Note 4: Given complex $Z(u,t)$ being WSS,

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$$\mathrm{Re}\{Z(u,t)\} = \frac{1}{2}\left[Z(u,t) + Z^*(u,t)\right]$$

① $$E\{\mathrm{Re}\{Z(u,t)\}\} = \frac{1}{2}\left[\eta_Z + \eta_Z^*\right], \text{ constant}$$

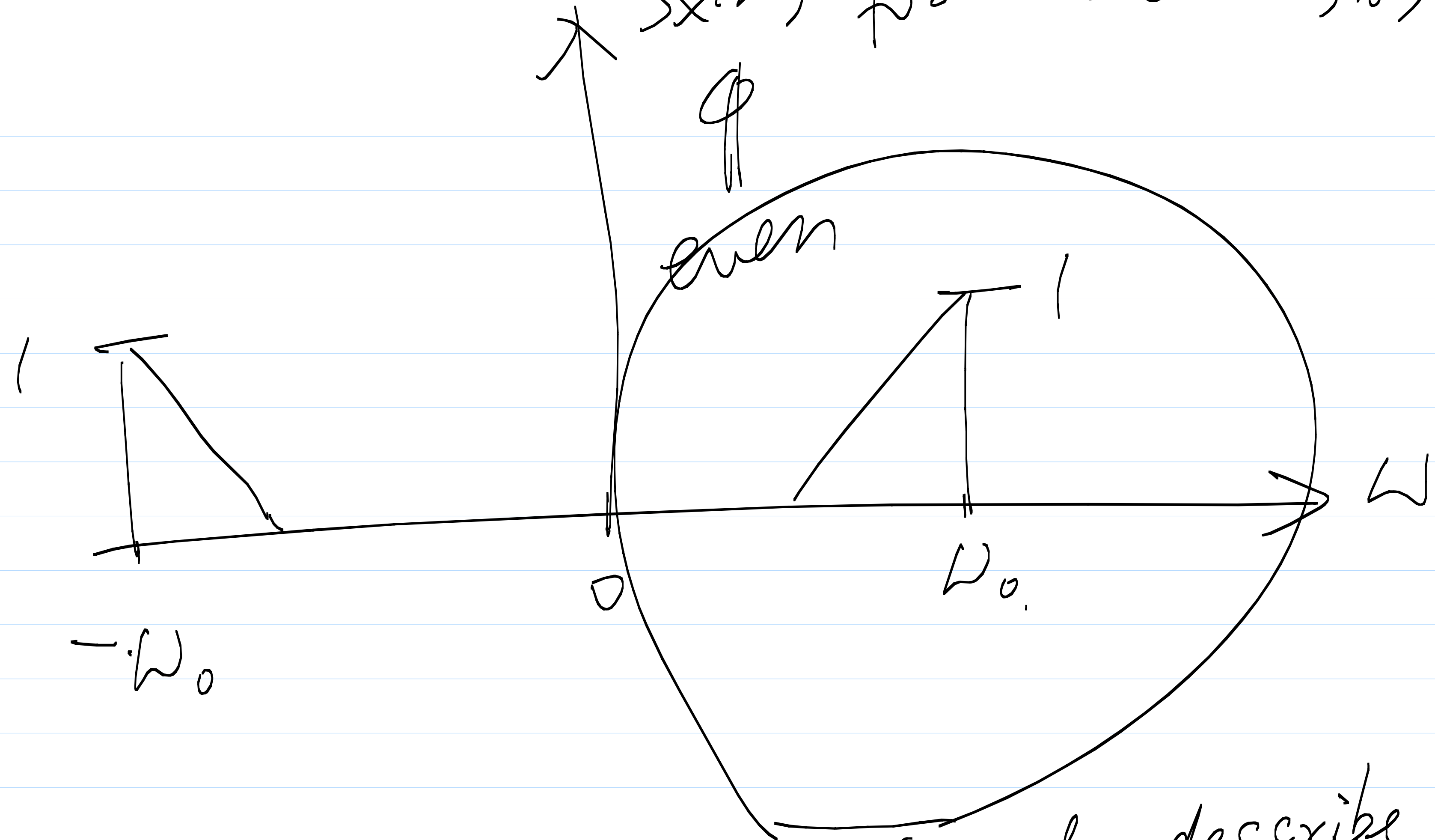
② $$E\{\mathrm{Re}\{Z(u,t_1)\}\,\mathrm{Re}\{Z(u,t_2)\}\}$$

$$= \frac{1}{4}\Big\{ E\{Z(u,t_1)Z^*(u,t_2)\} \longrightarrow R_Z(t_1-t_2)$$

$$+ E^*\{Z(u,t_1)Z^*(u,t_2)\} \longrightarrow R_Z^*(t_1-t_2)$$

$$\left.\begin{array}{l} + E\{Z(u,t_1)(Z^*(u,t_2))^*\} \\ + E^*\{Z(u,t_1)(Z^*(u,t_2))^*\}\Big\} \end{array}\right\} \rightarrow \text{a function of } t_1 - t_2\,?$$

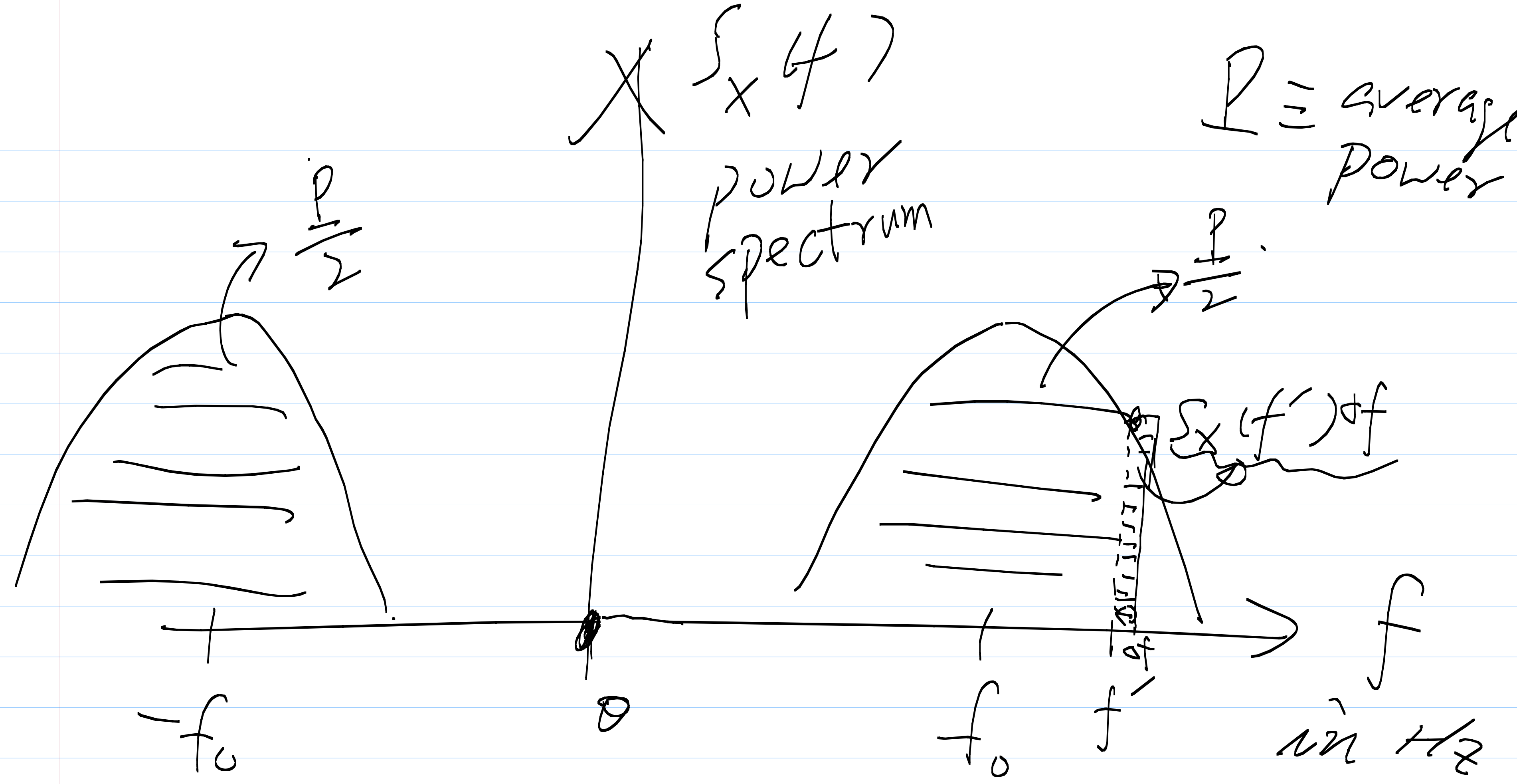
$S_x(\omega)$ for real $x(u,t)$



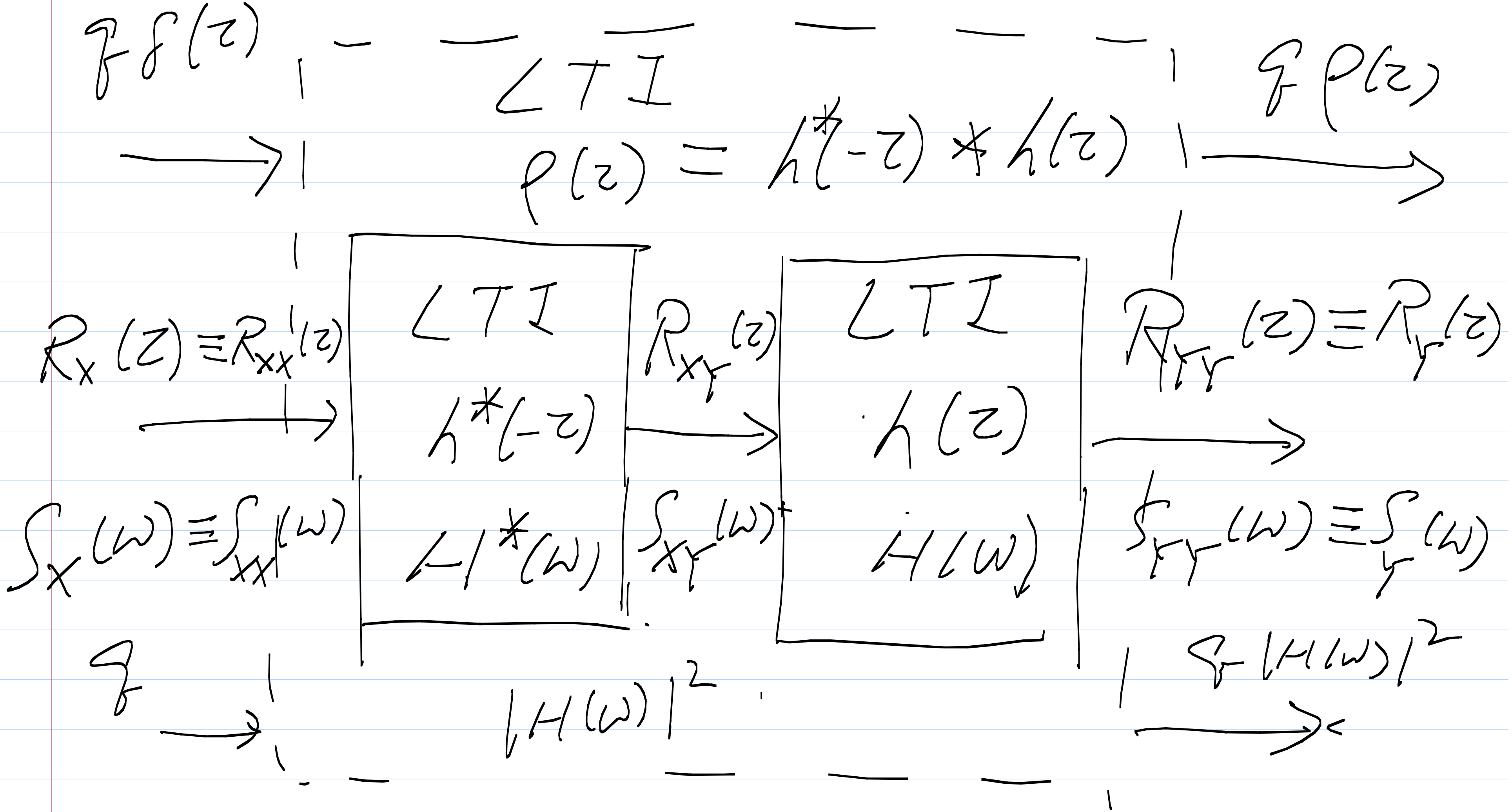
suffices to describe such $S_x(\omega)$

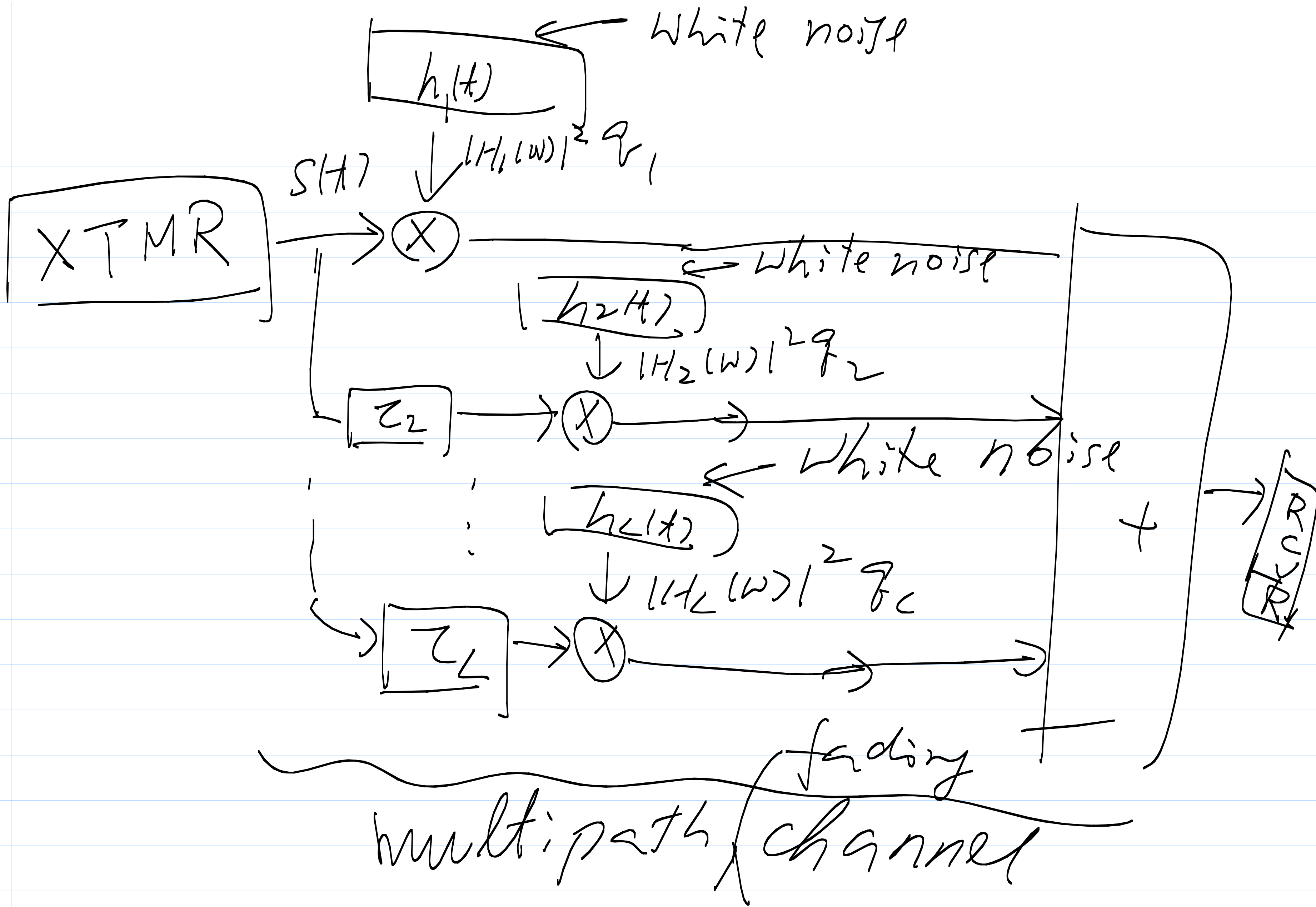
For any rv's $X(u)$ and $Y(u)$ and for any function $f_0(x, y)$,

$$E\{f_0(X(u), Y(u))\} = E\{\underbrace{E\{f_0(X(u), Y(u)) \mid X(u)\}}_{\substack{\| \\ \text{a function of } X(u) \\ \| \\ \int_{-\infty}^{\infty} f_0(X(u), y) f_{Y|X}(y \mid X(u)) dy}}\},$$



Power Spectrum $\equiv$ Power Spectral Density


$\mathcal{F}\,\delta(\tau)$
LTI
$\rho(\tau) = h^*(-\tau) * h(\tau)$
$\mathcal{F}\,\rho(\tau)$
$R_X(\tau) \equiv R_{XX}(\tau)$
LTI
$h^*(-\tau)$
$H^*(\omega)$
$R_{XY}(\tau)$
LTI
$h(\tau)$
$H(\omega)$
$R_{YY}(\tau) \equiv R_Y(\tau)$
$S_X(\omega) \equiv S_{XX}(\omega)$
$S_{XY}(\omega)$
$S_{YY}(\omega) \equiv S_Y(\omega)$
$\mathcal{F}$
$|H(\omega)|^2$
$\mathcal{F}\,|H(\omega)|^2$


White noise
$h_1(t)$
$|H_1(\omega)|^2 q_1$
$s(t)$
XTMR
White noise
$h_2(t)$
$|H_2(\omega)|^2 q_2$
τ_2
White noise
$h_L(t)$
$|H_L(\omega)|^2 q_L$
τ_L
+
RCVR
fading
multipath channel

An $n \times n$ matrix Λ is called positive definite iff, for any (nonnegative) $n \times 1$ vector $\underline{\delta}$, $\underline{\delta}^T \Lambda \underline{\delta} \underset{(-)}{>} 0$.

Properties:

Λ is positive definite

$\Rightarrow$ $|\Lambda| > 0$

iff Λ^{-1} exists

iff Λ has n positive eigenvalues

Let λ_i and $\underline{e}_i$ be the i^{th} eigenvalue and eigenvector, respectively, of $\underline{\Lambda}_Y$, $i = 1, 2, \cdots, n$. Thus,

$$\underline{\Lambda}_Y \underline{e}_i = \lambda_i \underline{e}_i \quad , i = 1, 2, \cdots, n$$

$$\Longrightarrow \underline{\Lambda}_Y \underbrace{\left[\underline{e}_1, \underline{e}_2, \cdots, \underline{e}_n \right]}_{\substack{E \\ \text{eigenmatrix}}} = \left[\underline{e}_1, \underline{e}_2, \cdots, \underline{e}_n \right] \cdot \underbrace{\begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ 0 & & \ddots & \\ & & & \lambda_n \end{bmatrix}}_{D}$$

$\Longleftrightarrow -\Lambda_Y E = ED$ where $EE^T = E^TE = I$

$\Longleftrightarrow -\Lambda_Y = EDE^T$ is the eigen decomposition of $-\Lambda_Y$.

Let $Z(u,t)$ be WSS. Then,

$$\text{"}X(u,t) + jY(u,t)\text{"}$$

① $E\{Z(u,t)\} = E\{X(u,t)\} + jE\{Y(u,t)\}$

$= \eta_X + j\eta_Y \equiv$ a constant

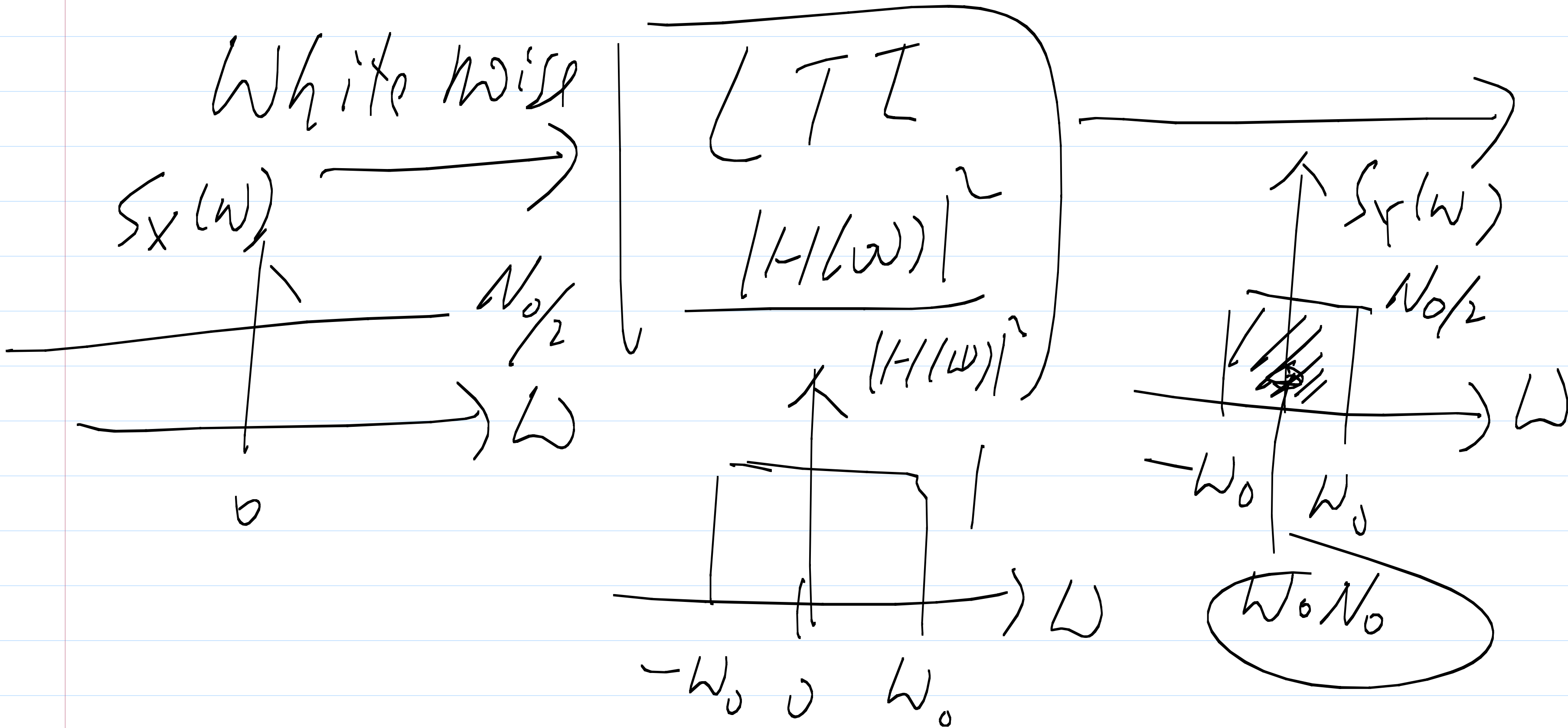
② $E\{Z(u,t_1)Z^*(u,t_2)\} = E\{X(u,t_1)X(u,t_2)\} - E\{Y(u,t_1)Y(u,t_2)\}$

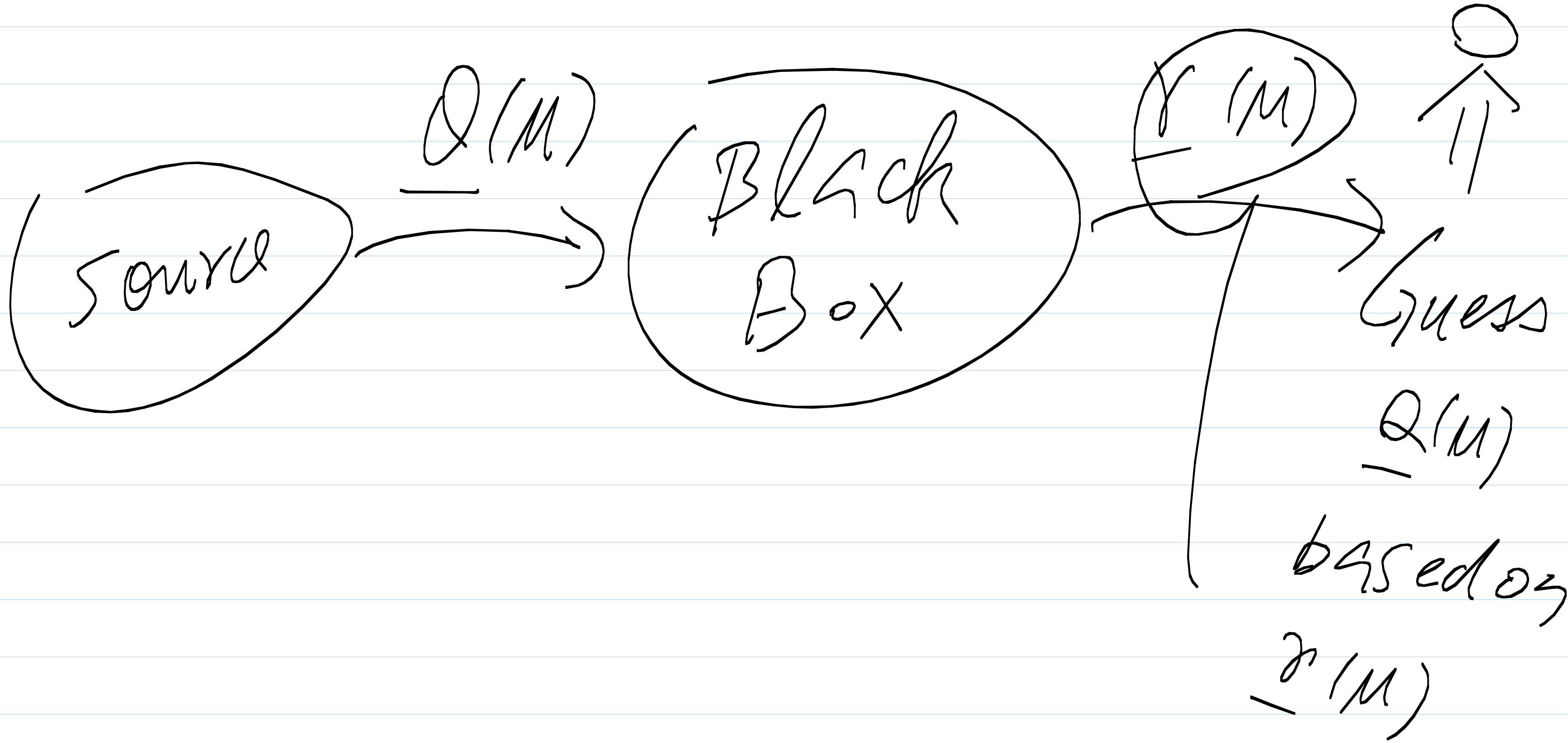
$+ j\left[-E\{X(u,t_1)Y(u,t_2)\} + E\{X(u,t_2)Y(u,t_1)\}\right]$

$= \left[R_X(t_1-t_2) - R_Y(t_1-t_2)\right] + j\left[-R_{XY}(t_1-t_2) + R_{YX}(t_1-t_2)\right].$

$\equiv$ a fx of $t_1 - t_2$,

if $X(u,t)$ and $Y(u,t)$ are jointly WSS.


White noise
Sx(w)
N0/2
w
0
LTI
|H(w)|^2
|H(w)|^2
1
-w0
0
w0
w
Sy(w)
N0/2
-w0
w0
w
w0N0


Source
$\underline{Q}(M)$
Black Box
$\underline{\gamma}(M)$
Guess $\underline{Q}(M)$ based on $\underline{\gamma}(M)$