Solutions to the Midterm of MATH4511

Quantitative Methods for Fixed-Income Securities

October 25, 2012

1. (a) The forward rates can be solved from price-forward-rate relationship:

$$
\begin{aligned}
B_T &= F\sum_{t=1}^{2T-1}\frac{c}{2}d(t/2)+F\left(1+\frac{c}{2}\right)d(T)\\
&= F\sum_{t=1}^{2T-1}\frac{c}{2}d(t/2)+F\frac{\left(1+\frac{c}{2}\right)d(T-0.5)}{1+\frac{r(T)}{2}}
\end{aligned}
$$

which gives

$$
r(T)=2\left[\frac{\left(1+\frac{c}{2}\right)d(T-0.5)}{B_T/F-\frac{c}{2}\sum_{t=1}^{2T-1}d(t/2)}-1\right].
$$

(b) For par bonds, $c=Y_T$ and $B_T=F=100$, we have

$$
r(T)=2\left[\frac{\left(1+\frac{y_T}{2}\right)d(T-0.5)}{1-\frac{y_T}{2}\sum_{t=1}^{2T-1}d(t/2)}-1\right].
$$

2. (a) The Formula for forward rates is given above.

(b) The discount factors can be calculated from forward rates recursively:

$$
\begin{cases}
d(0) &= 1,\\
d(T) &= \dfrac{d(T-0.5)}{1+\frac{r(T)}{2}}.
\end{cases}
$$

(c) The par yields can be calculated from:

$$
y_T=\frac{2(1-d(T))}{\sum_{t=1}^{2T}d(t/2)}.
$$

The results for this problem are summarized in the following table.

Table 1: Results for Problem 1.

Term	Coupon	YTM	Price	Fwd rate	Disc factor	Par yield
0.5	3.5%	1.5%	1.00993	0.01500	0.99256	1.5%
1	3%	2%	1.00985	0.02509	0.98026	2.001%
1.5	4.5%	2.25%	1.03300	0.02771	0.96686	2.255%
2	4%	2.5%	1.02909	0.03279	0.95127	2.505%

3. Today is Oct. 25, 2012

(a) The coupon dates are June 15 and December 15.

# of days between 6/15/2012 to 12/15/2012: 183;

# of days between 6/15/2012 to 10/25/2012: 132.

Thus,

$$AI = \frac{132}{183} \times 2.85/2 = 1.0279$$

And the full price equals:

$$101.25 + 1.0279 = 102.2779.$$

(b) The coupon dates are Jan. 31 and July 31.

(a) The number of days between 7/31/2012 to 1/31/2013 equals 184;

(b) the number of days between 10/25/2012 to 1/31/2013 equals 98.

Let $\tau = 98/184$, $c = 3.25\%$, $T = 5$. Then

$$\begin{aligned} B &= \frac{100}{(1+y/2)^\tau}\left[\sum_{t=0}^{2T}\frac{c/2}{(1+y/2)^t} + \frac{1}{(1+y/2)^{2T}}\right] \\ &= \frac{100}{(1+y/2)^\tau}\left[\frac{c}{y}(1+y/2)\left(1 - \frac{1}{(1+y/2)^{2T+1}}\right) + \frac{1}{(1+y/2)^{2T}}\right]. \end{aligned}$$

Plugging in the numerical values, we obtain $B = 103.1926$.

4. (a) For par bonds, $D_{\text{mod}}(y,T) = \frac{1}{y}\left(1 - \frac{1}{(1+y/2)^{2T}}\right)$. It follows that

$$\begin{cases} D(4\%, 6) = 5.2877, \\ D(2.5\%, 2) = 1.939. \end{cases}$$

(b) $P_2 = -P_1 \dfrac{D(4\%, 6)}{D(2.5\%, 2)} = -272.7$, or short 2.73 units.