MATH3423 - Statistical Inference
Assignment 2

1. (Bonus) Find $Var(M_2)$.

2. Find $Var(M_2)$ when $X_i \sim N(\mu, \sigma^2)$.

3. $X_1, X_2, ..., X_n$ are observations of a random sample of size n from the exponential distribution with mean $1/\theta$, i.e., $X_i \sim exp(\theta)$. Find the distribution of $\sum_{i=1}^{n} X_i$.

4. Using Central Limit Theorem, handle
   (a) Q16 in Exercise 1.
   (b) Q18 in Exercise 1.

5. Q3 in the midterm exam of 2013/2014
   Let $U_1, ..., U_n$ be a random sample from the U(0,1).
   (a) Let $X = -\log(U)$. Find the distribution of X.
   (b) Let $Y = \dfrac{1}{\prod_{i=1}^{n} U_i^{\frac{1}{n}}}$, where $U_1, ..., U_n$ be a random sample from the U(0,1) and n is very large. Using Central Limit Theorem and Delta method to find the approximate distribution of Y.

6. Q1 in the midterm exam of 2014/2015
   Let X_1, X_2 be random variables having the bivariate normal distribution with parameters μ_1, μ_2, σ_1, σ_2, ρ (correlation coefficient between X_1 and X_2), i.e.,

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \sim N_2\left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} \right).$$

   Set

$$Y_1 = \frac{X_1 - \mu_1}{\sigma_1} + \frac{X_2 - \mu_2}{\sigma_2}, \; Y_2 = \frac{X_1 - \mu_1}{\sigma_1} - \frac{X_2 - \mu_2}{\sigma_2}.$$

   Find the probability density functions of Y_1 and Y_2. Are they independent?