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Hypothesis testing

1. Wald test

   An approximate χ^2_1 statistic for testing

$$H_o : \beta_j = 0$$

$$H_i : \beta_j \neq 0$$

   can be used by computing

$$\chi^2 \;=\; \frac{\hat{\beta}_j^2}{c^{jj}} \quad (j \;=\; 0, 1, \ldots, p)$$

   where the c^{jj} is the $(j+1)^{th}$ diagonal element of $\boldsymbol{C}^{-1}$, where $\boldsymbol{C}^{-1}$ is the estimated variance-covariance matrix of estimates of regression coefficients and the $(i,j)^{th}$ element in $\boldsymbol{C}$ is defined as

$$c_{ij} \;=\; \frac{-\partial^2 \log L(\hat{\boldsymbol{\beta}})}{\partial\hat{\beta}_i \, \partial\hat{\beta}_j} \quad (i = 0, 1, \ldots, p;\; j = 0, 1, \ldots, p)$$

   Wald statistic for testing $\boldsymbol{L}^T\boldsymbol{\beta} = \boldsymbol{d}$ is defined by

$$(\boldsymbol{L}^T\hat{\boldsymbol{\beta}} - \boldsymbol{d})^T(\boldsymbol{L}^T\boldsymbol{C}^{-1}\boldsymbol{L})^{-1}(\boldsymbol{L}^T\hat{\boldsymbol{\beta}} - \boldsymbol{d})$$

   where $\hat{\boldsymbol{\beta}}$ is the maximum likelihood estimates and $\boldsymbol{C}^{-1}$ is its estimated covariance matrix. It is χ^2_r , where r is the rank of $\boldsymbol{L}$.

2. Likelihood ratio test

   For testing

$$H_o : \boldsymbol{\beta}_1 = 0$$

$$H_1 : \boldsymbol{\beta}_1 \neq 0$$

$$\begin{aligned}\text{test statistic} &= \lambda(\boldsymbol{\beta}_1|\boldsymbol{\beta}_2) \\ &= \lambda(\boldsymbol{\beta}_2) - \lambda(\boldsymbol{\beta}) \\ &\sim \chi^2_{(r)}\end{aligned}$$

   That is, reject H_o if $\lambda(\boldsymbol{\beta}_1|\boldsymbol{\beta}_2) > \chi^2_{\alpha,r}$

3. Score test

   Score statistic is defined to be

$$\boldsymbol{U}^T(\boldsymbol{\beta}_0)\boldsymbol{I}^{-1}(\boldsymbol{\beta}_0)\boldsymbol{U}(\boldsymbol{\beta}_0).$$

   Under $\boldsymbol{\beta} = \boldsymbol{\beta}_0$, score statistic $\sim \chi^2_r$ with r is the dimension of $\boldsymbol{\beta}_0$. $\boldsymbol{U}(\boldsymbol{\beta})$ is the vector of partial derivatives of the log-likelihood with respect to the parameter vector $\boldsymbol{\beta}$ and $\boldsymbol{I}(\boldsymbol{\beta})$ is the matrix of the negative second partial derivatives of the log-likelihood with respect to $\boldsymbol{\beta}$.