Solutions to Midterm of Math 4511
Quantitative Methods for Fixed-Income Securities
October 27, 2011

Solutions:

1 Let F be the par value of a bond

1.1 (2) $P(y) = F\left(\sum_{j=1}^{2T} \frac{c/2}{(1+y/2)^j} + \frac{1}{(1+y/2)^{2T}} \right)$

1.2 (2) $P(y) = F\left(\sum_{j=1}^{2T} \frac{c/2}{[1+\hat{r}(j/2)/2]^j} + \frac{1}{[1+r(T)/2]^{2T}} \right)$

1.3 (2) $P(y) = F\left(\sum_{j=1}^{2T} \frac{c/2}{\prod_{k=1}^{j}(1+r(k/2)/2)} + \frac{1}{\prod_{k=1}^{2T}(1+r(k/2)/2)} \right)$

1.4 (2) $r(T) = 2\left[\frac{\left(1+\hat{r}(T-1/2)/2\right)^{-2T+1}}{\left(1+\hat{r}(T)/2\right)^{-2T}} - 1 \right]$

1.5 (2) $\hat{r}(T) = 2\left\{ \left(\prod_{j=1}^{2T} \left(1+r(j/2)/2\right) \right)^{1/2T} - 1 \right\}.$

1.6 (2) $y(T) = \frac{1-d(T)}{\sum_{t=1}^{2T} \frac{1}{2} d(\frac{t}{2})}.$

1.7 (2) $d(T) = \frac{1-\frac{y(T)}{2}\sum_{t=1}^{2T-1} d(\frac{t}{2})}{1+\frac{y(T)}{2}} = \frac{1-\frac{y(T)}{y(T-\frac{1}{2})}\left[1-d(T-\frac{1}{2})\right]}{1+\frac{y(T)}{2}}.$

1.8 (2) Substitute $d(t) = \prod_{i=1}^{2t} \left(1+ {r(i)}/{2}\right)^{-1}$ into the last equation.

2 (4) From problem 1.4, $\hat{r}(T) = r(T) = const$, and thus the spot-rate curve is flat.

(4) Let $\hat{r}(T) = r = const$ and $z = 1/(1+r/2)$. Then, from problem 1.6, we have

$$y(T) = \frac{1-z^{2T}}{\sum_{t=1}^{2T} z^T/2} = \frac{1-z^{2T}}{\frac{z}{2}\frac{1-z^{2T}}{1-z}} = \frac{1-z}{z/2} = \frac{2\left(1-\frac{1}{1+r/2}\right)}{\frac{1}{1+r/2}} = r = const.$$

3 (4) Solve y by *trial and error* from

$$P = F\left(\sum_{j=1}^{2T} \frac{c/2}{(1+y/2)^{j-\tau}} + \frac{1}{(1+y/2)^{2T-\tau}} \right)$$

or

$$P = F(1+y/2)^{\tau}\left(\frac{c}{y}\left[1-\frac{1}{(1+y/2)^{2T}}\right]+\frac{1}{(1+y/2)^{2T}}\right)$$

with

$$F = 100$$
$$c = 0.055$$
$$T = 3$$
$$\tau = (30+12)/182 = 0.2308$$
$$AI = \frac{c}{2}F\tau = 0.635$$
$$P = 108 + 8/32 + 0.635 = 108.885.$$

(2) By numerical iteration, we obtain y=2.51%.

4. (10) The market value of the 10-yr and 30-yr bonds for hedging are

$$P_{10} = -P_{15}\frac{D_{15}}{D_{10}}\beta_{10} = -P_{15}\frac{D_{15}}{D_{10}}\frac{\sigma_{15}}{\sigma_{10}}\rho_{10,15} = -\$8.1263m;$$
$$P_{30} = -P_{15}\frac{D_{15}}{D_{30}}\beta_{30} = -P_{15}\frac{D_{15}}{D_{30}}\frac{\sigma_{15}}{\sigma_{30}}\rho_{15,30} = -\$3.4439m.$$

========================== END ==========================