Forward Rate Agreements (FRA)

- An financial contract to pay/receive the difference between a *fixed* interest rate and the *realized* interest rate, LIBOR in particular, applied to certain notional value.
- Initially the value of the FRA is zero.
- At maturity, the P&L to the receiver is

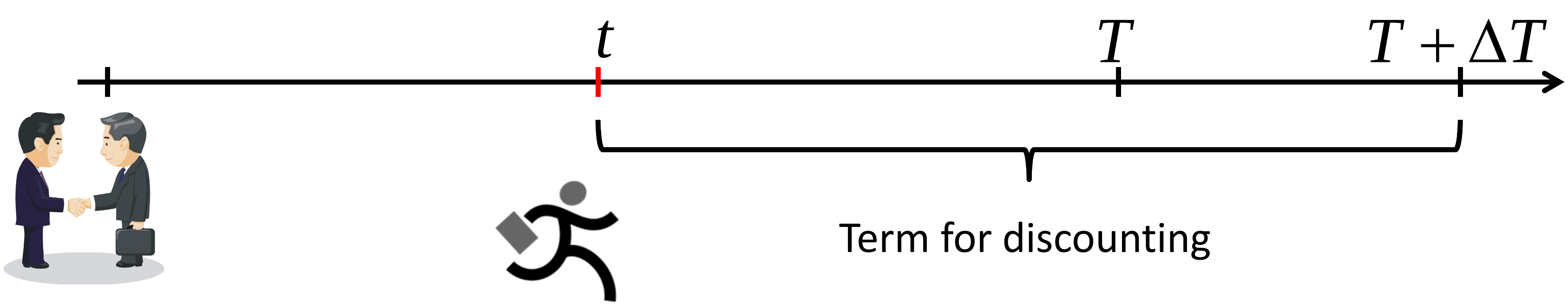
$$P\&L = d(T, T+\Delta T)\left[\$1m \times \Delta T \times (f_0 - f_T)\right]$$

where f_0 is the forward rate observed at t=0.

Marking-to-Market Value of FRA

- Let A receives fixed and pays LIBOR.
- Let f_t be the forward rate at a later time *t*, then the marking-to-market value of the FRA to the party who receives fixed and pays float is

$$P\,\&\,L \text{ to A} = d(t, T+\Delta T)\left[\$1m \times \Delta T \times (f_0 - f_t)\right]$$



Hedging an FRA

- A receives fixed rate (and short the FRA).
- A can offset the FRA by the following zero-net transactions
  - Short $\frac{d(0,T)}{d(0,T+\Delta T)}$- unit of $(T+\Delta T)$- unit zero
  - Long one unit of T-maturity zero

Hedging an FRA, cont'd

- At $0 \le t \le T$, the total P&L is

$$d(t,T+\Delta T)\Delta T\left(f_0 - f_t\right) + d(t,T) - \tfrac{d(0,T)}{d(0,T+\Delta T)} d(t,T+\Delta T)$$

$$= d(t,T+\Delta T)\left[\Delta T\left(f_0 - f_t\right) + \frac{d(t,T)}{d(t,T+\Delta T)} - \frac{d(0,T)}{d(0,T+\Delta T)}\right]$$

$$= d(t,T+\Delta T)\left[\left(\left(1+\Delta T f_0\right) - \frac{d(0,T)}{d(0,T+\Delta T)}\right)\right.$$

$$\left. - \left(\left(1+\Delta T f_t\right) - \frac{d(t,T)}{d(t,T+\Delta T)}\right)\right] = 0!!!$$

Examples

- An FRA between A and B
  - A pays B 3M LIBOR and receives 5% from B
  - Maturity: 1Y
  - Notional: $1m

P&L at Maturity

- Scenario 1
  - 1Y later, 3M LIBOR is 5.5%, then

$$P\,\&\,L \text{ to A}=\frac{\$1m\times 0.25\times(5\%-5.5\%)}{1+5.5\%/4}=-\$1,233.46$$

- Scenario 2
  - 1Y later, 3M LIBOR is 4.8%, then

$$P\,\&\,L \text{ to A}=\frac{\$1m\times 0.25\times(5\%-4.8\%)}{1+4.8\%/4}=\$494.07$$

P&L upon Early Closeout

- Suppose 3 months later,
  - the 3m LIBOR for the period becomes 5.5%,
  - The 1Y LIBOR rate is 5.25% so the discount factor is $d(1) = \left(1 + 5.25\%\right)^{-1}$
- So

$$
\begin{aligned}
P\,\&\,L \text{ to A} &= d(1) \times \$1m \times 0.25 \times (5\% - 5.5\%) \\
&= \frac{\$1m \times 0.25 \times (5\% - 5.5\%)}{1 + 5.25\%} \\
&= -\$1187.65
\end{aligned}
$$