Homework 6. Quantitative Methods for fixed Income Securities

Chapter 13 and 31, Hull

1. Prove that the solutions to the equations

$$\alpha S^d_{\Delta t} + (1 + r\Delta t)\beta = C^d_{\Delta t}$$
$$\alpha S^u_{\Delta t} + (1 + r\Delta t)\beta = C^u_{\Delta t}$$

are

$$\alpha = \frac{C^u_{\Delta t} - C^d_{\Delta t}}{S^u_{\Delta t} - S^d_{\Delta t}}, \quad \text{and} \quad \beta = \frac{S^u_{\Delta t} C^d_{\Delta t} - S^d_{\Delta t} C^u_{\Delta t}}{(1 + r\Delta t)(S^u_{\Delta t} - S^d_{\Delta t})}.$$

2. Let

$$S^d_{t+\Delta t} = S_t D$$
$$S^u_{t+\Delta t} = S_t U$$

with $0 < D < (1 + r\Delta t) < U$, and let

$$q = \frac{U - (1 + r\Delta t)}{U - D},$$

Verify that

$$S_t = \frac{1}{1 + r\Delta t}\left(q S^d_{t+\Delta t} + (1 - q) S^u_{t+\Delta t}\right).$$

3. For the one-period interest-rate model, the general option pricing formula is

$$C_{0,0} = (1 + r_{0,0}\Delta t)^{-1}\left(q_0 C_{0,1} + (1 - q_0) C_{1,1}\right)$$

for

$$q_0 = \frac{P_{1,1} - P_{0,0}(1 + r_{0,0}\Delta t)}{P_{1,1} - P_{0,1}},$$

with

$$P_{0,1} = \frac{1000}{1 + r_{0,1}\Delta t} \quad \text{and} \quad P_{1,1} = \frac{1000}{1 + r_{1,1}\Delta t},$$

Verify that

$$P_{0,0} = (1 + r_{0,0}\Delta t)^{-1}\left(q_0 P_{0,1} + (1 - q_0) P_{1,1}\right).$$

You can solve the following problems manually, but you are encouraged to use MATLAB, Excel or any other programming languages.

4. Consider the pricing of a put option on AAPL. The spot price is \$160, the strike price is \$160, and the maturity is six months. The growth rate of the AAPL is 2% and its annual volatility is 25%. The annualized interest rate for monthly compounding is fixed at 0.75%.

4.1. Calculate the option value and hedge ratio (i.e., alpha) using a six-step binomial tree.

4.2. If the path of the stock price for the next six months is {(0,0), (0,1), (1,2), (2,3), (3,4), (3,5), (4,6)}, provide the alpha's for delta hedging along the path.

5. Consider pricing a 6m-maturity call option on one unit of the 1yr-maturity zero-coupon bond for the strike price of $970 under the Ho-Lee model

$$\Delta r_t = \theta \Delta t + \sigma \sqrt{\Delta t}\ \varepsilon_B,$$

with $\Delta t = 0.5$, θ=0.01, σ=0.01 and $r_0 = \hat{r}(\frac{1}{2}) = 0.05$. Here, ε_B takes ± 1 with 50% equal probabilities. In addition, the one-year spot rate (for semi-annual compounding) is $\hat{r}(1) = 0.055$. Calculate the option price.