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Increase in Reg.S.S.

Distribution

After expressing Increase in $Reg.S.S.$ as the quadratic form of $\underset{\sim}{Y}$, i.e., $\underset{\sim}{Y}^T \underset{\sim}{A} \underset{\sim}{Y}$, it can be shown that Increase in $Reg.S.S. \sim \sigma^2\chi^2(r, \lambda_2)$, where

$$r = \text{number of unknown parameters in the full model} - \text{number of unknown parameters in the reduced model}$$

$$= p\,(\text{full}) - p\,(\text{reduced})$$

$$\lambda_2 = \text{non-centrality constant of Reg.S.S. (full) - non-centrality constant of Reg.S.S. (reduced)}$$

$$= \lambda - \lambda_1$$

$$= \frac{1}{\sigma^2}\left(\sum_{i=1}^{p}\sum_{j=1}^{p}\beta_i\beta_j S_{x_i,x_j} \; - \; \sum_{l}\sum_{m}\beta_l\beta_m S_{x_m,x_n}\right)$$

where x_l, x_m are the independent variables in the reduced model. Detailed proof is given in the file of "IncSS_a.pdf".

Note that: The discussion below is true only if Total S.S. for full model is equal to Total S.S. for reduced model (i.e., the model under H_0).

Test statistic

$$\mathrm{F} = \frac{\text{Increase in } Reg.S.S./r}{Res.S.S./(n-p')} = \frac{\text{Increase in } Reg.S.S./r}{\hat{\sigma}^2} \sim \mathrm{F}(r,\, n-p') \quad \text{under } H_0$$

$$E(F) \;\approx\; \frac{E(\text{Increase in } Reg.S.S./k)}{E(Res.S.S./(n-p'))} \;=\; 1 + \frac{\lambda_2}{r} > 1 \text{ if the null hypothesis is not true.}$$

Reject H_0 is $\mathrm{F}_{\text{obs}} > \mathrm{F}_\alpha(r,\, n-p')$

Calculation

$$\text{Increasing in } Reg\,S.S. = Reg\,S.S.|_{\text{full}} - Reg\,S.S.|_{\text{reduced}}$$

$$= Res\,S.S.|_{\text{reduced}} - Res\,S.S.|_{\text{full}}$$

if Total S.S. for full model = Total S.S. for reduced model.

Remarks

- For two-sided alternative
- Total S.S. for full model must be equal to Total S.S. for reduced model (i.e., the model under H_0)
- This test statistic is invalid if the null hypothesis involves the intercept or the right-hand side constant is a non-zero value.

General linear hypothesis

$H_0 : \underset{\sim}{C}\underset{\sim}{\beta} = \underset{\sim}{d}$

Under H_0,

$$F = \frac{(\underset{\sim}{C}\hat{\underset{\sim}{\beta}} - \underset{\sim}{d})^T [\underset{\sim}{C}(\underset{\sim}{X}^T \underset{\sim}{X})^{-1} \underset{\sim}{C}^T]^{-1} (\underset{\sim}{C}\hat{\underset{\sim}{\beta}} - \underset{\sim}{d})}{r\hat{\sigma}^2} \sim \mathrm{F}_{(r, n-p')}$$

where r is the rank of $\underset{\sim}{C}$. Reject H_0 is $\mathrm{F}_{\mathrm{obs}} > \mathrm{F}_{\alpha}(r,\, n - p')$