

MATH4511 Quantitive Methods for Fixed Income Derivatives, 2015-16 Fall

Quiz 04(T1C)

Name: _____

ID No.: _____

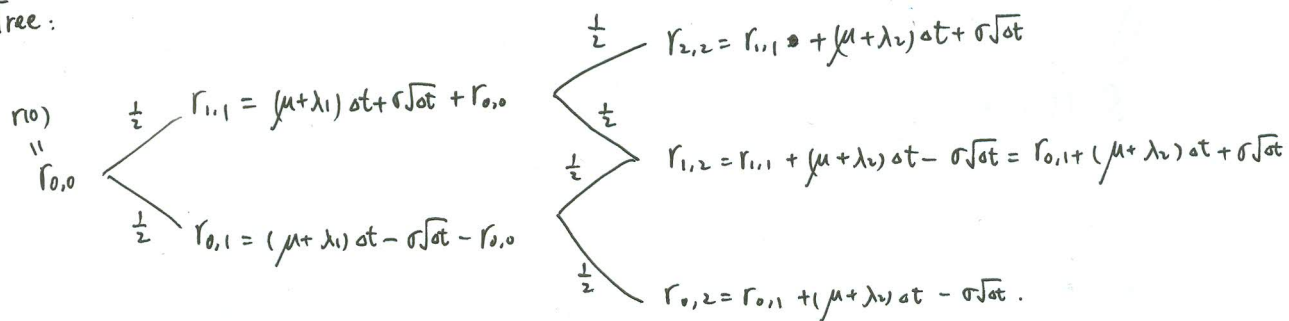
Tutorial Section: _____

1. (20 points) Assume the spot rate under the risk neutral measure follows the dynamic:

$$\Delta r_t = (\mu + \lambda_t) \Delta t + \sigma \sqrt{\Delta t} \epsilon_B,$$

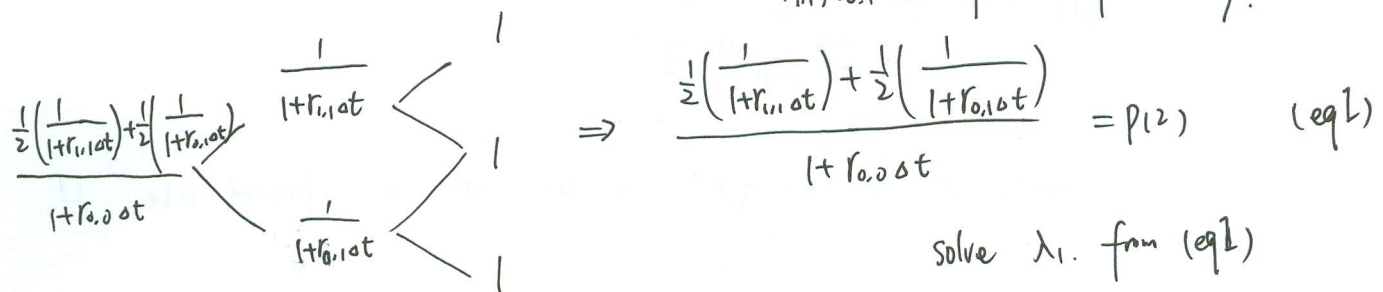
where ϵ_B takes +1 or -1 with equal probability. The current 1-year spot rate is $r(0)$ and the 2-year zero coupon bond price is $P(2)$ and the 3-year zero-coupon bond price is $P(3)$. Use a two step binomial tree to describe the evolution of the 1-year spot rate. Here, μ, σ are constant, risk neutral probability is $\{\frac{1}{2}, \frac{1}{2}\}$. State the procedure of calculating λ_t 's.

Tree:



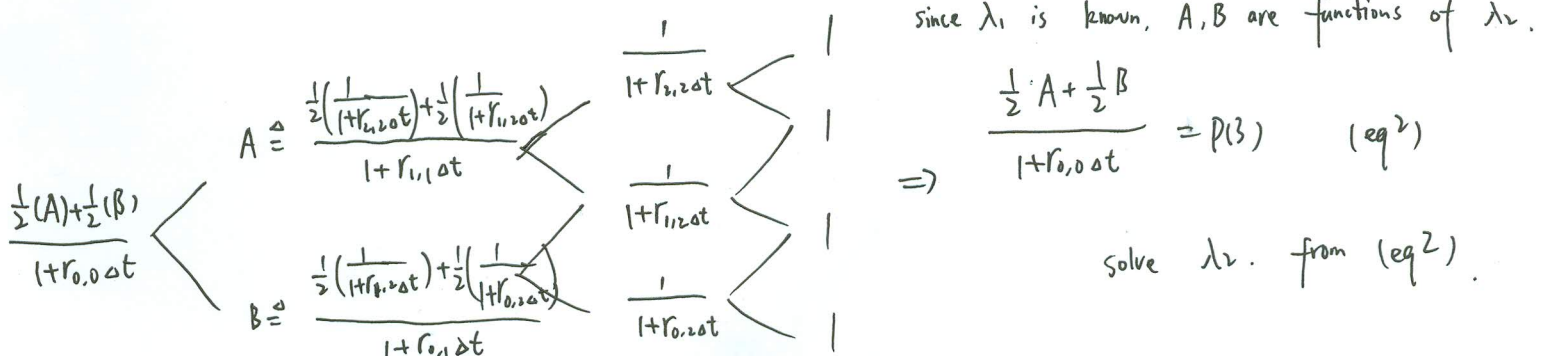
To calculate λ_1 :

Tree of 2-y zero-coupon bond (face = 1).



To calculate λ_2 :

Tree of 3-y zero-coupon bond. (now λ_1 is known)



2. (20 points) Given a risk-neutralized interest-rate tree

$$\Delta r_t = 0.0025\Delta t + 0.005\sqrt{\Delta t}\epsilon_B,$$

where $\Delta t=1$, $r_0=5\%$ and ϵ_B takes $+1$ or -1 with equal probability. Consider an interest rate product has a payoff \$1000 when the 1-year spot rate exceeds 5.5% every year. Assume the maturity is 2 years. Use the tree to price this product.

$$\frac{\frac{1}{2} \left(\frac{1000}{1+5.75\%} + 1000 \right) + \frac{1}{2} \left(\frac{500}{1+4.75\%} \right)}{1+5\%}$$

$$\parallel$$

$$\$1153.8$$

3. (10 points) The current forward-rate curve for semi-annual compounding is

$$f\left(\frac{i}{2}\right) = .02 + .001 * \frac{i}{2}, \quad i = 0, 1, 2, \dots$$

Calculate the in-5-to-10 swap rate (i.e., the swap rate for the swap for the period from 5 to 15 years).

$$S(0; 5, 15) = \frac{d(0, 5) - d(0, 15)}{\frac{1}{2} \sum_{i=11}^{30} d(0, \frac{i}{2})}$$

calculate all the $d(0, \frac{i}{2})$ from the curve of forward rate. \square .