15 September 2020

1. **Intercept is known**

$$\begin{aligned} y_i &= \beta_0 + \beta_1 x_{i1} + \ldots + \beta_p x_{ip} + e_i \qquad\qquad i = 1, \ldots, n, \\ \Rightarrow y_i - \beta_0 &= \beta_1 x_{i1} + \ldots + \beta_p x_{ip} + e_i \end{aligned}$$

2. **Regression coefficient is known**
   For example, , β_1 is known

$$\begin{aligned} y_i &= \beta_0 + \beta_1 x_{i1} + \ldots + \beta_p x_{ip} + e_i \qquad\qquad i = 1, \ldots, n, \\ \Rightarrow y_i - \beta_1 x_{i1} &= \beta_0 + \beta_2 x_{i2} + \ldots + \beta_p x_{ip} + e_i \end{aligned}$$

3. **Method of maximum likelihood**
   - MLE of β_j, for $j = 0, 1, \ldots, p$, are the same as their least squares estimates. It is true only when $f(y|x)$ follows normal distribution. If $f(y|x)$ follows other distributions, e.g. Bernoulli, estimation of β by the method of maximum likelihood is chosen.
   - $$\tilde{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

   $\sum_{i=1}^{n}\hat{e}_i^2 = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$ – define: Residual Sum of Squares (Res S.S.)

4. **Residual S.S.**

$$\begin{aligned} \text{Res.S.S.} &= (\underset{\sim}{Y} - \hat{\underset{\sim}{Y}})^T(\underset{\sim}{Y} - \hat{\underset{\sim}{Y}}) \\ &= \underset{\sim}{Y}^T(\underset{\sim}{I} - \underset{\sim}{X}(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{X}^T)\underset{\sim}{Y} \\ &= \underset{\sim}{Y}^T\underset{\sim}{Y} - \hat{\underset{\sim}{\beta}}^T\underset{\sim}{X}^T\underset{\sim}{Y} \end{aligned}$$

   - <u>For the model with intercept (β_0 is unknown)</u>
     Since $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x}_1 - \ldots - \hat{\beta}_p\bar{x}_p$,

$$\begin{aligned} \text{Res.S.S.} &= \sum_{i=1}^{n} {y_i}^2 - \hat{\beta}_0\sum_{i=1}^{n} y_i - \hat{\beta}_1\sum_{i=1}^{n} x_{i1}y_i - \ldots - \hat{\beta}_p\sum_{i=1}^{n} x_{ip}y_i \\ &= \mathrm{S}_{yy} - \hat{\beta}_1\mathrm{S}_{x_1y} - \ldots - \hat{\beta}_p\mathrm{S}_{x_py} \end{aligned}$$

   - <u>For the model with intercept and $p = 1$</u>

$$\begin{aligned} \text{Res.S.S.} &= S_{yy} - \hat{\beta}_1\mathrm{S}_{x_1y} \\ &= S_{yy} - \hat{\beta}_1^2 S_{x_1x_1} \quad \text{as } \hat{\beta}_1 = \frac{S_{x_1y}}{S_{x_1x_1}} \end{aligned}$$

- For the model with known intercept
  Write the model as $y_i' = y_i - \beta_0 = \beta_1 x_{i1} + \ldots + \beta_p x_{ip} + e_i$, then

$$
\begin{aligned}
\text{Res.S.S.} &= \sum_{i=1}^{n} {y_i'}^2 - \hat{\beta}_1 \sum_{i=1}^{n} x_{i1} y_i' - \ldots - \hat{\beta}_p \sum_{i=1}^{n} x_{ip} y_i' \\
&= \sum_{i=1}^{n} (y_i - \beta_0)^2 - \hat{\beta}_1 \sum_{i=1}^{n} x_{i1}(y_i - \beta_0) - \ldots - \hat{\beta}_p \sum_{i=1}^{n} x_{ip}(y_i - \beta_0)
\end{aligned}
$$