Final for Math 361

Quantitative Methods for Fixed-Income Securities

December 11, 2009

Problems (The numbers in parentheses are credits):

1. (10) Calculate the forward price of a coupon bond. The input information is
   1.1. Forward contract transaction date: Dec. 11, 2009
   1.2. Underlying security: 100 face amount of the 6s of 3/15/2013
   1.3. Forward date: Feb. 11, 2010
   1.4. Trade price of 6s of 3/15/2013 on Dec. 11, 2009: 105
   1.5. Repo rate from Dec. 12, 2009 to Feb. 11, 2010: 1%
2. Suppose that on December 11, 2009, a client of a trading desk wants to buy 6s of 10/11/2014 for \$1m face value. The desk makes market with the following transactions:
   2.1. On Dec. 11, 2009, the desk sold to the client the bond of face value \$1m at flat price 105-17 (for T+1 settlement)
   2.2. On Dec. 12, 2009, the desk lent out the payment to a third party who borrowed the money using the bond as a collateral, and the desk then delivered the bond to its client. The repo rate is 4.5%. On the same day, the desk buys the bond from open market at flat price 105-16 (again for T+1 settlement).
   2.3. On Dec 13, 2009, the repo matures: third party returns the money with interest, and the desk returns the bond to the third party.

   Do the following.

   2.4. (5) Calculate the P&L to the desk;
   2.5. (5) Calculate the *cost of carry*.
   2.6. (5) Find out the breakeven price.
3. (10) Given a forward-rate curve $r(i) = 0.01 + 0.003i,\ i = 1, \ldots, 10$, for annual compounding, compute the corresponding par swap-rate curve (with $\Delta T = 1$). Is a par swap rate the same as a par yield? Why and why not?
4. (15) Consider **continuous compounding**. Build a 2-period interest-rate tree for the Ho-Lee model $dr = \lambda dt + \sigma dW$ with parameters $\hat{r}(\Delta t) = 5\%, \sigma = 1\%$, and $\Delta t = 1/2$. Determine the drift $\lambda(t)$ according to spot rates $\hat{r}(2\Delta t) = 5.25\%$, and $\hat{r}(3\Delta t) = 5.5\%$. Keep branching probabilities to be {1/2,1/2} all along (Note: under continuous compounding, the discount factor is calculated according to $d(T) = \exp[-\hat{r}(T)T]$).
5. (10) [Continued from 4] Price a call option on $3\Delta t$ -maturity zero-coupon bond with face value \$1,000. The maturity of the option is $2\Delta t$ and the strike is $K = \$965$. Explain how to hedge the option.

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