**Solutions to Final for Math 361**

Quantitative Methods for Fixed-Income Securities

December 11, 2009

1. Note that there is no coupon payment occurring during the life of the forward contract. Hence the forward price observes:

$$F + AI(d) = [P(0) + AI(0)]\left(1 + \frac{rd}{360}\right)$$

It's ready to see $\mathrm{d} = 62$, $\mathrm{P}(0) = 105$, $\mathrm{r} = 1\%$, $\mathrm{AI}(0) = \frac{87\times 3}{181} = 1.441989$, $\mathrm{AI}(\mathrm{d}) = \frac{149\times 3}{181} = 2.469613$. Consequently, the forward price is 104.155692.

2. (1) On Dec. 12, 2009, the desk received the full price from the client:

$$\frac{\$1{,}000{,}000}{100} \times \left(105 + \frac{17}{32} + \frac{62}{182} \times 3\right) = \$1{,}065{,}532.2802.$$

The desk thereafter lent the amount to the third party and delivered the collateral to the client. On Dec. 13, 2009, the desk received the returning of the loan with interest overall equal to:

$$\$1{,}065{,}532.2802 \times \left(1 + \frac{4.5\%}{360}\right) = \$1{,}065{,}665.4718.$$

At the same time, the desk paid the full price of the bond and returned the purchased bond to the third party:

$$-\frac{\$1{,}000{,}000}{100} \times \left(105 + \frac{16}{32} + \frac{63}{182} \times 3\right) = -\$1{,}065{,}384.6154.$$

Therefore, the P&L of the desk is

$$\$1{,}065{,}665.4718 - \$1{,}065{,}384.6154 = \$280.8564.$$

(2) Carry is equal to the difference of interest earning and funding cost. Here, net coupon payment is $\frac{\$1{,}000{,}000}{100}\frac{1}{182} \times 3$ and repo earning is $\$1{,}065{,}532.28 \times \left(\frac{4.5\%}{360}\right)$.

Therefore, the carry is:

$$\$1{,}065{,}532.2802 \times \left(\frac{4.5\%}{360}\right) - \frac{\$1{,}000{,}000}{100}\frac{1}{182} \times 3 = -\$31.6436.$$

The cost of carry is \$31.6436.

(3) The breakeven price is

$$105 + \frac{17}{32} + (-\$31.6436) \times \frac{100}{\$1{,}000{,}000} = 105.5281.$$

3. (4) The formula for the par swap rates are

$$y(T)=\frac{1-d(T)}{\sum_{i=1}^{T} d(i)}$$

Where, in terms of forward rates,

$$d(1)=(1+r(1))^{-1},\ d(i)=d(i-1)(1+r(i))^{-1}$$

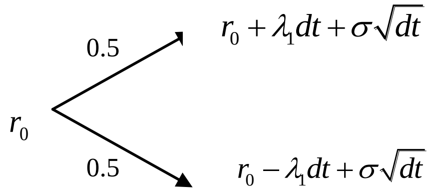
(3) It follows that the par swap rates are

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| 0.0130 | 0.0145 | 0.0160 | 0.0174 | 0.0189 | 0.0203 | 0.0217 | 0.0231 | 0.0245 | 0.0258 |

(3) The par swap rate is the same as a par yield, as it is the yield of a par bond.

4. We answer the questions one by one.

By considering the following interest-rate tree in p.252,



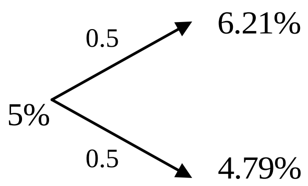

Since, we have

$$\begin{aligned} P(2\Delta t) &= e^{-2\hat{r}(2dt)dt} \\ &= e^{-r_0 dt}\times\frac{1}{2}\left[e^{-(r_0+\lambda_1 dt+\sigma\sqrt{dt})dt}+e^{-(r_0+\lambda_1 dt-\sigma\sqrt{dt})dt}\right] \\ &= e^{-2r_0 dt}\times\frac{1}{2}\left[e^{-\sigma dt\sqrt{dt}}+e^{\sigma dt\sqrt{dt}}\right]\times e^{-\lambda_1 dt^2} \end{aligned}$$

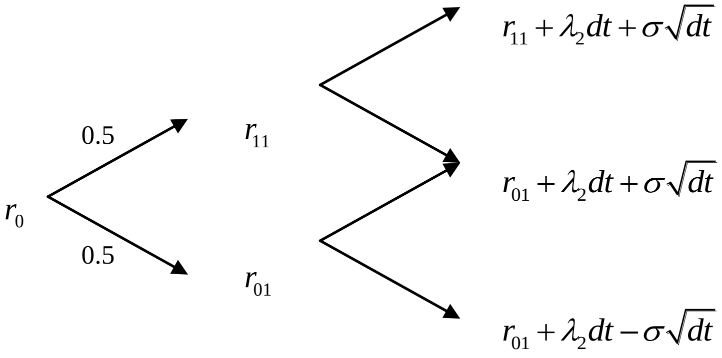
(5) It follows that

$$\lambda_1=\frac{2\left(\hat{r}(2dt)-\hat{r}(dt)\right)}{dt}+\frac{1}{dt^2}\ln\left[\frac{e^{-\sigma dt\sqrt{dt}}+e^{\sigma dt\sqrt{dt}}}{2}\right]=0.01$$

The resulted interest rate tree is




For the next time step, we have



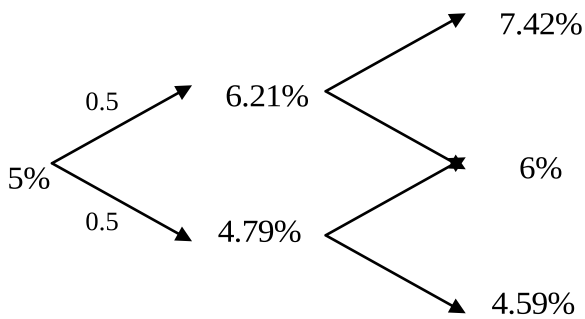

Since,

$$
\begin{aligned}
P(3\Delta t) &= e^{-3\hat{r}(3dt)dt} \\
&= e^{-r_0 dt}\left[\frac{e^{-r_{01}dt}}{2}\frac{\left(e^{-(r_{01}+\lambda_2 dt+\sigma\sqrt{dt})dt}+e^{-(r_{01}+\lambda_2 dt-\sigma\sqrt{dt})dt}\right)}{2}\right. \\
&\quad \left.+\frac{e^{-r_{11}dt}}{2}\frac{\left(e^{-(r_{11}+\lambda_2 dt+\sigma\sqrt{dt})dt}+e^{-(r_{11}+\lambda_2 dt-\sigma\sqrt{dt})dt}\right)}{2}\right] \\
&= e^{-r_0 dt}\frac{\left(e^{-2r_{01}dt}+e^{-2r_{11}dt}\right)}{2}\frac{\left(e^{-\sigma\sqrt{dt}dt}+e^{\sigma\sqrt{dt}dt}\right)}{2}\times e^{-\lambda_1 dt^2}
\end{aligned}
$$

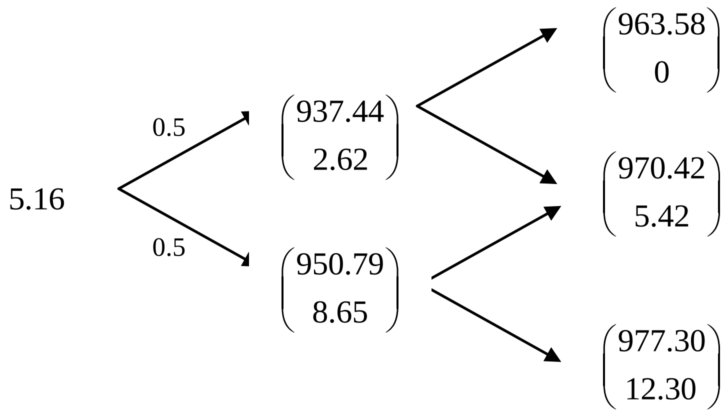
(5) It follows that

$$
\begin{aligned}
\lambda_2 &= \frac{3\hat{r}(3dt)-r(dt)}{dt}+\frac{1}{dt^2}\ln\frac{\left(e^{-2r_{01}dt}+e^{-2r_{11}dt}\right)}{2}+\frac{1}{dt^2}\ln\frac{\left(e^{-\sigma\sqrt{dt}dt}+e^{\sigma\sqrt{dt}dt}\right)}{2} \\
&= 0.0101
\end{aligned}
$$

(5) So the interest-rate tree is




5. (5) The price tree for the option is




(5) To hedge the option, we buy

$$\Delta = \frac{C_u - C_d}{P_u - P_d} = \frac{2.62 - 8.65}{937.44 - 950.79} = 0.4514 \tag{0.1}$$

unit of the $3\Delta t$-maturity zero-coupon bond.

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