MATH 4511

Quiz 4

Q1

$\hat{r}(0.5) = 5\%$ $\sigma r_t = \theta\Delta t + \sigma\sqrt{\Delta t}\,\varepsilon_t = 0.017071$ or 0.002929

$\therefore r_{0,1} = 0.052929$ $r_{1,1} = 0.067071$

$P_{1,1} = \frac{1000}{(1+\frac{r_{1,1}}{2})} = 967.5526$

$P_{0,1} = \frac{1000}{1+\frac{r_{0,1}}{2}} = 974.2178$

$P_{0,0} = \frac{1000}{(1+\hat{r}(1))^2} = \frac{1000}{(1+\frac{5.5\%}{2})^2} = 947.1883$

$q = \frac{P_{1,1} - P_{0,0}(1+r_{0,0}\Delta t)}{P_{1,1} - P_{0,1}} = 0.49742$

$C_{1,1} = \max(0, P_{1,1} - 970) = 0$

$C_{0,1} = \max(0, P_{0,1} - 970) = 4.2178$

$C_{0,0} = \frac{qC_{0,1} + (1-q)C_{1,1}}{1+\frac{r}{2}} = \$2.047$

Q2

$\hat{r}(\frac{1}{2}) = 0.02$ $\hat{r}(1) = 0.0205$ $\hat{r}(\frac{3}{2}) = 0.021$ $\hat{r}(2) = 0.0215$

$\hat{r}(\frac{5}{2}) = 0.022$ $\hat{r}(3) = 0.0225$

$d(\frac{1}{2}) = \frac{1}{1+\frac{0.02}{2}} = 0.990099$ $d(1) = \frac{1}{(1+\frac{0.0205}{2})^2} = 0.979811$

$d(\frac{3}{2}) = \frac{1}{(1+\frac{0.021}{2})^3} = 0.969150$ $d(2) = \frac{1}{(1+\frac{0.0215}{2})^4} = 0.958131$

$d(\frac{5}{2}) = \frac{1}{(1+\frac{0.022}{2})^5} = 0.946769$ $d(3) = \frac{1}{(1+\frac{0.0225}{2})^6} = 0.935080$

Ian-1-6-2 Swap rate $= \frac{d(1) - d(3)}{\frac{1}{2}(d(\frac{1}{2}) + d(2) + d(\frac{5}{2}) + d(3))}$

$= 2.349\%$

Q3

Q5

(a) Hedge ratio $\alpha = \phi(d_1)$

(b) $V_0 = S_0 - Ke^{-rT}$

(c) Long call - short put = Forward contract

$C_0 - P_0 = V_0 = S_0 - Ke^{-rT}$

(d) $P_0 = C_0 - S_0 + Ke^{-rT}$