Tutorial for 11-17 and 11-20.

Chapter 1 & 2: How to estimate unknown parameters
Chapter 3: More practical case (Verification)

- An example:

New medicine to treat Cancer.
Before it is allowed to widely use, effectiveness must be tested.
Find 2 group of patients. A. B ← the number of cancer cells.

A: take the new medicine
B: don't ... ...

A: $x_{A1} \rightarrow x_{A2}$
B: $x_{B1} \rightarrow x_{B2}$

Can we get the conclusion that the medicine works only with the evidence $x_{A2} < x_{B2}$?)) (Make sense or not ?)

↓

Need hypothesis testing.

It's common that we need to get our conclusion from samples, rather than population.
Is it convincing ?

HT is designed to tell us how much confidence we have to draw the conclusion.

• Hypothesis Testing.

Example 2: Weight Loss for Diet vs Exercise

Did dieters lose more fat than the exercisers?

Diet Only:

sample mean = 5.9 kg
sample standard deviation = 4.1 kg
sample size = n = 42
standard error = $SEM_1 = 4.1/\sqrt{42} = 0.633$

Exercise Only:

sample mean = 4.1 kg
sample standard deviation = 3.7 kg
sample size = n = 47
standard error = $SEM_2 = 3.7/\sqrt{47} = 0.540$

$\sigma = 0.025$

standard error $= \frac{S}{\sqrt{n}}$

$= \frac{\text{Sample deviation}}{\sqrt{\text{sample size}}}$

Step 1: Determine H_0, H_1. { ① Complete ② equal sign in H_0

$$\begin{cases} H_0: \mu = 0 \\ H_1: \mu \neq 0 \end{cases}$$

μ = SampleMean_Diet − SampleMean_exe.

Step 2: always skip it → "theorical proof."

Step 3: collect data

$$\begin{cases} \text{Diet } 100: x_1, x_2, \ldots, x_{42} \\ \text{Exercise } 100: y_1, y_2, \ldots, y_{47} \end{cases}$$

200 ppl in total.

x, y are weights for each person.

$$\bar{x} = \sum_{i=1}^{42} x_i / n = 5.9$$

$$\bar{y} = \sum_{i=1}^{47} y_i / n = 4.1$$

$\hat{\mu} = \bar{x} - \bar{y} = 5.9 - 4.1 = 1.8$ (SE = 0.633 − 0.54 = 0.83)

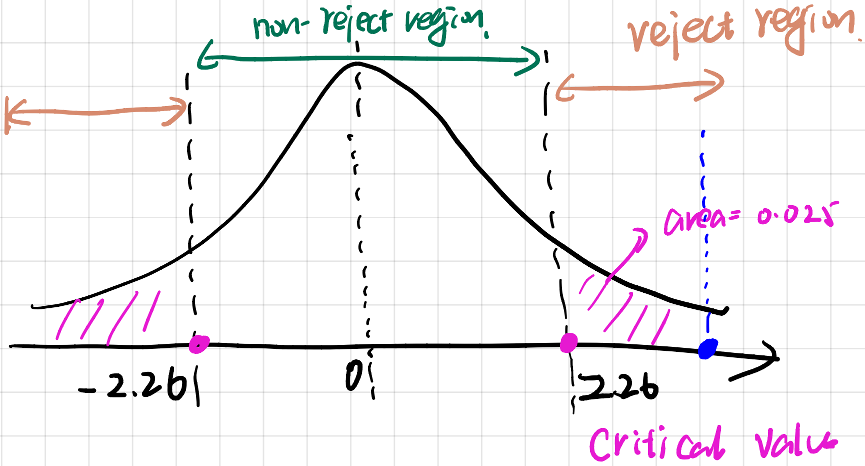
Futher, we don't know the true variance.

t-statistic $= \frac{\hat{\mu} - \mu}{SE} \sim t(100-1)$

SE → $\sigma/\sqrt{n}$ if σ is known.

At significant level $\alpha = 0.05$

$$t = \frac{\hat{\mu} - \mu}{SE} = \frac{1.8 - 0}{0.83} = 2.17$$



Conclusion: reject H_0

$\begin{cases} \text{Type I error: reject a true } H_0 \ (\alpha) \\ \text{Type II error: accept a false } H_0 \ (\beta) \end{cases}$ Cannot control both at the same time

the power of statistic test $= 1 - \beta$

- Likelihood Ratio test (LRT)

Definition: A likelihood ratio test (LRT) for tesing $H_0: \theta \in \Theta_0$ against $H_1: \theta \in \Theta_1$ at a significance level of α is a test with a rejection region

$$C_1 = \{x_1, \dots x_n: \lambda(x) \le k\},$$

→ work as the same function as critical value

where $k \in (0,1)$ satisfies $\sup_{\theta \in \Theta_0} P(\lambda(X) \le k \mid \theta) = \alpha$, and the LRT statistic

Likelihood ratio test statistic $\lambda(x) = \frac{L(\hat{\theta}_0)}{L(\hat{\theta})}$ $L(\hat{\theta}_0)$ → MLE over Θ_0; $L(\hat{\theta})$ → MLE over Θ

with an MLE $\hat{\theta}_0$ of θ over Θ_0 and an MLE $\hat{\theta}$ over $\Theta = \Theta_0 \cup \Theta_1$.

① $0 < \lambda(x) \le 1$

② $\lambda(x) \to 0$ not compatible with H_0 → reject H_0 ($\lambda < k$) Constant

$\lambda(x) \to 1$ compatible with H_0 → not reject H_0 ($\lambda > k$)

③ If it is simple hypothesis test, hypothesis value should be used. (Not MLE)