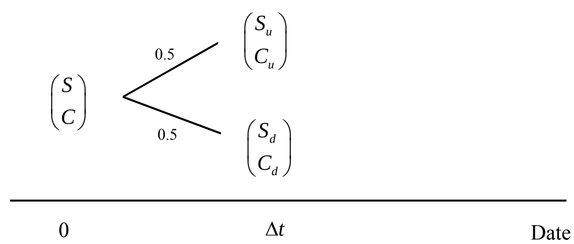
Final Exam for MATH4511

December 13, 2016

(The problem sheet is required to be returned with your booklet)

Problems (Numbers in brackets are credits, totaled to 80):

1. (8) Consider the regression-based hedging of a 20-year bond by a 30-year bond. Suppose the DV01s of the two bonds are 0.12 and 0.16, the daily volatilities of the two bond yields are 6.25 basis points and 5.93 basis points, respectively, and the correlation between the two bond yields is 90%. Let the face value of the 20-year bond be \$1m, how much face value you should long or short the 30-year bond?
2. Consider the following one-period equity option model



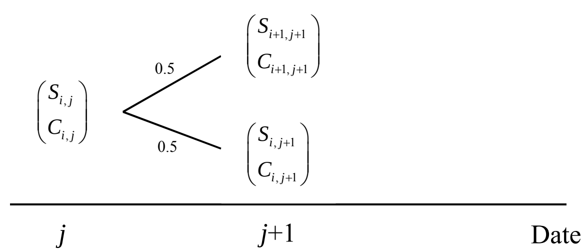
Let the interest rate for **simple compounding** be *r*. Provide solution or answer to the following sub-problems.

2.1. (2) Present the expectation pricing of C.

2.2. (4) Present the arbitrage pricing of C.

2.3. (4) Under what conditions the arbitrage pricing is equal to the expectation pricing?

3. In the (i, j)-state of a multi-period binomial equity option model ,



the corresponding delta-hedging ratio is

$$\alpha_{i,j} = \frac{C_{i+1,j+1} - C_{i,j+1}}{S_{i+1,j+1} - S_{i,j+1}}.$$

3.1. (4) Give the definition of a "self-financing hedging strategy".

3.2. (4) Justify that the delta-hedging strategy is a self-financing strategy.

4. The following questions are about the Black's formula.

4.1. (4) Give the definitions of a *T*-maturity forward contract and the corresponding forward price, and the expression of the latter under stochastic interest rates.

4.2. (4) Write down the Black's formula for call options.

4.3. (2) Under what circumstances the Black's formula becomes the Black-Scholes formula?

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5. The following questions are about swaps or swaptions.
   5.1. (4) Explain how to price a **payer's** swap with fixed rate k and tenor (T_0, T_N), and provide the expression.
   5.2. (4) Give the definitions of a swaption and an ATM swaption.
   5.3. (4) Show that a call option with the par strike price on a bond with coupon rate c can be priced like a receiver's swaption with payoff $\sum_{i=1}^{N} \frac{1}{2} d(T_0, T_i)\left(c - s(T_0; T_0, T_N)\right)^+$, where $s(T_0; T_0, T_N)$ is the market prevailing swap rate at T_0.
6. Let the **3m** forward rate for the period (T_{j-1}, T_j) at time-t be $f_{j-1}(t)$.
   6.1. (4) What is the time-t value of the long FRA initiated at $t = 0$ for the term (T_{j-1}, T_j) and for the fixed rate k? (Note: To long FRA means to receive $f_{j-1}(T_{j-1})$ and pay k.)
   6.2. (4) Present the Black's formula to price the **caplet** on $f_{j-1}(T_{j-1})$ for the strike rate of k. Let the notional value be \$1m and the forward-rate volatility be σ.
   6.3. (4) Explain how to hedge the caplet using the FRA only.
   6.4. (4) State the call-put parity for the pair of caplet and **floorlet** of strike rate k, and then derive the Black's formula for the floorlet based on the parity .
7. The following questions concern about the pricing of receiver's swaptions.
   7.1. (4) Write down the Black's formula for general receiver's swaptions.
   7.2. (4) Let the current term structure of **6m** forward rates be **flat** and be written as $\{f_{j-1}(0) = f,\ j = 1, \cdots, 60\}$. Find out a **succinct** formula of

$$A(0; T_0, T_N) = \sum_{i=1}^{N} \frac{1}{2} d(0, T_i)$$

   in terms of f, where $T_i = T_0 + i/2$ years.
   7.3. (4) For $f = 0.03$, find out the market prevailing swap rate for in-5-to-10 forward swap (i.e., $T_0 = 5$ and $T_N = 15$).
   7.4. (4) Price the in-5-to-10 **ATM** swaption on **receiver**'s swap for $f = 0.03$, swap-rate volatility $\sigma = 0.2$ and notional value \$1m (You may need this value: $\Phi(\frac{1}{2} \cdot 0.2\sqrt{5}) = 0.588468$).

==================== Good Luck to Everyone! =====================