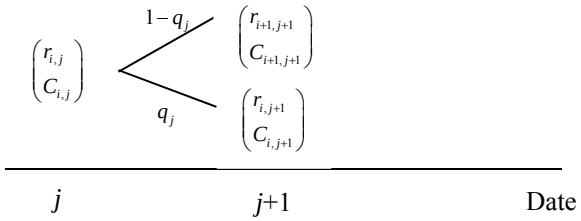
Final Exam for MATH4511, Paper C
8:30-11:30am, December 18, 2019

Problems (Numbers in brackets are credits, totaled to 50):

1. Consider key-rate hedging the long position of a 7-year bond with 100 million dollar face value. Let the par yield be flat at 3%, and the (2-, 5-, 10- and 30-year) kr01s of the 7-year bond be 0.02550, 0.04550, 0.01250 and 0.0000, respectively. Describe
   1.1. (2) the hedging instruments to be adopted and
   1.2. (6) how much each of these instruments to be long or short for hedging.
2. Answer the following questions
   2.1. (2) What is a forward contract?
   2.2. (6) Let S_t be the time-*t* price of a stock. Prove that the time-*t* price of a forward contract on the stock with fixed maturity $T \geq t$ and strike price K is $f_t = S_t - K \times P(t,T)$, where $P(t,T)$ is the time-*t* price of the *T*-maturity zero-coupon bond of \$1 notional value.
   2.3. (4) State the call-put parity for equity options and give a proof.
3. The (i, j)-state of a multi-period binomial **risk-neutral bond option tree** is given by



   where $\{q_j, 1-q_j\}$ are the risk-neutral probabilities. Let $B_{i,j}$ be the clean price of the bond at state (i, j), with coupon rate c.
   3.1. (4) Provide the formula for calculating $C_{i,j}$.
   3.2. (4) Describe the replication strategy for the bond option.
   3.3. (2) Give the definition of a "self-financing replication strategy".
   3.4. (4) Explain whether the strategy in Problem 3.2 is a self-financing one.
4. The following questions concern about the pricing of receiver's swaptions.
   4.1. (4) Let the current term structure of **6m** forward rates be given as
   $$f_j(0) = 0.0125 + 0.0003 \times (j-1), \quad j = 1, \cdots, 60.$$
   Calculate the in-5-to-5 market prevailing swap rate (i.e., for the tenor (5,10)).
   4.2. (4) Calculate the value of the in-5-to-5 ATM swaption (meaning in 5 years the swaption buyer can enter into the 5-year receiver's swap with the strike rate equal to the in-5-to-5 swap rate) using the Black's formula, with swap-rate volatility $\sigma = 0.2$ and notional value \$1m (Note: you may need this value: $\Phi(\frac{1}{2} \cdot 0.2\sqrt{5}) = 0.588468$).
   4.3. (4) Describe a hedging strategy for the swaption.
5. (4) Use the call-put parity and the results of Problem 4 to deduce the value of the in-5-to-5 swaption on the five-year payer's swap with the same strike rate and notional value.

========== Good Luck! ==========