24 September 2020

Confidence Interval & Hypothesis Testing

1. **T test**

   (a) For p=1
   $H_0 : \beta_0 = \beta_{00}$

   $$t_{\text{obs}} = \frac{\hat{\beta}_0 - \beta_{00}}{\hat{\sigma}\sqrt{\frac{1}{n} + \frac{\bar{x}_1^2}{S_{x_1x_1}}}} \sim t_{n-2}$$

   Reject H_0 if $|t_{\text{obs}}| > t_{\alpha/2,n-2}$ for two-sided alternative. Reject H_0 if $|t_{\text{obs}}| > t_{\alpha,n-2}$ for one-sided alternative.

   $(1-\alpha)100\%$ C.I. for β_0 is

   $$\left(\hat{\beta}_0 - t_{\alpha/2,(n-2)}\hat{\sigma}\sqrt{\frac{1}{n} + \frac{\bar{x}_1^2}{S_{x_1x_1}}},\quad \hat{\beta}_0 + t_{\alpha/2,(n-2)}\hat{\sigma}\sqrt{\frac{1}{n} + \frac{\bar{x}_1^2}{S_{x_1x_1}}}\right)$$

   (b) For any p
   $H_0 : \beta_j = \beta_{j0}$

   $$t_{\text{obs}} = \frac{\hat{\beta}_j - \beta_{j0}}{\hat{\sigma}\sqrt{c^{jj}}} \sim t_{(n-p')}$$

   where c^{jj} is the $(j+1)^{th}$ diagonal element in $(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}$ for $j = 0, 1, \ldots, p$.

   Reject H_0 if $|t_{\text{obs}}| > t_{\alpha/2,n-p'}$ for two-sided alternative. Reject H_0 if $|t_{\text{obs}}| > t_{\alpha,n-p'}$ for one-sided alternative.

   $(1-\alpha)100\%$ C.I. for β_j is

   $$\left(\hat{\beta}_j - t_{\alpha/2,(n-p')}\ \hat{\sigma}\sqrt{c^{jj}},\ \hat{\beta}_j + t_{\alpha/2,(n-p')}\ \hat{\sigma}\sqrt{c^{jj}}\right)$$

In general, test statistic is equal to

$$t = \frac{\text{point est. - value under } H_0}{\text{standard error}} \sim t_{\text{d.f. of Res.S.S.}}$$

and C.I. is equal to

$$\text{point est.} \pm t_{\alpha/2,\text{d.f. of Res.S.S.}} * \text{standard error}$$

Remarks

- $Var(aX \pm bY) = a^2Var(X) + b^2Var(Y) \pm 2abCov(X,Y)$
- $$\begin{aligned}\text{Var}(\underset{\sim}{c}\hat{\underset{\sim}{\beta}}) &= \underset{\sim}{c}\,\text{Var}(\hat{\underset{\sim}{\beta}})\underset{\sim}{c}^T \\ &= \sigma^2\underset{\sim}{c}\,(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{c}^T\end{aligned}$$
- For both one-sided and two-sided alternatives
- For ONE linear combination of regression coefficients (including intercept) only

2. **F test**

   (a) **Partitioning total variability**

$$\begin{pmatrix}\text{Total variability}\\ \text{in response}\end{pmatrix} = \begin{pmatrix}\text{Variability}\\ \text{explained by model}\end{pmatrix} + \begin{pmatrix}\text{Unexplained}\\ \text{variability}\end{pmatrix}$$

$$Total\,S.S. = Reg.\,S.S. + Residual\,S.S.$$

$$\sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2 + \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$