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MATH4511 Quantitive Methods for Fixed Income Derivatives, 2015-16 Fall
Quiz 03(T1C)

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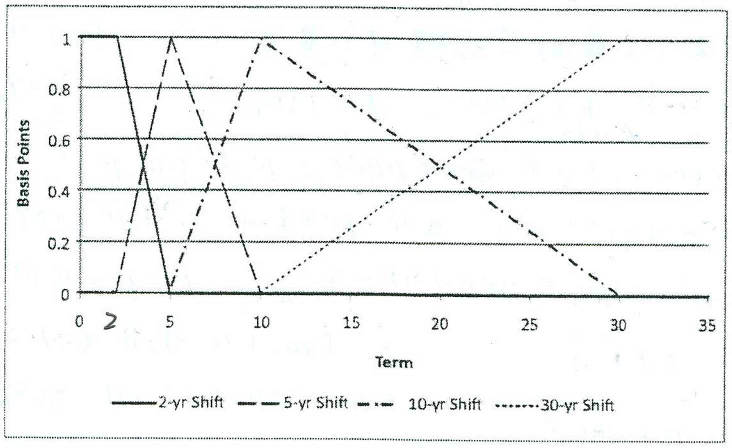
1. (15 points) (1) What is the major drawback when we use duration and DV01 to hedge yield-based securities?
(2) Write down the formula of each key rate shift according to the following table and explain why the summation of all the KV01 is equal to DV01.



(1) when using duration and DV01 to hedge. we assume parallel yield curve shift, that is the yield of all maturity shift by the same amount. But in reality, yield of different maturity often change by different amount, or may even in different direction. so it's not parallel shift.

(2)
$$\Delta y^2 = \begin{cases} 1 & 0 \le T \le 2 \\ \frac{-T+5}{3} & 2 < T \le 5 \\ 0 & T > 5. \end{cases} \qquad \Delta y^5 = \begin{cases} 0 & 0 \le T \le 2. \\ \frac{T-2}{3} & 2 < T \le 5. \\ \frac{-T+10}{5} & 5 < T \le 10 \\ 0 & T > 10. \end{cases} \qquad \Delta y^{10} = \begin{cases} 0 & 0 \le T \le 5 \\ \frac{T-5}{5} & 5 < T \le 10. \\ \frac{30-T}{20} & 10 < T \le 30 \\ 0 & T > 30. \end{cases}$$

$$\Delta y^{30} = \begin{cases} 0 & 0 \le T \le 10. \\ \frac{T-10}{20} & 10 < T \le 30 \\ 1 & T > 30. \end{cases}$$

from the formula we observe that

$$\Delta y^2 + \Delta y^5 + \Delta y^{10} + \Delta y^{30} = \begin{cases} 1+0=1 & 0 \le T \le 2 \\ \frac{-T+5+T-2}{3} = 1 & 2 < T \le 5 \\ \frac{-T+10+T-5}{5} = 1 & 5 < T \le 10. \\ \frac{30-T+T-10}{20} = 1 & 10 < T \le 30 \\ 1 & T > 30 \end{cases}$$

$\therefore \Delta y^2 + \Delta y^5 + \Delta y^{10} + \Delta y^{30} = 1$

so this is equivalent to the yield of that specific maturity change by 1 basic point.

The effect of one bp change in 2, 5, 10, 30 year key rate on a specific maturity is the same as one bp shift of yield of that maturity

price change are the same

so all KV01 sum up to DV01

2.5 2.5 2.5 102.5. DV01.

par yield flat at 5.01% for 0-2 year: spot rate = 5.01% yield. discount it.

par yield flat at 4.99% for 0-2 year. spot rate = 4.99% discount it.

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2. (15 points)

Par yields flat at:	5%	xxx	xxx	xxx	xxx
xxxxx	xxx	Key Rate 01s (100 Face)	xxx	xxx	xxx
Coupon	Term	2-Year	5-Year	10-Year	30-Year
F_1 5%	2	0.01881	0	0	0.
F_2 5%	5	0	.04375	0	0
F_3 5%	10	0	0	.0779	0
F_4 5%	30	0	0	0	.15444
Mortgage	xxx	1.0	4.0	43.0	67.0

~~bond~~ rate ↑ key.

Calculate all the missing KV01's in the table above. According to the KV01's, how to hedge a long position of the nonprepayable mortgage by using these bonds?

2-~~yield~~ year key rate change by 1 basic point. 5. 10. 30 year yield won't change. par yield is still 5%. still par bond, so price won't change. KV01^2 is 0. 0. 0 for 5. 10. 30 year ~~key rate shift~~ bond.

for key rate 2 year shift. KV01 = DV01 on 2-year bond. DV01 = $\frac{-P(5.01\%)+P(4.99\%)}{2} = \frac{-99.98119+100.01881}{2}$ = 0.01881

similarly. a five-year key rate shift won't affect 2. 10. 30 par yield. so KV01^5 of 2. 10. 30 year bond is 0. a 10-year key rate shift won't change 2, 5, 30 par yield. a 30-year ~~yield~~ key rate shift won't change 2. 5. 10 year par yield all remaining numbers are 0.

$\frac{0.01881}{100}F_1 = 1.$ $F_1 = 5316.3211$

$\frac{0.04375}{100}F_2 = 4$ $F_2 = 9142.8571$

$\frac{0.0779}{100}F_3 = 43$ $F_3 = 55198.9730$

$\frac{0.15444}{100}F_4 = 67$ $F_4 = 43382.5434.$

so I need to short 5316.3211 face value of 5% 2y bond. 9142.8571 5y bond. 55198.9730 10y bond and 43382.5434 30 year bond.

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3. (20 points)

A: a 9-year zero-coupon bond; $P=100(1+\frac{y}{2})^{-18}$

B: a portfolio of 2-year and 30-year zero-coupon bond with weights 0.75 and 0.25.

Suppose the current yield curve is flat at 6%. Compare the duration and convexity of A and B.

A: zero coupon bond. $P = \frac{100}{(1+\frac{y}{2})^{18}}$

$\frac{dP}{dy} = \frac{100\cdot(-18)}{(1+\frac{y}{2})^{19}}\cdot\frac{1}{2}$ $D = -\frac{1}{P}\frac{dP}{dy} = \frac{1800}{(1+\frac{y}{2})^{19}\cdot 2\cdot 100}(1+\frac{y}{2})^{18} = \frac{18}{2+y} = \frac{18}{2+0.06} = 8.7379$

$\frac{d^2P}{dy^2} = \frac{100\cdot(-18)(-19)}{2(1+\frac{y}{2})^{20}\cdot 2} = \frac{18\cdot 19\cdot 100}{4(1+\frac{y}{2})^{20}}$. $C = \frac{1}{P}\frac{d^2P}{dy^2} = \frac{18\cdot19\cdot100}{4(1+\frac{y}{2})^{20}}\cdot\frac{(1+\frac{y}{2})^{18}}{100} = \frac{18\cdot 19}{4(1.03)^2} = 80.59193$

B:

$D_2 = \frac{T}{1+\frac{y}{2}} = \frac{2}{1+0.03} = 1.9417$ $D_{30} = \frac{T}{1+\frac{y}{2}} = \frac{30}{1+0.03} = 29.1262.$

$D_P = 0.75D_2 + 0.25D_{30} = 8.7378$

$C_2 = \frac{T(T+0.5)}{(1+\frac{y}{2})^2} = \frac{2\cdot 2.5}{1.03^2} = 4.7130$ $C_{30} = \frac{T(T+0.5)}{(1+\frac{y}{2})^2} = \frac{30\cdot 30.5}{1.03^2} = 862.4753.$

$C_P = 0.75C_2 + 0.25C_{30} = 219.1536.$

duration are almost the same

$C_B > C_A$.