Solutions to the Midterm Exam of MATH4511, Fall 2020
October 28, 1200-1330

Problems (The numbers in brackets are credits, totalled to 50):

1. (a) (2) The government bond is less risky, because it has fixed cash flows.
   (b) (2) The bond market is bigger.
   (c) (2) The Treasury security market is based on bonds issued by the Treasury Department of the United States, while the LIBOR market is based on bonds issued by big international banks. The former has no credit risk.
   (d) (2) It means those derivatives that can be priced without a model.

2. (a) (6) Using central differencing formula we obtain

Rate Level	Bond Price	Duration	Convexity	Option	Duration	Convexity
3.95%	100.4098			10.1536		
4.00%	100.0000	8.1758	78.8981	9.9251	45.68215937	1450.8670
4.05%	99.5922			9.7002		

   (b) (4) Let P_b and P_o be the bond and option prices, respectively, then we long bond with dollar value
$$P_b = \frac{D_o}{D_b} P_o = \frac{45.6821}{8.1758} \times \$10m = \$55.8751m.$$
   (c) (2)
$$C_{port} = \frac{P_b}{P_b + P_2} C_b + \frac{P_o}{P_b + P_o} C_o = -220.1674.$$

3. Let $B_0 = \$7m, y = 3.25\%$ and $T = 30$.
   (a) (4)
$$X = \frac{B_0 \times y/12}{1 - (1 + y/12)^{-12T}} = \$30,464.4423$$
   (b) (4) After the 60^{th} payment, the outstanding value of the loan is
$$B_{60} = X \frac{12}{y} \left(1 - \frac{1}{(1 + y/12)^{300}} \right) = \$6,251,473.017.$$

4. (a) (6) With the notional payment at the end, the floating leg is just a rolling forward CD with notional equal to par. So, without the notional payment at the end, the value is the par subtracted by the PV of the notional at the end.
   (b) (2) Direct present valuing yields $s(0,T) \sum_{i=1}^{2T} \frac{1}{2} d(0, i/2)$.
   (c) (2) At initiation, the value of the swap is zero, meaning the value of the floating leg offsets the value of the fixed leg, thus yielding
$$s(0,T) = \frac{1 - d(0,T)}{\sum_{i=1}^{2T} \frac{1}{2} d(0, i/2)}.$$
   (d) (2) Rewrite the last equation, we have
$$s(0,T) \sum_{i=1}^{2T} \frac{1}{2} d(0, i/2) + d(0,T) = 1,$$
   meaning the bond with coupon rate $s(0,T)$ is a par bond.

5. Let $D_{TIPS} = 9.5, D_T = 7.5, \sigma_{TIPS} = 5, \sigma_T = 4.8$ and $\rho = 0.825$.

   (a) (4) The coupon rate remains fixed, yet the principal is adjusted by a factor of (1 + inflation rate over the last six months). Also, the principal cannot be lower than the par.

   (b) (6) It is calculated according to

$$P_T = P_{TIPS} \times \frac{D_{TIPS}}{D_T}\beta = P_{TIPS} \times \frac{D_{TIPS}}{D_T}\frac{\sigma_{TIPS}}{\sigma_T}\rho = \$10.8854m.$$

================= Good Luck! =================