MATH 243 Statistical Inference

Midterm Examination - Fall 98/99

---

1. (a) A candy maker produces mints that have a label weight of 20.4 grams. Assume that the distribution of the weights of these mints is $N(21.37, 0.16)$.

   (i) Let X denote the weight of a single mint selected at random from the production line. Find $P(X < 20.857)$.

   (ii) During a particular shift 100 mints are selected at random and weighed. Let Y equal the number of these mints that weigh less than 20.857 grams. Find approximately $P(Y \leq 5)$.

   (iii) Let $\bar{X}$ equal the sample mean of the 100 mints selected and weighed on a particular shift. Find $P(21.31 \leq \bar{X} \leq 21.39)$.

   (b) Let Y denote the sum of the items of a random sample of size 12 from a distribution having p.d.f. $f(x) = \frac{1}{6}, x = 1, 2, 3, 4, 5, 6$, zero elsewhere. Compute an approximate value of $Pr(36 \leq Y \leq 48)$.

2. Let $X_1, ..., X_n$ be iid exponential (λ), i.e. $f_{X_i}(x_i) = \dfrac{1}{\lambda} e^{-\frac{x_i}{\lambda}}$, $0 \leq x_i < \infty$

   (a) Find an unbiased estimator of λ based only on $Y = \min\{X_1, ..., X_n\}$.
   Hint: Find the distribution of Y and then $E(Y)$.

   (b) Guess an unbiased estimator for λ. Show that it is better than the one in part (a) using the Cramer-Rao lower bound.
   Hint: There is no need to find the variance of an unbiased estimator in part (a).

3. Suppose that the random variables $Y_1, ..., Y_n$ satisfy

$$Y_i = \beta x_i + \varepsilon_i \qquad i = 1, ..., n,$$

   where $x_1, ..., x_n$ are fixed constants, and $\varepsilon_1, ..., \varepsilon_n$ are iid $N(0, \sigma^2), \sigma^2$ unknown.

   (a) Find the MLE of β and show that it is an unbiased estimator of β.

   (b) Find the distribution of the MLE of β.

   (c) Show that $\Sigma Y_i / \Sigma x_i$ is an unbiased estimator of β.

   (d) Calculate the variance of $\Sigma Y_i / \Sigma x_i$ and compare it to the variance of the MLE.
   Hint: Write $\varepsilon_i = Y_i - \beta x_i$ and then find the likelihood function of β and σ^2 in terms of Y_i and x_i.