

**MATH3423 - Statistical Inference**  
**Assignment 5**

1. Q19 in Exercise 3
2. Q24 in Exercise 3
3. Consider a random sample  $\{X_i : i = 1, 2, \dots, n\}$  of size  $n > 2$  from  $N(\mu, \sigma^2)$ , where  $\mu$  and  $\sigma^2$  are unknown. Define  $S_{n-1}^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$ .
  - (a) Find the UMVUE of  $\frac{1}{\sigma}$ .
  - (b) Find the UMVUE of  $\frac{\mu}{\sigma}$ .
4. Q2 in the final exam of 2013/2014

Let  $X_1, \dots, X_n$  be a random sample from a location distribution family

$$f(x; \theta) = \frac{1}{\theta} \exp\left(-\frac{x - \delta}{\theta}\right) I(x \geq \delta).$$

Note that  $X_i - \delta \sim \exp\left(\frac{1}{\theta}\right)$ .

- (a) Assume that  $\delta$  is known.
  - i. Find the complete and sufficient statistic of the unknown parameter  $\theta$ . What is its distribution?
  - ii. Find the UMVUE of  $\theta$ .
  - iii. Find the UMVUE of  $Pr(X_1 > 1)$  when  $\delta < 1$ .
- (b) Assume that  $\theta$  is known.
  - i. Find the complete and sufficient statistic of the unknown parameter  $\delta$ . What is its distribution?
  - ii. Find the UMVUE of  $\delta$ .
  - iii. Find the UMVUE of  $Pr(X_1 > 1)$  when  $\delta < 1$ .
5. Q1 in the final exam of 2014/2015

Let  $X_1, \dots, X_n$  be a random sample from the Bernoulli( $\theta$ ), where  $\theta$  is the unknown parameter.

- (a) Find the complete and sufficient statistic for  $\theta$ . Find its distribution.
- (b) Find the UMVUE for  $\theta^2$ .
- (c) Find the CRLB for  $\theta^2$ . Is the variance of the UMVUE for  $\theta^2$  equal to its CRLB? Explain in details.
- (d) Find the limiting distribution of the maximum likelihood estimator for  $\theta^2$  as  $n \rightarrow \infty$  by Delta method. What phenomenon do you observe?
- (e) Find the UMVUE of  $P(X_1 + X_2 + X_3 = 1)$ .
- (f) Find the maximum likelihood estimator for the variance of  $\sum X_i$ , i.e.,  $n\theta(1 - \theta)$ . Is it unbiased? Hence or otherwise, find the UMVUE for the variance of  $\sum X_i$ .