Homework 1 Solution. Quantitative Methods for fixed Income Securities

CHAPTER 1 Bond Prices, Discount Factors, and Arbitrage (Tuckman)

For the following problems, we assume that today is May 15, 2001.

1.1 10/30/2001 $20
    4/30/2002 $20
    10/30/2002 $20
    4/30/2003 $1,020

1.2 a. $10,000*(112+2^5/_8/32)/100=$11,208.20
    b. $1,000*(99+14.5/32)/100=$994.53
    c. $1,000,000*(107+4/32)/100=$1,071,250.00

1.3 The discount factors must satisfy the following equations:

$$103.75\,d\!\left(.5\right)=101-25\tfrac{3}{4}$$

$$3.75\,d\!\left(.5\right)+103.75\,d\!\left(1\right)=103-12\tfrac{15}{16}$$

$$5.8125\,d\!\left(.5\right)+5.8125\,d\!\left(1\right)+105.8125\,d\!\left(1.5\right)=110-21\tfrac{1}{4}$$

The solution is
$$d\!\left(.5\right)=.981250$$
$$d\!\left(1\right)=.961201$$
$$d\!\left(1.5\right)=.939148$$

1.4 3.753*0.98125+3.753*0.961201+103.753*0.939148=104.720765.

1.5 Let F_1, F_2, and F_3 be the face amounts of the bonds listed in question 1.3, respectively. The portfolio that replicates $100 face amount of the 7.5s of November 15, 2002, satisfies the following equations:

$$103.75\%\times F_1+3.75\%\times F_2+5.8125\%\times F_3=3.75$$
$$103.75\%\times F_2+5.8125\%\times F_3=3.75$$
$$105.8125\%\times F_3=103.75$$

Solving,

$$F_1=-1.810843$$
$$F_2=-1.878749$$
$$F_3=98.050797$$

Since the 7.5s of November 15, 2002, are rich at 105, the arbitrage trade is to sell the 7.5s of November 15, 2002, and buy the replicating portfolio. Buying the replicating portfolio means selling 1.81 of the 7.5s of November 15, 2001, selling 1.88 of the 7.5s of May 15, 2002, and buying 98.05 of the 11.625s of November 15, 2002.

Since the arbitrage price of the 7.5s of November 15, 2002, is 104.720765, the price of the replicating portfolio must also be 104.720765. Therefore, the arbitrage profit of the trade is 105−104.720765=.279235 per $100 face amount of the 7.5s of November 15, 2002.