The Results

$$\sigma_{20} = 5.27, \quad \sigma_{30} = 4.94, \quad \rho = 0.9912$$

TABLE 8.1 Regression Analysis of Changes in 20-Year Yields on 30-Year Yields

Number of observations	1,680	
R-squared	98.25%	
Standard error	0.6973	
Regression Coefficients	**Value**	**t-Statistic**
Constant	0.0007	0.0438
Change in 30-year yield	1.0570	306.9951

Explanations

- R-square: ρ^2
- Standard error: $\mathrm{std}\left(\Delta y^{20} - \hat{\alpha} - \hat{\beta}\Delta y^{30}\right)$
- *t*-statistics: the significant indicator s.t. when t>2, trust the results.
- Prediction: if $\Delta y_t^{30} = 3$, then

$$\Delta y_t^{20} = 0.0007 + 1.057 \times 3 = 3.1717$$

R squared

- The total sum of squares (proportional to the variance of the data):

$$SS_{\text{tot}} = \sum_i (y_i - \bar{y})^2,$$

- The regression sum of squares, also called the regression sum of squares:

$$SS_{\text{reg}} = \sum_i (f_i - \bar{y})^2,$$

- The sum of squares of residuals, also called the residual sum of squares:

$$SS_{\text{res}} = \sum_i (y_i - f_i)^2 = \sum_i e_i^2$$

The most general definition of the coefficient of determination is

$$R^2 \equiv 1 - \frac{SS_{\text{res}}}{SS_{\text{tot}}}$$

Example

- Hedging: choose an appropriate amount of P_{30} to minimize the absolute value of P&L:

$$\begin{aligned}
\text{P\&L} &= P_{20}D_{20}\Delta y_t^{20} - P_{30}D_{30}\Delta y_t^{30} \\
&= P_{20}D_{20}\left(\hat{\alpha} + \hat{\beta}\Delta y_t^{30} + \varepsilon_t\right) - P_{30}D_{30}\Delta y_t^{30} \\
&= \left(P_{20}D_{20}\hat{\beta} - P_{30}D_{30}\right)\Delta y_t^{30} + P_{20}D_{20}(\hat{\alpha} + \varepsilon_t)
\end{aligned}$$

- We take

$$P_{30} = P_{20}\frac{D_{20}}{D_{30}}\hat{\beta}$$

Example, cont'd

- Let

$$
\begin{aligned}
P_{20} &= \$10,000,000 \\
D_{20} &= 11.8428 \\
D_{30} &= 14.2940 \\
\beta &= 1.0507
\end{aligned}
$$

- Then,

$$P_{30} = \$10,000,000 \times \frac{11.8428}{14.2940} \times 1.0507 = \$8,757,410$$

Face value and DV01

- The hedging equation

$$P_{30} \times D_{30} = P_{20} \times D_{20} \times \hat{\beta}$$

- Since

$$-\Delta P = D \times P \times 0.01\% = F \times \frac{DV01}{100}$$

- We have

$$D \times P = F \times \frac{DV01}{100} \times 10000 = F \times DV01 \times 100$$

- In terms of F and $DV01$, there is, after 100 is cancelled

$$F_{30} = F_{20} \times \frac{DV01_{20}}{DV01_{30}} \times \hat{\beta}$$

2-v regression-based hedging

- The market maker has bought a 20-year receiver's swap, relatively illiquid and needs to hedge the resulting interest rate exposure.
- The market maker chooses instead to sell a combination of 10- and 30-year swaps.

- The market maker relies on a two-variable regression model to describe the relationship between changes in 20-year swap rates and changes in 10- and 30-year swap rates.

$$\Delta y_t^{20} = \alpha + \beta^{10} \Delta y_t^{10} + \beta^{30} \Delta y_t^{30} + \epsilon_t$$

- α and β's can be estimated by least squares, by minimizing

$$\sum_t \left(\Delta y_t^{20} - \widehat{\alpha} - \widehat{\beta}^{10} \Delta y_t^{10} - \widehat{\beta}^{30} \Delta y_t^{30} \right)^2$$

System of Linear Equations

$$-2\sum_t\left(\Delta y_t^{20}-\hat{\alpha}-\hat{\beta}^{10}\Delta y_t^{10}-\hat{\beta}^{30}\Delta y_t^{30}\right)=0$$

$$-2\sum_t\left(\Delta y_t^{20}-\hat{\alpha}-\hat{\beta}^{10}\Delta y_t^{10}-\hat{\beta}^{30}\Delta y_t^{30}\right)\Delta y_t^{10}=0$$

$$-2\sum_t\left(\Delta y_t^{20}-\hat{\alpha}-\hat{\beta}^{10}\Delta y_t^{10}-\hat{\beta}^{30}\Delta y_t^{30}\right)\Delta y_t^{30}=0$$

- Or

$$+\hat{\alpha}+\hat{\beta}^{10}\,\overline{\Delta y^{10}}+\hat{\beta}^{30}\,\overline{\Delta y^{30}}=\overline{\Delta y^{20}}$$

$$\hat{\alpha}\,\overline{\Delta y_t^{10}}+\hat{\beta}^{10}\,\overline{(\Delta y^{10})^2}+\hat{\beta}^{30}\,\overline{\Delta y^{30}\Delta y^{10}}=\overline{\Delta y^{10}\Delta y^{20}}$$

$$\hat{\alpha}\,\overline{\Delta y^{30}}+\hat{\beta}^{10}\,\overline{\Delta y_t^{10}\Delta y^{30}}+\hat{\beta}^{30}\,\overline{(\Delta y_t^{30})^2}=\overline{\Delta y^{20}\Delta y^{30}}$$

- Solve them numerically we obtain $\hat{\alpha}$, $\hat{\beta}^{10}$ and $\hat{\beta}^{30}$.

Prediction

- The estimation of these parameters then provides a predicted change for the 20-year swap rate:

$$\Delta\widehat{y}_t^{20} = \widehat{\alpha} + \widehat{\beta}^{10}\Delta y_t^{10} + \widehat{\beta}^{30}\Delta y_t^{30}$$

Hedging using duration

- To hedge the 20-year bond of value P_{20}, we need P_{10} and P_{30} dollars of the 10- and 30-year bonds, such that

$$P_{10} = P_{20} \frac{D_{20}}{D_{10}} \beta_{10}$$

$$P_{30} = P_{20} \frac{D_{20}}{D_{30}} \beta_{30}$$

- Hedge error

$$\mathrm{P\&L} = P_{20} D_{20} (\hat{\alpha} + \varepsilon_t)$$

Hedging using Face value and DV01

- To hedge the 20-year bond of face value F_{20}, we need F_{10} and F_{30} face values the 10- and 30-year bonds, such that

$$
\begin{aligned}
& F_{20}\frac{DV01_{20}}{100}\Delta y_t^{20} - F_{10}\frac{DV01_{10}}{100}\Delta y_t^{10} - F_{30}\frac{DV01_{30}}{100}\Delta y_t^{30} \\
& = \left(F_{20}\frac{DV01_{20}}{100}\beta_{10} - F_{10}\frac{DV01_{10}}{100} \right)\Delta y_t^{10} \\
& + \left(F_{20}\frac{DV01_{20}}{100}\beta_{30} - F_{30}\frac{DV01_{30}}{100} \right)\Delta y_t^{30} + F_{20}\frac{DV01_{20}}{100}(\alpha + \varepsilon_t)
\end{aligned}
$$

- The hedge

$$F_{10} = F_{20} \frac{DV01_{20}}{DV01_{10}} \beta_{10}$$

$$F_{30} = F_{20} \frac{DV01_{20}}{DV01_{30}} \beta_{30}$$

2V regression for swap rates

TABLE 6.4 Regression Analysis of Changes in the Yield of the 20-Year EUR Swap Rate on Changes in the 10- and 30-Year EUR Swap Rates From July 2, 2001, to July 3, 2006

No. of Observations	**1281**	
R-Squared	**99.8%**	
Standard Error	**.14**	
Regression Coefficients	**Value**	**Std. Error**
Constant ($\widehat{\alpha}$)	−.0014	.0040
Change in 10-Year Swap Rate ($\widehat{\beta}^{10}$)	.2221	.0034
Change in 30-Year Swap Rate ($\widehat{\beta}^{30}$)	.7765	.0037

- The results in Table 6.4 say that 22.21% of the *DV*01 of the 20-year swap should be hedged with a 10-year swap and 77.65% with a 30-year swap.
- The sum of these weights, 99.86%, happens to be very close to one, meaning that the *DV*01 of the regression hedge very nearly matches the *DV*01 of the 20-year swap.

Hedging Errors

- Errors are computed as the realized change in the 20-year yield minus the predicted change for that yield based on the estimated regression in Table 6.4:

$$\widehat{\epsilon}_t = \Delta y_t^{20} - (-.0014 + .2221\,\Delta y_t^{10} + .7765\,\Delta y_t^{30})$$

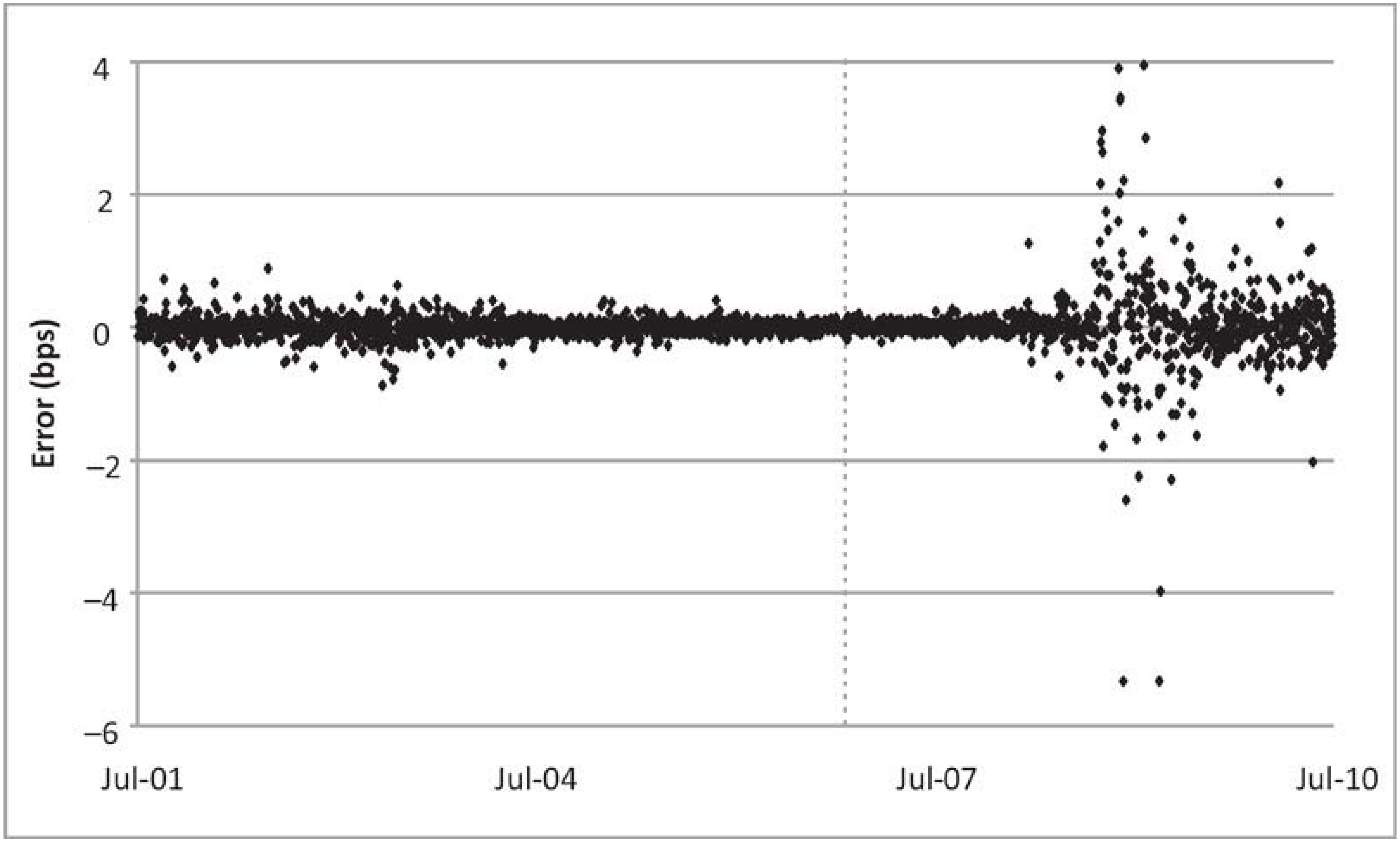


Figure 6.3: In- and Out-of-Sample Errors for a Regression of Changes of 20-Year and 10- and 30-Year EUR Swap Rates with Estimation Period July 2, 2001, to July 3, 2006

Error and Crisis

- The errors to the left of the vertical dotted line are in-sample, while, the errors to the right of the dotted line are out-of-sample.
- The size and behavior of these out-of-sample errors that provide evidence as to the stability for the estimated coefficients over time.
- The out-of-sample errors are small for the most part, until August and September 2008, a peak in the financial crisis of 2007–2009.

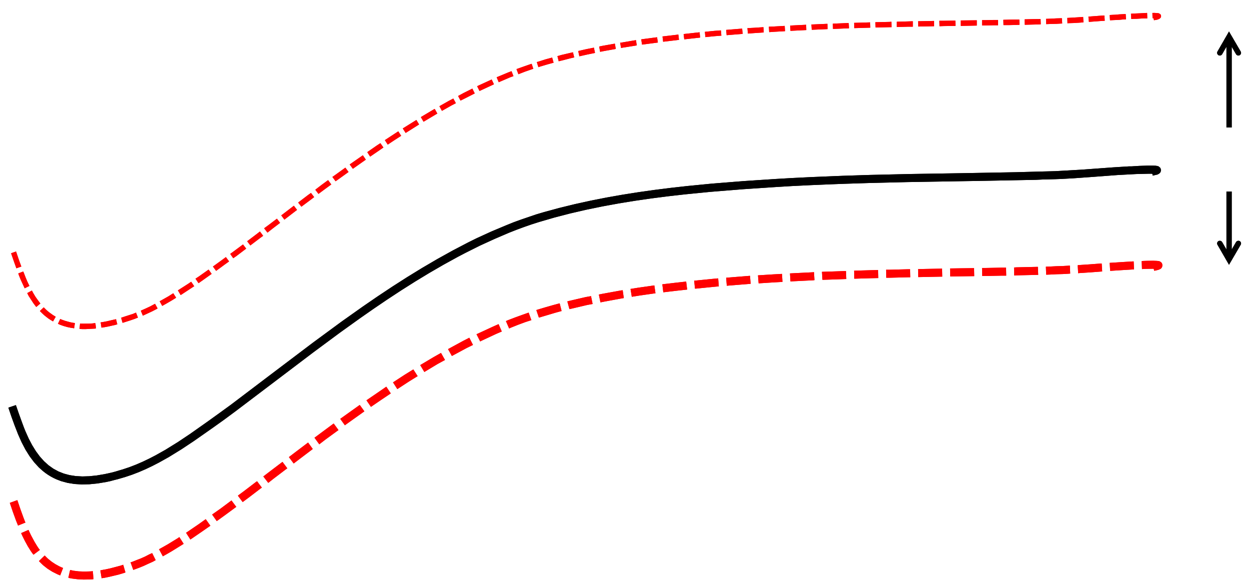
- It is obvious and easy to say that the market maker, during the turbulence of a financial crisis, should have replaced the regression of Table 6.4 and the resulting hedging rule.
- But replace these with what?
- What does the market maker do at that time, before there exist sufficient post-crisis data points?

- And what does the market maker do after the worst of the crisis: estimate a regression from data during the crisis or revert to some earlier, more stable period?
- These are the kinds of issues that make regression hedging an art rather than a science.

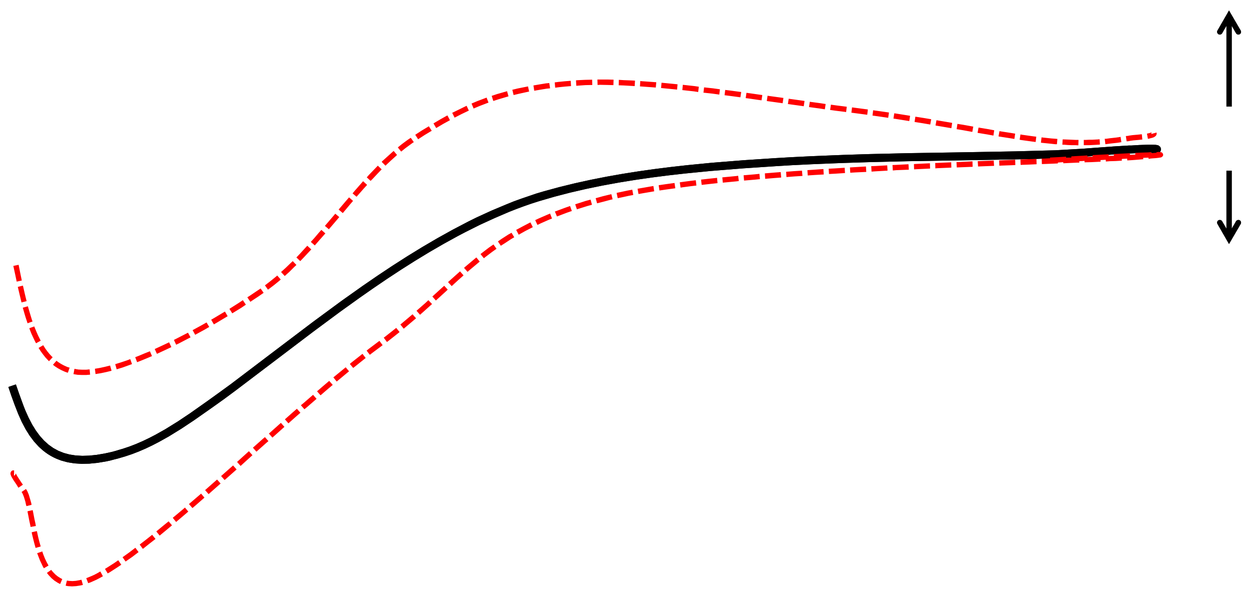
Chapter 7 of Tuckman
Key Rate Hedging

Limitation of the Duration/DV01

- A major weakness of the duration/DV01 based hedging is the assumption that yield curve does parallel shift.



The Reality



Curve Risk

- In reality, it is widely recognized that rates in different regions of the term structure are far from perfectly correlated.
- The risk that rates along the term structure move by different amounts is known as *curve risk*.

Driving force of the yield curve

- This chapter revises the theory based on the fact that some swap rates of particular maturities, so-called key rates, largely determine the shape of the yield curve.
- As a results, an interest-rate portfolio can be hedges by these swaps as well.