MATH 3423 Statistical Inference

Assignment 1

Please submit your solution (in pdf) to Canvas **before 4pm on Oct 9, 2020**.

Question 1: If $\{X_1, X_2, \dots, X_{100}\}$ is a r.s. from a distribution with mean μ and variance 16, find the approximate 95% C.I. for $\mu(\mu + 3)$ with $\bar{x} = 15$. Please show the details how you get the random and confidence intervals.

Question 2: Consider a r.s. of size n from a distribution with mean μ and variance σ^2, where $\mu = \sigma \in (0, \infty)$. Find an appropriate function g such that $g'(\mu)\sigma = 1$ in delta method. Then, use the function g to construct a 95% confidence interval for μ with $\bar{x} = 34.23$ and $n = 50$.

Question 3: If X and Y are *independent* random variables from the *standard normal* distribution, then show that $\frac{X}{Y} \sim t(1)$.

Remark that the *t* distribution with 1 degree of freedom is also known as a *Cauchy* distribution, where it is well-known that the mean of the Cauchy distribution does NOT exist.

Question 4: Consider a r.s. $\{X_1, X_2, \dots, X_n\}$ of size $n > 1$ from a distribution with mean μ and variance σ^2. We have already known that S_{n-1}^2 has a mean σ^2. Here, we look at its variance. Please show that the variance of S_{n-1}^2 is

$$\frac{1}{n}\left(\mu_4 - \frac{n-3}{n-1}\sigma^4\right).$$

Question 5: Consider a r.s. $\{X_1, X_2, \dots, X_n\}$ with size $n > 1$ from a uniform distribution over $[0, \theta]$, where $0 < \theta < \infty$. Find an order statistic(s) with the smallest variance. Please justify your answer.