MATH3423 - Statistical Inference
Assignment 3

1. $X_1, X_2, ..., X_n$ are observations of a random sample of size n from the exponential distribution with mean $1/\theta$, i.e., $X_i \sim exp(\theta)$. Find the distribution of $2\theta \sum_{i=1}^{n} X_i$.

2. Q55 in Exercise 1

3. Q56 in Exercise 1

4. Let $X_1, \ldots, X_n$ be i.i.d. r.v.'s from the $U(\theta, 2\theta)$, $\theta \in \Omega = (0, \infty)$ distribution.

   (a) Find the p.d.f. of Y_1, $E(Y_1)$ and $Var(Y_1)$, where $Y_1 = \min(X_1, \ldots, X_n)$.

   (b) Find the p.d.f. of Y_n, $E(Y_n)$ and $Var(Y_n)$, where $Y_n = \max(X_1, \ldots, X_n)$.

5. Q23 in Exercise 2

6. Q5 in the midterm exam of 2015/2016

   Consider a random sample $\{X_1, X_2\}$ from density

$$f_X(x|\theta) = \frac{3x^2}{\theta^3} I_{(0<x<\theta)},$$

   where $\theta > 0$.

   (a) Are $\hat{\theta}_1 = \frac{2}{3}(X_1 + X_2)$ and $\hat{\theta}_2 = \frac{7}{6}\max(X_1, X_2)$ unbiased for θ?

   (b) Find the mean squared errors (MSEs) of $\hat{\theta}_1$ and $\hat{\theta}_2$, and compare those estimators.

   (c) Prove that in the sense of MSE, $T_{8/7}$ is the best estimator of θ among the estimators in form of $T_c = c\max(X_1, X_2)$.