3 November 2020

Tests on the main effects are meaningful, i.e., test

1. No Difference in Means Due to Factor A
   $H_0^1 : \mu_{1.} = \mu_{2.} = \ldots = \mu_{a.} \;\Rightarrow\; H_0^1 : \alpha_1 = \ldots = \alpha_{a-1} = 0$

$$\begin{aligned} \text{SSA} &= R(\underset{\sim}{\alpha}|\underset{\sim}{\beta}, \beta_0) \\ &= R(\underset{\sim}{\alpha}|\underset{\sim}{\beta}, \beta_0) - R(\underset{\sim}{\beta}|\beta_0) \end{aligned}$$

2. No Difference in Means Due to Factor B
   $H_0^2 : \mu_{.1} = \mu_{.2} = \ldots = \mu_{.b} \;\Rightarrow\; H_0^2 : \beta_1 = \ldots = \beta_{b-1} = 0$

$$\begin{aligned} \text{SSB} &= R(\underset{\sim}{\alpha}|\underset{\sim}{\beta}, \beta_0) \\ &= R(\underset{\sim}{\alpha}|\underset{\sim}{\beta}, \beta_0) - R(\underset{\sim}{\beta}|\beta_0) \end{aligned}$$

<u>Balanced Design</u>: $n_{ij} = n$ for all i, j

Total S.S. = SSA + SSB + SS(AB) + SSE

where

$$\begin{aligned} \text{SSA} &= bn\sum_{i=1}^{a}(\bar{y}_{i..} - \bar{y}_{...})^2 \\ &= \frac{1}{bn}\sum_{i=1}^{a}(T_{i..} - \frac{T_{...}}{a})^2 \\ \text{SSB} &= an\sum_{j=1}^{b}(\bar{y}_{.j.} - \bar{y}_{...})^2 \\ &= \frac{1}{an}\sum_{j=1}^{b}(T_{.j.} - \frac{T_{...}}{b})^2 \\ \text{SS(AB)} &= n\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...})^2 \\ \text{SSE} &= \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(y_{ijk} - \bar{y}_{ij.})^2 \\ \text{Total S.S.} &= \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(y_{ijk} - \bar{y}_{...})^2 \end{aligned}$$

1. SS(AB) is equal to $\text{SS}_{\text{Total}} - (\text{SSE} + \text{SSA} + \text{SSB})$.

   The test statistic is
$$\frac{n\sum\limits_{i=1}^{a}\sum\limits_{j=1}^{b}(\bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...})^2}{\hat{\sigma}^2(a-1)(b-1)}$$

   where $\hat{\sigma}^2 = \dfrac{\sum\limits_{i=1}^{a}\sum\limits_{j=1}^{b}\sum\limits_{k=1}^{n}(y_{ijk} - \bar{y}_{ij.})^2}{N - ab}$.

   Then, reject H_0 if $f_{obs} > F(\alpha, (a-1)(b-1), N - ab)$.

2. Main effect hypothesis tests are based on

$$\begin{aligned} f_1 &= \frac{SSA}{(a-1)\hat{\sigma}^2_{\text{no int}}} \qquad \text{for Factor A} \\ f_2 &= \frac{SSB}{(b-1)\hat{\sigma}^2_{\text{no int}}} \qquad \text{for Factor B} \end{aligned}$$

<u>"Interaction" is significant</u>

If the interaction is significant, it is meaningless to purely test the significance of the main effects of the two factor variables, say A and B, because in the presence of the interaction term, A * B, affect each other. Thus, the data should be analyzed in a somewhat different manner and tests on the single effects are appropriate, i.e., test

1. No Difference in Means Due to Factor A for each level of Factor B

   $H_0^1 : \mu_{1j} = \mu_{2j} = \ldots = \mu_{aj} \quad \text{for } j = 1, \ldots, b$

2. No Difference in Means Due to Factor B for each level of Factor A

   $H_0^2 : \mu_{i1} = \mu_{i2} = \ldots = \mu_{ib} \quad \text{for } i = 1, \ldots, a$

Then, use the test statistic

$$\frac{(\underset{\sim}{C}\hat{\underset{\sim}{\beta}})^T[\underset{\sim}{C}(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{C}^T]^{-1}(\underset{\sim}{C}\hat{\underset{\sim}{\beta}})}{r\hat{\sigma}^2}$$

where $\underset{\sim}{\beta} = (\mu_{11}, \mu_{12}, \ldots, \mu_{ab})^T$.