Solutions to Final for Math 361

Quantitative Methods for Fixed-Income Securities

December 21, 2010

1.

a. The total money owed is

$$\begin{aligned} & 42000\times\left(1.05+1.05^2+1.05^3\right) \\ & =42000\times\frac{1.05}{0.05}\times\left(1.05^3-1\right) \\ & =139025.25 \end{aligned}$$

b. Solve the installment *MP* from

$$\begin{aligned} 139025.25 &= MP\times\sum_{i=1}^{60}\left(1+5\%/12\right)^{-i} \\ &= MP\times\frac{1-(1+5\%/12)^{-60}}{5\%/12} \end{aligned}$$

We obtain *MP*=2623.58

2. The hedge positions are

$$P_{10}=P_{20}\frac{D_{20}}{D_{10}}\beta_{10}=\$261678.07$$

$$P_{30}=P_{20}\frac{D_{20}}{D_{30}}\beta_{30}=\$706334.59$$

3. The risk-neutral probability is obtained from

$$\begin{aligned} P(2) &= \frac{1}{(1+r(1))(1+r(2))} \\ &= \frac{1}{1+r(1)}\left(q\frac{1}{1+6\%}+(1-q)\frac{1}{1+4\%}\right) \end{aligned}$$

It follows that q=60%. The price of the option is

$$C=\frac{1}{1+5\%}\left(0.6\times\$1,000,000+0.4\times\$0\right)=\$571428.57$$

For hedging, we should long

$$\begin{aligned} \beta &= \frac{1000000-0}{1000\times\left(\frac{1}{1+6\%}-\frac{1}{1+4\%}\right)} \\ &= -55120 \end{aligned}$$

units of $P(2)$.

4. The day counts are listed below.
   - Coupon dates: 11/21 & 5/21
   - No. of days b/w 11/21 – 12/22: 31
   - No. of days b/w 11/21 – 03/21: 120
   - No. of days b/w 11/21 – 05/21: 181
   - No. of days b/w 12/22 – 03/21: 89

   The forward price is then given by

$$
\begin{aligned}
P_{fwd} &= \left(P(0)+AI(0)\right)\left(1+\tfrac{d}{360}r\right)-AI(d) \\
&= \left(104+\tfrac{31}{181}\times 6/2\right)\left(1+\tfrac{89}{360}\times 2.5\%\right)-\tfrac{120}{181}\times 6/2 \\
&= 103.1708
\end{aligned} \tag{1.1}
$$

5. We answer the question one by one.

   5.1 The P&L for \$100 notional of bond is

$$
\begin{aligned}
&P\&L=(P(0)+AI(0))(1+rd/360)-(P(d)+AI(d)) \\
&=(104+1/32+\frac{31}{181}\times 6/2)(1+2.5\%/360)-(104+\frac{32}{181}\times 6/2) \\
&=0.0219355\ \mathit{millions} \\
&=\$21935.5
\end{aligned}
$$

   5.2 The carry is

$$
\begin{aligned}
&Carry = P\&L - \text{Price Change} \\
&= 21935.5-(P(0)-P(d)) \\
&= 21935.5-\frac{1}{32}\times 1000000 \\
&= -9315
\end{aligned}
$$

   5.3 The breakeven price is

$$
\begin{aligned}
&P(d)=(P(0)+AI(0))(1+rd/360)-AI(d) \\
&=(104+1/32+\frac{31}{181}\times 6/2)(1+2.5\%/360)-\frac{32}{181}\times 6/2 \\
&=104.0219
\end{aligned}
$$

========================= THE END =========================