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1. (1%) 請比較說明 generative model、logistic regression 兩者的異同為何？再分別列出本次使用的資料中五個分得正確/不正確的 sample, 並說明為什麼如此？

Logistic regression 是 discriminative 的方法，而 generative model 是 generative 的方法。在 logistic regression 中, 我們可以用 gradient descent 去迭代，算出 optimal w and b, 使到 loss function 達到 minimum. 而 generative model 的話，就是先算 u^1, u^2, 和 Σ ,然後算出 w 和 b.

兩者算出的 w 和 b 是不一樣的。

五個分的正確的 sample:

1. i:1028, y:0, yhat: 0.00670425
2. i:1029, y:0, yhat: 0.28461435
3. i:1030, y:0, yhat: 0.00968792
4. i:1031, y:0, yhat: 0.02517578
5. i:1032, y:0, yhat: 0.23454912

五個分得不正確的 sample:

1. i:3764, y:0, yhat: 0.7456637
2. i:3762, y:1, yhat: 0.28589513
3. i:3753, y:1, yhat: 0.44111478
4. i:3749, y:1: yhat: 0.15594321
5. i:3743, y:0, yhat: 0.66042516

他們在 logistic regression 下是分的不正確的，原因可能是 logistic regression 這個 model 的 complexity 不足，沒辦法 capture 到 feature 之間的重要性。而且，logistic regression 需要對 data 進行假設，他假設了 feature 之間是 independent 的，但這個在實際情況未必會發生。就比如在這幾個 sample 裡面，有一些 feature 可能中間有關聯，例如是 never-married 和 age。最後 logistic 的準確率只有大概 84%,過不了 strong baseline. 還有就是看了一下其他的 sample, 發現當 y=1 的時候，準確率不足，所以可能是最後算出的 weighting 和 bias 偏向 y=0 的 sample

2. (1%) 請實作兩種 feature scaling 的方法 (feature normalization, feature standardization), 並說明哪種方法適合用在本次作業？

如果使用 normalization, accuracy 是 0.8483979936533934

如果使用 standardization, accuracy 是 0.8288463507011977

normalization 把 value 會 scale 到[0,1]區間裡面，而 standardization 則是把 data rescale 至 mean 0, std = 1 的 normal distribution 裡面。從上述結果可見，本次作業

適合使用 normalization,而 standardization 的 accuracy 比較低，可能是因為 data 本身並不服從 normal distribution, 應該用 normalization 方法比較好。

3. (1%) 請說明你實作的 best model 及其背後「原理」為何？你覺得這次作業的 dataset 比較適合哪個 model？為什麼？

我的 best model 還是使用了 logistic regression，準確率只有 84%，過不了 strong baseline。這個 model 的原理就是使用 sigmoid function 去預測最後 P(Y=0)和 P(Y=1)的概率，並且使用 gradient descent 去更新每次的 w 和 b, 最後使用 w 和 b 給出預測的概率。我最後使用了 grid search 去尋找最佳的 threshold，發現最佳 threshold 在 53%左右，並不是 50%。 不過，這個是在 validation set 的結果，有可能會導致 overfit, 因此在最後提交的 result 並沒有超過 private leaderboard 的 strong baseline。我覺得這次作業的 Dataset 比較適合一些 deep learning model，比如 neural network。因為 feature 有很多，需要一個比較 complex 的 model 去 generalize data 中間的規律，而 logistic regression 相比還是稍微簡單，沒辦法超越 85%以上的準確率。

Homework 2

2021年10月30日 12:54

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HW2 - Handwritten Assignment

1.

Consider a generative classification model for K classes defined by prior class probabilities $p(C_k) = \pi_k$ and general class-conditional densities $p(x|C_k)$, where x is the input feature vector. (Note that $\pi_1 + \ldots + \pi_k = 1$)
Suppose we are given a training data set $\{x_n, \mathbf{t_n}\}$ where $n = 1, \ldots, N$,and $\mathbf{t_n}$ is a binary target vector of length K that uses the $1 - of - K$ coding scheme, so that it has components $t_{nk} = 1$ if pattern n is from class C_k, otherwise $t_{nj} = 0$. Assuming that the data points are drawn independently from this model, show that the maximum-likelihood solution for the prior probablities is given by

$$\pi_k = \frac{N_k}{N}$$

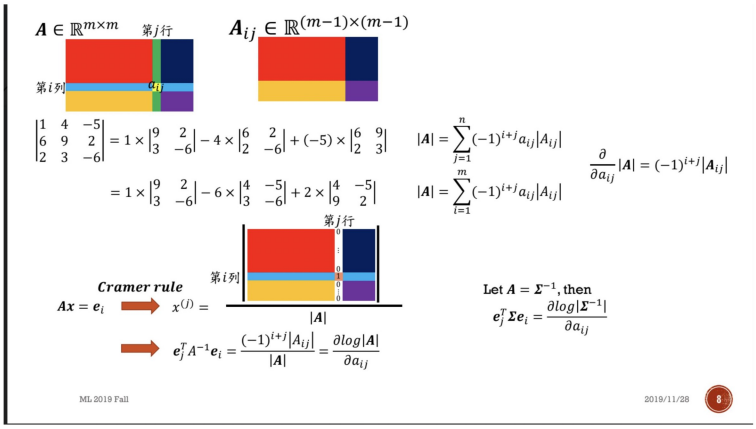
where N_k is the number of data points assigned to class C_k.

2.

Show that

$$\frac{\partial log(det\ \mathbf{\Sigma})}{\partial \sigma_{ij}} = \mathbf{e_j \Sigma^{-1}\ e_i^T}$$

where $\mathbf{\Sigma} \in \mathbb{R}^{m \times m}$ is a (non-singular) covariance matrix and $\mathbf{e_j}$ is a row vector(ex: $e_3 = [0, 0, 1, 0, \ldots, 0]$).
Hint:



3.

Consider the classification model of **problem 1** & result of **problem 2** and now suppose that the class-condition densities are given by Gaussian distributions with a shared convariance matrix, so that

$$p(x|C_k) = \mathcal{N}(x|\mu_\mathbf{k}, \mathbf{\Sigma})$$

Show that the maximum likelihood solution for the mean of the Gaussian distribution for class C_k is given by

$$\mu_{\mathbf{k}} = \frac{1}{N_k} \sum_{n=1}^{N} t_{nk} x_n$$

which represents the mean of those feature vectors assigned to class C_k. Similarly, show that the maximum likelihood solution for the shared covariance matrix is given by

$$\mathbf{\Sigma} = \sum_{k=1}^{K} \frac{N_k}{N} \mathbf{S_k}$$

where

$$\mathbf{S_k} = \frac{1}{N_k} \sum_{n=1}^{N} t_{nk} (\mathbf{x_n} - \mu_{\mathbf{k}})(\mathbf{x_n} - \mu_{\mathbf{k}})^T$$

Thus $\mathbf{\Sigma}$ is given by a weighted average of the covariance of the data associated with each class, in which the weighting coeffients are given by the prior probabilities of the classes.

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Consider a generative classification model for K classes defined by prior class probabilities $p(C_k) = \pi_k$ and general class-conditional densities $p(x|C_k)$, where x is the input feature vector. (Note that $\pi_1 + \ldots + \pi_k = 1$)
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where N_k is the number of data points assigned to class C_k.

Maximize log likelihood s.t. $\sum_{k=1}^{K} \pi_k = 1$

Using Langrange Multiplier.

$$\mathcal{L}(\pi, \lambda) = \sum_{n=1}^{N} \sum_{k=1}^{K} y_{n,k} [\log(P(x_n | C_k) + \log \pi_k] + \lambda \left(\sum_{k=1}^{K} \pi_k - 1 \right)$$

To maximize the above equation, we take $\nabla_{\pi_k} \mathcal{L}(\pi, k) = 0$:

$$\frac{\partial}{\partial \pi_k} \mathcal{L}(\pi, \lambda) = \frac{1}{\pi_k} \sum_{n=1}^{N} y_{n,k} + \lambda = 0$$

$$\sum_{n=1}^{N} y_{n,k} = -\lambda \pi_k$$

$$\frac{\partial}{\partial \pi_k} \ldots = 0$$

$$\sum_{n=1}^{N} y_{n,k} = -\lambda \pi_k$$

$$\pi_k = -\frac{\sum_{n=1}^{N} y_{n,k}}{\lambda}$$

$$\pi_k = -\frac{N_k}{\lambda} \quad -(*)$$

Taking $\nabla_\lambda L(\pi, \lambda) = 0$,

$$\frac{\partial}{\partial \lambda} L(\pi, \lambda) = \sum_{k=1}^{K} \pi_k - 1 = 0$$

$$\sum_{k=1}^{K} \pi_k = 1$$

$$\sum_{k=1}^{K} \pi_k = \sum_{k=1}^{K} -\frac{N_k}{\lambda}$$

$$\sum_{k=1}^{K} \pi_k = -\frac{N}{\lambda} = 1$$

$$\lambda = -N$$

$\Rightarrow$ From $(*)$,

$$\pi_k = \frac{N_k}{N}.$$

2. Show that

$$\frac{\partial log(det\ \Sigma)}{\partial \sigma_{ij}} = \mathbf{e_j} \Sigma^{-1}\, \mathbf{e_i^T}$$

where $\mathbf{\Sigma} \in \mathbb{R}^{m \times m}$ is a (non-singular) covariance matrix and $\mathbf{e_j}$ is a row vector(ex: $e_3 = [0, 0, 1, 0, \ldots, 0]$).
Hint:

$$\frac{\partial}{\partial \sigma_{ij}} \log|\Sigma| = \frac{1}{|\Sigma|} \frac{\partial}{\partial \sigma_{ij}} |\Sigma|$$

$$= \frac{(-1)^{i+j} |A_{ij}|}{|\Sigma|}$$

$$= e_j \Sigma^{-1} e_i^T$$

Hence proved.

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which represents the mean of those feature vectors assigned to class C_k. Similarly, show that

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$$\mathbf{\Sigma} = \sum_{k=1}^{K}\frac{N_k}{N}\mathbf{S_k}$$

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$$\mathbf{S_k} = \frac{1}{N_k}\sum_{n=1}^{N} t_{nk}(\mathbf{x_n} - \mu_{\mathbf{k}})(\mathbf{x_n} - \mu_{\mathbf{k}})^T$$

Thus $\mathbf{\Sigma}$ is given by a weighted average of the covariance of the data associated with each class, in which the weighting coeffients are given by the prior probabilities of the classes.

Likelihood $= p(t, x|\pi, \mu, \Sigma) = \prod_{n=1}^{N}\prod_{j=1}^{K}[\pi_j N(x|\mu_k, \Sigma)]^{t_{nj}}$

Taking log:

$$-\frac{1}{2}\sum_{n=1}^{N}\sum_{j=1}^{K} t_{nj}[k_n \ln|\Sigma| + (x_n-\mu_j)^T\Sigma^{-1}(x_n-\mu_j)] \quad -(*)$$

Taking $\nabla_{\mu_k}(*) = 0$, we have

$$\sum_{n=1}^{K} t_{nk}\,\Sigma^{-1}(x_n - \mu_k) = 0$$

$$N_k\,\Sigma^{-1}(x - \mu_k) = 0$$

$$\mu_k = \frac{1}{N_k}\sum_{n=1}^{N} t_{nk}\,x_n$$

Next, From (*), the terms dependent on Σ are:

$$-\frac{N}{2}\ln|\Sigma| - \frac{1}{2}\sum_{k}\sum_{n}(x_n-\mu_j)^T\Sigma^{-1}(x_n-\mu_j)$$

$$-\frac{N}{2}\ln|\Sigma| - \frac{1}{2}\sum_{j=1}^{K}\sum_{n\in C_j}(x_n-\mu_j)^T\Sigma^{-1}(x_n-\mu_j)$$

$$= -\frac{N}{2}\ln|\Sigma| - \frac{1}{2}\sum_{j=1}^{K}\sum_{n\in C_j}\mathrm{tr}\left(\Sigma^{-1}(x_n-\mu_j)(x_n-\mu_j)^T\right)$$

$$= -\frac{N}{2}\ln|\Sigma| - \frac{1}{2}\mathrm{tr}\left(\sum_{j=1}^{K}\sum_{n\in C_j}\Sigma^{-1}(x_n-\mu_j)(x_n-\mu_j)^T\right)$$

$$= -\frac{N}{2}\ln|\Sigma| - \frac{N}{2}\mathrm{tr}\left(\Sigma^{-1}\frac{1}{N}\sum_{j=1}^{K}\sum_{n\in C_j}(x_n-\mu_j)(x_n-\mu_j)^T\right)$$

$$= -\frac{N}{2}\ln|\Sigma| - \frac{N}{2}\mathrm{tr}\left(\Sigma^{-1}S\right) \quad -(*)$$

Taking $\nabla_\Sigma(*) = 0$, we have

$$\Sigma = \sum_{k=1}^{K}\frac{N_k}{N}S_k.$$

Hence proved.