PART TWO

ANSWERS

Chapter 1

1.1 What are the cash flow dates and the cash flows of $1,000 face value of the U.S. Treasury 2 3/4s of May 31, 2017, issued on May 31, 2010.

A. $13.75 every November 30 and May 31, starting on November 30, 2010, through May 31, 2010, plus a principal payment of $1,000 on May 31, 2017.

1.2 Use this table of U.S. Treasury bond prices for settle on May 15, 2010, to derive the discount factors for cash flows to be received in 6 months, 1 year, and 1.5 years.

BOND	PRICE
4 1/2s of 11/15/2010	102.15806
0s of 5/15/2011	99.60120
1 3/4s of 11/15/2011	101.64355

A. Solve the following equations

$$102.25d(.5) = 102.15806$$

$$100d(1) = 99.60120$$

$$.875\left[d(.5) + d(1) + d(1.5)\right] + 100d(1.5) = 101.64355$$

to get $d(.5) = .999101$; $d(1) = .996012$; $d(1.5) = .990313$

1.3 Suppose there existed a Treasury issue with a coupon of 2% maturing on November 15, 2011. Using the discount factors derived from Question 1.2, what would be the price of the 2s of November 15, 2011?

A. 102.01673:

$$1 \times \left[d(.5) + d(1) + d(1.5)\right] + 100d(1.5) = 102.01673$$

1.4 Say that the 2s of November 15, 2011, existed and traded at a price of 101 instead of the price derived from Question 1.3. How could an arbitrageur profit from this price difference using the bonds in the earlier table? What would that profit be?

A. First find the replicating portfolio by solving the following equations:

$$102.25F_1 + 0 \times F_2 + .875F_3 = 1$$

$$100F_2 + .875F_3 = 1$$

$$100.875F_3 = 101$$

Solving, $F_1 = .12119\%$; $F_2 = .12392\%$; $F_3 = 100.12392\%$. Hence, the arbitrageur should buy 100 face amount of the 2s for 101 and short the replicating portfolio for a proceeds of

$$\begin{aligned} &.12119\% \times 102.15806 + .12392\% \times 99.60120 \\ &+100.12392\% \times 101.64355 \\ =&102.01673 \end{aligned}$$

As the portfolio matures, the cash flows from the 2s will exactly offset the requirements for covering the short. This leaves the initial profit at the time of the trade of

$$102.01673 - 101 = 1.01673$$

per 100 face of the 2s bought.

1.5 Given the prices of the two bonds in the table as of May 15, 2010, find the price of the third by an arbitrage argument. Since the 3 1/2s of 5/15/2020 is the on-the-run 10-year, why might this arbitrage price not obtain in the market?

BOND	PRICE
0s of 5/15/2020	69.21
3 1/2s of 5/15/2020	?
8 3/4s of 5/15/2020	145.67

A. Verify that buying 40 face amount of the 8.75s and 60 face amount of the 0s replicates 100 face amount of the 3 1/2s. Therefore, the arbitrage price of the 3.5s is

$$40\% \times 145.67 + 60\% \times 69.21 = 99.794$$

The on-the-run 10-year might very well sell for more than this arbitrage price because of its superior liquidity and financing characteristics relative to the 0s and the 8 3/4s.

Chapter 2

2.1 You invest $100 for two years at 2% compounded semiannually. How much do you have at the end of the two years?

A. Since two years are 4 semiannual periods,:

$$100\left(1+\frac{2\%}{2}\right)^4 = 104.0604$$

2.2 You invested $100 for three years and, at the end of those three years, your investment was worth $107. What was your semiannually compounded rate of return?

A. Solve for the rate r in the equation

$$100\left(1+\frac{r}{2}\right)^6 = 107$$

$$r = 2.2681\%$$

2.3 Using the discount factors in the table, derive the corresponding spot and forward rates.

TERM	DISCOUNT FACTOR
.5	.998752
1	.996758
1.5	.993529

A. Solve the following equations for the forward rates

$$.998752 = \frac{1}{1+\frac{f(.5)}{2}}$$

$$.996758 = \frac{.998752}{1+\frac{f(1)}{2}}$$

$$.993529 = \frac{.996758}{1+\frac{f(1.5)}{2}}$$

to obtain $f(.5) = .25\%$; $f(1) = .4\%$; $f(1.5) = .65\%$.

Then solve the following equations for the spot rates

$$\hat{r}(.5) = f(.5)$$

$$\left(1+\frac{\hat{r}(1)}{2}\right)^2 = \left(1+\frac{\hat{r}(.5)}{2}\right)\left(1+\frac{f(1)}{2}\right)$$

$$\left(1+\frac{\hat{r}(1.5)}{2}\right)^3 = \left(1+\frac{\hat{r}(1)}{2}\right)^2\left(1+\frac{f(1.5)}{2}\right)$$

to get $\hat{r}(.5) = .25\%$; $\hat{r}(1) = .3250\%$; $\hat{r}(1.5) = .4333\%$.

Note that there are other mathematically equivalent ways to obtain these same results.

2.4 Are the forward rates above or below the spot rates in the answers to Question 2.3? Why is this the case?

A. Forward rates are above spot rates since the term structure is upward-sloping.

2.5 Using the discount factors from question 2.3, price a 1.5-year bond with a coupon of .5%. If over the subsequent six months the term structure remains unchanged, will the price of the .5% bond increase, decrease, or stay the same? Try to answer the question before calculating and then calculate to verify.

A. The price of the bond is

$$\frac{.5}{2}[.998752 + .996758 + .993529] + 99.3529 = 100.10016$$

If the term structure remains unchanged after six months, the price of this bond after six months will be higher: from year 1 to year 1.5 the bond is earning .5% which is below the market forward rate of .65% for that period. To verify this, price a .5% 1-year bond under an unchanged term structure:

$$\frac{.5}{2}\left[.998752+.996758\right]+99.6758=100.17468$$

Chapter 3

3.1 The price of the 3/4s of May 31, 2012 was 99.961 as of May 31, 2010. Calculate its price using the discount factors in Table 2.3. Is the bond trading cheap or rich to those discount factors? Then, using trial-and-error, express the price difference as a spread to the spot rate curve implied by those discount factors.

A. The price under those discount factors is

$$.375\times\left(.99925+.99648+.99135+.98532\right)+98.532=100.02165$$

so the bond is trading cheap.

The spot rates implied by these discount factors are .15011%, .35293%, .58001%, and .74081%.

The spread, s, relative to this curve, is determined by the following equation

$$99.961=\frac{.375}{\left(1+\frac{.15011\%+s}{2}\right)}+\frac{.375}{\left(1+\frac{.35293\%+s}{2}\right)^2}+\frac{.375}{\left(1+\frac{..58001\%+s}{2}\right)^3}+\frac{100.375}{\left(1+\frac{.74081\%+s}{2}\right)^4}$$

Solving by trial-and-error gives that $s=.03065\%$, or about 3 basis points.

3.2 The yield of the 3/4s of May 31, 2012, was .7697% as of May 31, 2010. Verify that this is consistent with the price in Question 3.1.

A. The price is given by

$$\frac{.375}{\left(1+\frac{.7697\%}{2}\right)}+\frac{.375}{\left(1+\frac{.7697\%}{2}\right)^2}+\frac{.375}{\left(1+\frac{.7697\%}{2}\right)^3}+\frac{100.375}{\left(1+\frac{.7697\%}{2}\right)^4}$$
$$=99.96098$$

which matches the price given in Question 3.1.

3.3 The price of the 4 3/4s of May 31, 2012, was 107.9531 as of May 31, 2010. What was the yield of the bond? Please solve by trial-and-error.

A. The yield y is given by the equation

$$\frac{2.375}{\left(1+\frac{y}{2}\right)}+\frac{2.375}{\left(1+\frac{y}{2}\right)^2}+\frac{2.375}{\left(1+\frac{y}{2}\right)^3}+\frac{102.375}{\left(1+\frac{y}{2}\right)^4}=107.9531$$

Solving for the yield by trial-and-error gives $y=.7368\%$.

3.4 Did you get a higher yield for the 4 3/4s from Question 3.3 than the yield of the 3/4s given in Question 3.2? Is that what you expected? Why or why not?

A. The yield of 4 3/4s is lower than the yield of the 1/4s. This was expected because the term structure is upward-sloping and $4.75>.75$.

3.5 An investor purchases the 4 3/4s of May 31, 2012 on May 31, 2010, at the yield given in Question 3.3. Exactly six months later the investor sells the bond at that same yield. What is the price of the bond on the sale date and what is the investor's total return from the bond over those six months?

A. The price of the bond on the sale date is

$$\frac{2.375}{\left(1+\frac{.7368\%}{2}\right)}+\frac{2.375}{\left(1+\frac{.7368\%}{2}\right)^2}+\frac{102.375}{\left(1+\frac{.7368\%}{2}\right)^3}=105.9757$$

The total return of the bond over the period is

$$\frac{105.9757+2.375-107.9531}{107.9531}=.3683\%$$

3.6 Interpret your answer to Question 3.5. a) In what way is the return significant or interesting? b) Explain why an investor would buy a premium bond when that bond is worth only par at maturity? How does this relate to your work in Question 3.5?

A. a) The return of .3683% is, to rounding error, one-half of the yield of .7368%. This is consistent with the discussion in the subsection "Unchanged

Yields." b) The price of a premium bond does fall to par, but the investor earns an above-market rate of interest in compensation. This is illustrated in the return calculation of Question 3.5 where the price falls from 107.9531 to 105.9757, but the investor still enjoys a positive rate of return.

3.7 Re-compute the sample return decomposition of Tables 3.2 and 3.3 of the text, replacing the assumption of realized forwards with the assumption of an unchanged term structure.

A. The revised versions of Tables 3.2 and 3.3 are as follows. Italicized entries have changed from the table in the text.

Start Period	**5/30/10**	**11/30/10**	**5/31/11**	
End Period	**11/30/10**	**5/31/11**	**11/30/11**	**Price**
Pricing Date 5/28/10				
Initial Forwards	.193%	.600%	1.080%	100.190
Term Structure	.149%	.556%	1.036%	
Spreads	.044%	.044%	.044%	
Pricing Date 11/30/10				
Carry-Roll-Down Forwards		*.193%*	*.600%*	*100.353*
Term Structure		*.149%*	*.556%*	
Spreads		.044%	.044%	
Rate-Change Forwards		*.093%*	*.500%*	*100.453*
Term Structure		*.049%*	*.456%*	
Spreads		.044%	.044%	
Spread-Change Forwards		*.049%*	*.456%*	*100.497*
Term Structure		*.049%*	*.456%*	
Spreads		0%	0%	

	$
Initial Price	101.19
Price Appreciation	+*.307*
Roll Down	+*.163*
Rates	+.100

Spread	+*.044*
Cash Carry	**.375**
Coupon	.375
Financing	0
P&L	+*.682*

3.8 Start with any upward-sloping term structure, e.g., from C-STRIPS prices or even some made up rates. Then replicate the zero-coupon, par, and 9% coupon curves in Figure 3.2. Add a curve for a security that makes equal fixed payments to various maturities, i.e., a mortgage.

3.9 In the subsection “News Excerpt: Sale of Greek Government Bonds in March, 2010,” approximately what is the yield on seven-year Spanish debt?

A. 3.27%. The seven-year Greek bonds sold at a yield of 6%. The excerpt says that seven-year German bunds trade 334 basis points below that, i.e., at 2.66%. The excerpt also says that seven-year Spanish debt trades 61 basis points over German bunds, i.e., at 3.27%.

3.10 Return to table 1.7 in the text, which shows that the 3 1/2s of May 15, 2020, are 2.076 per 100 face amount away from being correctly priced by C-STRIPS while the 8 3/4s maturing on the same date are .338 per 100 face amount away. According to the discussion of the text, this difference is due to the on-the-run premium of the 3 1/2s that is reflected in the price of its P-STRIPS. As of the same pricing date, however, the yields of the 3 1/2s and 8 3/4s were only a few basis points apart, i.e., nothing like the difference justified by the more than 2% premium on the price of the final principal payment. How is this possible?

A. With respect to the 3 1/2s, note that it sells at 101.896, a bit above par, and that its yield is significantly lower than C-STRIPS pricing would indicate because of the on-the-run premium. With respect to the 8 3/4s, its yield is significantly lower than the yield of a (hypothetical) par bond of equal maturity because of the coupon effect; at a price of 146.076 the 8 3/4s trades at a very large premium. As it happens, the effect of the on-the-run premium on the yield of the 3 1/2s is about 25 basis points while the coupon effect on the yield of the 8 3/4s is roughly 20 basis points. Hence, these two effects approximately cancel and the yields of the two bonds were not that different.

Chapter 4

4.1 The following table gives the prices of TYU0 and of TYU0C 120 as of May 2010 for a narrow range of the 7-year par rate. Please fill in the other columns, ignoring cells marked with an "X." Over the given range, which security's price-rate function is concave and which convex? How can you tell?

	TYU0					
RATE	PRICE	*DV*01	DURATION	CONVEXITY	1ST DERIV	2ND DERIV
3.320%	115.5712	X	X	X	X	X
3.412%	114.8731			X		X
3.504%	114.1715					
3.596%	113.4668			X		X
3.688%	112.7591	X	X	X	X	X

	TYU0C 120					
RATE	PRICE	*DV*01	DURATION	CONVEXITY	1ST DERIV	2ND DERIV
3.320%	.4564	X	X	X	X	X
3.412%	.3483			X		X
3.504%	.2619					
3.596%	.1940			X		X
3.688%	.1415	X	X	X	X	X

A. The results are given in the tables below. The first table is followed by some sample calculations.

The price rate function of TYU0 is concave while that for TYU0C 120 is convex. In the case of TYU0, you can tell from any of the following equivalent ways (the signs are reversed for the option):

- *DV*01 increases as rates increase.

- The convexity is negative.
- The magnitude of the first derivative increases as rates increase.
- The second derivative is negative.

	TYU0					
RATE	**PRICE**	***DV*01**	**DURATION**	**CONVEXITY**	**1ST DERIV**	**2ND DERIV**
3.320%	115.5712	X	X	X	X	X
3.412%	114.8731	.07607	6.62215	X	–760.70652	X
3.504%	114.1715	.07643	6.69426	–32.86	–764.29348	–3,751.18
3.596%	113.4668	.07676	6.76505	X	–767.60870	X
3.688%	112.7591	X	X	X	X	X

$$-\frac{1}{10,\ 000}\frac{114.1715-115.5712}{3.504\%-3.320\%}=.07607$$

$$-\frac{1}{114.8731}\frac{114.1715-115.5712}{3.504\%-3.320\%}=6.62215$$

$$\frac{114.1715-115.5712}{3.504\%-3.320\%}=-760.70652$$

$$\frac{-767.60870+760.70652}{3.596\%-3.412\%}=-3,\ 751.18$$

$$\frac{-3,\ 751.18}{114.1715}=-32.86$$

	TYU0C 120					
RATE	**PRICE**	***DV*01**	**DURATION**	**CONVEXITY**	**1ST DERIV**	**2ND DERIV**
3.320%	.4564	X	X	X	X	X
3.412%	.3483	.01057	303.49274	X	–105.70652	X
3.504%	.2619	.00839	320.19357	83,569.36	–83.85870	21,886.81
3.596%	.1940	.00654	337.29269	X	–65.43478	X
3.688%	.1415	X	X	X	X	X

4.2 Using the data in Question 4.1, how would a market maker hedge the purchase of \$50 million face amount of TYU0C with TYU0 when the 7-year par rate is 3.596%?

Check how well this hedge works by computing the change in the value of the position should the rate move instantaneously from 3.596% to 3.668%. What if the rate falls to 3.320%? Is the P&L of the hedged position positive or negative? Why is this the case?

A. The market maker would SELL the following face amount equivalent of TYU0:

$$\$50\text{mm} \times \frac{.00654}{.07676} = \$4.26\text{mm}$$

The value of this position at 3.596% is

$$\$50\text{mm} \times \frac{.1940}{100} - \$4.26\text{mm} \times \frac{113.4668}{100} = -\$4,\ 736,\ 686$$

The value at 3.688%

$$\$50\text{mm} \times \frac{.1415}{100} - \$4.26\text{mm} \times \frac{112.7591}{100} = -\$4,\ 732,\ 788$$

At 3.320% the value is

$$\$50\text{mm} \times \frac{.4564}{100} - \$4.26\text{mm} \times \frac{115.5712}{100} = -\$4,\ 695,\ 133$$

The hedge works extremely well moving to 3.688%, with a P&L of less than $4,000 on $4.7 million. But it works well to 3.320% also, with a P&L of about $41,500 on $4.7 million.

The P&L is positive whether rates increase or fall because the trader is long the option which is more convex. This exercise does not include any change in values due to the passage of time.

4.3 Using the data in Question 4.1, how much would an investment manager profit from $100mm of TYU0C if the rate instantaneously fell from 3.504% to 3.404%? Use a duration estimate.

A. The duration at 3.504% is about 320.194, which means that a 10-basis point fall in rate would result in a 32.02% return or, in this case, $32.02 million.

4.4 Using the data from the answer to Question 4.1, provide a 2nd order estimate of the price of TYU0C should the 7-year par rate be 3.75%.

A. The 2nd order Taylor approximation is

$$P(3.75\%) = P(3.504\%) + \frac{dP}{dy}(3.75\% - 3.504\%) + \frac{1}{2}\frac{d^2P}{dy^2}(3.75\% - 3.504\%)^2$$

$$= .2619 - 83.85870 \times (3.75\% - 3.504\%) + \frac{1}{2} \times 21,\ \ 886.81 \times (3.75\% - 3.504\%)^2$$

$$= .2619 - .206292 + .066225 = .1218$$

4.5 The table below gives the prices, durations, and convexities of three bonds. a) What is the duration and convexity of a portfolio that is long \$50mm face amount of each of the 5- and 10-year bonds? b) What portfolio of the 5- and 30-year bonds has the same price and duration as the portfolio of part a)? c) Which of the two portfolios has the greater convexity and why?

COUPON	MATURITY	PRICE	DURATION	CONVEXITY
2.50%	5 years	102.248	4.687	25.052
2.75%	10 years	100.000	8.691	86.130
3%	30 years	95.232	19.393	495.423

A a) The fraction of portfolio value in the 5-year bond is

$$\frac{50\text{mm} \times 102.248\%}{50\text{mm} \times 102.248\% + 50\text{mm} \times 100\%} = 50.556\%$$

and the fraction of portfolio value in the 10-year bond is $100\% - 50.556\% = 49.444\%$.

Hence, the duration of the portfolio is

$$50.556\% \times 4.687 + 49.444\% \times 8.691 = 6.667$$

and its convexity is

$$50.556\% \times 25.052 + 49.444\% \times 86.130 = 55.251$$

Note for the next part that the price of this portfolio is

$$50\text{mm} \times 102.248\% + 50\text{mm} \times 100\% = 101.124\text{mm}$$

b) Find face amounts such that

$$F^5 \times 102.248\% + F^{30} \times 95.232\% = 101.124\text{mm}$$

$$\frac{F^5 \times 102.248\%}{101.124} \times 4.687 + \frac{F^{30} \times 95.232\%}{101.124} \times 19.393 = 6.667$$

Solving, $F^5 = 85.587$ and $F^{10} = 14.295$, or 86.538% and 13.462% of portfolio value respectively. The convexity of the portfolio is

$$86.538\% \times 25.052 + 13.462\% \times 495.423 = 88.372$$

c) This portfolio has higher convexity because the values of its cash flows are much more spread out than those of the portfolio of the 5- and 10-year with equal price and duration.

4.6 The following table gives yields, *DV*01s, and durations for three 15-year bonds. The three coupon rates are 0%, 3.5%, and 7%. Which coupon rate belongs to which bond? What is the shape of the term structure of spot rates underlying the valuation of these bonds?

BOND	YIELD	*DV*01	DURATION
#1	3.50%	.1159	11.59
#2	3.50%	.0876	14.75
#3	3.50%	.1443	10.26

A The coupons for bonds #1, #2, and #3, are 3.5%, 0%, and 7%, respectively. The term structure of spot rates has to be flat for bonds of the same maturity and different coupons to have the same yield.

Chapter 5

5.1 Using the following instructions, complete a spreadsheet to compute the two-year and five-year key-rate duration profiles of four-year bonds. For the purposes of this question, key-rate shifts are in terms of spot rates.

a) In Column A put the coupon payment dates in years, from .5 to 5 in increments of .5. Put a spot rate curve, flat at 3%, in Column B. Put the discount factors corresponding to this spot rate curve in Column C. Now price a 3% and an 8% four-year coupon bond under this initial spot rate curve.

b) Create a new spot rate curve in Column D by adding a two-year key rate shift of 10 basis points. Compute the new discount factors in Column E. What are the new bond prices?

c) Create a new spot rate curve in Column F by adding a five-year key rate shift of 10 basis points. Compute the new discount factors in Column F. What are the new bond prices?

d) Use the results in parts a through c to calculate the key-rate duration profiles of each of the bonds.

e) Sum the key-rate durations for each bond to obtain the total durations. Calculate the percentage of the total duration attributed to each key rate for each bond. Comment on the results.

f) What would the key-rate duration profile of a four-year zero coupon bond look like relative to those of these coupon bonds? How about a five-year zero coupon bond?

A. The spreadsheet described has the following entries:

A	B	C	D	E	F	G
	INITIAL CURVE		**2-YEAR SHIFT**		**5-YEAR SHIFT**	
	SPOT	**DISCOUNT**	**SPOT**	**DISCOUNT**	**SPOT**	**DISCOUNT**
TERM	RATE	FACTOR	RATE	FACTOR	RATE	FACTOR
0.5	3.00%	.985222	3.100%	.984737	3.000%	.985222
1.0	3.00%	.970662	3.100%	.969706	3.000%	.970662
1.5	3.00%	.956317	3.100%	.954905	3.000%	.956317
2.0	3.00%	.942184	3.100%	.940330	3.000%	.942184
2.5	3.00%	.928260	3.083%	.926357	3.017%	.927879
3.0	3.00%	.914542	3.067%	.912742	3.033%	.913642
3.5	3.00%	.901027	3.050%	.899475	3.050%	.899475
4.0	3.00%	.887711	3.033%	.886546	3.067%	.885382
4.5	3.00%	.874592	3.017%	.873946	3.083%	.871368
5.0	3.00%	.861667	3.000%	.861667	3.100%	.857434

The pricing results per unit face amount are summarized in this table:

COUP	MAT	INITIAL	2-YEAR SHIFT		5-YEAR SHIFT		TOTAL	2YR	5YR
(%)	(YEARS)	PRICE	PRICE	DUR	PRICE	DUR	DUR	%DUR	%DUR
3	4	1.0000	.9987	1.332	.9976	2.406	3.738	35.6%	64.4%
8	4	1.1871	1.1855	1.356	1.1846	2.136	3.492	38.8%	61.2%
0	4	.8877	.8865	1.313	.8854	2.623	3.936	33.3%	66.7%
0	5	.8617	.8617	0.000	.8574	4.913	4.913	0.0%	100.0%

The higher the coupon, the more of its present value is paid earlier and the greater the percentage of its total duration in the two-year bucket. This applies across the three four-year coupon bonds in the pricing and duration table. Since the key-rate shifts here are defined in terms of spot rates, and since the five-year spot rate is a key rate, the key-rate duration profile of a five-year zero will be completely concentrated in the five-year shift.

5.2 Continue with the setting and results of Question 5.1. Verify that a 3% two-year bond has a duration of 1.925 that is completely concentrated as a two-year key-rate duration. How would one hedge the key-rate risk profile of the 8% four-year bond with the 3% two-year bond and the 3% four-year bond? Note that the total value of the 8% bond and of the hedge need not be the same. Comment on the result.

A. The row for the pricing of the 3% two-year bond, corresponding to the pricing and duration table in the answer to Question 5.1 is

COUP	MAT	INITIAL	2-YEAR SHIFT		5-YEAR SHIFT		TOTAL	2YR	5YR
(%)	(YEARS)	PRICE	PRICE	DUR	PRICE	DUR	DUR	%DUR	%DUR
3	2	1.0000	.9981	1.925	1.0000	0.000	1.925	100.0%	0.0%

Let F^2 and F^4 be the face amounts of the two-year and four-year bonds in the hedge against a unit face amount of the 8% bonds. Note that the prices of the two 3% bonds are 1 while the price of the 8% bond is 1.1871. To set the two-year key-rate '01 of the portfolio equal to the two-year key-rate '01 of the 8% bond,

$$1.925 \times F^2 + 1.335 \times F^4 = 1.1871 \times 1.356$$

To set the five-year key-rate '01 of the portfolio equal to that of the 8% bond,

$$2.406 \times F^4 = 1.1871 \times 2.136$$

Solving, $F^2 = 1.053884$ and $F^4 = .105336$. Intuitively, to hedge the risk of the 8% four-year bond one has to sell mostly 3% four-year bonds but then some 3% two-year bonds to take care of the relatively high exposure of the 8% bond to the short-end of the curve.

Note that the value of the hedging portfolio, $1 \times 1.053884 + 1 \times .015336$ or 1.15922, does not equal the 1.1871 value of the 8% bond being hedged.

5.3 Use Table 5.6 for this question. A trader constructs a butterfly portfolio that is short €100mm of the 10-year swap and long 50% of the 10-year swap's total '01 in 5-year swaps and 50% of the 10-year swap's total '01 in 15-year swaps. What are the forward-bucket exposures of the resulting portfolio?

A. The total '01 of short 10-year swap position is €100mm*.0857/100 = €85,700. To get 50% of that, or €42,850 in a 5-year swap position requires a face amount of €42,850 / (.0472/100) or €90.784mm. To get the other 50% in a 15-year swap position requires a face amount of €42,850 / (.1165/100) or €36,781mm. Multiplying each of the face amounts of these positions by its forward-bucket exposures gives the following table of EUR '01s. The last row gives the forward-bucket '01s of the resulting portfolio.

	Face (€mm)	**0–2**	**2–5**	**5–10**	**10–15**	**Total**
5-year	90.784	17,794	25,056	0	0	42,850
10-year	–100	–19,400	–26,900	–39,400	0	–85,700
15-year	36,781	7,136	9,747	14,087	11,880	42,850
Total		5,529	7,903	–25,313	11,880	

Chapter 6

The following introduction applies to Questions 6.1 through 6.5.

You are a market maker in long-term EUR interest rate swaps. You typically have to hedge the interest rate risk of having received from or paid to a customer on a 20-year interest rate swap. Given the transaction costs of hedging with both 10s and 30s and the relatively short time you wind up having to hold any such hedge, you consider hedging these 20-year swaps with either 10s or 30s but not both. To that end you run two single-variable regressions, both with changes in the 20-year EUR swap rates as the dependent variable, but one regression with changes in the 10-year swap rate as the independent variable and the other with changes in the 30-year swap rate as the independent variable. The results over the period July 1, 2009, to July 3, 2010, are given in the following table.

Number of Observations	259			
Independent variable	Change in 10-year		Change in 30-year	
R-squared	89.9%		96.3%	
Standard Error	1.105		.666	
Regression Coefficients	Value	Std. Error	Value	Std. Error
Constant	–.017	.069	–.008	.042
Independent variable	1.001	.021	.917	.011

6.1 What are the 95% confidence intervals around the constant and slope coefficients of each regression?

A. For the 10-year regression, the confidence interval corresponding to the constant and slope coefficients are

$$-.017 \pm 2 \times .069 = (-.155, .121)$$

$$1.001 \pm 2 \times .021 = (.959,\ 1.043)$$

For the 30-year regression, the confidence intervals are

$$-.008 \pm 2 \times .042 = (-.092, .076)$$

$$.917 \pm 2 \times .011 = (.895, .939)$$

6.2 Use the confidence intervals just derived. a) Can you reject the hypothesis that the constant in the 10-year regression equals 0? b) That the slope coefficient in the 30-year regression equals 1?

A. a) No; 0 is inside the relevant confidence interval. b) Yes; 1 is outside the relevant confidence interval.

6.3 As the swap market maker, you just paid fixed in 100 million notional of 20-year swaps. The *DV*01s of the 10-, 20-, and 30-year swaps are .0864, .1447, and .1911, respectively. Were you to hedge with 10-year swaps, what would you trade to hedge? And with 30-year swaps?

A. With 10-year swaps, the P&L variance-minimizing hedge is to receive fixed in 167.5 million swaps.

$$-\left(-100\text{mm}\right)\times\frac{.1447}{.0864}\times 1.001 = 167.64\text{mm}$$

With 30-year swaps, the hedge is to receive fixed in

$$-\left(-100\text{mm}\right)\times\frac{.1447}{.1911}\times .917 = 69.43\text{mm}$$

6.4 Approximately what would be the standard deviation of the P&L of a hedged position of 20-year swaps with 10-year swaps? And if hedged with 30-year swaps?

A. For the 10-year hedge, using the standard error of that regression,

$$100\text{mm}\frac{.1447}{100}\times 1.105 = 159,\ 894$$

For the 30-year, using the standard error of that regression,

$$100\text{mm}\frac{.1447}{100}\times .666 = 96,\ 370$$

6.5 If you were to hedge with one of either the 10- or 30-year swaps, which would it be and why?

A. The 30-year; the standard error of that regression is lower.

6.6 Use the principal components in Table 6.5 and the par swap data in Table 6.6 to hedge 100 face amount of 10-year swaps with 5- and 30-year swaps with respect to the first two principal components.

A. Solve the following two equations for the face amounts:

$$-F^5 \frac{.0468}{100} 6.85 - F^{30} \frac{.1731}{100} 5.38 - 100 \frac{.0842}{100} 6.35 = 0$$

$$-F^5 \frac{.0468}{100} (-1.53) - F^{30} \frac{.1731}{100} 1.09 - 100 \frac{.0842}{100} .06 = 0$$

The solution is $F^5 = -75.63$ and $F^{30} = -31.38$.

Chapter 7

7.1 A fixed income analyst needs to estimate the price of an interest rate caplet that pays $1,000,000 next year if the one-year Treasury rate exceeds 3% and pays nothing otherwise. Using a macroeconomic model developed in another area of the firm, the analyst estimates that the one-year Treasury rate will exceed 3% with a probability of 25%. Since the current one-year rate is 1%, the analyst prices the caplet as follows:

$$\frac{25\% \times \$1,\ 000,\ 000}{1.01} = \$247,\ 525$$

Comment on this pricing procedure.

A. It is the risk-neutral probabilities that are relevant for pricing, not the actual or real-world probabilities. This pricing procedure will almost certainly not yield the arbitrage-free price of the caplet.

7.2 Assume that the true six-month rate process starts at 5% and then increases or decreases by 100 basis points every six months. The probability of each increase or decrease is 50%. The prices of six-month, 1-year, and 1.5-year zeros are 97.5610, 95.0908, and 92.5069. Find the risk-neutral probabilities for the six-month rate process over the next year (i.e., two steps for a total of three dates, including today). Assume, as in the text, that the risk-neutral probability of an up move from date 1 to date 2 is the same from both date 1 states. As a check to your work, write down the price trees for the six-month, 1-year, and 1.5-year zeros.

A. The risk-neutral process is the following:

7%

70%

6%

60% 30%

5% 5%

40% 70%

4%

30%

3%

The price of the six-month zero is $\frac{100}{1+.05/2} = 97.5610$. The risk-neutral price tree for the 1-year zero is

100

70%

97.0874

60% 30%

95.0908 100

40% 70%

98.0392

30%

100

The risk-neutral price tree for the 1.5-year zero is

100

96.6184

70%

94.0788 100

60% 30%

92.5069 97.5610

40% 70%

95.9307 100

30%

98.5222

100

Note that the probabilities from date 2 to date 3 are not needed because the cash flow on date 3 is a certain 100.

7.3 Using the risk-neutral tree derived for Question 7.2, price \$100 face amount of the following 1.5-year *collared floater*. Payments are made every six months according to this rule. If the short rate on date i is r_i then the interest payment of the collared floater on date $i+1$ is $\frac{1}{2}3.50\%$ if $r_i < 3.50\%$; $\frac{1}{2}r_i$ if $6.50\% \geq r_i \geq 3.50\%$; $\frac{1}{2}6.50\%$ if $r_i > 6.50\%$. In addition, at maturity, the collared floater returns the \$100 principal amount.

A. The ex-coupon, risk-neutral price tree for the collared floater is

100

99.7585

70%

99.8358 100

60% 30%

99.9322 100

40% 70%

100.0724 100

30%

100.2463

100

Here are the calculations for each of the nodes:

Date 2, state 2:

$$\frac{100+100\times.5\times6.50\%}{1.035}=99.7585$$

Date 2, state 1:

$$\frac{100+100\times.5\times5.00\%}{1.025}=100$$

Date 2, state 0:

$$\frac{100+100\times.5\times3.50\%}{1.015}=100.2463$$

Date 1, state 1:

$$\frac{.7\times99.7585+.3\times100+100\times.5\times6\%}{1.03}=99.8358$$

Date 1, state 0:

$$\frac{.7\times100+.3\times100.2463+100\times.5\times4\%}{1.02}=100.0724$$

Date 0, state 0:

$$\frac{.6\times99.8358+.4\times100.0724+100\times.5\times5\%}{1.025}=99.9322$$

7.4 Using your answers to Questions 7.2 and 7.3, find the portfolio of the originally 1-year and 1.5-year zeros that replicates the collared floater from date 1, state 1, to date 2. Verify that the price of this replicating portfolio gives the same price for the collared floater at that node as derived for Question 7.3.

A. The payment on date 22 will be .5 $\times 6\%$. If the face amounts of the originally 1-year and 1.5-year zeros are F_1 and $F_{1.5}$ then the equations for replication are

$$F_1+.966184F_{1.5}=99.7585+3$$
$$F_1+.975610F_{1.5}=100+3$$

Solving, $F_1=78$ and $F_{1.5}=25.625$.

The price of the replicating portfolio is

$$.970874\times78+.940788\times25.625=99.8358$$

This equals the price of the collared floater on date 1, state 1 as derived using the risk-neutral pricing tree.

7.5 Using the risk-neutral tree from Question 7.2, price \$100 notional amount of a 1.5-year *participating cap* with a strike of 5% and a *participation rate* of 40%. Payments are made every six months according to the following rule. If the short rate on date i is r_i then the cash flow from the participating cap on date $i+1$ is, as a percent of par, $\frac{1}{2}(r_i - 5\%)$ if $r_i \geq 5\%$ and $\frac{1}{2}40\%(r_i - 5\%)$ if $r_i < 5\%$. There is no principal payment at maturity.

A. The ex-coupon, risk-neutral price tree for the participating cap is

					0
				.9662	
			70%		
		1.1421			0
	60%		30%		
.5468				0	
	40%		70%		
		-.3120			0
			30%		
				-.3941	
					0

Here are the calculations for each of the nodes:

Date 2, state 2:

$$\frac{.5 \times 100 \times (7\% - 5\%)}{1.035} = .9662$$

Date 2, state 1:

$$\frac{.5 \times 100 \times (5\% - 5\%)}{1.025} = 0$$

Date 2, state 0:

$$\frac{.4 \times .5 \times 100 \times (3\% - 5\%)}{1.015} = -.3941$$

Date 1, state 1:

$$\frac{.7\times.9662+.3\times0+.5\times100\times\left(6\%-5\%\right)}{1.03}=1.1421$$

Date 1, state 0:

$$\frac{.7\times0+.3\times\left(-.3941\right)+.4\times.5\times100\times\left(4\%-5\%\right)}{1.02}=-.3120$$

Date 0, state 0:

$$\frac{.6\times1.1421+.4\times\left(-.3120\right)+.5\times100\times\left(5\%-5\%\right)}{1.025}=.5468$$

7.6 Question 7.3 required the calculation of the price tree for a collared floater. Repeat this exercise under the same assumptions, but this time assume that the OAS of the collared floater is 10 basis points.

A. With an OAS of 10 basis points, the ex-coupon risk-neutral price tree for the collared floater is

					100
				99.7103	
			70%		
		99.7405			100
	60%		30%		
99.7898				99.9512	
	40%		70%		
		99.9755			100
			30%		
				100.1969	
					100

Here are the calculations for each of the nodes:

Date 2, state 2:

$$\frac{100+100\times.5\times 6.50\%}{1.0355}=99.7103$$

Date 2, state 1:

$$\frac{100+100\times.5\times 5.00\%}{1.0255}=99.9512$$

Date 2, state 0:

$$\frac{100+100\times.5\times 3.50\%}{1.0155}=100.1969$$

Date 1, state 1:

$$\frac{.7\times 99.7103+.3\times 99.9512+100\times.5\times 6\%}{1.0305}=99.7405$$

Date 1, state 0:

$$\frac{.7\times 99.9512+.3\times 100.1969+100\times.5\times 4\%}{1.0205}=99.9755$$

Date 0, state 0:

$$\frac{.6\times 99.7405+.4\times 99.9755+100\times.5\times 5\%}{1.0255}=99.7899$$

7.7 Using the price trees from Questions 7.3 and 7.6, calculate the return to a hedged and financed position in the collared floater from dates 0 to 1 assuming no convergence (i.e., the OAS on date 1 is also 10 basis points.) Hint #1: Use all of the proceeds from selling the replicating portfolio to buy collared floaters. Hint #2: You do not need to know the composition of the replicating portfolio to answer this question. Is your answer as you expected? Explain.

A. Refer to the answer to Question 7.3 for the price tree of the collared floater at an OAS of 0. By definition of the replicating portfolio, the replicating portfolio can be sold for the model value of the collared floater, i.e., 99.9322. These proceeds can be used to buy 100x99.9322/99.7898 or 100.1427 of the collared floater.

If rates go up, the collared floater at an OAS of 10 basis point is worth 99.7405 and a payment of 2.50 is collected for each 100 face. Hence the collared floater position is worth

$$100.1427\frac{99.7405+2.5}{100}=102.3864$$

By definition, the replicating portfolio at this node is worth the value of a collared floater at an OAS of zero, or 99.8358, plus the coupon payment of 2.50 for a total of 102.3358.

The financed and hedged position has experienced a return of

$$\frac{102.3864-102.3358}{99.9322}=.05\%$$

that is, of 5 basis points. This makes sense: over six months a financed and hedged position that did not converge earned half of the annualized OAS of 10 basis points.

If rates fall the collared floater position is worth

$$100.1427\frac{99.9755+2.5}{100}=102.6217$$

The replicating portfolio is worth 100.0724+2.5 or 102.5724. The return in this case, therefore, is

$$\frac{102.6217-102.5724}{99.9322}=.05\%$$

The answer is the same 5 basis points.

7.8 What is the return if the collared floater converges on date 1, i.e., its OAS equals zero on that date?

A. In this case, if rates rise the value of the position in the collared floater is

$$100.1427\frac{99.8358+2.5}{100}=102.4818$$

The return to the financed and hedged position increases to

$$\frac{102.4818-102.3358}{99.9322}=.1461\%$$

If rates fall the value of the position in the collared floater is

$$100.1427\frac{100.0724+2.5}{100}=102.7188$$

and the return is

$$\frac{102.7188-102.5724}{99.9322}=.1465\%$$

Chapter 8

8.1 Describe as fully as possible the qualitative effect of each of these changes on the instantaneous rates 10 and 30 years forward.

a) The market risk premium increases.

b) Volatility across the curve increases.

c) Rates are not expected to increase as much as previously.

d) The market risk premium falls and volatility falls in such a way as to keep the 10-year forward rate unchanged.

A: This question is based on equation (8.28).

1. a) Both rates increase, but the 30-year forward rate increases more because the duration of a 30-year zero is greater than that of a 10-year zero.

   b) Both rates fall, but the 30-year forward rate falls more because the convexity of a 30-year zero is greater than that of a 10-year zero.

   c) Both rates fall, but the 30-year forward rate falls more because the duration of a 30-year zero is greater than that of a 10-year zero.

   d) The 30-year forward rate increases. The ratio of convexity to duration is higher for a 30-year zero than for a 10-year zero. Therefore, if these effects cancel for the 10-year zero then the convexity effect will dominate for the 30-year zero. Since volatility declines, the 30-year forward rate increases.

Chapter 9

9.1 Assume an initial interest rate of 5%. Using a binomial model to approximate normally distributed rates with weekly time steps, no drift, and an annualized volatility of 100 basis points, what are the two possible rates on date 1?

A. The volatility over a week is $100\sqrt{\frac{1}{52}}$ or 13.87 basis points. Therefore, the two possible rates are 4.8613% and 5.1387%.

9.2 Add a drift of 20 basis points per year to the model described in Question 9.1. What are the two rates now?

A. The weekly drift is $20\times\frac{1}{52}$ or about .3846 basis points. Adding this to the results from question 9.1 gives rates of 5.2459% and 5.5233%.

9.3 Consider the following segment of a binomial tree with 6-month time steps. All transition probabilities equal .5.

		5.360%
	4.697%	
3.99%		4.648%
	3.964%	
		4.031%

Does this tree display mean reversion?

A. The drift from 4.697% on date 1 is 30.7 basis points:

$$.5\times5.360\%+.5\times4.648\%-4.697\%=.307\%$$

Similarly, the drift from 3.964% is 37.55 basis points. A higher drift from a lower rate level is indicative of mean reversion.

9.4 What mean reversion parameter is required to achieve a half-life of 15 years?

A. $\frac{\ln(2)}{15}=.046210$

Chapter 10

10.1 The yield volatility of a short-term interest rate is 20% at a level of 5%. Quote the basis point volatility and the CIR volatility parameter.

A. The basis point volatility is $20\% \times 5\% = .01$ or 100 basis points. The CIR volatility parameter is such that $\sigma^{CIR}\sqrt{.05} = .01$, which implies a parameter of about .0447.

10.2 You are told that the following tree was built with a constant volatility. All probabilities equal .5. Which volatility measure is, in fact, a constant?

		5.49723%
	4.69740%	
4.014%		4.63919%
	3.96420%	
		3.91507%

A. The basis point volatilities from date 0 and from date 1 are

	42.902
36.660	
	36.206

So the basis point volatility is not constant. Dividing each volatility by the short-term rate at the corresponding node gives a proportional volatility that equals 9.133% at each node. Hence, the proportional volatility is the measure being held constant.

10.3 Use the closed form solution for the Vasicek model in Appendix A in Chapter 10 to compute the spot rate of various terms with the parameters $\theta = 10\%$, $k = .035$, $\sigma = .02$, and $r_0 = 4\%$. Comment on the shape of the term structure.

A. The table below gives selected spot rates. The spot rate curve rises from its initial value of 4% toward its goal of 10%. But, because it approaches that goal rather slowly and because convexity reduces spot rates relatively quickly, the 30-year spot rate is less than 4%, at about 3.35%.

Term	Spot Rate (%)
.5	4.0506
1	4.0973
2	4.1799
3	4.2488
4	4.3049
5	4.3492
7	4.4055
10	4.4205
15	4.3014
20	4.0553
30	3.3521

Chapter 13

13.1 As of a spot settlement date of June 1, 2010, find the forward price of the U.S. Treasury 3 $\frac{5}{8}$ s of February 15, 2020, for delivery on September 30, 2010. The spot price is $102-21$ and the repo rate is .3%.

A. The following counts of (actual) days are necessary:

Start	End	Days
2/15/10	8/15/10	181
2/15/10	6/1/10	106
6/1/10	8/15/10	75
8/15/10	2/15/11	184
8/15/10	9/30/10	46

The accrued interest of the bond as of 6/1/10 is $\frac{1}{2}3.625\frac{106}{181}=1.061464$.

The accrued interest of the bond as of 9/30/10 is $\frac{1}{2}3.625\frac{46}{184}=.453125$.

As of 6/1/10, the present value of the coupon payment on 8/15/10 is $\frac{1}{2}3.625\left(1+\frac{.3\%\times 75}{360}\right)=1.81137$.

Then, using equation (13.3) of the text, the flat forward price is

$$\left(102\frac{21}{32}+1.061464-1.81137\right)\left(1+\frac{.3\%\times 121}{360}\right)-.453125=101.5560$$

13.2 Using your answer to Question 13.1, compute the forward yield of the $3\frac{5}{8}$s to September 30. Use equation (3.32) and a spreadsheet or other application.

A. There are 138 days from September 30, 2010, to February 15, 2011. Also, after the coupon payment of February 15, 2011, there are 18 left. Therefore, equation (3.32) is applied with $\tau=\frac{138}{181}$, with $T=9.5$, and with a full price for the left-hand side of $101.5560+.453125=102.009125$. Solving for the forward yield numerically gives 3.503%.

13.3. Use the risk-neutral tree, with annual steps, developed in the section "Arbitrage Pricing in a Multi-Period Setting" in Chapter 9. Consider a 5% 10-year bond that, two years from the starting date, takes on the values 104.701, 98.126, and 92.061, corresponding to the nodes 4%, 5%, and 6%, respectively. What is the forward price of the bond for delivery in two years? What is the futures price to that same delivery date?

A. Pricing the bond as of the starting date, using the three terminal values and the risk-neutral tree, gives a price of 95.598:

$$95.598=[.8024\times\frac{.6489\times 92.061+.3511\times 98.126+5}{1.055}$$
$$+.1976\times\frac{.6489\times 98.126+.3511\times 104.701+5}{1.045}+5]/1.05$$

The one-and two-year discount factors from the tree can be easily computed to be .952381 and .904438, respectively.

Hence, the forward price of the bond is

$$(95.598-.952381\times 5-.904438\times 5)/.904438=95.434$$

while the futures price is

$$\begin{aligned} 95.424 = .8024 \times (.6489 \times 92.061 + .3511 \times 98.126) \\ + .1976 \times (.6489 \times 98.126 + .3511 \times 104.701) \end{aligned}$$

Chapter 14

14.1 The conversion factor of the 4s of August 15, 2018, into TYU0 is .8774. If the price of TYU0 is 121.2039, what is the (flat) delivery price of the 4s at that time? If the price of the 4s is 107.1652, what is their cost of delivery at that time?

A. The delivery price is $.8774 \times 121.2039 = 106.3443$. The cost of delivery is $107.1652 - 106.3443 = .8209$.

14.2 The table below gives the prices of TYU0 and of its deliverable bonds in a particular term structure scenario on the delivery date that corresponds to a 7-year par yield of 3.32%. Conversion factors are also provided. Which bond is CTD in this scenario?

Futures Price:		117.2606	
			Conv.
Rate	**Maturity**	**Price**	**Factor**
$3\frac{1}{4}$	3/31/17	100.1567	.8538
$4\frac{1}{2}$	5/15/17	107.9783	.9202
$3\frac{1}{8}$	4/30/17	99.3447	.8471
$2\frac{3}{4}$	5/31/17	96.9980	.8272
$4\frac{3}{4}$	8/15/17	109.3542	.9314
$4\frac{1}{4}$	11/15/17	106.0361	.9012
$3\frac{7}{8}$	5/15/18	102.7277	.8732
4	8/15/18	103.2276	.8774
$3\frac{1}{2}$	2/15/18	100.6805	.8547

$3\frac{3}{4}$	11/15/18	101.0422	.8587
$3\frac{5}{8}$	8/15/19	98.9248	.8401
$3\frac{1}{8}$	5/15/19	95.5539	.8107
$2\frac{3}{4}$	2/15/19	93.3962	.7909
$3\frac{3}{8}$	11/15/19	96.8798	.8195
$3\frac{5}{8}$	2/15/20	98.6522	.8332
$3\frac{1}{2}$	5/15/20	97.6531	.8210

A: The 2 3/4s of May 31, 2017. Either compute the cost of delivery for each bond and see that this is 0 for the 2 3/4s or compute the ratio of price to conversion factor for each bond and note that the 2 3/4s give the minimum ratio (which equals the futures price). In searching for the CTD, it's best to start at the top of this deliverable list and work down: since the 7-year par rate is low relative to the notional coupon of the contract (6%), the CTD is most likely to be on the short-end of the maturity spectrum.

14.3 The 3/4s of May 31, 2012, are deliverable into TUU0, the September 2010 two-year note contract. Assume that the delivery date of the contract is September 30, 2010. The notional coupon of the contract is 6%. Approximately what is the conversion factor of the .75s for delivery into that contract?

A: Conversion factors are approximately equal to the price of 1 face amount of the bonds, as of the delivery date, at a yield of the notional coupon. Computing this price is done as follows. The accrued interest on this bond as of September 30, 2010, is $\frac{122}{183}\frac{.75}{2}=.25$. The next coupon is 2 months or 1/3 of a semi-annual period away. Hence, the full price of the bond at a yield of 6% is approximately

$$\frac{.5\times.75}{\left(1+\frac{6\%}{2}\right)^{1/3}}+\frac{.5\times.75}{\left(1+\frac{6\%}{2}\right)^{1+1/3}}+\frac{.5\times.75}{\left(1+\frac{6\%}{2}\right)^{2+1/3}}+\frac{100+.5\times.75}{\left(1+\frac{6\%}{2}\right)^{3+1/3}}=92.0386$$

Subtracting the .25 of accrued interest to get the flat price and normalizing to $1 face value gives a conversion factor of $\left(92.0386-.25\right)/100=.9179$.

14.4 The forward price of the 3 1/2s of February 15, 2018, to September 30, 2010, is 103.1303. Its conversion factor for delivery into TYU0 is .8547. If the price of TYU0 is 120, what is the net basis of the 3 1/2s in ticks?

A: The net basis is the forward price minus the conversion factor times the futures price. Here, $103.1303 - .8547 \times 120 = .5663$ or about 18 ticks.

14.5 A trader sells $50 million 3 1/2s net basis at the level you calculated in question 14.4. What is the trader's position in the bond and in TYU0? If the net basis of 3 1/2s is 10 ticks as of the delivery date, what is the trader’s profit or loss?

A: Selling $50 million 3 1/2s net basis means selling $50 million face amount of the 3 1/2s forward to the delivery date and purchasing a conversion-factor weighted position in futures. The conversion factor for the 3 1/2s is .8547, so this basis position requires the purchase of $.8547 \times \$50$ million or $42.735 million face amount of futures. Since each contract has $100,000 face value, this means buying 427 contracts.

If the net basis falls from .5663, as calculated for question 14.4 to 10 ticks or .3125, the short basis position wins in the amount of $\$50\text{mm} \times \frac{.5663 - .3125}{100} = \$126,\ 900$.

14.6 Figure 14.4 of the text graphs various net bases as option-like payoffs. Describe how each of the following deliverable bonds would look if added to this graph: the 3 1/4s of 3/31/2017; the 4s of 8/15/2018; and the 3 1/2s of 5/15/2022.

A: The net basis of the shorter-term bonds in the deliverable basket look like puts on bonds or calls on rates. The net basis of the intermediate-term bonds look like straddles. The net basis of the longer-term bonds look like calls on bonds or puts on rates. Of the bonds listed in this question, each pretty clearly falls into one of those maturity classifications.

14.7 How would the graphs in Figure 14.4 change if the curve steepened as the 7-year par rate increased?

A: The curve steepening means that long-term yields rise relative to short-term yields. This will make the long-term bonds cheaper as rates increase, closer to CTD, and, therefore, lower their net basis relative to shorter-term bonds. In the figure, the net basis curve of the 3 5/8s would decline faster and the net basis curve of the 4 1/2s would increase faster.

Chapter 15

15.1 As of May 28, 2010, you are financing $100 million worth of inventory of bonds in the repo market on an overnight basis. You plan to hold these bonds until mid-September 2010. Using both the Eurodollar and fed funds futures listed in Tables 15.3 and 15.11, what trades can you do to hedge against the risk that rates rise and increase your borrowing cost? Will you have to adjust the hedge at all between May 28 and mid-September?

A. You have to sell contracts, so as to gain on the hedge if rates increase. Selling 100 EDM0 will hedge the risk from mid-June to mid-September. Selling 20 FFM0 would hedge the risk over all of June, but, since only half that exposure is needed, sell only 10 FFM0. This is not ideal because its half the risk of the whole of June rather than all of the risk over the first half of June, but that is the price of trading standardized contracts. The hedge will have to be changed when EDM0 expires in mid-June. The expiring contracts will have to be replaced with June through September fed funds contracts. Also, if the value of the portfolio changes by much at any time, it might be worthwhile to adjust the hedge accordingly.

15.2 Approximately what borrowing rate is locked in by the hedge in Question 15.1?

A. Hedging with fed funds and Eurodollar contracts approximately locks in the rates of those contracts as of the time the hedge was implemented. In this case this means approximately locking in a rate of .220% for half of June (the rate on FFM0) and a rate of .6% from mid-June to mid-September. Assigning the first segment 15 days and the second 92 days gives a weighted average rate of

$$\frac{15\times.220\%+92\times.6\%}{107}=.547\%$$

15.3 Instead of the hedge constructed in Question 15.1, you decide to use only Eurodollar contracts. How does the hedge change? How does the locked-in rate compare with the previous hedge? Is this new hedge riskier in any way than the previous hedge?

A. "Stacking" the stub risk on EDM0 means increasing that position to get another 15 days of exposure. Each EDM0 covers 90 days of exposure, so you need an extra $\frac{15}{90}$ contracts, for a new position of $100\left(1+\frac{15}{90}\right)$ or about 117 contracts. You can get the same result by turning the exposure of the 10 FFM0 contracts into $\frac{10\times41.67}{25}$ or about 17 contracts.

This would seem to lock in a rate of .6% over the entire period. The risk is that the overnight rate rises more over the first half of June than are incorporated into the forward rate from mid-June to mid-September. In that case you would have to pay higher borrowing costs in the first half of June but EDM0 would not profit accordingly.

15.4 As of May 28, 2010, a 5% U.S. Treasury bond maturing on September 15, 2010, had a full price of 102.4055. Using the dates and rates of Table 15.8, calculate the TED spread of the bond.

A. The formula for the TED spread s in this case is

$$102.4055=\frac{102.50}{\left(1+\frac{(.3407\%-s)\times15}{360}\right)\left(1+\frac{(.6\%-s)\times91}{360}\right)}$$

Solving, $s=.25\%$.

15.5 As of the end of July 2004, the fed funds target rate stood at 1.25%. Say that the August fed fund futures rate at that time was 1.3516%. What is the market implied probability of a 25 basis-point increase at the August 10 meeting? If you're willing to assume a 50% chance of no change in policy, what are the implied probabilities of 25 and 50 basis-point increases?

A. As for the first question

$$\frac{10\times1.25\%+21\times\left[(1-p)\times1.25\%+p\times1.50\%\right]}{31}=1.3516\%$$
$$p=60\%$$

As for the second, verify that

$$\frac{10\times 1.25\% + 21\left[50\%\times 1.25\% + 40\%\times 1.50\% + 10\%\times 1.75\%\right]}{31} = 1.3516\%$$

Chapter 16

16.1 Recalculate swap cash flows as in Table 16.1 for a 1.5% swap rate and a LIBOR rate that starts at 50 basis points on June 2, 2010, but then increases by 50 basis points every three months, reaching 4% by March 2, 2012.

A. This table gives the new cash flows:

LIBOR	DATE	DAYS	FIXED	FLOATING
.50%	6/2/10			
1%	9/2/10	92		127,778
1.5%	12/2/10	91	750,000	252,778
2%	3/2/11	90		375,000
2.5%	6/2/11	92	750,000	511,111
3%	9/2/11	92		638,889
3.5%	12/2/11	91	750,000	758,333
4%	3/2/12	91		884,722
	6/4/12	94	758,333	1,044,444

16.2 Under the same simplifying assumptions used to price the CMS swap in Table 16.3, what is the fair fixed rate against the 10-year swap rate paid annually for four years starting in year 2?

A. Under the simplifying assumptions of Table 16.3, the only part of the pricing that changes is the term in parentheses on the right-hand side of equation (16.9). The value of this term for a 10-year, rather than a 5-year swap rate, is

$$\frac{1}{4\%} - 10\frac{\left(1+\frac{4\%}{2}\right)^{-21}}{1-\left(1+\frac{4\%}{2}\right)^{-20}} = 4.825138$$

Hence, the convexity correction incrases by the multiplier $\frac{4.825138}{2.616047} = 1.844439$, i.e., from 3.25 basis points to 5.99 basis points. With this correction, the fair fixed rate against the 10-year swap rate 4.0599%.

Chapter 17

17.1 Consider one- and two-year swaps of annal-paying .xed vs. annual-paying LIBOR with par rates of 2% and 2.75%, respectively. The investable and collateral rates, given by the OIS curve, are 1% for one year and 2% for one year one year forward. What is the NPV of receiving 3% for two years against LIBOR?

A. Using the two-curve pricing method, the first projected LIBOR rate, L_1', is such that

$$\frac{1+L_1'}{1.01}=\frac{1.02}{1.01}$$

so $L_1'=2\%$. The second projected LIBOR rate, L_2', is such that

$$\frac{L_1'}{1.01}+\frac{1+L_2'}{1.01\times1.02}=\frac{.0275}{1.01}+\frac{1.0275}{1.01\times1.02}$$

with $L_1'=2\%$, it can be solved that $L_2'=3.515\%$.

Finally, then, the NPV of receiving in a 3% two-year swap is

$$\frac{.03}{1.01}+\frac{1.03}{1.01\times1.02}-\frac{L_1'}{1.01}+\frac{1+L_2'}{1.01\times1.02}$$

which, using the calculated values for projected LIBOR equals .004902 per unit face amount.

17.2 Using the data from Question 17.1, what is the implied term structure of basis swap spreads of OIS vs. LIBOR?

A. The one-year basis swap spread is clearly 100 basis points: paying one-year OIS of 1% plus a spread of 100 basis points is fair against one-year LIBOR at 2%. The two-year basis swap spread, *X* (2), solves the following equation

$$\frac{L_1'}{1.01}+\frac{1+L_2'}{1.01\times1.02}=1+X(2)\left[\frac{1}{1.01}+\frac{1}{1.01\times1.02}\right]$$

Given the values of projected LIBOR from the answer to Question 17.1, namely, $L_1' = 2\%$ and $L_2' = 3.515\%$, $X(2) = 1.255\%$, meaning that two-year OIS plus 125.5 basis points is fair against two-year LIBOR.

Chapter 18

18.1 Using the formulae in the text, recalculate the value of the caplet in Table 18.1 by changing only the volatility from 77.22 basis points to 120 basis points.

A. The value of ξ^N changes to .01210, so the value of the caplet is .3036.

18.2 Using the formulae in the text, calculate the value of an at-the-money 2y5y receiver swaption on $100 million notional when the term structure is flat at 4% and the appropriate volatility is 100 basis points.

A. The parameters in this case are $S_0 = 4\%$; $T = 2$; $\tau = 5$; $K = 4\%$; $\sigma = 1\%$.

The required annuity value is

$$A_0(2,\ 2+5) = \frac{1}{\left(1+\frac{4\%}{2}\right)^4}\left[\frac{1}{4\%}\left(1-\frac{1}{\left(1+\frac{4\%}{2}\right)^{10}}\right)\right] = 4.14926$$

Note that the term in the square brackets is the value of the annuity after two years, which is then discounted for two years to give the desired quantity.

Using these parameters and the formulae in the text,

$$\pi^N(4\%,\ 2,\ 4\%,\ 1\%) = .005642$$

Finally then, the value of the swaption is

$$V_0^{\text{Receiver}} = \$100\text{mm} \times 4.14926 \times .005642 = \$2.341\text{mm}$$

Chapter 19

19.1 Consider a 10-year corporate bond with a coupon of 6%. The semi-annual compounded swap curve is flat at 4% and the corporate bond is trading at a LIBOR

OAS of 3%. Calculate the par-par asset swap spread and the market value asset swap spread.

A. The price of the bond (per 100 face amount) is found by discounting at a yield of 7%:

$$\frac{6}{7\%}\left[1-\left(1+\frac{7\%}{2}\right)^{-20}\right]+100\left(1+\frac{7\%}{2}\right)^{-20}=92.8938$$

The fixed annuity factor is

$$\frac{1}{4\%}\left[1-\left(1+\frac{4\%}{2}\right)^{-20}\right]=8.1757$$

Taking the quarterly compounded rate to as 3.9802%, so that $\left(1+\frac{3.9802\%}{4}\right)^2=1+\frac{.04}{2}$, the floating annuity factor is

$$\frac{1}{3.9802\%}\left[1-\left(1+\frac{3.9802\%}{4}\right)^{-40}\right]=8.2164$$

The semiannual discount factor for the final bond payment is

$$\frac{1}{\left(1+\frac{4\%}{2}\right)^{20}}=.672971$$

Then, using equation (19.1),

$$s^{PAR}=\frac{6\times 8.1757+67.2971-92.8938}{100\times 8.2164}=2.855\%$$

Using equation (19.2),

$$s^{Mkt}=\frac{100\times 2.855\%}{92.8938}=3.073\%$$

19.2 Say that the cumulative default rate over a 10-year horizon for some category of corporate bonds is 5%. If the recovery rate is 40%, what is the spread that just compensates investors for expected losses?

A. There are two way to calculate this. First, use equation (19.12) directly:

$$s = -\frac{1-40\%}{10}\ln(1-5\%) = .308\%$$

Second, calculate the instantaneous default rate from (19.11)

$$5\% = 1 - e^{-10\lambda}$$
$$\lambda = .5129\%$$

and then calculate the spread from (19.10):

$$s = .5129\%(1-40\%) = .308\%$$

19.3 The quoted spread on a one-year quarterly paying CDS is 110 basis points while the standardized coupon is 100 basis points. Let the assumed recovery rate be 40% and let the quarterly compounded term structure of swap rates be flat at 3%. What is the up-front payment for $10 million notional of the CDS? You will need to construct a spreadsheet to perform these calculations.

A. The answer is $9,726. Here is a a copy of the spreadsheet after the problem has been solved:

	A	**B**	**C**	**D**
1	Swap	3%	V Fee	1.0699%
2	Hazard	1.833%	V Cont	1.0699%
3	Quoted	110	diff	0
4	Coupon	100	Up-front	.09726%
5	R	40%	10mm	9,726
6				
7	time	CS(t[i])	CS(t[i–1])–CS(t[i])	df
8	.25	.995427	.004573	.992556
9	.5	.990875	.004552	.985167
10	.75	.986344	.004531	.977833
11	1	.981833	.004510	.970554

The order of computations is as follows. Fill in the parameters in cells B1, B3, B4, and B5. Make up a starting value for the hazard rate in B2.

Calculate CS in B8:B11 from (19.13). Calculate C8:C11 as differences of B8:B11. This means that C8=1-B8; C9=B8-B9; C10=B9-B10, etc. Calcluate the discount factors in D8:D11 from the swap rate in B1. Calculate the value of the fee leg and the value of the contingent leg using (19.14) and (19.15).

Now use the solver to find the hazard rate such that the difference between the fee and contingent legs (D3) is zero.

Finally, use equation (19.16) to calculate the up-front payment per unit notional in D4 and multiply by the $10mm notional to get the dollar payment in D5.

19.4. Create a spreadsheet to recreate the duration calculations in the subsection "The DV01 or Duration of a Bond with Credit Risk." Use this spreadsheet to compute the duration in the example of that subsection with a coupon of 8% instead of 6%, keeping the yield at 14%.

A. Here is a copy of the spreadsheet for the example of the text. The price of the bond at a yield of 14% is 57.6239 (which is to four places instead of the two in the text). Put an initial guess in for the hazard rate, which value will turn out to be 23.0%. Set the recovery rate at 40%. Next calculate the columns $CS(t)$ and $CS(t-1) - CS(t)$; see the text and the answer to Question 19.3. Then compute the model price of the bond using equation (19.19), which depends, of course, on the hazard rate. Use the solver to find the hazard rate such that this model price is equal to the market price of 57.6239. Then compute the column df'with a discount rate of one basis point below the discount rate, which become 3.99% in this example. Next recompute the bond price using (19.19) but this shifted discount function. The shifted price turns out to be 57.5467. The calculate the duration.

With a coupon of 8%, the market price at a yield of 14% is 68.281. The implied hazard rate is 19.6% and the resulting duration is 4.2736.

Coupon		6%		
Discount Rate		4%		
Market Price		57.6239		
Hazard Rate		23.0%		
Recovery Rate		40%		
Model Price		57.6239		
Shifted Price		57.5467		
Duration		3.9533		
Term	df	$CS(t)$	$CS(t-1) - CS(t)$	df'
.5	.9804	.8913	.1087	.9804
1	.9612	.7944	.0969	.9613
1.5	.9423	.7081	.0863	.9425
2	.9238	.6311	.0770	.9240
2.5	.9057	.5625	.0686	.9060
3	.8880	.5014	.0611	.8882
3.5	.8706	.4469	.0545	.8709
4	.8535	.3983	.0486	.8538
4.5	.8368	.3550	.0433	.8371
5	.8203	.3164	.0386	.8208
5.5	.8043	.2821	.0344	.8047
6	.7885	.2514	.0307	.7890
6.5	.7730	.2241	.0273	.7735
7	.7579	.1997	.0244	.7584
7.5	.7430	.1780	.0217	.7436
8	.7284	.1587	.0193	.7290
8.5	.7142	.1414	.0172	.7148
9	.7002	.1260	.0154	.7008
9.5	.6864	.1123	.0137	.6871
10	.6730	.1001	.0122	.6736

Chapter 20

20.1 Assume that the term structure of monthly compounded rates is flat at 6%. Find the monthly payment of a $100,000 15-year level-pay mortgage.

A. From equation (20.2),

$$X\frac{12}{y}\left[1-\frac{1}{\left(1+\frac{y}{12}\right)^{12T}}\right]=100,\ \ 000$$

With $y=6\%$ and $T=15$, $X=843.86$.

20.2 For the mortgage in Question 20.1, what is the interest component of the monthly payment after five years?

A. Along the lines of equation (20.5), the principal outstanding after 5 years, with 10 year remaining in the mortgage, is

$$843.86\frac{12}{6\%}\left[1-\frac{1}{\left(1+\frac{6\%}{12}\right)^{12\times10}}\right]=76,\ \ 009$$

The interest rate component is the outstanding balance times $\frac{6\%}{12}$ or 380.

20.3 An adjustable-rate mortgage (ARM) resets the interest rate periodically. How does the refinancing option of an ARM compare with the option to prepay a fixed rate mortgage?

A. Since the interest rate on an ARM floats, the value of a hypothetical non-prepayable ARM will not deviate from face value as much as the value of a hypothetical non-prepayable fixed rate mortgage. Hence, the refinancing option will be worth less for an ARM. Total prepayment differences across asset classes will depend on differences in default characteristics as well as differences in refinancing behavior.

20.4 Explain the intuition for each of the following results

(a) When interest rates fall, holding all else equal, POs outperform 30-year fixed rate securities.

(b) When interest rates rise by 100 basis points, mortgage pass-throughs fall by about 7%. When interest rates fall by 100 basis points, pass-throughs rise by 4%.

(c) When interest rates decline, IOs and inverse IOs decline in price, but IOs suffer more severely. (Like an IO, an inverse IO receives no principal payments but receives interest payments that float inversely with the level of rates.)

A. (a) Without prepayments, POs resemble zeros and, consequently, have longer durations than coupon bonds. In addition, because of prepayments, POs get principal back early as rates fall. Since the PO collects no interest payments, the early return of principal is unabmiguously desirable. These two factors explain the outperformance of POs in falling rate environments.

(b) This follows from the negative convexity genearated by the prepayment option.

(c) When rates decline and prepayments accelerate, IOs lose value because interest payments vanish. Inverse IOs receive some benefit in that, at the same time, the size of the payments they do receive increases.

20.5. Recompute the value of the roll in the example of the text for a coupon of 6%, a paydown percentage of 3%, and an August TBA price of 102.1. Keep all other quantities the same.

A. The value of the roll is $5,744.

Accrued interest is $100 \times (12/30) \times 6\%/12$ or .2. Sellting the July TBA gives proceeds of $\$10mm \times (102.50 + .2)/100$ or $10,270,000. Investing this for the month at 1% gives interest of $\$10,270,000 \times (31/360) \times 1\%$ or $8,844. Purchasing the August TBA, with the 3% paydown, costs $\$10mm \times (1-3\%) \times (102.1 + .2)/100 \text{ or } \$9,923,100$. The net proceeds from the role are $\$10,270,000 + \$8,844 - \$9,923,100 \, or \, \$355,744.$

The proceeds from not rolling are $\$10mm \times (6\%/12 + 3\%)$ or $350,000. Hence, the value of the roll is $\$355,744 - \$350,000 \, or \, \$5,744$.