MATH 3423 Statistical Inference

Assignment 3

Please submit your solution (in pdf) on Canvas before 4pm on Nov 27, 2020.

Question 1: Consider a random sample $\{X_1, \dots, X_n\}$ of size $n > 1$ from $N(\mu, \sigma^2)$, where $\mu \in (-\infty, \infty)$ and $\sigma^2 \in (0, \infty)$ are both unknown. Show that $\bar{X}$ and S_{n-1}^2 are the respective UMVUE of μ and σ^2.

Question 2: Show that if $T(X_1, \dots, X_n)$ is the *UMVUE* of $g(\theta)$, then $aT(X_1, \dots, X_n) + b$ is the *UMVUE* of $ag(\theta) + b$, where b is a constant and a is a non-zero constant.

Remark that on the lecture note, we know that if $T(X_1, \dots, X_n)$ and $g(\theta)$ satisfy the required form stated in Theorem 1, then the above result is definitely true. Here we want to show that it is true more generally. That is, for this question, $T(X_1, \dots, X_n)$ and $g(\theta)$ are not necessary to satisfy the required form in Theorem 1.

Question 3: Consider a random sample $\{X_1, \dots, X_n\}$ of size $n > 1$ from a distribution with a pdf given by

$$f(x|\theta) = e^{-(x-\theta)} I_{(\theta,\infty)}(x),$$

where θ is real and unknown.

a) Show that the smallest order statistic $X_{(1)}$ is a sufficient statistic for θ.
b) Show that $X_{(1)}$ is also complete.
c) Find the UMVUE of θ.

Question 4: Let a random sample of size n be taken from a uniform discrete distribution with its pmf given by

$$p(x|\theta) = 1/\theta$$

for $x = 1,2, \dots, \theta$ and $p(x|\theta) = 0$ elsewhere, where θ is an unknown positive integer.

a) Show that the largest order statistic, say Y, of the sample is a complete and sufficient statistic for θ.
b) Prove that

$$\frac{Y^{n+1} - (Y-1)^{n+1}}{Y^n - (Y-1)^n}$$

is the UMVUE for θ.

Hint: $P(Y = y) = P(Y \le y) - P(Y < y)$= $P(Y \le y) - P(Y \le y - 1)$.