Summary

8 Sept 2020

Model

$y_i = \beta_0 + \beta_1 x_{i1} + \ldots + \beta_p x_{ip} + e_i \quad$ for $i = 1, ..., n$

Or,

$\underset{\sim}{Y} = \underset{\sim}{X}\,\underset{\sim}{\beta} + \underset{\sim}{e}$

where $\underset{\sim}{Y}$ is a $n \times 1$ column vector;

$\underset{\sim}{X}$ is a $n \times p'$ matrix, called "Design Matrix";

$\underset{\sim}{\beta}$ is a $p' \times 1$ column vector;

n is the number of observations

$p' (= p + 1)$ is the number of unknown parameters in the model.

Assumptions

$$\left.\begin{array}{lll} (1) & \mathrm{E}(e_i) = 0 & \\ (2) & \mathrm{Var}(e_i) = \sigma^2 & \\ (3) & \mathrm{Cov}(e_i, e_j) = 0 & (\text{for } i \neq j) \\ (4) & e_i \sim N & \end{array}\right\} \quad e_i \quad \underset{\sim}{iid} \quad N(0, \sigma^2)$$

Or,

$$\underset{\sim}{e} \sim MN(\underset{\sim}{0}, \sigma^2 \underset{\sim}{I})$$

$$\Rightarrow \quad \underset{\sim}{Y} \sim MN(\underset{\sim}{X}\underset{\sim}{\beta}, \sigma^2 \underset{\sim}{I})$$

Method of least squares

$\underline{p = 1}$

Find $\hat{\beta}_0, \hat{\beta}_1$ such that $\sum_{i=1}^{n} \hat{e}_i^2$ is minimized, $\sum_{i=1}^{n} \hat{e}_i^2 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$, where

- $\hat{e}_i$ – define: Residual

- $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_{i1} \quad$ for $i = 1, ..., n$ – define: Fitted value