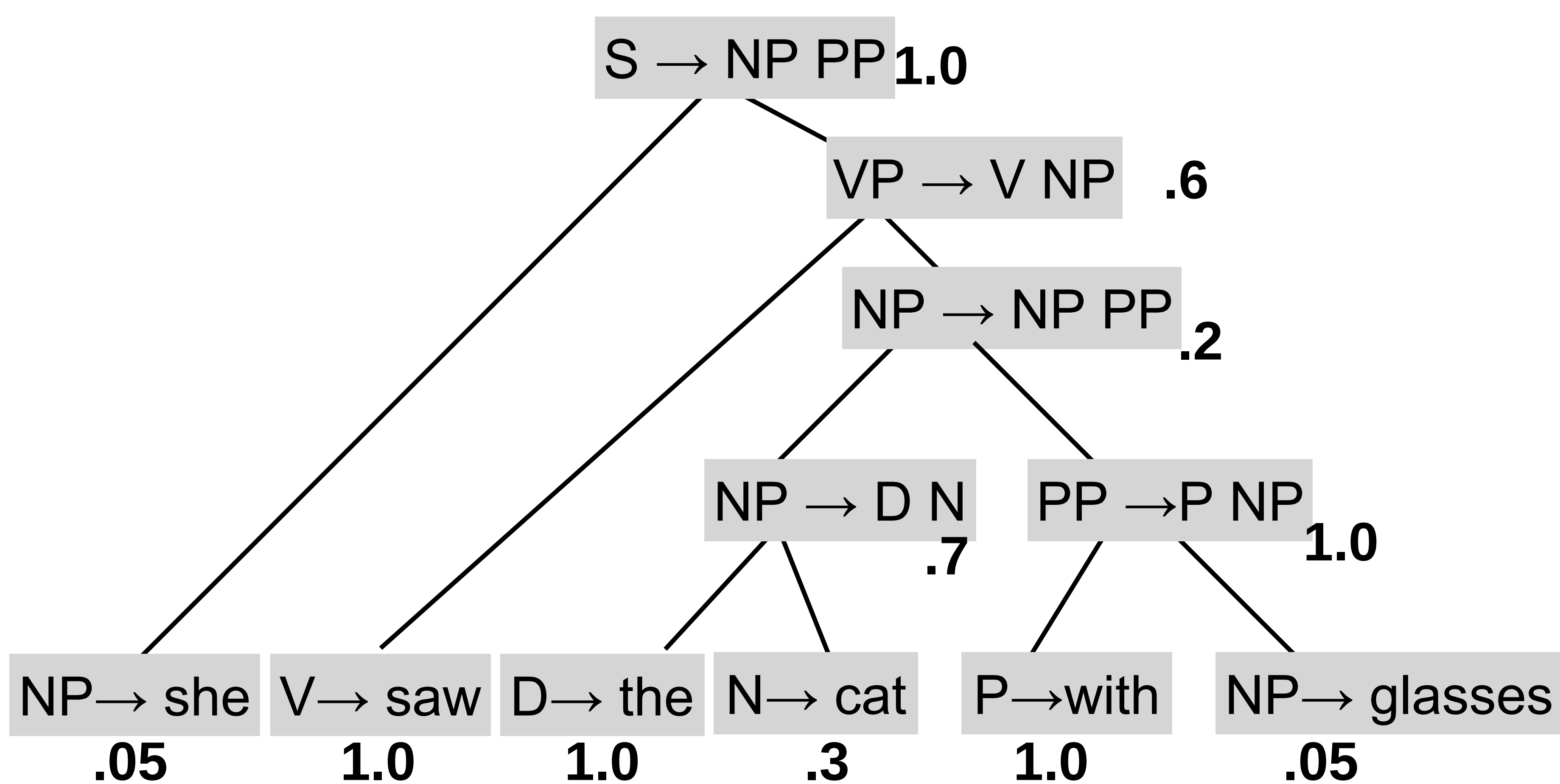


1 x .05 x .6 x 1 x .2 x .7 x 1 x .3 x 1 x 1 x .05 = 0.000063 < 0.000126

Estimating PCFG probabilities

- Supervised training: We can estimate PCFG probabilities from a *treebank,* a corpus manually annotated with constituency structure using maximum likelihood estimates:

$$P(A \rightarrow \beta) = \frac{count(A \rightarrow \beta)}{count(A)}$$

- Unsupervised training:
  - What if we have a grammar and a corpus, but no annotated parses?
  - Can use the **inside-outside** algorithm for parsing and do EM estimation of the probabilities (not discussed in this course)

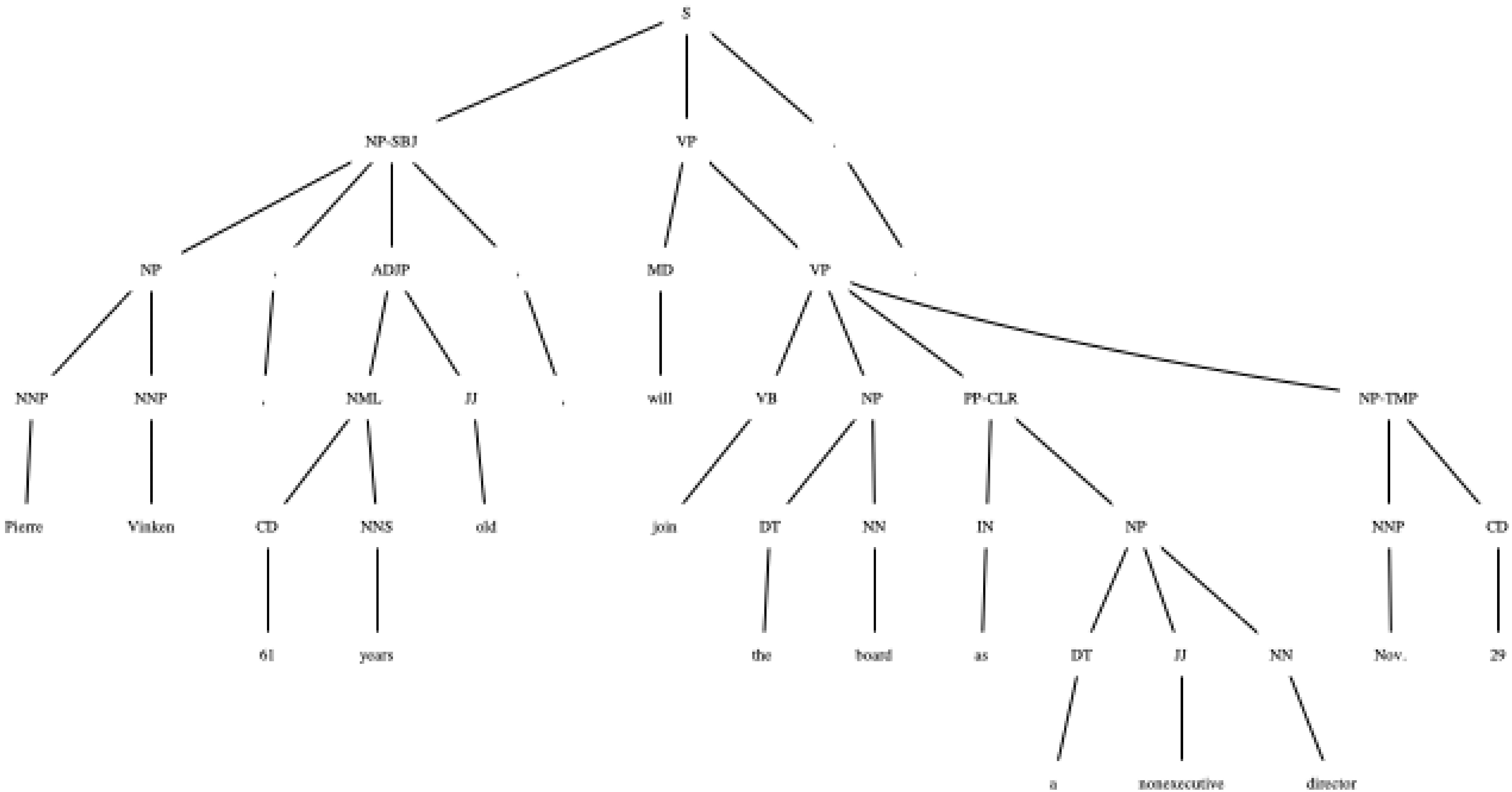
The Penn Treebank

- Syntactically annotated corpus of newspaper text (1989 Wall Street Journal Articles).
- The source text is naturally occurring but the treebank is not:
  - Assumes a specific linguistic theory (although a simple one).
  - Very flat structure (NPs, Ss, VPs).

PTB Example

```
( (S (NP-SBJ (NP (NNP Pierre) (NNP Vinken))
         (, ,)
         (ADJP (NML (CD 61) (NNS years))
         (JJ old))
         (, ,))
      (VP (MD will)
   (VP (VB join)
         (NP (DT the) (NN board))
         (PP-CLR (IN as)
           (NP (DT a) (JJ nonexecutive) (NN director)))
         (NP-TMP (NNP Nov.) (CD 29))))
      (. .)))
```

PTB Example


S
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.
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,
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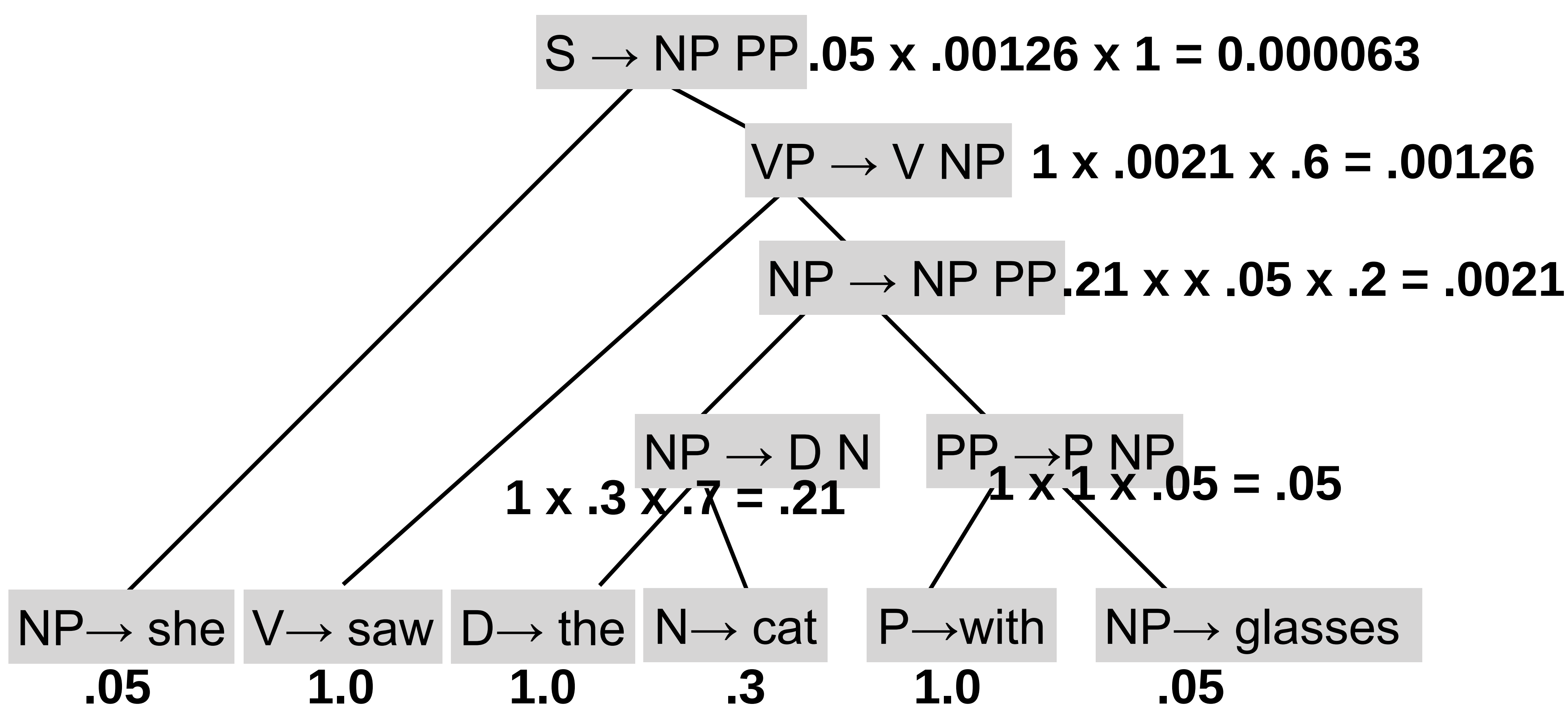
Parsing with PCFG

- We want to use PCFG to answer the following questions:
  - What is the total probability of the sentence under the PCFG?
  - What is the most probable parse tree for a sentence under the PCFG? *(decoding/parsing)*
- We can modify the CKY algorithm.
  Basic idea: Compute these probabilities bottom-up using dynamic programming.

Computing Probabilities Bottom-Up



CKY for PCFG Parsing

- Let $T_{\mathcal{G}}(s, A)$ be the set of trees generated by grammar G starting at nonterminal A, whose *yield* is string s

- Use a chart π so that $\pi[i,j,A]$ contains the probability of the highest probability parse tree for string s[i,j] starting in nonterminal A.

$$\pi[i,j,A] = \max_{t \in T_{\mathcal{G}}(s[i,j],A)} P(t)$$

- We want to find $\pi[0,lenght(s),S]$ -- the probability of the highest-scoring parse tree for s rooted in the start symbol S.

CKY for PCFG Parsing

- To compute *π[0,lenght(s),S]* we can use the following recursive definition:

Base case:

$$\pi[i, i+1, A] = \begin{cases} P(A \rightarrow s_i) & \text{if } A \rightarrow s_i \in R \\ 0 & \text{otherwise} \end{cases}$$

$$\pi[i, j, A] = \max_{\substack{k=i+1\ldots j-1, \\ A \rightarrow BC \in R}} P(A \rightarrow BC) \cdot \pi[i, k, B] \cdot \pi[k, j, C]$$

- Then fill the chart using dynamic programming.

CKY for PCFG Parsing

- **Input:** PCFG $G=(N, \Sigma, R, S)$, input string s of length n.

- for i=0…n-1: initialization

$$\pi[i, i+1, A] = \begin{cases} P(A \rightarrow s_i) & \text{if } A \rightarrow s_i \in R \\ 0 & \text{otherwise} \end{cases}$$

- for $length=2...n$: main loop
  for $i=0...(n\text{-}length)$:
    $j = i+length$
    for $k=i+1...j\text{-}1$:
      for A ∈ N:

$$\pi[i, j, A] = \max_{\substack{k=i+1...j-1, \\ A \rightarrow BC \in R}} P(A \rightarrow BC) \cdot \pi[i, k, B] \cdot \pi[k, j, C]$$

Use **backpointers** to retrieve the highest-scoring parse tree (see previous lecture).

Probability of a Sentence

- What if we are interested in the probability of a sentence, **not** of a single parse tree (for example, because we want to use the PCFG as a language model).
- Problem: Spurious ambiguity. Need to sum the probabilities of **all** parse trees for the sentence.
- How do we have to change CKY to compute this?

$$\pi[i,j,A] = \sum_{\substack{k=i+1\ldots j-1, \\ A \rightarrow BC \in R}} P(A \rightarrow BC) \cdot \pi[i,k,B] \cdot \pi[k,j,C]$$

Earley Parser

- CKY parser starts with words and builds parse trees bottom-up; requires the grammar to be in CNF.
- The Earley parser instead starts at the start symbol and tries to “guess” derivations top-down.
  - It discards derivations that are incompatible with the sentence.
  - The early parser sweeps through the sentence left-to-right only once. It keeps partial derivations in a table (“chart”).
  - Allows arbitrary CFGs, no limitation to CNF.

Parser States

- Earley parser keeps track of partial derivations using parser **states / items.**
- State represent hypotheses about constituent structure based on the grammar, taking into account the input.
- Parser states are represented as **dotted rules with spans.**
  - The constituents to the left of the · have already been seen in the input string *s* (corresponding to the span)

S → · NP VP [0,0] *“According to the grammar, there may be an NP starting in position 0. “*

NP → D A · N [0,2] *"There is a determiner followed by an adjective in s[0,2]“*

NP → NP PP · [3,8] *"There is a complete NP in s[3,8], consisting of an NP and PP”*

Earley Parser (sketch)

S	$\rightarrow$ NP VP	V	$\rightarrow$ saw
VP	$\rightarrow$ V NP	P	$\rightarrow$ with
VP	$\rightarrow$ VP PP	D	$\rightarrow$ the
PP	$\rightarrow$ P NP	N	$\rightarrow$ cat
NP	$\rightarrow$ D N	N	$\rightarrow$ tail
NP	$\rightarrow$ NP PP	N	$\rightarrow$ student

S $\rightarrow$ · NP VP [0,0]

NP $\rightarrow$ · NP PP [0,0]

NP $\rightarrow$ · D N [0,0]

D $\rightarrow$ · the [0,0]

Three parser operations:

1. **Predict** new subtrees top-down.

$_0$ *the* $_1$ *student* $_2$ *saw* $_3$ *the* $_4$ *cat* $_5$ *with* $_6$ *the* $_7$ *tail*

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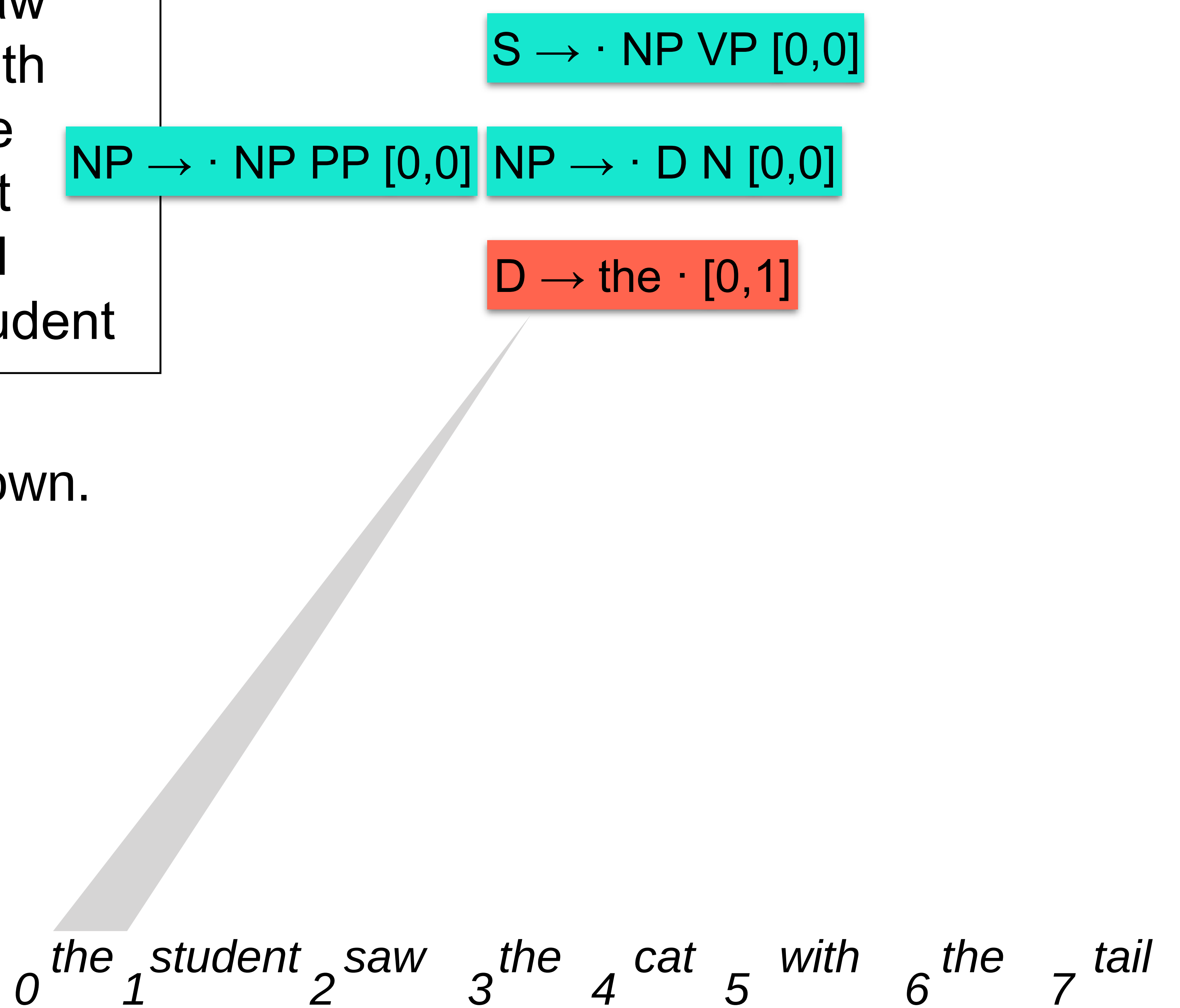

Three parser operations:

1. Predict new subtrees top-down.

2. **Scan** input terminals.

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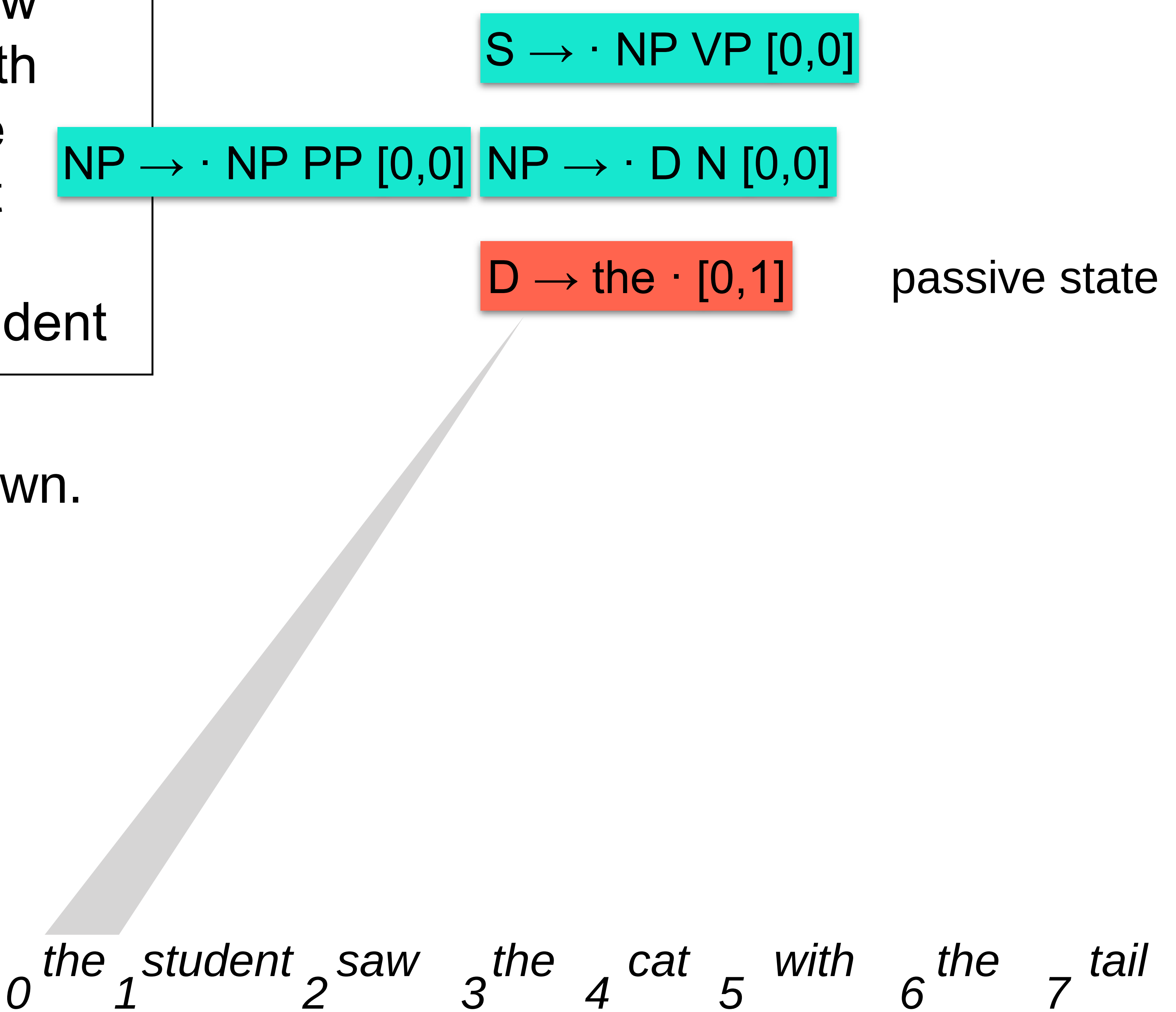

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Three parser operations:

1. Predict new subtrees top-down.
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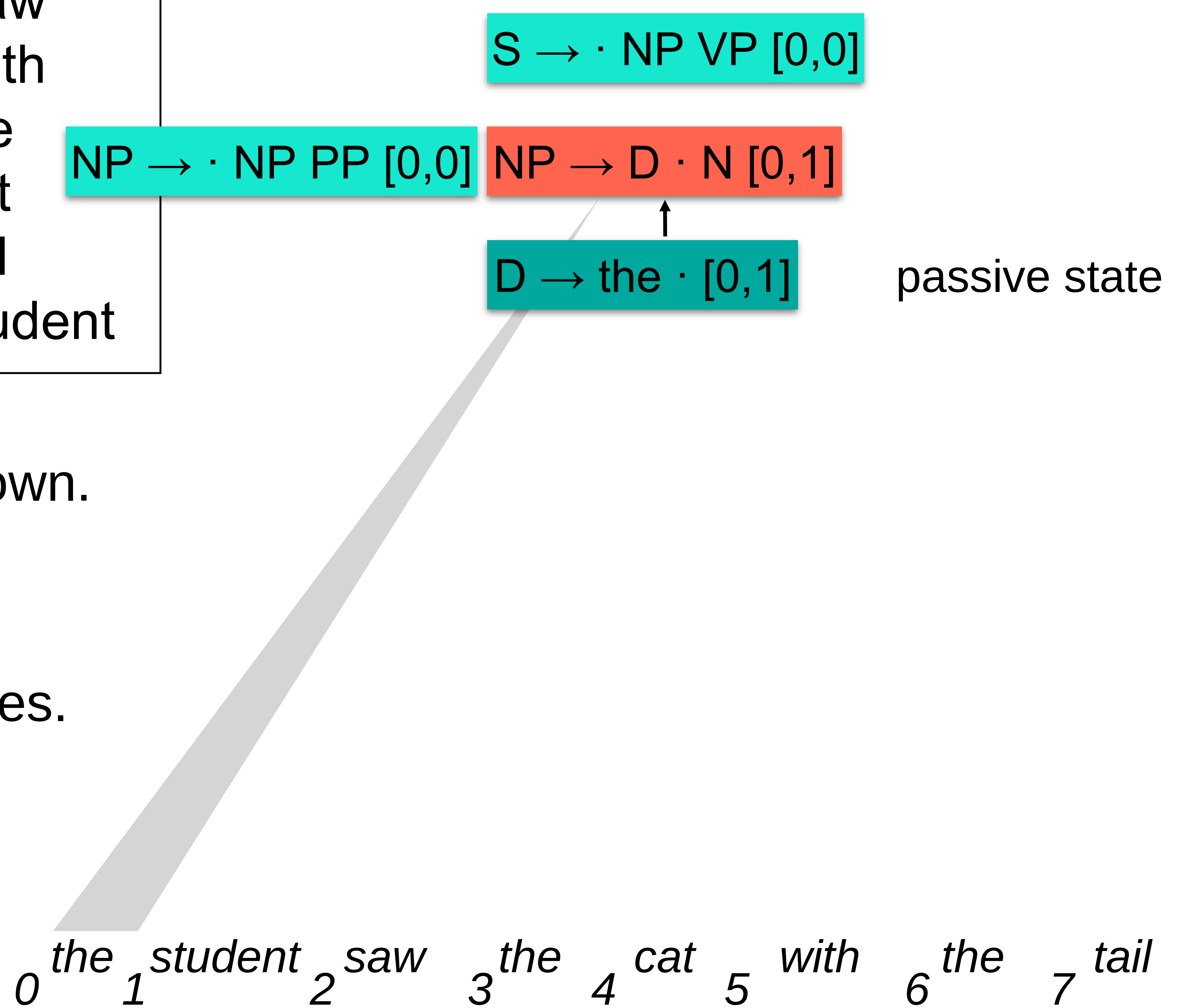
3. **Complete** with passive states.



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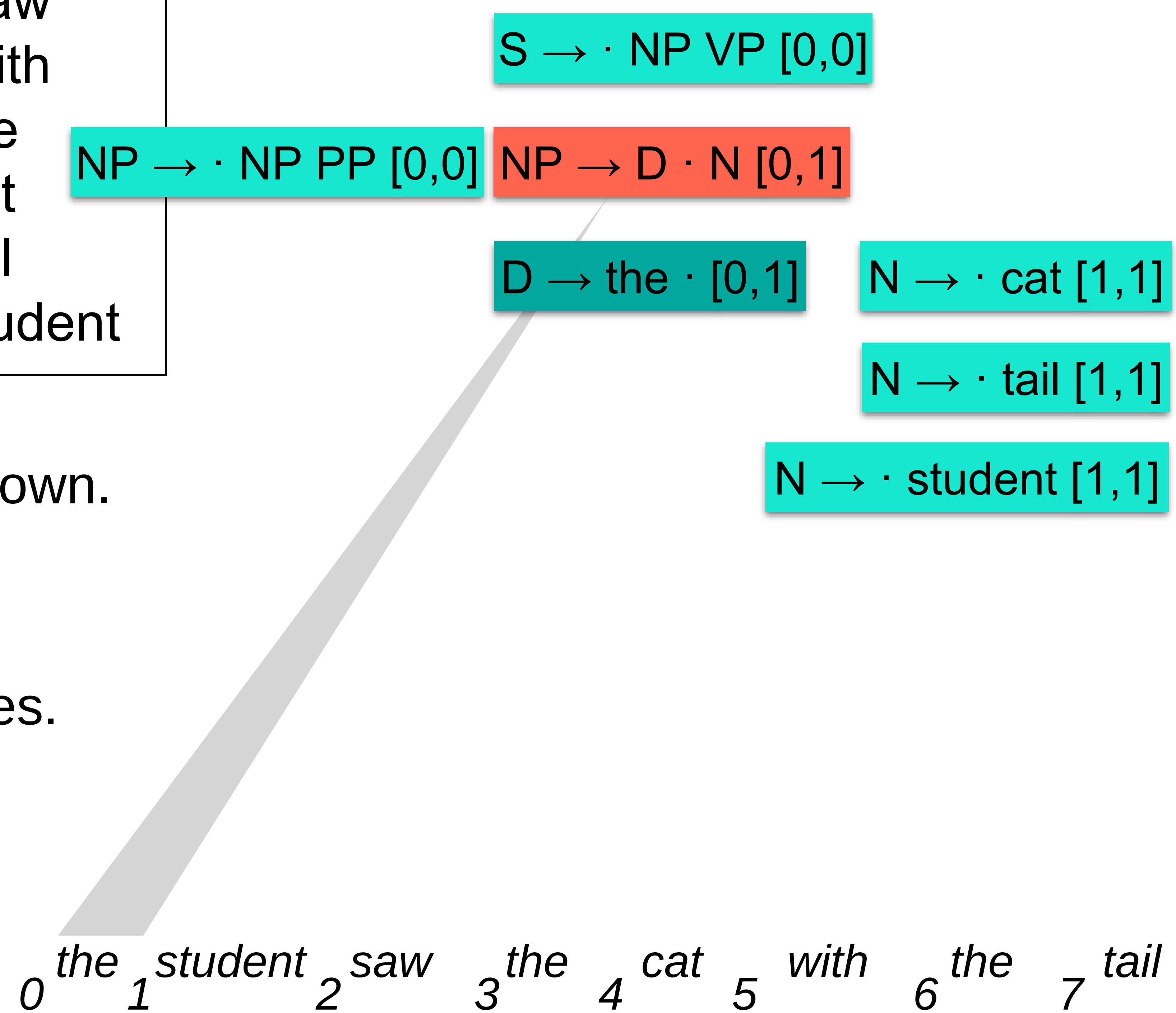

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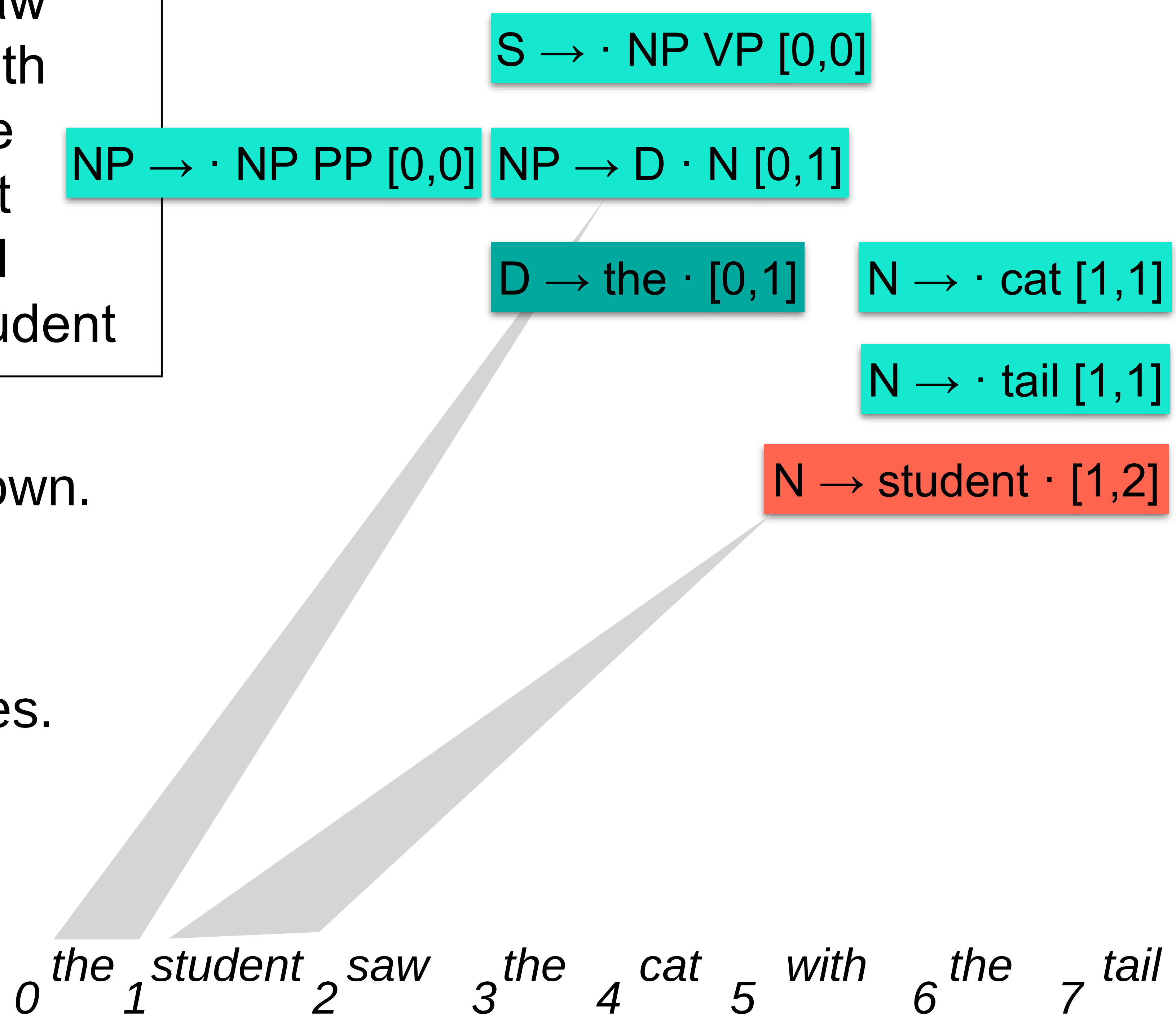

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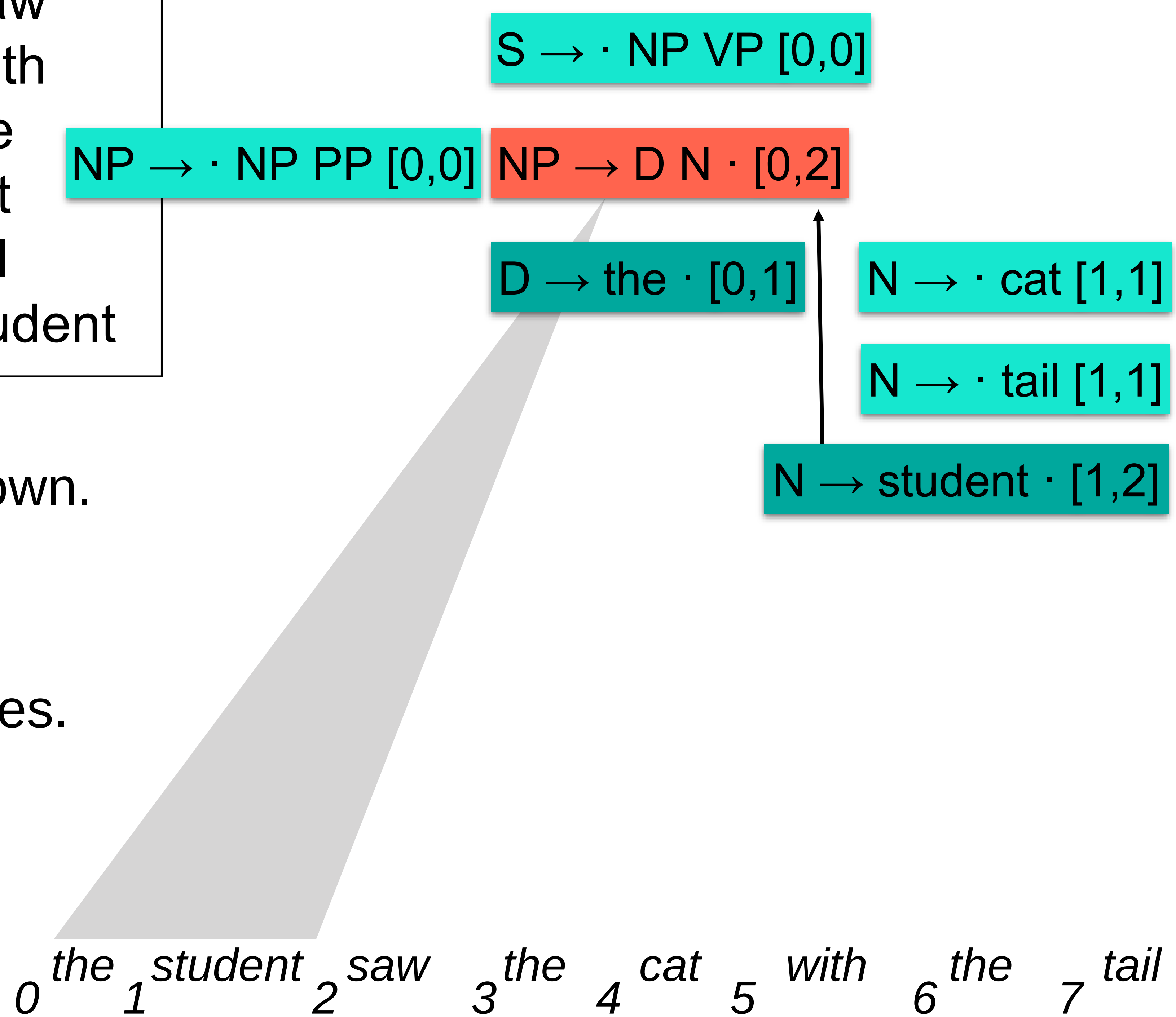

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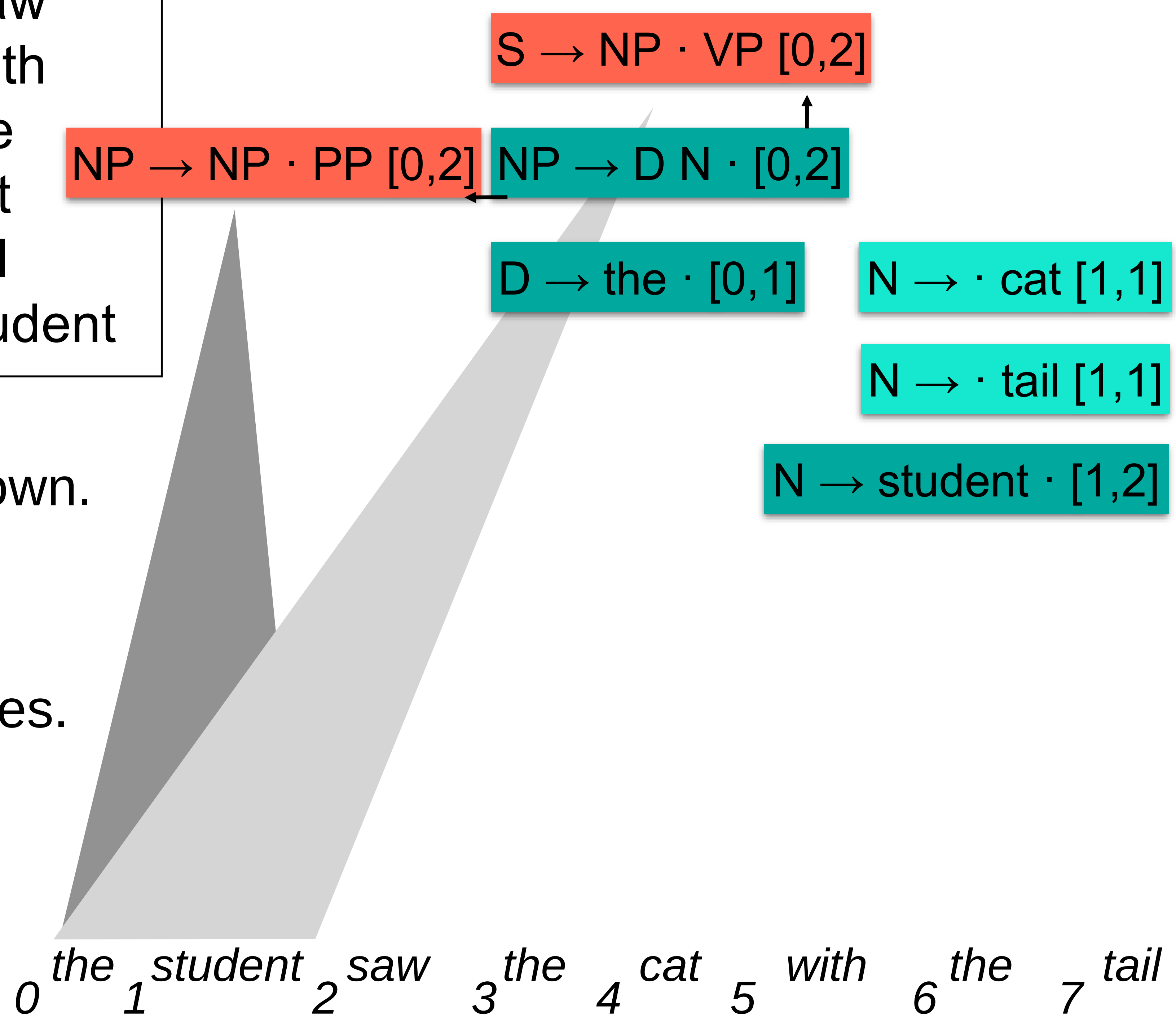

Earley Algorithm

- Keep track of parser states in a table ("chart"). *Chart[k]* contains a set of all parser states that end in position k.

- **Input:** Grammar $G=(N, \Sigma, R, S)$, input string s of length n.
- **Initialization:** For each production $S \rightarrow \alpha \in R$
  add a state $S \rightarrow \cdot \alpha [0,0]$ to Chart[0].
- for i = 0 to n:
  - for each *state* in Chart[i]:
    - if *state* is of form $A \rightarrow \alpha \cdot s[i]\ \beta\ [k,i]$:
      scan(*state*)
    - elif *state* is of form $A \rightarrow \alpha \cdot B\ \beta\ [k,i]$:
      predict(*state*)
    - elif *state* is of form $A \rightarrow \alpha \cdot [k,i]$
      complete(*state*)

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      complete(*state*)

else then is states of form
$A \rightarrow \alpha \cdot \beta\ [k,i]$, i.e.

β is not $s[i]$, in which case we don't want to do anything

Earley Algorithm - Scan

- The scan operation can only be applied to a state if the dot is in front of a terminal symbol that matches the next input terminal.

- function **scan(**state**):** // *state* is of form $A \rightarrow \alpha \cdot s[i]\ \beta\ [k,i]$
  - Add a new state $A \rightarrow \alpha\ s[i] \cdot \beta\ [k,i+1]$ *to* $Chart[i+1]$

Earley Algorithm - Predict

- The predict operation can only be applied to a state if the dot is in front of a non-terminal symbol.

- function **predict(**state**):** // *state* is of form $A \rightarrow \alpha \cdot B\ \beta\ [k,i]$:
  - Add a new state $B \rightarrow \cdot\ \gamma\ [i,i]$ *to* $Chart[i]$

- Note that this modifies Chart[i] ***while*** the algorithm is looping through it.

- No duplicate states are added (Chart[i] is a set)

Earley Algorithm - Complete

- The complete operation may only be applied to a passive item.

- function **complete(**state**):** // *state* is of form $A \rightarrow \alpha \cdot [k,j]$
  - for each state $B \rightarrow \beta \cdot A\ \gamma\ [i,k]$ add a new state $B \rightarrow \beta\ A \cdot \gamma [i,j]$ to Chart[j]

- Note that this modifies Chart[i] **while** the algorithm is looping through it.

- Note that it is important to make a copy of the old state before moving the dot.

- This operation is similar to the combination operation in CKY!

Earley Algorithm - Runtime

- The runtime depends on the number of items in the chart (each item is “visited” exactly once).
- We proceed through the input exactly once, which takes O(N).
  - For each position on the chart, there are O(N) possible split points where the dot could be.
  - Each complete operation can produce O(N) possible new items (with different starting points).
- Total: $O(N^3)$

Earley Algorithm - Some Observations

- How do we recover parse trees?
  - What happens in case of ambiguity?
    - Multiple ways to Complete the same state.
  - Keep back-pointers in the parser state objects.
  - Or use a separate data structure (CKY-style table or hashed states)
- How do we make the algorithm work with PCFG?
  - Easy to compute probabilities on Complete. Follow back pointer with max probability.