

Solution:

$$1. R^2 = \frac{RegS.S.}{TotalS.S.} = \frac{96.0794}{101.6} = 0.94566.$$

Then multiple correlation coefficient is $\sqrt{R^2} = \sqrt{0.94566} = 0.9725$

$$2. \hat{y}(x_0) = 0.81937 + 1.93414 * 2.2 + 1.01989 * 5 = 10.17393$$

$$\hat{\sigma}^2 = RSS/7 = 0.78866$$

$$x_{0c}^T = (1, 2.2 - \bar{x}_1, 5 - \bar{x}_2) = (1, 2.2 - \frac{12}{10}, 5 - \frac{30}{10}) = (1, 1, 2)$$

$$x_{0c}^T (X_c^T X_c)^{-1} x_{0c} = 0.31911$$

$$t_{0.025,7} = 2.365$$

The 0.95 prediction interval for y at $x_1 = 2.2$ and $x_2 = 5$ is $10.17393 \pm 2.365 \sqrt{0.78866 \times (1 + 0.31911)}$
 $= 10.17393 \pm 2.41222 = [7.76171, 12.58615]$.

Remarks:

We make use of centered model to compute $(X^T X)^{-1}$, so the corresponding x_0^T is $(1, 2.2 - \bar{x}_1, 5 - \bar{x}_2) = (1, 1, 2)$

$$3. F \text{ test for testing } H_0 : C\beta = d.$$

$$C = [0 \ 1 \ -1]$$

$$d = 0$$

$$C\hat{\beta} - d = 0.91425$$

In this case, since the first element of C is 0, so

$$[C(X^T X)^{-1} C^T] = [C(X_c^T X_c)^{-1} C^T] = 0.13303$$

$$\text{Value of test statistic} = 7.96652$$

$$\text{Critical value} = F_{0.05,1,7} = 5.59$$

Since $F_{obs} > F_{1,7,0.05}$, reject the null hypothesis.

$$4. \text{ "Increase in Regression Sum of Squares". The reduced model is } y = \beta_0 + \beta_1(x_1 + x_2) + e_i$$

$$\text{Take } x' = x_1 + x_2, \text{ and under reduced model, } \hat{\beta}_1 = \frac{S_{x'y}}{S_{x'x'}} = \frac{S_{x_1y} + S_{x_2y}}{S_{x_1x_1} + 2S_{x_1x_2} + S_{x_2x_2}} = 1.04902$$

$$RSS_{reduced} = S_{yy} - \hat{\beta}_1 S_{x'y} = 11.80392$$

$$\text{Increase in Regression Sum of Squares} = RSS_{reduced} - RSS_{full} = 11.80392 - 5.52062 = 6.28331$$

$$F_{obs} = \frac{RSS_{reduced} - RSS_{full}}{1 * \hat{\sigma}^2} = \frac{6.28331}{1 * 0.7887} = 7.9666 > F_{0.05,1,7} = 5.59$$

Reject the null hypothesis.