- Suppose the CD rates stay unchanged over the year, then the total returns of the CDs are

1.021305255
1.021369136
1.021413423
1.021400000

- Compounding frequency matters!

Popular Compounding Frequencies

- $\omega=1$, annual compounding (Euro bonds)
- $\omega=2$, semiannual compounding (USD bonds)
- $\omega=4$, quarterly compounding (floating-rate notes)
- $\omega=12$, monthly compounding (mortgages)
- $\omega=365$, daily compounding (saving accounts)
- $\omega=\infty$, continuous compounding (for theoretical modeling)

Interest rates and Compounding Frequencies

- Investing F at a rate of r compounded semiannually for T years generates

$$F\left(1+\frac{r}{2}\right)^{2T}$$

- Let ω be the compounding frequency, then the final balance is

$$F\left(1+\frac{r}{\omega}\right)^{\omega T}$$

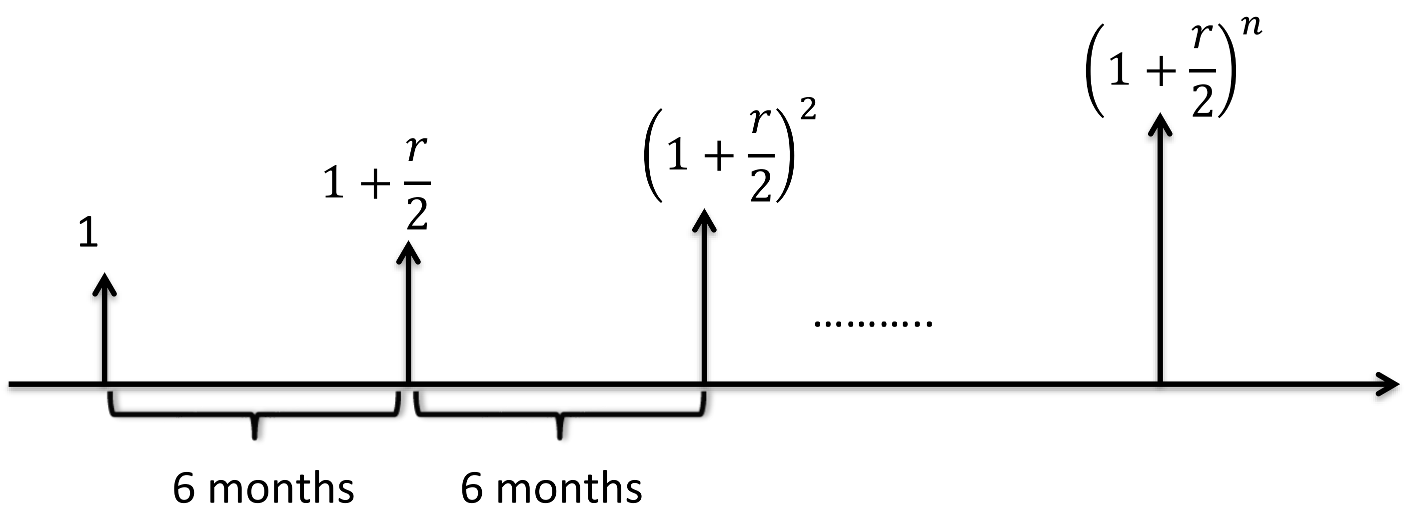
Continuous Compounding

- When ω→∞,

$$\lim_{\omega \to \infty} F \times \left(1 + \frac{\hat{r}}{\omega}\right)^{\omega T} = F e^{\hat{r} T}$$

Semiannual Compounding

- One of the most common definition of return rate is associated with *semi-annual compounding*



Investment Comparison

- Two example
  - a $100m 1-year loan that pays $102,521,281 in one year
  - a $100m 1.5-year loan that pays $103,797,070 in 1.5 years
- Which one is a better investment?

The Rate of Return

- For the first loan, the rate of return is implied by

$$100{,}000{,}000 \times \left(1 + {r_1}/{2}\right)^2 = 102{,}521{,}281$$

- Solve it and obtain r_1=2.5125%.

- For the second loan, the rate of return is implied by

$$100{,}000{,}000 \times \left(1 + {r_{1.5}}/{2}\right)^3 = 103{,}797{,}070$$

- Solve it, we obtain $r_{1.5}$=2.5%.
- The 1Y loan is more attractive.

Rate of return and discount rate

- We can also write

$$100,000,000 = \frac{102,521,281}{\left(1 + {r_1}/{2}\right)^2}$$

and

$$100,000,000 = \frac{103,797,070}{\left(1 + {r_{1.5}}/{2}\right)^3}$$

- A return rate is also the discount rate.

Present Value vs. Future Value

- For a future cash flow, I_T , its corresponding present value is

Future value

$$I_0 = d(T) \times I_T,$$

Present value

- Relative to I_0 , I_T is called the future value.

Important Linear Securities

Swaps

From Chapter 18

LIBOR Rates

- LIBOR is a set of reference interest rates at which banks lend unsecured loans to other banks in the London wholesale money market.
- The LIBOR rates are benchmark rates for certificates of deposit (CDs).

Table 6.1. LIBOR rates of 4/16/2008

Term	Rate (%)
1 month	2.72
3 months	2.72
6 months	2.72
12 months	2.64

(Source: www.bankrate.com)

The day-count convention for USD and Euro CDs is actual/360.

- Bank Rates

Interest Rate Swaps

- Parties A & B agree, on May 28, 2010, to enter into an *interest rate swap* with the following terms. Starting in two business days, on June 2, 2010,
  - party A agrees to pay a fixed rate of 1.235% on a notional amount of $100 million to party B for two years,
  - party B, in return, agrees to pay *three-month LIBOR* (*London Interbank Offered Rate)* on this same notional to Party A.

Interest Rate Swaps, cont'd

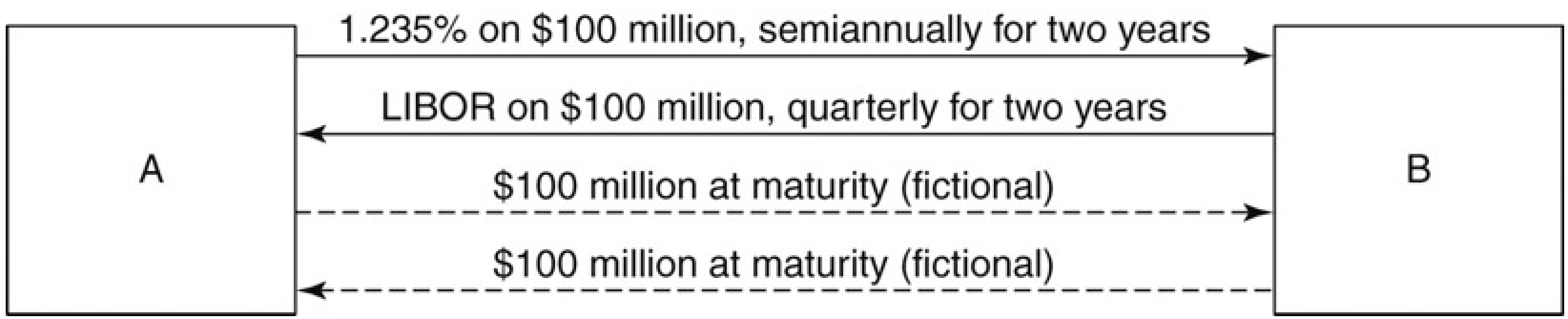
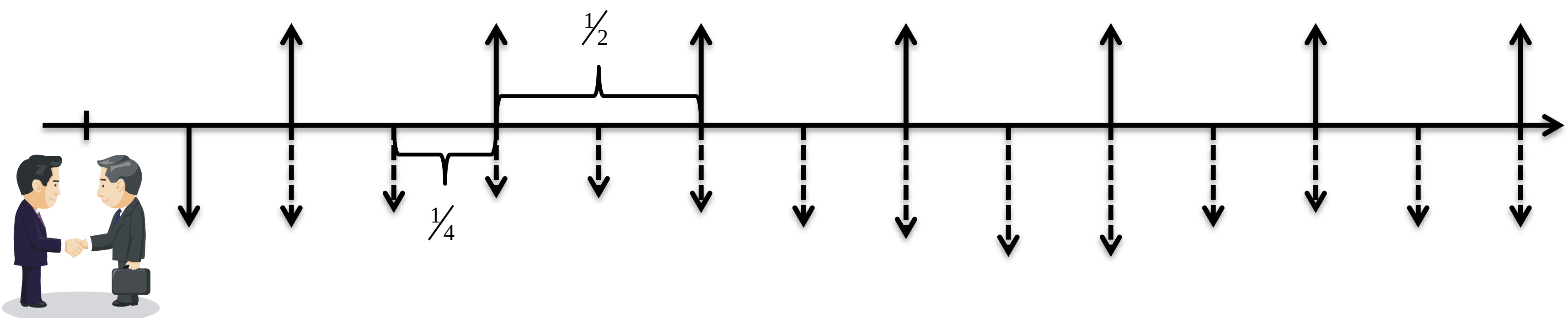


Figure 2.1: An Example of an Interest Rate Swap

Cash-Flow Pattern

- The dash-line arrow means uncertainty
- The red-line is the artificial principal



Cash Flow Calculation

- Fixed periodic payment from the fixed-leg

$$\frac{1.235\%}{2} \times \$100{,}000{,}000 = \$617{,}500$$

- Assume the current 3-mo LIBOR is 0.75%, then the next floating-leg payment is

$$\frac{0.75\%}{4} \times \$100{,}000{,}000 = \$187{,}500$$

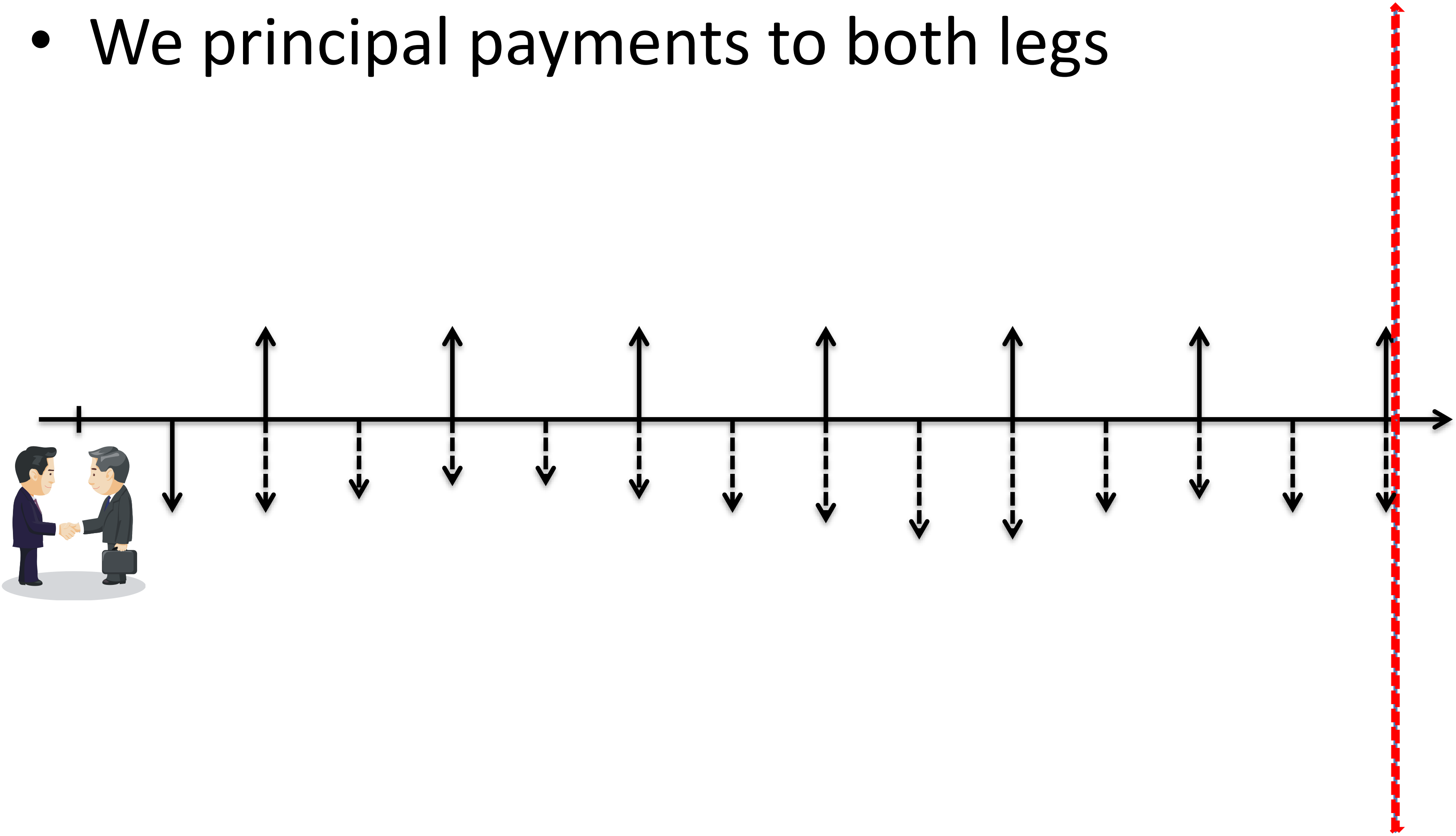
- The second floating-leg payment is not yet known.

Pricing of Swaps

- The cash flows are generated from
  - A fixed-rate bond with semiannual payments
  - A floating-rate bond with quarterly payments
- To price the swap, we may present value the cash flows.
- The fixed-leg is about a usual bond pricing.

Artificial Principal for Swaps

- We principal payments to both legs



Pricing of Floating Leg

- Proposition: Initially and right after a payment, the FRN is worth par.
  - You start with $100m, save it in three-month certificate of deposits (CDs).
  - Three month later, you collect and pay the interest, and save the remaining $100m to another CD.
  - Repeat this transaction until the maturity of the swap.

Pricing of Swaps

- So the price of a swap is the difference between a coupon bond and its par value
- At initiation, all swap have zero value, meaning the values of the fixed legs are also par.
- The fixed-leg is a par bond, and the swap rate is a par yield.

Par Bond Recalled

- Price-yield relationship

$$P = 1 + \left(\frac{c}{y} - 1\right)\left(1 - \frac{1}{\left(1 + {y}/{2}\right)^{2T}}\right)$$

- Par bond: when $c = y$, $P(T) = 1$.

Determination of PAR Yields

- Let $s(T)$ be the T-year par rate, then

$$s(T)\sum_{i=1}^{2T}\tfrac{1}{2}d(\tfrac{i}{2})+d(T)=1$$

- Swap rate formula

$$s(T)=\frac{1-d(T)}{\sum_{i=1}^{2T}\tfrac{1}{2}d(\tfrac{i}{2})}$$

Determination of PAR Rates, cont'd

- Let

$$A(T) = \sum_{i=1}^{2T} \tfrac{1}{2} d\left(\tfrac{i}{2}\right)$$

- Then

$$s(T) = \frac{1 - d(T)}{A(T)}$$

- Alternative formula for general bond price

$$P = 1 + \left(c - s(T)\right) A(T)$$

The Calculations

- Equations for the discount factors

$$\left(100+\frac{.705}{2}\right)d\left(.5\right)=100$$

$$\frac{.875}{2}d\left(.5\right)+\left(100+\frac{.875}{2}\right)d\left(1\right)=100$$

- Table 2.1

Spot Rates and Discount Factors

- Table 2.1 also gives the spot rates from the USD swap curve as of May 28, 2010.

TABLE 2.1 Discount Factors, Spot Rates, and Forward Rates Implied by Par USD Swap Rates as of May 28, 2010

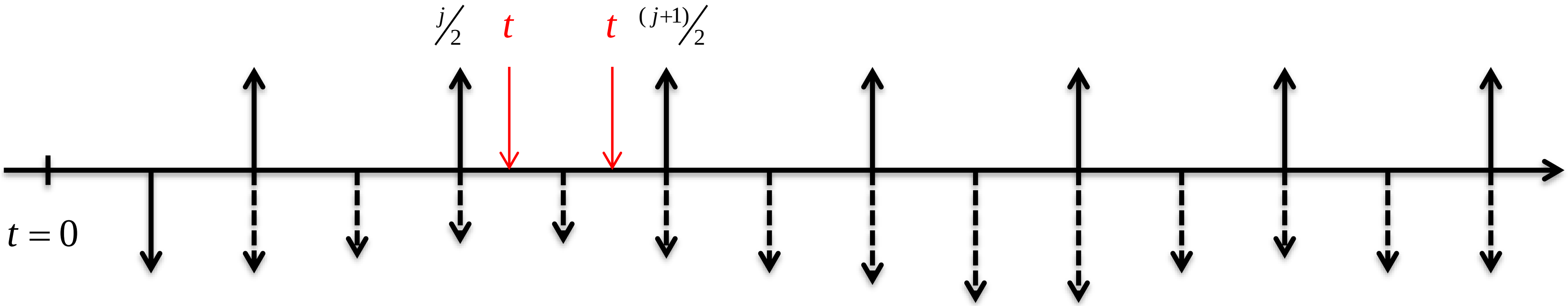
Term in Years	Swap Rate	Discount Factor	Spot Rate	Forward Rate
0.5	.705%	.996489	.705%	.705%
1.0	.875%	.991306	.875%	1.046%
1.5	1.043%	.984494	1.045%	1.384%
2.0	1.235%	.975616	1.238%	1.820%
2.5	1.445%	.964519	1.450%	2.301%

Why Swaps?

- To lock in long-term low interest rate to pay to high interest rate to receive.
- To speculate.
- To hedge other interest-rate sensitive securities/portfolios.

MtM Swaps

- What is the price of the swap at a later time $t > 0$?



PV the two legs

- At time $t = 0$, the swap rate is $s(0, T)$. At a later time $\frac{j}{2} \leq t < \frac{j+1}{2}$, what should be the MTM value of the swap?
- Answer: For the fixed leg, the value is

$$\mathrm{V}_{fixed}(t) = s(0,T) \times \sum_{i=1}^{2T} \tfrac{1}{2} d(t, i/2)$$

PV the two legs

- For the floating leg, when $\frac{j}{2} \le t < \frac{j}{2} + \frac{1}{4}$, the value is

$$V_{float}(t) = d(t, \tfrac{j}{2} + \tfrac{1}{4}) \times \frac{1}{d(0, \tfrac{j}{2} + \tfrac{1}{4})} - d(t, T)$$

- When $\frac{j}{2} + \frac{1}{4} \le t < \frac{j+1}{2}$, the value is

$$V_{float}(t) = d(t, \tfrac{j}{2} + \tfrac{1}{2}) \times \frac{1}{d(\tfrac{j}{2} + \tfrac{1}{4}, \tfrac{j}{2} + \tfrac{1}{2})} - d(t, T)$$

MtM Swaps, cont'd

- At other later time $j/2 \le t < (j+1)/2$, the MtM value of the swap is

$$MtM = \mathrm{V}_{float}(t) - \mathrm{V}_{fixed}(t)$$

General Swap Rate Formula

- Define the general swap rate formula

$$
s(t,T) = \frac{V_{float}(t)}{\sum_{i=j+1}^{2T} \frac{1}{2} d(t, i/2)} = \begin{cases} \dfrac{\dfrac{d(t,\frac{j}{2}+\frac{1}{4})}{d(\frac{j}{2},\frac{j}{2}+1/4)} - d(t,T)}{\sum_{i=j+1}^{2T} \frac{1}{2} d(t, i/2)}, & \frac{j}{2} \le t \le \frac{j}{2} + \frac{1}{4} \\ \dfrac{\dfrac{d(t,\frac{j}{2}+\frac{1}{2})}{d(\frac{j}{2}+1/4,\frac{j}{2}+\frac{1}{2})} - d(t,T)}{\sum_{i=j+1}^{2T} \frac{1}{2} d(t, i/2)}, & \frac{j}{2} + \frac{1}{4} \le t \le \frac{j+1}{2} \end{cases}
$$

General MtM value of Swaps

- So the value of the swap becomes

$$MtM = \left(s(t,T) - s(0,T) \right) \sum_{i=j+1}^{2T} \tfrac{1}{2} d(t, \tfrac{i}{2})$$