$$\begin{split} P_0 &= e^{-rT} E[(K - S_0 e^{\left(r - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T} \times z})^+] \\ S_T &< K \Leftrightarrow z \leq -d_2 \\ E\left[\left(K - S_0 e^{\left(r - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T} \times z}\right)^+\right] = \int_{-\infty}^{-d_2} K - S_0 e^{\left(r - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T} \times x} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \\ &= K \int_{-\infty}^{-d_2} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx - S_0 e^{rT} \int_{-\infty}^{-d_2} \frac{1}{\sqrt{2\pi}} e^{\left(\frac{-\sigma^2}{2}\right)T + \sigma\sqrt{T} \times x - \frac{x^2}{2}} dx \\ &= K \Phi(-d_2) - S_0 e^{rT} \int_{-\infty}^{-d_2} \frac{1}{\sqrt{2\pi}} e^{-\frac{(x - \sigma\sqrt{T})^2}{2}} dx \\ &= K \Phi(-d_2) - S_0 e^{rT} \int_{-\infty}^{-d_2 - \sigma\sqrt{T}} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy = K \Phi(-d_2) - S_0 e^{rT} \int_{-\infty}^{-d_1} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy \\ &= K \Phi(-d_2) - S_0 e^{rT} \Phi(-d_1) \\ P_0 &= e^{-rT} K \Phi(-d_2) - S_0 \Phi(-d_1) \\ \frac{\partial P(S_0, 0)}{\partial S_0} &= e^{-rT} K \frac{n(-d_2)}{S_0 \sigma \sqrt{T}} - \frac{S_0 n(d_1)}{S_0 \sigma \sqrt{T}} - \Phi(-d_1) = \frac{e^{-rT} \frac{K}{S_0} n(-d_2) - n(-d_1)}{S_0 \sigma \sqrt{T}} - \Phi(-d_1) \end{split}$$

Q3

Price of the Call 32.3250

Price of the Put 30.9625

Price of Forward contract 1.3625=32.3250-30.9625

```
function x=call(F,K,T,siq,r)
                                          function x=put(F,K,T,sig,r)
        if T*sig == 0
                                                   if T*sig == 0
            x=max(F-K,0);
                                                       x=max(K-F,0);
            return;
                                                       return;
        d1 = log(F/K) + 0.5*(sig^2)*T;
                                                   d1 = log(F/K) + 0.5*(sig^2)*T;
        d1=d1/(sig*sqrt(T));
                                                   d1=d1/(sig*sqrt(T));
        d2=d1-sig*sqrt(T);
                                                   d2=d1-sig*sqrt(T);
        nd1=0.5*(1+erf(d1/sqrt(2)));
                                                   nd1=0.5*(1+erf(-d1/sqrt(2)));
        nd2=0.5*(1+erf(d2/sqrt(2)));
                                                   nd2=0.5*(1+erf(-d2/sqrt(2)));
                                                   x=exp(-r*T)*K*nd2-F*nd1;
        x=F*nd1-exp(-r*T)*K*nd2;
                                                   return;
```

```
dK=5;
K=50:dK:150; CP=zeros(size(K));
DCF=0.98;
for i=1:length(K)
    Ki=K(i);
    CP(i) = call2(F, Ki, T, sig, DCF);
end
plot(K,CP)
    function x=call2(F,K,T,sig,DCF)
    if T*sig == 0
        x=max(F-K,0);
        return;
    end
    d1 = log(F/K) + 0.5*(sig^2)*T;
    d1=d1/(sig*sqrt(T));
    d2=d1-sig*sqrt(T);
    nd1=0.5*(1+erf(d1/sqrt(2)));
    nd2=0.5*(1+erf(d2/sqrt(2)));
    x=F*nd1-DCF*K*nd2;
    return;
for i=1:length(K)
    Ki=K(i);
    PP(i) = put2(F, Ki, T, sig, DCF);
plot(K, PP)
    function x=put2(F,K,T,sig,DCF)
    if T*sig == 0
        x=max(K-F,0);
        return;
    end
    d1 = log(F/K) + 0.5*(sig^2)*T;
    d1=d1/(sig*sqrt(T));
    d2=d1-sig*sqrt(T);
    nd1=0.5*(1+erf(-d1/sqrt(2)));
    nd2=0.5*(1+erf(-d2/sqrt(2)));
    x=DCF*K*nd2-F*nd1;
    return;
```



