1). a). Government bond is less risky because the probability of default of government is lower than the probability of default of corporations.

b). Bond market has bigger size.

c). Treasury securities are those short-term and long-term securities offered by the government. LIBOR refers to the London Interbank offered Rate, which is the benchmark rate for determining interest rate for different debt instruments.

So, LIBOR market refers to those securities with interest rate following LIBOR rate.

b). linear derivatives are the derivatives with linear payoff function. For example, futures contract has linear payoff with the payoff links to the underlying asset price movement.

Q2 a).

$$D = -\frac{1}{P}\frac{\Delta P}{\Delta y}$$

$$\text{Bond duration} = -\frac{1}{100}\left(\frac{99.5922 - 100.4098}{4.05\% - 3.95\%}\right)$$

$$= 8.176$$

$$\text{Bond convexity} = \frac{1}{100}\left(\frac{99.5922 - 2(100) + 100.4098}{(4.05\% - 3.95\%)^2}\right)$$

$$= 20$$

$$\text{option duration} = -\frac{1}{9.9251}\left(\frac{9.7002 - 10.1536}{4.05\% - 3.95\%}\right)$$

$$= 45.6822$$

$$\text{option convexity} = \frac{1}{9.9251}\left(\frac{9.7002 - 2(9.9251) + 10.1536}{(4.05\% - 3.05\%)^2}\right)$$

$$= 362.7167$$

2b). $P_2 = -\frac{D_1}{D_2} P_1$

$= -\frac{45.6822}{8.176}(10,000,000)$

$= -55,873,532$

$\therefore$ long $55,873,532/100$

$= 558,735$ unit of bond.

c). Convexity of portfolio:

$$\frac{20(558,735) - 362.7167\left(\frac{10,000,000}{9.9251}\right)}{55,873,532 - 10,000,000}$$

$= -7.5033$

3a). Monthly payment X:

$$X = \frac{B_0 \cdot \times \frac{y}{12}}{1-\left(1+\frac{y}{12}\right)^{-12T}}$$

where $B_0 = 7m$, $y = 3.25\%$, $T = 30$

$$X = \frac{7m \times \frac{3.25\%}{12}}{1-\left(1+\frac{3.25\%}{12}\right)^{-12\times 30}}$$

$$X = 30464.4423$$

b).

$$B_n = X\left(\frac{12}{y}\right)\left(1 - \frac{1}{\left(1+\frac{y}{12}\right)^{12T-n}}\right)$$

where B_n is remaining principal value of the mortgage.

$$B_{60} = (30464.44234)\left(\frac{12}{3.258}\right)\left(1 - \frac{1}{\left(1+\frac{3.258}{12}\right)^{360-60}}\right)$$

$B_{60} = \$6,251,473.017$

$\therefore$ The payment is $\$6,251,473.017$.

4). a).

Initially, the value of swap is worth at par.

For the floating leg, it is same as investing \$1 in 3 month CD, after 3 month,

we can collect $\$\frac{1}{d(0,\frac{1}{4})}$ and pay the interest, invest \$1 again in 3 month CD, repeat this instruction until the maturity of swap, so we can replicate the same cash flow for the floating leg.

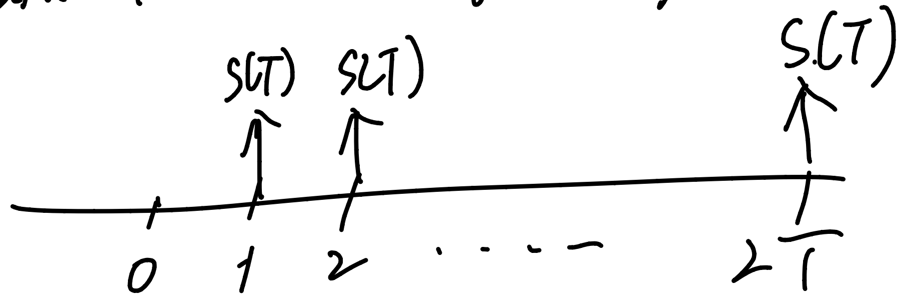
So the cash flow in general is \$1 at T_0, and get back \$1 at T_n, the PV at time $t = d(t,T_0) - d(t,T_n)$

After each payment, the floating leg is price at par value.

$\therefore$ The value of floating leg at $t=0$ will be

$1 - d(0,T)$

Cash flow for fixed leg:

b). 

At each time of fixed-leg payment, the fixed leg receive $S(0,T)$

$\therefore$ The value of fixed leg:

$$S(0,T)\left(\frac{1}{2}d(0,\tfrac{1}{2}) + \frac{1}{2}d(0,\tfrac{2}{2}) + \cdots + \frac{1}{2}d(0,T)\right)$$

$$= S(0,T)\sum_{i=1}^{2T}\frac{1}{2}d(0,\tfrac{i}{2})$$

c). After adding the principal to the last payment, both floating and fixed leg are priced at par.

$\therefore$

$$S(0,T)\frac{1}{2}\sum_{i=1}^{2T} d(0,\tfrac{i}{2}) + d(0,T) = 1$$

$$S(0,T) = \frac{1-d(0,T)}{\sum_{i=1}^{2T}\frac{1}{2}d(0,\frac{i}{2})}$$

d). After each fixed-floating payment exchange, the floating leg is priced at par value to reflect the fact that it will receive the floating interest rate. Therefore, the swap rate is the par yield.

(a).

For example, person A invests in \$1000 TIPS with 4% fixed interest rate, and 12% inflation rate from consumer price index in year 1. So, at the beginning of year 2, the face value becomes $1000 \times (1+12\%) = 1120$, and now the coupon payment is $1120 \times \frac{4\%}{2} = 22.4$

If the inflation rate is negative, we still use \$1000 to calculate coupon payment.

5b). Market value of the Treasury note:

$$\$10m \times \left(\frac{D_{TIPS}}{D_{note}}\right)\left(\frac{Volatility_{note}}{Volatility_{TIPS}}\right)\left(\frac{1}{corr(TIPS, note)}\right)$$

$$= 10,000,000 \left(\frac{9.5}{7.5}\right)\left(\frac{4.8}{5}\right)\left(\frac{1}{0.825}\right)$$

$$= 14,739,394$$