Bond Quotes

TABLE 1.1 Selected Treasury Bond Prices for Settlement on February 15, 2001

Coupon	Maturity	Price
7.875%	8/15/01	101-12$^3/_4$
14.250%	2/15/02	108-31+
6.375%	8/15/02	102-5
6.250%	2/15/03	102-18$^1/_8$
5.250%	8/15/03	100-27

- The price of coupon bonds are quoted as a percentage of the principal value, using a price tick of one-32nd of a percentage point:

$$101\text{-}12\tfrac{3}{4} = 101 + \frac{12.75}{32} = 101.3984$$

More on Quote Prices

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5.250%	8/15/03	100-27

- "+" means 0.5
- The actual tick size can be $\frac{1}{32}\times\frac{1}{8}=\frac{1}{256}$

More on Quote Prices

- So

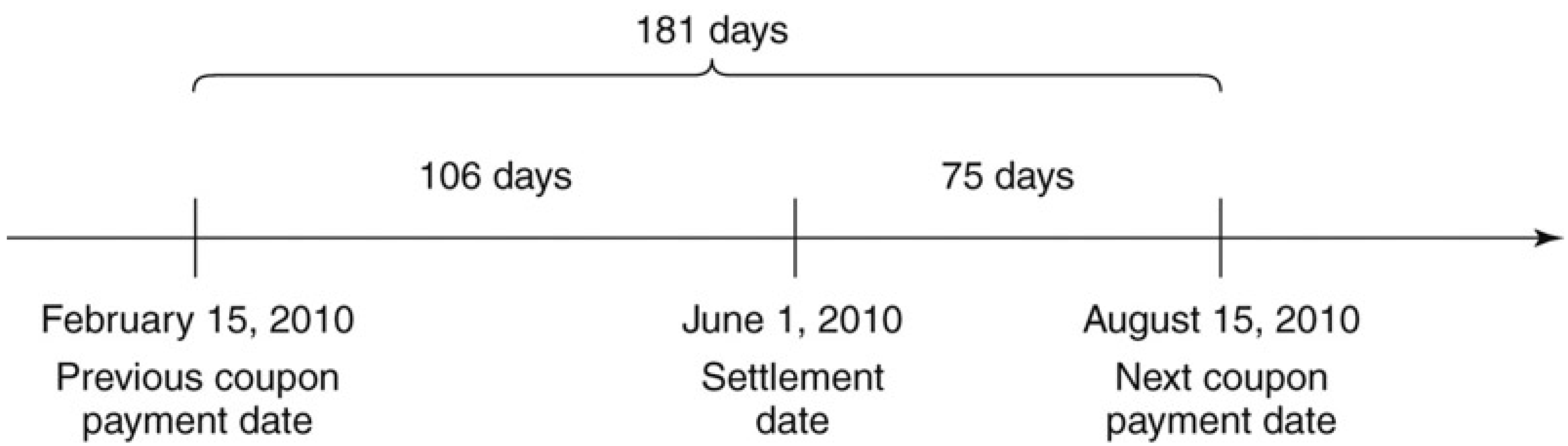
$$108\text{-}31+ = 108 + \frac{31.5}{32} = 108.9843$$

$$102\text{-}18\tfrac{1}{8} = 102 + \frac{18.125}{32} = 102.5664$$

ACCRUED INTEREST

- Consider an investor who purchases \$10,000 face amount of the U.S. Treasury 3⅝s of August 15, 2019 on May 29, 2010 (for T+1 settlement).
- The quote price of the 3⅝s is 102-26, meaning $100 + \frac{26}{32} = 102.8125.$
- The transaction price = quote price+accured Interest (AI).

ACCRUED INTEREST, cont'd



- The seller of the bond deserves the coupon value accrued from 2/15 to 6/1.

ACCRUED INTEREST

- For \$100 notional, the coupon value is

$$\frac{3.625}{2} = 1.8125$$

- The accrued value from 15/2 to 6/1 is

$$\frac{106}{181} \times 1.8125 = 1.0615$$

ACCRUED INTEREST, cont'd

- So the *Transaction Price* of the bond per 100 face amount is

$$102.8125+1.0615=103.8740.$$

- For \$10,000 face amount, the invoice price is

$$100*\$103.8740=\$10,387.40.$$

ACCRUED INTEREST, cont'd

- Denoting
  - the flat price by *p*,
  - accrued interest by *AI*,
  - the present value of the cash flows by *PV*,
  - and the full price by *P*,

Then,

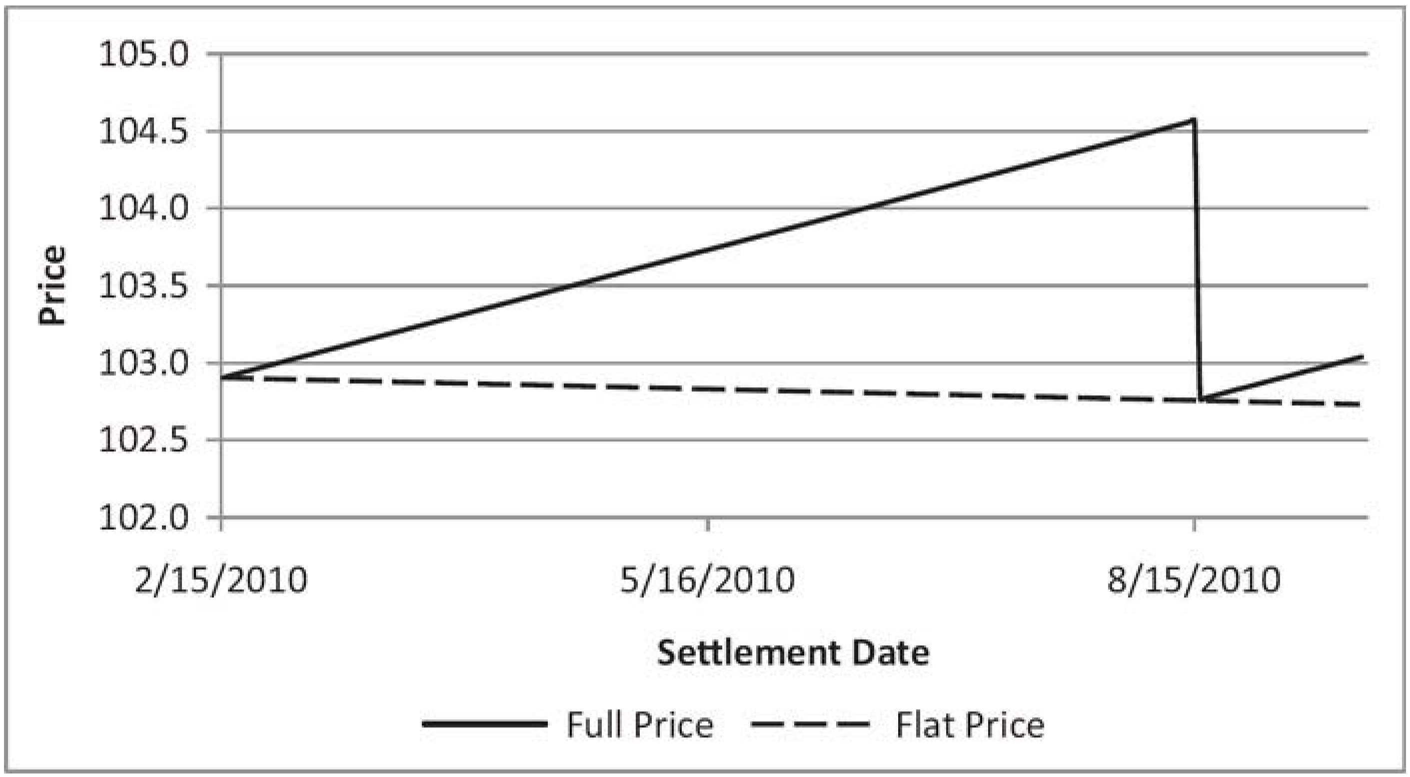
$$P = p + AI = PV$$

Market Jargons

- The flat price is also called
  - Clean price
  - Quote price
- Full price is also called
  - Invoice price
  - Transaction price
  - Dirty price

Advantage to Quote using Flat Price

- The full price jump across a coupon date



Day-Count Conventions

- There are a number of conventions for day count.
  - *Actual/actual* (government bonds)
  - Actual/360: the actual number of days between two dates divided by 360 (money markets, floating-leg of swaps)
  - 30/360: assume 30 days in any month (corporate bonds and fixed-leg of swaps)
    - Ex: the no. of days b/w June 1 and August 15 is 74 (29 days left in June, 30 days in July, and 15 days in August)

DISCOUNT FACTORS

- The *discount factor* for a particular term gives the value today, or the *present value,* of one unit of currency to be received at the end of that term.
- Denote the discount factor for t years by $d(t)$.
- Let $d(.5)$ =0.99925, then the PV of \$1 to be received in six months is 99.925 cents.
- The present value of the cash flow \$1,050,000 in six month is

$$0.99925 \times \$1{,}050{,}000 = 1{,}049{,}213.$$

Bootstrapping the Discount Factors

- A coupon bond can be considered as a portfolio of zero-coupon bonds.
- Given the prices of coupon bonds of consecutive maturities with six months gap, one can bootstrap the prices of zero-coupon bonds

Mid-Prices of Selected Bonds

TABLE 1.2 Selected U.S. Treasury Bond Prices as of May 28, 2010

Coupon	Maturity	Price
$1\frac{1}{4}\%$	11/30/2010	100.550
$4\frac{7}{8}\%$	5/31/2011	104.513
$4\frac{1}{2}\%$	11/30/2011	105.856
$4\frac{3}{4}\%$	5/31/2012	107.966
$3\frac{3}{8}\%$	11/30/2012	105.869
$3\frac{1}{2}\%$	5/31/2013	106.760
2%	11/30/2013	101.552
$2\frac{1}{4}\%$	5/31/2014	101.936
$2\frac{1}{8}\%$	11/30/2014	100.834

Bootstrapping the Discount Factors

- Each row of Table 1.2 implies an equation for discount factors:

$$100.550 = \left(100 + \frac{1\frac{1}{4}}{2}\right) d(.5) \quad \rightarrow d(.5) = .99925$$

$$104.513 = \frac{4\frac{7}{8}}{2} \times d(.5) + \left(100 + \frac{4\frac{7}{8}}{2}\right) d(1) \quad \rightarrow d(1) = .99648$$

$$105.856 = \frac{4\frac{1}{2}}{2} \times d(.5) + \frac{4\frac{1}{2}}{2} \times d(1) + \left(100 + \frac{4\frac{1}{2}}{2}\right) d(1.5)$$

$$\rightarrow d(1.5) = .99135$$

Discount Factors

Coupon	Maturity	Price	Discount Factors
1.250%	"11/30/2010"	100.55	0.999254658
4.8750%	"5/31/2011"	104.513	0.996483873
4.500%	"11/30/2011"	105.856	0.991350497
4.75%	"5/31/2012"	107.966	0.985315395
3.3750%	"11/30/2012"	105.869	0.975199189
3.500%	"5/31/2013"	106.76	0.96414441
2%	"11/30/2013"	101.552	0.946933188
2.25%	"5/31/2014"	101.936	0.931718009
2.1250%	"11/30/2014"	100.834	0.915836248

Table 1.2

THE LAW OF ONE PRICE

- Consider another U.S. bond, the ¾s of November 30, 2011, which is NOT included in Table 1.2. How should it be priced?
- Using the discount factors just obtained, we can present value the cash flow:

$$.375 \times .99925 + .375 \times .99648 + 100.375 \times .99135 = 100.255$$

- But, the market price is 100.19!
- An opportunity!

Market Prices vs. Fair Prices

Table 1.4: Market Prices for Three U.S. Treasury Notes as of May 28, 2010

Bond	**$\frac{7}{8}$s 5/31/11**	**$\frac{3}{4}$s 11/30/11**	**$\frac{3}{4}$s 5/31/12**
PV	100.521	100.255	100.022
Price	100.549	100.19	99.963
PV−Price	−.028	0.065	0.059

rich

cheap

ARBITRAGE TRADE

TABLE 1.5 The Replicating Portfolio of the $\frac{3}{4}$s of November 30, 2011, with Prices as of May 28, 2010

	(1)	(2)	(3)	(4)	(5)	(6)
(i)	Coupon	$1\frac{1}{4}$s	$4\frac{7}{8}$s	$4\frac{1}{2}$s		$\frac{3}{4}$s
(ii)	Maturity	11/30/10	5/31/11	11/30/11	Portfolio	11/30/11
(iii)	Face Amount	−1.779	−1.790	98.166		100
	Date			**Cash Flows**		
(iv)	11/30/10	−1.790	−.044	2.209	.375	.375
(v)	5/31/11		−1.834	2.209	.375	.375
(vi)	11/30/11			100.375	100.375	100.375
(vii)	Price	100.550	104.513	105.856		100.190
(viii)	Cost	−1.789	−1.871	103.915	100.255	100.190
(ix)	Net Proceeds	.065				

Construction Procedure

- Let the face values of 1¼s, 4⅞s and 4½s be F_1, F_2 and F_3, respectively. Then,

$$0\times F^1+0\times F^2+\left(1+\frac{4\frac{1}{2}\%}{2}\right)\times F^3=100+\frac{3/4}{2}$$

$$0\times F^1+\left(1+\frac{4\frac{7}{8}\%}{2}\right)\times F^2+\left(\frac{4\frac{1}{2}\%}{2}\right)\times F^3=\frac{3/4}{2}$$

$$\left(1+\frac{1\frac{1}{4}\%}{2}\right)\times F^1+\left(\frac{4\frac{7}{8}\%}{2}\right)\times F^2+\left(\frac{4\frac{1}{2}\%}{2}\right)\times F^3=\frac{3/4}{2}$$

Working with Spreadsheet

- The solutions are

$$F_3 = 98.166, F_2 = -1.790, F_1 = -1.779$$

- Go check out the spread sheet Table 1.5 .

Profit and Loss

- If you
  - Long the ¾s at the market price of 100.19 for the face value of 100
  - Short the portfolio of the three bonds at their market prices with face value F_1, F_2 and F_3 , at the cost of 100.255
- You make a profit of $0.065, or ¢6.5
- Congratulations, but ….

P&L

- However, if you trade \$500m face value, then

$$P\&L = \$500{,}000{,}000 \times \frac{0.065}{100} = \$325{,}000$$

That is likely to be exciting.

Law of One Price

- This example introduces the *law of one price*:
  - Absent complicating factors (e.g., liquidity, financing, taxes, credit risk), identical sets of cash flows should sell for the same price.
- Violation of the law means an arbitrage opportunity.
- In an efficient market, arbitrage opportunities disappear quickly.