

Homework 3 Solution. Quantitative Methods for fixed Income Securities

CHAPTER 3 Yield-to-Maturity

$$3.9 \hat{r}\left(\frac{i}{4}\right) = 0.0175 + 0.00125 \times \frac{i-1}{4} \quad (\text{Refer to [Swap P&L 1](#)})$$

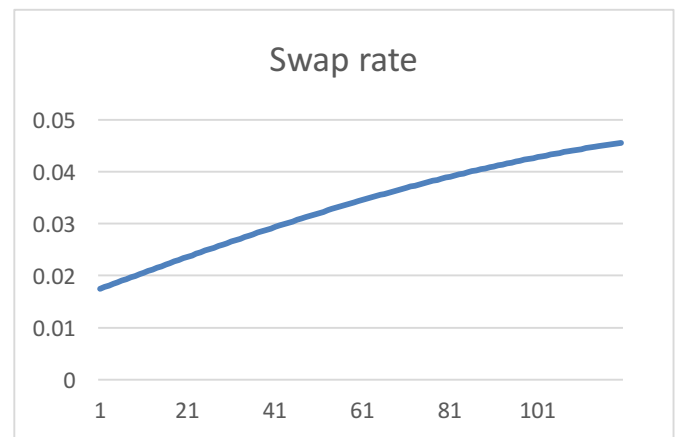
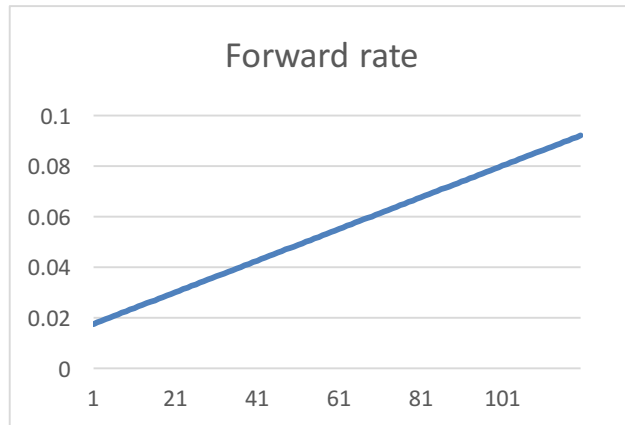
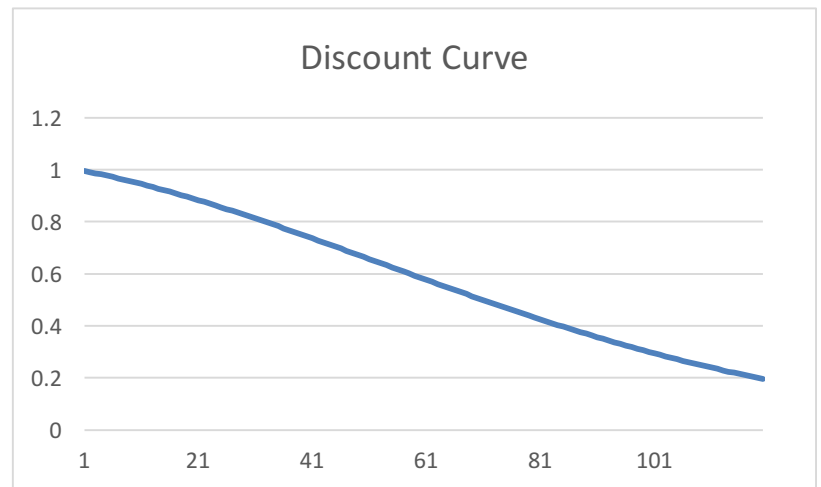
(a) Recall $d(t) = \frac{1}{(1+\frac{\hat{r}(t)}{4})^{4t}}$ for quarterly

compounding, we can calculate $d\left(\frac{i}{4}\right)$ based on the above function, i.e.

$$d\left(\frac{i}{4}\right) = \frac{1}{(1+\frac{\hat{r}(\frac{i}{4})}{4})^i}.$$

$$(b) f\left(\frac{i}{4}\right) = 4\left(\frac{d(\frac{i-1}{4})}{d(\frac{i}{4})} - 1\right)$$

$$(c) s(T) = \frac{1-d(T)}{\sum_{i=1}^{2T} \frac{1}{2} d(\frac{i}{2})}$$



(d) For 10-year swap: $s(20) = 0.02936791$, and after one year, it becomes 9-year swap, the updated $s(18)_{new} = 0.027473682$.

The P&L of the payer's swap (for a buyer): $(s(18)_{new} - s(20)) * \frac{1}{2} * \sum_{i=1}^{18} d\left(\frac{i}{2}\right) = -0.015191265$. (now we are at the time $t=1$ year)

3.10 (a) $f(1.25) = 4\left(\frac{d(1)}{d(1.25)} - 1\right) = 0.02051282$, the fair fixed rate for the trade should be the corresponding forward rate, i.e. 2.05%.

(b) $2.85\% > 2.05\%$. The cash flow of A is that it pays fixed rate at 2.05% of \$100m and receives 3m LIBOR rate at 2.85% of \$100m, so the P&L of A is $d(1,1.25) \cdot (2.85\% - 2.05\%) \cdot 100m \cdot 1/4 = 1/(1+2.85\%/4) \cdot (2.85\% - 2.05\%) \cdot 100m \cdot 1/4 = \$0.1986m$.

(c) The cash flow of B is that it pays 3m LIBOR rate at 2.85% of \$100m but receives a fixed rate at 2.05% of \$100m, so the P&L of B is $d(1,1.25) \cdot (2.05\% - 2.85\%) \cdot 100m \cdot 1/4 = -\$0.1986m$.

3.11 (a) 6 months later, $f(0.75) = 4 \left(\frac{d(0.5)}{d(0.75)} - 1 \right) = 0.06185567$, the updated fair fixed rate for the trade should be around 6.19%. The MtM value of FRA: $d(0.75) \cdot 100m \cdot 1/4 \cdot (6.19\% - 2.05\%) = \$1.00395m$.

(b) If A chooses to close out the FRA, the P&L of A is $d(0.75) \cdot 100m \cdot 1/4 \cdot (6.19\% - 2.05\%) = \$1.00395m$.

3.12 (a) $f(2.25) = 4 \left(\frac{d(2)}{d(2.25)} - 1 \right) = 0.0225009$

(b) $f(1.25)_{new} = 4 \left(\frac{d(1)}{d(1.25)} - 1 \right) = 0.02100016$

(c) If A chooses to close out the FRA, the P&L of A is $d(1.25) \cdot 100m \cdot 1/4 \cdot (2.1\% - 2.25\%) = -\$0.0365764m$.