**Final of Math 361:** Quantitative Methods for Fixed-Income Securities

December 21, 2010

**Problems** (The numbers in brackets are credits)**:**

1. Suppose that you finance your college education with a student loan and pay back after graduation. Assume that the annualized interest rates for various compounding frequencies stay at 5% all along.
   a. (4) In the beginning of each of the three college years, you borrow $42,000 from the university. Upon graduation (i.e. at the end of the third year), how much money, principal plus interest, will you owe the university?
   b. (4) You will pay back the loan over the five years after graduation with equal monthly payments, starting from the (end of the) first month. How much money should each payment be?
2. (6) Suppose that by regression analysis we obtain

$$\Delta y_t^{20} = \alpha + \beta_{10}\Delta y_t^{10} + \beta_{30}\Delta y_t^{30} + \varepsilon_t,$$

   where $\beta_{10} = 0.1613,\ \beta_{30} = 0.8774$ and $\varepsilon_t$ is the noise term. If the dollar value of the 20-year bond is one million, how much 10-year and 30-year bonds in dollar terms should be purchased for hedging? Assume the modified duration of the three bonds are $D_{10} = 7.89,\ D_{20} = 12.8$ and $D_{30} = 15.9$, respectively.
3. (10) A fixed income analyst needs to calculate the price of a digital interest-rate option that pays $1,000,000 next year if the one-year Treasury rate exceeds 5.5% and pays nothing otherwise. Suppose the current *forward rates* are $r(1) = 5\%$ and $r(2) = 5.19\%$, and in one year, the one-year rate (i.e. $r(1)$) becomes either 6% or 4% with 25% and 75% of probabilities, respectively, what should be the no-arbitrage price of the option? How to hedge it?
4. (6) Calculate the forward price (for quotation) of a coupon bond. The inputs are
   4.1. Forward contract transaction date: December 21, 2010
   4.2. Underlying security: 100 face amount of the 6s of 05/21/2015
   4.3. Forward date: March 21, 2011
   4.4. Clean price of 6s of 05/21/2015 on December 21, 2010: 104
   4.5. Repo rate from Dec. 22, 2010 to March 21, 2011: 2.5%
5. Suppose that on December 21, 2010, a client of yours wants to buy 6s of 05/21/2015 for $100m face value. You will then process through a set of transactions involving a repo. Suppose that the repo rate is 2.5%; and
   5.1. you sell the bond to the client for (the clean price of) 104-1, and,
   5.2. tomorrow, you buy (from the market) the same bond for 104.

   Please figure out
   5.3. (3) your *P&L*;
   5.4. (3) the cost of carry of your trades;
   5.5. (4) the breakeven price of your buying with which your *P&L*=0.

========== THE END ==========