The Hong Kong University of Science & Technology

MATH243 – Statistical Inference

Midterm Examination – Fall 01/02

Answer ALL questions **Date: 29 October 2001 (Mon)**

All Equal Marks **Time allowed: 2 Hours**

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1. Let $X_1, \ldots, X_m$ and $Y_1, \ldots, Y_n$ be two independent random samples with the same mean θ and known variances σ_1^2 and σ_2^2, respectively. Then show that for every $c \in [0, 1]$, $U = c\bar{X} + (1 - c)\bar{Y}$ is an unbiased estimator of θ. Also find the value of c for which the variance of U is minimum.

2. Let $X_1, \ldots, X_n$ be i.i.d. r.v.'s from Uniform (α, β). Find
   (i) the maximum likelihood estimators of α and β, $\hat{\alpha}$ and $\hat{\beta}$.
   (ii) the distributions of $\hat{\alpha}$ and $\hat{\beta}$ and their joint distribution.

3. Let $X_1, \ldots, X_n$ be independent r.v.'s distributed as Uniform $(\theta - a,\ \theta + b)$, where $a,\ b > 0$ are known and $\theta \in R$. Find the moment estimator for θ and calculate its variance.

4. (i) Let $X_j,\ Y_j,\ j = 1, \ldots, n,$ be independent r.v.'s such that the X's are identically distributed with $E(X_j) = \mu_1$, $Var(X_j) = \sigma^2$, both finite, and the Y's are identically distributed with $E(Y_j) = \mu_2$, $Var(Y_j) = \sigma^2$, both finite. Write down the asymptotic distribution of $\bar{X}_n - \bar{Y}_n$. No proof is needed.
   (ii) From a large collection of bolts which is known to contain 3% defective bolts, 1000 bolts are chosen at random. If X is the number of the defective bolts among those chosen, what is the approximated probability that this number does not exceed 5% of 1000?

5. Let $X_1,\ X_2,\ X_3$ be independent r.v.'s distributed as $N(0, 1)$ and set

$$Y_1 = -\frac{1}{\sqrt{2}} X_1 + \frac{1}{\sqrt{2}} X_2 \ ,$$

$$Y_2 = -\frac{1}{\sqrt{3}} X_1 - \frac{1}{\sqrt{3}} X_2 + \frac{1}{\sqrt{3}} X_3 \ ,$$

$$Y_3 = \frac{1}{\sqrt{6}} X_1 + \frac{1}{\sqrt{6}} X_2 + \frac{2}{\sqrt{6}} X_3 \ .$$

   Then show that the r.v.'s $Y_1,\ Y_2,\ Y_3$ are independent and as a consequence of it write down the mean vector and the variance and covariance matrix of $\underset{\sim}{Y} = (Y_1,\ Y_2,\ Y_3)$. What is the distribution of $\underset{\sim}{Y}$?

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