Solutions for Final. 06/07

Q1 a) $m.g.f$ of $T = E(e^{2\lambda \Sigma X_i \cdot t}) = E(e^{2\lambda t \Sigma X_i})$

$= E(e^{2\lambda t \cdot X_1}) \cdots E(e^{2\lambda t \cdot X_n}) = (\frac{\lambda}{\lambda - 2\lambda t})^n$

$= (\frac{1}{1-2t})^n = (\frac{1}{1-2t})^{-2n/2}$, $m.g.f$ of $\chi^2(2n)$

ie $T \sim \chi^2(2n)$

b) $X_1, \cdots, X_m \sim exp(\lambda)$, $Y_1, \cdots, Y_n \sim exp(\lambda)$

by a) $2\lambda \sum_{i=1}^m X_i \sim \chi^2(2m)$, $2\lambda \sum_{j=1}^n Y_j \sim \chi^2(2n)$

$S = \frac{2\lambda \sum_{i=1}^m X_i / 2m}{2\lambda \sum_{j=1}^n Y_j / 2n} \sim F(2m, 2n)$

c) i) $(X, Y) \sim N(1, 1, 4, 1, \frac{1}{2})$

Then $X+2Y \sim N(3, 12)$

$P(X+2Y \le 4) = P(Z \le \frac{4-3}{\sqrt{12}}) = P(Z \le 0.2887)$

$= 0.6136$

ii) $X+Y$ and $X-Y$ are independent

$\Leftrightarrow E(X+Y-E(X+Y))(X-Y-E(X-Y)) = 0$

$\Leftrightarrow E(X-EX+Y-EY)(X-EX-(Y-EY)) = 0$

$\Leftrightarrow E[(X-EX)^2-(Y-EY)^2] = 0$ ie $\sigma_1^2 - \sigma_2^2 = 0$

d) $E((\check{\sigma}^2 - \sigma^2)^2) = Var(\check{\sigma}^2 - \sigma^2) + [E(\check{\sigma}^2 - \sigma^2)]^2$

$= Var(\check{\sigma}^2) + (\frac{n-1}{n+1} - 1)^2 \sigma^4$

$= \frac{\sigma^4}{(n+1)^2} Var(\chi^2(n-1)) + \frac{4}{(n+1)^2}\sigma^4$

$= \frac{\sigma^4}{(n+1)^2}(2n+2) = \frac{2\sigma^4}{n+1}$

$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$, $\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{\sigma^2} \sim \chi^2(n-1)$

Q2. a) $\bar{X} = \tilde{E}X = \frac{1}{\tilde{\lambda}}$ ie $\tilde{\lambda} = \bar{X}^{-1}$

b) $f(\underset{\sim}{x}, \lambda) = \prod_{i=1}^{n} f(x_i) = \prod_{i=1}^{n} \lambda e^{-\lambda x_i} = \lambda^n e^{-\lambda \Sigma x_i}$

$\log f(\underset{\sim}{x}, \lambda) = n \log \lambda - \lambda \sum_{i=1}^{n} x_i$

$\frac{\partial}{\partial \lambda} \log f(\underset{\sim}{x}, \lambda) = \frac{n}{\lambda} - \Sigma x_i = 0$

$\Rightarrow$ MLE of λ is $\hat{\lambda} = \frac{n}{\Sigma X_i} = \bar{X}^{-1}$

c) ~~$\hat{\lambda}$ ... is the UMVUE of~~, $S = \Sigma X_i \sim \text{Gamma}(n, \lambda)$

$f(x, \lambda) = \exp(\log \lambda - \lambda x)$

$a(\lambda) = \log \lambda$, $b(x) = 0$, $c(\lambda) = -\lambda$, $d(X) = X$

belongs to the exponential family. and $\Sigma d(X_i) = \Sigma X_i$. ie $\bar{X}$ is complete and sufficient statistic for λ

$$E(\bar{X}^{-1}) = \int_0^{+\infty} \frac{n}{s} \frac{s^{n-1} e^{-\lambda s}}{\Gamma(n) \lambda^{-n}} ds$$

$$= \int_0^{+\infty} \frac{s^{n-2} e^{-\lambda s}}{\Gamma(n-1) \lambda^{-n+1}} \cdot \frac{n \Gamma(n-1)}{\Gamma(n) \lambda^{-1}} ds$$

$$= n \cdot \frac{\Gamma(n-1)}{\Gamma(n)} \cdot \lambda = \frac{n}{n-1} \lambda$$

$\Rightarrow \hat{\lambda} = \bar{X}^{-1}$ is not UMVUE

d) $P_r(X_1 \geq 1) = \int_1^{+\infty} \lambda e^{-\lambda x} dx = -e^{-\lambda x}\Big|_1^{+\infty} = e^{-\lambda}$

by b) MLE of $P(X_1 \geq 1)$ is $e^{-\bar{X}^{-1}}$

e) by c) $S = \sum_{i=1}^{n} X_i \sim \text{Gamma}(n, \lambda)$ is a complete and sufficient statistic Let $h(s)$ be the UMVUE for $e^{-\lambda}$:

$$\int_0^{+\infty} h(s) \frac{s^{n-1} e^{-\lambda s}}{\Gamma(n) \lambda^{-n}} ds = e^{-\lambda}$$

$$\Leftrightarrow \int_0^{+\infty} h(s) \cdot \frac{s^{n-1} e^{-\lambda(s-1)}}{\Gamma(n) \lambda^{-n}} ds = 1$$

$$\Leftrightarrow \int_0^{+\infty} \frac{h(s) s^{n-1} (s-1)^{n-1} e^{-\lambda(s-1)}}{(s-1)^{n-1} \Gamma(n) \lambda^{-n}} ds = 1$$

$$\Rightarrow \frac{h(s) \cdot s^{n-1}}{(s-1)^{n-1}} I(1 < s < +\infty) = 1$$

ie. $h(s) = \left(\frac{s-1}{s}\right)^{n-1} I(1 < s < +\infty) = \left(1 - \frac{1}{n\bar{x}}\right)^{n-1} I(1 < n\bar{x} < +\infty)$

is the UMVUE of $Pr(X_1 \geq 1)$

f) $\frac{\partial}{\partial\lambda} \log f(x_i, \lambda) = \frac{1}{\lambda} - x_i$, $\frac{\partial^2}{\partial\lambda^2} \log f(x_i, \lambda) = -\frac{1}{\lambda^2}$.

then $n E\left(\frac{\partial^2}{\partial\lambda^2} \log f(x, \lambda)\right) = -n \cdot \frac{1}{\lambda^2}$

CRLB of $e^{-\lambda} = \frac{-[(e^{-\lambda})']^2}{-n\frac{1}{\lambda^2}} = \frac{\lambda^2 e^{-2\lambda}}{n}$

g) The CRLB isn't attainable. Since $\frac{\partial}{\partial\lambda} \log f(x, \lambda) = \frac{n}{\lambda} - \sum x_i = -n(\bar{x} - \frac{1}{\lambda})$

Q3. a) i) $X_1, \dots, X_n \sim U(\theta_1, \theta_2)$

$f_X(x, \theta_1, \theta_2) = \prod_{i=1}^{n} f(x_i, \theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\theta_2 - \theta_1} I(\theta_1 < x_i < \theta_2)$

$= \frac{1}{(\theta_2 - \theta_1)^n} I(\theta_1 < x_{(1)}) I(x_{(n)} < \theta_2)$

MLE of θ_1 : $\hat{\theta}_1 = X_{(1)} = X_{min}$.

MLE of θ_2 : $\hat{\theta}_2 = X_{(n)} = X_{max}$

ii) $f_{X(n)}(x) = \frac{n!}{(n-1)!(n-n)!} F(x)^{n-1} (1 - F(x))^{n-n} f(x)$

$= n \frac{(x - \theta_1)^{n-1}}{(\theta_2 - \theta_1)^n}$.

$E(X_{max}) = E(X_{(n)}) = \int_{\theta_1}^{\theta_2} x \cdot n \frac{(x - \theta_1)^{n-1}}{(\theta_2 - \theta_1)^n} dx$

$= \frac{n}{n+1} \theta_2 + \frac{1}{n+1} \theta_1$

$f_{X(1)}(x) = \frac{n!}{(n-1)!(1-1)!} F(x)^{1-1} (1 - F(x))^{n-1} f(x)$

$= n \frac{(\theta_2 - x)^{n-1}}{(\theta_2 - \theta_1)^n}$

$E(X_{min}) = E(X_{(1)}) = \int_{\theta_1}^{\theta_2} x\, n \frac{(\theta_2 - x)^{n-1}}{(\theta_2 - \theta_1)^n} dx = \frac{n}{n+1} \theta_1 + \frac{1}{n+1} \theta_2$

iii) By factorization theorem, we can see that $T=(X(n), X(1))$ is joint sufficient for (θ_2, θ_1)

Now If $E\{g(X(n), X(1))\} = E(g(T)) = 0$, $\forall g(T)$ statistic

then $f_{X(n),X(1)}(x_n, x_1) = n(n-1)\frac{(x_n - x_1)^{n-2}}{(\theta_2-\theta_1)^n}$, $\theta_2 > x_n > x_1 > \theta_1$

$E(g(T)) = 0 \Leftrightarrow \int_{\theta_1}^{\theta_2}\int_{x_1}^{\theta_2} n(n-1)\frac{(x_n-x_1)^{n-2}}{(\theta_2-\theta_1)^n} g(T(x_n, x_1))\,dx_n\,dx_1 = 0$

ie $\int_{\theta_1}^{\theta_2}\int_{x_1}^{\theta_2}(x_n - x_1)^{n-2} g(T(x_n - x_1))\,dx_n\,dx_1 = 0$

Differentiating LHS w.r.t θ_1 and θ_2 we get:

$(\theta_2-\theta_1)^{n-2} g(T(\theta_2, \theta_1)) = 0$ for any (θ_2, θ_1)

$\Rightarrow g(T) \equiv 0$ for any θ_2, θ_1 ie T is complete.

Also, $\frac{E(X(n)) + E(X(1))}{2} = \frac{\theta_1+\theta_2}{2}$ ie $\frac{X(1)+X(n)}{2}$ is UMVUE.

b) i) $(n-1)V/\sigma^2 \sim \chi^2(n-1)$.

$E(V) = \frac{\sigma^2}{n-1}E(\chi^2(n-1)) = \sigma^2$.

$Var(V) = \frac{\sigma^4}{(n-1)^2} Var(\chi^2(n-1)) = \frac{2\sigma^4}{n-1}$

ii) $f(x, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma}\exp\{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2\}$

$= \exp(-\frac{\mu^2}{2\sigma^2} - \frac{1}{2}\log\sigma^2 - \frac{1}{2}\log 2\pi + \frac{\mu}{\sigma^2}x - \frac{1}{2\sigma^2}x^2)$

$a(\mu, \sigma) = -\frac{\mu^2}{2\sigma^2} - \frac{1}{2}\log\sigma^2$, $b(x) = -\frac{1}{2}\log 2\pi$

$c_1(\mu, \sigma) = \frac{\mu}{\sigma^2}$, $d_1(x) = x$

$c_2(\mu, \sigma) = -\frac{1}{2\sigma^2}$, $d_2(x) = x^2$ belongs to exponential family

Then $T = (\Sigma X_i, \Sigma X_i^2)$ is sufficient and complete statistic.

$\Rightarrow (\Sigma X_i, \Sigma(X_i - \bar{X})^2)$ is also S-C statistic for (μ, σ^2)

We already know that $E(V) = \sigma^2$.

So V is the UMVUE for σ^2.

iii) $S=\sum_{i=1}^{n}(X_i-\bar{X})^2$ and $Y=\frac{(n-1)V}{\sigma^2}=\frac{S}{\sigma^2}\sim\chi^2(n-1)$

$E(\frac{1}{S})=E(\frac{1}{Y\sigma^2})$

$$=\frac{1}{\sigma^2}\int_0^{\infty}\frac{1}{y}\cdot\left[y^{\frac{(n-1)}{2}-1}\cdot e^{-\frac{y}{2}}\Big/2^{\frac{n-1}{2}}\Gamma(\frac{n-1}{2})\right]dy$$

$$=\frac{1}{\sigma^2}\int_0^{\infty}\frac{y^{\frac{n-2-1}{2}-1}\cdot e^{-\frac{y}{2}}}{2^{\frac{n-2-1}{2}}\Gamma(\frac{n-2-1}{2})}\cdot\frac{2^{\frac{n-3}{2}}\Gamma(\frac{n-3}{2})}{2^{\frac{n-1}{2}}\Gamma(\frac{n-1}{2})}\,dy$$

$$=\frac{1}{\sigma^2}\cdot\frac{1}{2}\cdot\frac{\Gamma(\frac{n-3}{2})}{\Gamma(\frac{n-1}{2})}$$

$(\frac{n-3}{2})\Gamma(\frac{n-3}{2})$

$\Rightarrow\ \frac{2\Gamma(\frac{n-1}{2})}{\Gamma(\frac{n-3}{2})\cdot S}$ is UMVUE For $\frac{1}{\sigma^2}$

$E(\frac{1}{S^2})=\frac{1}{6^4}\frac{2^{\frac{n-5}{2}}\Gamma(\frac{n-5}{2})}{2^{\frac{n-1}{2}}\Gamma(\frac{n-1}{2})}$

$=\frac{1}{6^4}\frac{1}{(n-3)(n-5)}$

$Var(\frac{1}{S})=\frac{1}{6^4}*\frac{2}{(n-3)^2(n-5)}$

$Variance=\frac{1}{6^4}\frac{2}{n-5}$

4 a) i) $\delta_c(\underline{X})=1$ if $X_{max}\geq C$ the critical function

$=0$, otherwise.

It means $C_1=\{\underline{x}:x_{max}\geq C\}$ critical region.

$Q(\theta)=P(X_{max}\geq C\mid\theta)=\int_C^{\theta}f_{max}(x)\,dx$

$=\int_C^{\theta}n\cdot\frac{x^{n-1}}{\theta^n}dx=1-(\frac{C}{\theta})^n$, $(\theta>C)$.

It's a monotone increasing function of θ.

ii) $\begin{cases}H_0:\theta=1/2\\H_1:\theta=3/4\end{cases}$

$Q(\theta_0=\frac{1}{2})=P(X_{max}\geq C\mid\theta=\frac{1}{2})=1-2^nC^n=0.05$.

$\Rightarrow C^n=0.95/2^n$ ie $C=0.95^{\frac{1}{n}}/2$. $(*)$

It is the UMP test for testing $\begin{cases}H_0:\theta\leq1/2\\H_1:\theta>1/2\end{cases}$ at $\alpha=0.05$

iii) $Q(\frac{3}{4})=P(\underline{x}\in C_1\mid\theta=\frac{3}{4})=0.98$.

$\Rightarrow 1-\frac{C^n}{(\frac{3}{4})^n}=0.98\Rightarrow 0.02*(\frac{3}{4})^n=C^n$ $(**)$

$(*,**)\Rightarrow n=\frac{\log 95/2}{\log 3/2}\simeq9.5217\simeq10$

iv) $X_{max} = 0.48$.

when $n=20$, then $p\text{-value} = 1 - \left(\frac{0.48}{0.5}\right)^{20} = 0.558$

then we don't reject H_0.

b) $\begin{cases} H_0 : \theta_{01} = \theta_{02} = 1/2 \\ H_1 : \text{otherwise} \end{cases}$

Since n is large enough, then we use Pearson goodness-of-fit-test

$$G = \frac{(560 - 1000 \times \frac{1}{2})^2}{1000 \times 1/2} + \frac{(440 - 1000 \times \frac{1}{2})^2}{1000 \times \frac{1}{2}} = 14.4$$

$\chi^2(1, 0.05) = 3.841 < G = 14.4$.

then reject H_0 and conclude the coin is not fair.

Q5. a)

i) $L(\mu_1, \mu_2, \mu_3, \sigma^2) = (2\pi\sigma^2)^{-\frac{3n}{2}} \exp\left\{-\frac{\Sigma(y_{1i}-\mu_1)^2 + \Sigma(y_{2i}-\mu_2)^2 + \Sigma(y_{3i}-\mu_3)^2}{2\sigma^2}\right\}$

$\log L(\mu_1, \mu_2, \mu_3, \sigma^2) = -\frac{3n}{2}\log 2\pi\sigma^2 - \frac{\Sigma(y_{1i}-\mu_1)^2 + \Sigma(y_{2i}-\mu_2)^2 + \Sigma(y_{3i}-\mu_3)^2}{2\sigma^2}$.

under $H_0 : \mu_1 = \mu_2 = \mu_0$.

$$\left.\begin{aligned} &\frac{\partial}{\partial \mu_0} \log L(\hat{\mu}_0, \hat{\mu}_3, \hat{\sigma}_0^2) = 0 \\ &\frac{\partial}{\partial \mu_3} \log L(\hat{\mu}_0, \hat{\mu}_3, \hat{\sigma}_0^2) = 0 \\ &\frac{\partial}{\partial \sigma^2} \log L(\hat{\mu}_0, \hat{\mu}_3, \hat{\sigma}_0^2) = 0 \end{aligned}\right\} \Rightarrow \begin{aligned} \hat{\mu}_0 &= \frac{\bar{Y}_1 + \bar{Y}_2}{2} \\ \hat{\mu}_3 &= \bar{Y}_3 \\ \hat{\sigma}_0^2 &= \frac{\Sigma(y_{1i}-\hat{\mu}_0)^2 + \Sigma(y_{2i}-\hat{\mu}_0)^2 + \Sigma(y_{3i}-\hat{\mu}_3)^2}{3n} \end{aligned}$$

Under H_1 : otherwise.

$$\left.\begin{aligned} &\frac{\partial}{\partial \mu_1} \log L(\hat{\mu}_1, \hat{\mu}_2, \hat{\mu}_3, \hat{\sigma}^2) = 0 \\ &\frac{\partial}{\partial \mu_2} \log L(\hat{\mu}_1, \hat{\mu}_2, \hat{\mu}_3, \hat{\sigma}^2) = 0 \\ &\frac{\partial}{\partial \mu_3} \log L(\hat{\mu}_1, \hat{\mu}_2, \hat{\mu}_3, \hat{\sigma}^2) = 0 \\ &\frac{\partial}{\partial \sigma^2} \log L(\hat{\mu}_1, \hat{\mu}_2, \hat{\mu}_3, \hat{\sigma}^2) = 0 \end{aligned}\right\} \Rightarrow \begin{aligned} \hat{\mu}_1 &= \bar{Y}_1 \\ \hat{\mu}_2 &= \bar{Y}_2 \\ \hat{\mu}_3 &= \bar{Y}_3 \\ \hat{\sigma}^2 &= \frac{\Sigma(y_{1i}-\bar{y}_1)^2 + \Sigma(y_{2i}-\bar{y}_2)^2 + \Sigma(y_{3i}-\bar{y}_3)^2}{3n} \end{aligned}$$

$$\lambda(\underset{\sim}{y_1}, \underset{\sim}{y_2}, \underset{\sim}{y_3}) = \left(\frac{\hat{\sigma}_0^2}{\hat{\sigma}^2}\right)^{-3n/2} = \left(\frac{\Sigma(y_{1i}-\hat{\mu}_0)^2 + \Sigma(y_{2i}-\hat{\mu}_0)^2 + \Sigma(y_{3i}-\hat{\mu}_3)^2}{\Sigma(y_{1i}-\bar{y}_1)^2 + \Sigma(y_{2i}-\bar{y}_2)^2 + \Sigma(y_{3i}-\bar{y}_3)^2}\right)^{-\frac{3n}{2}}$$

ii) $-2\log\lambda(\underset{\sim}{y}_1, \underset{\sim}{y}_2, \underset{\sim}{y}_3) = 3n\log\left(\frac{\Sigma(y_{1i}-\hat{\mu}_0)^2+\Sigma(y_{2i}-\hat{\mu}_0)^2+\Sigma(y_{3i}-\bar{y}_3)^2}{\Sigma(y_{1i}-\bar{y}_1)^2+\Sigma(y_{2i}-\bar{y}_2)^2+\Sigma(y_{3i}-\bar{y}_3)^2}\right)$

$\sim \chi^2(1)$

So $C_1 = \{(\underset{\sim}{y}_1, \underset{\sim}{y}_2, \underset{\sim}{y}_3) : -2\log\lambda(\underset{\sim}{y}_1, \underset{\sim}{y}_2, \underset{\sim}{y}_3) \geq \chi^2(1, \alpha)\}$

iii) $\Sigma(y_{1i}-\hat{\mu}_0)^2+\Sigma(y_{2i}-\hat{\mu}_0)^2 = \Sigma(y_{1i}-\bar{y}_1)^2+\Sigma(y_{2i}-\bar{y}_2)^2+n(\bar{y}_1-\hat{\mu}_0)^2+n(\bar{y}_2-\hat{\mu}_0)^2$

Then $\lambda(\underset{\sim}{y}_1, \underset{\sim}{y}_2, \underset{\sim}{y}_3) = \left(1+\frac{n(\bar{y}_1-\bar{y}_2)^2/2}{\Sigma(y_{1i}-\bar{y}_1)^2+\Sigma(y_{2i}-\bar{y}_2)^2+\Sigma(y_{3i}-\bar{y}_3)^2}\right)^{-\frac{3}{2}n}$

$C_1 = \{(\underset{\sim}{y}_1, \underset{\sim}{y}_2, \underset{\sim}{y}_3) : \lambda(\underset{\sim}{y}_1, \underset{\sim}{y}_2, \underset{\sim}{y}_3) \leq k_0\}$

$\Rightarrow C_1 = \{(\underset{\sim}{y}_1, \underset{\sim}{y}_2, \underset{\sim}{y}_3) : \frac{n(\bar{y}_1-\bar{y}_2)^2/2}{\Sigma(y_{1i}-\bar{y}_1)^2+\Sigma(y_{2i}-\bar{y}_2)^2+\Sigma(y_{3i}-\bar{y}_3)^2} \geq k.\}$

$\Rightarrow C_1 = \{(\underset{\sim}{y}_1, \underset{\sim}{y}_2, \underset{\sim}{y}_3) : \frac{n(\bar{y}_1-\bar{y}_2)^2/2}{[\Sigma(y_{1i}-\bar{y}_1)^2+\Sigma(y_{2i}-\bar{y}_2)^2+\Sigma(y_{3i}-\bar{y}_3)^2]/(3n-3)} \geq F_\alpha(1, 3n-3)\}$

b) i) $\begin{cases} H_0: \mu_A = \mu_B \\ H_1: \mu_A \neq \mu_B \end{cases}$ at $\alpha = 0.05$

$-2\log\lambda(\underset{\sim}{y}_1, \underset{\sim}{y}_2, \underset{\sim}{y}_3) = 3\cdot5\log\left(1+\frac{1.764}{34.928}\right) = 0.739 < \chi^2(1, 0.05) = 3.841$

22.804 + 13.888

18.672 + 2.368 + 13.888

can't reject H_0.

ii) $\frac{A}{\sigma^2} = \frac{\Sigma(y_{1i}-\bar{y}_1)^2+\Sigma(y_{2i}-\bar{y}_2)^2+\Sigma(y_{3i}-\bar{y}_3)^2}{\sigma^2} \sim \chi^2(3n-3)$

then 90% confidence interval for σ

$= \left[\sqrt{\frac{A}{\chi^2(12, 0.05)}}, \sqrt{\frac{A}{\chi^2(12, 0.95)}}\right] = [1.289, 2.585]$

iii) $\frac{\left[\left(\frac{\bar{Y}_1+\bar{Y}_2}{2}-\bar{Y}_3\right)-\left(\frac{\mu_1+\mu_2}{2}-\mu_3\right)\right]/\sqrt{\frac{3}{2}\frac{\sigma^2}{n}}}{\sqrt{A/(3n-3)\sigma^2}} \sim t(3n-3)$

estimate $= \frac{61.96+61.12}{2} - 56.12 = 5.42$

std. error $= \sqrt{\hat{\sigma}^2\left(\frac{1}{4n}+\frac{1}{4n}+\frac{1}{n}\right)} = \sqrt{\frac{34.928}{2*5*4}} = 0.9345$

So the 95% c.I for $\frac{\mu_A+\mu_B}{2}-\mu_C$ is

$\left[\frac{\bar{y}_1+\bar{y}_2}{2}-\bar{y}_3 \pm t_{\frac{\alpha}{2}}\sqrt{\frac{3A}{2(n-1)n}}\right] = [3.3837, 7.456]$

2.179

$E\left(\frac{\bar{Y}_1+\bar{Y}_2}{2}-\bar{Y}_3\right) = \frac{\mu_1+\mu_2}{2}-\mu_3$, $\frac{\bar{Y}_1+\bar{Y}_2}{2}-\bar{Y}_3$ is ~~the~~ an unbiased estimator