

MATH 3424 Tutorial

September 18, 2020

1 Review

Preliminaries, Chapter 1 (Sec. 1 Introduction, Sec. 2 Estimation).

2 Exercises

Using the following data set

x_1	x_2	y	x_1	x_2	y
1	-1	-3	8	4	15
2	2	7	8	4	18
2	2	8	8	4	15
2	2	9	9	6	18
3	2	9	9	6	20
4	7	20	10	9	32
5	7	19	11	10	31
5	8	19	11	10	33
5	8	18	11	11	33
6	8	20	12	12	37

with summary statistics:

$$\begin{aligned}n &= 20, & \sum_{i=1}^{20} x_{i1} &= 132, & \sum_{i=1}^{20} x_{i2} &= 121, & \sum_{i=1}^{20} y_i &= 378, \\ \sum_{i=1}^{20} x_{i1}^2 &= 1110, & \sum_{i=1}^{20} x_{i1}x_{i2} &= 987, & \sum_{i=1}^{20} x_{i2}^2 &= 973, & \sum_{i=1}^{20} x_{i1}y_i &= 3109, \\ \sum_{i=1}^{20} x_{i2}y_i &= 2953, & \sum_{i=1}^{20} y_i^2 &= 9160, \\ S_{x_1x_1} &= 238.8000, & S_{x_1x_2} &= 188.4000, & S_{x_2x_2} &= 240.9500, & S_{x_1y} &= 614.2000, \\ S_{x_2y} &= 666.1000, & S_{yy} &= 2015.8000.\end{aligned}$$

and

$$\begin{pmatrix} 238.8000 & 188.4000 \\ 188.4000 & 240.9500 \end{pmatrix}^{-1} = \begin{pmatrix} 0.0109303 & -0.0085464 \\ -0.0085464 & 0.0108327 \end{pmatrix},$$

to fit a model of y on x_1 and x_2 , i.e., do the following regression model,

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + e_i, \quad e_i \sim N(0, \sigma^2).$$

Give all your answers in 6 decimal points.

(1) **Assume that $\beta_0 = 1$.**

1. Find the least squares estimates of the unknown parameters β_1 and β_2 . Then, write down the fitted line.
2. Find the Residual Sum of Squares and the unbiased estimate of the unknown parameter σ^2 .

(2) **Assume that $\beta_1 = 1$.**

1. Find the least square estimates of the unknown parameters β_0 and β_2 . Then, write down the fitted line.
2. Find the unbiased estimate of the unknown parameter σ^2 .