

Final for Math 361
Quantitative Methods for Fixed-Income Securities
May 24, 2005

Problems:

1. Build a two-period (recombining) binomial tree for the normal model (Page 219, equation (11.1)) with $r_0 = 4.5\%$, $\sigma = 1\%$, $\Delta t = 1/2$ and $P(2\Delta t) = 955$.
 - 1.1 Calculate the *risk-neutral probabilities* for branching out (in the first period).
 - 1.2 Calculate a Δt -maturity call option on $2\Delta t$ -maturity zero-coupon bond with strike price \$978.
 - 1.3 Explain how to hedge the option.
2. Build a 1-period interest-rate tree under the Salomon Brothers model (Page 251, equation (12.10)), with parameters $r_0 = \hat{r}(\Delta t) = 5\%$, $\sigma = 20\%$, $\Delta t = 1/2$. Determine the drift a_1 according to spot rate $\hat{r}(2\Delta t) = 5.25\%$.
3. Let $a_2 = a_1$ and extend the tree in Problem 2 to two periods. Then price an *interest-rate floor* with cash flows

$$\frac{\$1,000,000}{2} \max(5\% - r(i\Delta t), 0), \text{ at } T_i = i\Delta t, i = 2, 1.$$

4. Suppose that on May 20, 2005, a client of a trading desk wants to buy 6s of 8/15/2010 for \$1m face value. The desk makes market with the following transactions:
 - 4.1. On May 20, 2005, the desk sold to the client the bond of face value \$1m at flat price 104-13 (for T+1 settlement).
 - 4.2. On May 23, 2005, the desk lent out the payment to a third party who borrowed the money using the bond as a collateral, and the desk then delivered the bond to its client. The repo rate is 4.5%. On the same day, the desk buys the bond from open market at flat price 104-12 (again for T+1 settlement).
 - 4.3. On May 24, 2005, the repo matures: third party returns the money with interest, and the desk returns the bond to the third party.

You are asked to

- 4.5. Calculate the P&L to the desk;
- 4.6. Calculate the *cost of carry*.
- 4.7. Find out the breakeven price.

===== THE END =====