

Final Exam for Math 4511
Quantitative Methods for Fixed-Income Securities
1230-1530, December 20, 2013

Problems (Total of six. The numbers of credits are given in brackets):

1. (10) Consider the regression hedging of a 20-year bond using a 30-year bond. Let the DV01s of the 20-yr and 30-yr bonds be 0.10 and 0.14, the yield volatilities be 5.5 bps and 5 bps, respectively, and the yield correlation be 0.98. Suppose the face value of the 20-year bond is \$100m, calculate the face value of the 30-year bond for hedging.
2. (10) Consider a forward contract for buying 5s of 11/15/2023 **today**. Let the maturity of the contract be 12/20/2015 and the current spot-rate curve be flat at 5%, calculate the **clean** forward price.
3. (10) An investor buys a forward-rate agreement (FRA) with the following characteristics:

Underlying rate	3-month LIBOR
Notional value	\$1m
Maturity	1 year
Current spot-rate curve	5% flat

Suppose that three months from now, the spot-rate curve makes a parallel shift to 4.5% and stays there afterwards, and six months from now the investors decides to close out the FRA, calculate the P&L (**Note**: “Long FRAs” means “short rates”).

4. (10) Suppose that the investor buys instead a Eurodollar futures with the same characteristics as that of **the FRA above** at the futures rate of 5.1%. Suppose that three months from now, the futures rate for the same contract changes to 4.58% (while the spot-rate curve makes a parallel shift to 4.5%) and stays unchanged afterwards, and six months from now the investor decides to close out the futures, calculate the P&L (**Note**: “Long futures” means “short rates”).
5. Suppose that an investor long a 5-year **payer’s swap** when the forward-rate curve is

$$r(i/2) = 0.01 + i \times 0.001, \quad i = 1, 2, \boxed{?}.$$

The notional value of the swap is \$1m.

- 5.1. (5) Calculate the fair swap rate.
- 5.2. (5) Suppose that one year later the forward-rate curve remains the same, i.e.,

$$r(j/2) = 0.01 + j \times 0.001, \quad j = 1, 2, \boxed{?}.$$

If the investor closes out the swap then, what will be the P&L?

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6. Consider the pricing of a **put** option on zero-coupon bond using the **Vasicek model**. The characteristics of the option are given below.

Underlying	1-year maturity zero-coupon bond
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Maturity	0.5 year
Strike	\$975
Spot rates	$\hat{r}_{0.5} = 5\%$, $r_1 = 5.15\%$
Spot rate volatility	$\sigma = 1\%$
Strength of mean reversion	$\kappa = 0.25$

- 6.1. (10) Build a one-step interest-rate tree and calibrate to the spot rates, with step size $\Delta t = \frac{1}{2}$, while keeping the branching probabilities to be $\{\frac{1}{2}, \frac{1}{2}\}$.
- 6.2. (5) Calculate the option value.
- 6.3. (5) How will you hedge the option?

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