MATH4511 Quantitive Methods for Fixed Income Derivatives, 2017-18 Fall Quiz 04(T1B)

Name:	ID No.:	Tutorial Section:

1. (20 points) Consider pricing a 6m-maturity call option on one unit of the 1yr-maturity zero-coupon bond for the strike price of \$970 under the Ho-Lee model

$$\Delta r_t = \theta \Delta t + \sigma \sqrt{\Delta t} \epsilon_B,$$

where ϵ_B takes +1 or -1 with equal probability. Here $r_0 = \hat{r}(0.5) = 5\%$, $\theta = 0.02$, $\sigma = 0.01$ and $\Delta t = 0.5$. In addition, the one-year spot rate (for semi-annual compounding) is $\hat{r}(1) = 5.5\%$. Calculate the option price.

2. (10 points) In November 15, 2017, the spot-rate curve for semi-annual compounding is

$$\hat{r}(i/2) = 0.02 + 0.001 * \frac{i-1}{2}, i = 1, ..., 20.$$

Calculate the in-1-to-2 swap rate (i.e., the swap rate for the swap for the period from 1 to 3 years).

3. (20 points)

Denote the current price of the asset by S_0 , strike price by K and the delivery time by T. We have the Black-Scholes call formula

$$C_0 = S_0 \Phi(d_1) - K e^{-rT} \Phi(d_2),$$

where
$$d_1 = \frac{\ln(S_0 e^{rT}/K) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$$
, $d_2 = d_1 - \sigma\sqrt{T}$. (a) What is the hedge ratio α ?

- (b) What is the fair value of the forward contract V_0 in terms of S_0 , K, r and T.
- (c) Write down the Call and Put Parity using the notations above.
- (d) Derive the formula for put option P_0 in terms of all the notations above.