1.

1.1

You only need to compare the payoffs for LHS and RHS, which is identical to put-call parity.

1.2

Based on the results in 1.1, it suffices to show that sum of FRA is equivalent to a swap.

2.

2.1

At time t, the value of the long position is $d(t,T) * g(F_t)$, which is equal to P_t; the value of an forward contract is 0. Therefore the time-t value of this portfolio is $P_t+0=P_t$.

2.2

The proof is extremely similar to “differentiation” in lecture note 23, page 41. You should replace the C by P. Note that the partial derivative of $g(F_t)$ with respect to F_t is $-\varphi(-d_1)$.

3

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%   This code is to calculate prices of swaptions with the Black's formula%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear

    format long

%   Forward-rate curve

    f = 0.01+0.0003*(0:1:119);

%   Model parameters

    sig=0.25;

%   Option and bond parameters

    TC=5; TB=15;

%   Parameters for the tree

    dt=0.25;
    N=TB/dt;                % index for bond's maturity
```

```
    M=TC/dt;                  % index for option's maturity

%    Cy=zeros(NC,1);

%   calculate the swap rate

    P0=zeros(N,1);
    P0(1)=1/(1+dt*f(1));
    for j=2:N
        P0(j)=P0(j-1)/(1+dt*f(j));
    end

%   calculate the ATM swap rate;
%  3.1: 0.015722196453958
    s0  =  (1-P0(N-M))/sum(2*dt*P0(2:2:(N-M)))

%   calculate the ATM swap rate, in 5-to-10 forward;
%  3.3:  0.021690685873849
    sum=0;
    for j=(M+2):2:N
        sum=sum+2*dt*P0(j);
    end
    k=(P0(M)-P0(N))/sum


%   calculate cap price;
%  3.2:  3.417105925284749e+04
    BCap=0;
    for j=1:(N-M)
        ff=f(j);
        Tj=(j-1)*dt;
        BCap=BCap+P0(j)*dt*call(ff,s0,Tj,sig);
    end
    BCap*1000000

%   calculate swaption price;
%  3.4:  4.042355838857720e+04
    BC=sum*call(k,k,TC,sig);
    BC*1000000

%    plot(y,Cy)
```