MATH4511 Quantitative Methods for Fixed Income Derivatives

Tutorial 7

Not a good assumption: parallel shifts in yield

1-V Least-Squares Regression

$$\Delta y_t^{20} = \alpha + \beta \Delta y_t^{30} + \epsilon_t$$

Use least-square estimation to estimate α and β

$$\hat{\alpha} = \frac{\overline{\Delta y^{20}}\,\overline{\left(\Delta y^{30}\right)^2} - \overline{\Delta y^{30}}\ \overline{\Delta y^{30}\Delta y^{20}}}{\overline{\left(\Delta y^{30}\right)^2} - \left(\overline{\Delta y^{30}}\right)^2}, \qquad \hat{\alpha} = \frac{\mu_{20}\left(\sigma_{30}^2 + \mu_{30}^2\right) - \mu_{30}\left(\rho\sigma_{20}\sigma_{30} + \mu_{20}\mu_{30}\right)}{\sigma_{30}^2}$$

$$\hat{\beta} = \frac{\overline{\Delta y^{20}\Delta y^{30}} - \overline{\Delta y^{20}}\ \overline{\Delta y^{30}}}{\overline{\left(\Delta y^{30}\right)^2} - \left(\overline{\Delta y^{30}}\right)^2} \qquad \hat{\beta} = \rho\frac{\sigma_{20}}{\sigma_{30}}$$

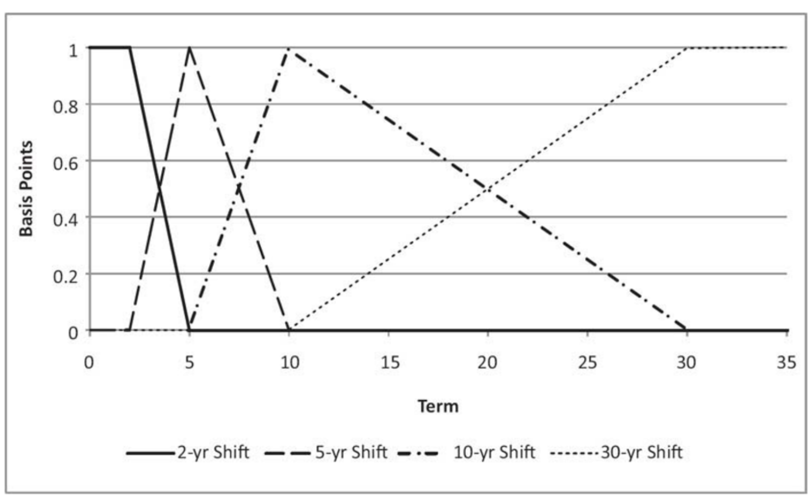
Hedging: $P_{30} = P_{20}\frac{D_{20}}{D_{30}}\hat{\beta}$ or $F_{30} = F_{20}\frac{DV01_{20}}{DV01_{30}}\hat{\beta}$

2-V Least-Squares Regression $\quad \Delta y_t^{20} = \alpha + \beta^{10}\Delta y_t^{10} + \beta^{30}\Delta y_t^{30} + \epsilon_t$

Hedging: $\quad P_{10} = P_{20}\dfrac{D_{20}}{D_{10}}\beta_{10}$ **or** $\quad F_{10} = F_{20}\dfrac{DV01_{20}}{DV01_{10}}\beta_{10}$

$\quad P_{30} = P_{20}\dfrac{D_{20}}{D_{30}}\beta_{30} \qquad F_{30} = F_{20}\dfrac{DV01_{20}}{DV01_{30}}\beta_{30}$

Key rate Shifts



Exercise 1

You consider hedging FNMA 6.5s of August 15, 2004, with FNMA 6s of May 15, 2011. Taking changes in the yield of the 6s of May 15, 2011, as the independent variable and changes in the yield of the 6.5s of August 15, 2004, as the dependent variable from July 2001 to January 2002 gives the following regression results:

Number of observations	131
R-squared	77.93%
Standard error	4.0861

Regression Coefficients	*Value*	*t-Stat*
Constant	–.0007549	–2.1126
Change in yield of 6s of 5/15/2011	.9619	21.3399

The DV01 of the 6.50s of August 15, 2004, is 2.796, and the DV01 of the 6s of May 15, 2011, is 7.499. Using the regression results given, how much face value of the 6s of May 15, 2011, would you sell to hedge a $10,000,000 face value position in the 6.50s of August 15, 2004?

Since the yield of the 6.5s of August 15, 2004, is assumed to move .9619 basis points for every basis point move of the 6s of May 15, 2011, the face amount of the latter, F_{6s}, must satisfy the following equation:

$$10{,}000{,}000 * \frac{2.796}{100} * 0.9619 = F_{6s} * \frac{7.499}{100}$$

Solving, approximately $3.59 million of the 6s of May 15, 2011, must be sold.

Exercise 2

Column A should contain the coupon payment dates from .5 to 5 years in increments of .5 years. Let column B hold a spot rate curve flat at 4.50%. Put the discount factors corresponding to the spot rate curve in column C. Price a 12% and a 6.50% four-year bond under this initial spot rate curve.

Term	Spot0	Disc0	CF1	Bond1	CF2	Bond2
0.5	4.50%	0.97799511	6	5.86797066	3.25	3.178484108
1	4.50%	0.956474435	6	5.738846611	3.25	3.108541915
1.5	4.50%	0.935427321	6	5.612563923	3.25	3.040138792
2	**4.50%**	0.914843345	**6**	5.489060072	**3.25**	2.973240872
2.5	4.50%	0.894712318	6	5.368273909	3.25	2.907815034
3	4.50%	0.875024272	6	5.250145632	3.25	2.843828884
3.5	4.50%	0.855769459	6	5.134616755	3.25	2.781250742
4	4.50%	0.836938346	106	88.71546471	103.25	86.41388426
4.5	4.50%	0.81852161				
5	**4.50%**	0.800510132				
				127.1769423		107.2471846

Create a new spot rate curve, by adding a two-year key rate shift of 10 basis points, in column D.

Compute the new discount factors in column E. What are the new bond prices?

Spot1	Disc1	Bond1	Bond2
4.60%	0.977517107	5.865102639	3.176930596
4.60%	0.955539694	5.733238162	3.105504004
4.60%	0.934056396	5.604338379	3.035683289
4.60%	0.913056106	5.478336636	2.967432345
4.58%	0.892891577	5.357349464	2.901897626
4.57%	0.873314684	5.239888105	2.838272724
4.55%	0.854306248	5.12583749	2.776495307
4.53%	0.835847784	88.59986513	86.30128373
4.52%	0.817921472		
4.50%	0.800510132		
		127.003956	107.1034996

Create a new spot rate curve, by adding a five-year key rate shift of 10 basis points, in column F. Compute the new discount factors in column G. What are the new bond prices?

Spot2	Disc2	Bond1	Bond2
4.50%	0.97799511	5.86797066	3.178484108
4.50%	0.956474435	5.738846611	3.108541915
4.50%	0.935427321	5.612563923	3.040138792
4.50%	0.914843345	5.489060072	2.973240872
4.52%	0.894347814	5.366086883	2.906630395
4.53%	0.874168991	5.245013943	2.841049219
4.55%	0.854306248	5.12583749	2.776495307
4.57%	0.834758821	88.48443499	86.18884823
4.58%	0.815525804		
4.60%	0.796606165		
		126.9298146	107.0134288

Use the results above to calculate the key rate durations of each of the bonds.
The results may be summarized as follows:

	Initial Price	*Two-Year 10bp Shift*	*Two-Year Key Rate Duration*	*Five-Year 10bp Shift*	*Five-Year Key Rate Duration*
12% bond	127.1769	127.0040	1.3602	126.9298	1.9432
6.5% bond	107.2472	107.1035	1.3398	107.0134	2.1796

Sum the key rate durations to obtain the total duration of each bond. Calculate the percentage of the total duration accounted for by each key rate for each bond. Comment on the results.

The results are as follows:

	Two-Year Key Rate Duration	*Five-Year Key Rate Duration*	*Total Duration*	*Percent in Two-Year*	*Percent in Five-Year*
12% bond	1.3602	1.9432	3.3034	41.2%	58.8%
6.5% bond	1.3398	2.1796	3.5194	38.1%	61.9%