The Hong Kong University of Science & Technology
MATH3423 - Statistical Inference
Midterm Examination - Fall 2013/2014

Answer ALL Questions | **Date: 17 October 2013**

Full marks: 30 + 3 marks for Bonus | **Time Allowed: 75 minutes**

- DO NOT open the exam paper until instructed to do so.
- It is a closed-book examination.
- Only the calculator approved by H.K.E.A. is allowed in the examination.
- Three questions are included in the paper.
- Last page of each question is blank. You may write down your answer for this question if the provided spaces are not enough.

Name : ________________________

Student Number : _______________

Signature : _____________________

1. (**8 marks, 1 mark each**) Let X_1, ...,X_n be a random sample from N(μ,σ^2). Define

$$\bar{X}_k = \frac{1}{k}\sum_{i=1}^{k} X_i$$

$$\bar{X}_{n-k} = \frac{1}{n-k}\sum_{i=k+1}^{n} X_i$$

and

$$S_k^2 = \frac{1}{k-1}\sum_{i=1}^{k}(X_i - \bar{X}_k)^2,$$

$$S_{n-k}^2 = \frac{1}{n-k-1}\sum_{i=k+1}^{n}(X_i - \bar{X}_{n-k})^2,$$

Answer the following:

(a) What is the distribution of $((k-1)S_k^2 + (n-k-1)S_{n-k}^2)/\sigma^2$?

(b) What is the distribution of S_k^2/S_{n-k}^2 ?

(c) What is the distribution of $(\bar{X}_k + \bar{X}_{n-k})/2$?

(d) What is the distribution of $(\bar{X}_n - \mu)/(S_n/\sqrt{n})$?

If $\mu = 0$ $\sigma = 1$,

(e) What is the distribution of $k\bar{X}_k^2 + (n-k)\bar{X}_{n-k}^2$?

(f) What is the distribution of X_1^2/X_2^2 ?

(g) What is the distribution of X_1/X_2 ?

(h) What is the distribution of $(X_1+X_2)^2/(X_1-X_2)^2$?

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2. (**14 marks + 3 marks for Bomus**) Let $U_1, ..., U_n$ be a random sample from the $U(0, \theta)$, where θ is the unknown parameter.

   (a) (**2 marks**) Find the moment estimator of θ. Is it unbiased? Hence or otherwise, find an unbiased estimator for θ.

   (b) (**3 marks**) Find the maximum likelihood estimator of θ. Is it unbiased? Hence or otherwise, find an unbiased estimator for θ.

(c) (**4 marks**) Find the variances of unbiased estimators from (a) and (b). Which unbiased estimator for θ is more efficient?

(d) (**2 marks**) Suppose a random sample with sample size of six is drawn. The values are as follow:

$$0.3, \quad 1.2, \quad 1.8, \quad 2.4, \quad 4.1, \quad 5.5$$

Calculate estimates from methods of moment and maximum likelihood. Hence or otherwise, state one problem of moment estimator **other than efficiency**

(e) (**2 marks**) Given that $\theta > 1$, find the maximum likelihood estimator of θ.

(f) (**1 mark**) Find the MLE for θ^3.

(g) (**Bonus: 3 marks**) Calculate the variance of MLE for θ^3. Hence or otherwise, find its Mean Squared Error (MSE).

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3. (**8 marks**) Let $U_1, ..., U_n$ be a random sample from the U(0,1).

   (a) (**2 marks**) Let $X = -\log(U)$. Find the distribution of X.

   (b) (**6 marks**) Let $Y = \dfrac{1}{\prod_{i=1}^{n} U_i^{\frac{1}{n}}}$, where $U_1, ..., U_n$ be a random sample from the U(0,1) and n is very large. Using Central Limit Theorem and Delta method to find the approximate distribution of Y.

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