MATH4511 Quantitative Methods for Fixed Income Derivatives

Tutorial 5

Swap Position

Long a payer's swap: pay fixed and receive floating

Short a payer's swap: pay floating and receive fixed

Long a receiver's swap: pay floating and receive fixed

Short a receiver's swap: pay fixed and receive floating

FRA Position

Long FRA: pay fixed and receive floating

Short FRA: pay floating and receive fixed

Call/Put option

A contract which gives its holder the right to **buy/sell** a prescribed asset, known as the underlying asset, at a certain date for a predetermined price.

DV01: one way to measure risk

$$DV01 = -\frac{P(y + .01\%) - P(y - .01\%)}{2}$$

DV01 based hedging: to make the portfolio "DV01 neutral" the relationship between the face value if we use security A to hedge security B:

$$F_A \cdot DV01_A = F_B \cdot DV01_B$$

Duration: percentage change of price with respect to the yield

$$\frac{\Delta P}{P} = \frac{P(y + \Delta y) - P(y)}{P(y)} = \frac{1}{P}\frac{\Delta P}{\Delta y}\Delta y = -D\Delta y$$

$$D = -\frac{1}{P}\frac{dP}{dy}, \qquad DV01 = -\Delta P = DP\Delta y = \frac{P}{10000}D$$

Duration of a portfolio:

$$D_P = \frac{P_1}{P_1 + P_2}D_1 + \frac{P_2}{P_1 + P_2}D_2 = \sum \frac{P^i}{P}D^i$$

Hedge based on duration:

$$P_2 = -\frac{D_1}{D_2} P_1$$

Convexity

$$C = \frac{1}{P}\frac{d^2P}{dy^2} = \frac{1}{P}\frac{P(y+\Delta y) - 2P(y) + P(y-\Delta y)}{\Delta y^2}$$

Conveity of a portfolio:

$$C_P = \sum \frac{P^i}{P} C^i$$

Yield-based

$$D = \frac{1}{P}[\frac{100c}{y^2}(1 - \frac{1}{(1+\frac{y}{2})^{2T}}) + T(1-\frac{c}{y})\frac{100}{(1+\frac{y}{2})^{2T+1}}]$$

$$DV01 = \frac{1}{10000}[\frac{100c}{y^2}(1 - \frac{1}{(1+\frac{y}{2})^{2T}}) + T(1-\frac{c}{y})\frac{100}{(1+\frac{y}{2})^{2T+1}}]$$

$$C = \frac{1}{P}\frac{1}{(1+\frac{y}{2})^2}[\frac{100c}{2}\sum_{t=1}^{2T}\frac{t}{2}\frac{t+1}{2}\frac{1}{(1+\frac{y}{2})^t} + T(T+.5)\frac{100}{(1+\frac{y}{2})^{2T}}]$$

A major weakness of duration and convexity based hedging is the assumption that the yield curve shift in parallel.

1. Given the full prices of four Treasury bonds at October 21, 2009 as follows:

Bond	Price
3s of 4/21/2010	100+
3.5s of 10/21/2010	100-8
4s of 4/21/2011	100
4.5s of 10/21/2011	101

Assume semiannual compounding. Do the following.

1.1 (4) Compute the discount factors from a half to two years.

1.2 (4) Compute the forward rates from a half to two years.

1.3 (4) Calculate the par yields from a half to two years.

Bond	Coupon rates	maturity	Price
3s of 4/21/2010	3.00%	0.5	100.50
3.5s of 10/21/2010	3.50%	1	100.25
4s of 4/21/2011	4.00%	1.5	100.00
4.5s of 10/21/2011	4.50%	2	101.00

Cash flows	0.5	1	1.5	2	Price	Discount Factor	Forward rate	Par yield
3s of 4/21/2010	101.5	0	0	0	100.5	0.990148	0.0199	0.019900498
3.5s of 10/21/2010	1.75	101.75	0	0	100.25	0.968228	0.045277	0.032446864
4s of 4/21/2011	2	2	102	0	100	0.941993	0.055703	0.04
4.5s of 10/21/2011	2.25	2.25	2.25	102.25	101	0.923953	0.039049	0.039770314

2. (10) (Continued from problem 1) Let the yield to maturity of the 4.5s bond be 3.974%. Suppose that you long one unit of 4.5s bond today (10/21/2009). Form a duration neutral portfolio by trading the one-year par bond. Calculate how many units of the par bond should be long or shorted.

- Yield-based measures assume that the yield of a security is the interest rate factor

$$P(y) = \frac{100c}{2}\sum_{t=1}^{2T}\frac{1}{(1+\frac{y}{2})^t} + \frac{100}{(1+\frac{y}{2})^{2T}}$$

- or

$$P(y) = \frac{100c}{y}\left(1 - \frac{1}{(1+\frac{y}{2})^{2T}}\right) + \frac{100}{(1+\frac{y}{2})^{2T}}$$

- Taking the negative of the derivative of the two pricing expressions

$$D = \frac{1}{P}\frac{1}{1+\frac{y}{2}}\left[\frac{100c}{2}\sum_{t=1}^{2T}\frac{t}{2}\frac{1}{(1+\frac{y}{2})^t} + T\frac{100}{(1+\frac{y}{2})^{2T}}\right]$$

- or

$$D = \frac{1}{P}\left[\frac{100c}{y^2}\left(1 - \frac{1}{(1+\frac{y}{2})^{2T}}\right) + T\left(1 - \frac{c}{y}\right)\frac{100}{(1+\frac{y}{2})^{2T+1}}\right]$$

Bond	maturity	coupon	Yield	price	Duration
4.5s of 10/21/2011	2	4.500%	3.974%	101	1.897646
1 year par bond	1	3.245%	3.245%	100	0.976181

$$xP_{par} = -P_{4.5s} * 1 \times \frac{D_{4.5s}}{D_{par}} = -1.01 \times \frac{1.897646}{0.976181} = -1.96338841$$

As $P_{par} = 1$, so $x = -1.96338841$ units (short).