Midterm for Math 361

Quantitative Methods for Fixed-Income Securities

April 14, 2008

Solutions:

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1.1 (2) A spot rate is the yield of a zero-coupon bond. A spot-rate curve is the curve formed by spot rates of various maturities.

1.2 (2) A forward rate is the arbitrage free interest rate for a half-year period in the future. A forward-rate curve is the curve formed by forward rates of various maturities.

1.3 (2) A par yield is a coupon rate of a par bond. A par-yield curve is the curve formed by par yields of various maturities.

1.4 (2) $P(y) = F\left(\sum_{j=1}^{2T}\frac{c/2}{(1+y/2)^j} + \frac{1}{(1+y/2)^{2T}}\right)$

1.5 (2) $P(y) = F\left(\sum_{j=1}^{2T}\frac{c/2}{[1+\hat{r}(j/2)/2]^j} + \frac{1}{[1+\hat{r}(T)/2]^{2T}}\right)$

1.6 (2) $P(y) = F\left(\sum_{j=1}^{2T}\frac{c/2}{\prod_{k=1}^{j}(1+r(k/2)/2)} + \frac{1}{\prod_{k=1}^{2T}(1+r(k/2)/2)}\right)$

1.7 (2) $r(T) = 2\left[\frac{\left(1+\hat{r}(T-1/2)/2\right)^{-2T+1}}{\left(1+\hat{r}(T)/2\right)^{-2T}} - 1\right]$

1.8 (2) $\hat{r}(T) = 2\left\{\left(\prod_{j=1}^{2T}\left(1+r(j/2)/2\right)\right)^{1/2T} - 1\right\}.$

1.9 (2) $y(T) = \dfrac{1-d(T)}{\sum_{t=1}^{2T} d(t)\big/2}.$

1.10 (2) Substitute $d(t) = \prod_{i=1}^{2t}\left(1+r(i)\big/2\right)^{-1}$ into the last equation.

2 (4) Solve y by *trial and error* from

$$P = F\left(\sum_{j=0}^{2T}\frac{c/2}{(1+y/2)^{j+\tau}} + \frac{1}{(1+y/2)^{2T+\tau}}\right)$$

or

$$P = \frac{F}{(1+y/2)^{\tau}}\left(\frac{c}{2}+\frac{c}{y}\left[1-\frac{1}{(1+y/2)^{2T}}\right]+\frac{1}{(1+y/2)^{2T}}\right)$$

with

$$\begin{aligned}
F &= 100 \\
c &= 0.055 \\
T &= 3 \\
\tau &= (31+28+31)/181 = 0.4972 \\
AI &= \frac{c}{2}F\frac{\tau}{181} = 1.367 \\
P &= 110+1.367 = 111.367.
\end{aligned}$$

3.

3.1 (4) Differentiating bond price formula

$$P(y) = F\left\{\frac{c}{y}\left[1-\frac{1}{(1+y/2)^{2T}}\right]+\frac{1}{(1+y/2)^{2T}}\right\}$$

w. r. t. y we get

$$\frac{dP}{dy} = F\left\{-\frac{c}{y^2}\left[1-\frac{1}{(1+y/2)^{2T}}\right]+\left(\frac{c}{y}-1\right)\frac{T}{(1+y/2)^{2T+1}}\right\}$$

We calculate DV01s using

$$DV01 = \frac{-1}{10,000}\frac{dP}{dy}$$

For 5.5s of 04/07/2008, we have

$$DV01 = 0.027312.$$

For 5.0s of 04/07/2006, we have

$$DV01 = 0.009637.$$

3.2 (2) Modified durations are calculated according to

$$D = -\frac{1}{P}\frac{dP}{dy}$$

For 5.5s of 04/14/2011, we have

$$D = 2.7312.$$

For 5.0s of 04/14/2009, we have

$$D = 0.9637.$$

3.3 (2) We should short sell

$$\frac{D_{5.5}}{D_{5.0}} = \frac{2.7312}{0.9637} = 2.8340$$

units of 5s 04/14/2009 to form a duration neutral portfolio.

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