Midterm Exam of MATH4511, Fall 2020
October 28, 1200-1330

Problems (The numbers in brackets are credits, totalled to 50):

1. Answer the following questions
   (a) (2) Between a government bond and an ordinary stock, which one is less risky? Why?
   (b) (2) Between the bond market and the equity market of a developed country, which one has bigger size?
   (c) (2) What are the major differences between the Treasury security market and the LIBOR market?
   (d) (2) What do we mean by "linear derivatives"?

2. Consider the following table of bond and option prices as functions of yield:

Rate Level	Bond Price	Duration	Convexity	Option	Duration	Convexity
3.95%	100.4098			10.1536		
4.00%	100.0000	?	?	9.9251	?	?
4.05%	99.5922			9.7002		

   (a) (6) Calculate the duration and convexity of the bond and the option at 4% yield.
   (b) (4) Suppose you short \$10m dollar value of the option, how many dollar value of the bond you should long or short for duration hedging?
   (c) (2) What is the convexity of the hedged portfolio?

3. Suppose that your family borrows a 30-year mortgage from Hang Seng Bank to buy an apartment, for the amount of \$7m with the mortgage rate 3.25%.
   (a) (4) What should be the amount of monthly mortgage repayment?
   (b) (4) Suppose that exactly five years later, your family will sell the apartment and thus **pay off** the mortgage. How much will be payment?

4. Consider the pricing of the T-year vanilla swaps with swap rate $s(0,T)$ and \$1 notional value.
   (a) (6) Argue that the value of the floating leg (without paying the notional at the end) is $1-d(0,T)$, where $d(0,T)$ is the discount factor seen at $t=0$.
   (b) (2) Show that the value of the fixed leg is $s(0,T)\sum_{i=1}^{2T}\frac{1}{2}d(0,i/2)$.
   (c) (2) Argue that the swap rate is given by
   $$s(0,T)=\frac{1-d(0,T)}{\sum_{i=1}^{2T}\frac{1}{2}d(0,i/2)}.$$
   (d) (2) Prove that a swap rate is also a par yield.

5. Consider hedging the 10-year maturity *Treasury Inflation Protected Security* (TIPS) using the regular 10-year maturity Treasury note. Let the durations of the TIPS and the note be 9.5 years and 7.5 years, respectively, the daily volatilities of their yields be 5 basis points and 4.8 basis point, respectively, and let the correlation of the two yields be 0.825.
   (a) (4) How are the cash flows of the TIPS calculated?
   (b) (6) Suppose you long the TIPS for a **market value** of \$10m, how much **market value** of the Treasury note should be shorted for hedging?

========== Good Luck! ==========