MACs, Authenticated Encryption, and PRNGs

CS 161 Spring 2022 - Lecture 9

Message Authentication Codes (MACs)

Textbook Chapter 8.1–8.3 & 8.5–8.6

Cryptography Roadmap

	Symmetric-key	Asymmetric-key
Confidentiality	● One-time pads ● Block ciphers with chaining modes (e.g. AES-CBC)	● RSA encryption ● ElGamal encryption
Integrity, Authentication	● MACs (e.g. HMAC)	● Digital signatures (e.g. RSA signatures)

- Hash functions
- Pseudorandom number generators
- Public key exchange (e.g. Diffie-Hellman)

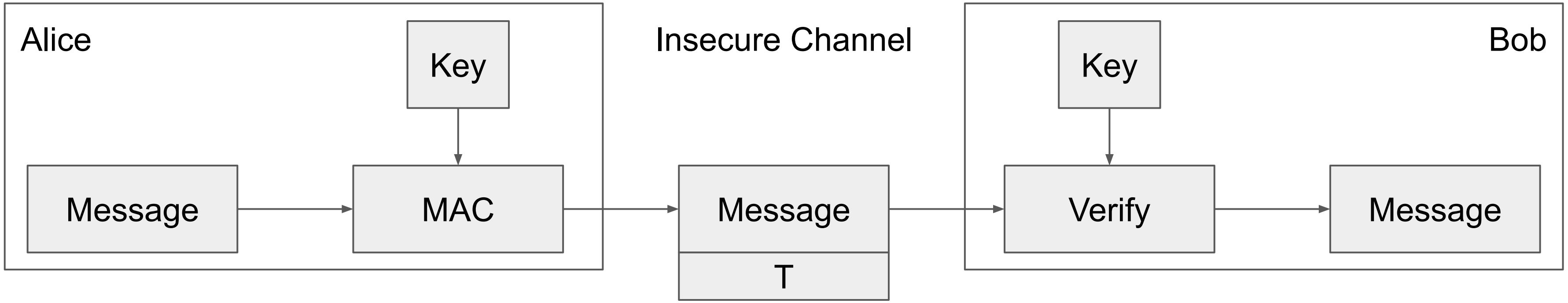
- Key management (certificates)
- Password management

How to Provide Integrity

- Reminder: We’re still in the symmetric-key setting
  - Assume that Alice and Bob share a secret key, and attackers don’t know the key
- We want to attach some piece of information to **prove** that someone with the key sent this message
  - This piece of information can only be generated by someone with the key

MACs: Usage

- Alice wants to send *M* to Bob, but doesn't want Mallory to tamper with it
- Alice sends *M* and *T* = MAC(*K*, *M*) to Bob
- Bob receives *M* and *T*
- Bob computes MAC(*K*, *M*) and checks that it matches *T*
- If the MACs match, Bob is confident the message has not been tampered with (integrity)



MACs: Definition

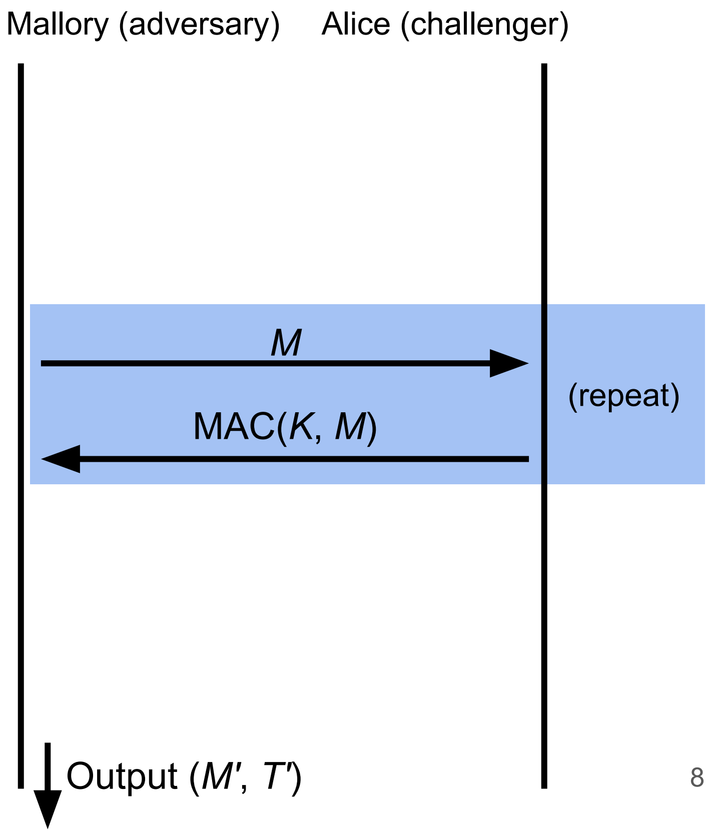
- Two parts:
    - KeyGen() → *K*: Generate a key *K*
    - MAC(*K*, *M*) → *T*: Generate a tag *T* for the message *M* using key *K*
        - Inputs: A secret key and an arbitrary-length message
        - Output: A fixed-length **tag** on the message
- Properties
    - **Correctness**: Determinism
        - Note: Some more complicated MAC schemes have an additional Verify(*K*, *M*, *T*) function that don't require determinism, but this is out of scope
    - **Efficiency**: Computing a MAC should be efficient
    - **Security**: EU-CPA (existentially unforgeable under chosen plaintext attack)

Defining Integrity: EU-CPA

- A secure MAC is **existentially unforgeable**: without the key, an attacker cannot create a valid tag on a message
  - Mallory cannot generate MAC(K, M') without K
  - Mallory cannot find any $M' \neq M$ such that MAC(K, M') = MAC(K, M)
- Formally defined by a security game: existential unforgeability under chosen-plaintext attack, or EU-CPA

Defining Integrity: EU-CPA

1. Mallory may send messages to Alice and receive their tags
2. Eventually, Mallory creates a message-tag pair (*M'*, *T'*)
    - *M'* cannot be a message that Mallory requested earlier
    - If *T'* is a valid tag for *M'*, then Mallory wins. Otherwise, she loses.
    - A scheme is EU-CPA secure if for **all** polynomial time adversaries, the probability of winning is 0 or negligible



Example: NMAC

- Can we use secure cryptographic hashes to build a secure MAC?
  - Intuition: Hash output is unpredictable and looks random, so let's hash the key and the message together
- KeyGen():
  - Output two random, *n*-bit keys K_1 and K_2, where *n* is the length of the hash output
- NMAC(K_1, K_2, M):
  - Output $H(K_1 \mid\mid H(K_2 \mid\mid M))$
- NMAC is EU-CPA secure if the two keys are different
  - Provably secure if the underlying hash function is secure
- Intuition: Using two hashes prevents a length extension attack
  - Otherwise, an attacker who sees a tag for M could generate a tag for $M \mid\mid M'$

Example: HMAC

- Issues with NMAC:
  - Recall: NMAC(K_1, K_2, M) = $H(K_1 \| H(K_2 \| M))$
  - We need two different keys
  - Key must be of the hash output size
  - Can we use NMAC to design a scheme that uses one key?
- HMAC(K, M):
  - If key is longer than the desired size, we can hash it first, but be careful with using keys that are too much smaller, they have to have enough randomness in them
  - Output $H((K \oplus opad) \| H((K \oplus ipad) \| M))$

Example: HMAC

- HMAC(K, M):
  - Output $H((K \oplus opad) \,||\, H((K \oplus ipad) \,||\, M))$
- Use K to derive two different keys
  - *opad* (outer pad) is the hard-coded byte **`0x5c`** repeated until it's the same length as K
  - *ipad* (inner pad) is the hard-coded byte **`0x36`** repeated until it's the same length as K
  - As long as *opad* and *ipad* are different, you'll get two different keys
  - For paranoia, the designers chose two very different bit patterns, even though they theoretically need only differ in one bit

HMAC Properties

- HMAC(K, M) = $H((K \oplus opad) \mid\mid$ H$((K \oplus ipad) \mid\mid M))$
- HMAC is a hash function, so it has the properties of the underlying hash too
  - It is collision resistant
  - Given HMAC(K, M) and K, an attacker can't learn M
  - If the underlying hash is secure, HMAC doesn't reveal M, but it is still deterministic
- You can't verify a tag T if you don't have K
  - This means that an attacker can't brute-force the message M without knowing K

Do MACs provide integrity?

- Do MACs provide integrity?
  - Yes. An attacker cannot tamper with the message without being detected
- Do MACs provide authenticity?
  - It depends on your threat model
  - If a message has a valid MAC, you can be sure it came from *someone with the secret key*, but you can't narrow it down to one person
  - If only two people have the secret key, MACs provide authenticity: it has a valid MAC, and it's not from me, so it must be from the other person
- Do MACs provide confidentiality?
  - MACs are deterministic ⇒ No IND-CPA security
  - MACs in general have no confidentiality guarantees; they can leak information about the message
    - HMAC doesn't leak information about the message, but it's still deterministic, so it's not IND-CPA secure

Authenticated Encryption

Textbook Chapter 8.7 & 8.8

Cryptography Roadmap

	Symmetric-key	Asymmetric-key
Confidentiality	● One-time pads ● Block ciphers with chaining modes (e.g. AES-CBC)	● RSA encryption ● ElGamal encryption
Integrity, Authentication	● MACs (e.g. HMAC)	● Digital signatures (e.g. RSA signatures)

- Hash functions
- Pseudorandom number generators
- Public key exchange (e.g. Diffie-Hellman)
- Key management (certificates)
- Password management

Authenticated Encryption: Definition

- **Authenticated encryption** (**AE**): A scheme that simultaneously guarantees confidentiality and integrity (and authenticity, depending on your threat model) on a message
- Two ways of achieving authenticated encryption:
  - Combine schemes that provide confidentiality with schemes that provide integrity
  - Use a scheme that is designed to provide confidentiality and integrity

Combining Schemes: Let's design it together

- First method for authenticated encryption: Combining schemes that provide confidentiality with schemes that provide integrity
- You can use:
  - An IND-CPA encryption scheme (e.g. AES-CBC): Enc(K, M) and Dec(K, M)
  - An unforgeable MAC scheme (e.g. HMAC): MAC(K, M)
- First attempt: Alice sends Enc(K_1, M) and MAC(K_2, M)
  - Integrity? Yes, attacker can't tamper with the MAC
  - Confidentiality? No, the MAC is not IND-CPA secure
- Idea: Let's compute the MAC on the *ciphertext* instead of the plaintext: Enc(K_1, M) and MAC(k_2, Enc(K_1, M))
  - Integrity? Yes, attacker can't tamper with the MAC
  - Confidentiality? Yes, the MAC might leak info about the ciphertext, but that's okay
- Idea: Let's encrypt the MAC too: Enc(K_1, M || MAC(K_2, M))
  - Integrity? Yes, attacker can't tamper with the MAC
  - Confidentiality? Yes, everything is encrypted

MAC-then-Encrypt or Encrypt-then-MAC?

- MAC-then-encrypt
  - First compute MAC(K_2, M)
  - Then encrypt the message and the MAC together: Enc(K_1, M || MAC(K_2, M))
- Encrypt-then-MAC
  - First compute Enc(K_1, M)
  - Then MAC the ciphertext: MAC(K_2, Enc(K_1, M))
- Which is better?
  - In theory, both are IND-CPA and EU-CPA secure if applied properly
  - MAC-then-encrypt has a flaw: You don’t know if tampering has occurred until after decrypting
    - Attacker can supply arbitrary tampered input, and you always have to decrypt it
    - Passing attacker-chosen input through the decryption function can cause side-channel leaks
- **Always use encrypt-then-MAC** because it’s more robust to mistakes

Key Reuse

- **Key reuse**: Using the same key in two different use cases
  - Note: Using the same key multiple times for the same use (e.g. computing HMACs on different messages in the same context with the same key) is not key reuse
- Reusing keys can cause the underlying algorithms to interfere with each other and affect security guarantees
  - Example: If you use a block-cipher-based MAC algorithm and a block cipher chaining mode, the underlying block ciphers may no longer be secure
  - Thinking about these attacks is hard

Key Reuse

- Simplest solution: Do not reuse keys! One key per **use**.
  - Encrypt a piece of data and MAC a piece of data?
    - Different use; different key
  - MAC one of Alice's messages to Bob and MAC one of Bob's messages to Alice?
    - Different use; different key
  - Encrypt one of Alice's files and encrypt another one of Alice's files?
    - It's *probably* fine to use the same key, but cryptographic design is tricky to get right!
  - Encrypt user metadata, encrypt file metadata, and encrypt file data?
    - You'll have to think about this in Project 2!

TLS 1.0 “Lucky 13” Attack

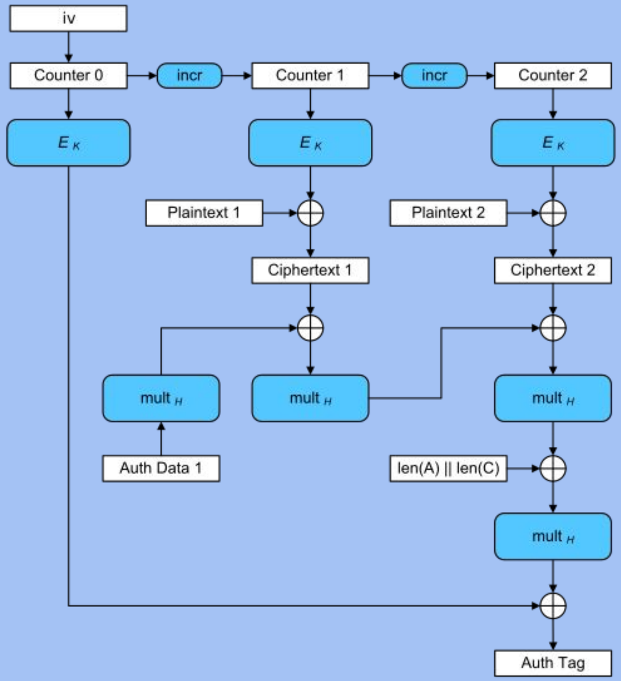
- TLS: A protocol for sending encrypted and authenticated messages over the Internet (we’ll study it more in the networking unit)
- TLS 1.0 uses MAC-then-encrypt: Enc(K_1, M || MAC(K_2, M))
  - The encryption algorithm is AES-CBC
- The Lucky 13 attack abuses MAC-then-encrypt to read encrypted messages
  - Guess a byte of plaintext and change the ciphertext accordingly
  - The MAC will error, but the time it takes to error is different depending on if the guess is correct
  - Attacker measures how long it takes to error in order to learn information about plaintext
  - TLS will send the message again if the MAC errors, so the attacker can guess repeatedly
- Takeaways
  - Side channel attack: The algorithm is proved secure, but poor implementation made it vulnerable
  - Always encrypt-then-MAC
  - You’ll try a similar attack in Homework 2!

AEAD Encryption

- Second method for authenticated encryption: Use a scheme that is designed to provide confidentiality, integrity, and authenticity
- **Authenticated encryption with additional data** (**AEAD**): An algorithm that provides both confidentiality and integrity over the plaintext and integrity over *additional data*
  - Additional data is usually context (e.g. memory address), so you can't change the context without breaking the MAC
- Great if used correctly: No more worrying about MAC-then-encrypt
  - If you use AEAD incorrectly, you lose *both* confidentiality and integrity/authentication
  - Example of correct usage: Using a crypto library with AEAD

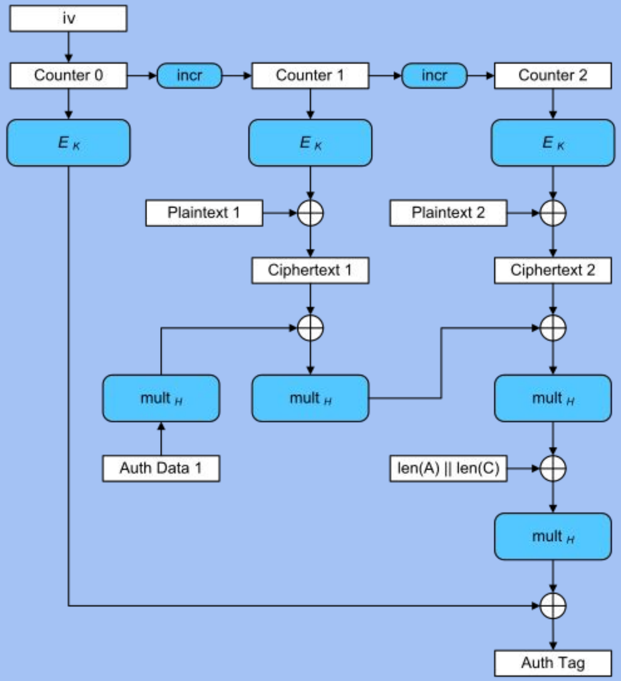
AEAD Example: Galois Counter Mode (GCM)

- **Galois Counter Mode** (**GCM**): An AEAD block cipher mode of operation
- E_K is standard block cipher encryption
- mult$_H$ is 128-bit multiplication over a special field (Galois multiplication)
  - Don't worry about the math



AEAD Example: Galois Counter Mode (GCM)

- Very fast mode of operation
  - Fully parallel encryption
  - Galois multiplication isn't parallelizable, but it's very fast
- Drawbacks
  - IV reuse leads to loss of confidentiality, integrity, and authentication
  - This wouldn't happen if you used AES-CTR and HMAC-SHA256
  - Implementing Galois implementation is difficult and easy to screw up
- **Takeaway**: GCM provides integrity and confidentiality, but if you misuse it, it's even worse than CTR mode



Hashes: Summary

- Map arbitrary-length input to fixed-length output
- Output is deterministic and unpredictable
- Security properties
  - One way: Given an output y, it is infeasible to find any input x such that $H(x) = y$.
  - Second preimage resistant: Given an input x, it is infeasible to find another input $x' \neq x$ such that $H(x) = H(x')$.
  - Collision resistant: It is infeasible to find another any pair of inputs $x' \neq x$ such that $H(x) = H(x')$.
- Some hashes are vulnerable to length extension attacks
- Application: Lowest hash scheme
- Hashes don’t provide integrity (unless you can publish the hash securely)

MACs: Summary

- Inputs: a secret key and a message
- Output: a tag on the message
- A secure MAC is unforgeable: Even if Mallory can trick Alice into creating MACs for messages that Mallory chooses, Mallory cannot create a valid MAC on a message that she hasn't seen before
    - Example: HMAC(K, M) = $H((K' \oplus opad) \| H((K' \oplus ipad) \| M))$
- MACs do not provide confidentiality

Authenticated Encryption: Summary

- Authenticated encryption: A scheme that simultaneously guarantees confidentiality and integrity (and authenticity) on a message
- First approach: Combine schemes that provide confidentiality with schemes that provide integrity and authenticity
  - MAC-then-encrypt: $\text{Enc}(K_1, M \,||\, \text{MAC}(K_2, M))$
  - Encrypt-then-MAC: $\text{Enc}(K_1, M) \,||\, \text{MAC}(K_2, \text{Enc}(K_1, M))$
  - Always use Encrypt-then-MAC because it's more robust to mistakes
- Second approach: Use AEAD encryption modes designed to provide confidentiality, integrity, and authenticity
  - Drawback: Incorrectly using AEAD modes leads to losing *both* confidentiality and integrity/authentication

Next: PRNGs

- Symmetric-key encryption schemes need randomness. How do we securely generate random numbers?

Pseudorandom Number Generators (PRNGs)

Textbook Chapter 9

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Randomness

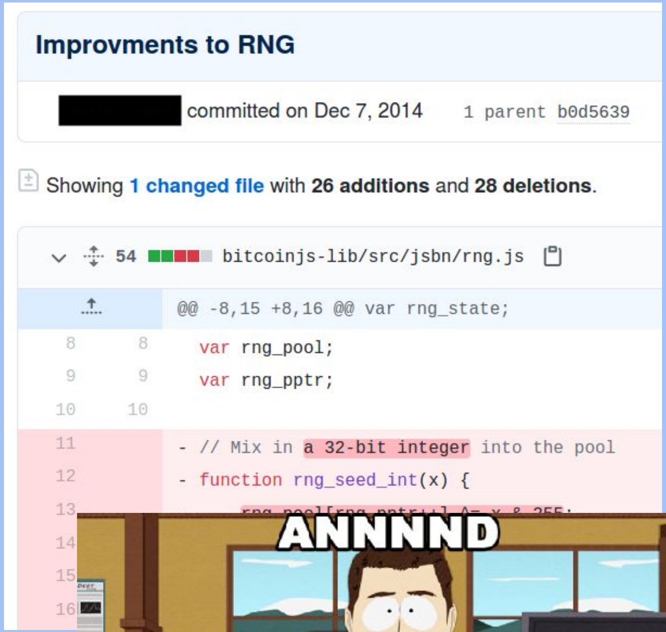
- Randomness is essential for symmetric-key encryption
  - A random key
  - A random IV/nonce
  - Universally unique identifiers (we’ll see this shortly)
  - We’ll see more applications later
- If an attacker can predict a random number, things can catastrophically fail
- How do we securely generate random numbers?

Entropy

- In cryptography, “random” usually means “random and unpredictable”
- Scenario
  - You want to generate a secret bitstring that the attacker can't guess
  - You generate random bits by tossing a fair (50-50) coin
  - The outcomes of the fair coin are harder for the attacker to guess
- **Entropy**: A measure of uncertainty
  - In other words, a measure of how unpredictable the outcomes are
  - High entropy = unpredictable outcomes = desirable in cryptography
  - The uniform distribution has the highest entropy (every outcome equally likely, e.g. fair coin toss)
  - Usually measured in bits (so 3 bits of entropy = uniform, random distribution over 8 values)

Breaking Bitcoin Wallets

- What happens if we use a poor source of entropy?
- Bitcoin users use a randomly-generated private key to access their account (and money)
  - An attacker who learns the key can access the money
  - We'll learn more about Bitcoin later
- An "improvment" [sic] to the algorithm reduced the entropy used to generate the private keys
  - Any private key created with this "improvment" could be brute-forced



Improvments to RNG

[redacted] committed on Dec 7, 2014 1 parent b0d5639

Showing **1 changed file** with **26 additions** and **28 deletions**.

54 bitcoinjs-lib/src/jsbn/rng.js

```
@@ -8,15 +8,16 @@ var rng_state;
  var rng_pool;
  var rng_pptr;

- // Mix in a 32-bit integer into the pool
- function rng_seed_int(x) {
```

True Randomness

- To generate truly random numbers, we need a physical source of entropy
  - An unpredictable circuit on a CPU
  - Human activity measured at very fine time scales (e.g. the microsecond you pressed a key)
- Unbiased entropy usually requires combining multiple entropy sources
  - Goal: Total number of bits of entropy is the sum of all the input numbers of bits of entropy
    - Many poor sources + 1 good source = good entropy
- Issues with true randomness
  - It’s expensive and slow to generate
  - Physical entropy sources are often biased

Exotic entropy source: Cloudflare has a wall of lava lamps that are recorded by an HD video camera that views the lamps through a rotating prism

Pseudorandom Number Generators (PRNGs)

- True randomness is expensive and biased
- **Pseudorandom number generator** (**PRNGs**): An algorithm that uses a little bit of true randomness to generate a lot of random-looking output
    - Also called **deterministic random bit generators** (**DRBGs**)
- Usage
    - Generate some expensive true randomness (e.g. noisy circuit on your CPU)
    - Use the true randomness as input to the PRNG
    - Generate random-looking numbers quickly and cheaply with the PRNG
- PRNGs are deterministic: Output is generated according to a set algorithm
    - However, for an attacker who can't see the internal state, the output is *computationally indistinguishable* from true randomness

PRNG: Definition

- A PRNG has three functions:
  - PRNG.Seed(randomness): Initializes the internal state using the entropy
    - Input: Some truly random bits
  - PRNG.Reseed(randomness): Add in the additional entropy
    - Input: More truly random bits
    - Never reduces entropy, only adds to it!
  - PRNG.Generate(*n*): Generate *n* pseudorandom bits
    - Input: A number *n*
    - Output: *n* pseudorandom bits
    - Updates the internal state as needed
- Properties
  - **Correctness**: Deterministic
  - **Efficiency**: Efficient to generate pseudorandom bits
  - **Security**: Indistinguishability from random
  - **Additional security**: Rollback resistance

PRNG: Security

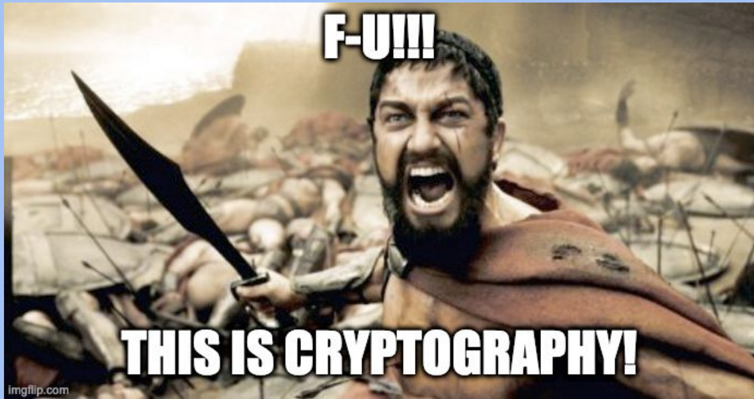
- Can we design a PRNG that is truly random?
- A PRNG cannot be truly random
  - The output is deterministic given the initial seed
  - If the initial seed is *s* bits long, there are only 2^s possible output sequences
- A secure PRNG is computationally indistinguishable from random to an attacker
  - Game: Present an attacker with a truly random sequence and a sequence outputted from a secure PRNG
  - An attacker should not be able to determine which is which with probability $> 1/2 + \varepsilon$
- Equivalent definition: An attacker cannot predict future output of the PRNG

PRNG: Rollback Resistance

- **Rollback resistance**: If the attacker learns the internal PRNG state, they cannot learn anything about previous states or outputs
  - Game: An attacker knows the current internal state of the PRNG and is given a sequence of truly random bits and a sequence of previous output from the PRNG
  - The attacker cannot determine which is which with probability > 1/2
- Rollback resistance is not required in a secure PRNG, but it is a useful property
  - Consider:
    - Alice uses the same PRNG to generate her secret key and the IVs for encryption
    - Mallory compromises the internal state of the PRNG
    - If the PRNG is not rollback resistant, Mallory can derive previous PRNG output… such as the secret key

Breaking Slot Machines

- What happens if PRNGs are used improperly?
- A casino in St. Louis experienced unusual bad “luck”
  - Suspicious players would hover over the lever and then spin at a specific time to win
- Vulnerability: Slot machines used predictable PRNGs
  - The PRNG output was based on the current time and a low-entropy seed
- Strategy:
  - Set up a smartphone to watch you play a couple rounds at the slot machine
    - Learning the output of the PRNG!
  - Then, the smartphone predicts future PRNG outputs and alerts you to when to “spin” to be more likely to win
- Oh, and this never affected Las Vegas!
  - Evaluation standards for Nevada slot machines are specifically designed to address this sort of issue



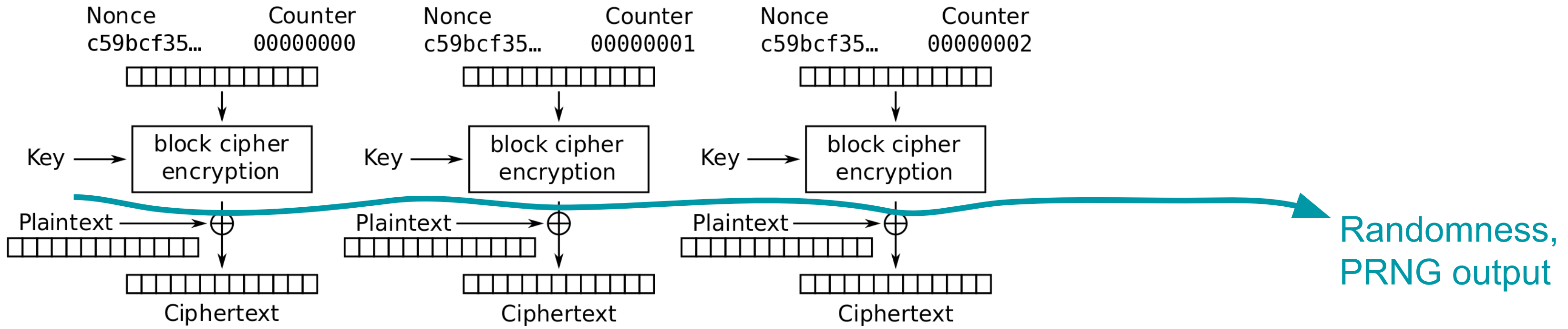
Insecure PRNGs: OpenSSL PRNG bug

- What happens if we don't use enough entropy?
- Debian OpenSSL CVE-2008-0166
  - Debian: A Linux distribution
  - OpenSSL: A cryptographic library
  - In "cleaning up" OpenSSL (Debian "bug" #363516), the author "fixed" how OpenSSL seeds random numbers
  - The existing code caused Purify and Valgrind to complain about reading uninitialized memory
  - The cleanup caused the PRNG to only be seeded with the process ID
  - There are only 2^{15} (32,768) possible process IDs, so the PRNG only has 15 bits of entropy
- Easy to deduce private keys generated with the PRNG
  - Set the PRNG to every possible starting state and generate a few private/public key pairs
  - See if the matching public key is anywhere on the Internet

HOW DEBIAN BUG #363516 WAS REALLY FIXED:
YOU'RE USING UNINITIALIZED MEMORY THERE, GAIUS.
AH, RIGHT. LET ME FIX THAT.

CTR-DRBG

- Using block cipher in CTR mode:
  - If you want m random bits, and a block cipher with E_k has n bits, apply the block cipher *m*/*n* times and concatenate the result:
- PRNG.Seed(K || IV) = E_K(IV || 1) || E_K(IV || 2) || … || E_K(IV || ceil(m/n))
- Security:
  - **Secure**: The attacker can't predict outputs because that would break the unpredictability of the block cipher's random permutation
  - **Not rollback resistant**: If the adversary learns the key (internal state), they can encrypt previous counters to learn previous output!



Counter (CTR) mode encryption

HMAC-DRBG

- Idea: HMAC output looks unpredictable. Let’s use HMAC to build a PRNG!
- HMAC takes two arguments (key and message). Let’s keep two values, *K* (key) and *V* (value) as internal state

HMAC-DRBG

```
Seed($s$):
    $K = 0$
    $V = 0$

    $K = \text{HMAC}(K, V \mathbin{||} \text{0x00} \mathbin{||} s)$
    $V = \text{HMAC}(K, V)$
    $K = \text{HMAC}(K, V \mathbin{||} \text{0x01} \mathbin{||} s)$
    $V = \text{HMAC}(K, V)$
```

Initialize internal state

Update internal state with provided entropy

HMAC-DRBG

```
Reseed(s):
    $K$ = HMAC($K$, $V$ || 0x00 || $s$)
    $V$ = HMAC($K$, $V$)
    $K$ = HMAC($K$, $V$ || 0x01 || $s$)
    $V$ = HMAC($K$, $V$)
```

Update internal state with provided entropy

HMAC-DRBG

Generate(n):

```
output = ''
while len(output) < $n$ do
        $V$ = HMAC($K$, $V$)
        output = output || $V$
end while

$K$ = HMAC($K$, $V$ || 0x00)
$V$ = HMAC($K$, $V$)

return output[:$n$]
```

Call HMAC repeatedly to generate random-looking output

Update internal state with no extra entropy

HMAC-DRBG: Security

- Assuming HMAC is secure, HMAC-DRBG is a secure, rollback-resistant PRNG
  - **Secure**: If you can distinguish PRNG output from random, then you've distinguished HMAC from random
  - **Rollback-resistant**: If you can derive old output from the current state, then you've reversed the hash function or HMAC
  - The full proof is out of scope
  - In other words: if you break HMAC-DRBG, you've either broken HMAC or the underlying hash function
- Generally considered the best DRBG
  - **Accept no substitutes!**

Insecure PRNGs: CVE-2019-16303

- Relevant if you wrote an app in JHipster before 2019
- Password reset functions
  - When you forget your password, receive an email with a special link to reset your password
  - The special link should contain a randomly-generated code (so attackers can't make their own link)
- Vulnerability: Bad PRNG
  - You can figure out the PRNG’s internal state from the reset link
  - Request password reset links for other people's accounts
  - Predict the “random” reset link and take over any account you want!

Insecure PRNGs: Rust Rand_Core

- A Rust library has an interface for “secure” random number generators… but it isn’t actually secure!
- Example: ChaCha8Rng
  - A stream cipher PRNG
  - No reseed function: no way of adding extra entropy after the initial seed
  - Seed only takes 32 bits: no way to combine entropy
  - No rollback resistance
- None of the “secure” RNGs are cryptographically secure
  - None have a reseed function to add extra entropy
  - None take arbitrarily long seeds
- **Takeaway**: Always make sure you use a secure PRNG
  - Consider human factors? Use fail-safe defaults?

Application: Universally Unique Identifiers (UUIDs)

- Scenario
  - You have a set of objects (e.g. files)
  - You need to assign a unique name to every object
  - Every name must be unique and unpredictable
- Solution: Choose a random value
  - If you use enough randomness, the probability of generating the same random value twice are astronomically small (basically 0)
- Universally Unique Identifiers (UUIDs)
  - 128-bit unique values
  - To generate a new UUID, seed a secure PRNG properly, and generate a random value
  - Often written in hexadecimal: **`00112233-4455-6677-8899-aabbccddeeff`**
  - You'll work with UUIDs in Project 2

PRNGs: Summary

- True randomness requires sampling a physical process
  - Slow, expensive, and biased (low entropy)
- PRNG: An algorithm that uses a little bit of true randomness to generate a lot of random-looking output
  - Seed(entropy): Initialize internal state
  - Reseed(entropy): Add additional entropy to the internal state
  - Generate(n): Generate n bits of pseudorandom output
  - Security: Computationally indistinguishable from truly random bits
- CTR-DRBG: Use a block cipher in CTR mode to generate pseudorandom bits
- HMAC-DRBG: Use repeated applications of HMAC to generate pseudorandom bits
- Application: UUIDs

Stream Ciphers

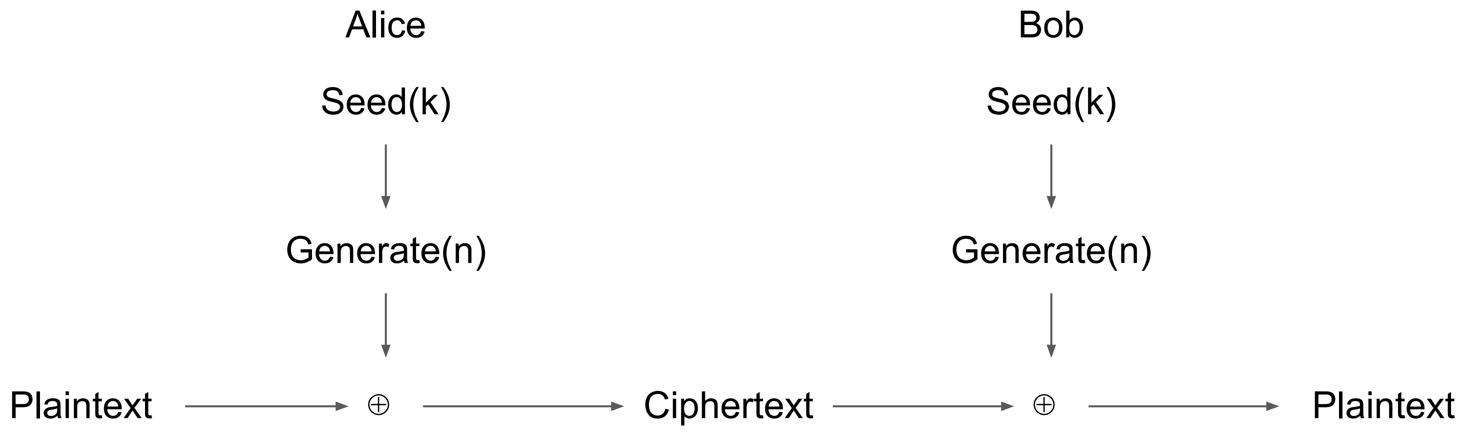
Textbook Chapter 9.5

Stream Ciphers

- Another way to construct symmetric key encryption schemes
- Idea
  - A secure PRNG produces output that looks indistinguishable from random
  - An attacker who can't see the internal PRNG state can't learn any output
  - What if we used PRNG output as the key to a one-time pad?
- **Stream cipher**: A symmetric encryption algorithm that uses pseudorandom bits as the key to a one-time pad

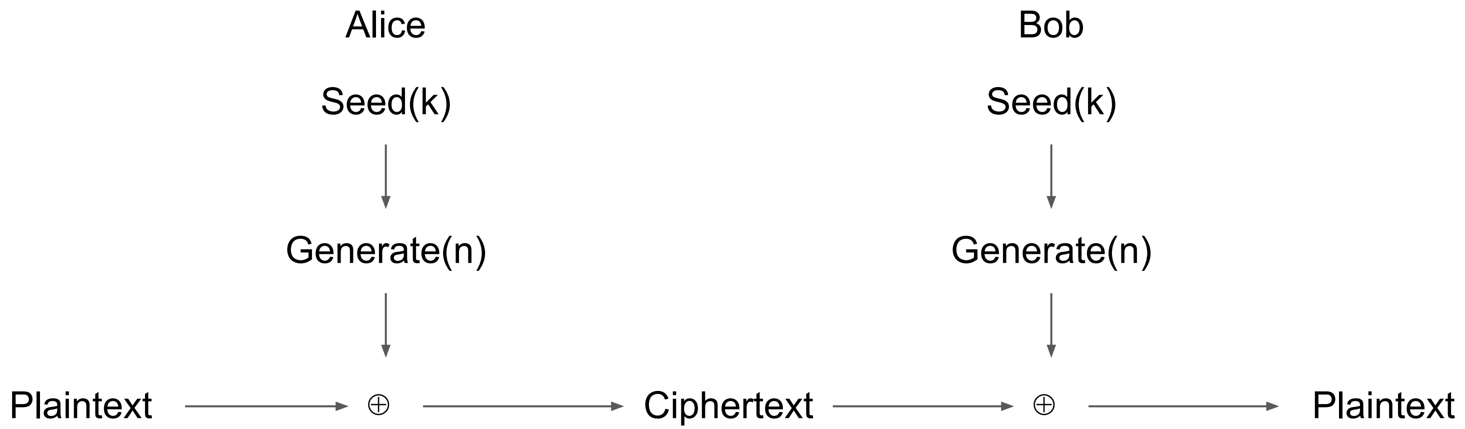
Stream Ciphers

- Protocol: Alice and Bob both seed a secure PRNG with their symmetric secret key, and then use the output as the key for a one-time pad



Stream Ciphers: Encrypting Multiple Messages

- Recall: One-time pads are insecure when the key is reused. How do we encrypt multiple messages without key reuse?



Stream Ciphers: Encrypting Multiple Messages

- Solution: For each message, seed the PRNG with the key and a random IV, concatenated(“|”). Send the IV with the ciphertext

$$\begin{tikzcd}
 & \text{Alice} & & \text{Bob} & \\
 & \text{IV} \arrow[rr] \arrow[d] & & \text{IV} \arrow[d] & \\
 & \text{Seed}(k \mid \text{IV}) \arrow[d] & & \text{Seed}(k \mid \text{IV}) \arrow[d] & \\
 & \text{Generate}(n) \arrow[d] & & \text{Generate}(n) \arrow[d] & \\
\text{Plaintext} \arrow[r] & \oplus \arrow[r] & \text{Ciphertext} \arrow[r] & \oplus \arrow[r] & \text{Plaintext}
\end{tikzcd}$$

Application of PRNG: Stream ciphers

- Similar in spirit to one-time pad: it XORs the plaintext with some random bits
- But random bits are not the key (as in one-time pad) but are output of a pseudorandom generator PRG

Stream Ciphers: Security

- Stream ciphers are IND-CPA secure, assuming the pseudorandom output is secure
- In some stream ciphers, security is compromised if too much plaintext is encrypted
  - Example: In AES-CTR, if you encrypt so many blocks that the counter wraps around, you'll start reusing keys
  - In practice, if the key is *n* bits long, usually stop after $2^{n/2}$ bits of output
  - Example: In AES-CTR with 128-bit counters, stop after 2^{64} blocks of output

Stream Ciphers: Encryption Efficiency

- Stream ciphers can continually process new elements as they arrive
  - Only need to maintain internal state of the PRNG
  - Keep generating more PRNG output as more input arrives
- Compare to block ciphers: Need modes of operations to handle longer messages, and modes like AES-CBC need padding to function, so doesn’t function well on streams

Stream Ciphers: Decryption Efficiency

- Suppose you received a 1 GB ciphertext (encryption of a 1 GB message) and you only wanted to decrypt the last 128 bytes
- Benefit of some stream ciphers: You can decrypt one part of the ciphertext without decrypting the entire ciphertext
  - Example: In AES-CTR, to decrypt only block *i*, compute E_K(nonce || *i*) and XOR with the *i*th block of ciphertext
  - Example: ChaCha20 (another stream cipher) lets you decrypt arbitrary parts of ciphertext
  - What about HMAC-DRBG? You have to generate all the PRNG output up until the block you want to decrypt

Next: Diffie-Hellman Key Exchange

- When discussing symmetric-key schemes, we assumed Alice and Bob managed to share a secret key. How can Alice and Bob share a symmetric key over an insecure channel?

Diffie-Hellman Key Exchange

Textbook Chapter 10

Cryptography Roadmap

	Symmetric-key	Asymmetric-key
Confidentiality	● One-time pads ● Block ciphers with chaining modes (e.g. AES-CBC)	● RSA encryption ● ElGamal encryption
Integrity, Authentication	● MACs (e.g. HMAC)	● Digital signatures (e.g. RSA signatures)

- Hash functions
- Pseudorandom number generators
- Public key exchange (e.g. Diffie-Hellman)

- Key management (certificates)
- Password management

Secure Color Sharing

- Suppose Alice and Bob want a secret *paint color*, but Eve can see paint colors sent between Alice and Bob
- Alice generates a secret color **amber A**, and Bob generates a secret color **blue B**
- Alice and Bob agree on a common, public color **green G**
- They both mix their secret colors with **G**, so Alice has **green-amber GA**, and Bob has **green-blue GB**
- Alice sends **GA** to Bob, and Bob sends **GB** to Alice
    - Note: Eve now knows the colors **GA** and **GB**! Assume that it is hard to separate colors.
- Alice knows **GB**, so she can mix in **A** to form green-amber-blue **GAB**. Bob knows **GA**, so he can mix in **B** to form **GAB**, as well!
    - Eve only knows **G**, **GA**, and **GB**, so she can only form **green-amber-green-blue GAGB**, which is not the same!

Discrete Log Problem and Diffie-Hellman Problem

- Recall our paint assumption: Separating a paint mixture is hard
  - Is there a mathematical version of this? Yes!
- Assume everyone knows a large prime p (e.g. 2048 bits long) and a generator g
  - Don't worry about what a generator is
- **Discrete logarithm problem** (**discrete log problem**): Given g, p, $g^a \bmod p$ for random a, it is computationally hard to find a
- **Diffie-Hellman assumption**: Given g, p, $g^a \bmod p$, and $g^b \bmod p$ for random a, b, no polynomial time attacker can distinguish between a random value R and $g^{ab} \bmod p$.
  - Intuition: The best known algorithm is to first calculate a and then compute $(g^b)^a \bmod p$, but this requires solving the discrete log problem, which is hard!
  - Note: Multiplying the values doesn't work, since you get $g^{a+b} \bmod p \neq g^{ab} \bmod p$

Diffie-Hellman Key Exchange

Public: g, p

Secret key → Generate a

Public key → Calculate $g^a \bmod p$

Alice	Eve	Bob
Generate a		Generate b
Calculate $g^a \bmod p$	g^a g^b	Calculate $g^b \bmod p$
Receive $g^b \bmod p$		Receive $g^a \bmod p$
Calculate $(g^b)^a \bmod p$		Calculate $(g^a)^b \bmod p$
$a, g^a, g^b \Rightarrow g^{ab}$	$g^a, g^b \not\Rightarrow g^{ab}$	$b, g^a, g^b \Rightarrow g^{ab}$

Shared symmetric key is g^{ab}

Ephemerality of Diffie-Hellman

- Diffie-Hellman can be used ephemerally (called Diffie-Hellman ephemeral, or DHE)
  - **Ephemeral**: Short-term and temporary, not permanent
  - Alice and Bob discard *a*, *b*, and $K = g^{ab} \bmod p$ when they're done
  - Because you need *a* and *b* to derive *K*, you can never derive *K* again!
  - Sometimes *K* is called a **session key**, because it's only used for a an ephemeral session
- Benefit of DHE: **Forward secrecy**
  - Eve records everything sent over the insecure channel
  - Alice and Bob use DHE to agree on a key $K = g^{ab} \bmod p$
  - Alice and Bob use *K* as a symmetric key
  - After they're done, discard *a*, *b*, and *K*
  - Later, Eve steals all of Alice and Bob's secrets
  - Eve can't decrypt any messages she recorded: Nobody saved *a*, *b*, or *K*, and her recording only has $g^a \bmod p$ and $g^b \bmod p$!