

**MATH4511**

Quantitative Methods for Fixed-  
income Securities

By

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# Prerequisite

- This course requires background of
  - Multivariable calculus
  - Linear algebra
  - Probability
  - Background in finance and financial markets is *desirable*

# Grading Rules

- Performance will be judged by
  - weekly or biweekly homework assignments,
  - 4 quizzes
  - one midterm, and
  - one final exam.
- Total course scores will be calculated according to the weights of  
(20%, 10%, 20%, 50%)?

# Guideline for Grade Assignments

- University statistics
  - A 10-20%
  - B 25-40%
  - C 35-45%
  - D 5-10%
  - F 0-5%

# Reference Books

- Reference books
  - Bruce, T. (2002) Fixed Income Securities: Tools for Today's Markets, 2<sup>nd</sup> edition (Wiley Finance)
  - Hull, John (2014). Options, Futures, and Other Derivatives, 9th ed. Prentice Hall.
  - McDonald, Robert, L. (2014). Derivatives Markets, 3rd ed. Pearson.

# Other Reference Books

- Veronsi, P. (2010). Fixed-income securities: valuation, risk and risk management. Wiley.
- Fabozzi, Frank J. (1997). Fixed income mathematics: analytical & statistical techniques, 3rd ed., McGraw-Hill (ISBN: 0-7863-1121-5).
- Fabozzi, Frank J. (2003). Bond markets, analysis and strategies, 5th ed. Pearson/Prentice Hall (ISBN: 0-13-049782-7).
- Zipf Robert (1996). How the Bond Market Works, the 2nd edition, New York Institute of Finance (ISBN:0-13-124306-3).

# Course Outlines

Introduction to financial markets
Bond and interest-rate derivatives markets
Yields, forward rate and swap rates
Yield-based hedging and regression-based hedging
Mortgage mathematics.
Binomial models for equity and fixed-income derivatives.
Arbitrage pricing and risk-neutral valuation principle
Eurodollar futures
Lognormal models
Black formula for caps and swaptions.

# **Chapter 0**

## **An Overview of Global Financial Markets**





New York



London



Chicago



Singapore



Tokyo



Zurich





Frankfurt



Dubai





Hong Kong

# What People Trade?

- Stocks
- Bonds
- Currencies
- Commodities
- and a combination of all of them with optionality involved (and the securities can be very complex!)
- [Welcome to Bloomberg!](#)

# What people do in an exchange, like this?





# Or like this?



Traders



Sales

# How to make a deal or trade?

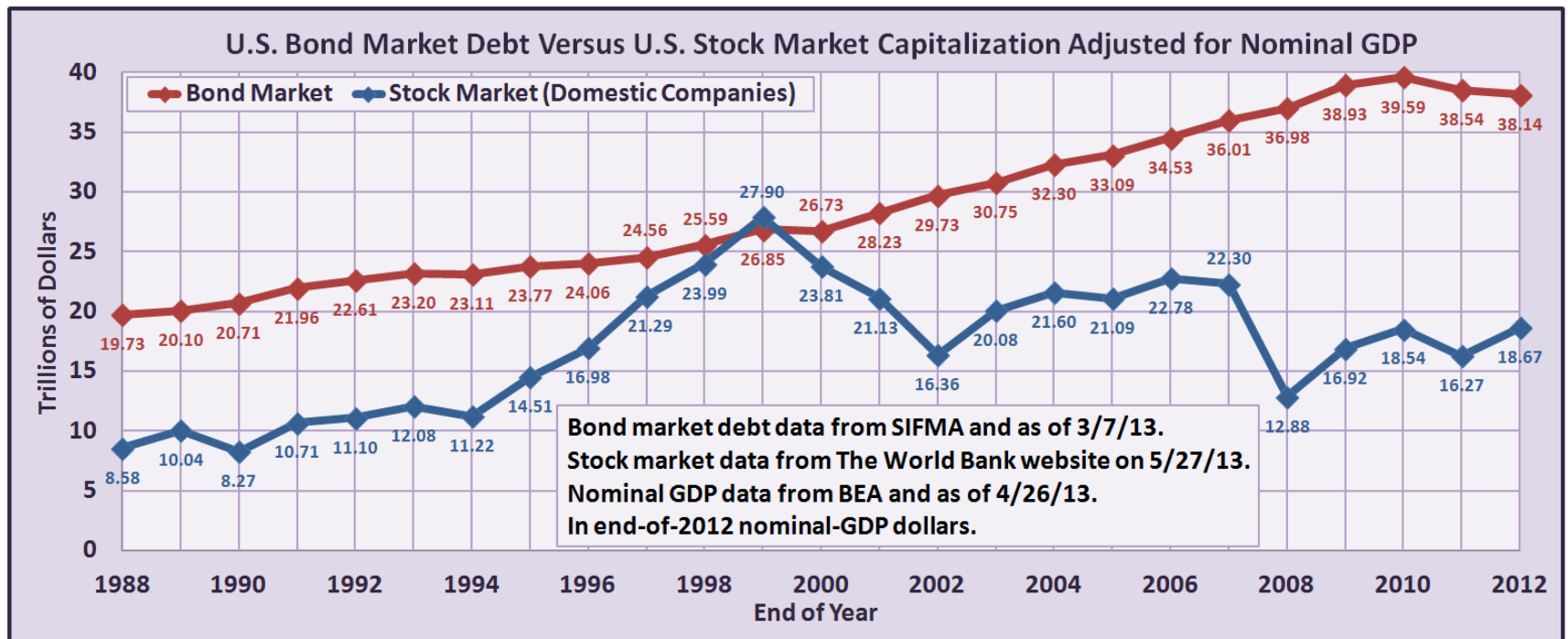
- Go through an exchange
  - Standardized products
- Go through Over-The-Counter (OTC) markets
  - Trade can be standardized or non-standardized.
  - Used to be less regulated

# Bond Note



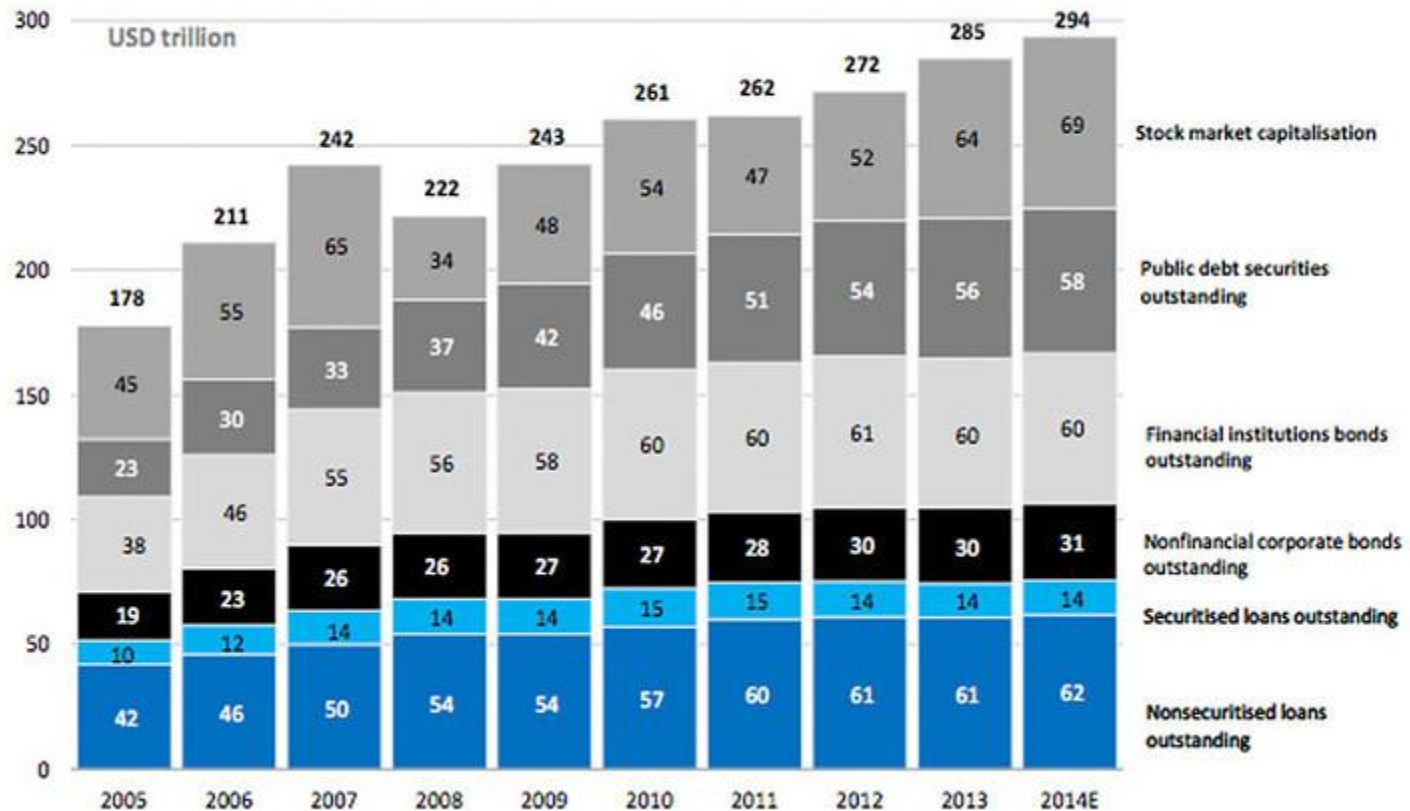


# Which is Bigger, Bond vs. Equity?



# Debt vs. Equity: Global prospective

## Global financial assets



McKinsey Global Institute, Hover, BIS, Deutsche Bank estimates, 2015



# Securities Industry and Financial Markets Association (sifma.org)



## EQUITY MARKETS



**\$221.2 BILLION**  
of equity was issued  
in the U.S., including  
common and preferred  
shares

**\$49.9 BILLION**  
initial public offering  
(IPO) volume, excluding  
closed-end funds

Sources: World Federation of  
Exchanges, Dealogic

## BOND MARKETS



**\$2.2 TRILLION**  
corporate debt,  
asset-backed securities  
and non-agency  
mortgage-backed  
securities was issued  
in the U.S.

Sources: BIS, Refinitiv,  
Bloomberg

## U.S. CAPITAL MARKETS

U.S. merger and acquisition announced deals totaled **\$1.7 trillion** in 2018, a **23.7% increase** from 2017, while the value of completed M&A deals rose by **16.7% to \$1.9 trillion**.

Source: Dealogic



## INVESTOR PARTICIPATION

**\$42.2 TRILLION**  
The value of U.S. households' liquid assets decreased by **1.6%**.

Source: Federal Reserve Board



## RETIREMENT

**\$34.6 TRILLION**

The total value of U.S. retirement assets decreased **1.4%** year-over-year but is the second highest on record.

Source: Federal Reserve Flow of Funds  
Accounts



## SAVINGS

**6.4%**

The U.S. household savings rate is projected to be **6.4%**, middle-of-the-pack for major nations

Source: OECD Economic Outlook



## INDUSTRY

**970K**

The securities industry employs 970,100 individuals, a **2.7% increase** year-over-year.

Source: U.S. Department of Labor



# **Chapter 1**

## **Bond Prices, Discount Factors, and Arbitrage**

# Starting with Coupon Bonds

- Three aspects: In May 2010 the U.S. Treasury sold a bond with
  - a coupon rate of  $2 \frac{1}{8}\%$  and
  - a maturity date of May 31, 2015
  - a payment frequency of two a year, six months apart
- This bond is called “ $2 \frac{1}{8}s$  of May 31, 2015”

Coupon rate

Coupon frequency, “s”  
is for “semi-annual”

maturity

# Cash Flow of the Bond

- The unit for bond purchasing is \$1,000.
- Suppose that an investor purchases \$1m face value of the bond, i.e., 1,000 units.
- The the coupon payment is calculated according to

$$\frac{1}{2} \times 2\frac{1}{8}\% \times \$1,000,000 = \$10,625$$

Year fraction

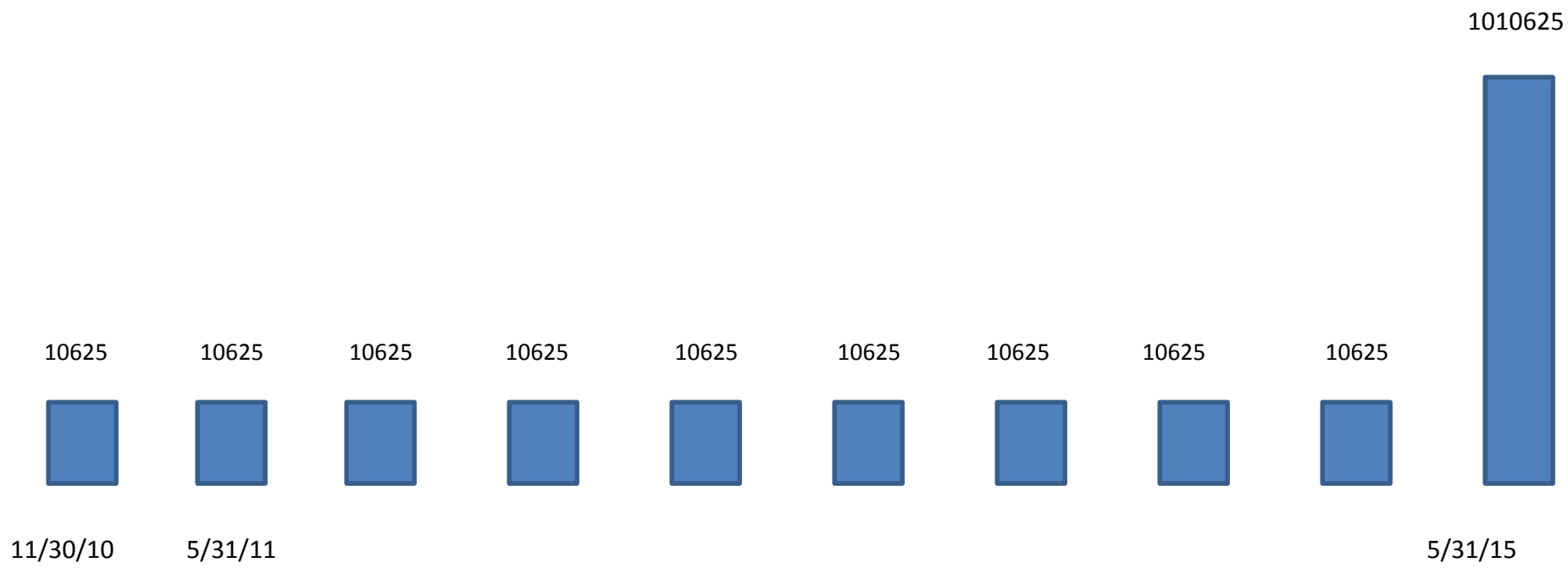
Coupon rate

Face value

# Cash Flow of the Bond, cont'd

Table 1.1: Cash Flows of the U.S.  $2\frac{1}{8}$  from books of May 31, 2015

	<b>Coupon Payment</b>	<b>Principal Payment</b>
<b>Date</b>		
"11/30/2010"	\$10, 625	
"5/31/2011"	\$10, 625	
"11/30/2011"	\$10, 625	
"5/31/2012"	\$10, 625	
"11/30/2012"	\$10, 625	
"5/31/2013"	\$10, 625	
"11/30/2013"	\$10, 625	
"5/31/2014"	\$10, 625	
"11/30/2014"	\$10, 625	
"5/31/2015"	\$10, 625	\$1, 000, 000



# Government Bonds

- [US Treasury](#)
- [Exchange Fund Bills & Notes Fixings](#) (Hong Kong)