27 October

Source of Variation of	Sum of Squares	Degrees of freedom	Mean Square	Computed f
Model	$\sum_{i=1}^{m} n_i(\bar{y}_{i.} - \bar{y}_{..})^2$	$m-1$	$\frac{\sum_{i=1}^{m} n_i(\bar{y}_{i.}-\bar{y}_{..})^2}{m-1}$	$\frac{(\sum_{i=1}^{m} n_i - m)\sum_{i=1}^{m} n_i(\bar{y}_{i.}-\bar{y}_{..})^2}{(m-1)\sum_{i=1}^{m}\sum_{j=1}^{n_i}(y_{ij}-\bar{y}_{i.})^2}$
Error	$\sum_{i=1}^{m}\sum_{j=1}^{n_i}(y_{ij} - \bar{y}_{i.})^2$	$\sum_{i=1}^{m} n_i - m$	$\frac{\sum_{i=1}^{m}\sum_{j=1}^{n_i}(y_{ij}-\bar{y}_{i.})^2}{\sum_{i=1}^{m} n_i - m}$	
Total	$\sum_{i=1}^{m}\sum_{j=1}^{n_i}(y_{ij} - \bar{y}_{..})^2$	$\sum_{i=1}^{m} n_i - 1$		

The advantages of choosing equal sample sizes over the choice of unequal sample sizes are: 1) the f ratio is insensitive to slight departures from the assumption of equal variances for the m populations when the sample are of equal sizes; and 2) the choice of equal sample size minimizes the probability of committing a type II error.

Example

	Group					
	1	2	3	4	5	
	551	595	639	417	563	
	457	580	615	449	631	
	450	508	511	517	522	
	731	583	573	438	613	
	499	633	648	415	656	
	632	517	677	555	679	
Total	3320	3416	3663	2791	3664	16854
Mean	553.33	569.33	610.50	465.19	610.67	561.80

$\mu_4 < \frac{\mu_1 + \mu_2 + \mu_3 + \mu_5}{4}$?

Solution

$S_1^2 = 12,133.8667$, $S_2^2 = 2,302.6667$, $S_3^2 = 3593.5$, $S_4^2 = 3,318.5667$, $S_5^2 = 3,455.4667$, $S_T^2 = 7,219.8897$

Source of Variation of	Sum of Squares	Degrees of freedom	Mean Square	Computed f
Group	85,356	4	21,339	4.30
Error	124,021	25	4,961	
Total	209,377	29		

Reject Ho! Why?

The critical value $f_{0.05}(4, 25) = 2.76$. Thus, $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$ is rejected.

1

$$H_0: \frac{\mu_1+\mu_2+\mu_3+\mu_5}{4} = \mu_4$$

Define

$$\underbrace{W}_{\text{contrast}} = \mu_1+\mu_2+\mu_3-4\mu_4+\mu_5$$

$\Rightarrow H_0: W = 0$

Contrast

$W = \sum_{i=1}^{m} c_i \mu_i$ where $\sum_{i=1}^{m} c_i = 0$ e.g. $c_1=1, c_2=1, c_3=1, c_4=-4, c_5=1$

pt. est. $\hat{W} = \sum_{i=1}^{m} c_i \hat{\mu}_i = \sum_{i=1}^{m} c_i \bar{y}_{i\cdot}$ $\leftarrow N(\mu_i, \frac{\sigma^2}{n_i})$

dist. of $\hat{W} \sim N\left(\sum_{i=1}^{m} c_i \mu_i, \underbrace{\sum_{i=1}^{m} c_i^2 \frac{\sigma^2}{n_i}}_{= \sigma^2 \sum_{i=1}^{m} \frac{c_i^2}{n_i}}\right)$

$$\Rightarrow \frac{\sum_{i=1}^{m} c_i \hat{\mu}_i - \sum_{i=1}^{m} c_i \mu_i}{\sigma \sqrt{\sum_{i=1}^{m} \frac{c_i^2}{n_i}}} \sim N(0,1)$$

Under H_0 $\Rightarrow$ $$\frac{\sum_{i=1}^{m} c_i \hat{\mu}_i - 0}{\sigma \sqrt{\sum_{i=1}^{m} \frac{c_i^2}{n_i}}} \sim N(0,1)$$

$$\Rightarrow \left(\frac{\sum_{i=1}^{m} c_i \hat{\mu}_i}{\sigma \sqrt{\sum_{i=1}^{m} \frac{c_i^2}{n_i}}}\right)^2 \sim \chi^2_{(1)}$$

$$\Rightarrow \frac{\dfrac{\left(\sum_{i=1}^{m} c_i \hat{\mu}_i\right)^2}{\cancel{\sigma^2}\left(\sum_{i=1}^{m} \frac{c_i^2}{n_i}\right)} \Big/ 1}{\dfrac{\text{Res S.S.}}{\cancel{\sigma^2}} \Big/ \sum_{i=1}^{m}(n_i-1)} \sim F\left(1, \sum_{i=1}^{m}(n_i-1)\right)$$

indep.

$\frac{\text{Res S.S.}}{\sigma^2} \sim \chi^2\left(\sum_{i=1}^{m} n_i - m\right)$

$\sum_{i=1}^{m} n_i - m = \sum_{i=1}^{m}(n_i-1)$

$\frac{\text{Res S.S.}}{\sum_{i=1}^{m}(n_i-1)} = \hat{\sigma}^2$

2

$$\Rightarrow \frac{\boxed{\left(\sum_{i=1}^{m} c_i \hat{\mu}_i\right)^2}}{\hat{\sigma}^2 \boxed{\sum_{i=1}^{m} \frac{c_i^2}{n_i}}} \sim F\left(1, \sum_{i=1}^{m}(n_i - 1)\right)$$

s.s. for contrast

Reg S.S. = $SSW_1 + SSW_2 + SSW_3 + SSW_4$

provided that W_1, W_2, W_3, W_4 are orthogonal

Now, $W_1 = \mu_1 + \mu_2 + \mu_3 - 4\mu_4 + \mu_5$

Orthogonal contrast $\quad W_1 = \sum_{i=1}^{m} b_i \mu_i \qquad \sum_{i=1}^{m} b_i = 0$

$W_2 = \sum_{i=1}^{m} c_i \mu_i \qquad \sum_{i=1}^{m} c_i = 0$

Orthogonal if $\sum_{i=1}^{m} \frac{b_i c_i}{n_i} = 0$

if $n_1 = n_2 = \cdots = n_m = n \Rightarrow \sum_{i=1}^{m} b_i c_i = 0$

$W_1 = \mu_1 + \mu_2 + \mu_3 - 4\mu_4 + \mu_5 \rightarrow$

$W_2 = \mu_1 + \mu_2 - (\mu_3 + \mu_5)$

$$\begin{array}{ccccccc} & & \mu_1 & \mu_2 & \mu_3 & \mu_4 & \mu_5 \\ \underset{\sim}{b} & = & (1, & 1, & 1, & -4, & 1) \\ \underset{\sim}{c} & = & (1, & 1, & -1, & 0, & -1) \\ & & 1 & 1 & -1 & 0 & -1 \end{array}$$

$W_3 = \mu_1 - \mu_2$

$W_4 = \mu_3 - \mu_5$

$$\text{S.S. for contrast} = \frac{\left(\sum_{i=1}^{m} c_i \hat{\mu}_i\right)^2}{\sum_{i=1}^{m} \frac{c_i^2}{n_i}}$$

Source of Variation of	Sum of Squares	Degrees of freedom	Mean Square	Computed f
Model	$\sum_{i=1}^{m} n_i(\bar{y}_{i.} - \bar{y}_{..})^2$	$m-1$	$\frac{\sum_{i=1}^{m} n_i(\bar{y}_{i.}-\bar{y}_{..})^2}{m-1}$	$\frac{(\sum_{i=1}^{m} n_i - m)\sum_{i=1}^{m} n_i(\bar{y}_{i.}-\bar{y}_{..})^2}{(m-1)\sum_{i=1}^{m}\sum_{j=1}^{n_i}(y_{ij}-\bar{y}_{i.})^2}$
Error	$\sum_{i=1}^{m}\sum_{j=1}^{n_i}(y_{ij} - \bar{y}_{i.})^2$	$\sum_{i=1}^{m} n_i - m$	$\frac{\sum_{i=1}^{m}\sum_{j=1}^{n_i}(y_{ij}-\bar{y}_{i.})^2}{\sum_{i=1}^{m} n_i - m}$	
Total	$\sum_{i=1}^{m}\sum_{j=1}^{n_i}(y_{ij} - \bar{y}_{..})^2$	$\sum_{i=1}^{m} n_i - 1$		

The advantages of choosing equal sample sizes over the choice of unequal sample sizes are: 1) the f ratio is insensitive to slight departures from the assumption of equal variances for the m populations when the sample are of equal sizes; and 2) the choice of equal sample size minimizes the probability of committing a type II error.

Example

	Group					
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	450	508	511	517	522	
	731	583	573	438	613	
	499	633	648	415	656	
	632	517	677	555	679	
Total	3320	3416	3663	2791	3664	16854
	$\bar{y}_{1.}$	$\bar{y}_{2.}$	$\bar{y}_{3.}$	$\bar{y}_{4.}$	$\bar{y}_{5.}$	
Mean	553.33	569.33	610.50	465.19	610.67	561.80

? ?

Solution

$S_1^2 = 12,133.8667$, $S_2^2 = 2,302.6667$, $S_3^2 = 3593.5$, $S_4^2 = 3,318.5667$, $S_5^2 = 3,455.4667$, $S_T^2 = 7,219.8897$

$SSW_2 + SSW_3 + SSW_4 = \text{Reg } S.S. - SSW_1 = 15,321$

Source of Variation of	Sum of Squares	Degrees of freedom	Mean Square	Computed f
Group	85,356	4	21,339	4.30
$\boxed{\mu_1+\mu_2+\mu_3-4\mu_4+\mu_5 = W_1}$	70,035	1 $4-1=3$	70,035	$\frac{70,035}{4,961} = 14.12 > F_{0.05,1,25} = 4.24$
Error	124,021	25	4,961	
Total	209,377	29		

The critical value $f_{0.05}(4, 25) = 2.76$. Thus, $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$ is rejected.

Mean Square for $(W_2, W_3, W_4) = \frac{15,321}{3} = 5,107$

$c_1 = 1, c_2 = 1, c_3 = 1, c_4 = -4, c_5 = 1$

S.S. for contrast

$$= \frac{\left(\sum_{i=1}^{5} c_i \hat{\mu}_i\right)^2}{\sum_{i=1}^{5} \frac{c_i^2}{n_i}} = \frac{\left(\sum_{i=1}^{5} c_i \bar{y}_{i.}\right)^2}{\sum_{i=1}^{5} \frac{c_i^2}{n_i}}$$

$F = \frac{5107}{4961} = 1.xxxx < F_{0.05,3,25}$

Can't reject Ho ⇒ W_2, W_3 & W_4 are not significant

4

$$\Rightarrow \frac{\boxed{\left(\sum_{i=1}^{m} c_i \hat{\mu}_i\right)^2}}{\hat{\sigma}^2 \boxed{\sum_{i=1}^{m} \frac{c_i^2}{n_i}}} \sim F\left(1, \sum_{i=1}^{m}(n_i - 1)\right)$$

s.s. for contrast

~~Reg S.S. = $SSW_1 + SSW_2 + SSW_3 + SSW_4$~~

~~provided that W_1, W_2, W_3, W_4 are orthogonal~~

~~Now, $W_1 = \mu_1 + \mu_2 + \mu_3 - 4\mu_4 + \mu_5$~~

---

orthogonal contrast $\quad W_1 = \sum_{i=1}^{m} b_i \mu_i \qquad \sum_{i=1}^{m} b_i = 0$

$$W_2 = \sum_{i=1}^{m} c_i \mu_i \qquad \sum_{i=1}^{m} c_i = 0$$

Orthogonal if $\sum_{i=1}^{m} \frac{b_i c_i}{n_i} = 0$

If $n_1 = n_2 = \cdots = n_m = n \Rightarrow \sum_{i=1}^{m} b_i c_i = 0$

$$\begin{array}{ll} & \quad\ \ \mu_1 \quad \mu_2 \quad \mu_3 \quad \mu_4 \quad \mu_5 \\ W_1 = \mu_1 + \mu_2 + \mu_3 - 4\mu_4 + \mu_5 \longrightarrow & \underset{\sim}{b} = (1 \quad 1 \quad 1 \quad -4 \quad 1) \\ W_2 = \mu_1 + \mu_2 - (\mu_3 + \mu_5) & \underset{\sim}{c} = (1 \quad 1 \quad -1 \quad 0 \quad -1) \\ & \quad\ \ 1 \quad\ 1 \quad -1 \quad\ 0 \quad -1 \end{array}$$

$W_3 = \mu_1 - \mu_2$

$W_4 = \mu_3 - \mu_5$

$$\text{S.S. for contrast} = \frac{\left(\sum_{i=1}^{m} c_i \hat{\mu}_i\right)^2}{\sum_{i=1}^{m} \frac{c_i^2}{n_i}}$$

If ω_1 and ω_2 are orthogonal, then the quantities SSW_1 and SSW_2 are components of SSA (i.e., S.S. for group in our example), each with a single degree of freedom. The treatment sum of squares with $m-1$ degrees of freedom can be partitioned into at most $m-1$ independent single-degree-of-freedom contrast sum of squares satisfying the identity

$$SSA = SSW_1 + SSW_2 + \ldots + SSW_{m-1}$$

if the contrasts are orthogonal to each other.

Example

Find the contrast sum of squares corresponding to the orthogonal contrasts

$$\omega_1 = \mu_1 + \mu_2 + \mu_3 + \mu_5 - 4\mu_4$$

$$\omega_2 = \mu_1 + \mu_2 - \mu_3 - \mu_5$$

and carry out appropriate tests of significance.

Solution

One can write down two additional contrasts orthogonal to the first two such as

$$\omega_3 = \mu_1 - \mu_2$$

$$\omega_4 = \mu_3 - \mu_5$$

Source of Variation of	Sum of Squares	Degrees of freedom	Mean Square	Computed f
Groups	85,356	4	21,339	4.30
$\mu_1+\mu_2+\mu_3-4\mu_4+\mu_5 = W_1 =$ (1,2,3,5) vs 4	70,035	1	70,035	14.12
$= W_2 =$ (1,2) vs (3,5)	14,553	1	14,553	2.93 $\simeq$
$\mu_1+\mu_2-\mu_3-\mu_5 = W_3 =$ (1) vs (2)	768	1	768	0.15
$\mu_1-\mu_2$ $W_4 =$(3) vs (5)	0.08	1	0.08	0.00
Error	124,021	25	4,961	
Total	209,377	29		

$\mu_3-\mu_5$

$\frac{14,553}{4,961} < 4.24$

Can't reject Ho

The contrast ω_1 is not significant when compared to the critical value $\boxed{f_{0.05}(1, 25) = 4.24.}$ However, the f value of 14.12 for ω_2 is significant and the hypothesis

$$H_0 : \mu_1 + \mu_2 + \mu_3 + \mu_5 = 4\mu_4$$

is rejected.

Reject Ho: $\mu_1 = \mu_2 = \cdots = \mu_m = \mu$

1. Find the most significant contrast
2. Test whether the remaining S.S. is significant

If no, STOP

If yes, CONTINUE

Two-way ANOVA

two categoral varables e.g. "method" ~~on~~ 1, 2 ⇒ one dummy m_1

"variety" 1, 2, 3 ⇒ two dummy v_1, v_2

Model I Regression model

$\boxed{\alpha_2 * m_{2,k}}$ $\boxed{\beta_3 * v_{3,k}}$

$$y_k = \beta_0 + \alpha_1 * m_{1,k} + \beta_1 * v_{1,k} + \beta_2 * v_{2,k} + e_k \qquad k = 1, \dots, \sum_{i=1}^{a}\sum_{j=1}^{b} n_{ij}$$

level of method

level of variety

$\gamma_{11} * m_{1,k} * v_{1,k}$

$+ \gamma_{12} * m_{1,k} * v_{2,k}$

interaction terms

Method	Variety	$E(y_k)$
1	1	$\beta_0 + \alpha_1 + \beta_1$
	2	$\beta_0 + \alpha_1 + \beta_2$
	3	$\beta_0 + \alpha_1$
2	1	$\beta_0 + \beta_1$
	2	$\beta_0 + \beta_2$
	3	β_0

$+\gamma_{11}$ $+\gamma_{12}$ $\alpha_1 + \gamma_{11}$ $\alpha_1 + \gamma_{12}$ α_1

⇒ Not a general model

$\mu_{11} - \mu_{21} = \mu_{12} - \mu_{22} = \mu_{13} - \mu_{23}$ *

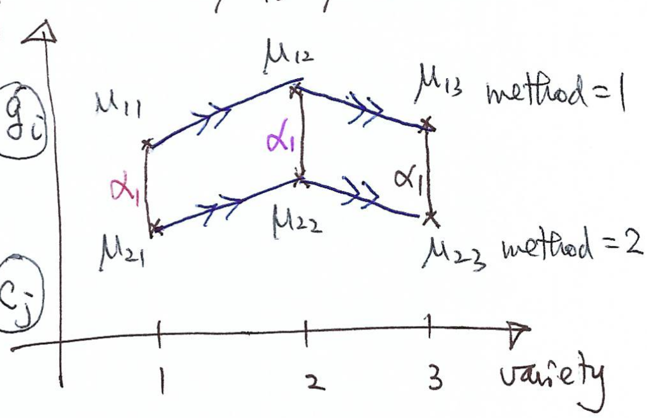
In general

Categoral varable 1 (Factor A)

a levels ⇒ (a−1) dummy g_i

Categoral varable 2 (Factor B)

b levels ⇒ (b−1) dummy c_j



Parallel curves

$$y_k = \beta_0 + \sum_{i=1}^{a-1} \alpha_i g_{i,k} + \sum_{j=1}^{b-1} \beta_j c_{j,k} + \sum_{i=1}^{a-1}\sum_{j=1}^{b-1} \gamma_{ij} * g_{i,k} * c_{j,k} + e_k .$$

for $i = 1, \dots, a-1$

$j = 1, \dots, b-1$

$k = 1, \dots, \sum_{i=1}^{a}\sum_{j=1}^{b} n_{ij}$

$g_{a,k} = 0$ — (a−1) terms ~~columns~~ columns in $\underset{\sim}{X}$

$c_{b,k} = 0$ — (b−1) terms columns in $\underset{\sim}{X}$

interaction terms between two categoral varables

~~Ex~~ Example in p.11 $\quad a = 2, \ b = 3$

$\Rightarrow \alpha_1, \beta_1, \beta_2, \gamma_{11}, \gamma_{12}$

Model II (ANOVA table)

~~If~~ For ~~i, j~~ fixed i, j

$$y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + e_{ijk} \qquad i = 1, \ldots, a; \quad j = 1, \ldots, b; \quad k = 1, \ldots, n_{ij}$$

$\alpha_a = 0$ $\quad \beta_b = 0$ $\quad \gamma_{aj} = 0$ $\quad \gamma_{ib} = 0$

e.g

Method	Variety	$E(y)$	$\Rightarrow$ $E(y)$
1	1	$\mu + \alpha_1 + \beta_1 + \gamma_{11}$	μ_{11}
	2	$\mu + \alpha_1 + \beta_2 + \gamma_{12}$	μ_{12}
	3	$\mu + \alpha_1$	μ_{13}
2	1	$\mu + \beta_1$	μ_{21}
	2	$\mu + \beta_2$	μ_{22}
	3	μ	μ_{23}

New categorical variable with 6 levels (11, 12, 13, 21, 22, 23)

$\mu, \alpha_1, \beta_1, \beta_2, \gamma_{11}, \gamma_{12}$

6 unknown parameters

6 unknown para.

$\Rightarrow$ In ~~gen~~ general,

$$y_{ijk} = \mu_{ij} + e_{ijk} \qquad i = 1, \ldots, a; \quad j = 1, \ldots, b; \quad k = 1, \ldots, n_{ij}$$

$$\underset{\sim}{\beta} = \begin{pmatrix} \mu_{11} \\ \mu_{12} \\ \mu_{13} \\ \mu_{21} \\ \mu_{22} \\ \mu_{23} \end{pmatrix} \qquad \underset{n \times 6}{\underset{\sim}{X}} \Rightarrow \underset{6 \times 6}{\underset{\sim}{X}^T \underset{\sim}{X}} = \begin{pmatrix} n_{11} & & & & & \\ & n_{12} & & & \underset{\sim}{0} & \\ & & n_{13} & & & \\ & & & n_{21} & & \\ & \underset{\sim}{0} & & & n_{22} & \\ & & & & & n_{23} \end{pmatrix}$$

$$(\underset{\sim}{X}^T \underset{\sim}{X})^{-1} = \begin{pmatrix} \frac{1}{n_{11}} & & & & & \\ & \frac{1}{n_{12}} & & & \underset{\sim}{0} & \\ & & \frac{1}{n_{13}} & & & \\ & & & \frac{1}{n_{21}} & & \\ & \underset{\sim}{0} & & & \frac{1}{n_{22}} & \\ & & & & & \frac{1}{n_{23}} \end{pmatrix}$$

$$\underset{\sim}{X}^T \underset{\sim}{Y} = \begin{pmatrix} \sum_{k=1}^{n_{11}} y_{11k} \\ \sum_{k=1}^{n_{12}} y_{12k} \\ \vdots \\ \sum_{k=1}^{n_{23}} y_{23k} \end{pmatrix} \Rightarrow \hat{\mu}_{ij} = \bar{y}_{ij.}$$

$$Var(\hat{\mu}_{ij}) = Var(\bar{y}_{ij.}) = \frac{\sigma^2}{n_{ij}}$$

$$Cov(\hat{\mu}_{ij}, \hat{\mu}_{kl}) = 0$$

$$Res\ S.S. = \underset{\sim}{Y}^T \underset{\sim}{Y} - \underset{\sim}{\beta}^T \underset{\sim}{X}^T \underset{\sim}{Y}$$

$$\sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{n_{ij}} y_{ijk}^2 \qquad (\bar{y}_{11.}, \bar{y}_{12.} \ \dots \ \bar{y}_{23.}) \begin{pmatrix} \sum_{k=1}^{n_{11}} y_{11k} \\ \sum_{k=1}^{n_{12}} y_{12k} \\ \vdots \\ \sum_{k=1}^{n_{23}} y_{23k} \end{pmatrix}$$

$i=1, j=1$

$$\sum_{k=1}^{n_{11}} y_{11k}^2 - \bar{y}_{11.} \sum_{k=1}^{n_{11}} y_{11k} \longrightarrow n_{11} \bar{y}_{11.}$$

$$= \sum_{k=1}^{n_{11}} (y_{11k} - \bar{y}_{11.})^2$$

$$Res\ S.S. = \sum_{i=1}^{a} \sum_{j=1}^{b} \boxed{\sum_{k=1}^{n_{ij}} (y_{ijk} - \bar{y}_{ij.})^2} / \sigma^2 \sim \chi^2(n_{ij}-1)$$

$$d.f. = \sum_{i=1}^{a} \sum_{j=1}^{b} (n_{ij} - 1)$$

$$= \sum_{i=1}^{a} \sum_{j=1}^{b} n_{ij} - a * b$$

$$= N - ab$$

e.g. $a = 2,\ b = 3,\ n_{ij} = 6$

$$\Rightarrow d.f. = 30$$

Example

Method	Variety 1	Variety 2	Variety 3	Sum	CSS
1	22.3	19.8	20		
	25.8	28.3	17		
	22.8	26.8	24		
	28.3	27.3	22.5		
	21.3	26.8	28		
	18.3	26.8	22.5		
Sum	138.8	155.8	134	428.6	
Corrected S.S.	61.333333	47.333333	68.833333	221.237778	
2	16.4	24.5	11.8		
	14.4	16	14.3		
	21.4	11	21.3		
	19.9	7.5	6.3		
	10.4	14.5	7.8		
	21.4	15.5	13.8		
Sum	103.9	89	75.3	268.2	
Corrected S.S.	97.208333	163.833333	143.375	472.62	
Sum	242.7	244.8	209.3	696.8	
Corrected S.S.	260.0425	583.02	499.349167	1408.53	

$\sum_{k=1}^{6}(y_{11k}-\bar{y}_{11.})^2$

Res S.S. $= \sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{n_{ij}}(y_{ijk}-\bar{y}_{ij.})^2$

$= 61.333333 + 47.333333 + \cdots + 143.375$

$= 581.916667$

d.f. $= 30$

$\Rightarrow \hat{\sigma}^2 = \frac{\text{Res S.S.}}{\text{d.f}} = 19.397222$

Source of Variation of	Sum of Squares	Degrees of freedom	Mean Square	Computed f
Method	714.671111	1	714.671111	36.84
Variety	66.117222	2	33.058611	1.71
Interaction	45.823889	2	22.911944	1.18
Error	581.916667	30	19.397222	
Total	1408.528889	35		

Test "interaction" effect is equivalent to test $H_0 : \mu_{11} - \mu_{21} = \mu_{12} - \mu_{22} = \mu_{13} - \mu_{23}$.

As the interaction terms are not significant, we re-construct the ANOVA table.

– $(1-\alpha)100\%$ C.I. for μ_{ij} : $\bar{y}_{ij\cdot} \pm t_{\alpha/2,\ \sum_{i=1}^{a}\sum_{j=1}^{b}(n_{ij}-1)} \dfrac{\hat{\sigma}}{\sqrt{n_{ij}}}$

– $H_0 : \mu_{ij} = \mu_{ij0}$

$$t_{obs} = \frac{\bar{y}_{ij\cdot} - \mu_{ij}}{\hat{\sigma}/\sqrt{n_{ij}}}$$

Reject H_0 if $|t_{obs}| > t_{\alpha/2,\ \sum_{i=1}^{a}\sum_{j=1}^{b}(n_{ij}-1)}$

– $H_0 : \mu_{ij} = \mu$ for i, j.

Model under H_0 : $y_{ijk} = \mu + e_{ijk}$ (ANOVA model)

$y_k = \beta_0 + e_k$ (Regression model)

$$\Updownarrow$$

$$H_0 : \underbrace{\alpha_i = 0,\ \beta_j = 0,\ \gamma_{ij} = 0}_{\text{all reg. coeff} = 0}$$

From Chapter 1, Section 4

$$F_{obs} = \frac{\text{Reg S.S.}/\text{df reg s.s}}{\text{Res S.S.}/\text{df res s.s.}} = \frac{\text{Reg S.S} / \text{d.f. reg s.s}}{\hat{\sigma}^2}$$

d.f. of total S.S. − d.f. of Res S.S.

$= (N-1) - (N-ab)$

$ab - 1$

total S.S. − Res S.S.

$$\hat{\sigma}^2 = \frac{\text{Res S.S.}}{\sum_{i=1}^{a}\sum_{j=1}^{b}(n_{ij}-1)}$$

Reject H_0 if $F_{obs} > F_{\alpha,\ ab-1,\ N-ab}$

– Can't reject H_0 STOP!

– Reject H_0 $\Rightarrow$ continue to do analysis

Example

	Variety				
Method	1	2	3	Sum	CSS
1	22.3	19.8	20		
	25.8	28.3	17		
	22.8	26.8	24		
	28.3	27.3	22.5		
	21.3	26.8	28		
	18.3	26.8	22.5		
Sum	138.8	155.8	134	428.6	
Corrected S.S.	61.333333	47.333333	68.833333	221.237778	
2	16.4	24.5	11.8		
	14.4	16	14.3		
	21.4	11	21.3		
	19.9	7.5	6.3		
	10.4	14.5	7.8		
	21.4	15.5	13.8		
Sum	103.9	89	75.3	268.2	
Corrected S.S.	97.208333	163.833333	143.375	472.62	
Sum	242.7	244.8	209.3	696.8	
Corrected S.S.	260.0425	583.02	499.349167	$\boxed{1408.53}$ ← total S.S.	

$$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n_{ij}}(y_{ijk}-\bar{y}_{...})^2$$

overall sample mean

Source of Variation of	Sum of Squares	Degrees of freedom	Mean Square	Computed f
Method	714.671111	1	714.671111	36.84
Variety	66.117222	2	33.058611	1.71
Interaction	45.823889	2	22.911944	1.18
Error	581.916667	30	19.397222	
Total	1408.528889	35		

Test "interaction" effect is equivalent to test $H_0 : \mu_{11} - \mu_{21} = \mu_{12} - \mu_{22} = \mu_{13} - \mu_{23}$.

As the interaction terms are not significant, we re-construct the ANOVA table.

Total S.S. = 1408.53 $\quad N-1 = 35$

Res S.S. = 581.916667 $\quad N-ab = 30$

⇒ Reg. SS. = 826.612222 $\quad$ diff. = 5

$$F = \frac{826.612222/5}{19.397222} = 8.522996 > F_{0.05,5,30}$$

Reject $H_0 : \mu_{ij} = \mu$ for all i, j.