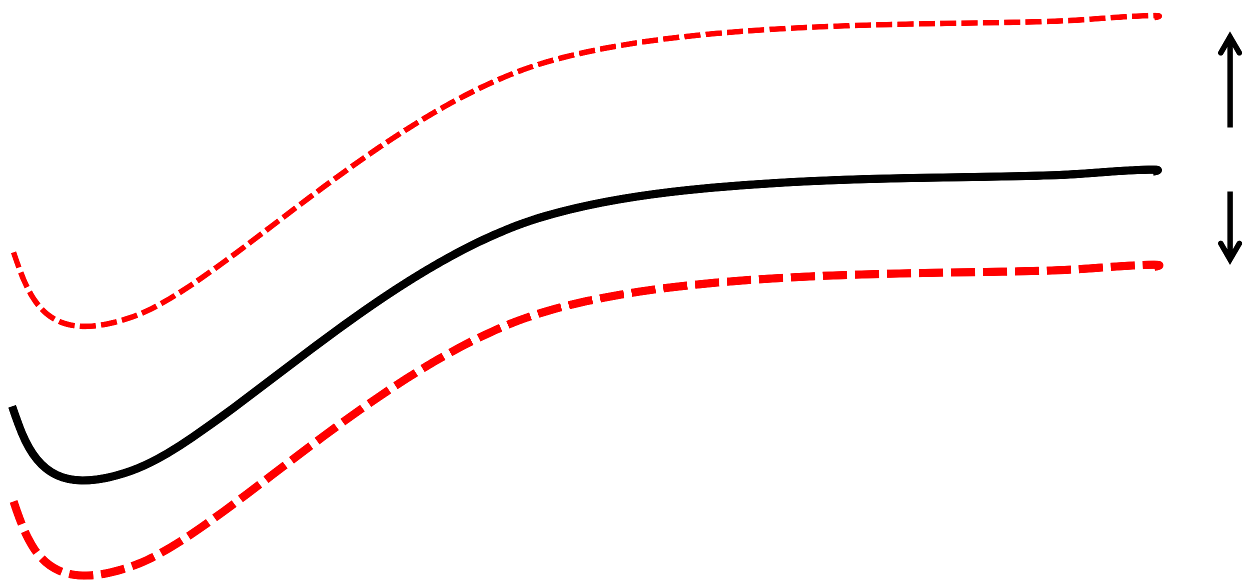
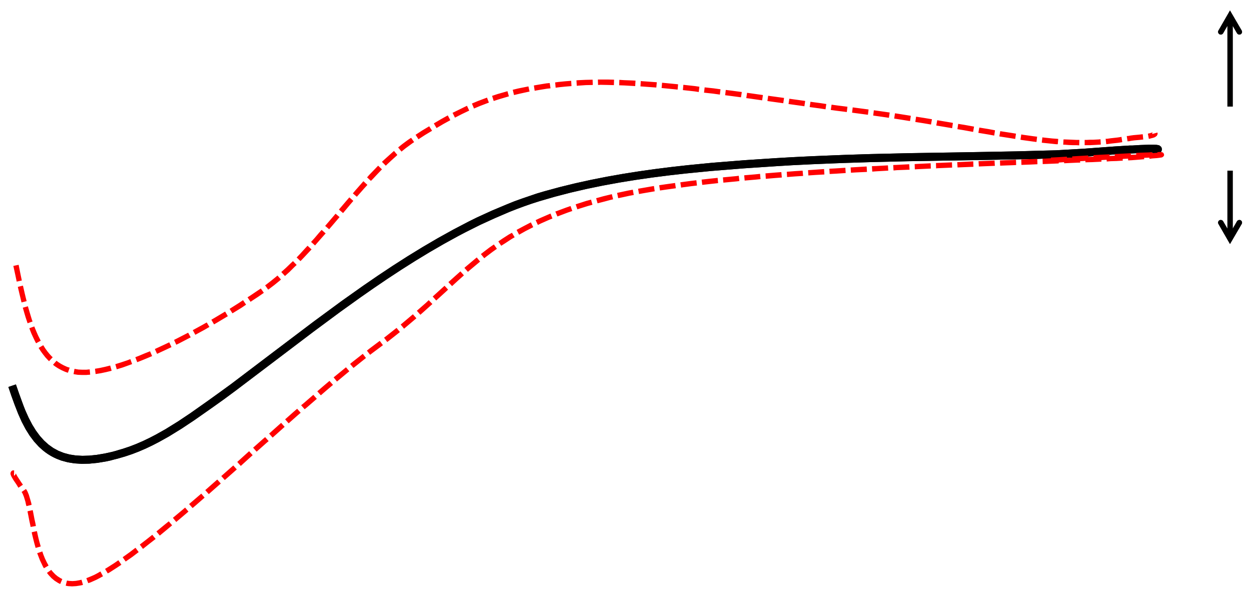
Chapter 7 of Tuckman
Key Rate Hedging

Limitation of the Duration/DV01

- A major weakness of the duration/DV01 based hedging is the assumption that yield curve does parallel shift.



The Reality



Curve Risk

- In reality, it is widely recognized that rates in different regions of the term structure are far from perfectly correlated.
- The risk that rates along the term structure move by different amounts is known as *curve risk*.

Driving force of the yield curve

- This chapter revises the theory based on the fact that some swap rates of particular maturities, so-called key rates, largely determine the shape of the yield curve.
- As a results, an interest-rate portfolio can be hedges by these swaps as well.

Key-Rate Shifts

- One popular choice of key rates for the U.S. Treasury and related markets are the
  - 2-,
  - 5-,
  - 10-, and
  - 30-year swap rates or par yields.

- This set of choices
  - cover risk across the term structure,
  - keep the number of key rates as few as reasonable, and
  - rely only on the most liquid government securities.

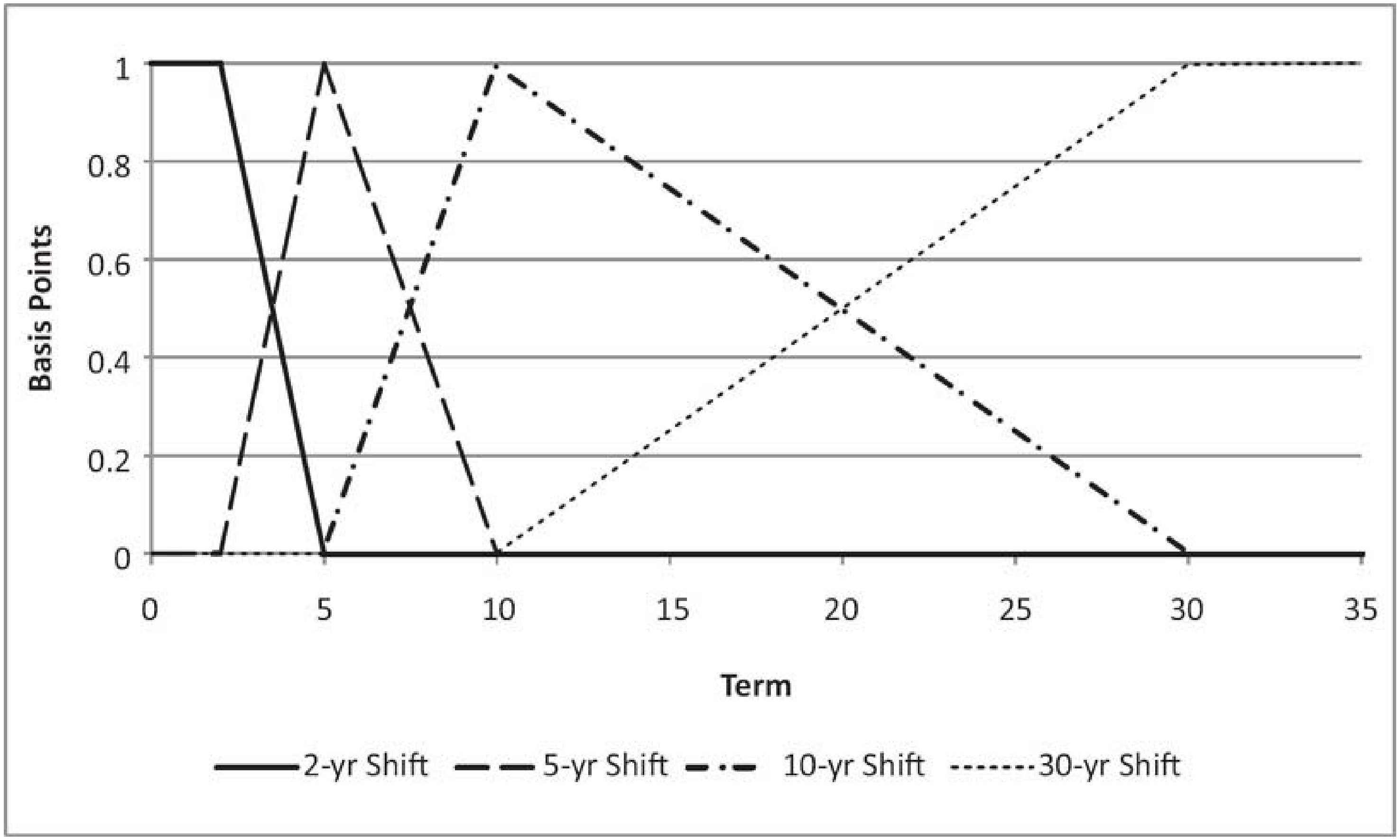


Figure 5.1: A Specification of Key-Rate Shifts

2-year Key-rate Shifts, Δy_2

$$\Delta y_2 = \begin{cases} 1 & T \le 2 \\ \dfrac{5-T}{5-2} & 2 \le T \le 5 \\ 0 & 5 \le T \end{cases}$$

5-yr Key-rate Shifts, Δy_5

$$\Delta y_5 = \begin{cases} 0 & T \le 2 \\ \dfrac{T-2}{5-2} & 2 \le T \le 5 \\ \dfrac{10-T}{10-5} & 5 \le T \le 10 \\ 0 & 10 \le T \end{cases}$$

10-yr Key-rate Shifts, Δy_{10}

$$\Delta y_{10} = \begin{cases} 0 & T \leq 5 \\ \dfrac{T-5}{10-5} & 5 \leq T \leq 10 \\ \dfrac{30-T}{30-10} & 10 \leq T \leq 30 \end{cases}$$

30-yr Key-rate Shifts, Δy_{30}

$$\Delta y_{30} = \begin{cases} 0 & T \le 10 \\ \dfrac{T-10}{30-10} & 10 \le T \le 30 \end{cases}$$

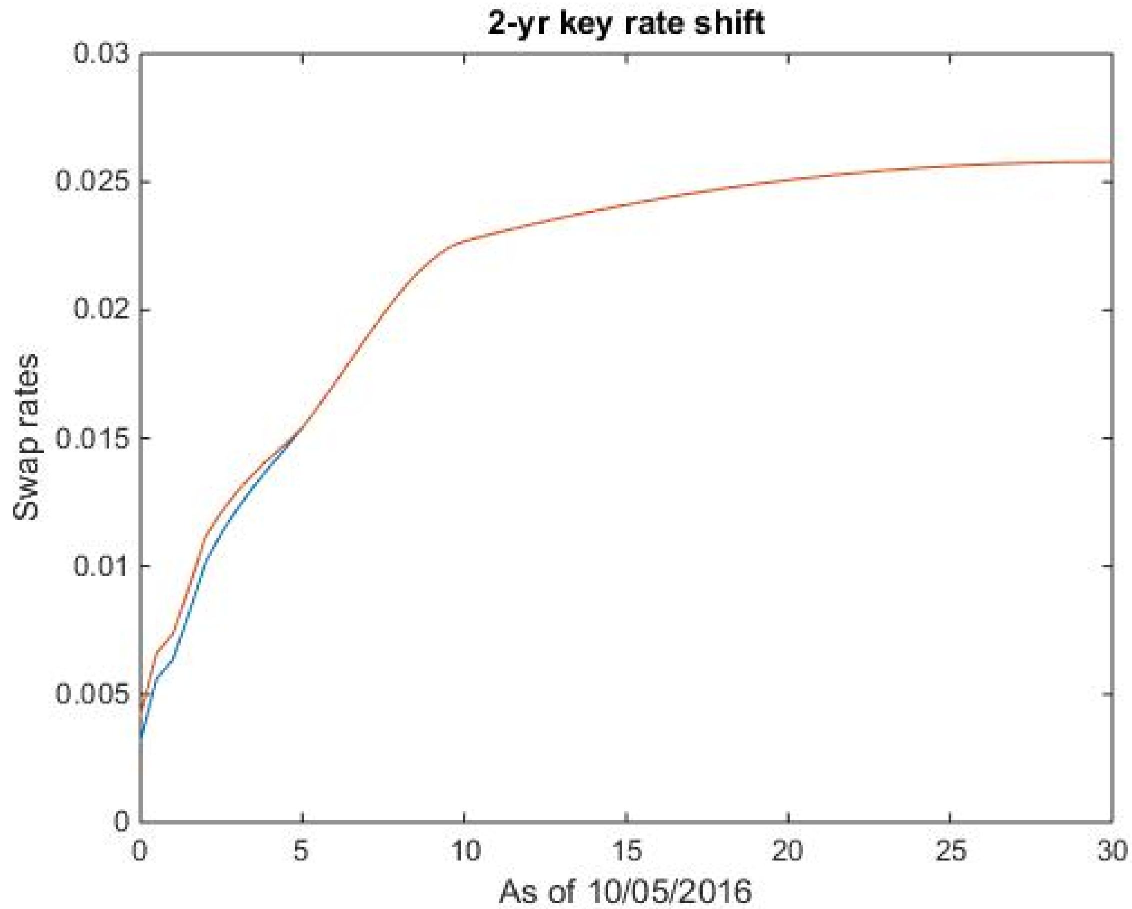
Key-rate shifts add up to parallel shift

- One can verify that

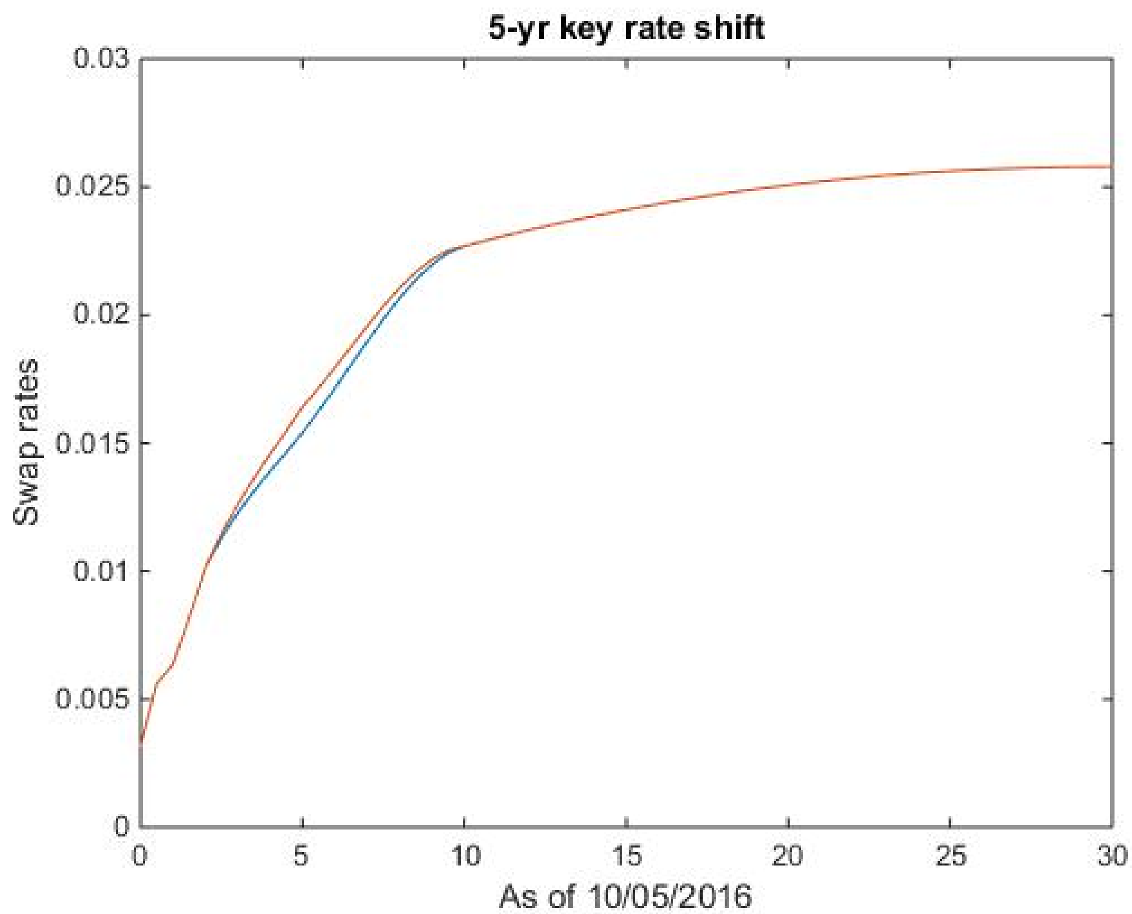
$$\Delta y_2 + \Delta y_5 + \Delta y_{10} + \Delta y_{30} = 1$$

- --- key-rate shift add up to parallel shift of 1bp!

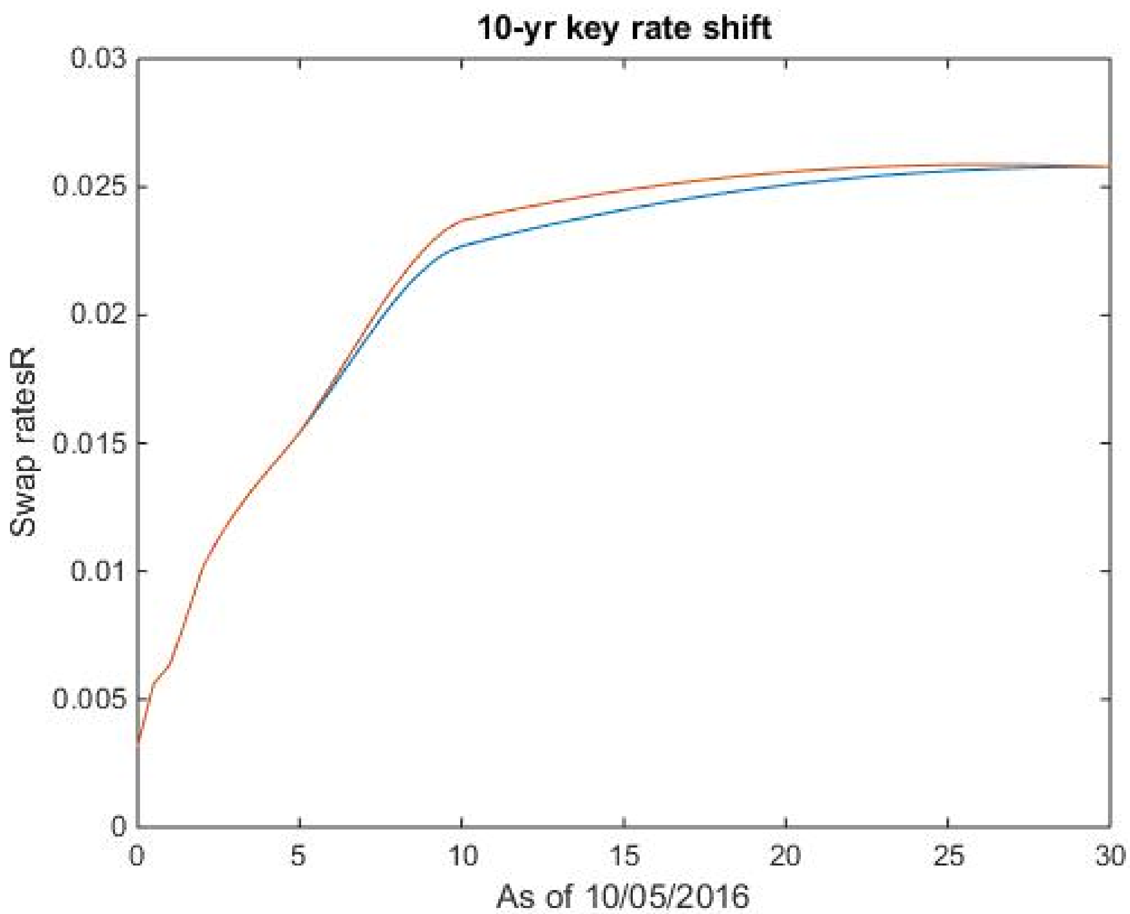
2-yr key rate shift



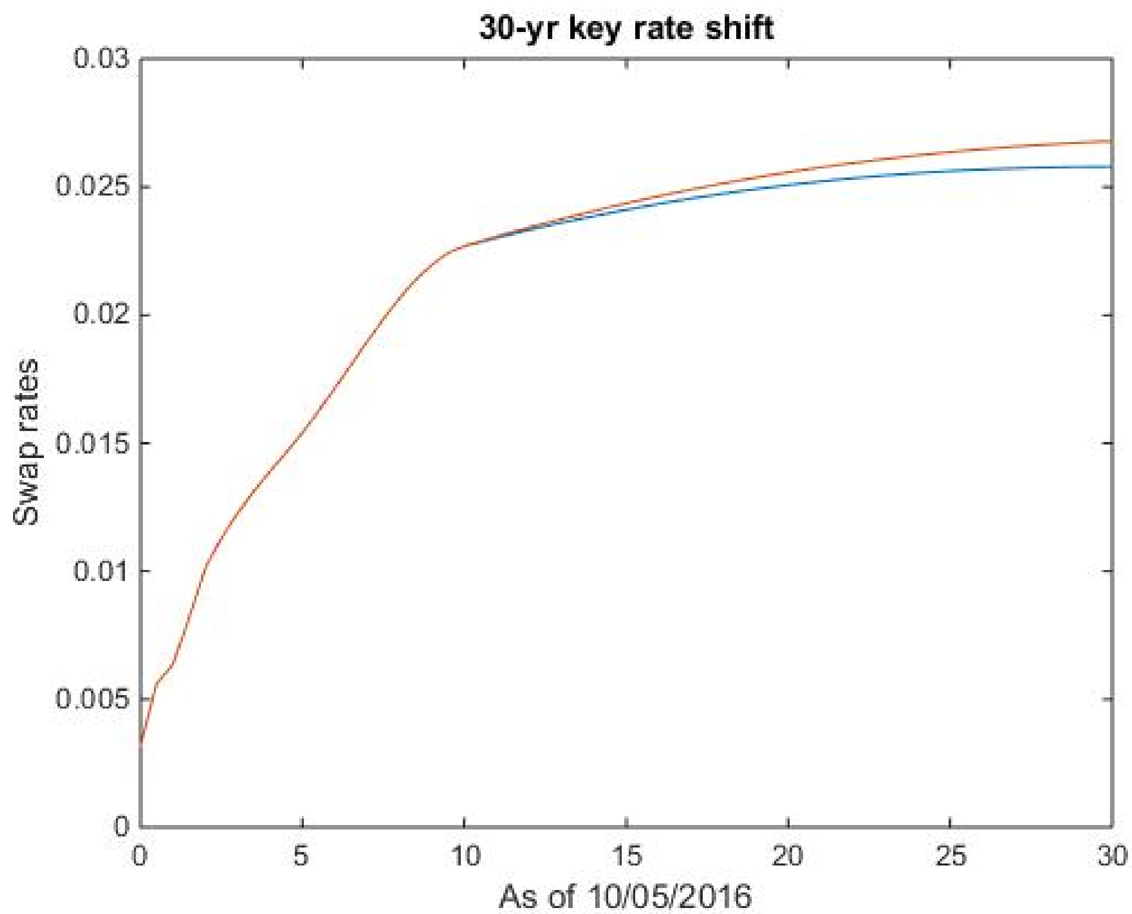
5-yr key rate shift



10-yr key rate shift



30-yr key rate shift



Kr01s add up to DV01

- By construction,
  - the four key-rate shifts sum to a constant shift of one basis point.
  - This allows for the interpretation of key-rate exposures as a decomposition of the total *DV*01 or duration of a security or a portfolio into exposures to four different regions of the term structure.

Key-rate ’01 and Key-rate Duration

- The key-rate one-basis point shifts

$$DV01_k = -P(y + \Delta y_k) + P(y)$$

- The key-rate durations are defined analogously to duration:

$$D_k = \frac{10{,}000}{P} DV01_k$$

EX: Key-rate Hedging of a Mortgage

Monthly payment:	\$3,250			
Par yields flat at:	5%			
	(\$)	Key Rate 01(\$)	Key Rate Duration	Percent of Total
Initial value	100,453.13			
After 2-year shift	100,452.15	0.98	0.10	0.9%
After 5-year shift	100,449.36	3.77	0.38	3.3%
After 10-year shift	100,410.77	42.37	4.22	37.0%
After 30-year shift	100,385.88	67.26	6.70	58.8%
Total:		114.38	11.39	

- Key-rate ’01 of the mortgage

Key-rate ’01 of Bonds for Hedging

Key Rate Exposures of Four key-rate swaps

Par yields flat at:		**5%**			
		Key Rate 01s (100 Face)			
Coupon	**Term**	**2-Year**	**5-Year**	**10-Year**	**30-Year**
5%	2	0.01881	0	0	0
5%	5	0	0.04375	0	0
5%	10	0	0	0.0779	0
5%	30	0	0	0	0.15444
Nonprepayable					
mortage:		0.98129	3.77314	42.36832	67.25637

The Face Amounts

- Let F_2, F_5, F_{10}, and F_{30} be the face amounts of the payer's swaps in the hedging portfolio bought against the nonprepayable mortgage:

$$\frac{0.01811}{100} F_2 = 0.98129 \quad \Rightarrow \quad F_2 = 5216.85$$

$$\frac{0.04375}{100} F_5 = 3.77314 \quad \Rightarrow \quad F_5 = 8624.32$$

$$\frac{0.0779}{100} F_{10} = 42.36823 \quad \Rightarrow \quad F_{10} = 54388.08$$

$$\frac{0.15444}{100} F_{30} = 67.25637 \quad \Rightarrow \quad F_{30} = 5216.85$$