

PART ONE

EXERCISES

Chapter 1

1.1 What are the cash flow dates and the cash flows of \$1,000 face value of the U.S.

Treasury $2\frac{3}{4}$ s of May 31, 2017, issued on May 31, 2010?

1.2 Use this table of U.S. Treasury bond prices for settle on May 15, 2010, to derive the discount factors for cash flows to be received in 6 months, 1 year, and 1.5 years.

BOND	PRICE
$4\frac{1}{2}$ s of 11/15/2010	102.15806
0s of 5/15/2011	99.60120
$1\frac{3}{4}$ s of 11/15/2011	101.64355

1.3 Suppose there existed a Treasury issue with a coupon of 2% maturing on November 15, 2011. Using the discount factors derived from Question 1.2, what would be the price of the 2s of November 15, 2011?

1.4 Say that the 2s of November 15, 2011, existed and traded at a price of 101 instead of the price derived from Question 1.3. How could an arbitrageur profit from this price difference using the bonds in the earlier table? What would that profit be?

1.5 Given the prices of the two bonds in the table as of May 15, 2010, find the price of the third by an arbitrage argument. Since the $3\frac{1}{2}$ s of 5/15/2020 is the on-the-run 10-year, why might this arbitrage price not obtain in the market?

BOND	PRICE
0s of 5/15/2020	69.21
$3\frac{1}{2}$ s of 5/15/2020	?
$8\frac{3}{4}$ s of 5/15/2020	145.67

Chapter 2

2.1 You invest \$100 for two years at 2% compounded semiannually. How much do you have at the end of the two years?

2.2 You invested \$100 for three years and, at the end of those three years, your investment was worth \$107. What was your semiannually compounded rate of return?

2.3 Using the discount factors in the table, derive the corresponding spot and forward rates.

TERM	DISCOUNT FACTOR
.5	.998752
1	.996758
1.5	.993529

2.4 Are the forward rates above or below the spot rates in the answers to Question 2.3? Why is this the case?

2.5 Using the discount factors from question 2.3, price a 1.5-year bond with a coupon of .5%. If over the subsequent 6 months the term structure remains unchanged, will the price of the .5% bond increase, decrease, or stay the same? Try to answer the question before calculating and then calculate to verify.

Chapter 3

3.1 The price of the $\frac{3}{4}$ s of May 31, 2012 was 99.961 as of May 31, 2010. Calculate its price using the discount factors in Table 2.3. Is the bond trading cheap or rich to those discount factors? Then, using trial-and-error, express the price difference as a spread to the spot rate curve implied by those discount factors.

3.2 The yield of the $\frac{3}{4}$ s of May 31, 2012, was .7697% as of May 31, 2010. Verify that this is consistent with the price in Question 3.1.

3.3 The price of the $4\frac{3}{4}$ s of May 31, 2012, was 107.9531 as of May 31, 2010. What was the yield of the bond? Please solve by trial-and-error.

3.4 Did you get a higher yield for the $4\frac{3}{4}$ s from Question 3.3 than the yield of the $\frac{3}{4}$ s given in Question 3.2? Is that what you expected? Why or why not?

3.5 An investor purchases the $4\frac{3}{4}$ s of May 31, 2012 on May 31, 2010, at the yield given in Question 3.3. Exactly six months later the investor sells the bond at that same yield. What is the price of the bond on the sale date and what is the investor's total return from the bond over those six months?

3.6 Interpret your answer to Question 3.5. a) In what way is the return significant or interesting? b) Explain why an investor would buy a premium bond when that bond is worth only par at maturity? How does this relate to your work in Question 3.5?

3.7 Re-compute the sample return decomposition of Tables 3.2 and 3.3 of the text, replacing the assumption of realized forwards with the assumption of an unchanged term structure.

3.8 Start with any upward-sloping term structure, e.g., from C-STRIPS prices or even some made-up rates. Then replicate the 0-coupon, par, and 9% coupon curves in Figure 3.2. Add a curve for a security that makes equal fixed payments to various maturities, i.e., a mortgage.

3.9 In the subsection "News Excerpt: Sale of Greek Government Bonds in March, 2010," approximately what is the yield on seven-year Spanish debt?

3.10 Return to Table 1.7 in the text, which shows that the $3\frac{1}{2}$ s of May 15, 2020, are

2.076 per 100 face amount away from being correctly priced by C-STRIPS while the $8\frac{3}{4}$ s maturing on the same date are .338 per 100 face amount away. According to the discussion of the text, this difference is due to the on-the-run premium of the $3\frac{1}{2}$ s that is reflected in the price of its P-STRIPS. As of the same pricing date, however, the yields

of the $3\frac{1}{2}$ s and $8\frac{3}{4}$ s were only a few basis points apart, i.e., nothing like the difference justified by the more than 2% premium on the price of the final principal payment. How is this possible?

Chapter 4

4.1 The following tables give the prices of TYU0 and of TYU0C 120 as of May 2010 for a narrow range of the 7-year par rate. Please fill in the other columns, ignoring cells marked with an “X.” Over the given range, which security’s price-rate function is concave and which convex? How can you tell?

TYU0						
Rate	Price	DV01	Duration	Convexity	1st Deriv	2nd Deriv
3.32%	115.5712	X	X	X	X	X
3.41%	114.8731			X		X
3.50%	114.1715					
3.60%	113.4668			X		X
3.69%	112.7591	X	X	X	X	X
TYU0C 120						
Rate	Price	DV01	Duration	Convexity	1st Deriv	2nd Deriv
3.32%	0.4564	X	X	X	X	X
3.41%	0.3483			X		X
3.50%	0.2619					
3.60%	0.194			X		X
3.69%	0.1415	X	X	X	X	X

4.2 Using the data in Question 4.1, how would a market maker hedge the purchase of \$50 million face amount of TYU0C with TYU0 when the 7-year par rate is 3.596%?

Check how well this hedge works by computing the change in the value of the position should the rate move instantaneously from 3.596% to 3.668%. What if the rate falls to 3.320%? Is the P&L of the hedged position positive or negative? Why is this the case?

4.3 Using the data from the answer to Question 4.1, how much would an investment manager make from \$100mm of TYU0C if the rate instantaneously fell from 3.504% to 3.404%? Use a duration estimate.

4.4 Using the data in Question 4.1, provide a 2nd order estimate of the price of TYU0C should the 7-year par rate be 3.75%.

4.5 The table below gives the prices, durations, and convexities of three bonds. a) What is the duration and convexity of a portfolio that is long \$50mm face amount of each of the 5- and 10-year bonds? b) What portfolio of the 5- and 30-year bonds has the same price and duration as the portfolio of part a)? c) Which of the two portfolios has the greater convexity and why?

COUPON	MATURITY	PRICE	DURATION	Convexity
2.50%	5 years	102.248	4.687	25.052
2.75%	10 years	100.000	8.691	86.130
3%	30 years	95.232	19.393	495.423

4.6 The following table gives yields, *DV01*s, and durations for three 15-year bonds. The three coupon rates are 0%, 3.5%, and 7%. Which coupon rate belongs to which bond? What is the shape of the term structure of spot rates underlying the valuation of these bonds?

BOND	Yield	DV01	Duration
#1	3.50%	.1159	11.59
#2	3.50%	.0876	14.75
#3	3.50%	.1443	10.26

Chapter 5

5.1 Using the following instructions, complete a spreadsheet to compute the two-year and five-year key-rate duration profiles of four-year bonds. For the purposes of this question, key-rate shifts are in terms of spot rates.

- (a) In Column A put the coupon payment dates in years, from .5 to 5 in increments of .5. Put a spot rate curve, flat at 3%, in Column B. Put the discount factors corresponding to this spot rate curve in

Column C. Now price a 3% and an 8% four-year coupon bond under this initial spot rate curve.

- (b) Create a new spot rate curve in Column D by adding a two-year key rate shift of 10 basis points. Compute the new discount factors in Column E. What are the new bond prices?
- (c) Create a new spot rate curve in Column F by adding a five-year key rate shift of 10 basis points. Compute the new discount factors in Column F. What are the new bond prices?
- (d) Use the results in parts (a) through (c) to calculate the key-rate duration profiles of each of the bonds.
- (e) Sum the key-rate durations for each bond to obtain the total durations. Calculate the percentage of the total duration attributed to each key rate for each bond. Comment on the results.
- (f) What would the key-rate duration profile of a four-year zero coupon bond look like relative to those of these coupon bonds? How about a five-year zero coupon bond?

5.2 Continue with the setting and results of Question 5.1. Verify that a 3% two-year bond has a duration of 1.925 that is completely concentrated as a two-year key-rate duration. How would one hedge the key-rate risk profile of the 8% four-year bond with the 3% two-year bond and the 3% four-year bond? Note that the total value of the 8% bond and of the hedge need not be the same. Comment on the result.

5.3 Use Table 5.6 for this question. A trader constructs a butterfly portfolio that is short €100mm of the 10-year swap and long 50% of the break; 10-year swap's total '01 in 5-year swaps and 50% of the 10-year swap's total '01 in 15-year swaps. What are the forward-bucket exposures of the resulting portfolio?

Chapter 6

The following introduction applies to Questions 6.1 through 6.5.

You are a market maker in long-term EUR interest rate swaps. You typically have to hedge the interest rate risk of having received from or paid to a customer on a 20-year interest rate swap. Given the transaction costs of hedging with both 10s and 30s and the relatively short time you wind up having to hold any such hedge, you consider hedging these 20-year swaps with either 10s or 30s but not both. To that end you run two single-variable regressions, both with changes in the 20-year EUR swap rates as the dependent variable, but one regression with changes in the 10-year swap rate as the independent variable and the other with changes in the 30-year swap rate as the independent variable. The results over the period July 1, 2009, to July 3, 2010, are given in the following table.

NUMBER OF OBSERVATIONS	259			
Independent variable	Change in 10-year	Change in 30-year		
R-squared	89.9%	96.3%		
Standard Error	1.105	.666		
Regression Coefficients	Value	Std. Error	Value	Std. Error
Constant	-.017	.069	-.008	.042
Independent variable	1.001	.021	.917	.011

6.1 What are the 95% confidence intervals around the constant and slope coefficients of each regression?

6.2 Use the confidence intervals just derived. Can you reject a) the hypothesis that the constant in the 10-year regression equals 0? b) That the slope coefficient in the 30-year regression equals 1?

6.3 As the swap market maker, you just paid fixed in €100 million notional of 20-year swaps. The *DV01*s of the 10-, 20-, and 30-year swaps are .0864, .1447, and .1911, respectively. Were you to hedge with 10-year swaps, what would you trade to hedge? And with 30-year swaps?

6.4 Approximately what would be the standard deviation of the P&L of a hedged position of 20-year swaps with 10-year swaps? And if hedged with 30-year swaps?

6.5 If you were to hedge with one of either the 10- or 30-year swaps, which would it be and why?

6.6 Use the principal components in Table 6.5 and the par swap data in Table 6.6 to hedge 100 face amount of 10-year swaps with 5- and 30-year swaps with respect to the first 2 principal components.

Chapter 7

7.1 A fixed income analyst needs to estimate the price of an interest rate caplet that pays \$1,000,000 next year if the one-year Treasury rate exceeds 3% and pays nothing otherwise. Using a macroeconomic model developed in another area of the firm, the analyst estimates that the one-year Treasury rate will exceed 3% with a probability of 25%. Since the current 1-year rate is 1%, the analyst prices the caplet as follows:

$$\frac{25\% \times \$1,000,000}{1.01} = \$247,525$$

Comment on this pricing procedure.

7.2 Assume that the true 6-month rate process starts at 5% and then increases or decreases by 100 basis points every 6 months. The probability of each increase or decrease is 50%. The prices of 6-month, 1-year, and 1.5-year zeros are 97.5610, 95.0908, and 92.5069. Find the risk-neutral probabilities for the six-month rate process over the next year (i.e., two steps for a total of three dates, including today). Assume, as in the text, that the risk-neutral probability of an up move from date 1 to date 2 is the same from both date 1 states. As a check to your work, write down the price trees for the 6-month, 1-year, and 1.5-year zeros.

7.3 Using the risk-neutral tree derive for Question 7.2, price \$100 face amount of the following 1.5-year *collared floater*. Payments are made every six months according to this rule. If the short rate on date i is r_i then the interest payment of the collared floater on date $i+1$ is $\frac{1}{2}3.50\%$ if $r_i < 3.50\%$; $\frac{1}{2}r_i$ if $6.50\% \geq r_i \geq 3.50\%$; $\frac{1}{2}6.50\%$ if $r_i > 6.50\%$. In addition, at maturity, the collared floater returns the \$100 principal amount.

7.4 Using your answers to Questions 7.2 and 7.3, find the portfolio of the originally 1-year and 1.5-year zeros that replicates the collared floater from date 1, state 1, to date

2. Verify that the price of this replicating portfolio gives the same price for the collared floater at that node as derived for Question 7.3

7.5 Using the risk-neutral tree from Question 7.2, price \$100 notional amount of a 1.5-year *participating cap* with a strike of 5% and a *participation rate* of 40%. Payments are made every six months according to the following rule. If the short rate on date i is r_i then the cash flow from the participating cap on date $i+1$ is, as a percent of par,

$\frac{1}{2}(r_i - 5\%)$ if $r_i \geq 5\%$ and $\frac{1}{2}40\%(r_i - 5\%)$ if $r_i < 5\%$. There is no principal payment at maturity.

7.6 Question 7.3 required the calculation of the price tree for a collared floater. Repeat this exercise under the same assumptions, but this time assume that the OAS of the collared floater is 10 basis points.

7.7 Using the price trees from Questions 7.3 and 7.6, calculate the return to a hedged and financed position in the collared floater from dates 0 to 1 assuming no convergence (i.e., the OAS on date 1 is also 10 basis points.) Hint #1: Use all of the proceeds from selling the replicating portfolio to buy collared floaters. Hint #2: You do not need to know the composition of the replicating portfolio to answer this question. Is your answer as you expected? Explain.

7.8 What is the return if the collared floater converges on date 1, i.e., its OAS equals 0 on that date?

Chapter 8

8.1 Describe as fully as possible the qualitative effect of each of these changes on the instantaneous rates 10 and 30 years forward.

1. The market risk premium increases.
2. Volatility across the curve increases.
3. Rates are not expected to increase as much as previously.
4. The market risk premium falls and volatility falls in such a way as to keep the 10-year forward rate unchanged.

Chapter 9

9.1 Assume an initial interest rate of 5%. Using a binomial model to approximate normally distributed rates with weekly time steps, no drift, and an annualized volatility of 100 basis points, what are the two possible rates on date 1?

9.2 Add a drift of 20 basis points per year to the model described in Question 9.1. What are the two rates now?

9.3 Consider the following segment of a binomial tree with 6-month time steps. All transition probabilities equal .5.

		5.360%
	4.697%	
3.99%		4.648%
	3.964%	
		4.031%

Does this tree display mean reversion?

9.4 What mean reversion parameter is required to achieve a half-life of 15 years?

Chapter 10

10.1 The yield volatility of a short-term interest rate is 20% at a level of 5%. Quote the basis point volatility and the CIR volatility parameter.

10.2 You are told that the following tree was built with a constant volatility. All probabilities equal .5. Which volatility measure is, in fact, a constant?

		5.49723%
	4.69740%	
4.014%		4.63919%
	3.96420%	

3.91507%

10.3 Use the closed form solution for the Vasicek model in Appendix A in Chapter 10 to compute the spot rate of various terms with the parameters $\theta = 10\%$, $k = .035$, $\sigma = .02$, and $r_0 = 4\%$. Comment on the shape of the term structure.

Chapter 13

13.1 As of a spot settlement date of June 1, 2010, find the forward price of the U.S. Treasury $3\frac{5}{8}$ s of February 15, 2020, for delivery on September 30, 2010. The spot price is 102 – 21 and the repo rate is .3%.

13.2 Using your answer to Question 13.1, compute the forward yield of the $3\frac{5}{8}$ s to September 30. Use equation (3.32) and a spreadsheet or other application.

13.3 Use the risk-neutral tree, with annual steps, developed in the section “Arbitrage Pricing in a Multi-Period Setting” in Chapter 9. Consider a 5% 10-year bond that, 2 years from the starting date, takes on the values 104.701, 98.126, and 92.061, corresponding to the nodes 4%, 5%, and 6%, respectively. What is the forward price of the bond for delivery in two years? What is the futures price to that same delivery date?

Chapter 14

14.1 The conversion factor of the 4s of August 15, 2018, into TYU0 is .8774. If the price of TYU0 is 121.2039, what is the (flat) delivery price of the 4s at that time? If the price of the 4s is 107.1652, what is their cost of delivery at that time?

14.2 The following table gives the prices of TYU0 and of its deliverable bonds in a particular term structure scenario on the delivery date that corresponds to a 7-year par yield of 3.32%. Conversion factors are also provided. Which bond is CTD in this scenario?

FUTURES PRICE:		117.2606	
RATE	MATURITY	PRICE	CONV. FACTOR
$3\frac{1}{4}$	3/31/17	100.1567	.8538
$4\frac{1}{2}$	5/15/17	107.9783	.9202
$3\frac{1}{8}$	4/30/17	99.3447	.8471
$2\frac{3}{4}$	5/31/17	96.9980	.8272
$4\frac{3}{4}$	8/15/17	109.3542	.9314
$4\frac{1}{4}$	11/15/17	106.0361	.9012
$3\frac{7}{8}$	5/15/18	102.7277	.8732
4	8/15/18	103.2276	.8774
$3\frac{1}{2}$	2/15/18	100.6805	.8547
$3\frac{3}{4}$	11/15/18	101.0422	.8587
$3\frac{5}{8}$	8/15/19	98.9248	.8401
$3\frac{1}{8}$	5/15/19	95.5539	.8107
$2\frac{3}{4}$	2/15/19	93.3962	.7909
$3\frac{3}{8}$	11/15/19	96.8798	.8195

FUTURES PRICE:		117.2606	
RATE	MATURITY	PRICE	CONV. FACTOR
$3\frac{5}{8}$	2/15/20	98.6522	.8332
$3\frac{1}{2}$	5/15/20	97.6531	.8210

14.3 The $3\frac{3}{4}$ s of May 31, 2012, are deliverable into TUU0, the September 2010 2-year note contract. Assume that the delivery date of the contract is September 30, 2010. The notional coupon of the contract is 6%. Approximately what is the conversion factor of the $3\frac{3}{4}$ s for delivery into that contract?

14.4 The forward price of the $3\frac{1}{2}$ s of February 15, 2018, to September 30, 2010, is 103.1303. Its conversion factor for delivery into TYU0 is .8547. If the price of TYU0 is 120, what is the net basis of the $3\frac{1}{2}$ s in ticks?

14.5 A trader sells \$50 million $3\frac{1}{2}$ s net basis at the level you calculated in question

14.4. What is the trader's position in the bond and in TYU0? If the net basis of $3\frac{1}{2}$ s is 10 ticks as of the delivery date, what is the trader's profit or loss?

14.6 Figure 14.4 of the text graphs various net bases as option-like payoffs. Describe how each of the following deliverable bonds would look if added to this graph: the $3\frac{1}{4}$ s of 3/31/2017; the 4s of 8/15/2018; and the $3\frac{1}{2}$ s of 5/15/2022.

14.7 How would the graphs in Figure 14.4 change if the curve steepened as the 7-year par rate increased?

Chapter 15

15.1 As of May 28, 2010, you are financing \$100 million worth of inventory of bonds in the repo market on an overnight basis. You plan to hold these bonds until mid-September 2010. Using both the Eurodollar and Fed funds futures listed in Tables 15.3 and 15.11, what trades can you do to hedge against the risk that rates rise and increase your borrowing cost? Will you have to adjust the hedge at all between May 28 and mid-September?

15.2 Approximately what borrowing rate is locked in by the hedge in Question 15.1?

15.3 Instead of the hedge constructed in Question 15.1, you decide to use only Eurodollar contracts. How does the hedge change? How does the locked-in rate compare with the previous hedge? Is this new hedge riskier in any way than the previous hedge?

15.4 As of May 28, 2010, a 5% U.S. Treasury bond maturing on September 15, 2010, had a full price of 102.4055. Using the dates and rates of Table 15.8, calculate the TED spread of the bond.

15.5 As of the end of July 2004, the fed funds target rate stood at 1.25%. Say that the August fed fund futures rate at that time was 1.3516%. What is the market implied probability of a 25 basis-point increase at the August 10 meeting? If you're willing to assume a 50% chance of no change in policy, what are the implied probabilities of 25 and 50 basis-point increases?

Chapter 16

16.1 Recalculate swap cash flows as in Table 16.1 for a 1.5% swap rate and a LIBOR rate that starts at 50 basis points on June 2, 2010, but then increases by 50 basis points every three months, reaching 4% by March 2, 2012.

16.2 Under the same simplifying assumptions used to price the CMS swap in Table 16.3, what is the fair fixed rate against the 10-year swap rate paid annually for four years starting in year 2?

Chapter 17

17.1 Consider 1- and 2-year swaps of annually-paying fixed vs. annually-paying LIBOR with par rates of 2% and 2.75%, respectively. The investable and collateral rates, given by the OIS curve, are 1% for 1 year and 2% for 1 year, 1 year forward. What is the NPV of receiving 3% for two years against LIBOR?

17.2 Using the data from Question 17.1, what is the implied term structure of basis swap spreads of OIS vs. LIBOR?

Chapter 18

18.1 Using the formulae in the text, recalculate the value of the caplet in Table 18.1 by changing only the volatility from 77.22 basis points to 120 basis points.

18.2 Using the formulae in the text, calculate the value of an at-the-money 2y5y receiver swaption on \$100 million notional when the term structure is flat at 4% and the appropriate volatility is 100 basis points.

Chapter 19

19.1 Consider a 10-year corporate bond with a coupon of 6%. The semi-annual compounded swap curve is flat at 4% and the corporate bond is trading at a LIBOR OAS of 3%. Calculate the par-par asset swap spread and the market value asset swap spread.

19.2 Say that the cumulative default rate over a 10-year horizon for some category of corporate bonds is 5%. If the recovery rate is 40%, what is the spread that just compensates investors for expected losses?

19.3 The quoted spread on a one-year quarterly paying CDS is 110 basis points while the standardized coupon is 100 basis points. Let the assumed recovery rate be 40% and let the quarterly compounded term structure of swap rates be flat at 3%. What is the up-front payment for \$10 million notional of the CDS? You will need to construct a spreadsheet to perform these calculations.

19.4 Create a spreadsheet to recreate the duration calculations in the subsection “The DV01 or Duration of a Bond with Credit Risk.” Use this spreadsheet to compute the duration in the example of that subsection with a coupon of 8% instead of 6%, keeping the yield at 14%.

Chapter 20

20.1 Assume that the term structure of monthly compounded rates is flat at 6%. Find the monthly payment of a \$100,000 15-year level-pay mortgage.

20.2 For the mortgage in Question 20.1, what is the interest component of the monthly payment after five years?

20.3 An adjustable-rate mortgage (ARM) resets the interest rate periodically. How does the refinancing option of an ARM compare with the option to prepay a fixed rate mortgage?

20.4 Explain the intuition for each of the following results:

- (a) When interest rates fall, holding all else equal, POs outperform 30-year fixed rate securities.
- (b) When interest rates rise by 100 basis points, mortgage pass-throughs fall by about 7%. When interest rates fall by 100 basis points, pass-throughs rise by 4%.
- (c) When interest rates decline, IOs and inverse IOs decline in price, but IOs suffer more severely. (Like an IO, an inverse IO receives no principal payments but receives interest payments that float inversely with the level of rates.)

20.5 Recompute the value of the roll in the example of the text for a coupon of 6%, a paydown percentage of 3%, and an August TBA price of 102.1. Keep all other quantities the same.