MATH3423 Statistical Inference

Classwork 2

1. (**6 marks**)

   Let X_1, ...,X_n be a random sample from N(μ,σ^2). Define

$$\overline{X}_k = \frac{1}{k}\sum_{i=1}^{k} X_i$$

$$\overline{X}_{n-k} = \frac{1}{n-k}\sum_{i=1+k}^{n} X_i$$

   and

$$S_k^2 = \frac{1}{k-1}\sum_{i=1}^{k}(X_i - \overline{X}_k)^2,$$

$$S_{n-k}^2 = \frac{1}{n-k-1}\sum_{i=k+1}^{n}(X_i - \overline{X}_{n-k})^2,$$

   Answer the following:

   (a) What is the distribution of $((k-1)S_k^2 + (n-k-1)S_{n-k}^2)/\sigma^2$?

   (b) What is the distribution of S_k^2/S_{n-k}^2 ?

   (c) What is the distribution of $(\overline{X}_k + \overline{X}_{n-k})/2$?

   (d) What is the distribution of $(\overline{X}_n - \mu)/(S_n/\sqrt{n})$?

   If $\mu = 0$ $\sigma = 1$,

   (e) What is the distribution of X_1/X_2 ?

   (f) What is the distribution of $(X_1 + X_2)^2/(X_1 - X_2)^2$?

   Answers:

   (a) ________________ (b) ________________

   (c) ________________ (d) ________________

   (e) ________________ (f) ________________

2. (**6 marks**)

Let $X_1, \ldots, X_n$ be i.i.d. r.v.'s from the $U(0, \theta)$, $\theta \in \Omega = (0, \infty)$, distribution. Answer the following questions.

(a) Find the p.d.f. of $X_{(n)}$ and $E(X_{(n)})$, where $X_{(n)} = \max(X_1, \ldots, X_n)$;

(b) Find the p.d.f. of $X_{(1)}$ and $E(X_{(1)})$, where $X_{(1)} = \min(X_1, \ldots, X_n)$;

(c) Find two unbiased estimators for θ.