MATH4511 Quantitative Methods for Fixed Income Derivatives

Tutorial 8

Tree of interest rate

1. the value of the underlying asset should be determined by the interest rate curve: bonds (not stocks)
2. dynamic of the interest rate

$$\Delta r_t = (\mu_t + \lambda_t)\Delta t + \sigma\sqrt{\Delta t}\epsilon$$

3. λ_t: the tree must reproduce the discount curve

$$\begin{tikzcd}
 & r_{1,1} = r_{0,0} + (\mu_0 + \lambda_0)\Delta t + \sigma\sqrt{\Delta t} \\
r_{0,0} \arrow[ur, "\frac{1}{2}"] \arrow[dr, "\frac{1}{2}"'] & \\
 & r_{0,1} = r_{0,0} + (\mu_0 + \lambda_0)\Delta t - \sigma\sqrt{\Delta t}
\end{tikzcd}
\qquad
\begin{tikzcd}
 & & 1000 \\
 & P_{1,1} \arrow[ur, "\frac{1}{2}"] \arrow[dr, "\frac{1}{2}"'] & \\
P_{0,0} \arrow[ur, "\frac{1}{2}"] \arrow[dr, "\frac{1}{2}"'] & & 1000 \\
 & P_{0,1} \arrow[ur, "\frac{1}{2}"] \arrow[dr, "\frac{1}{2}"'] & \\
 & & 1000
\end{tikzcd}$$

$$P_{1,1}(\lambda_0) = \frac{1000}{1 + r_{1,1}\Delta t} \quad P_{0,1}(\lambda_0) = \frac{1000}{1 + r_{0,1}\Delta t} \quad P_{0,0}(\lambda_0) = \frac{\frac{1}{2}(P_{1,1} + P_{0,1})}{1 + r_{0,0}\Delta t}$$

From these three equations, once we get the current price $P_{0,0}$ from the market, λ_0 can be solved.

Option pricing

$$\begin{tikzcd}
 & C_{1,1} = PayOff(P_{1,1}) \\
C_{0,0} = \frac{\frac{1}{2}(C_{0,1}+C_{1,1})}{1+r_{0,0}\Delta t} \arrow[ur, "\frac{1}{2}"] \arrow[dr, "\frac{1}{2}"'] & \\
 & C_{0,1} = PayOff(P_{0,1})
\end{tikzcd}$$

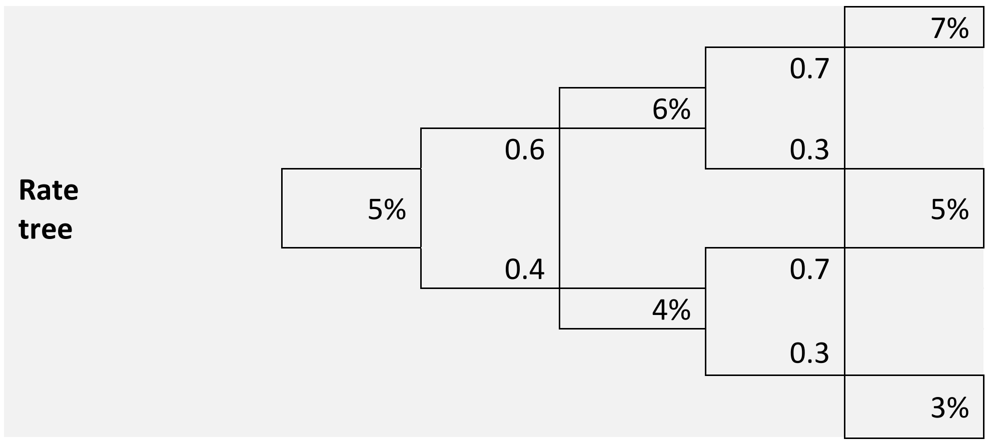
hedge ratio $\quad \alpha = \dfrac{C_{1,1} - C_{0,1}}{P_{1,1} - P_{0,1}}$

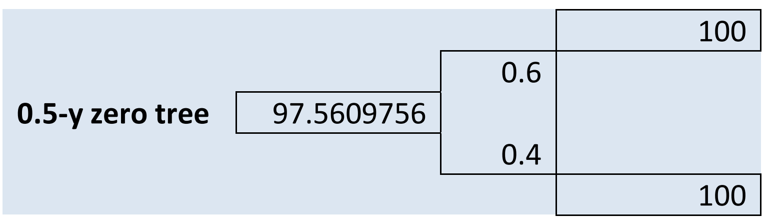
pricing coupon bond option:

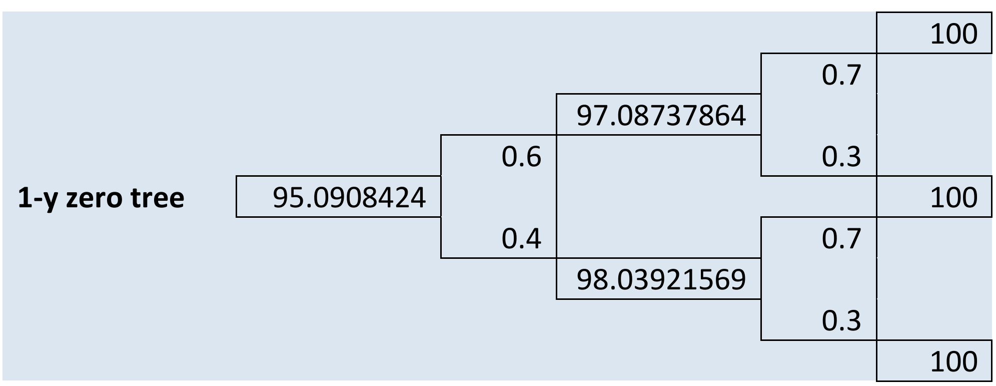
1. build an interest rate tree to reproduce the whole discount curve
2. obtain the price distribution $P_{i,j}$
3. calculate the payoff $C_{i,j}$
4. calclulate the PV (the value at the node (0,0)) of the option

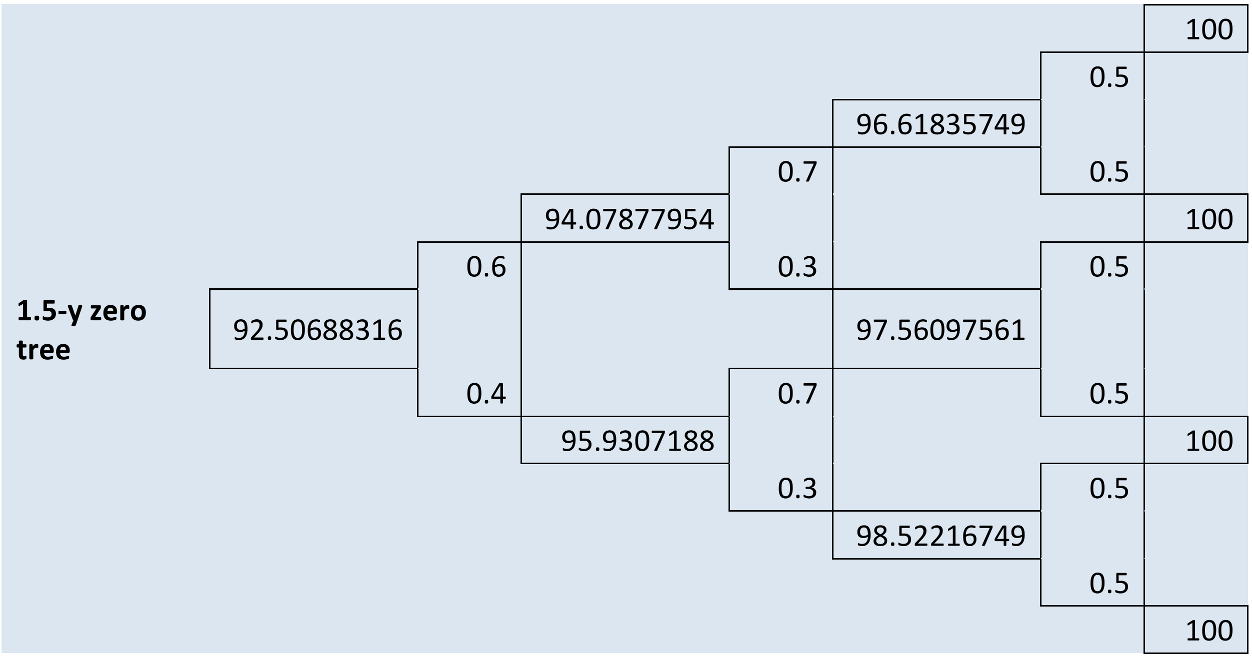
Example 1

Risk-neutral Probability		0.6	0.7	0.5
Time	0	0.5	1	1.5
Date	0	1	2	3

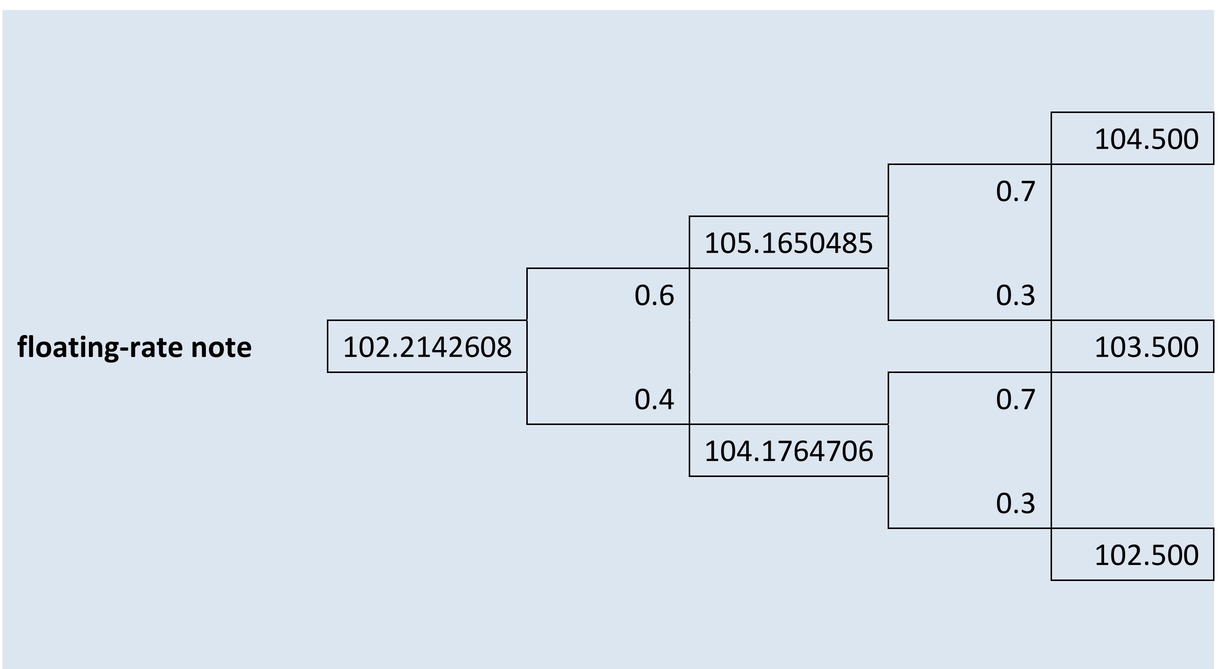








1. Price a floating-rate note with the maturity of one year that at date i, $i = 1, 2$, pays the coupon in the amount of $\$100 \times (r_i + 2\%)/2$. In addition, at maturity, the principal \$100 is paid.



2. Using the risk-neutral process tree built above to price \$100 notional amount of a 1.5-year *participating cap* with a strike of 5% and a participation rate of 40%. Payments are made every six months according to this rule: If the short rate on date i is r_i, then the cash flow from the participating cap on date $i+1$ is, as a percent of par,

$$\begin{cases} (r_i - 5\%)/2, & \text{if } r_i \geq 5\% \\ 40\% \times (r_i - 5\%)/2, & \text{if } r_i < 5\%. \end{cases}$$

There is no principal payment at maturity.

							0
						0.5	
					0.966183575		
				0.7		0.5	
			1.142066507				0
		0.6		0.3		0.5	
participating cap	0.546775765				0		
		0.4		0.7		0.5	
			-0.311986864				0
				0.3		0.5	
					-0.39408867		
						0.5	
							0

3. (1) Price \$100 face amount of the following 1.5-year *collared floater*. Payments are made every six months according to this rule: If the short rate on date i is r_i, then the interest payment of the collared floater on date $i+1$ is

$$\begin{cases} 3.5\%/2, & \text{if } r_i < 3.5\%, \\ r_i/2, & \text{if } 3.5\% \leq r_i \leq 6.5\%, \\ 6.5\%/2, & \text{if } r_i > 6.5\%. \end{cases}$$

							100
						0.5	
					99.75845411		
				0.7		0.5	
			99.8358426				100
		0.6		0.3		0.5	
floater	99.93217821				100		
		0.4		0.7		0.5	
			100.0724428				100
				0.3		0.5	
					100.2463054		
						0.5	
							100

Example 2

Time	0	0.5	1	1.5
Spot		5.0%	5.5%	6.0%

lambda		0.020083127	0.020165565
sigma	1.10%		
dt	0.5		

			8.57%
		6.78%	
Rate tree	5.00%		7.01%
		5.23%	
			5.46%

		1
0.5y zero	0.975610	
		1

			1
		0.967202	
1y zero	0.947188		1
		0.974534	
			1

				1
			0.958920	
		0.930954		1
1.5y zero	0.915142		0.966126	
		0.945086		1
			0.973441	
				1

Call on 1.5-y zero

Strike	965
face value	1000

			0
		0.544312	
1-y call	2.53932		1.125539148
		4.661286	
			8.440646883

Hedge ratio $\boxed{0.291315224}$