Midterm Exam of Math 4511, Fall 2019

(Please return the problem sheet with your booklet)

Problems (The numbers in brackets are credits, summed up to 50)**:**

1. This problem concerns with interest-rate swaps.
   1.1 (4) Explain how a vanilla interest-rate swap works.
   1.2 (8) Prove that, at time 0, the swap rate for the *T*-year maturity vanilla swap is

$$s(0,T)=\frac{1-d(0,T)}{\frac{1}{2}\sum_{i=1}^{2T} d(0,i/2)},$$

   where $d(0,i/2),\ i=1,\cdots,2T$ are discount factors seen at $t=0$ and $2T$ is a positive integer.
   1.3 (4) Provide the formula for the MtM value of the vanilla swap at a later time $t<T$.
2. Given the **6m** swap-rate curve, $s(0,j/2),\ j=1,\cdots,60,$ answer with formulae on
   2.1 (4) how to calculate the 6m discount curve, $d(j/2),\ j=1,\cdots,60,$
   2.2 (4) how to calculate the 6m spot-rate curve, $\hat{r}(j/2),\ j=1,\cdots,60,$ and
   2.3 (4) how to calculate the 6m forward-rate curve, $f(j/2),\ j=1,\cdots,60.$
3. The **full** prices of four Treasury bonds in October 24, 2019 are given below:

Bond	Price	Discount Factor	Par Yield
0.5s of 4/24/2020	99-30		
0.75s of 10/24/2020	99-28		
1s of 4/24/2021	100-2		
1.25s of 10/24/2021	100-8		

   Take **semiannual** compounding and
   3.1 (4) find out the discount factors from a half to two years, and
   3.2 (4) calculate the par yields from a half to two years.
4. [Continued from Problem 3] Use the two-year vanilla swap to duration-hedge the long position of the 1.25s two-year bond of $1m face value, given that the yield to maturity of the latter is 1.123%.
   4.1 (2) Indicate the type of the swap, payer's or receiver's, is to be long or short for hedging.
   4.2 (4) Calculate the face value of the swap for hedging.
5. Suppose that your family has just purchased an apartment for $8m with 20% down payment and 80% loan. Let the loan rate be 2.5% and the loan maturity be 20 years,
   5.1 (4) find out the monthly payment.
   5.2 (4) What will be the outstanding value of the loan five years later?

========== THE END ==========