Chapter 2
Bond Yields

Bond Yields

- Yield-to-maturity (YTM) or *Yield* is the single rate such that discounting a security's cash flows at that rate gives that security's market price.
- YTM is a practical and intuitive way to quote price and is used extensively for quick insight and analysis.
- For bonds of the same maturity, the yields can be used as a measure of relative price cheapness or richness.

Yield-to-Maturity

- E.g, the price of the 4½s of November 30, 2011 (Table 1.2), was 105.856. The yield-to-maturity, *y*, of this bond is therefore defined such that

$$105.856 \equiv \frac{2.25}{\left(1+\frac{y}{2}\right)} + \frac{2.25}{\left(1+\frac{y}{2}\right)^2} + \frac{102.25}{\left(1+\frac{y}{2}\right)^3}$$

- Solving 3.11 for *y* by trial-and-error or some numerical method shows that the yield of the 4½s is about .574% (See Demo 3.1).
- Yield is often used as an alternate way to quote price: a trader could bid to buy the 4½s of November 30, 2011, at a price of 105.856 or at a yield of .574%.

Yield-to-Maturity

- The definition of yield for a coupon bond for settlement on a coupon payment date is

$$P = \frac{\frac{1}{2}c}{\left(1+\frac{y}{2}\right)} + \frac{\frac{1}{2}c}{\left(1+\frac{y}{2}\right)^2} + \cdots + \frac{1+\frac{1}{2}c}{\left(1+\frac{y}{2}\right)^{2T}}$$

$$P = \frac{c}{2}\sum_{t=1}^{2T} \frac{1}{\left(1+\frac{y}{2}\right)^t} + \frac{1}{\left(1+\frac{y}{2}\right)^{2T}}$$

- The concise formula

$$P = \frac{c}{y}\left(1 - \frac{1}{\left(1+\frac{y}{2}\right)^{2T}}\right) + \frac{1}{\left(1+\frac{y}{2}\right)^{2T}}$$

Price-yield Relationship

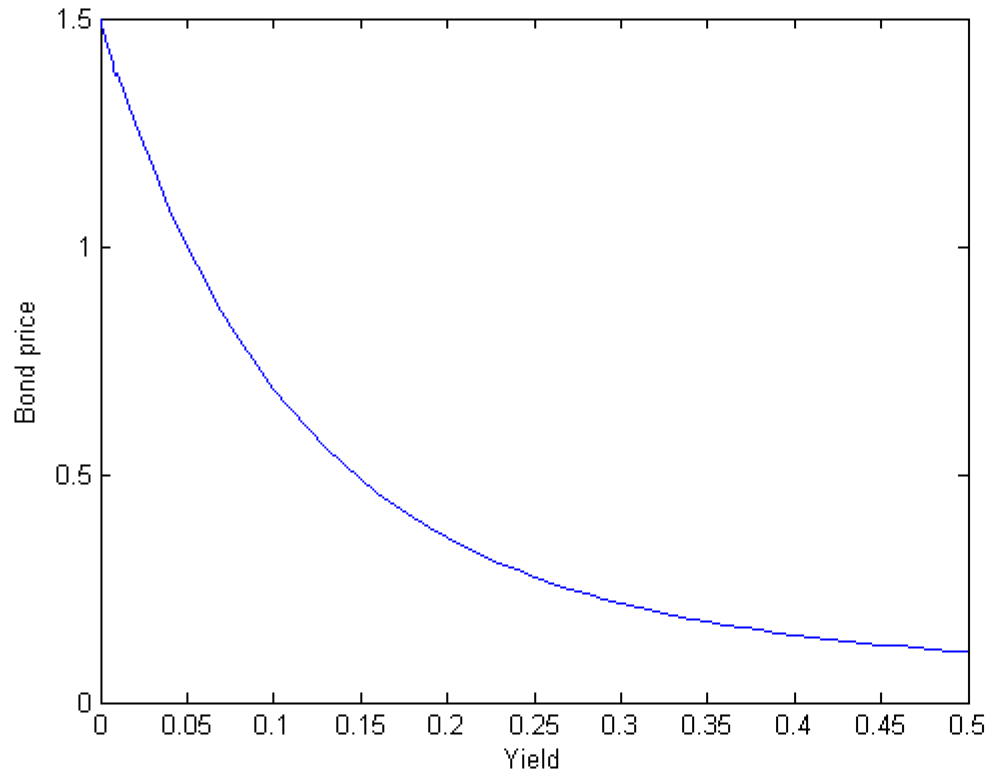


Figure: price of a 10y 5s bond vs. yield

Yield-to-Maturity

- The concise formula is invalid for $y = 0$, but the limit for $y \to 0$ exists since

$$\lim_{y\to 0}\frac{1}{y}\left(1-\frac{1}{\left(1+\frac{y}{2}\right)^{2T}}\right)=T$$

- implying

$$\lim_{y\to 0}P(y)=cT+1$$

Coupon-Yield Relationship

- Par bond: when $c = y$, $P(T) = 1$.
- Premium bond: when $c > y$, $P(T) > 1$.
- Bond sold at discount: when $c < y$, $P(T) < 1$.

Pull-to-Par

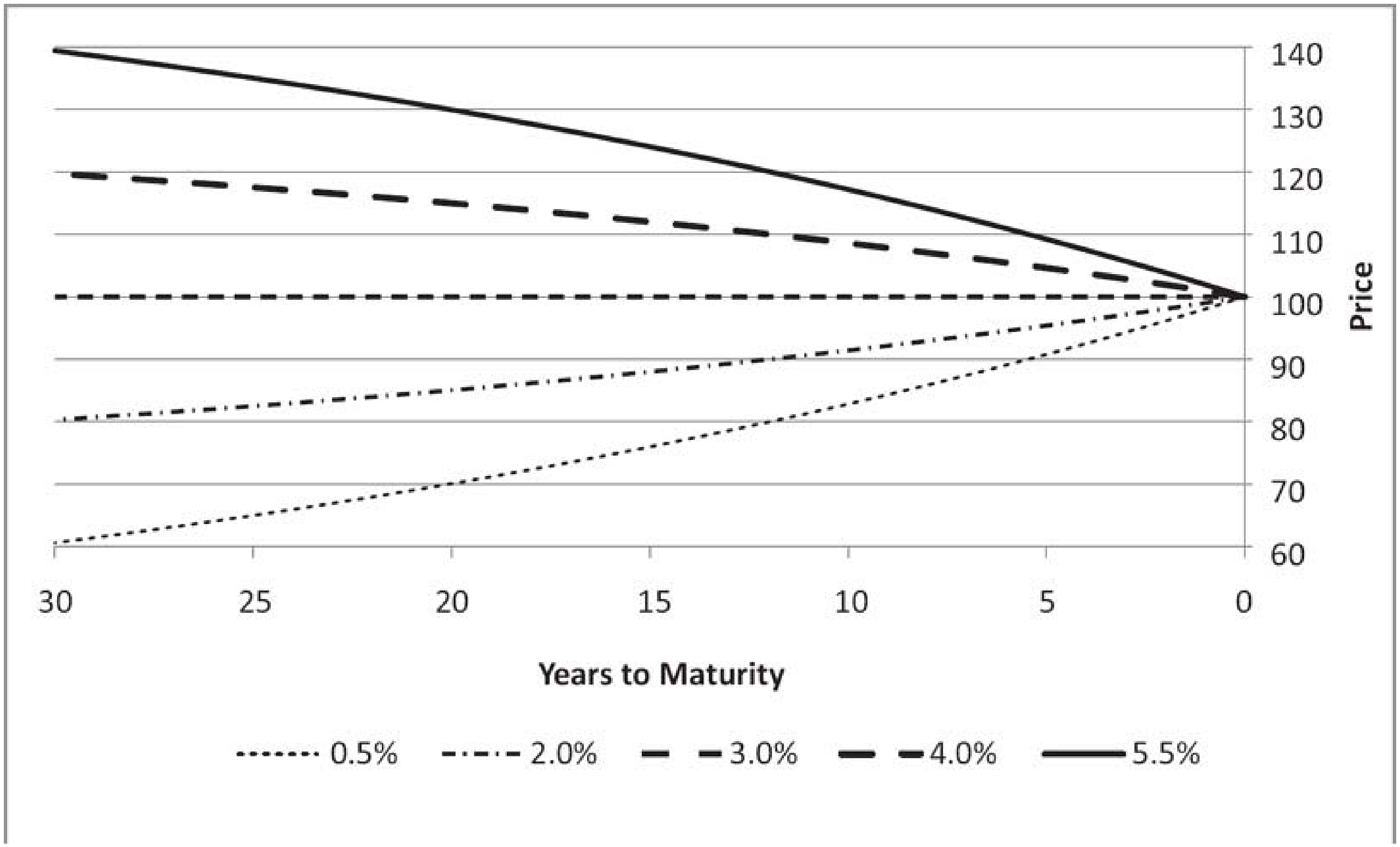
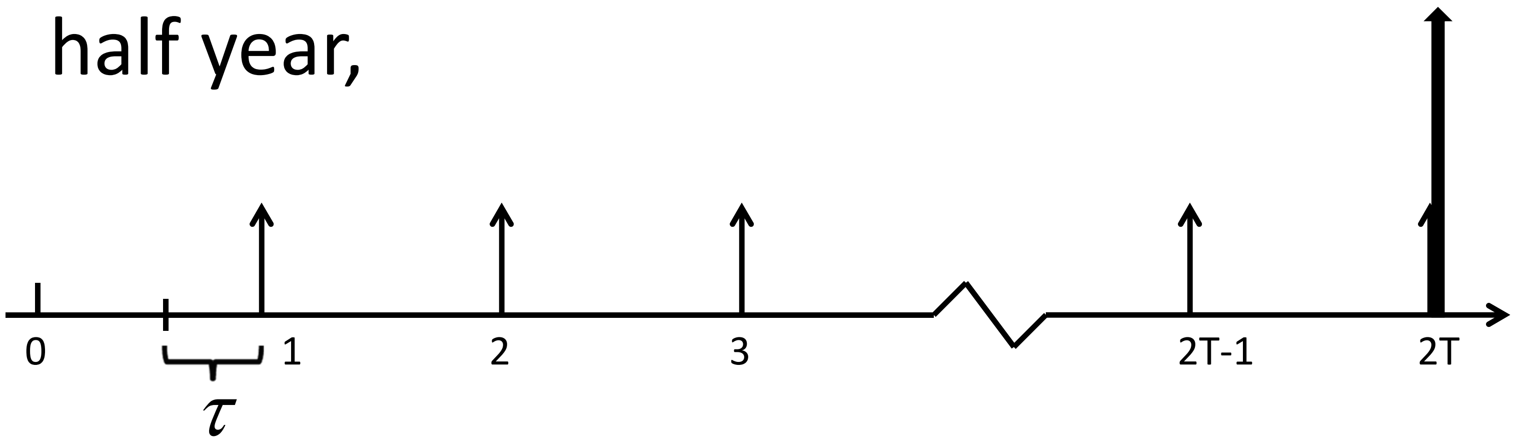


Figure 3.1: Prices of Bonds with Varying Coupons Over Time with Yields Fixed at 3%

General Bond Price Formula

- When the first coupon is due in τ fraction of a half year,



- The bond price formula becomes

$$P = \left(1+\frac{y}{2}\right)^{1-\tau}\left[\frac{c}{y}\left(1-\frac{1}{\left(1+\frac{y}{2}\right)^{2T}}\right)+\frac{1}{\left(1+\frac{y}{2}\right)^{2T}}\right]$$

Formula for Annuities

- An annuity makes annual payments of *c* until date *T* with no final principal payment.
- The Annuity Formula

$$A(T) = \frac{c}{y}\left(1 - \frac{1}{\left(1+\frac{y}{2}\right)^{2T}}\right)$$

- The Perpetual Formula (for $T = \infty$)

$$Perp(T) = \frac{c}{y}$$

Zero-coupon Yields or SPOT Rates

- A spot rate is the rate on a *spot loan*, an agreement between a lender and a borrower at the time of the agreement, to be repaid at a later time.
- Denote the semiannually compounded *t*-year spot rate by $\widehat{r}(t)$. Then, investing 1 unit of currency from now to year *t* will generate

$$\left(1 + \frac{\widehat{r}(t)}{2}\right)^{2t}$$

SPOT Rates, cont'd

- So the PV of $\left(1+\frac{\widehat{r}(t)}{2}\right)^{2t}$ is 1, meaning

$$\left(1+\frac{\widehat{r}(t)}{2}\right)^{2t} d(t) = 1$$

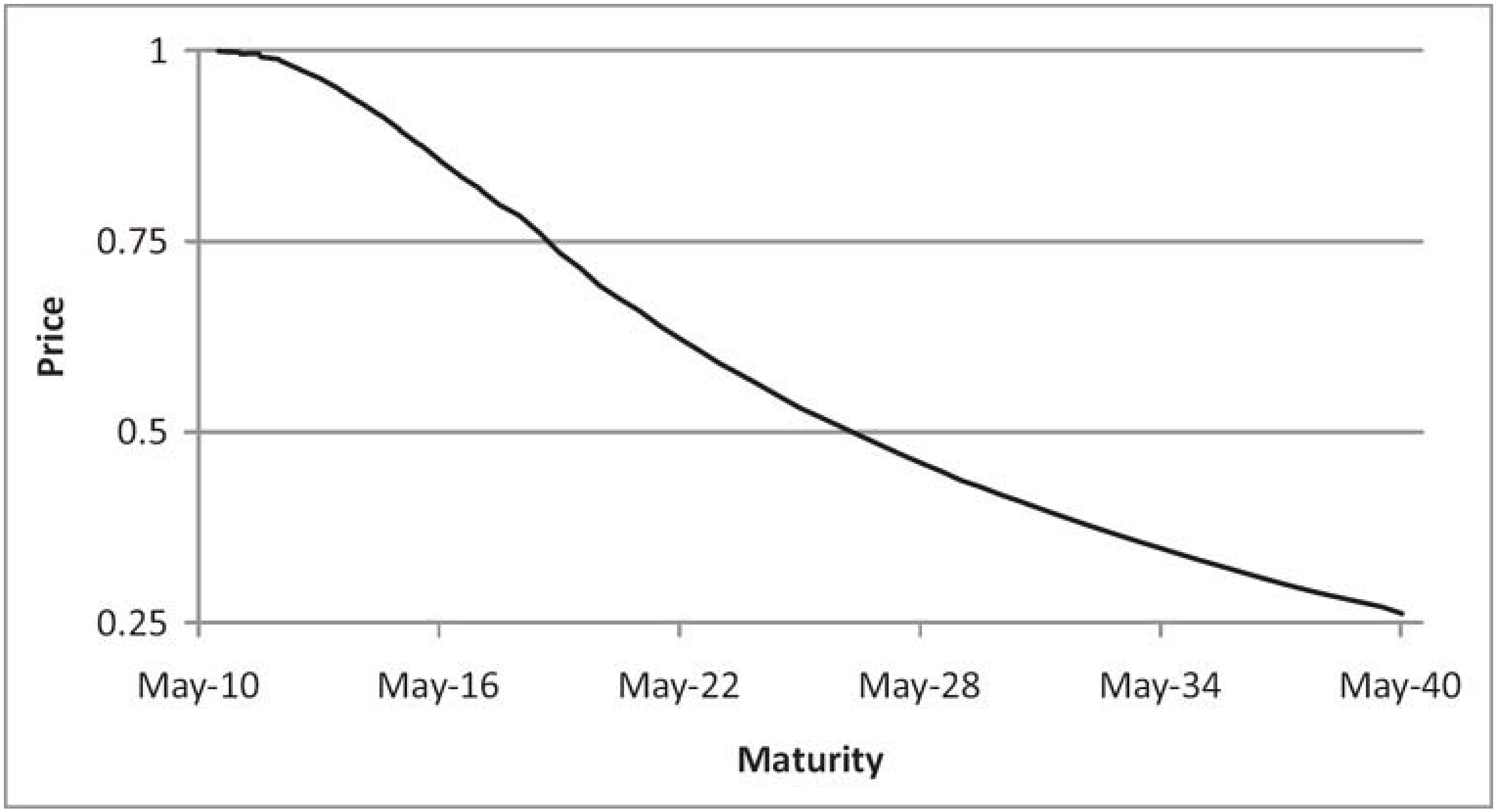
or

$$d(t) = \frac{1}{\left(1+\frac{\widehat{r}(t)}{2}\right)^{2t}}$$

- So the spot rates are directly linked to the discount factors.

Discount Curve

- We have got a discount curve.



Spot Rates and Discount Factors

- Given the discount factors, we can derive corresponding spot rates from

$$\hat{r}(t) = 2\left(\frac{1}{d(t)^{1/2t}} - 1\right)$$

which can be realized in spreadsheets.

Discount Yields for Bills

Table 3.1. Quotes for U.S. Treasuries as of 3/7/2008

U.S. Treasuries

Bills

	Maturity date	Discount/Yield	Discount/Yield change
3-Month	06/05/2008	1.42 / 1.44	-0.01 / .087
6-Month	09/04/2008	1.51 / 1.54	0.03 / -.031

Notes/Bonds

	Coupon	Maturity date	Current price/Yield	Price/Yield change
2-Year	2	02/28/2010	100-29¾ / 1.52	-0-00¾ / .012
3-Year	4.75	03/31/2011	103-21¾ /1.42	-0-02 / .018
5-Year	2.75	02/28/2013	101-16 / 2.43	0-06 / -.040
10-Year	3.5	02/15/2018	99-23+ / 3.53	0-14 / -.053
30-Year	4.375	02/15/2038	97-08½ / 4.54	0-08+ / -.017

Source: http://www.bloomberg.com/markets/rates/index.html

Discount Yield and Price

The dollar value of a Treasury bill is calculated using the discount yield according to the formula

$$V = \Pr\left(1 - \frac{\tau}{360} Y_d\right), \tag{3.11}$$

where τ is the number of days remaining to maturity. Suppose, for instance, that the six-month Treasury bill has a time to maturity of $\tau = 100$ days. Then, its price is

$$P = 100 \times \left(1 - \frac{100}{360} \times 1.51\%\right) = \$99.5806.$$

Note that the discount yield is a quoting mechanism rather than a good measure of returns on an investment in a Treasury bill.

Other fixed-income securities

Saving Account

- Saving account accrued daily, according to the formula

$$B_{t+\Delta t} = B_t\left(1 + r_t \Delta t\right)$$

where

Δt --- 1/365

r_t --- short rate of date t (now ≈ 0)

Certificate of Deposits (CDs)

- CDs are deposits for fixed terms, typical terms are 1m, 3m, 6m and 1Y.
- Let $r_{\Delta t}$ by the annualized interest rate for the CD of term ΔT , then the interest gain for $1 is

$$\Delta T \cdot r_{\Delta t}$$

- A CD can be rolled over, say *n* times. Suppose that the interest rate stays unchanged, then the total return is

$$(1 + \Delta T \cdot r_{\Delta t})^n$$

Certificate of Deposits (CDs)

- Ex:
  - 2.11% for 1m
  - 2.12% for 3m
  - 2.13% for 6m
  - 2.14% for 1Y
- Which one to go for?

Effective Annual Yield

- The effective annual yield (EAY) is define as

$$EAY = (1 + r_{\Delta t}\Delta t)^{1/\Delta t} - 1$$

- EAY is used to compare returns for different CDs if invested for a year.