Midterm Review

What Have We Learned So Far?

I. Interest rates and bond yields
II. Yield-based risk management
III. Regression-based hedging
IV. Linear interest-rate derivatives

I. Interest Rates and Bond Yields

1. PV & FV
2. Discount factors or discount curve
3. Law of one price and arbitrage
4. Replicating portfolio
5. Bond quotes, AI and full prices
6. Yield to maturity
7. Discount curve, spot-rate curve, forward rate curve, par-yield curve or swap curve.
8. Bootstrapping discount factors or forward rates.

II. Yield-Based Risk Management

1. DV01
2. Duration (modified and MaCaulay)
3. Convexity
4. Duration neutral hedging
5. Kr01
6. Key-rate duration
7. Key-rate hedging

III. Regression-Based Hedging

- Linear relationship b/w the changes of yields:

$$\Delta y_t^{20} = \alpha + \beta \Delta y_t^{30} + \varepsilon_t$$

- Here β, the volatility ratio, is given by

$$\beta = \rho \frac{\sigma_{20}}{\sigma_{30}}$$

- Let F^{20} be the face amount of the 20-year bond, then for hedging we choose

$$P^{30} = -P^{20} \frac{D^{20}}{D^{30}} \beta \qquad \text{or} \qquad F^{30} = -F^{20} \frac{DV01^{20}}{DV01^{30}} \beta$$

IV. Linear interest-rate derivatives

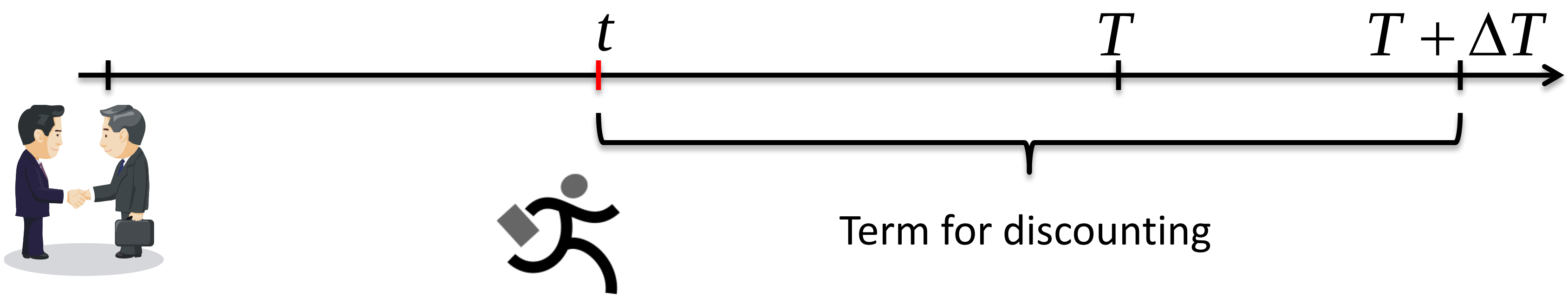
- Linear derivatives
  - Bonds
  - FRA
  - Swaps

FRA

- A bet on interest rates b/w two parties, fixed for floating, indexed to LIBOR.
- Initially the value of the FRA is zero.
- Typically,
  - Three month LIBOR (or CD rates)
  - At least $1m notional.
- The arbitrage free fixed rate is f_0, the forward rate observed at t=0.

MtM Value of FRA

- Let A long the FRA (i.e. pays fixed).

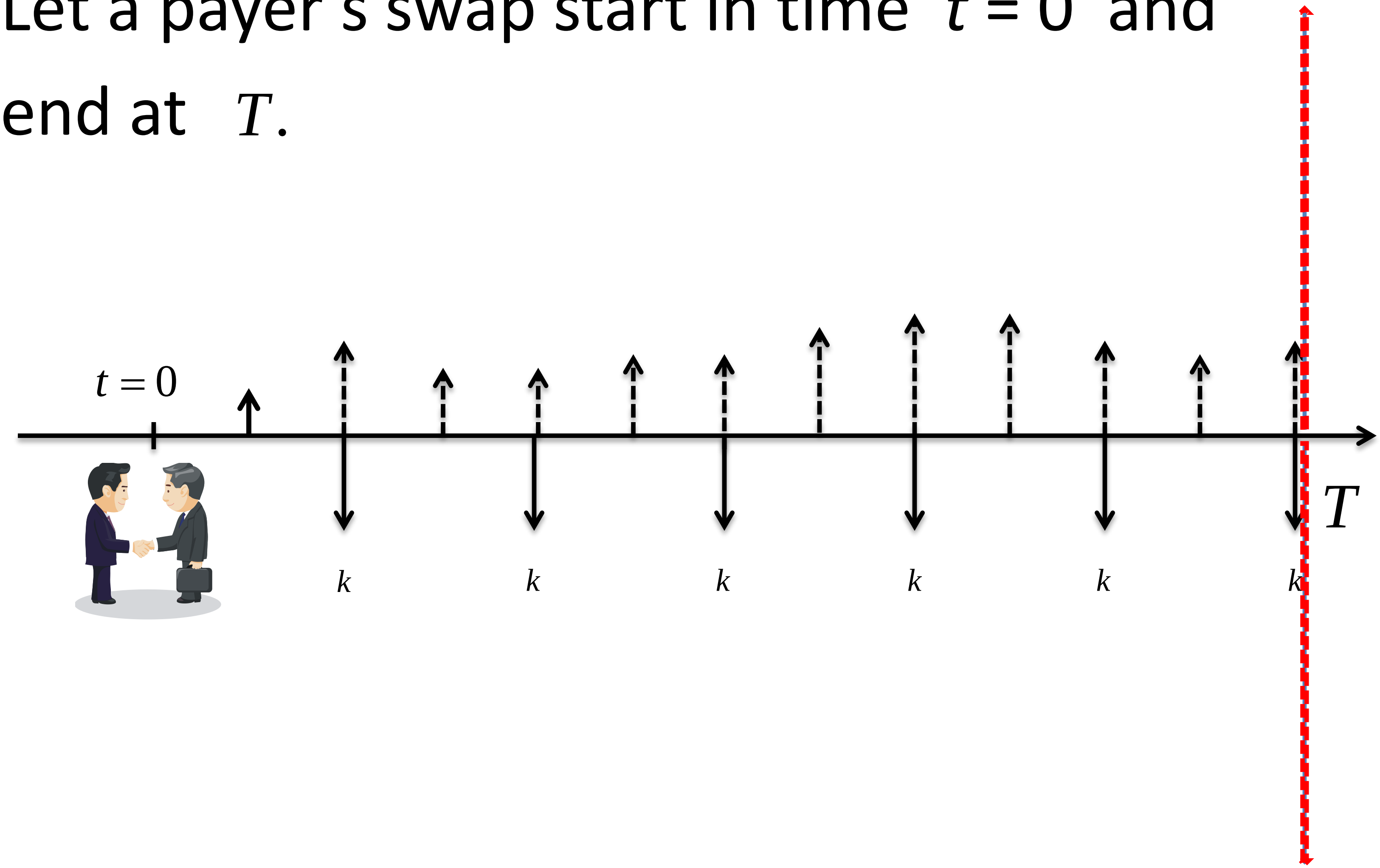


- Let f_t be the forward rate at a later time *t*, then the marking-to-market value of the FRA to the party who pays fixed is

$$P\,\&\,L \text{ to A} = d(t,T+\Delta T)\left[\$1m \times \Delta T \times (f_t - f_0)\right]$$

Swaps

- Let a payer's swap start in time $t = 0$ and end at T.



Determination of the swap rate

– Floating leg: par at $t = 0$, so that

$$V_{float} = 1$$

– Fixed leg: let $s(0,T)$ be the swap rate, then

$$V_{fixed} = \sum_{i=1}^{2T} \tfrac{1}{2} \times s(0,T) d(0, i/2) + d(0,T)$$

- Set

$$0 = V_{float} - V_{fix}$$

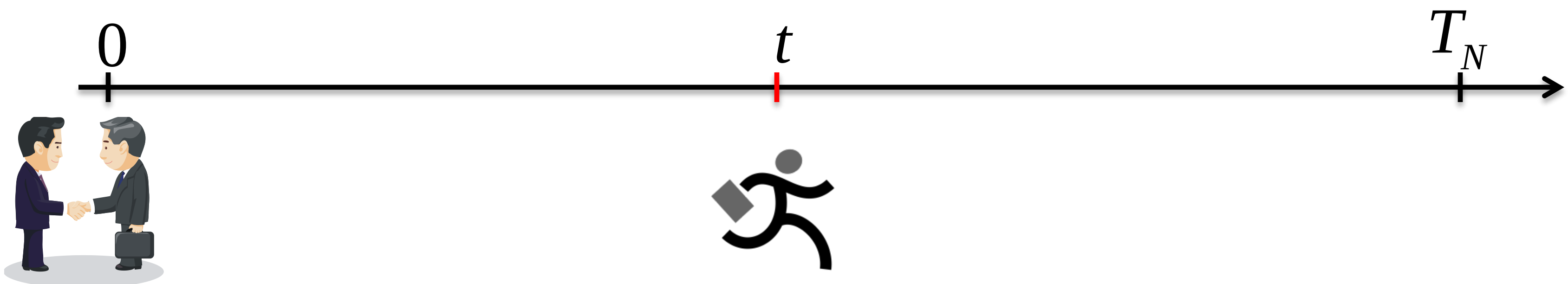
- We obtain

$$s(0,T) = \frac{1 - d(0,T)}{\sum_{i=1}^{N} \frac{1}{2} \times d(0, i/2)}$$

- The above swap rate is often taken as the strike rate for swaptions (so-called ATM swaptions).

MtM Value of a swap

- Let A pays fixed and B pays LIBOR.



- At a later time $j/4 \le t \le (j+1)/4$, the swap rate becomes

$$s(t,T) = \frac{\dfrac{d\left(t, \frac{j+1}{4}\right)}{d\left(\frac{j}{4}, \frac{j+1}{4}\right)} - d(t,T)}{\displaystyle\sum_{i=j+1}^{2T} \tfrac{1}{2}\, d\left(t, \tfrac{i}{2}\right)}$$

- So the value of the swap becomes

$$MtM = \frac{d(t, {}^{j+1}\!/_4)}{d({}^{j}\!/_4, {}^{j+1}\!/_4)} - d(t,T) - s(0,T)\sum_{i=j+1}^{2T} \tfrac{1}{2} d(t, \tfrac{i}{2})$$

$$= \left(s(t,T) - s(0,T)\right)\sum_{i=j+1}^{2T} \tfrac{1}{2} d(t, \tfrac{i}{2})$$