# The Hong Kong University of Science & Technology

## MATH243 – Statistical Inference

## Mid-term Examination – Fall 04/05

**Answer ALL questions** **Date: 11 Nov 2004 (Thu)**

**Time allowed: 1 hour 20 minutes**

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1. **(6 marks)** Let $X_1$, $X_2$ be r.v.'s having the bivariate normal distribution with parameters $\mu_1$ (mean of $X_1$), $\mu_2$ (mean of $X_2$), $\sigma_1$ (standard deviation of $X_1$), $\sigma_2$ (standard deviation of $X_2$), $\rho$ (correlation coefficient between $X_1$ and $X_2$). Set

$$Y_1 = \frac{X_1-\mu_1}{\sigma_1} + \frac{X_2-\mu_2}{\sigma_2}, \; Y_2 = \frac{X_1-\mu_1}{\sigma_1} - \frac{X_2-\mu_2}{\sigma_2}.$$

Find the p.d.f.'s of the r.v.'s $Y_1$ and $Y_2$. Are they independent?

2. Let $X_1, \ldots, X_n$ be a random sample from $f(x;\lambda) = \dfrac{e^{-\lambda}\lambda^x}{x!}$ for $x = 0, 1, 2, \ldots$

(a) Find Cramer Rao lower bound for the variance of an unbiased estimator for $\dfrac{e^{-\lambda}\lambda^k}{k!}$ for any $k = 0, 1, 2, \ldots$ **(3 marks)**

(b) Is the proportion of observations in the sample equal to $k$, i.e., $\dfrac{1}{n}\sum_{i=1}^{n} I_{\{k\}}(X_i)$, unbiased? Note that $I_{\{k\}}(x_i) = 1$ if $x_i = k$; $I_{\{k\}}(x_i) = 0$ otherwise. **(2 marks)**

(c) Find the variance of the estimator in (b). **(3 marks)**

(d) Find the sufficient statistic for $\lambda$. Hence and otherwise, find the UMVUE of $\dfrac{e^{-\lambda}\lambda^k}{k!}$. **(5 marks)**.

3. Let $X_1, \ldots, X_n$ be independent r.v.'s distributed as Uniform $(\theta - a,\ \theta + b)$, where $a$, $b > 0$ are known and $\theta \in R$.

(a) Find the method of moments estimator for $\theta$. Is it unbiased? **(3 marks)**

(b) Find the variance of the estimator in (a). **(2 marks)**

(c) Find the maximum likelihood estimator for $\theta$. **(2 marks)**

(d) Is the mid-range $Y = (Y_n - b + Y_1 + a)/2$ where $Y_1 = \min(X_i)$ and $Y_n = \max(X_i)$ an unbiased estimator for $\theta$. **(4 marks)**

(e) Find the variance of the estimator $Y$ in (d). Is the variance of the estimator in (a) greater? **(Bonus: 4 marks)**

Hint: $f_{Y_1,Y_n}(y_1,y_n) = n(n-1)(y_n - y_1)^{n-2}\big/(a+b)^n$ for $\theta - a \le y_1 \le y_n \le \theta + b$.

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