

Quiz3 T1A

2020年11月20日 9:32

2. (15 points) 2-v regression-based hedging

$$\Delta y_t^{20} = \alpha + \beta^{10} \Delta y_t^{10} + \beta^{30} \Delta y_t^{30} + \epsilon_t$$

$$\Delta \hat{y}_t^{20} = \hat{\alpha} + \hat{\beta}^{10} \Delta \hat{y}_t^{10} + \hat{\beta}^{30} \Delta \hat{y}_t^{30}$$

What is the face amount for 10- and 30-year bonds (F_{10} and F_{30}) if you want to hedge for the 20-year bond of value F_{20} ? And what is the hedge error? Here we assume $DV01_{20}$, $DV01_{10}$ and $DV01_{30}$ are known, and you can use notations $\hat{\alpha}$, $\hat{\beta}^{10}$, $\hat{\beta}^{30}$ and ϵ_t to represent the DV01 hedging and corresponding error.

We want to choose an appropriate amount of F_{10} and F_{30} to minimize the absolute of PHL:

$$\begin{aligned} PHL &= F_{20} \times \frac{DV01_{20}}{100} \times \Delta \hat{y}_t^{20} - F_{10} \times \frac{DV01_{10}}{100} \Delta y_t^{10} - F_{30} \times \frac{DV01_{30}}{100} \Delta y_t^{30} \\ &= F_{20} \times \frac{DV01_{20}}{100} \left(\hat{\alpha} + \hat{\beta}_{10} \Delta \hat{y}_t^{10} + \hat{\beta}_{30} \Delta \hat{y}_t^{30} + \epsilon_t \right) \\ &\quad - F_{10} \times \frac{DV01_{10}}{100} \Delta y_t^{10} - F_{30} \times \frac{DV01_{30}}{100} \Delta y_t^{30} \\ &= \left(F_{20} \times \frac{DV01_{20}}{100} \hat{\beta}_{10} - F_{10} \times \frac{DV01_{10}}{100} \right) \Delta y_t^{10} + \left(F_{20} \times \frac{DV01_{20}}{100} \hat{\beta}_{30} - F_{30} \times \frac{DV01_{30}}{100} \right) \Delta y_t^{30} \\ &\quad + F_{20} \times \frac{DV01_{20}}{100} (\hat{\alpha} + \epsilon_t) \end{aligned}$$

To hedge 20-year bond using 10 & 30 year bonds, we want to set coefficient of Δy_t^{10} , Δy_t^{30} to 0.

$$\Rightarrow F_{20} \times \frac{DV01_{20}}{100} \hat{\beta}_{10} - F_{10} \times \frac{DV01_{10}}{100} = 0$$

$$F_{20} \times \frac{DV01_{20}}{100} \hat{\beta}_{10} = F_{10} \times \frac{DV01_{10}}{100}$$

$$F_{20} \times \frac{DV01_{20}}{DV01_{10}} \hat{\beta}_{10} = F_{10}$$

$$+ DV01_{10} \wedge$$

$$DVol_{1,0} \quad \text{---}$$

$$F_{1,0} = F_{2,0} \times \frac{DVol_{2,0}}{DVol_{1,0}} \hat{\beta}_{1,0}$$

$$\text{Similarly, } F_{3,0} = F_{2,0} \times \frac{DVol_{2,0}}{DVol_{3,0}} \hat{\beta}_{3,0}$$

and the hedging error would be

$$F_{2,0} \times \frac{DVol_{2,0}}{DVol_{3,0}} (\Delta + \epsilon_c)$$

3. (20 points) Consider the model

$$\Delta r_t = \mu(t) \Delta t + \sigma \sqrt{\Delta t} \epsilon_B,$$

where ϵ_B takes +1 or -1 with equal probability. Here $r_0 = 5\%$, $\sigma = 2\%$, $\mu(0.5) = 0.02008$, $\mu(1) = 0.02017$ and $\Delta t = 1/2$.

(a) Build a two-step binomial tree for the short rate process.

(b) price a one-year call option on a 1.5 year zero-coupon bond with face value \$1,000, with strike price \$960. Explain how to hedge the call.

Time	0	0.5	1	1.5
Spot		5%	5.5%	6%
μ		0.02008	0.02017	
$\sigma = 0.02 \cdot \Delta t = 0.5$				



$$r_{1,1} = r_{0,0} + \mu(0.5) \Delta t + \sigma \sqrt{\Delta t}$$

$$r_{0,1} = r_{0,0} + \mu(0.5) \Delta t - \sigma \sqrt{\Delta t}$$

$$\therefore r_{1,1} = 0.0742, \quad r_{0,1} = 0.0459$$

$$r_{2,2} = r_{1,1} + \mu(1) \Delta t + \sigma \sqrt{\Delta t}$$

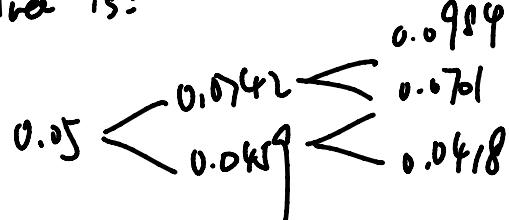
$$r_{2,1} = r_{1,1} + \mu(1) \Delta t + \sigma \sqrt{\Delta t}$$

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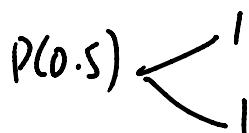
$$r_{0,2} = r_{0,1} + \mu(1) \Delta t - \sigma \sqrt{\Delta t}$$

$$r_{2,1} = 0.1984, r_{1,2} = 0.0701, r_{0,2} = 0.0418$$

\therefore Rate tree is:

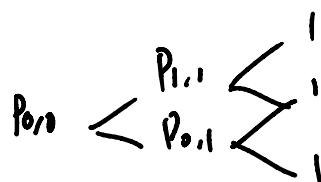


0.5y ZCB:
tree



$$P(0.5) = \frac{1+1}{(1+5\%/2)} = 0.975610$$

1y ZCB:
tree



$$\text{where } P_{1,1} = \frac{1+1}{2} \times \left(\frac{1}{1+r_{1,1}/2} \right)$$

$$P_{1,1} = 0.964235$$

$$P_{0,1} = \frac{1+1}{2} \times \left(\frac{1}{1+r_{0,1}/2} \right)$$

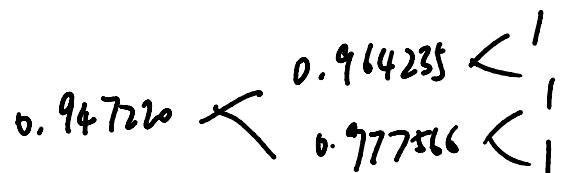
$$P_{0,1} = 0.977566$$

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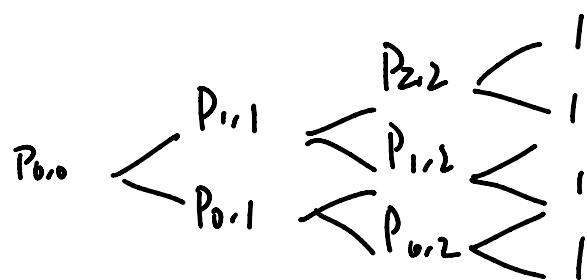
$$P_{0,0} = \frac{P_{0,1} + P_{1,1}}{2} \times \left(\frac{1}{1+r_{0,0}/2} \right)$$

$$P_{0,0} = 0.947220$$

Tree:



For 1.5 year ZCB:



$$\text{where } P_{2,2} = \frac{1+1}{2} \times \left(\frac{1}{1+r_{2,2}/2} \right)$$

$$P_{2,2} = 0.953103$$

$$P_{1,2} = \frac{1+1}{2} \times \left(\frac{1}{1+r_{1,2}/2} \right)$$

$$P_{1,2} = 0.966125$$

$$P_{0,2} = \frac{1+1}{2} \times \left(\frac{1}{1+r_{0,2}/2} \right)$$

$$P_{0,2} = 0.979508$$

$$P_{1,1} = \frac{P_{2,2} + P_{1,2}}{2} \times \left(\frac{1}{1+r_{1,1}/2} \right)$$

$$P_{1,1} = 0.925294$$

$$P_{0,1} = \frac{P_{0,2} + P_{1,2}}{2} \times \left(\frac{1}{1+r_{0,1}/2} \right)$$

$$P_{0,1} = 0.950993$$

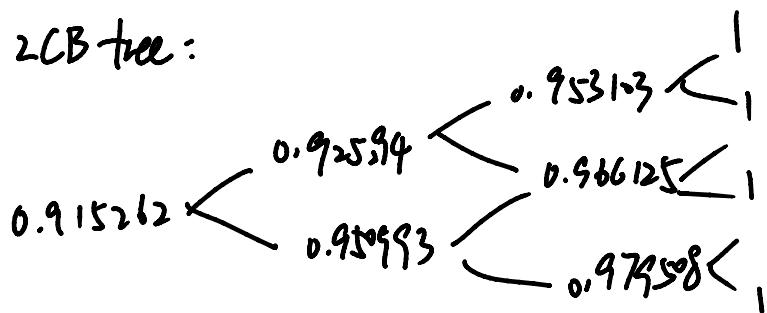
$$D_{min} = (D_0 + D_1 + \dots + D_n) \rightarrow$$

$$P_{0,1} = 0.1211$$

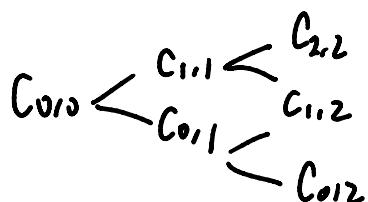
$$P_{0,0} = \left(\frac{P_{0,1} + P_{0,1}}{2} \right) \times \left(\frac{1}{1+r_{0,0}/2} \right)$$

$$P_{0,0} = 0.915262$$

\therefore 1.5 year 2CB tree:

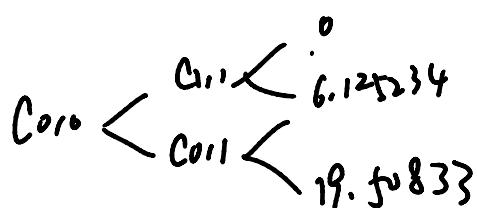


$$b). K = 960, F = 1000$$



$$\text{where } C_{i,j} = \max(F \times P_{i,j} - K, 0)$$

\therefore option tree:



$$C_{1,1} = \left(\frac{C_{2,2} + C_{1,2}}{2} \right) \times \left(\frac{1}{1+r_{0,1}/2} \right)$$

$$C_{0,1} = \left(\frac{C_{0,2} + C_{1,2}}{2} \right) \times \left(\frac{1}{1+r_{0,1}/2} \right)$$

$$C_{0,0} = \left(\frac{C_{0,1} + C_{1,1}}{2} \right) \times \left(\frac{1}{1+r_{0,0}/2} \right)$$

$$\Rightarrow C_{1,1} = 2.953084$$

$\sim \dots \sim 0.8768$

$$\Rightarrow C_{1,1} = 2.71510^+$$

$$C_{0,1} = 12.51924 \}$$

$$C_{0,2} = 7.55236$$

$$\therefore \text{Hedge ratio} = \frac{C_{1,1} - C_{0,1}}{P_{1,1} - P_{0,1}} \times \left(\frac{1}{1000} \right)$$

$$\text{Hedge ratio} = 0.37624.$$

\therefore one could long 0.37624 unit of bond
and short 1 unit of option.