$$\Rightarrow \hat{\sigma}^2 = \frac{\text{Res S.S.}}{n-p'}$$

e.g. β_0 is known

$$\text{Res S.S.} = \sum_{i=1}^{n} (y_i - \beta_0)^2 - \hat{\beta}_1 \sum_{i=1}^{n} x_{i1}(y_i - \beta_0) - \cdots - \hat{\beta}_p \sum_{i=1}^{n} x_{ip}(y_i - \beta_0)$$

$$\hat{\sigma}^2 = \frac{\text{Res S.S.}}{n-p}$$

Distribution of Res S.S.

$p=1$ dist. of $\hat{\beta}_0, \hat{\beta}_1, \hat{\sigma}^2 \Leftarrow$ joint dist. of y_i $i=1, \cdots, n$

$$\text{let } \hat{\theta}_1 = \frac{\hat{\beta}_0 - \beta_0}{\sigma}, \quad \hat{\theta}_2 = \frac{\hat{\beta}_1 - \beta_1}{\sigma}, \quad \hat{\theta}_3 = \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sigma^2}$$

$$\text{m.g.f.} = m(t_1, t_2, t_3) = E\left(e^{t_1\hat{\theta}_1 + t_2\hat{\theta}_2 + t_3\hat{\theta}_3}\right)$$

$$= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} e^{t_1\hat{\theta}_1 + t_2\hat{\theta}_2 + t_3\hat{\theta}_3} \times \frac{1}{(2\pi)^{n/2}} \frac{1}{(\sigma^2)^{n/2}} \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_{i1})^2\right\} dy_1 \cdots dy_n$$

$$= \cdots$$

$$= \exp\left\{\frac{1}{2}\left(t_1^2 \frac{\sum_{i=1}^{n} x_{i1}^2/n}{\sum_{i=1}^{n}(x_{i1}-\bar{x}_1)^2} + 2t_1t_2\left(-\frac{\bar{x}_1}{\sum_{i=1}^{n}(x_{i1}-\bar{x}_1)^2}\right) + t_2^2 \frac{1}{\sum_{i=1}^{n}(x_{i1}-\bar{x}_1)^2}\right)\right\}(1-2t_3)^{-(n-2)/2}$$

$\Downarrow$ $\Downarrow$

$$\begin{pmatrix}\hat{\beta}_0\\ \hat{\beta}_1\end{pmatrix} \sim MN \qquad \frac{\text{Res S.S.}}{\sigma^2} \sim \chi^2_{(n-2)}$$

independent

For any p

Model $\underset{\sim}{Y} \sim MN(\underset{\sim}{\mu}, \underset{\sim}{\Sigma})$ $\qquad \underset{\sim}{\mu} = \underset{\sim}{X}\underset{\sim}{\beta} \quad \underset{\sim}{\Sigma} = \sigma^2 \underset{\sim}{I}$

non-zero mean

Theorem 3.4 $\quad \underset{\sim}{Y} \sim MN(\underset{\sim}{\mu}, \underset{\sim}{I})$

$\underset{\sim}{Y}^T \underset{\sim}{A} \underset{\sim}{Y}$ has a non-central chi-square with

k d.f. and the non-centrality constant

$$\lambda = \underset{\sim}{\mu}^T \underset{\sim}{A} \underset{\sim}{\mu}$$

iff $\underset{\sim}{A}$ is a symmetric idempotent matrix of rank k

MATH 3423 $\quad Z \sim N(0, 1) \qquad Z^2 \sim \chi^2_{(1)}$

(central) chi-square with d.f. = 1

Theorem 3.3

$$E(\underset{\sim}{Y}^T \underset{\sim}{A} \underset{\sim}{Y}) = \text{trace}(\underset{\sim}{A} \boxed{\underset{\sim}{\Sigma}}) + \underset{\sim}{\mu}^T \underset{\sim}{A} \underset{\sim}{\mu}$$

$\underset{\sim}{I}$

$$= \text{trace}(\underset{\sim}{A}) + \underset{\sim}{\mu}^T \underset{\sim}{A} \underset{\sim}{\mu}$$

$$= k + \lambda$$

when $\lambda = 0$

$\Rightarrow$ (central) chi-square with d.f. $= k$

$$Var(\underset{\sim}{Y}^T \underset{\sim}{A} \underset{\sim}{Y}) = 2(k + 2\lambda)$$

- p.d.f. $= \displaystyle\sum_{i=0}^{\infty} \underbrace{\frac{e^{-(\lambda/2)} \left(\frac{\lambda}{2}\right)^i}{i!}}_{P_o\left(\frac{\lambda}{2}\right)} \underbrace{f_{Y_{k+2i}}(y)}_{Y \sim \chi^2_{(k+2i)}}$

- Poisson weighted mixture of (central) chi-squares r.v.s

$- \; J \sim Po\left(\frac{\lambda}{2}\right)$

Conditional dist. of Y given $J = \quad \sim \chi^2_{(k+2j)}$

(unconditional)

$\Rightarrow$ distribution of $Y \sim \chi^2_{\cancel{(k+)}}(k, \lambda)$

$$\underset{\sim}{Y} \sim MN(\underset{\sim}{X}\underset{\sim}{\beta}, \sigma^2 \underset{\sim}{I}) \Rightarrow \underset{\sim}{Y}^* = \frac{Y}{\sigma} \sim MN(\underbrace{\boxed{\frac{1}{\sigma}\underset{\sim}{X}\underset{\sim}{\beta}}}_{\underset{\sim}{\mu}^*}, \underset{\sim}{I})$$

$$\text{Res. S.S.} = \underset{\sim}{Y}^T \underbrace{(\underset{\sim}{I} - \underbrace{\underset{\sim}{X}(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{X}^T}_{\underset{\sim}{H}})}_{\underset{\sim}{A}} \underset{\sim}{Y}$$

$\underset{\sim}{A}$ – symmetric ~~idemp~~ idempotent

$$\frac{\text{Res S.S.}}{\sigma^2} = \frac{\underset{\sim}{Y}^T(\underset{\sim}{I} - \underset{\sim}{X}(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{X}^T)\underset{\sim}{Y}}{\sigma * \sigma}$$

$\underset{\sim}{Y}^* \qquad \underset{\sim}{Y}^*$

$$= \underset{\sim}{Y}^{*T}(\underset{\sim}{I} - \underset{\sim}{X}(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{X}^T)\underset{\sim}{Y}^*$$

By Theorem 3.4

$$d.f. = \text{trace}(\underset{\sim}{I} - \underset{\sim}{X}(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{X}^T)$$

$$= n - p'$$

$$\lambda = \underset{\sim}{\mu}^{*T}(\underset{\sim}{I} - \underset{\sim}{X}(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{X}^T)\underset{\sim}{\mu}^* \qquad \underset{\sim}{\mu}^* = \frac{\underset{\sim}{X}\underset{\sim}{\beta}}{\sigma}$$

$$= \frac{1}{\sigma^2}\underset{\sim}{\beta}^T\underset{\sim}{X}^T(\underset{\sim}{I} - \underset{\sim}{X}(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{X}^T)\underset{\sim}{X}\underset{\sim}{\beta}$$

$$= \cancel{\frac{1}{\sigma^2}\underset{\sim}{\beta}^T\underset{\sim}{X}^T\underset{\sim}{X}\underset{\sim}{\beta}}$$

$$= \frac{1}{\sigma^2}\underset{\sim}{\beta}^T(\underset{\sim}{X}^T\underset{\sim}{X} - \underset{\sim}{X}^T\underset{\sim}{X}(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{X}^T\underset{\sim}{X})\underset{\sim}{\beta}$$

$$= 0.$$

$$\Rightarrow \frac{\text{Res S.S.}}{\sigma^2} \sim \chi^2_{(n-p')}$$ — (central) chi-square dist with d.f. $= n-p'$

③ <u>Independence of $\hat{\beta}$ & Res S.S.</u> $= \sum_{i=1}^n (y_i - \hat{y}_i)^2 = \sum_{i=1}^n \hat{e}_i^2$

<u>MATH 3423</u> $\quad X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$

sample variance S^2_{n-1} & sample mean $\bar{X}$ are indep.

S^2_{n-1}: est. of σ^2 $\qquad$ $\bar{X}$: est. of μ

<u>MATH 3424</u> $\quad \frac{\text{Res S.S.}}{n-p'} \longrightarrow \sigma^2 \qquad \underset{\sim}{X}\hat{\underset{\sim}{\beta}} \longrightarrow \underset{\sim}{\mu} = \underset{\sim}{X}\underset{\sim}{\beta}$

indep.

<u>$p=1$</u> $\quad \hat{e}_i = \sum_{j=1}^n (\delta_{ij} - (c_j + d_j x_{i1}))y_j \qquad \delta_{ij} = 1$ if $i=j$, $=0$ otherwise

$y_i - \hat{y}_i$

$$\hat{\beta}_0 = \sum_{j=1}^n \underbrace{\left(\frac{1}{n} - \frac{(x_{j1} - \bar{x}_1)\bar{x}_1}{S_{x_1x_1}}\right)}_{c_j} y_j$$

$$\hat{\beta}_1 = \sum_{j=1}^n \underbrace{\left(\frac{x_{j1} - \bar{x}_1}{S_{x_1x_1}}\right)}_{d_j} y_j$$

<u>By Theorem 3.1</u> $\quad cov\left(\sum_{i=1}^n c_i y_i, \sum_{i=1}^n d_i y_i\right) = \sigma^2 \left(\sum_{i=1}^n c_i d_i\right)$

$$cov(\hat{e}_i, \hat{\beta}_0) = \sigma^2 \sum_{j=1}^n (\delta_{ij} - (c_j + d_j x_{i1}))c_j$$
$$= \sigma^2 \left\{ c_i - \sum_{j=1}^n (c_j + d_j x_{i1}) c_j \right\}$$
$$= \sigma^2 \left\{ c_i - \left(\boxed{\sum_{j=1}^n c_j^2} + \boxed{\sum_{j=1}^n c_j d_j} x_{i1} \right) \right\}$$

from the proof of $Var(\hat{\beta}_0)$ $\qquad$ from the proof of $cov(\hat{\beta}_0, \hat{\beta}_1)$

$$= \sigma^2 \left\{ c_i - \left(\frac{1}{n} + \frac{\bar{x}_1^2}{S_{x_1x_1}}\right) + \frac{\bar{x}_1}{S_{x_1x_1}} x_{i1} \right\}$$

$$c_i = \frac{1}{n} - \frac{(x_{i1} - \bar{x}_1)\bar{x}_1}{S_{x_1x_1}}$$

$$= 0$$

$\Rightarrow$ $\hat{e}_i$, $\hat{\beta}_0$ are indep. $\Leftarrow$ See p.18 & p.19 of Chapter 1

$\text{cov}(\hat{e}_i, \hat{\beta}_1) = 0 \Leftarrow$ ~~proof~~ prove it!

$p > 1$ Theorem 3.2 (p.14)

$$\text{cov}(\underset{\sim}{c}\, \underset{\sim}{z}, \underset{\sim}{d}\, \underset{\sim}{z}) = \underset{\sim}{c}\, \text{Var}(\underset{\sim}{z})\, \underset{\sim}{d}^T$$

$$\text{cov}(\underset{\sim}{\hat{e}}, \underset{\sim}{\hat{\beta}}) = \underbrace{(\underset{\sim}{I} - \underset{\sim}{H})}_{\underset{\sim}{c}} \underbrace{\text{Var}(\underset{\sim}{Y})}_{\sigma^2 \underset{\sim}{I}} \underbrace{((\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{X}^T)^T}_{\underset{\sim}{d}}$$

$\underset{\sim}{\hat{e}} = \underset{\sim}{Y} - \underset{\sim}{\hat{Y}}$, $\quad \underset{\sim}{\hat{\beta}} = \underbrace{(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{X}^T}_{\underset{\sim}{d}}\underset{\sim}{Y}$

$$= \underset{\sim}{Y} - \underset{\sim}{X}\underset{\sim}{\hat{\beta}}$$

$$= \underset{\sim}{Y} - \underset{\sim}{X}(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{X}^T\underset{\sim}{Y}$$

$$= (\underset{\sim}{I} - \underset{\sim}{X}(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{X}^T)\underset{\sim}{Y}$$

$$= \underbrace{(\underset{\sim}{I} - \underset{\sim}{H})}_{\underset{\sim}{c}}\underset{\sim}{Y}$$

$$= \sigma^2 (\underset{\sim}{I} - \underset{\sim}{H})(\underset{\sim}{X}(\underset{\sim}{X}^T\underset{\sim}{X})^{-1})$$

$$= \sigma^2 (\underset{\sim}{X}(\underset{\sim}{X}^T\underset{\sim}{X})^{-1} - \underbrace{\underset{\sim}{X}\boxed{(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}}}_{\underset{\sim}{H}}\boxed{\underset{\sim}{X}^T\underset{\sim}{X}}(\underset{\sim}{X}^T\underset{\sim}{X})^{-1})$$

$$= \underset{\sim}{0}$$

$$\text{cov}(\underset{\sim}{\hat{e}}, \underset{\sim}{\hat{\beta}})$$

$\underset{\sim}{\hat{e}}$ $n \times 1$, $\quad \underset{\sim}{\hat{\beta}}$ $p' \times 1$

$$= \begin{pmatrix} \text{cov}(\hat{e}_1, \hat{\beta}_0) & \cdots & \text{cov}(\hat{e}_1, \hat{\beta}_p) \\ \text{cov}(\hat{e}_2, \hat{\beta}_0) & \cdots & \text{cov}(\hat{e}_2, \hat{\beta}_p) \\ \vdots & & \vdots \\ \text{cov}(\hat{e}_n, \hat{\beta}_0) & \cdots & \text{cov}(\hat{e}_n, \hat{\beta}_p) \end{pmatrix}_{n \times p'} = \underset{\sim}{0}$$

$\Rightarrow$ $\underset{\sim}{\hat{e}}$ & $\underset{\sim}{\hat{\beta}}$ are indep.

$E(y_i) = \underset{\sim}{x}_i^T \underset{\sim}{\beta} = \beta_0 + \beta_1 x_{i1} + \cdots + \beta_p x_{ip} \Leftarrow$ diff. mean

Section 4 Confidence interval & hypothesis testing of $\underset{\sim}{\beta}$

$$\hat{\underset{\sim}{\beta}} \sim MN(\underset{\sim}{\beta}, (\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\sigma^2)$$

$$\frac{Res\,S.S.}{n-p'} \sim \chi^2(n-p')$$

4.1 T-test

$p=1$

$$\hat\beta_1 \sim N\left(\beta_1, \frac{\sigma^2}{S_{x_1x_1}}\right)$$

$$\Rightarrow \frac{\hat\beta_1-\beta_1}{\sigma/\sqrt{S_{x_1x_1}}} \sim N(0,1)$$

$$\Rightarrow \frac{\overbrace{\frac{\hat\beta_1-\beta_1}{\sigma/\sqrt{S_{x_1x_1}}}}^{\sim N(0,1)}}{\sqrt{\underbrace{\frac{Res\,S.S.}{\sigma^2}}_{\sim\chi^2}\Big/(n-p')}} \sim t(n-p')$$

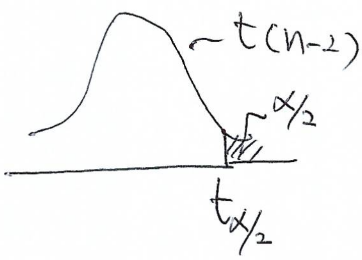
indep

$\leftarrow \hat\sigma^2$

$$\Rightarrow \frac{\hat\beta_1-\beta_1}{\hat\sigma/\sqrt{S_{x_1x_1}}} \sim t(n-p') \leftarrow t(n-2)$$

$(1-\alpha)100\%$ C.I.

$$Pr\left(-t_{\alpha/2,n-2} \le \frac{\hat\beta_1-\beta_1}{\hat\sigma/\sqrt{S_{x_1x_1}}} \le t_{\alpha/2,n-2}\right) = 1-\alpha$$



$\vdots$

$$\hat\beta_1 - t_{\alpha/2,n-2}\frac{\hat\sigma}{\sqrt{S_{x_1x_1}}} \le \beta_1 \le \hat\beta_1 + t_{\alpha/2}\frac{\hat\sigma}{\sqrt{S_{x_1x_1}}}$$

random interval

$$\sum_{i=1}^{n}(y_i - (\hat\beta_0 + \hat\beta_1 x_{i1})) = 0$$

$$\sum_{i=1}^{n} x_{i1}(y_i - (\hat\beta_0 + \hat\beta_1 x_{i1})) = 0$$

$$\Rightarrow \left.\begin{aligned} \sum_{i=1}^{n}\hat e_i = 0 \\ \sum_{i=1}^{n} x_{i1}\hat e_i = 0 \end{aligned}\right\} \quad \text{Properties of } \hat e_i$$

Example 1: Example in Simple Linear Regression

i	1	2	3	4	5	6	7	8	9
x_i	1.5	1.8	2.4	3.0	3.5	3.9	4.4	4.8	5.0
y_i	4.8	5.7	7.0	8.3	10.9	12.4	13.1	13.6	15.3

Summary statistics:

$$\textstyle\sum_{i=1}^{9} x_i = 30.3 \qquad \sum_{i=1}^{9} y_i = 91.1 \qquad \sum_{i=1}^{9} x_i y_i = 345.09$$

$$\textstyle\sum_{i=i}^{9} x_i^2 = 115.11 \qquad \bar x = 3.3667 \qquad \bar y = 10.1222$$

$\Rightarrow \hat\beta_1 = 2.9303$ and $\hat\beta_0 = 0.2568.$

Thus, the estimated regression line (or the fitted line) is given by

$$\hat y = 0.2568 + 2.9303x$$

$S_{x_1x_1} = \sum_{i=1}^{n} X_{i1}^2 - n\bar X_1^2$

$= 13.1$

$Res\,S.S. = S_{yy} - \hat\beta_1 S_{x_1 y}$

$\Uparrow$

$Res\,S.S. = S_{yy} - \hat\beta_1^2 S_{x_1x_1}$

$\frac{S_{x_1y}}{S_{x_1x_1}} = \hat\beta_1$

$S_{yy} = \sum_{i=1}^{n} y_i^2 - n\bar y^2$

$= 114.52$

$Res\,S.S. = 2.0258$

$\hat\sigma^2 = \frac{Res\,S.S}{n-2}$

$= 0.2894$

$\hat\sigma = 0.538$

$C.I.$ for β_1 : $\hat\beta_1 \pm t_{\alpha/2,7} \frac{\hat\sigma}{\sqrt{S_{x_1x_1}}}$

$= (2.579, 3.282)$

For any p

$$\text{Min} \sum_{i=1}^{n}(y_i - \hat y_i)^2$$

$$\Rightarrow \text{Min} \sum_{i=1}^{n}(y_i - (\hat\beta_0 + \hat\beta_1 x_{i1} + \ldots + \hat\beta_p x_{ip}))^2$$

$$= \sum_{i=1}^{n}\hat e_i^2$$

$$= (\hat e_1 \ \ldots \ \hat e_n)\begin{pmatrix}\hat e_1 \\ \vdots \\ \hat e_n\end{pmatrix}$$

$$= (y_i - (\hat\beta_0 + \hat\beta_1 x_{i1} + \ldots + \hat\beta_p x_{ip}) \ \ldots \ y_n - (\hat\beta_0 + \hat\beta_1 x_{n1} + \ldots + \hat\beta_p x_{np}))\begin{pmatrix} y_i - (\hat\beta_0 + \hat\beta_1 x_{i1} + \ldots + \hat\beta_p x_{ip}) \\ \vdots \\ y_n - (\hat\beta_0 + \hat\beta_1 x_{n1} + \ldots + \hat\beta_p x_{np})\end{pmatrix}$$

$$= (\underset{\sim}{Y}^T - \underset{\sim}{\hat\beta}^T \underset{\sim}{X}^T)(\underset{\sim}{Y} - \underset{\sim}{X}\underset{\sim}{\hat\beta})$$

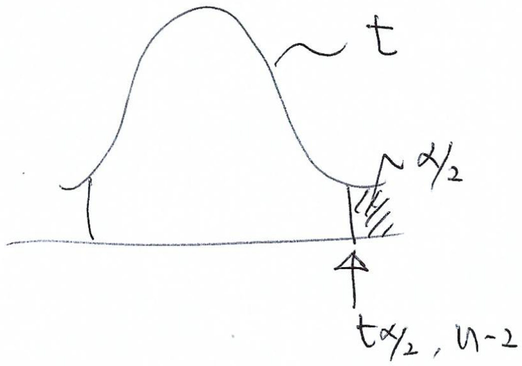
$$= (\underset{\sim}{Y} - \underset{\sim}{X}\underset{\sim}{\hat\beta})^T(\underset{\sim}{Y} - \underset{\sim}{X}\underset{\sim}{\hat\beta})$$

$H_0 : \beta_1 = \beta_{10}$ e.g. $H_0 : \beta_1 = 2.5$

$$\frac{\hat{\beta}_1 - \beta_{10}}{\hat{\sigma}/\sqrt{S_{x_1x_1}}} \sim t(n-2)$$ ← dist. under Ho

$$\Rightarrow t_{obs} = \frac{\hat{\beta}_1 - \boxed{\beta_{10}}}{\hat{\sigma}/\sqrt{S_{x_1x_1}}} =$$

↑ test stat.



$H_1 : \beta \neq \beta_{10}$

Reject Ho if $|t_{obs}| > t_{\alpha/2, n-2}$

$H_1 : \beta > \beta_{10}$ or $H_1 : \beta < \beta_{10}$

Reject Ho if $|t_{obs}| > t_{\alpha, n-2}$

Example 1: Example in Simple Linear Regression (cont.)

Summary statistics:

$$
\begin{aligned}
\sum_{i=1}^{9} y_i^2 &= 1036.65 \\
S_{xx} &= 115.11 - \frac{(30.3)^2}{9} = 13.10 \\
S_{yy} &= 1036.65 - \frac{(91.1)^2}{9} = 114.52 \\
S_{xy} &= 345.09 - \frac{(30.3)(91.9)}{9} = 38.39 \\
\hat{\beta}_1 &= 2.9303 \\
\hat{\sigma}^2 &= \frac{S_{yy} - \hat{\beta}_1 S_{xy}}{n-2} \\
&= \frac{114.52 - (2.9303)(38.39)}{n-2} = 0.2894 \\
\Rightarrow \quad \hat{\sigma} &= 0.538 \\
t_{0.05/2,7} &= 2.365
\end{aligned}
$$

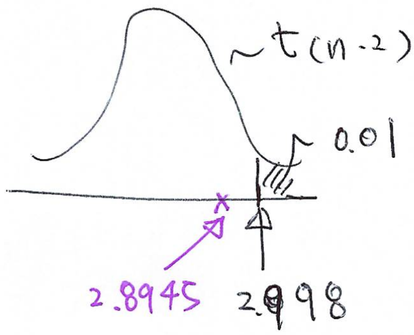
$H_0 : \beta_1 = 2.5$ vs $H_1 : \beta_1 > 2.5$

$t = \dfrac{\hat{\beta}_1 - 2.5}{0.538/\sqrt{13.1}}$

$= 2.8945$

value of test stat.

critical value $= t_{0.01,7} = 2.998$



$H_0 : \beta_1 = 2.5, \qquad H_1 : \beta_1 > 2.5$

$$
\begin{aligned}
t &= \frac{2.9303 - 2.5}{0.538/\sqrt{13.10}} \\
&= 2.8945 \\
&< 2.998 = t_{0.01,7}
\end{aligned}
$$

Can't reject H_0 $\Rightarrow \beta_1$ does not significantly differ from 2.5

OR There is no enough evidence to support that β_1 is greater than 2.5

95% C.I. of β_1:

$$
(2.9305 - \frac{2.365 * 0.538}{\sqrt{13.10}}, \quad 2.9305 - \frac{2.365 * 0.538}{\sqrt{13.10}})
$$

$$\Rightarrow 2.579 < \beta_1 < 3.282$$

$H_0 : \beta_0 = 0$

$$
\frac{\hat{\beta}_0 - \beta_0}{\hat{\sigma}\sqrt{\frac{1}{n} + \frac{\bar{x}_1^2}{S_{x_1x_1}}}} = \frac{0.2568 - 0}{0.538\sqrt{\frac{1}{9} + \frac{3.3667^2}{13.10}}} = 0.4831 < 1 \text{ (can't reject } H_0)
$$

95% C.I. of β_0 :

$$
(0.2568 - (2.365)0.538\sqrt{\frac{1}{9} + \frac{3.3667^2}{13.10}}, \quad 0.2568 + (2.365)(0.538)\sqrt{\frac{1}{9} + \frac{3.3667^2}{13.10}})
$$

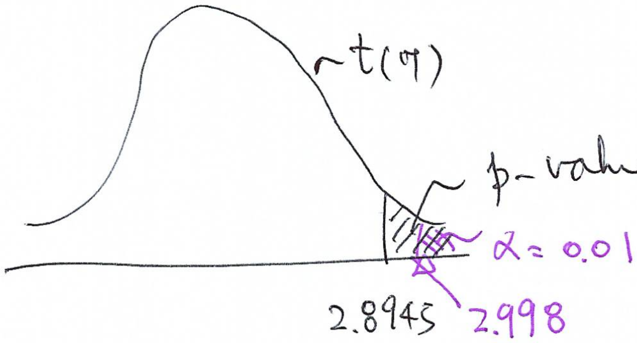
$$\Rightarrow -1.0005 < \beta_0 < 1.514$$

Example 4: Intercept is known (cont.)

$H_0 : \beta_1 = \beta_{10}$

$$
\hat{\beta}_1 \sim \mathrm{N}(\beta_1, \frac{\sigma^2}{\sum_{i=1}^{n} x_i^2})
$$

① p-value



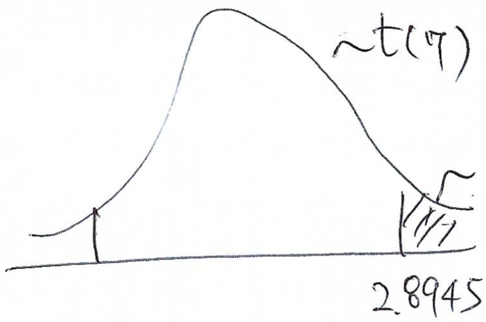
value of test stat.

p-value = $Pr(T_7 \geq \underline{2.8945}) \geq \alpha = 0.01$

Can't reject Ho

if p-value < α, then reject Ho.

② $H_1 : \beta_1 \neq 2.5$



p-value = $Pr(T_7 \geq 2.8945) * 2$

③ p-value in computer output is normally for two-sided alternative

For one-sided alternative, p-value = (p-value from computer-output)/2

$H_0 : \beta_1 \geq 2.5$

④ $H_1 : \beta_1 < 2.5$ ← No.

← meaningless

$\hat{\beta}_1 = 2.9303 \Rightarrow$ data set supports that $\beta_1 > 2.5$

⇒ Can't reject Ho