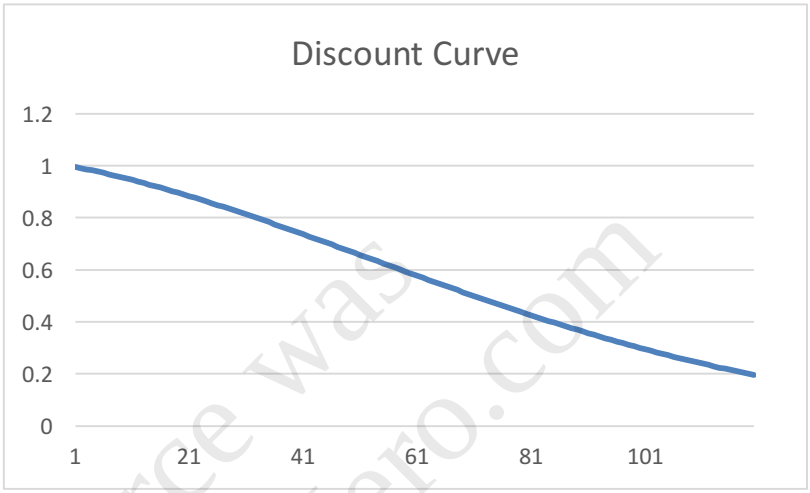
Homework 3 Solution. Quantitative Methods for fixed Income Securities

CHAPTER 3 Yield-to-Maturity

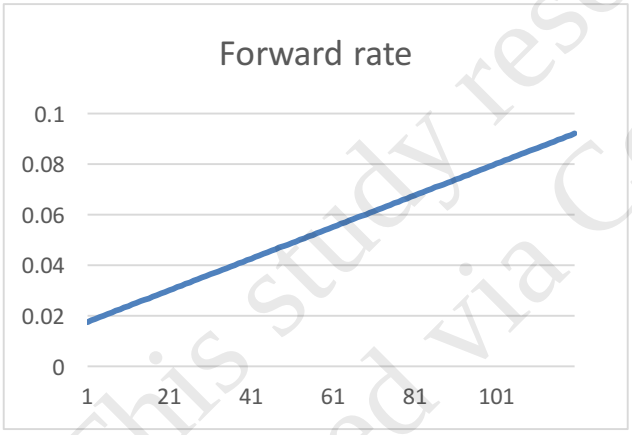
3.9 $\hat{r}\left(\frac{i}{4}\right) = 0.0175 + 0.00125 \times \frac{i-1}{4}$ (Refer to Swap P&L 1)

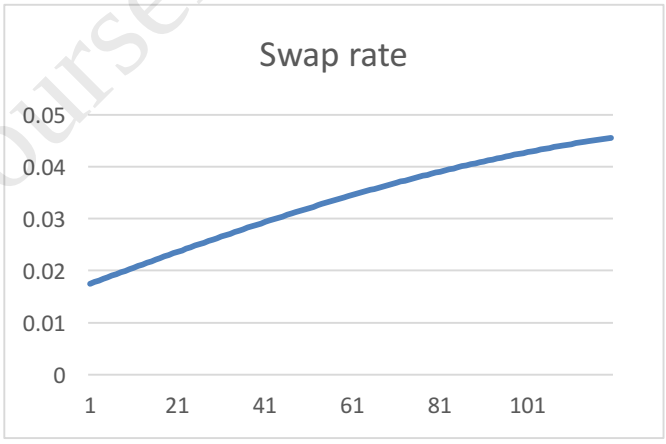
(a) Recall $d(t) = \frac{1}{(1+\frac{\hat{r}(t)}{4})^{4t}}$ for quarterly compounding, we can calculate $d\left(\frac{i}{4}\right)$ based on the above function, i.e. $d\left(\frac{i}{4}\right) = \frac{1}{(1+\frac{\hat{r}\left(\frac{i}{4}\right)}{4})^{i}}$.



(b) $f\left(\frac{i}{4}\right) = 4\left(\frac{d\left(\frac{i-1}{4}\right)}{d\left(\frac{i}{4}\right)} - 1\right)$

(c) $s(T) = \frac{1-d(T)}{\sum_{i=1}^{2T}\frac{1}{2}d\left(\frac{i}{2}\right)}$





(d) For 10-year swap: $s(20) = 0.02936791$, and after one year, it becomes 9-year swap, the updated $s(18)_{new} = 0.027473682$.
The P&L of the payer's swap (for a buyer): $\left(s(18)_{new} - s(20)\right) * \frac{1}{2} * \sum_{i=3}^{20} d\left(\frac{i}{2}\right) = -0.015191265$.

3.10 (a) $f(1.25) = 4\left(\frac{d(1)}{d(1.25)} - 1\right) = 0.02051282$, the fair fixed rate for the trade should be the corresponding forward rate, i.e. 2.05%.

(b) 2.85%>2.05%. The cash flow of A is that it pays fixed rate at 2.05% of $100m and receives 3m LIBOR rate at 2.85% of $100m, so the P&L of A is d(1,1.25)*(2.85%-2.05%)*100m*1/4 = 1/(1+2.85%/4)* (2.85%-2.05%)*100m*1/4 = $0.1986m.

(c) The cash flow of B is that it pays 3m LIBOR rate at 2.85% of $100m but receives a fixed rate at 2.05% of $100m, so the P&L of B is d(1,1.25)*(2.05%-2.85%)*100m*1/4 = -$0.1986m.

3.11 (a) 6 months later, $f(0.75) = 4\left(\frac{d(0.5)}{d(0.75)} - 1\right) = 0.06185567$, the updated fair fixed rate for the trade should be around 6.19%. The MtM value of FRA: d(0.75)*100m*1/4*(6.19%-2.05%) = $1.00395m.

(b) If A chooses to close out the FRA, the P&L of A is d(0.75)*100m*1/4*(6.19% - 2.05%) = $1.00395m.

3.12 (a) $f(2.25) = 4\left(\frac{d(2)}{d(2.25)} - 1\right) = 0.0225009$

(b) $f(1.25)_{new} = 4\left(\frac{d(1)}{d(1.25)} - 1\right) = 0.02100016$

(c) If A chooses to close out the FRA, the P&L of A is d(1.25)*100m*1/4*(2.1%-2.25%) =

-$0.0365764m.