Hw12

2022年4月22日 10:23

HW12 Q1.

Proof: any closed differential 2-form in $\mathbb{R}^3$ is exact.

exact: Let w be a 2-form in an open set $E \subset \mathbb{R}^3$. if there is a 1-form λ in E s.t. $w = d\lambda$, then w is exact in E.

closed: if w is of class C' and $dw = 0$, then w is closed.

Consider a counter example:

a closed unit sphere $S^2 \subset \mathbb{R}^3$ and a map $w_p : T_pS^2 \times T_pS^2 \to \mathbb{R}$ defined by

$$w_p(u,v) = (u \times v) \cdot p$$

The volume for $\mathbb{R}^3$ is $dx \wedge dy \wedge dz$. then define V_{S_2} to be the volume of S^2, where $V_{S_2} = i_N dx dy dz$, N is unit normal.

if e_i is orthonormal frame for T_pS^2, then

$$V_{S_2}(e_1, e_2) = i_N dx\, dy\, dz\, (e_1, e_2) = (p \times e_1) \cdot e_2$$

$$= e_1 \times e_2 \cdot p$$

$$= w_p(e_1, e_2)$$

w_p is the volume for S^2,

if w is exact, let b is 1-form in $\mathbb{R}^3$, then $db = w$

$\int_{S^2} w = 4\pi$

$\int_{S^2} w = \int_{S^2} b = 0$

So it is not exact.