Q1 a) $f(x;\theta) = \exp(-\log\theta - \frac{x}{\theta})$

$a(\theta) = -\log\theta$, $b(x) = 0$, $c(\theta) = -\frac{1}{\theta}$, $d(x) = x$

which belongs to the exponential family.

$\therefore \sum d(x_i) = \sum_{i=1}^{n} x_i$ is complet and sufficient for θ.

b) $L(\theta) = f(\underline{x},\theta) = \prod_{i=1}^{n} \frac{1}{\theta} e^{-x_i/\theta} = \frac{1}{\theta^n} e^{-\frac{1}{\theta}\sum_{i=1}^{n} x_i}$

$\operatorname{Log} L(\theta) = -n\log\theta - \frac{1}{\theta}\sum_{i=1}^{n} x_i$

$\frac{\partial}{\partial\theta}\log L(\theta) = -\frac{n}{\theta} + \frac{1}{\theta^2}\sum_{i=1}^{n} x_i$

Let $\frac{\partial}{\partial\theta}\log L(\theta) = 0 \Rightarrow \hat{\theta} = \bar{x}$, MLE of θ

c) $E(\bar{x}) = E(X_i) = \theta$

By (a), $\bar{x}$ is complete and sufficient for θ

$\therefore \bar{x}$ is UMVUE for θ

d) $\operatorname{Var}(\bar{x}) = \frac{1}{n}\operatorname{Var}(X) = \frac{1}{n}\cdot\theta^2$

$\therefore$ MLE of $\frac{\theta^2}{n}$ is $\frac{\bar{x}^2}{n}$

e) $\Pr(X_1 \geq 1) = \int_1^{+\infty} \frac{1}{\theta} e^{-\frac{1}{\theta}x}\,dx = e^{-\frac{1}{\theta}}$

MLE of $e^{-\frac{1}{\theta}}$ is $e^{-\frac{1}{\bar{x}}}$

f) $\log f(x,\theta) = -\log\theta - \frac{x}{\theta}$, $\frac{\partial}{\partial\theta}\log L(x,\theta) = \frac{-1}{\theta} + \frac{x}{\theta^2}$

$\frac{\partial^2}{\partial\theta^2}\log f(x,\theta) = \frac{1}{\theta^2} - 2\cdot\frac{x}{\theta^3}$

$E\left(\frac{\partial^2}{\partial\theta^2}\log f(x,\theta)\right) = \frac{1}{\theta^2} - 2\frac{EX}{\theta^3} = \frac{1}{\theta^2} - 2\frac{1}{\theta^2} = \frac{-1}{\theta^2}$

$$CRLB = \frac{[(e^{-\frac{1}{\theta}})']^2}{-nE\left(\frac{\partial^2}{\partial\theta^2}\log f(x,\theta)\right)} = \frac{e^{-2/\theta}}{n\theta^2}$$

g) $S=\sum_{i=1}^{n} X_i \sim \text{Gamma}(n, \frac{1}{\theta})$ is a complete and sufficient ~~statistic~~ for $e^{-1/\theta}$.

Let $h(s)$ be the UMVUE for $e^{-\frac{1}{\theta}}$, then

$$\int_0^{+\infty} h(s) \frac{s^{n-1} e^{-\frac{s}{\theta}}}{\Gamma(n)\,\theta^n}\, ds = e^{-\frac{1}{\theta}} \qquad \text{ie } E(h(s)) = e^{-\frac{1}{\theta}}.$$

$$\int_0^{+\infty} h(s) \frac{s^{n-1} e^{-\frac{s-1}{\theta}}}{\Gamma(n)\,\theta^n}\, ds$$

$$\int_0^{+\infty} \frac{h(s)\, s^{n-1} (s-1)^{n-1} e^{-\frac{s-1}{\theta}}}{(s-1)^{n-1}\,\Gamma(n)\,\theta^n}\, ds = 1$$

Let $\frac{h(s)\, s^{n-1}}{(s-1)^{n-1}} \cdot I(1<s<+\infty) = 1$

$\therefore h(s) = \left(\frac{s-1}{s}\right)^{n-1} I(1<s<+\infty)$, UMVUE for $e^{-\frac{1}{\theta}}$.

(h) $\bar{x} = 0.37401$

est. of $\frac{1}{\theta} = 2.6737$

Q2. a) $X_1, \cdots, X_n \sim U(\theta_1, \theta_2)$

$$f(\underline{x}, \theta_1, \theta_2) = \prod_{i=1}^{n} f(x_i, \theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\theta_2 - \theta_1} I(\theta_1 < x_i < \theta_2)$$

$$= \frac{1}{(\theta_2-\theta_1)^n} I(\theta_1 < x(1)) I(x(n) < \theta_2)$$

MLE of θ_1, $\hat{\theta}_1 = X(1) = X_{min}$.

MLE of θ_2, $\hat{\theta}_2 = X(n) = X_{max}$

b) $$f_{X(n)}(x) = \frac{n!}{(n-1)!(n-n)!} F(x)^{n-1} (1-F(x))^{n-n} f(x)$$

$$= n \frac{(x-\theta_1)^{n-1}}{(\theta_2-\theta_1)^n}.$$

$$f_{X(1)}(x) = \frac{n!}{(n-1)!(1-1)!} F(x)^{1-1} (1-F(x))^{n-1} f(x)$$

$$= n \frac{(\theta_2 - x)^{n-1}}{(\theta_2-\theta_1)^n}$$

$$E(X(n)) = \int_{\theta_1}^{\theta_2} x \cdot n \frac{(x-\theta_1)^{n-1}}{(\theta_2-\theta_1)^n} dx = \frac{n}{n+1}\theta_2 + \frac{1}{n+1}\theta_1$$

$$E(X(1)) = \int_{\theta_1}^{\theta_2} x n \frac{(\theta_2 - x)^{n-1}}{(\theta_2-\theta_1)^n} dx = \frac{n}{n+1}\theta_1 + \frac{1}{n+1}\theta_2.$$

c) by a) and the factorization theorem, we know that $T = (X(n), X(1))$ is sufficient for (θ_2, θ_1)

Now, if $E(g(X(n), X(1))) = E(g(T)) = 0$, $\forall g$

$f_{X(n), X(1)}(x_n, x_1) = n(n-1) \frac{(x_n - x_1)^{n-2}}{(\theta_2-\theta_1)^n}$, $\theta_2 > x_n > x_1 > \theta_1$

then $E(g(T)) = 0 \Leftrightarrow \int_{\theta_1}^{\theta_2} \int_{x_1}^{\theta_2} n(n-1) \frac{(x_n - x_1)^{n-2}}{(\theta_2-\theta_1)^n} g(T(x_n, x_1)) dx_n dx_1 = 0$

i.e. $\int_{\theta_1}^{\theta_2} \int_{x}^{\theta_2} (x_n - x_1)^{n-1} g(T(x_n, x_1)) dx_n dx_1 = 0$

Differentiating LHS w.r.t θ_1 and θ_2; we get

$(\theta_2-\theta_1)^{n-2} g(T(\theta_2, \theta_1)) = 0$ for $\forall (\theta_2, \theta_1)$

$\therefore g(T) \equiv 0$ for any θ_1, θ_2. i.e. T is complete.

d) We know $\overset{\text{by b)}}{}$ $E\left(\frac{X(n) + X(1)}{2}\right) = \frac{\theta_1 + \theta_2}{2}$

$\therefore$ By c) $\frac{X(n) + X(1)}{2}$ is UMVUE for $\frac{\theta_1 + \theta_2}{2}$

Q3 a) $Y_i = \alpha + \beta x_i + \varepsilon_i$, $E(Y_i) = E(\alpha+\beta x_i + \varepsilon_i) = \alpha+\beta x_i$

$Var(Y_i) = Var(\alpha+\beta x_i + \varepsilon_i) = \sigma^2$

$Y_i \sim N(\alpha+\beta x_i, \sigma^2)$

b)

$\varepsilon_i = y_i - \alpha - \beta x_i$.

$L(\alpha, \beta) = (2\pi\sigma^2)^{-\frac{n}{2}} \exp\left(-\frac{\Sigma(y_i-\alpha-\beta x_i)^2}{2\sigma^2}\right)$

$\log L(\alpha,\beta) = -\frac{n}{2}\log 2\pi\sigma^2 - \frac{\Sigma(y_i-\alpha-\beta x_i)^2}{2\sigma^2}$.

$$\begin{cases} \frac{\partial}{\partial\alpha}\log L(\alpha,\beta) = -\frac{2}{2\sigma^2}\Sigma(y_i-\alpha-\beta x_i) = 0 \\ \frac{\partial}{\partial\beta}\log L(\alpha,\beta) = -\frac{2}{2\sigma^2}\Sigma(y_i-\alpha-\beta x_i)x_i = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \Sigma(y_i-\alpha-\beta x_i) = 0 \\ \Sigma(y_i-\alpha-\beta x_i)x_i = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \hat\alpha = \bar y - \hat\beta \bar x & \text{, MLE for } \alpha \\ \hat\beta = \frac{\Sigma(x_i-\bar x)(y_i-\bar y)}{\Sigma(x_i-\bar x)^2} = \frac{\Sigma(x_i-\bar x)y_i}{\Sigma(x_i-\bar x)^2} & \text{, MLE for } \beta. \end{cases}$$

c) $E(\hat\beta) = E\left(\frac{\Sigma(x_i-\bar x)(y_i-\bar y)}{\Sigma(x_i-\bar x)^2}\right) = \frac{1}{\Sigma(x_i-\bar x)^2}\cdot \Sigma\left((x_i-\bar x)E(y_i-\bar y)\right)$

$E(y_i-\bar y) = Ey_i - E\bar y = \alpha+\beta x_i - (\alpha+\beta\bar x) = \beta(x_i-\bar x)$

$\therefore E(\hat\beta) = \beta\cdot\frac{\Sigma(x_i-\bar x)^2}{\Sigma(x_i-\bar x)^2} = \beta$, unbiased.

$E(\hat\alpha) = E\bar y - E(\hat\beta\bar x) = \alpha+\beta\bar x - (E\hat\beta)\bar x = \alpha$, unbiased.

d) $(2\pi\sigma^2)^{-\frac{n}{2}}\exp\left(-\frac{\Sigma(y_i-\alpha-\beta x_i)^2}{2\sigma^2}\right)$

$= \exp\left(-\frac{n}{2}\log 2\pi\sigma^2 + \frac{n\alpha^2}{2\sigma^2} - \frac{\beta^2\Sigma x_i^2}{2\sigma^2} - \frac{2\alpha\beta\Sigma x_i}{2\sigma^2} - \frac{\Sigma y_i^2}{2\sigma^2}\right.$
$\left. + \frac{\alpha\Sigma y_i + \beta\Sigma x_i y_i}{\sigma^2}\right)$

$\therefore \sum_{i=1}^n y_i$, $\sum_{i=1}^n x_i y_i$ are complete and sufficient for (α, β)

And $\hat\alpha$, $\hat\beta$ are functions of $(\sum_{i=1}^n y_i, \sum_{i=1}^n x_i y_i)$

$\therefore \hat\alpha$, $\hat\beta$ are complete and sufficient for (α, β)

e) By (c) and (d), we know that $\hat{\alpha}$, $\hat{\beta}$ are UMVUE for α, β. i.e. the best estimators

f) $\hat{\beta} = \dfrac{\sum_{i=1}^{n}(x_i-\bar{x})y_i}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$, $Y_i \sim N(\alpha+\beta_i, \sigma^2)$, $i=1,\dots,n$

$$Var(\hat{\beta}) = Var\left(\frac{\sum_{i=1}^{n}(x_i-\bar{x})y_i}{\sum_{i=1}^{n}(x_i-\bar{x})^2}\right)$$

$$= \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2 Var(y_i)}{\left(\sum_{i=1}^{n}(x_i-\bar{x})^2\right)^2}$$

$$= \frac{\sigma^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$$

$$\therefore \hat{\beta} \sim N\left(\beta, \frac{\sigma^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}\right)$$

g) $$\frac{\hat{\beta}-\beta}{\sigma^2/\sum_{i=1}^{n}(x_i-\bar{x})^2} \sim N(0,1)$$

$$95\%\ \text{C.I of } \beta = \left[\hat{\beta} - \underbrace{Z_{0.025}}_{1.96}\sqrt{\frac{\sigma^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}},\ \hat{\beta} + \underbrace{Z_{0.025}}_{1.96}\sqrt{\frac{\sigma^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}}\right]$$

Q4 $L(\mu_1,\mu_2,\mu_3,\sigma^2) = (2\pi\sigma^2)^{-\frac{3n}{2}} \exp\left(-\frac{\Sigma(x_{1i}-\mu_1)^2+\Sigma(x_{2i}-\mu_2)^2+\Sigma(x_{3i}-\mu_3)^2}{2\sigma^2}\right)$

$\log L(\mu_1,\mu_2,\mu_3,\sigma^2) = -\frac{3n}{2}\log(2\pi\sigma^2) - \frac{\Sigma(x_{1i}-\mu_1)^2+\Sigma(x_{2i}-\mu_2)^2+\Sigma(x_{3i}-\mu_3)^2}{2\sigma^2}$

a) Under Θ_0 ie $\mu_1=\mu_2=\mu_3=\mu_0$

$$\begin{cases} \frac{\partial}{\partial\mu_0}\log L(\mu_0,\mu_0,\mu_0,\sigma_0^2) = 0 \\ \frac{\partial}{\partial\sigma_0^2}\log L(\mu_0,\mu_0,\mu_0,\sigma_0^2) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \hat{\mu}_0 = \frac{\sum_{i=1}^n x_{1i}+\sum_{i=1}^n x_{2i}+\sum_{i=1}^n x_{3i}}{3n} = (\bar{x}_1+\bar{x}_2+\bar{x}_3)/3 \\ \hat{\sigma}_0^2 = \frac{\sum_{i=1}^n (x_{1i}-\hat{\mu}_0)^2+\sum_{i=1}^n (x_{2i}-\hat{\mu}_0)^2+\sum_{i=1}^n (x_{3i}-\hat{\mu}_0)^2}{3n} \end{cases}$$

b) Under Θ : otherwise.

$$\begin{cases} \frac{\partial}{\partial\mu_1}\log L(\mu_1,\mu_2,\mu_3,\sigma^2) = 0 \\ \frac{\partial}{\partial\mu_2}\log L(\mu_1,\mu_2,\mu_3,\sigma^2) = 0 \\ \frac{\partial}{\partial\mu_3}\log L(\mu_1,\mu_2,\mu_3,\sigma^2) = 0 \\ \frac{\partial}{\partial\sigma^2}\log L(\mu_1,\mu_2,\mu_3,\sigma^2) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \hat{\mu}_1 = \bar{x}_1, \quad \hat{\mu}_2 = \bar{x}_2, \quad \mu_3 = \bar{x}_3 \\ \hat{\sigma}^2 = \frac{\sum_{i=1}^n (x_{1i}-\bar{x}_1)^2+\sum_{i=1}^n (x_{2i}-\bar{x}_2)^2+\sum_{i=1}^n (x_{3i}-\bar{x}_3)^2}{3n} \end{cases}$$

c) $$\lambda(\underset{\sim}{x_1},\underset{\sim}{x_2},\underset{\sim}{x_3}) = \left(\frac{\hat{\sigma}_0^2}{\hat{\sigma}^2}\right)^{-\frac{3n}{2}}$$

$$= \left(\frac{\sum_{i=1}^n (x_{1i}-\hat{\mu}_0)^2+\sum_{i=1}^n (x_{2i}-\hat{\mu}_0)^2+\sum_{i=1}^n (x_{3i}-\hat{\mu}_0)^2}{\sum_{i=1}^n (x_{1i}-\bar{x}_1)^2+\sum_{i=1}^n (x_{2i}-\bar{x})^2+\sum_{i=1}^n (x_{3i}-\bar{x}_3)^2}\right)^{-\frac{3n}{2}}$$

$$-2\log\lambda(\underset{\sim}{x_1},\underset{\sim}{x_2},\underset{\sim}{x_3}) = 3n\log\left(\frac{\sum_{i=1}^n (x_{1i}-\hat{\mu}_0)^2+\sum_{i=1}^n (x_{2i}-\hat{\mu}_0)^2+\sum_{i=1}^n (x_{3i}-\hat{\mu}_0)^2}{\sum_{i=1}^n (x_{1i}-\bar{x}_1)^2+\sum_{i=1}^n (x_{2i}-\bar{x}_2)^2+\sum_{i=1}^n (x_{3i}-\bar{x}_3)^2}\right)$$

$$\sim \chi^2(2).$$

d) $X_{i1}, \dots, X_{in} \sim N(\mu_i, \sigma^2)$

$\Rightarrow \sum_{j=1}^{n}(X_{ij} - \bar{X}_i)^2/\sigma^2 \sim \chi^2(n-1)$, $i=1,2,3$.

$\therefore \sum_{i=1}^{3}\sum_{j=1}^{n}(X_{ij} - \bar{X}_i)^2/\sigma^2 \sim \chi^2(3n-3)$ (1)

e) (pf.) $\operatorname{cov}(X_{ij} - \bar{X}_i, \bar{X}_{i'} - \bar{\bar{X}})$, $i=1,2,3$, $i'=1,2,3$, $j=1,\dots,n$.

$= \operatorname{cov}(X_{ij}, \bar{X}_{i'}) + \operatorname{cov}(\bar{X}_i, \bar{\bar{X}}) - \operatorname{cov}(X_{ij}, \bar{\bar{X}}) - \operatorname{cov}(\bar{X}_i, \bar{X}_{i'})$

If $i = i'$,

$\operatorname{cov}(X_{ij} - \bar{X}_i, \bar{X}_{i'} - \bar{\bar{X}}) = \frac{\sigma^2}{n} + \frac{\sigma^2}{3n} - \frac{\sigma^2}{n} - \frac{\sigma^2}{3n} = 0$

If $i \neq i'$

$\operatorname{cov}(X_{ij} - \bar{X}_i, \bar{X}_{i'} - \bar{\bar{X}}) = 0 + \frac{\sigma^2}{3n} - \frac{\sigma^2}{3n} + 0 = 0$

$\therefore \sum_{i=1}^{3}\sum_{j=1}^{n}(X_{ij} - \bar{X}_i)^2$ and $\sum_{i=1}^{3}(\bar{X}_i - \bar{\bar{X}})^2$ are independent (*)

$\sum_{i=1}^{3}\sum_{j=1}^{n}(X_{ij} - \bar{\bar{X}})^2 \sim \chi^2(3n-1)$ (**)

And $\sum_{i=1}^{3}\sum_{j=1}^{n}(X_{ij} - \bar{\bar{X}})^2 = \sum_{i=1}^{3}\sum_{j=1}^{n}(X_{ij} - \bar{X}_i + \bar{X}_i - \bar{\bar{X}})^2$

$= \sum_{i=1}^{3}\sum_{j=1}^{n}(X_{ij} - \bar{X}_i)^2 + \sum_{i=1}^{3}\sum_{j=1}^{n}(\bar{X}_i - \bar{\bar{X}})^2 + 2\sum_{i=1}^{3}\sum_{j=1}^{n}(X_{ij} - \bar{X}_i)(\bar{X}_i - \bar{\bar{X}})$

$= \sum_{i=1}^{3}\sum_{j=1}^{n}(X_{ij} - \bar{X}_i)^2 + n\sum_{i=1}^{3}(\bar{X}_i - \bar{\bar{X}})^2$

ie $n\sum_{i=1}^{3}(\bar{X}_i - \bar{\bar{X}})^2 = \sum_{i=1}^{3}\sum_{j=1}^{n}(X_{ij} - \bar{\bar{X}}) - \sum_{i=1}^{3}\sum_{j=1}^{n}(X_{ij} - \bar{X}_i)^2$, (***)

By (*), (**), (***) and (1), $n\sum_{i=1}^{3}(\bar{X}_i - \bar{\bar{X}})^2 \sim \chi^2(2)$

$3n(n-1)\sum_{i=1}^{3}(\bar{X}_i - \bar{\bar{X}})^2 / 2\sum_{i=1}^{3}\sum_{j=1}^{n_i}(X_{ij} - \bar{X}_i)^2$

$= \left[n\sum_{i=1}^{3}(\bar{X}_i - \bar{\bar{X}})^2/2\right] / \left[\sum_{i=1}^{3}\sum_{j=1}^{n_i}(X_{ij} - \bar{X}_i)^2/3(n-1)\right]$

$\sim F(2, 3n-3)$

g) $\sum_{i=1}^{n}(x_{1i}-\hat{\mu}_0)^2+\sum_{i=1}^{n}(x_{2i}-\hat{\mu}_0)^2+\sum_{i=1}^{n}(x_{3i}-\hat{\mu}_0)^2$

$=\sum_{i=1}^{n}(x_{1i}-\bar{x}_1)^2+\sum_{i=1}^{n}(x_{2i}-\bar{x}_2)^2+\sum_{i=1}^{n}(x_{3i}-\bar{x}_3)^2+n[(\bar{x}_1-\hat{\mu}_0)^2+(\bar{x}_2-\hat{\mu}_0)^2+(\bar{x}_3-\hat{\mu}_0)^2]$

And, $n[(\bar{x}_1-\hat{\mu}_0)^2+(\bar{x}_2-\hat{\mu}_0)^2+(\bar{x}_3-\hat{\mu}_0)^2]/\sigma^2 \sim \chi^2(2)$

$$\Lambda(\underset{\sim}{x_1},\underset{\sim}{x_2},\underset{\sim}{x_3})=\left(1+\frac{n[(\bar{x}_1-\hat{\mu}_0)^2+(\bar{x}_2-\hat{\mu}_0)^2+(\bar{x}_3-\hat{\mu}_0)^2]}{\sum_{i=1}^{n}(x_{1i}-\bar{x}_1)^2+\sum_{i=1}^{n}(x_{2i}-\bar{x}_2)^2+\sum_{i=1}^{n}(x_{3i}-\bar{x}_3)^2}\right)^{-\frac{3n}{2}}$$

$$C_1=\{(\underset{\sim}{x_1},\underset{\sim}{x_2},\underset{\sim}{x_3}):\Lambda(\underset{\sim}{x_1},\underset{\sim}{x_2},\underset{\sim}{x_3})\le k_1\}$$

$$\Rightarrow C_1=\{(\underset{\sim}{x_1},\underset{\sim}{x_2},\underset{\sim}{x_3}):\frac{n[(\bar{x}_1-\hat{\mu}_0)^2+(\bar{x}_2-\hat{\mu}_0)^2+(\bar{x}_3-\hat{\mu}_0)^2]}{\sum_{i=1}^{n}(x_{1i}-\bar{x}_1)^2+\sum_{i=1}^{n}(x_{2i}-\bar{x}_2)^2+\sum_{i=1}^{n}(x_{3i}-\bar{x}_3)^2}\ge k_1'\}$$

$$\Rightarrow C_1=\{(\underset{\sim}{x_1},\underset{\sim}{x_2},\underset{\sim}{x_3}):\frac{n[(\bar{x}_1-\hat{\mu}_0)^2+(\bar{x}_2-\hat{\mu}_0)^2+(\bar{x}_3-\hat{\mu}_0)^2]/2}{[\sum_{i=1}^{n}(x_{1i}-\bar{x}_1)^2+\sum_{i=1}^{n}(x_{2i}-\bar{x}_2)^2+\sum_{i=1}^{n}(x_{3i}-\bar{x}_3)^2]/(3n-3)}\ge F(2,3n-3,\alpha)\}$$

h). (c): $-2\log\Lambda(\underset{\sim}{x_1},\underset{\sim}{x_2},\underset{\sim}{x_3})=3\times5\log(1+\frac{5\times0.1592}{49.82})\approx \cancel{0.103}$ 0.2378

$<\chi^2(2,\alpha)=\chi^2(2,0.05)=5.991$

Can't reject H_0.

(g): $\frac{n[(\bar{x}_1-\hat{\mu}_0)^2+(\bar{x}_2-\hat{\mu}_0)^2+(\bar{x}_3-\hat{\mu}_0)^2]/2}{[\sum_{i=1}^{n}(x_{1i}-\bar{x}_1)^2+\sum_{i=1}^{n}(x_{2i}-\bar{x}_2)^2+\sum_{i=1}^{n}(x_{3i}-\bar{x}_3)^2]/(3n-3)}=\frac{5\times0.1592/2}{49.82/12}\doteq0.096$

$<F(2,3n-3,\alpha)=F(2,12,0.05)=3.89$

Can't reject. H_0.

∴ The conclusions are the same

$\frac{0.796/2}{49.82/12}=\frac{0.398}{4.1517}=$

$\bar{x}_1=81.2$, $\bar{x}_2=81.54$, $\bar{x}_3=80.98$, $\hat{\mu}_0=81.24$

Total S.S. = 50.616 d.f. = 14

0.0016 + 0.09 + 0.0676 = 0.1592

28.42 + 6.892 + 14.508 = 49.82

28.428 + 7.342 + 14.846 = 50.616