The Hong Kong University of Science & Technology

MATH243 – Statistical Inference

Midterm Examination – Fall 03/04

Answer <u>ALL</u> questions **Date: 5 November 2003 (Wed)**

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1. (5 marks) Let X_j, $j = 1, \ldots, n$ be i.i.d. r.v.'s with p.d.f. f of the continuous type. If m is a median of f, calculate the probability that all X's exceed m. Also, calculate the probability $\Pr(Y_n \le m)$, where Y_n is the n^{th} order statistic of a random sample of size n.

2. (5 marks) Let X_1, X_2 be a random sample from a distribution having the p.d.f. $f(x) = e^{-x}$, $0 < x < \infty$, zero elsewhere. Find the distribution of $Z = X_1 / X_2$.

3. (5 marks) Let $X_1, \ldots, X_n, X_{n+1}$ be a random sample of size $n + 1$, $n > 1$, from a distribution which is $N(\mu, \sigma^2)$. Let $\overline{X} = \sum_1^n X_i \Big/ n$, and $S^2 = \sum_1^n (X_i - \overline{X})^2 \Big/ n$. Find the constant c so that the statistic $c(\overline{X} - X_{n+1})/S$ has a t distribution. If $n = 8$, determine k such that $\Pr(\overline{X} - kS < X_9 < \overline{X} + kS) = 0.80$.

4. (5 marks) Let Y_n be the n^{th} order statistic of a random sample of size n from a continuous-type uniform distribution on the interval $(0, \theta)$. Find c such that $\Pr(c\theta < Y_4 < \theta) = 0.95$. What then is a 95 per cent confidence interval for θ?

5. (10 marks)

   (a) Let $X_1, \ldots, X_n$ are independently uniformly distributed on $(0, \theta)$. Find the estimators from the method of moment and the method of maximum likelihood.

   (b) Find the expectation and variance of the estimator from the method of moments.

   (c) Find the expectation and variance of the estimator from the method of maximum likelihood.

   (d) Hence or otherwise, construct two unbiased estimators of θ based on the two estimators in part (a).

   (e) Compare the variances of the two unbiased estimates in (d) and comment briefly.

6. (10 marks)

Suppose that (X_1, …, X_n), $n > 2$, is a random sample from an exponential distribution with p.d.f. of the form

$$f(x;\lambda) = \lambda e^{-\lambda x}$$

where $\lambda \in (0,\infty)$.

(a) Find the Cramer-Rao lower bound for estimating λ.

(b) Find the expectation of the statistic $1/\overline{X}$. Hence or otherwise, find an unbiased estimator as a function of $\overline{X}$ for λ.

(c) Compare the variance of the unbiased estimator in part (b) with the Cramer-Rao lower bound.

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