MATH 243 Statistical Inference

Final Examination - Fall 2000/2001

Answer ALL Questions | **Date: 15 December 2000**
All Equal Marks | **Time Allowed: 3 hours**

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1. Let X be a r.v. distributed as $Bi(n,\theta)$ and set

$$\begin{aligned} g(\theta) = \Pr(X \le 2) &= \sum_{x=0}^{2} \binom{n}{x} \theta^x (1-\theta)^{n-x} \\ &= (1-\theta)^n + n\theta(1-\theta)^{n-1} + \binom{n}{2} \theta^2 (1-\theta)^{n-2} \end{aligned}$$

On the basis of r independent r.v.'s $X_1, \ldots, X_r$ distributed as X, find the UMVU estimator of $g(\theta)$.

Hint: Find the UMVUE of each term in $g(\theta)$ separately.

2. Let $X_1, \ldots, X_n$ be i.i.d. r.v.'s from the Negative Binomial distribution with parameter $\theta \in \Omega = (0,1)$, i.e.

$$F_X(x,\theta) = \binom{k+x-1}{x} \theta^k (1-\theta)^x .$$

(a) Find the UMVU estimator of $g(\theta) = 1/\theta$ and determine its variance.
(b) Investigate whether the Cramer-Rao lower bound is attained.
(c) What is the MLE of θ?

3. Let $X_1, \ldots, X_n$ be i.i.d. r.v.'s from $N(\mu, \theta)$ with μ known.

(a) Construct the UMP test for $H_o : \theta = \theta_o$ against $H_A : \theta > \theta_o$ at level of significance α.
(b) Calculate the power at $\theta = 12$ when $\theta_o = 4$, $\alpha = 0.05$ and $n = 25$.

4. Let $X_1, \ldots, X_m$ and $Y_1, \ldots, Y_n$ be two independent random samples with p.d.f.'s f_1 and f_2, respectively, given below

$$\begin{aligned} f_1(x;\theta_1) &= \frac{1}{\theta_1} \exp\left(-\frac{x}{\theta_1}\right) I_{(0,\infty)}(x) \qquad \theta_1 \in \Omega = (0,\infty) , \\ f_2(y;\theta_2) &= \frac{1}{\theta_2} \exp\left(-\frac{y}{\theta_2}\right) I_{(0,\infty)}(y) \qquad \theta_2 \in \Omega . \end{aligned}$$

Derive the likelihood ratio test in the form of an F test for testing the hypothesis $H_o : \theta_1/\theta_2 = \Delta_o$ against $H_A : \theta_1/\theta_2 \ne \Delta_o$ at level of significance α.

Hint: If $W \sim Gamma(\alpha, \beta)$, then $2\beta W \sim \chi^2_{2\alpha}$.

5. A coin, with probability P of falling heads, is tossed 100 times and 60 heads are observed. At the level of significance $\alpha = 0.1$.

   (a) Test the hypothesis $H_o : P = \frac{1}{2}$ against the alternative $H_A : P \neq \frac{1}{2}$ by using the likelihood ratio test and employ the normal approximation to determine the critical point. Is the null hypothesis rejected?

   (b) Test the same hypothesis by means of the chi-squared goodness-of-fit test and determine the critical point. Is the null hypothesis rejected?

   Hint: Use the formula directly. No proof is needed.