Summary $\quad H_0: \underset{\sim}{\alpha} = 0 \Leftrightarrow H_0 = \mu_{1\cdot} = \mu_{2\cdot} = \cdots = \mu_{a\cdot}$ $\quad \alpha_1 = \alpha_2 = \cdots = \alpha_{a-1} = 0$

5 Nov

$$SSA = R(\underset{\sim}{\alpha} \mid \underset{\sim}{\beta}, \beta_0)$$

$$= \underbrace{R(\underset{\sim}{\alpha}, \underset{\sim}{\beta} \mid \beta_0)}_{\text{Total SS.}\,\phi\, -\, \text{Res S.S.}\,|\,\text{model without interaction}} - \underbrace{R(\underset{\sim}{\beta} \mid \beta_0)}_{\text{Total S.S.} - \text{Res S.S.}\,|\,\text{Factor B}}$$

$$F_{obs} = \frac{SSA/(a-1)}{\hat{\sigma}^2_{\text{no interaction}}}$$

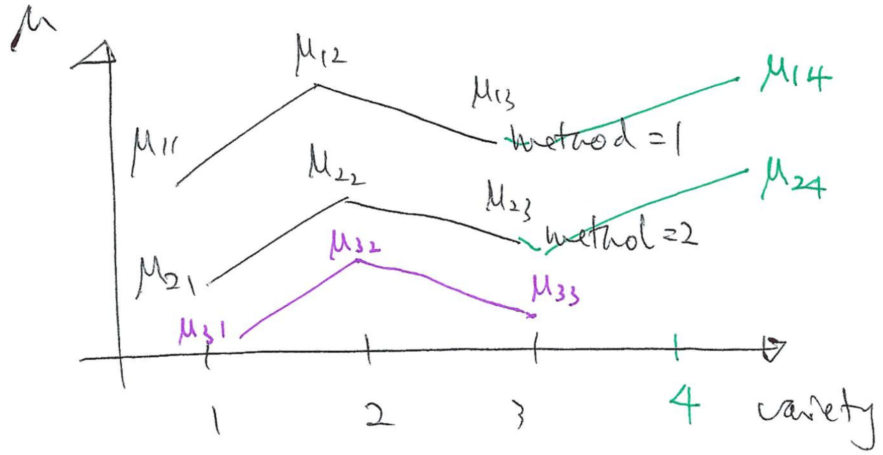
Reject H_0 if $F_{obs} > F_{\alpha,\, a-1,\, \sum_{i=1}^{a}\sum_{j=1}^{b}(n_{ij}-1)+(a-1)+(b-1)}$

$H_0: \beta_1 = \beta_2 = \cdots = \beta_{b-1} = 0 \Leftrightarrow H_0 = \mu_{\cdot 1} = \mu_{\cdot 2} = \cdots = \mu_{\cdot b}$

$$SSB = R(\underset{\sim}{\alpha}, \underset{\sim}{\beta} \mid \beta_0) - R(\underset{\sim}{\alpha} \mid \beta_0)$$

$H_0: \underset{\sim}{C}\,\underset{\sim}{\beta} = \underset{\sim}{0}$

$$F = \frac{(\underset{\sim}{C}\hat{\underset{\sim}{\beta}})^T \boxed{(\underset{\sim}{C}\boxed{(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}}\underset{\sim}{C}^T)^{-1}} (\underset{\sim}{C}\hat{\underset{\sim}{\beta}})/r}{\hat{\sigma}^2}$$



$$H_0 : \mu_{11} - \mu_{21} = \mu_{12} - \mu_{22} = \mu_{13} - \mu_{23} = \mu_{14} - \mu_{24}$$

$$\mu_{21} - \mu_{31} = \mu_{22} - \mu_{32} = \mu_{23} - \mu_{33}$$

$$\Rightarrow H_0 : \underset{\sim}{C}\, \underset{\sim}{\beta} = \underset{\sim}{0}$$

interaction terms

3. ~~ANO~~ ANCOVA - Analysis of covariance

One categorical variable + one quantitative variable

e.g. group ⇒ 2 dummy variables g_1, g_2

Regression model $y_i = \beta_0 + \underline{\beta_{g_1} * g_{1i} + \beta_{g_2} * g_{2i}} + \beta_1 x_{i1} + e_i$

$+ \beta_{1g_1} * g_{1i} * x_{i1} + \beta_{1g_2} * g_{2i} * x_{i1}$

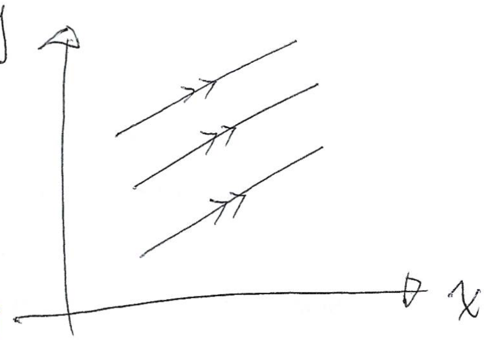
$+ \beta_2 x_{i2} + \dots + \beta_p x_{ip}$

$+ \beta_{2g_1} * g_{1i} * x_{i2} + \beta_{2g_2} * g_{2i} * x_{i2} + \dots + \beta_{pg_1} * g_{1i} * x_{ip} + \beta_{pg_2} * g_{2i} * x_{ip}$

Group	Model	
1	$y_i = \beta_0 + \beta_{g_1} + \beta_1 x_i + \beta_{g_1} * x_i + e_i$	$i = 1, \dots, n_1$
2	$y_i = \beta_0 + \beta_{g_2} + \beta_1 x_i + \beta_{g_2} * x_i + e_i$	$i = n_1 + 1, \dots, n_1 + n_2$
3	$y_i = \beta_0 + \beta_1 x_i + e_i$	$i = n_1 + n_2 + 1, \dots, n_1 + n_2 + n_3$

It is not a general model!

⇒ ANCOVA model



Group	Model	$\gamma_{21} x_{i2} + \dots + \gamma_{p1} x_{ip}$
1	$y_i = \gamma_{01} + \gamma_{11} x_{i1} + e_i$	$i = 1, \dots, n_1$
2	$y_i = \gamma_{02} + \gamma_{12} x_{i1} + e_i$	$i = n_1 + 1, \dots, n_1 + n_2$
3	$y_i = \gamma_{03} + \gamma_{13} x_{i1} + e_i$	$i = n_1 + n_2 + 1, \dots, n_1 + n_2 + n_3$

$$\Rightarrow \quad \beta_0 + \beta_{g1} = \gamma_{01} \qquad \beta_1 + \beta_{1g_1} = \gamma_{11}$$

$$\beta_0 + \beta_{g2} = \gamma_{02} \qquad \beta_1 + \beta_{1g_2} = \gamma_{12}$$

$$\beta_0 = \gamma_{03} \qquad \beta_1 = \gamma_{13}$$

<u>Regression model</u> $\quad \underset{\sim}{\beta}^T = (\beta_0, \beta_{g1}, \beta_{g2}, \beta_1, \beta_{1g_1}, \beta_{1g_2})$

$$\underset{n\times 6}{\underset{\sim}{X}} = \begin{array}{c} \left(\begin{array}{cccccc} 1 & 1 & 0 & x_1 & x_1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 0 & x_{n_1} & x_{n_1} & 0 \\ \hline 1 & 0 & 1 & x_{n_1+1} & 0 & x_{n_1+1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 1 & x_{n_1+n_2} & 0 & x_{n_1+n_2} \\ \hline 1 & 0 & 0 & x_{n_1+n_2+1} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & x_{n_1+n_2+n_3} & 0 & 0 \end{array}\right) \begin{array}{l} \\ \text{group}=1 \\ \\ \\ \text{group}=2 \\ \\ \\ \text{group}=3 \\ \\ \end{array} \\ \begin{array}{cccccc} \beta_0 & \beta_{g1} & \beta_{g2} & \beta_1 & \beta_{1g_1} & \beta_{1g_2} \end{array} \end{array}$$

$$\underset{6\times 6}{\underset{\sim}{X}^T \underset{\sim}{X}}$$

<u>ANCOVA</u> $\quad \underset{\sim}{\beta}^T = (\underbrace{\gamma_{01}, \gamma_{11}}_{\text{group}=1}, \underbrace{\gamma_{02}, \gamma_{12}}_{\text{group}=2}, \underbrace{\gamma_{03}, \gamma_{13}}_{\text{group}=3})$

$$\underset{n\times 6}{\underset{\sim}{X}} = \begin{array}{c} \left(\begin{array}{cccccc} 1 & x_1 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n_1} & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & x_{n_1+1} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 1 & x_{n_1+n_2} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & x_{n_1+n_2+1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 & x_{n_1+n_2+n_3} \end{array}\right) \begin{array}{l} \\ \text{group}=1 \\ \\ \\ \text{group}=2 \\ \\ \\ \text{group}=3 \\ \\ \end{array} \\ \begin{array}{cccccc} \gamma_{01} & \gamma_{11} & \gamma_{02} & \gamma_{12} & \gamma_{03} & \gamma_{13} \end{array} \end{array}$$

model without intercept

$$\underset{6\times 6}{\underset{\sim}{X}^T \underset{\sim}{X}} = \left(\begin{array}{cc|cc|cc} n_1 & \sum_{i=1}^{n_1} x_i & 0 & 0 & 0 & 0 \\ \sum_{i=1}^{n_1} x_i & \sum_{i=1}^{n_1} x_i^2 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & n_2 & \sum_{i=n_1+1}^{n_1+n_2} x_i & 0 & 0 \\ 0 & 0 & \sum_{i=n_1+1}^{n_1+n_2} x_i & \sum_{i=n_1+1}^{n_1+n_2} x_i^2 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & n_3 & \sum_{i=n_1+n_2+1}^{n_1+n_2+n_3} x_i \\ 0 & 0 & 0 & 0 & \sum x_i & \sum_{i=n_1+n_2+1}^{n_1+n_2+n_3} x_i^2 \end{array}\right)$$

2×2 $\quad$ 2×2 $\quad$ 2×2

$$= \left(\begin{array}{c|c|c} \underset{\sim}{X}_1^T \underset{\sim}{X}_1 & \underset{\sim}{0} & \underset{\sim}{0} \\ \hline \underset{\sim}{0} & \underset{\sim}{X}_2^T \underset{\sim}{X}_2 & \underset{\sim}{0} \\ \hline \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{X}_3^T \underset{\sim}{X}_3 \end{array}\right)$$

design matrix for group = 1

$$\underset{n_1\times 2}{\underset{\sim}{X}_1} = \begin{pmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_{n_1} \end{pmatrix} \qquad \underset{n_2\times 2}{\underset{\sim}{X}_2} = \begin{pmatrix} 1 & x_{n_1+1} \\ \vdots & \vdots \\ 1 & x_{n_1+n_2} \end{pmatrix} \qquad \underset{n_3\times 2}{\underset{\sim}{X}_3} = \begin{pmatrix} 1 & x_{n_1+n_2+1} \\ \vdots & \vdots \\ 1 & x_{n_1+n_2+n_3} \end{pmatrix}$$

$$\underset{\sim}{X}^T \underset{\sim}{Y} = \begin{pmatrix} \underset{\sim}{X}_1^T \underset{\sim}{Y}_1 \\ \hline \underset{\sim}{X}_2^T \underset{\sim}{Y}_2 \\ \hline \underset{\sim}{X}_3^T \underset{\sim}{Y}_3 \end{pmatrix}$$

$$\underset{\sim}{Y} = \begin{pmatrix} y_1 \\ \vdots \\ y_{n_1} \\ \hline y_{n_1+1} \\ \vdots \\ y_{n_1+n_2} \\ \hline y_{n_1+n_2+1} \\ \vdots \\ y_{n_1+n_2+n_3} \end{pmatrix} \begin{array}{l} \underset{n_1\times 1}{\underset{\sim}{Y}_1} \\ \\ \underset{n_2\times 1}{\underset{\sim}{Y}_2} \\ \\ \underset{n_3\times 1}{\underset{\sim}{Y}_3} \end{array}$$

$$(\underset{\sim}{X}^T \underset{\sim}{X})^{-1} \underset{\sim}{X}^T \underset{\sim}{Y} = \left(\begin{array}{c|c|c} (\underset{\sim}{X}_1^T \underset{\sim}{X}_1)^{-1} & \underset{\sim}{0} & \underset{\sim}{0} \\ \hline \underset{\sim}{0} & (\underset{\sim}{X}_2^T \underset{\sim}{X}_2)^{-1} & \underset{\sim}{0} \\ \hline \underset{\sim}{0} & \underset{\sim}{0} & (\underset{\sim}{X}_3^T \underset{\sim}{X}_3)^{-1} \end{array}\right) \begin{pmatrix} \underset{\sim}{X}_1^T \underset{\sim}{Y}_1 \\ \underset{\sim}{X}_2^T \underset{\sim}{Y}_2 \\ \underset{\sim}{X}_3^T \underset{\sim}{Y}_3 \end{pmatrix}$$

$$\hat{\underset{\sim}{\beta}} = \begin{pmatrix} \gamma_{01} \\ \gamma_{11} \\ \hline \gamma_{02} \\ \gamma_{12} \\ \hline \gamma_{03} \\ \gamma_{13} \end{pmatrix} \begin{array}{l} \hat{\underset{\sim}{\beta}}_1 \\ \\ \hat{\underset{\sim}{\beta}}_2 \\ \\ \hat{\underset{\sim}{\beta}}_3 \end{array} = \begin{pmatrix} \boxed{\underbrace{(\underset{\sim}{X}_1^T \underset{\sim}{X}_1)^{-1}}_{2\times 2} \underbrace{\underset{\sim}{X}_1^T \underset{\sim}{Y}_1}_{2\times 1}} \\ \hline (\underset{\sim}{X}_2^T \underset{\sim}{X}_2)^{-1} \underset{\sim}{X}_2^T \underset{\sim}{Y}_2 \\ \hline (\underset{\sim}{X}_3^T \underset{\sim}{X}_3)^{-1} \underset{\sim}{X}_3^T \underset{\sim}{Y}_3 \end{pmatrix}$$

simple linear regression for group 1

$$\Rightarrow \quad \hat{\gamma}_{0j} = \bar{y}_j - \hat{\gamma}_{1j} \bar{x}_j$$

$$\hat{\gamma}_{1j} = \frac{S_{x_j y_j}}{S_{x_j x_j}} \qquad j = 1, 2, 3$$

$$\underline{\text{Res S.S.}} = \underset{\sim}{Y}^T \underset{\sim}{Y} - \hat{\underset{\sim}{\beta}}^T \underset{\sim}{X}^T \underset{\sim}{Y}$$

$$= \left(\underset{\sim}{Y}_1^T \underset{\sim}{Y}_1 + \underset{\sim}{Y}_2^T \underset{\sim}{Y}_2 + \underset{\sim}{Y}_3^T \underset{\sim}{Y}_3 \right) - \left(\hat{\underset{\sim}{\beta}}_1^T ; \hat{\underset{\sim}{\beta}}_2^T , \hat{\underset{\sim}{\beta}}_3^T \right) \begin{pmatrix} \underset{\sim}{X}_1^T \underset{\sim}{Y}_1 \\ \hline \underset{\sim}{X}_2^T \underset{\sim}{Y}_2 \\ \hline \underset{\sim}{X}_3^T \underset{\sim}{Y}_3 \end{pmatrix}$$

$$= \left(\underline{\underset{\sim}{Y}_1^T \underset{\sim}{Y}_1} + \underline{\underset{\sim}{Y}_2^T \underset{\sim}{Y}_2} + \underline{\underset{\sim}{Y}_3^T \underset{\sim}{Y}_3} \right) - \left(\underline{\hat{\underset{\sim}{\beta}}_1^T \underset{\sim}{X}_1^T \underset{\sim}{Y}_1} + \underline{\hat{\underset{\sim}{\beta}}_2^T \underset{\sim}{X}_2^T \underset{\sim}{Y}_2} + \underline{\hat{\underset{\sim}{\beta}}_3^T \underset{\sim}{X}_3^T \underset{\sim}{Y}_3} \right)$$

$$= \underbrace{\left(\underset{\sim}{Y}_1^T \underset{\sim}{Y}_1 - \hat{\underset{\sim}{\beta}}_1 \underset{\sim}{X}_1^T \underset{\sim}{Y}_1 \right)}_{\text{Res S.S. for group 1}} + \underbrace{\left(\underset{\sim}{Y}_2^T \underset{\sim}{Y}_2 - \hat{\underset{\sim}{\beta}}_2 \underset{\sim}{X}_2^T \underset{\sim}{Y}_2 \right)}_{\text{Res S.S. for group 2}} + \underbrace{\left(\underset{\sim}{Y}_3^T \underset{\sim}{Y}_3 - \hat{\underset{\sim}{\beta}}_3 \underset{\sim}{X}_3^T \underset{\sim}{Y}_3 \right)}_{\text{Res S.S. for group 3}}$$

$$= \sum_{i=1}^{3} \text{Res S.S.} \Big|_{\text{group} = i}$$

$$\text{d.f.} = \sum_{i=1}^{3} (n_i - 2)$$

\# of unknown para. in the model

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{3} \text{Res S.S.} \big|_{\text{group} = i}}{\sum_{i=1}^{3} (n_i - 2)}$$

$$\cancel{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}$$

$\Rightarrow$ C.I. of γ_{0j} & γ_{ij} $\Leftarrow$ $Var(\hat{\underset{\sim}{\beta}}) = (\underset{\sim}{X}^T \underset{\sim}{X})^{-1} \sigma^2$

Hypothesis testing

$$Var(\hat{\underset{\sim}{\beta}}_j) = (\underset{\sim}{X}_j^T \underset{\sim}{X}_j)^{-1} \sigma^2$$

3. ANCOVA

Regression model

3.1 One categorical variables + one cont. variable

$y_i = \beta_0 + \beta_{g_1} * \cdots + \boxed{e_i}$

$e_i \overset{iid}{\sim} N(0, \sigma^2)$

Table 3.3 Data involving three sets of subjects

	Group	Weight (lb)	HDL Cholesterol (mg/decaliter)
Control	1	163.5	75
	1	180	72.5
	1	178.5	62
	1	161.5	60
	1	127	53
	1	161	53
	1	165	65
	1	144	63.5
Running	2	141	49
	2	162	53.5
	2	134	30
	2	121	40.5
	2	145	51.5
	2	106	57.5
	2	134	49
	2	216.5	74
Running and Weightlifting	3	136.5	54.5
	3	142.5	79.5
	3	145	64
	3	165	69
	3	226	50.5
	3	122	58
	3	193	63.5
	3	163.5	76
	3	154	55.5
	3	139	68

$\Rightarrow \hat{\gamma}_{1j} = \dfrac{S_{x_j y_j}}{S_{x_j x_j}}$

$\hat{\gamma}_{0j} = \bar{y}_j - \hat{\gamma}_{1j} \bar{x}_j$

$\boxed{\hat{\sigma}_j^2} = \dfrac{Res.S.S.|_j}{n_j - \cancel{2}}$

est.

σ_j^2 C.I. for γ_{0j} & γ_{1j}

but not σ^2

Model I

$$y = \beta_0 + \beta_{g_1} * g_1 + \beta_{g_2} * g_2 + \beta_1 * x + \beta_{1g_1} * g_1 * x + \beta_{1g_2} * g_2 * x + e$$

Model II

$$A: y_i = \gamma_{01} + \gamma_{11} x_i + e_i, \qquad i = 1, \ldots, 8$$
$$B: y_i = \gamma_{02} + \gamma_{12} x_i + e_i, \qquad i = 9, \ldots, 16$$
$$C: y_i = \gamma_{03} + \gamma_{13} x_i + e_i, \qquad i = 17, \ldots, 26$$

$$\underset{\sim}{Y} = \begin{pmatrix} 75 \\ 72.5 \\ \vdots \\ 68 \end{pmatrix} \qquad \underset{\sim}{\beta} = \begin{pmatrix} \gamma_{01} \\ \gamma_{11} \\ \gamma_{02} \\ \gamma_{12} \\ \gamma_{03} \\ \gamma_{13} \end{pmatrix}$$

$$
\begin{array}{c}
\qquad\qquad \underset{\sim}{X_1} \qquad\qquad \underset{\sim}{X_2} \\
\underset{\sim}{X} = \begin{pmatrix}
1 & 163.5 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 144 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 141 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 1 & 216.5 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 136.5 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 1 & 139
\end{pmatrix} \underset{\sim}{X_3}
\end{array}
$$

ANCOVA

$\Rightarrow$ group 1 $\gamma_{01} = 23.0543$

$\gamma_{11} = 0.24956$

group 2 $\gamma_{02} = 14.255$

$\gamma_{12} = 0.25094$

group 3 $\gamma_{03} = 76.88002$

$\gamma_{13} = -0.08213$

Control

$$E(y) = \beta_0 + \beta_{g_1} + (\beta_1 + \beta_{1g_1})\text{weight} = \gamma_{01} + \gamma_{11} \text{ weight}$$

Running

$$E(y) = \beta_0 + \beta_{g_2} + (\beta_1 + \beta_{1g_2})\text{weight} = \gamma_{02} + \gamma_{12} \text{ weight}$$

Running and Weighting

Regression model

$$E(y) = \beta_0 + \beta_1\text{weight} = \gamma_{03} + \gamma_{13} \text{ weight}$$

$\hat{\gamma}_{03}$ " $\quad \hat{\gamma}_{01} - \hat{\beta}_0 \quad \hat{\gamma}_{02} - \hat{\beta}_0$

$\hat{\beta}_0 = 76.88002 \quad \hat{\beta}_{g1} = 23.5431 - 76.88002 = -53.82571 \quad \hat{\beta}_{g2} = -62.62502$

$\hat{\gamma}_{13} \rightarrow \hat{\beta}_1 = -0.08213 \quad \hat{\beta}_{1g_1} = 0.24956 - (-0.08213) = 0.33169 \quad \hat{\beta}_{1g_2} = 0.33307$

$\Rightarrow y = 76.88 - 53.83g_1 - 62.63g_2 - 0.08213\text{weight} + 0.33169(g_1 * \text{weight}) + 0.033307(g_2 * \text{ weight}) + e$

$\hat{\gamma}_{11} - \hat{\beta}_1 \quad \hat{\gamma}_{12} - \hat{\beta}_1$

- Estimate of σ^2

- F-test

- $H_0 : \beta_{1g1} = \beta_{1g2} = 0$ (no interaction) $\Rightarrow H_0 : \gamma_{11} = \gamma_{12} = \gamma_{13}$ (parallel lines)

$\hat{\sigma}^2 = 85.61794$

3.2 Two categorical variables + one cont. variable

Model I

$$
\begin{aligned}
y = {} & \beta_0 + \beta_{g_1} * g_1 + \beta_{g_2} * g_2 + \beta_{c_1} * c_1 + \beta_{c_2} * c_2 + \beta_{g_1,c_1}(g_1 * c_1) + \\
& \beta_{g_1,c_2}(g_1 * c_2) + \beta_{g_2,c_1}(g_2 * c_1) + \beta_{g_2,c_2}(g_2 * c_2) + \beta_1 * x + \beta_{g_1,x}(g_1 * x) + \beta_{g_2,x}(g_2 * x) + \\
& \beta_{c_1,x}(c_1 * x) + \beta_{c_2,x}(c_1 * x) + \beta_{g_1,c_1,x}(g_1 * c_1 * x) + \\
& \beta_{g_1,c_2,x}(g_1 * c_2 * x) + \beta_{g_2,c_1,x}(g_2 * c_1 * x) + \beta_{g_2,c_2,x}(g_2 * c_2 * x) + e
\end{aligned}
$$

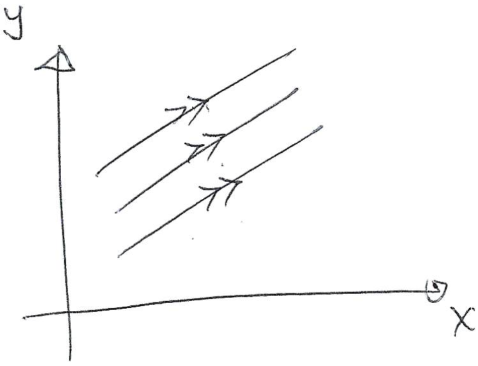
Model II

$y_{ijk} = \gamma_{0ij} + \gamma_{1ij} + e$ for $i, j = 1, 2, 3$; $k = 1, \ldots, n_{ij}$

$H_0 : \beta_{g_1,x} = \beta_{g_2,x} = \beta_{c_1,x} = \beta_{c_2,x} = \beta_{g_1,c_1,x} = \beta_{g_1,c_2,x} = \beta_{g_2,c_1,x} = \beta_{g_2,c_2,x} = 0$ (no interaction)
$\Rightarrow H_0 : \gamma_{111} = \gamma_{112} = \gamma_{113} = \gamma_{121} = \gamma_{122} = \gamma_{123} = \gamma_{131} = \gamma_{132} = \gamma_{133}$ (parallel curves)

H_0 : No interaction term

$\Rightarrow H_0 : \beta_{1g_1} = \beta_{1g_2} = 0 \quad \Rightarrow$



$\Rightarrow H_0$: reg. coeff. of x are the same

$\Rightarrow H_0 : \gamma_{11} = \gamma_{12} = \gamma_{13}$

$\Rightarrow H_0 : \gamma_{11} = \gamma_{12}, \ \gamma_{12} = \gamma_{13}$

$$\Rightarrow H_0 : \underbrace{\begin{pmatrix} 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{pmatrix}}_{r=2} \begin{pmatrix} \gamma_{01} \\ \gamma_{11} \\ \gamma_{02} \\ \gamma_{12} \\ \gamma_{03} \\ \gamma_{13} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$\cancel{F = (C\hat{\beta})^T (C(X^TX)^{-1}}$

$$F_{obs} = \frac{(\underset{\sim}{C}\hat{\beta})^T (\underset{\sim}{C}(X^T X)^{-1} \underset{\sim}{C}^T)^{-1} C\hat{\beta}}{2\hat{\sigma}^2}$$

Reject H_0 if $F_{obs} \geq F_{\alpha, r, \sum_{i=1}^{m}(n_i - p')}$

Can't reject $H_0 \Rightarrow \beta_{1g_1} = \beta_{1g_2} = 0$

~~Ho~~ Regression model

$$y_i = \beta_0 + \beta_{1g_1} * g_{1i} + \beta_{1g_2} * g_{2i} + \beta_1 \cdot X_i + e_i$$

$i = 1, \text{—}, n_1 + n_2 + n_3$

4 unknown parameters

ANCOVA

~~Group~~ ~~Model~~

$$y_i = \gamma_{0j} + \gamma X_{i} + e_i$$

$j = 1, 2, 3$

$i = 1, \text{—}, n_1$ for $j = 1$;

$i = n_1 + 1 \text{---} n_1 + n_2$ for $j = 2$;

$i = n_1 + n_2 + 1 \text{---} n_1 + n_2 + n_3$ for $j = 3$.

$$\underset{n\times 4}{X} = \begin{array}{c} \begin{pmatrix} 1 & 0 & 0 & x_1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & x_{n_1} \\ \hline 0 & 1 & 0 & x_{n_1+1} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & 0 & x_{n_1+n_2} \\ \hline 0 & 0 & 1 & x_{n_1+n_2+1} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 1 & x_{n_1+n_2+n_3} \end{pmatrix} \\ \begin{matrix} \gamma_{01} & \gamma_{02} & \gamma_{03} & \gamma \end{matrix} \end{array} \quad \begin{matrix} \text{group} = 1 \\ \\ \\ \text{group} = 2 \\ \\ \\ \text{group} = 3 \end{matrix}$$

model without intercept

$$\underset{4\times 4}{X^T X} = \left(\begin{array}{ccc|c} n_1 & 0 & 0 & \sum_{i=1}^{n_1} x_i \\ 0 & n_2 & 0 & \sum_{i=n_1+1}^{n_1+n_2} x_i \\ 0 & 0 & n_3 & \sum_{i=n_1+n_2}^{n_1+n_2+n_3} x_i \\ \hline & & & \end{array} \right)$$

$$\sum_{i=1}^{n_1+n_2+n_3} x_i^2$$

Regression model $\quad y_i = \boxed{\beta_0} + \beta_{g_1} * g_{1i} + \beta_{g_2} * g_{2i} + \beta_1 x_i + e_i$

$i = 1, \text{—}, n_1+n_2+n_3$

$\Rightarrow$ centred model

$$\Rightarrow y_i = \beta_0' + \beta_{g_1} * (g_{1i} - \bar{g}_1) + \beta_{g_2} * (g_{2i} - \bar{g}_2) +$$

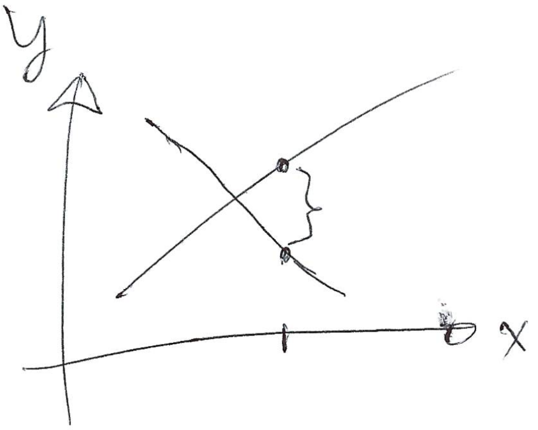
group effect $\quad \beta_1 (x_i - \bar{x}) + e_i \quad$ weight effect

Consider $H_0 : \beta_{g_1} = \beta_{g_2} = 0 \quad$ or $\quad H_0 : \beta_1 = 0$

Reject $H_0 \Rightarrow \beta_{1g_1} \neq 0, \ \beta_{1g_2} \neq 0$

Model

Group	Model
1	$\gamma_{01} + \gamma_{11} X_i + e_i$
2	$\gamma_{02} + \gamma_{12} X_i + e_i$
3	$\gamma_{03} + \gamma_{13} X_i + e_i$



$H_0 = \gamma_{11} = \gamma_{12}$