4.2 F test.

4.2.1 All coefficients = 0.

① Partitioning total variability

$$TSS = RegSS + ResSS$$

$$\Sigma(y_i-\bar{y})^2 = \Sigma(\hat{y}_i-\bar{y})^2 + \Sigma(y_i-\hat{y}_i)^2$$

② Distributions.

$$\frac{ResSS}{\sigma^2} \sim \chi^2_{n-p'} \perp\!\!\!\perp \frac{RegSS}{\sigma^2} \sim \chi^2_{p,\lambda} \Rightarrow \frac{TSS}{\sigma^2} \sim \chi^2_{n-1,\lambda}, \quad \lambda = \frac{1}{\sigma^2}\sum_{i,j=1}^{p} \beta_i\beta_j S_{x_ix_j}$$

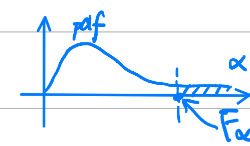
Oct. 16

③ Test.

H_0: $\beta_1 = \cdots = \beta_p = 0$.

Test stat: $F = \dfrac{Reg.SS/p}{RSS/(n-p-1)} \overset{H_0}{\sim} F(p, n-p-1)$

Reject H_0 if $F > F_\alpha(p, n-p-1)$. $P(F>F_\alpha)=\alpha$



ANOVA table:

Source	Sum of squares (S.S.)	d.f.	Mean Squares (M.S.)	F
Regression	Reg.S.S $= \sum_{i=1}^n(\hat{y}_i-\bar{y})^2$	p	SS_{reg}/p	$F = \dfrac{SS_{reg}/p}{SS_{res}/(n-p')} = \dfrac{MS_{reg}}{\hat{\sigma}^2}$
Residual	Res. S.S. $= \sum_{i=1}^n(y_i-\hat{y}_i)^2$	$n-p'$	$SS_{res}/(n-p')$	
Total	Total S.S. $= \sum_{i=1}^n(y_i-\bar{y})^2$	$n-1$		

4.2.2 Subset of regression coefficients

- Whether a reduced model (under H_0) is good enough.
- Partitioning reg.SS

$$\underset{\sim\sigma^2\chi^2(p,\lambda)}{Reg.SS|_F} = \underset{\sim\sigma^2\chi^2(p-r,\lambda_1)}{Reg.SS|_R} + \underset{\sim\sigma^2\chi^2(r,\lambda_2)}{\text{Increase in } Reg.SS}$$

r: # free parameters reduced by H_0.
known params specified by H_0.

H_0: $\beta_1=\beta_2=0$

H_0: $\beta_1=\beta_2$

- Test

H_0: a reduced model.

Test stat: $F = \dfrac{\text{Increase in Reg.SS}}{\hat{\sigma}^2} \overset{H_0}{\sim} F(r, n-p-1)$.

Reject H_0 if $F > F_\alpha(r, n-p-1)$.

4.2.3 Generalized linear hypothesis (including above as special cases)

H_0: $C\beta = d$

Test stat: $F = \dfrac{(C\hat{\beta}-d)^T [C(X^TX)^{-1}C^T]^{-1}(C\hat{\beta}-d)}{r\hat{\sigma}^2} \overset{H_0}{\sim} F(r, n-p-1)$, $r = \text{rank}(C)$.

Reject H_0 if $F > F_\alpha(r, n-p-1)$