6 Oct

$$\text{Reg S.S. (full model)} = \text{Increase in Reg S.S.} + \text{Reg S.S. (reduced)}$$

$\sim \chi^2_{(4, \lambda)}$ $\quad\Downarrow\quad$ $\sim \chi^2_{(2, \lambda_1)}$

$\lambda = \frac{1}{\sigma^2} \sum_{i=1}^{4} \sum_{j=1}^{4} \beta_i \beta_j S_{x_i x_j}$ $\quad \sim \chi^2_{(\underbrace{4-2}_{r}, \lambda - \lambda_1)}$ $\quad \lambda_1 = \frac{1}{\sigma^2} \sum_{i=1}^{2} \sum_{j=1}^{2} \beta_i \beta_j S_{x_i x_j}$

e.g. $H_0 : \beta_3 = \beta_4 = 0$

Under H_0 : $\lambda = \lambda_1 \Rightarrow \lambda - \lambda_1 = 0$

Under H_1 : $\lambda - \lambda_1 > 0$

$H_0 : \underset{\sim}{\beta_r} = \underset{\sim}{0}$

$$F = \frac{\text{Increase in Reg S.S.}/r}{\text{Res S.S.}/(n-p')} \overset{\cancel{\text{under } H_0}}{\sim} F(r, n-p', \lambda - \lambda_1)$$

$$E(F) \approx \frac{E(\text{Increase in Reg S.S.}/r)}{E(\text{Res S.S.}/(n-p'))}$$

$E\left(\frac{\text{Increase in Reg S.S.}}{\sigma^2}\right) = r + (\lambda - \lambda_1)$

$\frac{\text{Res S.S.}}{\sigma^2} \sim \chi^2_{(n-p')}$

$E\left(\frac{\text{Res S.S.}}{\sigma^2}\right) = n - p'$

$$= \frac{\frac{1}{r}(r + (\lambda - \lambda_1))\sigma^2}{\sigma^2}$$

$$= 1 + \frac{1}{r}(\lambda - \lambda_1)$$

If $\lambda - \lambda_1 \approx 0 \Rightarrow E(F) \approx 1$

$\Rightarrow$ F is not a large value

If $\lambda - \lambda_1 \neq 0 \Rightarrow E(F) > 1$

$\Rightarrow$ Reject H_0 if F is a large value

Reject H_0 if $F_{obs} = \dfrac{\text{Increase in Reg S.S.}/r}{\hat{\sigma}^2} > F_\alpha(r, n-p')$

e.g.

1. Example in multiple linear regression p.7

(a) $H_0 : \beta_3 = 0$

(i) t - test

(ii) F - test

Increase in Reg S.S. = Reg S.S. | full − Reg S.S. | reduced

- β_0
- reduced model
- full model

= (Total S.S. | full − Res S.S. | full)

− (total S.S. | reduced − ~~tota~~ Res S.S. | reduced)

= Res S.S | reduced − Res S.S. | full

$S_{yy} - (\tilde{\beta}_1 S_{x_1 y} + \tilde{\beta}_2 S_{x_2 y})$ $\qquad (n-p')\hat{\sigma}^2$

est. from the reduced model

i.e. $y_i = \tilde{\beta}_0 + \tilde{\beta}_1 x_{i1} + \tilde{\beta}_2 x_{i2} + e_i$

Note that $\tilde{\beta}_1 \neq \hat{\beta}_1$, $\tilde{\beta}_2 \neq \hat{\beta}_2$

~~$E(\tilde{\beta}_1)$ under~~ Under H_1 : $E(\tilde{\beta}_1) \neq \beta_1$

under the model of

$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + e_i$

$$(\underset{\sim}{X}_c^T \underset{\sim}{X}_c)_R^{-1} = \begin{pmatrix} 1/13 & 0 & 0 \\ 0 & 123.039 & -13.3812 \\ 0 & & 61.7639 \end{pmatrix}^{-1} \qquad (\underset{\sim}{X}_c^T \underset{\sim}{Y})_R = \begin{pmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n (x_{i1}-\bar{x}_1) y_i \\ \sum_{i=1}^n (x_{i2}-\bar{x}_2) y_i \end{pmatrix}$$

$\tilde{\beta}_1 = 1.031$, $\tilde{\beta}_2 = 1.87$ $\Rightarrow$ Res S.S. | reduced

= 40.01

Example 5: Example in Multiple Linear Regression

The percent survival of a certain type of animal semen after storage was measured at various combinations of concentrations of three materials used to increase chance of survival. The data are as follows:

y (% survival)	x_1 (weight %)	x_2 (weight %)	x_3 (weight %)
25.5	1.74	5.30	10.80
31.2	6.32	5.42	9.40
25.9	6.22	8.41	7.20
38.4	10.52	4.63	8.50
18.4	1.19	11.60	9.40
26.7	1.22	5.85	9.90
26.4	4.10	6.62	8.00
25.9	6.32	8.72	9.10
32.0	4.08	4.42	8.70
25.2	4.15	7.60	9.20
39.7	10.15	4.83	9.40
35.7	1.72	3.12	7.60
26.5	1.70	5.30	8.20

Summary statistics:

$$\sum_{i=1}^{13} y_i = 377.5 \qquad \sum_{i=1}^{13} y_i^2 = 11,400.15 \qquad \sum_{i=1}^{13} x_{i1} = 59.43$$

$$\sum_{i=1}^{13} x_{i2} = 81.82 \qquad \sum_{i=1}^{13} x_{i3} = 115.40 \qquad \sum_{i=1}^{13} x_{i1}^2 = 394.7255$$

$$\sum_{i=1}^{13} x_{i2}^2 = 576.7264 \qquad \sum_{i=1}^{13} x_{i3}^2 = 1035.96 \qquad \sum_{i=1}^{13} x_{i1}y_i = 1877.567$$

$$\sum_{i=1}^{13} x_{i2}y_i = 2246.661 \qquad \sum_{i=1}^{13} x_{i3}y_i = 3337.78 \qquad \sum_{i=1}^{13} x_{i1}x_{i2} = 360.6621$$

$$\sum_{i=1}^{13} x_{i1}x_{i3} = 522.078 \qquad \sum_{i=1}^{13} x_{i2}x_{i3} = 728.31 \qquad n = 13$$

$H_0: \beta_3 = 0$

t - test

$$\begin{pmatrix} 13 & 59.43 & 81.82 & 115.40 \\ 59.43 & 394.7255 & 360.6621 & 522.078 \\ 81.82 & 360.6621 & 576.7264 & 728.31 \\ 115.40 & 522.078 & 728.31 & 1035.96 \end{pmatrix}^{-1} = \begin{pmatrix} 8.06479 & -0.0825927 & -0.0941951 & -0.790527 \\ -0.0825927 & 0.00847982 & 0.00171669 & 0.00372002 \\ -0.0941951 & 0.00171669 & 0.0166294 & -0.00206331 \\ -0.790527 & 0.00372002 & -0.00206331 & \boxed{0.0886013} \end{pmatrix}$$

$$t = \frac{\hat{\beta}_3 - 0}{\text{s.e. of } \hat{\beta}_3} = \frac{-0.3433}{\hat{\sigma}\sqrt{0.0886013}} = -0.556$$

Or

$$(\underset{\sim}{X}_c^T \underset{\sim}{X}_c)^{-1} = \begin{pmatrix} 13 & 0 & 0 & 0 \\ 0 & 123.039 & -13.3812 & -5.4775 \\ 0 & -13.3812 & 61.7639 & 2.0002 \\ 0 & -5.4775 & 2.0002 & 11.5631 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 0.0769231 & 0 & 0 & 0 \\ 0 & 0.00847981 & 0.00171669 & 0.00371998 \\ 0 & 0.00171669 & 0.0166294 & -0.00206338 \\ 0 & 0.00371998 & -0.00206338 & 0.0886011 \end{pmatrix}$$

$\Rightarrow \hat{\beta}_0 = 39.1574,\ \hat{\beta}_1 = 1.0161,\ \hat{\beta}_2 = -1.8616,\ \hat{\beta}_3 = -0.3433.$

F-test

8

d.f. of Reg S.S. | full − d.f. of Reg S.S. | reduced
= 3 − 2 = 1

Res. S.S. | full = 38.68

Increase in Reg S.S. = 40.01 − 38.68 = 1.33

$$F = \frac{\text{Increase in Reg S.S.}/1}{\hat{\sigma}^2_{full}} = \frac{1.33/1}{4.298} = 0.30946 < 1$$

$$\hat{\sigma}^2_{full} = \frac{\text{Res S.S.}\,|\,\text{full}}{n-4}$$

Can't reject Ho

(b) $H_0: \beta_1 = \beta_2 = 0$

H_1: at least one of β_1, β_2 is equal to zero

- Can't use t-test

- F-test

Reduced model : $y_i = \beta_0 + \beta_3 X_{i3} + e_i$

β_0 — $\beta_3 +$ reduced model β_3 — $\beta_1, \beta_2 +$ full model $\beta_1, \beta_2, \beta_3$

Res S.S. | reduced

$= S_{yy} - \tilde{\beta}_3 S_{x_3 y}$ $\qquad \tilde{\beta}_3 = \dfrac{S_{x_3 y}}{S_{x_3 x_3}}$

$= 422.92834$

Res S.S. | full $= 38.68$

$\Rightarrow$ Increase in Reg S.S. $= 422.92834 - 38.68$

$= 384.252$

$$F = \frac{\text{Increase in Reg S.S.} / 2}{\hat{\sigma}^2_{full}} = 44.7077$$

Reject H_0

(c) $H_0: \beta_1 = \boxed{1}, \ \beta_2 = \boxed{-1}$

~~How~~ Can I find the increase in Reg S.S. ? NO

Total S.S. | full $\overset{?}{=}$ Total S.S. | reduced X

Full model : $y_i = \boxed{\beta_0} + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + e_i$

$\Rightarrow$ Total S.S. $= \sum_{i=1}^{n} (y_i - \bar{y})^2$

Reduced model : $y_i = \boxed{\beta_0} + X_{i1} - X_{i2} + \beta_3 X_{i3} + e_i$

$\Rightarrow \underbrace{y_i - X_{i1} + X_{i2}}_{y_i'} = \boxed{\beta_0} + \beta_3 X_{i3} + e_i$

$\Rightarrow$ total S.S. $= \sum_{i=1}^{n} (y_i' - \bar{y}')^2$

$= \sum_{i=1}^{n} \left((y_i - X_{i1} + X_{i2}) - (\bar{y} - \bar{X}_1 + \bar{X}_2) \right)^2$

(d) $H_0 : \beta_0 = 40$

Can I find an increase in Reg S.S. ? <u>No</u>

$$\text{Total S.S.}\,|\,\text{full} \overset{?}{=} \text{Total S.S.}\,|\,\text{reduced} \quad X$$

$$\| $$

$$\sum_{i=1}^{n} (y_i - \bar{y})^2 .$$

Reduced model: $y_i = 40 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + e_i$

$$\Rightarrow \underbrace{y_i - 40}_{y_i'} = \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + e_i$$

$$\cancel{\text{Total S.S.}} = \cancel{\sum_{i=1}^{n} (y_i' - \bar{y}')^2}$$

model without intercept

$$\Rightarrow \text{Total S.S.} = \sum_{i=1}^{n} y_i'^2 .$$

$$= \sum_{i=1}^{n} (y_i - 40)^2$$

---

§4.2.3 <u>general linear hypothesis</u>

p.31 $H_0 : \underset{\sim}{C}\,\underset{\sim}{\beta} = \underset{\sim}{d}$

e.g. (a) $H_0 : \beta_3 = 0 \Rightarrow H_0 : \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = 0$

(b) $H_0 : \beta_1 = \beta_2 = 0 \Rightarrow H_0 : \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\Updownarrow$

$\beta_1 = 0, \beta_2 = 0$

Can't find the Increase in Reg S.S.

(c) $H_0 : \beta_1 = \boxed{1}, \beta_2 = \boxed{-1} \Rightarrow H_0 : \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

r.h.s. $\neq 0$

(d) $H_0 : \boxed{\beta_0} = 40 \Rightarrow H_0 : \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = 40$

involve intercept.

Theorem 4.3 (p.31) $\underset{\sim}{Y} \sim MN(\underset{\sim}{\mu}, \underset{\sim}{\Sigma})$ where $|\underset{\sim}{\Sigma}| > 0$

then $(\underset{\sim}{Y} - \underset{\sim}{\mu})^T \underset{\sim}{\Sigma}^{-1} (\underset{\sim}{Y} - \underset{\sim}{\mu}) \sim \chi^2_r$

r: rank of $\underset{\sim}{\Sigma}$

$H_0 : \underset{\sim}{C} \underset{\sim}{\beta} = \underset{\sim}{d}$

$$\hat{\underset{\sim}{\beta}} \sim MN(\underset{\sim}{\beta}, (\underset{\sim}{X}^T \underset{\sim}{X})^{-1} \sigma^2)$$

$$\underset{\sim}{C} \hat{\underset{\sim}{\beta}} \sim MN(\underset{\sim}{C}\underset{\sim}{\beta}, \underline{\sigma^2 \underset{\sim}{C} (\underset{\sim}{X}^T \underset{\sim}{X})^{-1} \underset{\sim}{C}^T})$$

Under $H_0 : \underset{\sim}{C}\underset{\sim}{\beta} = \underset{\sim}{d}$ — Theorem 3.2 in p.14

By Theorem 4.3
$$(\underset{\sim}{C} \hat{\underset{\sim}{\beta}} - \underset{\sim}{d})^T (\sigma^2 \underset{\sim}{C} (\underset{\sim}{X}^T \underset{\sim}{X})^{-1} \underset{\sim}{C}^T)^{-1} (\underset{\sim}{C} \hat{\underset{\sim}{\beta}} - \underset{\sim}{d}) \sim \chi^2_r$$

r: rank of $\underset{\sim}{C}$

$$\Rightarrow \frac{(\underset{\sim}{C} \hat{\underset{\sim}{\beta}} - \underset{\sim}{d})^T (\underset{\sim}{C} (\underset{\sim}{X}^T \underset{\sim}{X})^{-1} \underset{\sim}{C}^T)^{-1} (\underset{\sim}{C} \hat{\underset{\sim}{\beta}} - \underset{\sim}{d})}{\sigma^2} \sim \chi^2_r$$

indep.

$$\frac{\text{Res S.S.}}{\sigma^2} \sim \chi^2_{(n-p')} \leftarrow \text{p.17}$$

$$\Rightarrow \frac{(\underset{\sim}{C} \hat{\underset{\sim}{\beta}} - \underset{\sim}{d})^T (\underset{\sim}{C} (\underset{\sim}{X}^T \underset{\sim}{X})^{-1} \underset{\sim}{C}^T)^{-1} (\underset{\sim}{C} \hat{\underset{\sim}{\beta}} - \underset{\sim}{d}) / r}{\boxed{\text{Res S.S.} / (n-p')} = \hat{\sigma}^2}$$

$$\sim F(r, n-p')$$

e.g. ⓒ $H_0: \beta_1 = 1,\ \beta_2 = -1$

$$\Rightarrow H_0 : \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\underset{\sim}{c}\,\hat{\underset{\sim}{\beta}} - \underset{\sim}{d} = \begin{pmatrix} \hat\beta_1 - 1 \\ \hat\beta_2 + 1 \end{pmatrix}$$

$$\underset{\sim}{c}\,(\underset{\sim}{X}^T \underset{\sim}{X})^{-1} \underset{\sim}{c}^T = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \underset{\substack{\uparrow \\ \text{p.7}}}{(\underset{\sim}{X}_c^T \underset{\sim}{X}_c)^{-1}} \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0.00847981 & 0.00171669 \\ 0.00171669 & 0.0166294 \end{pmatrix}$$

$$\left(\underset{\sim}{c}\,(\underset{\sim}{X}^T \underset{\sim}{X})^{-1} \underset{\sim}{c}^T\right)^{-1} = \left(\qquad \right)^{-1}$$

$$F = \frac{(\underset{\sim}{c}\,\hat{\underset{\sim}{\beta}} - \underset{\sim}{d})^T (\underset{\sim}{c}\,(\underset{\sim}{X}^T \underset{\sim}{X})^{-1} \underset{\sim}{c}^T)^{-1} (\underset{\sim}{c}\,\hat{\underset{\sim}{\beta}} - \underset{\sim}{d}) / 2}{\underset{\substack{\uparrow \\ 4.298}}{\hat\sigma^2}}$$

$$(\hat\beta_1 - 1,\ \hat\beta_2 + 1) \begin{pmatrix} 0.00847981 & 0.00171669 \\ & 0.0166294 \end{pmatrix}^{-1} \begin{pmatrix} \hat\beta_1 - 1 \\ \hat\beta_2 + 1 \end{pmatrix}$$

$$= \frac{22.98765}{4.298} = 5.35 > F_{0.05}(2, 13-4)$$

Reject H_0

e.g. ⓓ $H_0 : \beta_0 = 40$

- t-test

- F-test

$$H_0 : (1 \quad 0 \quad 0 \quad 0) \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = 40$$

$$F = \frac{(\underset{\sim}{c}\hat{\underset{\sim}{\beta}} - \underset{\sim}{d})^T \boxed{(\underset{\sim}{c}(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{c}^T)^{-1}} (\underset{\sim}{c}\hat{\underset{\sim}{\beta}} - \underset{\sim}{d}) / 1}{\hat{\sigma}^2}$$

Example 5: Example in Multiple Linear Regression

The percent survival of a certain type of animal semen after storage was measured at various combinations of concentrations of three materials used to increase chance of survival. The data are as follows:

y (% survival)	x_1 (weight %)	x_2 (weight %)	x_3 (weight %)
25.5	1.74	5.30	10.80
31.2	6.32	5.42	9.40
25.9	6.22	8.41	7.20
38.4	10.52	4.63	8.50
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26.7	1.22	5.85	9.90
26.4	4.10	6.62	8.00
25.9	6.32	8.72	9.10
32.0	4.08	4.42	8.70
25.2	4.15	7.60	9.20
39.7	10.15	4.83	9.40
35.7	1.72	3.12	7.60
26.5	1.70	5.30	8.20

$$\underset{\sim}{c}\hat{\underset{\sim}{\beta}} = (1\ 0\ 0\ 0)\begin{pmatrix}\hat\beta_0\\ \hat\beta_1\\ \hat\beta_2\\ \hat\beta_3\end{pmatrix} = \hat\beta_0$$

Summary statistics:

$\sum_{i=1}^{13} y_i = 377.5$ $\quad \sum_{i=1}^{13} y_i^2 = 11,400.15$ $\quad \sum_{i=1}^{13} x_{i1} = 59.43$

$\sum_{i=1}^{13} x_{i2} = 81.82$ $\quad \sum_{i=1}^{13} x_{i3} = 115.40$ $\quad \sum_{i=1}^{13} x_{i1}^2 = 394.7255$

$\sum_{i=1}^{13} x_{i2}^2 = 576.7264$ $\quad \sum_{i=1}^{13} x_{i3}^2 = 1035.96$ $\quad \sum_{i=1}^{13} x_{i1}y_i = 1877.567$

$\sum_{i=1}^{13} x_{i2}y_i = 2246.661$ $\quad \sum_{i=1}^{13} x_{i3}y_i = 3337.78$ $\quad \sum_{i=1}^{13} x_{i1}x_{i2} = 360.6621$

$\sum_{i=1}^{13} x_{i1}x_{i3} = 522.078$ $\quad \sum_{i=1}^{13} x_{i2}x_{i3} = 728.31$ $\quad n = 13$

Original model

$$\underset{\sim}{c}(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{c}^T = (1,0,0,0)$$

$$\begin{pmatrix} 13 & 59.43 & 81.82 & 115.40 \\ 59.43 & 394.7255 & 360.6621 & 522.078 \\ 81.82 & 360.6621 & 576.7264 & 728.31 \\ 115.40 & 522.078 & 728.31 & 1035.96 \end{pmatrix}^{-1} = \begin{pmatrix} 8.06479 & -0.0825927 & -0.0941951 & -0.790527 \\ -0.0825927 & 0.00847982 & 0.00171669 & 0.00372002 \\ -0.0941951 & 0.00171669 & 0.0166294 & -0.00206331 \\ -0.790527 & 0.00372002 & -0.00206331 & 0.0886013 \end{pmatrix}\begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

$$= 8.06479 \qquad F = \frac{(\hat\beta_0 - 40)^2}{8.06479 \times 4.298}$$

Or

Centered model $y_i = \beta_0' + \beta_1(x_{i1}-\bar{x}_1) + \beta_2(x_{i2}-\bar{x}_2) + \beta_3(x_{i3}-\bar{x}_3) + e_i$

$$(\underset{\sim}{X}_c^T\underset{\sim}{X}_c)^{-1} = \begin{pmatrix} 13 & 0 & 0 & 0 \\ 0 & 123.039 & -13.3812 & -5.4775 \\ 0 & -13.3812 & 61.7639 & 2.0002 \\ 0 & -5.4775 & 2.0002 & 11.5631 \end{pmatrix}^{-1}$$

$$(1, -\bar{x}_1, -\bar{x}_2, -\bar{x}_3)$$

$$= \begin{pmatrix} 0.0769231 & 0 & 0 & 0 \\ 0 & 0.00847981 & 0.00171669 & 0.00371998 \\ 0 & 0.00171669 & 0.0166294 & -0.00206338 \\ 0 & 0.00371998 & -0.00206338 & 0.0886011 \end{pmatrix}\begin{pmatrix}1\\-\bar{x}_1\\-\bar{x}_2\\-\bar{x}_3\end{pmatrix} = 8.06479$$

$\Rightarrow \hat\beta_0 = 39.1574$, $\hat\beta_1 = 1.0161$, $\hat\beta_2 = -1.8616$, $\hat\beta_3 = -0.3433$.

$\cancel{H_0}$ $H_0: \beta_0 = \beta_0' - \beta_1\bar{x}_1 - \beta_2\bar{x}_2 - \beta_3\bar{x}_3 = 40$

$$\underset{\sim}{c}(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{c}^T$$

$$\Rightarrow H_0: \underbrace{\boxed{(1\ \ -\bar{x}_1\ \ -\bar{x}_2\ \ -\bar{x}_3)}}_{\underset{\sim}{c}}\begin{pmatrix}\beta_0'\\ \beta_1\\ \beta_2\\ \beta_3\end{pmatrix} = 40$$

F-test

d.f. of Reg S.S. | − d.f. of Reg S.S. | reduced

$$F = \frac{\boxed{\text{Increase in Reg S.S.}}/r}{\hat{\sigma}^2} = F = \frac{\boxed{(\underset{\sim}{C}\hat{\underset{\sim}{\beta}} - \underset{\sim}{d})^T (\underset{\sim}{C}(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{C}^T)^{-1}(\underset{\sim}{C}\hat{\underset{\sim}{\beta}} - \underset{\sim}{d})}/r}{\hat{\sigma}^2}$$

Can be proved

rank of $\underset{\sim}{C}$

Res S.S. | reduced

$\boxed{\underset{\sim}{C}(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{C}^T}$

$$\tilde{\beta}_R = \boxed{(\underset{\sim}{X}_R^T\underset{\sim}{X}_R)^{-1}}\underset{\sim}{X}_R^T\underset{\sim}{Y}$$

(a) $H_0: \beta_3 = 0$

(b) $H_0: \beta_1 = \beta_2 = 0$