22 September 2020

1. **Residual Sum of Squares**

   Distribution

   As

$$\text{Res.S.S.} = \underset{\sim}{Y}^T(\underset{\sim}{I} - \underset{\sim}{X}(\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\underset{\sim}{X}^T)\underset{\sim}{Y}$$

   By Theorem 3.4, it can be proved that

$$\frac{\text{Res. S. S.}}{\sigma^2} \sim \chi^2(n-p', \lambda).$$

   where $\lambda = 0$.

2. **Independence of $\hat{\underset{\sim}{\beta}}$ and $\hat{\sigma}^2$**

   For $p = 1$

   $Cov(\hat{e}_i, \hat{\beta}_0) = 0$ and $Cov(\hat{e}_i, \hat{\beta}_1) = 0$

   $\Rightarrow \hat{\sigma}^2$ and $(\hat{\beta}_0, \hat{\beta}_1)$ are independent.

   For any p

   $Cov(\hat{\underset{\sim}{e}}, \hat{\underset{\sim}{\beta}}) = 0$

   $\Rightarrow \hat{\sigma}^2$ and $\hat{\underset{\sim}{\beta}}$ are independent.

3. **Confidence Interval & Hypothesis Testing**

$$\hat{\underset{\sim}{\beta}} \sim \text{N}(\underset{\sim}{\beta}, \quad (\underset{\sim}{X}^T\underset{\sim}{X})^{-1}\sigma^2)$$

$$\frac{(n-p')\hat{\sigma}^2}{\sigma^2} = \frac{RSS}{\sigma^2} \sim \chi_{(n-p')}$$

   Res S.S. and $\hat{\underset{\sim}{\beta}}$ are independent.

   For p=1

   $H_0 : \beta_1 = \beta_{10}$

$$t_{\text{obs}} \;\; = \;\; \frac{\hat{\beta}_1 - \beta_{10}}{\hat{\sigma}/\sqrt{S_{x_1x_1}}} \sim t_{n-2}$$

   Reject H_0 if $|t_{\text{obs}}| > t_{\alpha/2,n-2}$ for two-sided alternative. Reject H_0 if $|t_{\text{obs}}| > t_{\alpha,n-2}$ for one-sided alternative.

   $(1-\alpha)100\%$ C.I. for β_1 is

$$\left(\hat{\beta}_1 - t_{\alpha/2,(n-2)}\frac{\hat{\sigma}}{\sqrt{S_{x_1x_1}}},\ \hat{\beta}_1 + t_{\alpha/2(n-2)}\frac{\hat{\sigma}}{\sqrt{S_{x_1x_1}}}\right)$$