<u>8 October 2020</u>

When Total S.S. for full model is equal to Total S.S. for reduced model (i.e., the model under H_0):

1. It can be proved that Increase in $Reg.S.S.$ and numerator of the test for general linear hypothesis, i.e., $(\underset{\sim}{C}\hat{\underset{\sim}{\beta}} - \underset{\sim}{d})^T[\underset{\sim}{C}(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{C}^T]^{-1}(\underset{\sim}{C}\hat{\underset{\sim}{\beta}} - \underset{\sim}{d})$, are equivalent. See Supplement for the details.

2. Choose the test statistic for general linear hypothesis if $r < p'$ (reduced).

Prediction

<u>For $p = 1$</u>

<u>Mean Prediction</u>

$$\widehat{\mathrm{E}(y_0)} = \hat{\beta}_0 + \hat{\beta}_1 x_{01} \ \sim \ N\left(\beta_0 + \beta_1 x_{01}, \sigma^2(\frac{1}{n} + \frac{(x_{01} - \bar{x}_1)^2}{S_{x_1x_1}})\right)$$

$(1-\alpha)$ C.I. for mean value of y at x_{01} ($\mu_{y|x_{01}}$):

$$(\hat{\beta}_0 + \hat{\beta}_1 x_{01}) \pm t_{\alpha/2,(n-2)}\hat{\sigma}\sqrt{\frac{1}{n} + \frac{(x_{01} - \bar{x}_1)^2}{S_{x_1x_1}}}$$

For testing $H_0 : \mu_{y|x_{01}} = a$,

$$t = \frac{(\hat{\beta}_0 + \hat{\beta}_1 x_{01}) - a}{\hat{\sigma}\sqrt{\frac{1}{n} + \frac{(x_{01} - \bar{x}_1)^2}{S_{x_1x_1}}}}$$

<u>Individual Prediction</u>

We consider

$$\hat{y}_0 - y_0 \ \sim \ N\left(0, \sigma^2\left(1 + \frac{1}{n} + \frac{(x_{01} - \bar{x}_1)^2}{S_{x_1x_1}}\right)\right)$$

$(1-\alpha)$ C.I. for individual value of y at x_{01} (prediction interval):

$$(\hat{\beta}_0 + \hat{\beta}_1 x_{01}) \pm t_{\alpha/2,(n-2)}\hat{\sigma}\sqrt{1 + \frac{1}{n} + \frac{(x_{01} - \bar{x}_1)^2}{S_{x_1x_1}}}$$

<u>For any p</u>

New observation: $\underset{\sim}{x}_0^T = (1, x_{01}, \ldots, x_{0p})$

C.I. for mean value of y at x_0 ($\mu_{y_0|\underset{\sim}{x}_0}$):

$$\underset{\sim}{x}_0^T\hat{\underset{\sim}{\beta}} - t_{\alpha/2,(n-p')}\hat{\sigma}\sqrt{\underset{\sim}{x}_0^T(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{x}_0} < \mu_{y_0|\underset{\sim}{x}_0} < \underset{\sim}{x}_0^T\hat{\underset{\sim}{\beta}} + t_{\alpha/2,(n-p')}\hat{\sigma}\sqrt{\underset{\sim}{x}_0^T(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{x}_0}$$

C.I. for individual value of y at x_0 (prediction interval) :

$$\underset{\sim}{x}_0^T\hat{\underset{\sim}{\beta}} - t_{\alpha/2,(n-p')}\hat{\sigma}\sqrt{1 + \underset{\sim}{x}_0^T(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{x}_0} < y_0 < \underset{\sim}{x}_0^T\hat{\underset{\sim}{\beta}} + t_{\alpha/2,(n-p')}\hat{\sigma}\sqrt{1 + \underset{\sim}{x}_0^T(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{x}_0}$$