## MATH4511 Quantitive Methods for Fixed Income Derivatives, 2017-18 Fall Quiz 04(T1A)

Name:	ID No.:	Tutorial Section:

1. (20 points) Consider pricing a 6m-maturity put option on one unit of the 1yr-maturity zero-coupon bond for the strike price of \$970 under the Ho-Lee model

$$\Delta r_t = \theta \Delta t + \sigma \sqrt{\Delta t} \epsilon_B,$$

where  $\epsilon_B$  takes +1 or -1 with equal probability. Here  $r_0 = \hat{r}(0.5) = 5\%$ ,  $\theta = 0.02$ ,  $\sigma = 0.01$  and  $\Delta t = 0.5$ . In addition, the one-year spot rate (for semi-annual compounding) is  $\hat{r}(1) = 5.5\%$ . Calculate the option price.

2. (10 points) In November 15, 2017, the forward-rate curve for semi-annual compounding is

$$f(i/2) = 0.018 + 0.001 * \frac{i-1}{2}, i = 1, ..., 20.$$

Calculate the in-1-to-2 swap rate (i.e., the swap rate for the swap for the period from 1 to 3 years).

3. (20 points) Denote the current price of the asset by  $S_0$ , strike price by K and the delivery time by T. We have the Black-Scholes put formula

$$P_0 = Ke^{-rT}\Phi(-d_2) - S_0\Phi(-d_1),$$

where 
$$d_1 = \frac{\ln(S_0 e^{rT}/K) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$$
,  $d_2 = d_1 - \sigma\sqrt{T}$ . (a) What is the hedge ratio  $\alpha$ ?

- (b) What is the fair value of the forward contract  $V_0$  in terms of  $S_0$ , K, r and T.
- (c) Write down the Call and Put Parity using the notations above.
- (d) Derive the formula for call option  $C_0$  in terms of all the notations above.