

MATH4511 Quantitative Methods for Fixed Income Derivatives Tutorial 2

Yield-to-maturity (yield): all cash flows are discounted by a uniform rate y

$$\begin{aligned} \text{(dirty price) } P &= \text{principle} * \left(\frac{\frac{1}{2}c}{(1+\frac{y}{2})} + \frac{\frac{1}{2}c}{(1+\frac{y}{2})^2} + \dots + \frac{\frac{1}{2}c}{(1+\frac{y}{2})^{2T}} + \frac{1}{(1+\frac{y}{2})^{2T}} \right) \\ &= \text{principle} * \left[\frac{c}{y} \left(1 - \frac{1}{(1+\frac{y}{2})^{2T}} \right) + \frac{1}{(1+\frac{y}{2})^{2T}} \right] \end{aligned}$$

Par bond: when $c = y$, $P(T) = 1$.

Premium bond: when $c > y$, $P(T) > 1$.

Bond sold at discount: when $c < y$, $P(T) < 1$.

Reference

<https://www.thebalance.com/premium-vs-discount-bonds-417066>

Spot Rate (zero-coupon yield)

$\hat{r}(t)$: investing 1 unit of currency from now to year t will generate $(1 + \frac{\hat{r}(t)}{2})^{2t}$ future value of 1.

So we have $d(t) = \frac{1}{(1+\frac{\hat{r}(t)}{2})^{2t}}$ and $\hat{r}(t) = 2(\frac{1}{d(t)^{\frac{1}{2t}}} - 1)$.

Discount yield

Discount yield is a measure of a bond's rate of return to an investor and it is used to calculate the yield on municipal notes and treasury bills sold **at a discount**.

$Y_d = \frac{\text{Par value} - \text{purchase price}}{\text{Par value}} * \frac{360}{\tau}$, where τ is time to maturity, and the formula uses a 30-day month and 360-day year to simplify the calculation. Note that the discount yield is a quoting mechanism (different from yield).

Ex 1 Replication

Bond information

T2M	Maturity	Coupon(%)	Freq	Price(%)
0.5y	15/11/2010	4.5	2	102.15806
1y	15/05/2011	0	2	99.6012
1.5y	15/11/2011	1.75	2	101.64355
1.5y	15/11/2011	2	2	?

Replication, matching of cash flows

Dates	15/05/2001	15/11/2001	15/05/2002	15/11/2002	Face value
Bond1	102.15806	102.25	0	0	0.121188985
Bond2	99.6012	0	100	0	0.123915737
Bond3	101.64355	0.875	0.875	100.875	100.1239157
Bond4	102.0167282	1	1	101	100

Arbitrage

Ex 2

Bond information

	Maturity	Coupon	Freq	Price
B1	15/05/2002	0	2	96.375
B2	15/05/2002	7.5	2	103.4042969
B3	15/05/2002	15	2	106.0625

Q: Are there any arbitrage opportunities?

A: the portfolio consisting of \$50 face amount of B1 and B3 has the same cash flows of \$100 face amount of B2, hence replicates B2.

Price of the Portfolio: 101.21875 < 103.4042969
proceeds by short B2 and long the portfolio: 2.185546875

Spot Rate

Ex 3

Bond information

T2M	Maturity	Coupon	Freq	Price
0.5y	15/11/2001	7.5	2	101.8046875
1y	15/05/2002	7.5	2	103.4042969
1.5y	15/11/2002	11.625	2	110.6640625

The cash flows of the bonds

Time	15/05/2001	15/11/2001	15/05/2002	15/11/2002
Bond1	101.8046875	103.75	0	0
Bond2	103.4042969	3.75	103.75	0
Bond3	110.6640625	5.8125	5.8125	105.8125
T2M	0	0.5	1	1.5
Disc. Factors	1	0.98125	0.961201054	0.939147698
Spot rates	0	3.8217%	3.9966%	4.2296%