MATH 243 Statistical Inference

Midterm Examination - Fall 1999/2000

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1. Let $X_1, \ldots X_n$ be a random sample from $N(0,1)$. Define

$$\bar{X}_k = \frac{1}{k}\sum_{i=1}^{k} X_i \quad \text{and} \quad \bar{X}_{n-k} = \frac{1}{n-k}\sum_{i=k+1}^{n} X_i$$

   Answer the following:

   (a) What is the distribution of $\frac{1}{2}(\bar{X}_k + \bar{X}_{n-k})$?

   (b) What is the distribution of $k\bar{X}_k^2 + (n-k)\bar{X}_{n-k}^2$?

   (c) What is the distribution of X_1^2/X_2^2?

2. Let X_1, X_2, X_3 be a random sample from a distribution of the continuous type having p.d.f. $f(x) = 2x$, $0 < x < 1$, zero elsewhere. Compute the probability that the smallest of these X_i exceeds the median of the distribution.

3. Let X be a single observation from the Bernoulli density $f(x;\theta) = \theta^x(1-\theta)^{1-x}I_{\{0,1\}}(x)$, where $0 < \theta < 1$. Let $t_1(X) = X$ and $t_2(X) = \frac{1}{2}$.

   (a) Are both $t_1(X)$ and $t_2(X)$ unbiased for θ? Is either?

   (b) Compare the mean-squared error of $t_1(X)$ with that of $t_2(X)$.

4. Let $X_1, \ldots, X_n$ be a random sample from the geometric density

$$f(x;\theta) = \theta(1-\theta)^x \, I_{\{0,1,\ldots\}}(x)$$

where $0 < \theta < 1$.

(a) Find a method of moments estimator of θ.

(b) Find a maximum-likelihood estimator of θ.

(c) Find a maximum-likelihood estimator of the mean.

(d) Find the Cramer-Rao lower bound for $1 - \theta$.