**MATH4511 Quantitative Methods for Fixed Income Derivatives**

**Tutorial 11**

**Stock price**

$$dS(t) = rS(t)dt + \sigma S(t)dW(t)$$

$$S(t) = S(0)e^{(r-\frac{1}{2}\sigma^2)t+\sigma W(t)}$$

$W(t)$: Brownian motion.

Black-Scholes model

B-S formula for Call Option and Put Option

$$C_0 = d(0,T)\left[F_0\Phi(d_1) - K\Phi(d_2)\right] = S_0\Phi(d_1) - d(0,T)K\Phi(d_2)$$

where

$$d_1 = \frac{\ln\frac{F_0}{K} + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}.$$

Hedge ratio with the underlying

$$\alpha = \Phi(d_1)$$

$$P_0 = d(0,T)\left[K\Phi(-d_2) - F_0\Phi(-d_1)\right] = d(0,T)K\Phi(-d_2) - S_0\Phi(-d_1)$$

Hedge ratio with the underlying

$$\alpha = -\Phi(-d_1)$$

**Call-Put Parity**

Long call – Short Put = Forward Contract

$$\left(S_T - K\right)^+ - \left(K - S_T\right)^+ = S_T - K$$

Swaption

$$s(0;T_0,T) = \frac{d(0,T_0) - d(0,T)}{\sum_{i=T_0/\Delta T+1}^{2T} \Delta T \times d(0,T_i)}$$

It is often taken as the strike rate for swaptions.

Payer's swaption: Receive float (long Par product) – Pay fixed (short bond)

Receiver's swaption: Receive fixed (long bond) – Pay float (short Par product)