MATH 3424 Tutorial

October 30, 2020

1 Review

Chapter 2 Sec 1 one categorical variable.

2 Exercises

Using the following data set

group									number	mean	variance
A_1	1600	1610	1650	1680	1700	1700	1780		n_1 7	1674.286	3795.238
A_2	1500	1640	1400	1700	1750				n_2 5	1598	21020
A_3	1640	1550	1600	1620	1640	1600	1740	1800	n_3 8	1648.75	6669.643
A_4	1510	1520	1530	1570	1640	1600			n_4 6	1561.667	2616.667

The overall sample variance is 8633.385.

1. Write down a one-way classification model (a model in terms of population means of group) for the analysis of the above data set. Define the variables clearly.

Let y_{ij} be the j^{th} observation in the i^{th} level. e_{ij} : experiment error.

$$y_{ij} = \mu_i + e_{ij}, \quad i = 1,2,3,4, \quad j = 1, \cdots, n_i, \quad e_{ij} \sim N(0, \sigma^2).$$

μ_i : population mean of A_i group.

2. Estimate the unknown parameters in (1).

$$\hat{\mu}_i = \bar{y}_{i\cdot}$$

$$RSS = \sum_{i=1}^{4} (n_i - 1) \cdot S_i^2 = 166622.3.$$

$$\hat{\sigma}^2 = \frac{RSS}{26-4} = 7573.739.$$

3. Test all population means of groups are equal at $\alpha = 0.05$. Write down the test statistic, the critical value and your conclusion clearly.

$H_0: \mu_1 = \cdots = \mu_4$.

$TSS = (\Sigma n_i - 1) \cdot S_y^2 = 215834.6$.

$SSA = TSS - RSS = 49212.35$.

$F_{obs} = \frac{SSA/3}{RSS/22} = 2.165921$.

$F_{0.05}(3, 22) = 3.049125$

$F_{obs} < f_{0.05}(3, 22)$. We cannot reject H_0.

4. Test population means of group A_1 and A_4 are equal at $\alpha = 0.05$. Write down the test statistic, the critical value and your conclusion clearly.

$H_0: \mu_1 = \mu_4$.

$\hat{\mu}_1 - \hat{\mu}_4 = 112.619$.

$\widehat{Var}(\hat{\mu}_1 - \hat{\mu}_4) = \hat{\sigma}^2(\frac{1}{n_1} + \frac{1}{n_4}) = 2344.253$.

$t_{obs} = \frac{\hat{\mu}_1 - \hat{\mu}_4}{se(\hat{\mu}_1 - \hat{\mu}_4)} = 2.326$.

$t_{0.025,22} = 2.073873$. $|t_{obs}| > t_{0.025,22}$

Reject H_0.