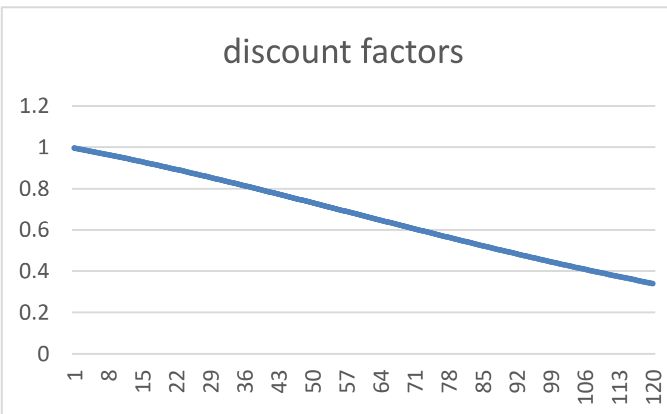
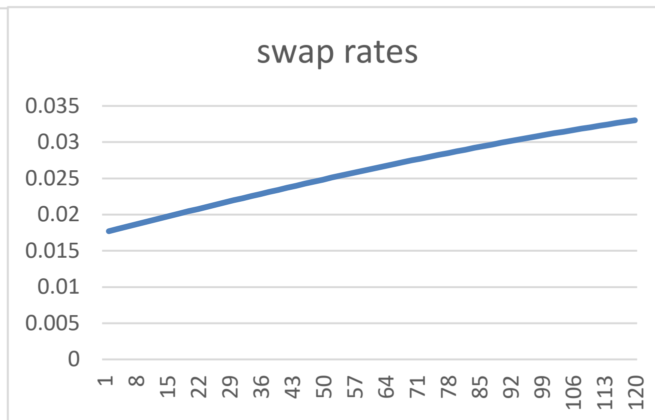
Homework 4 Solution. Quantitative Methods for fixed Income Securities

CHAPTER 3 Yield to Maturity (Tuckman)

3.13 $f\left(\frac{i}{4}\right) = 0.0175 + 0.00125 \times \frac{i-1}{4}$ (Refer to [Swap_P&L 2])

(a) Recall that $f\left(\frac{i}{4}\right) = 4\left(\frac{d\left(\frac{i-1}{4}\right)}{d\left(\frac{i}{4}\right)} - 1\right)$, we have $d\left(\frac{i}{4}\right) = \frac{d\left(\frac{i-1}{4}\right)}{1+\frac{1}{4}f\left(\frac{i}{4}\right)}$.





(b) $s(T) = \frac{1-d(T)}{\sum_{i=1}^{2T}\frac{1}{2}d\left(\frac{i}{2}\right)}$.

(c) For 10-year receiver's swap: $s(20) = 0.023417091$, and after one year, it becomes 9-year swap, the updated $s(18)_{new} = 0.023285246$.

The P&L of the receiver's swap (for a buyer): $(s(20) - s(18)_{new}) * \frac{1}{2} * \sum_{i=1}^{18} d\left(\frac{i}{2}\right)$ =-0.001071379 (now we are at time t=1).

3.14 $f(2.25) = 0.02$ and $f(1.25)_{new} = 0.02$, If A chooses to close out the FRA, the P&L of A is d(1.25)*1m*1/4*(2%-2%) = $0.

3.15 $P = \sum_{i=1}^{40} \frac{25000}{(1+6\%)^i} = 25000 * 15.04629687 = \$376157.4218.$

3.16 $X = \frac{B(0)\times\frac{y}{12}}{1-(1+\frac{y}{12})^{-12T}} = \frac{500000\times\frac{3.25\%}{12}}{1-(1+\frac{3.25\%}{12})^{-12\times 30}} = 21760.32.$

CHAPTER 6

6.3 Using equations of the text, the results are as follows:

Coupon	*Maturity*	*Yield*	*Price*	*DV01*	*Modified Duration*
8.75	5/15/2020	5.9653%	131–12$^7/_8$	.1372	10.44
8.125	5/15/2021	5.9857%	124–24$^1/_8$	.1357	10.88

Note that the 8.75s of May 15, 2020, have the larger DV01 but the 8.125s of May 15, 2021, have the longer duration. The 8.125s of May 15, 2021, have the longer duration because they are of longer maturity and lower coupon. The 8.75s of May 15, 2020, nevertheless have a higher DV01 because their dollar price is so much higher.

6.14 On May 15, 2001, the price of the 6.75s of May 15, 2005, is 106-21$\frac{1}{8}$. The yield is 4.8964%.

(a) DV01

$$DV01 = -\frac{P(y+.01\%) - P(y-.01\%)}{2} = -\frac{106.6228 - 106.6975}{2} = 0.03735$$

(b) Duration $\Delta y = 10\ bps$

$$D = -\frac{1}{P}\frac{\Delta P}{\Delta y} = -\frac{1}{P}\frac{P(y+\Delta y) - P(y-\Delta y)}{2\Delta y} = -\frac{1}{P}\frac{P(y+0.001) - P(y-0.001)}{0.002}$$
$$= -\frac{1}{106.6602}\frac{106.2874 - 107.0345}{0.002} = 3.5022$$

(c) Convexity

$$C = \frac{1}{P}\frac{d^2P}{dy^2} = \frac{1}{P}\frac{P(y+\Delta y) - 2P(y) + P(y-\Delta y)}{\Delta y^2} = \frac{1}{106.6602}\frac{106.2874 - 2 * 106.6602 + 107.0345}{0.001^2}$$
$$= 14.06335$$

For a yield change of 25 bps, the actual price for $P(y+\Delta y)$ is 105.7313, $P(y-\Delta y)$ is 107.5989.

If we estimate price using duration and convexity, we will get $\Delta P \approx -DP\Delta y + \frac{1}{2}CP\Delta y^2 = -3.5022 * 106.6602 * 0.0025 + \frac{1}{2} * 14.06335 * 106.6602 * 0.0025^2 = -0.929176$.

The estimation is quite close to the actual price.