MATH4511 Quantitive Methods for Fixed Income Derivatives, 2017-18 Fall
Quiz 03(T1B)

Name: _____ **ID No.:** _____ **Tutorial Section:**_____

1. (20 points)

Par yields flat at:	5%	xxx	xxx	xxx	xxx
xxxxx	xxx	Key Rate 01s (100 Face)	xxx	xxx	xxx
Coupon	Term	2-Year	5-Year	10-Year	30-Year
5%	2				
5%	5		.04375		
5%	10			.0779	
5%	30				.15444
Mortgage	xxx	1.0	4.0	43.0	67.0

Calculate all the missing KV01's in the table above. According to the KV01's, how to hedge a long position of the nonprepayable mortgage by using these bonds?

2. (10 points) 2-v regression-based hedging

$$\Delta y_t^{20} = \alpha + \beta^{10}\Delta y_t^{10} + \beta^{30}\Delta y_t^{30} + \epsilon_t$$
$$\Delta \hat{y_t}^{20} = \hat{\alpha} + \hat{\beta}^{10}\Delta \hat{y_t}^{10} + \hat{\beta}^{30}\Delta \hat{y_t}^{30}$$

What is the value for 10- and 30-year bonds (P_{10} and P_{30}) if you want to hedge for the 20-year bond of value P_{20}? And what is the hedge error? Here we assume D_{20}, D_{10} and D_{30} are known, and you can use notations $\hat{\alpha}$, $\hat{\beta}^{10}$, $\hat{\beta}^{30}$ and ϵ_t to represent the duration hedging and corresponding error.

3. (20 points)

Assume the dynamics of the risk-neutral process of the short rate is given by

$$\Delta r_t = \mu \Delta t + \sigma \sqrt{\Delta t} \epsilon_B,$$

where ϵ_B takes +1 or -1 with equal probability. Here $r_0 = 5\%$, $\mu = 0.229\%$, $\sigma = 1.10\%$ and $\Delta t = 1/2$.
(a) Build a two-step binomial tree for the short rate.
(b) Using the risk-neutral tree built above to price \$100 notional amount of a 1.5-year *participating cap* with a strike of 5% and a participation rate of 40%. Payments are made every six months according to this rule: If the short rate on date i is r_i, then the cash flow from the participating cap on date i+1 is, as a percent of par,

$$\begin{cases} (r_i - 5\%)/2, & \text{if } r_i \geq 5\% \\ 40\% \times (r_i - 5\%)/2, & \text{if } r_i < 5\%. \end{cases}$$

There is no principal payment at maturity.