

## Solutions to the Midterm of Math 4511, Fall 2019

### 1. Solution:

1.1 (4) A vanilla interest-rate swap is a financial contract between two parties such that, out of the same principal value, one party makes semiannual interest payments with a fixed interest rate, while the other party makes quarterly interest payments according to the three-month LIBOR rate.

1.2 (8) After adding the principal to the last payments, the floating leg is priced at par, so the fixed is also priced at par, leading to the equality

$$s(0, T) \frac{1}{2} \sum_{i=1}^{2T} d(0, i/2) + d(0, T) = 1.$$

The formula for the swap rate then follows

1.3 (4) The MtM value is  $MtM = (s(t, T) - s(0, T)) \frac{1}{2} \sum_{\frac{i}{2} > t}^{2T} d(t, i/2)$ . Let  $T_j \leq t < T_{j+1}$ ,

$$\text{then } s(t, T) = \left( \frac{d(t, T_{k+1})}{d(T_k, T_{k+1})} - d(t, T) \right) / \frac{1}{2} \sum_{i=k+1}^{2T} d(t, i/2).$$

### 2. Solution:

$$2.1 \quad (4) \quad d(0) = 1, \quad d(i/2) = \frac{1 - \frac{s(0, i/2)}{2} \sum_{j=1}^{2i-1} d(j/2)}{1 + s(0, i/2)/2}, \quad i = 1, \dots, 60.$$

$$2.2 \quad (4) \quad \hat{r}(i/2) = 2 \left( \frac{1}{[d(i/2)]^{1/i}} - 1 \right).$$

$$2.3 \quad (4) \quad f(i/2) = 2 \left( \frac{d((i-1)/2)}{d(i/2)} - 1 \right).$$

### 3. Solution:

Maturity	Coupon	Price	DC (4)	SW rate (4)
0.5	0.50%	99.9375	0.99688	0.625%
1	0.75%	99.875	0.99129	0.876%
1.5	1%	100.063	0.98576	0.958%
2	1.25%	100.25	0.9778	1.123%

### 4. Solution:

4.1 (2) Either long payer's swap or short receiver's swap.

4.2 (4) The DV01's are

$$DV01_b = 0.019753419, \quad DV01_{sw} = 0.019722372.$$

It then follows that the face value is \$1,001,574.22.

### 5. Solution:

5.1 (4) With  $B_0 = \$6,400,000$ ,  $y = 2.5\%$  and  $T = 20$ , we obtain

$$X = \frac{B_0 \times \frac{y}{12}}{1 - (1 + \frac{y}{12})^{-12T}} = \$33,913.7852.$$

5.2 (4) Let  $T = 15$ ,  $X = 33913.7852$ , then the outstanding value of the loan is

$$B_{60} = \frac{12X}{y} \left( 1 - (1 + y/12)^{-12T} \right) = \$5,086,132.88.$$