Final for Math 361

Quantitative Methods for Fixed-Income Securities

May 30, 2008

Problems:

1. Suppose that you finance your tuition with a student loan such that in the beginning of each of the three college years, you borrow $42,000 from the university. Assume that the annualized interest rates for various compounding frequencies stay at 5% all along.
   a. (4) Upon graduation (i.e. at the end of the third year), how much money, principal plus interest, will you owe the university?

   Solution:

   $$42000\times\left(1.05+1.05^2+1.05^3\right)=139025.25$$

   b. (4) You will pay back the loan over the five years after graduation with equal monthly payments, starting from the (end of the) first month. How much money should each payment be?

   Solution: Solve the installment *MP* from

   $$\begin{aligned}139025.25 &= MP\times\sum_{i=1}^{60}(1+5\%/12)^{-i}\\ &= MP\times\frac{1-(1+5\%/12)^{-N}}{5\%/12}\end{aligned}$$

   We obtain *MP*=2623.58

2. Given the prices of the following benchmark bonds:

Bond	Price
$3\frac{1}{8}$s of 11/30/2008	99-4
5.5s of 5/30/2009	100-15+
6s of 11/30/2009	101-22
$6\frac{3}{4}$s of 5/30/2010	103-17

   a. (4) Calculate the discount factors $d(i/2),\ i=1,2,3,4$
   b. (2) Price the 5s of 5/30/2010 based on the discount curve just obtained.
   c. (3) Derive the (half-year) forward rates $r(i/2),\ i=1,2,3,4$.
   d. (3) Compute the par yield for the term from 0 year to 2 years.

   Solution: For a, c and d, we have

discount factor	forward rates	par yield
0.976	0.049180328	0.04918
0.951828467	0.050789683	0.049975
0.931106938	0.044509451	0.048195
0.908172605	0.050506551	0.048752

   For b, the answer is 100.235.

3. (8) Consider the DV01 hedge of 5s of 5/30/2010 using interest-rate futures. The notional value of the bond is $100 millions. Suppose that the yield curve is flat at 4.87%, how many contracts of interest-rate futures should be long or short to hedge the bond? The notional value for each futures contract is $1 million.

   Solution: The DV01 of the bond is $159199.5022, and the DV01 of the futures contract is $25. Hence, we need 159199.5022/25=6368 units of futures contract.

4. (6) There are two portfolios to choose from.
   a. A 15-year zero-coupon bond, and
   b. A basket of 5-, 10- and 30-year zero-coupon bonds with weights (20%,50%,30%).

   Which one will you pick and why?

   Solution: the durations of the portfolios are the same (15yr) but the convexities are 120 vs. 195. We should take the one with bigger convexity.

5. (6) Suppose that by regression analysis we obtain

$$\Delta y_t^{20} = \alpha + \beta_{10}\Delta y_t^{10} + \beta_{30}\Delta y_t^{30} + \varepsilon_t,$$

   where $\beta_{10} = 0.1613$, $\beta_{30} = 0.8774$ and ε_t is the noise term. If the dollar value of the 20-year bond is one million, how much 10-year and 30-year bonds in dollar terms should be purchased for hedging? Assume the modified duration of the three bonds are $D_{10} = 7.89$, $D_{20} = 12.8$ and $D_{30} = 15.9$, respectively.

   Solution: The hedge positions are

$$P_{10} = P_{20}\frac{D_{20}}{D_{10}}\beta_{10} = \$265377.9$$

$$P_{30} = P_{20}\frac{D_{20}}{D_{30}}\beta_{30} = \$706334.6$$

6. (10) A fixed income analyst needs to calculate the price of a digital interest-rate option that pays $1,000,000 next year if the one-year Treasury rate exceeds 5.5% and pays nothing otherwise. Suppose the current forward rates are $r(1) = 5\%$ and $r(2) = 5.19\%$, and in one year, the one-year rate (i.e. $r(1)$)becomes either 6% or 4% with 25% and 75% of probabilities, respectively, you are asked to price the option.

   Solution: The risk-neutral probability is obtained from

$$\begin{aligned} P(2) &= \frac{1}{(1+r(1))(1+r(2))} \\ &= \frac{1}{1+r(1)}\left(q\frac{1}{1+6\%} + (1-q)\frac{1}{1+4\%}\right) \end{aligned}$$

   It follows that q=60%. The price of the option is

$$C = \frac{1}{1+5\%}\left(0.6 \times \$1,000,000 + 0.4 \times \$0\right) = \$571428.57$$

========================== THE END ==========================