29 September 2020

2. **F test**

(b) **Distributions**

$$\begin{array}{ccccc} \dfrac{\text{Total S. S.}}{\sigma^2} & = & \dfrac{\text{Reg. S. S.}}{\sigma^2} & + & \dfrac{\text{Res. S. S.}}{\sigma^2} \\ \sim\ \chi^2\,(n-1,\lambda) & & \sim\ \chi^2\,(p,\ \lambda) & & \sim\ \chi^2(n-p') \end{array}$$

where $\lambda = \dfrac{1}{\sigma^2}\sum_{i=1}^{p}\sum_{j=1}^{p}\beta_i\beta_j S_{x_i,x_j}$.

(c) **All regression coefficients equal to zero**

For testing $H_0 : \beta_1 = \beta_2 = \ldots = \beta_p = 0$

Model with intercept (Intercept is unknown)

Source	Sum of squares (S.S.)	d.f.
Regression	$\hat\beta_1 \mathrm{S}_{x_1y} + \ldots + \hat\beta_p \mathrm{S}_{x_py}$	p
Residual	$\mathrm{S}_{yy} - \hat\beta_1 \mathrm{S}_{x_1y} - \ldots - \hat\beta_p \mathrm{S}_{x_py}$	$n-p' = n-(p+1)$
Total	$\sum_{i=1}^n (y_i-\bar y)^2 = \mathrm{S}_{yy}$	$n-1$

$$\mathrm{F} = \frac{RegS.S./p}{Res.S.S./(n-p')} = \frac{Reg.\,M.S.}{\hat\sigma^2} \sim \mathrm{F}(p,\,n-p') \quad \text{under } H_0$$

Reject H_0 is $\mathrm{F}_{\text{obs}} > \mathrm{F}_\alpha(p,\,n-p')$

Model without intercept (Intercept is known)

Source	Sum of squares (S.S.)	d.f.
Regression	$\hat\beta_1 \sum_{i=1}^n x_{i1}(y_i-\beta_0) + \ldots + \hat\beta_p \sum_{i=1}^n x_{ip}(y_i-\beta_0)$	p
Residual	$\sum_{i=1}^n (y_i-\beta_0)^2 - \hat\beta_1 \sum_{i=1}^n x_{i1}(y_i-\beta_0) - \ldots - \hat\beta_p \sum_{i=1}^n x_{ip}(y_i-\beta_0)$	$n-p$
Total	$\sum_{i=1}^n (y_i-\beta_0)^2$	n

$$\mathrm{F} = \frac{RegS.S./p}{Res.S.S./(n-p)} = \frac{Reg.\,M.S.}{\hat\sigma^2} \sim \mathrm{F}(p,\,n-p) \quad \text{under } H_0$$

Reject H_0 is $\mathrm{F}_{\text{obs}} > \mathrm{F}_\alpha(p,\,n-p)$

- For testing $H_0 : \beta_1 = \ldots = \beta_p = 0$, it is incorrect to test $H_0 : \beta_i = 0$ for $i = 1\ldots p$ separately.