HW8:

Q1a).

What do we mean by “replication pricing”?

Replication pricing is used in option pricing to determine the price of other market instruments that can guarantee to replicate the cash flow of the option in all possible states. If an option is worth as much as a portfolio of other instruments in a future time for all possible states, then the portfolio of these instruments and the option must have equal value today, otherwise, there is an arbitrage opportunity.

Q1b).

What do we mean by “risk-neutral valuation”?

When valuing a derivative, we can make the assumption that investors are risk-neutral. This means that investors do not increase the expected return they require from an investment to compensate for increased risk.

Assuming a risk neutral principle can give us the right option price for the real situation.

To apply risk-neutral valuation to the pricing of a derivative, we first calculate what the probabilities of different outcomes would be if the world were risk-neutral. We then calculate the expected payoff from the derivative and discount that expected payoff at the risk-free rate of interest.

Q1c). Why do we require a stock-price binomial tree or an interest-rate tree to price the stock or zero-coupon bonds correctly?

We would use binomial tree because of it allows a multiple-period view and be able to incorporate probabilities in calculation. Also, it is transparent for user to visualize and understand the pricing.

Q2). Answer = 1.977174420034350

(Matlab code provided at the end of document)

Q3). Answer = 13.79821699837658

(Matlab code provided at the end of document)

Q4a). d(k) = d(k-1)/(1+f(k)/2);

Chart, line chart

Description automatically generated

Q4b). s(k) = (1-d(k))/(1/2\*sum(d(1:k)));

Chart, line chart

Description automatically generated

q4c). st = (d(10)-d(30))/(1/2\*sum(d(11:30))

st = 0.029503653118719

q4d). Using the code provided in course page, we have

C2(1,1) = 6.688720220423199

The value of payer’s swap = 6.6887

(Matlab code provided at the end of document)

q5). C(1,1) = 6.688720220423121

The value of receiver’s swap = 6.6887

======= Matlab Code =========

q2.m:

format long;  
  
% zero-coupon yields  
  
r0=0.018:0.0005:0.0475;  
  
% Model parameters  
  
th=0.004; sig=0.01;  
  
% Option and bond parameters  
  
c=2; TC=5; TB=15; K=99;  
  
% Parameters forthe tree  
  
dt=0.5; rdt=sqrt(dt);N=TB/dt;  
  
% index for bond's maturity  
  
M=TC/dt;  
  
% index for option's maturity  
  
P0=zeros(N,1);  
  
P0(1)=100/(1+dt\*r0(1));  
  
for j=2:N  
  
P0(j)=100/(1+dt\*r0(j))^(j);  
  
%spot rate  
  
%P0(j)=P0(j-1)/(1+dt\*r0(j));  
  
%forward rate  
  
end  
  
r=zeros(N,N);q=zeros(N,1);r(1,1)=r0(1);  
  
for j=2:N  
  
for i=1:j-1  
  
r(i,j)=r(i,j-1)+th\*dt-sig\*rdt;  
  
end  
  
r(j,j)=r(j-1,j-1)+th\*dt+sig\*rdt;  
  
end  
  
OPTIONS=[];  
  
for j=2:N  
  
j2=j; q0=0.5;q1=get\_q1(P0,r,q,j2,dt);q(j-1)=q1;  
  
end  
  
%plot(q);  
  
%pause  
  
B=zeros(N+1,N+1);B(:,N+1)=100\*ones(N+1,1);  
  
for j=N:-1:M+1  
  
for i=1:j  
  
B(i,j)=(q(j)\*B(i,j+1)+(1-q(j))\*B(i+1,j+1)+c\*dt)/(1+r(i,j)\*dt);  
  
end  
  
end  
  
C=zeros(M+1,M+1); % option value tree  
  
for i=1:M+1  
  
C(i,M+1)=max(B(i,M+1)-K,0);  
  
end  
  
for j=M:-1:1  
  
for i=1:j  
  
C(i,j)=(q(j)\*C(i,j+1)+(1-q(j))\*C(i+1,j+1))/(1+r(i,j)\*dt);  
  
end  
  
end  
  
C(1,1)

q3.m

P=zeros(M+1,M+1);  
  
% option value tree  
  
for i=1:M+1  
  
P(i,M+1)=max(K-B(i,M+1),0);  
  
end  
  
for j=M:-1:1  
  
for i=1:j  
  
P(i,j)=(q(j)\*P(i,j+1)+(1-q(j))\*P(i+1,j+1))/(1+r(i,j)\*dt);  
  
end  
  
end  
  
P(1,1)

q4&5.m

f=0.02:0.0005:0.0495;  
  
n=length(f);  
  
O=zeros(n,1);  
  
d=zeros(n,1);  
  
d(1)=1/(1+0.02/2);  
  
s=zeros(n,1);  
  
for k=2:n  
  
d(k)=d(k-1)/(1+f(k)/2);  
  
%Q4a  
  
end  
%plot(d);  
%pause;  
  
for k=1:n  
  
s(k)=(1-d(k))/(1/2\*sum(d(1:k)));  
  
%Q4b  
  
end  
%plot(s);  
  
st = (d(10)-d(30))/(1/2\*sum(d(11:30)));  
%pause;  
%Q4c  
  
%price the swap Q4c and Q5  
  
% Model parameters  
  
th=0.005; sig=0.01; TC=5; TB=15; K=100;  
  
% Parameters for the tree  
  
dt=0.5; rdt=sqrt(dt);N=TB/dt;  
  
% index for bond's maturity  
  
M=TC/dt;  
  
% index for option's maturity  
  
P0=zeros(N,1);  
  
P0(1)=100/(1+dt\*f(1));  
  
for j=2:N  
  
P0(j)=P0(j-1)/(1+dt\*f(j));  
  
end  
  
% Calculate the swap rate  
  
sum=0;  
  
for j=M+1:N  
  
sum=sum+P0(j);  
  
end  
  
c=2\*(P0(M)-P0(N))/sum;  
  
c=c\*100;  
  
% Build tree  
  
q=zeros(N,1);  
  
r=zeros(N,N);  
  
r(1,1)=f(1);  
  
for j=2:N  
  
for i=1:j-1  
  
r(i,j)=r(i,j-1)+th\*dt-sig\*rdt;  
  
end  
  
r(j,j)=r(j-1,j-1)+th\*dt+sig\*rdt;  
  
end  
  
for j=2:N  
  
jj=j; q0=0.5;q1=get\_q1(P0,r,q,jj,dt);  
  
q(j-1)=q1;  
  
end  
  
% Calculate bond price at option's maturity  
  
P=zeros(N+1,N+1);  
  
P(:,N+1)=100\*ones(N+1,1);  
  
for j=N:-1:M+1  
  
for i=1:j  
  
P(i,j)=(q(j)\*P(i,j+1)+(1-q(j))\*P(i+1,j+1)+c\*dt)/(1+r(i,j)\*dt);  
  
end  
  
end  
  
% Calculate option's value (receiver's swap)  
  
C=zeros(M+1,M+1);  
  
for i=1:M+1  
  
C(i,M+1)=max(P(i,M+1)-K,0);  
  
%V\_fix -V\_float  
  
end  
  
for j=M:-1:1  
  
for i=1:j  
  
C(i,j)=(q(j)\*C(i,j+1)+(1-q(j))\*C(i+1,j+1))/(1+r(i,j)\*dt);  
  
end  
  
end  
  
C(1,1); %receiver’s swap value is 6.6887  
pause;  
% Calculate option's value (payer's swap)  
  
C2=zeros(M+1,M+1);  
  
for i=1:M+1  
  
C2(i,M+1)=max(K-P(i,M+1),0);  
  
%V\_float -V\_fix  
  
end  
  
for j=M:-1:1  
  
for i=1:j  
  
C2(i,j)=(q(j)\*C2(i,j+1)+(1-q(j))\*C2(i+1,j+1))/(1+r(i,j)\*dt);  
  
end  
  
end  
  
C2(1,1)  
  
%payer’s swap value is 6.6887