1. Suppose X_1, ...,X_n be a random sample with Bernoulli distribution with unknown parameter p.
   (a) **(2 marks)** Show that $X_1 - X_2$ is not a complete statistic.
   (b) **(4 marks)** Find the complete and sufficient statistic for p. What is its distribution?
   (c) **(4 marks)** Find the maximum likelihood estimator of $p(1-p)$. Is it unbiased? Give the detailed explanation. No mark will be given for an answer of "Yes" or "No".
   (d) **(6 marks)** Find the UMVUE of $\theta = (1-p)^2$. Find the Cramer Rao Lower Bound of all unbiased estimators for θ. Is the variance of the UMVUE equal to the CRLB? Explain. No need to calculate the variance of the UMVUE for θ. No mark will be given for an answer of "Yes" or "No".
   (e) **(4 marks)** Find the UMVUE for p^m, where m is a positive integer less than or equal to n.
2. Let X_1, ...,X_n be a random sample from a location distribution family
$$f(x;\theta) \;=\; \frac{1}{\theta}\exp\left(-\frac{x-\delta}{\theta}\right) I(x \ge \delta)\ .$$
   Note that $X_i - \delta \sim \exp\left(\frac{1}{\theta}\right)$.
   (a) Assume that δ is known.
      i. **(4 marks)** Find the complete and sufficient statistic of the unknown parameter θ. What is its distribution?
      ii. **(2 marks)** Find the UMVUE of θ.
      iii. **(4 marks)** Find the UMVUE of $Pr(X_1 > 1)$ when $\delta < 1$.
   (b) Assume that θ is known.
      i. **(6 marks)** Find the complete and sufficient statistic of the unknown parameter δ. What is its distribution?
      ii. **(4 marks)** Find the UMVUE of δ.
      iii. **(Bonus: 4 marks)** Find the UMVUE of $Pr(X_1 > 1)$ when $\delta < 1$.
3. Suppose that we have two independent random samples: $X_1, \ldots, X_n$ are exponential(θ), with density
$$f(x|\theta) \;=\; \theta e^{-\theta x}, \quad x > 0$$
   and $Y_1, \ldots, Y_n$ are exponential (μ).
   (a) **(4 marks)** Find the UMP test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta < \theta_0$ at $\alpha = 0.05$, $\theta_0 = 1$ and $n = 10$.
   (b) **(2 marks)** Based on the test in part (a), calculate the power of test at $\theta_1 = 0.3$.
   (c) **(4 marks)** Find the expression of likelihood ratio, $\lambda(X_1, \ldots, X_n, Y_1, \ldots, Y_n)$, for testing $H_0 : \theta = \mu$ against $H_1 : \theta \neq \mu$.
   (d) **(6 marks)** Hence or otherwise, find the likelihood ratio test for testing $H_0 : \theta = \mu$ against $H_1 : \theta \neq \mu$ at $\alpha = 0.1$ and $n = 10$.
   (e) **(4 marks)** Derive the approximate large sample likelihood ratio test for testing $H_0 : \theta = \mu$ against $H_1 : \theta \neq \mu$ at the significance level of α and a large value of n. Make your conclusion at $\alpha = 0.05$ if $\sum x_i = 100$, $\sum y_i = 50$ and $n = 50$. Write down the value of test statistic and critical value clearly.
4. (a) **(7 marks)** A baker, Mr. C, baked THREE large chocolate cakes each day. Those not sold on the same day were given away to a food bank. In 300 days, there was one day that no cake was sold, there were 6 days that only one cake was sold and so on. The data were given below.

Number of cakes sold per day, X	0	1	2	3
Numbers of days	1	6	65	228

   By Pearson goodness-of-fit test, test whether X follows the binomial distribution, i.e., $X \sim Bi(3, p)$, where p is the probability of a cake being sold, at the 0.05 level of significance.
   (b) From part (a), it is noted that 810 of chocolate cakes were sold during 300 days. In addition to baking THREE chocolate cakes each day, Mr. C also baked THREE large cheese cakes per day. In 300 days, 720 of cheese cakes were sold. Test whether the probability of a chocolate cake being sold is different from that of a cheese cake significantly by the following tests:
      i. **(4 marks)** z test;
      ii. **(4 marks)** approximate large sample likelihood ratio test;
      iii. **(4 marks)** Pearson goodness-of-fit test. No need to make the Yates's Correction.

   (1 mark) What recommendation do you make to Mr. C?

   State clearly the hypothesis statements, value of test statistic, critical value and your conclusion for each test.