MATH4511 Quantitive Methods for Fixed Income Derivatives, 2017-18 Fall
Quiz 03(T1A)

Name: ____________________ **ID No.:** ____________________ **Tutorial Section:**______

1. (15 points) Calculate the 2-year and 5-year KV01 of a 10-year swap (face value \$1m). Assume the par yield curve is flat at 6%.

2. (15 points) 2-v regression-based hedging

$$\Delta y_t^{20} = \alpha + \beta^{10}\Delta y_t^{10} + \beta^{30}\Delta y_t^{30} + \epsilon_t$$
$$\Delta \hat{y_t}^{20} = \hat{\alpha} + \hat{\beta}^{10}\Delta \hat{y_t}^{10} + \hat{\beta}^{30}\Delta \hat{y_t}^{30}$$

What is the face amount for 10- and 30-year bonds (F_{10} and F_{30}) if you want to hedge for the 20-year bond of value F_{20}? And what is the hedge error? Here we assume $DV01_{20}$, $DV01_{10}$ and $DV01_{30}$ are known, and you can use notations $\hat{\alpha}$, $\hat{\beta}^{10}$, $\hat{\beta}^{30}$ and ϵ_t to represent the DV01 hedging and corresponding error.

3. (20 points) Consider the model

$$\Delta r_t = \mu(t)\Delta t + \sigma\sqrt{\Delta t}\epsilon_B,$$

where ϵ_B takes +1 or -1 with equal probability. Here $r_0 = 5\%$, $\sigma = 2\%$, $\mu(0.5) = 0.02008$, $\mu(1) = 0.02017$ and $\mathrm{d}t = 1/2$.

(a) Build a two-step binomial tree for the short rate process.

(b) price a one-year call option on a 1.5 year zero-coupon bond with face value \$1,000, with strike price \$960. Explain how to hedge the call.