MATH 3424 Tutorial

October 16, 2020

1   Review

Chapter 1 Sec 4.2 F test.

2   Exercises

1. Using the following data set

x_1	x_2	y_1	x_1	x_2	y_1
2	-2	2	1	-1	2
0	1	3	1	3	5
2	3	7	3	3	11
1	4	6	0	5	6
1	6	9	1	8	11

with summary statistics:

Overall

$n = 10,$ $\qquad \sum_{i=1}^{10} x_{i1} = 12,$ $\qquad \sum_{i=1}^{10} x_{i2} = 30,$ $\qquad \sum_{i=1}^{10} y_i = 62,$

$\sum_{i=1}^{10} x_{i1}^2 = 22,$ $\qquad \sum_{i=1}^{10} x_{i1}x_{i2} = 31,$ $\qquad \sum_{i=1}^{10} x_{i2}^2 = 174,$ $\qquad \sum_{i=1}^{10} x_{i1}y_i = 84,$

$\sum_{i=1}^{10} x_{i2}y_i = 262,$ $\qquad \sum_{i=1}^{10} y_i^2 = 486,$

$S_{x_1x_1} = 7.6000,$ $\qquad S_{x_1x_2} = -5.0000,$ $\qquad S_{x_2x_2} = 84.0000,$ $\qquad S_{x_1y} = 9.6,$

$S_{x_2y} = 76.0000,$ $\qquad S_{yy} = 101.6000.$

and

$$\begin{pmatrix} 7.6000 & -5.0000 \\ -5.0000 & 84.0000 \end{pmatrix}^{-1} = \begin{pmatrix} 0.136942 & 0.008151 \\ 0.008151 & 0.012390 \end{pmatrix},$$

When β_0, β_1 and β_2 are all unknown, to fit the following model

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + e_i, \qquad e_i \sim_{iid} N(0, \sigma^2)$$

Note that estimations of **centered model** are $\hat{\beta}_0' = 6.20000$, $\hat{\beta}_1' = 1.93414$, $\hat{\beta}_2' = 1.01989$
Give **all** your answers in **4** decimal points.

(a) Find the Regression Sum of Squares, Residual Sum of Squares, Total Sum of Squares and the unbiased estimate of the unknown parameter σ^2

(b) Fill the ANOVA table for H_0: $\beta_1 = \beta_2 = 0$ at significance level $\alpha = 0.05$. Write down your conclusion clearly.

Source	Sum of Squares	D.F.	Mean Squares	F value
Regression				
Residual				-
Total			-	-

(c) Test the hypothesis $H_0 : \beta_1 = 1.5$ *vs* $H_1 : \beta_1 > 1.5$ at the significant level of $\alpha = 0.05$

(d) Find a 95% confidence interval of β_0.

(e) Test the hull hypothesis $H_0 : \beta_1 = \beta_2$ vs $H_\alpha : \beta_1 \neq \beta_2$ at the significant level of $\alpha = 0.05$.

    i. t-test. Write down the test statistic, the critical value and your conclusion clearly.

ii. F test for testing $H_0 : \underset{\sim}{C}\beta = \underset{\sim}{d}$. Write down the test statistic, the critical value and your conclusion clearly.

iii. F test in terms of "Increase in Regression Sum of Squares". Write down the test statistic, the critical value and your conclusion clearly.