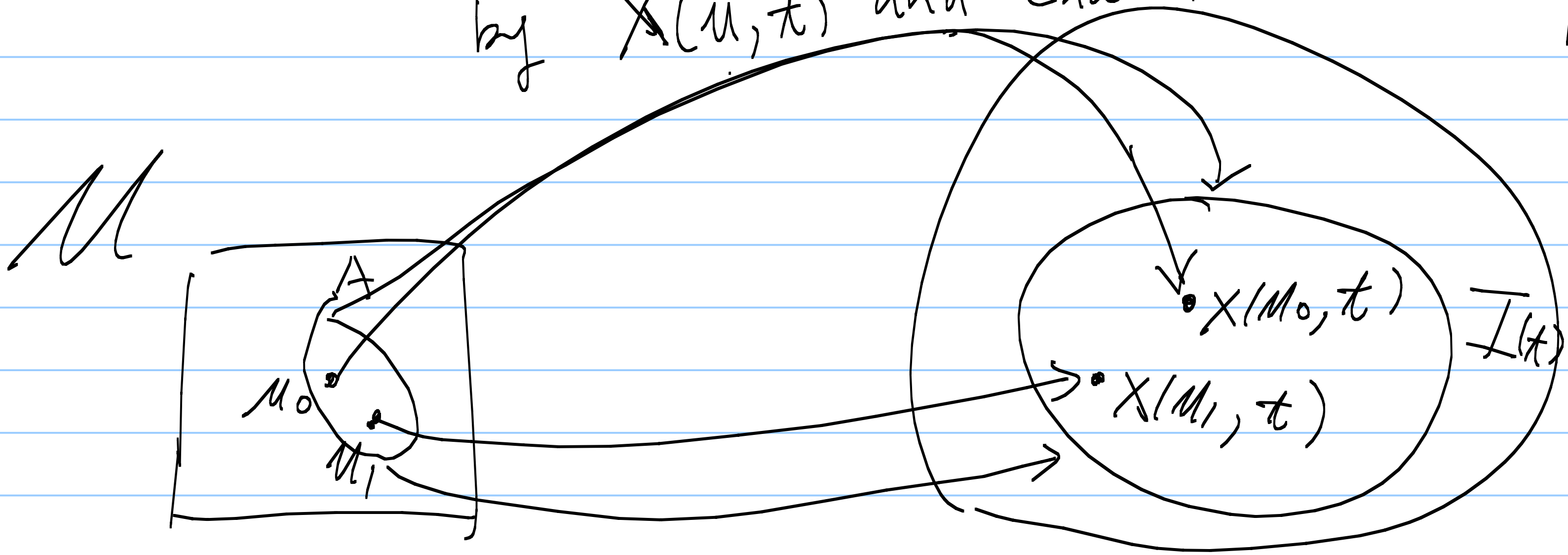
Note 3

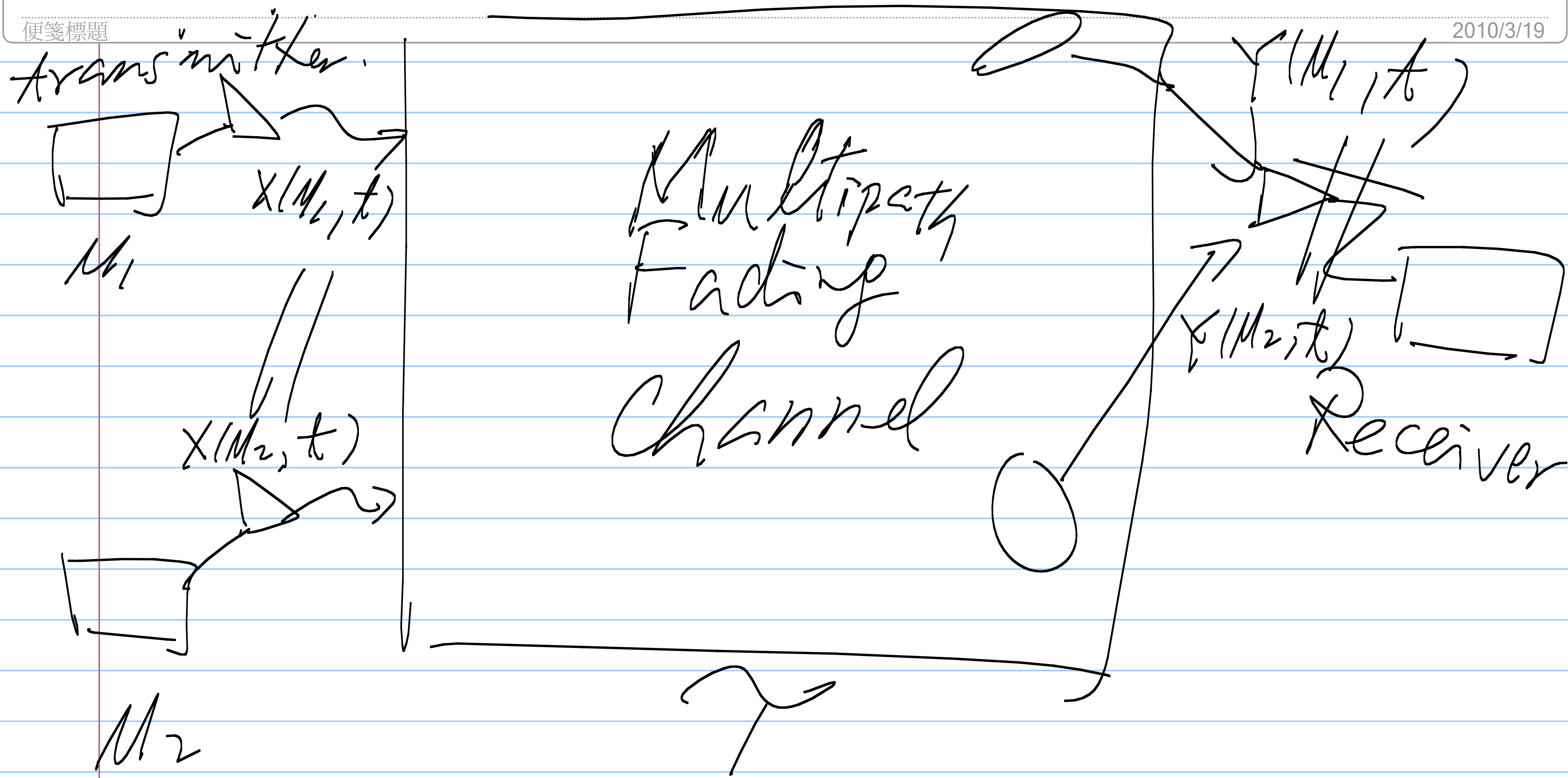
The mapping is denoted by $X(u,t)$ and called a random process.



domain

range (a set of functions of t)

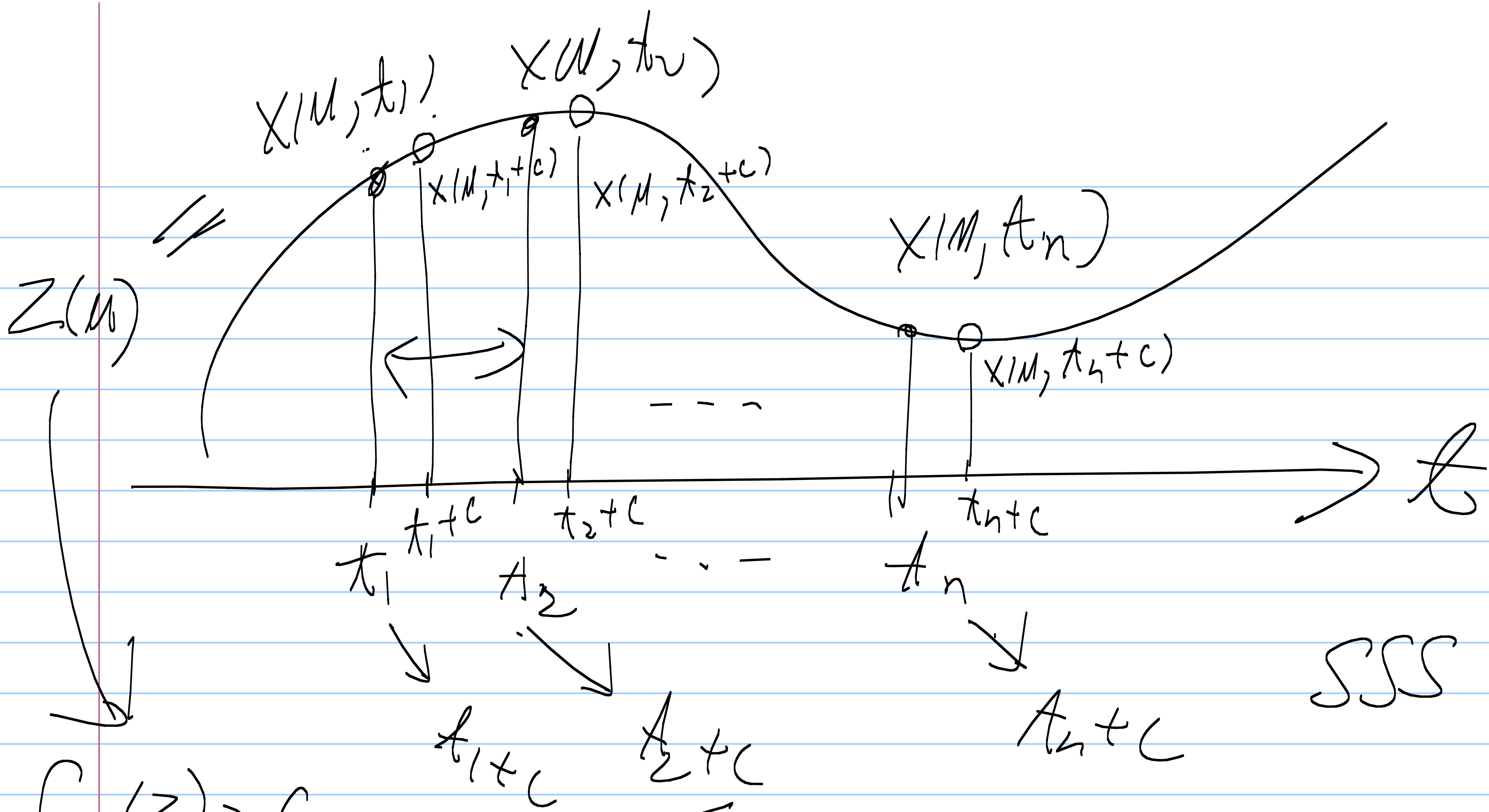
$$A = \{X(u,t) \in I(t)\}$$


transmitter.
X(M1,t)
M1
X(M2,t)
M2
Multipath
Fading
Channel
Y(M1,t)
Y(M2,t)
Receiver

$Y(u,t) = T[X(u,t)]$ is a linear system, if, for any n rp's $X_1(u,t), X_2(u,t), \cdots, X_n(u,t)$ and for any n constants $a_1, a_2, \cdots, a_n$, then

$$\overset{\text{linear}}{T}\left[\overset{\text{linear}}{\sum_{i=1}^{n}} a_i X_i(u,t)\right] = \sum_{i=1}^{n} a_i \, T\left[X_i(u,t)\right]$$

Principle of Superposition



$$f_Z(z) = f_Z(z; t_1) \Rightarrow \int z f(z; t_1)\, dz = \text{constant}$$

Recall that $E\{f(X(u))g(Y(u))\}$

$$= E\{f(X(u))\}E\{g(Y(u))\}$$

if $X(u)$ and $Y(u)$ are independent.

$P1 := P(A|B) = \dfrac{P(A\cap B)}{P(B)}$ given B

Test ① $P(A|B) \geq 0$ for any A,

② $P(U|B) = 1$ for space U.

③ $P(A\cup C|B) = P(A|B) + P(C|B)$

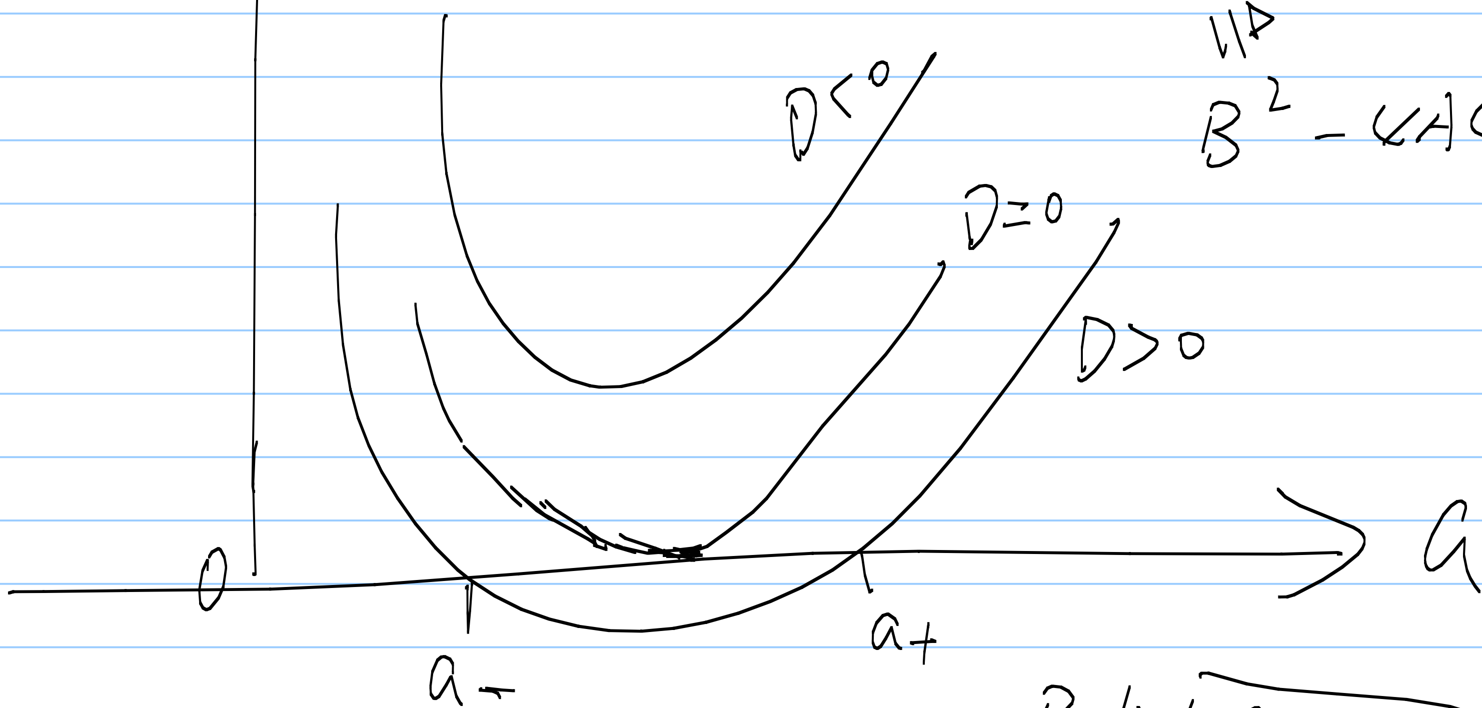
when $A \cap C = \emptyset$, for any A and C,

$$C_z(t_1, t_2) = E\left\{ \left(z(u, t_1) - E\{z(u, t_1)\}\right)\left(z(u, t_2) - E\{z(u, t_2)\}\right)^* \right\}$$

$$\gamma_z(t_1, t_2) = E\left\{ \left(\frac{z(u, t_1) - E\{z(u, t_1)\}}{\sqrt{C_z(t_1, t_1)}} \right) \left(\frac{z(u, t_2) - E\{z(u, t_2)\}}{\sqrt{C_z(t_2, t_2)}} \right)^* \right\}$$

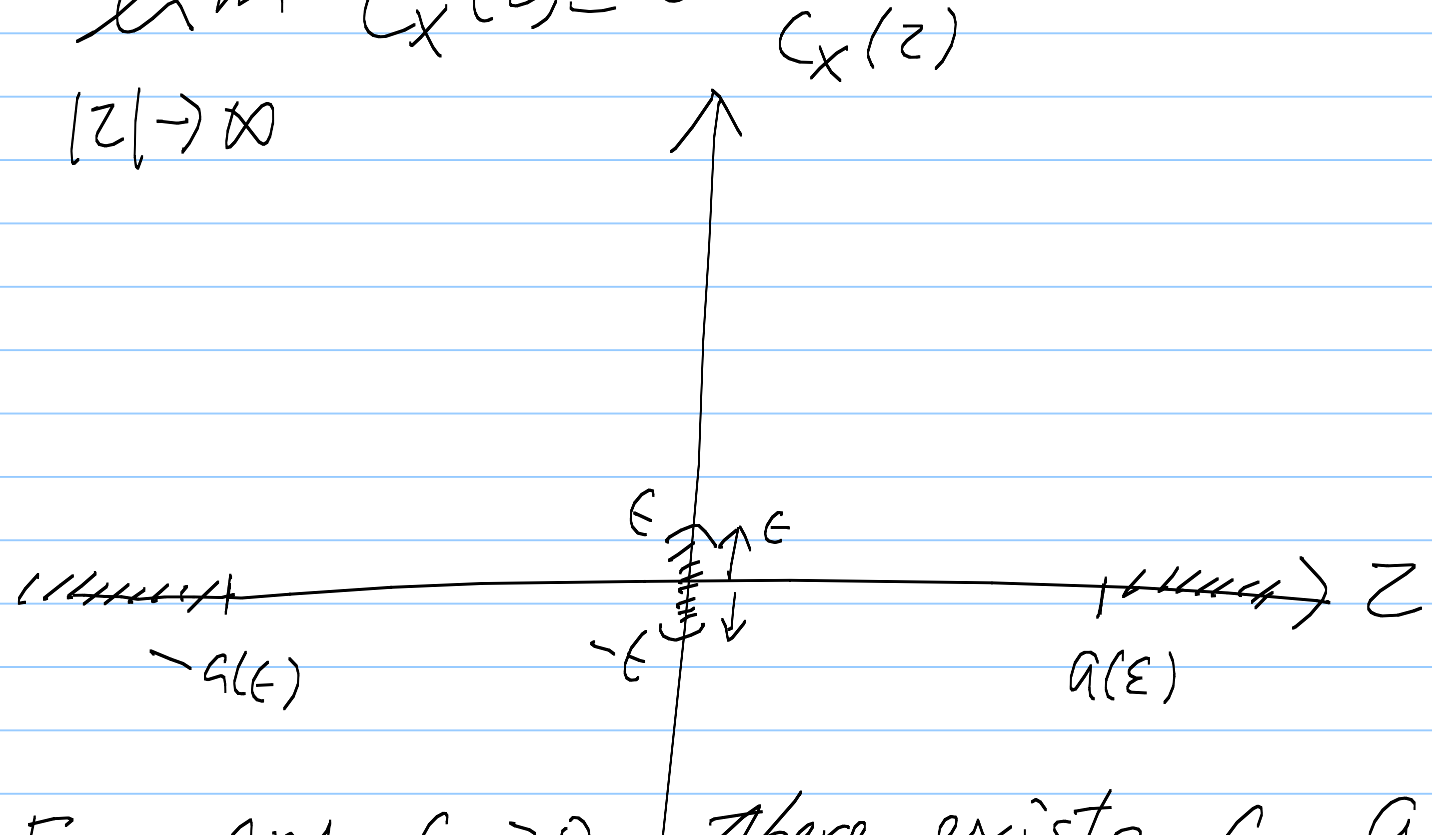
$$Aa^2 + Ba + C = f(a) \qquad A > 0$$

$D \equiv$ Discrimant $= B^2 - 4AC$



$f(a) = 0$ has the roots $\dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$$\lim_{|z|\to\infty} C_X(z) = 0$$



For any $\epsilon > 0$, there exists a a so that $|C_X(z)| < \epsilon$ for all $|z| > a$.

LTI

$$x(t) \longrightarrow \boxed{\text{System } h(t)} \longrightarrow y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(t-\alpha)\, h(\alpha)\, d\alpha$$

$$= \int_{-\infty}^{\infty} x(\alpha)\, h(t-\alpha)\, d\alpha$$

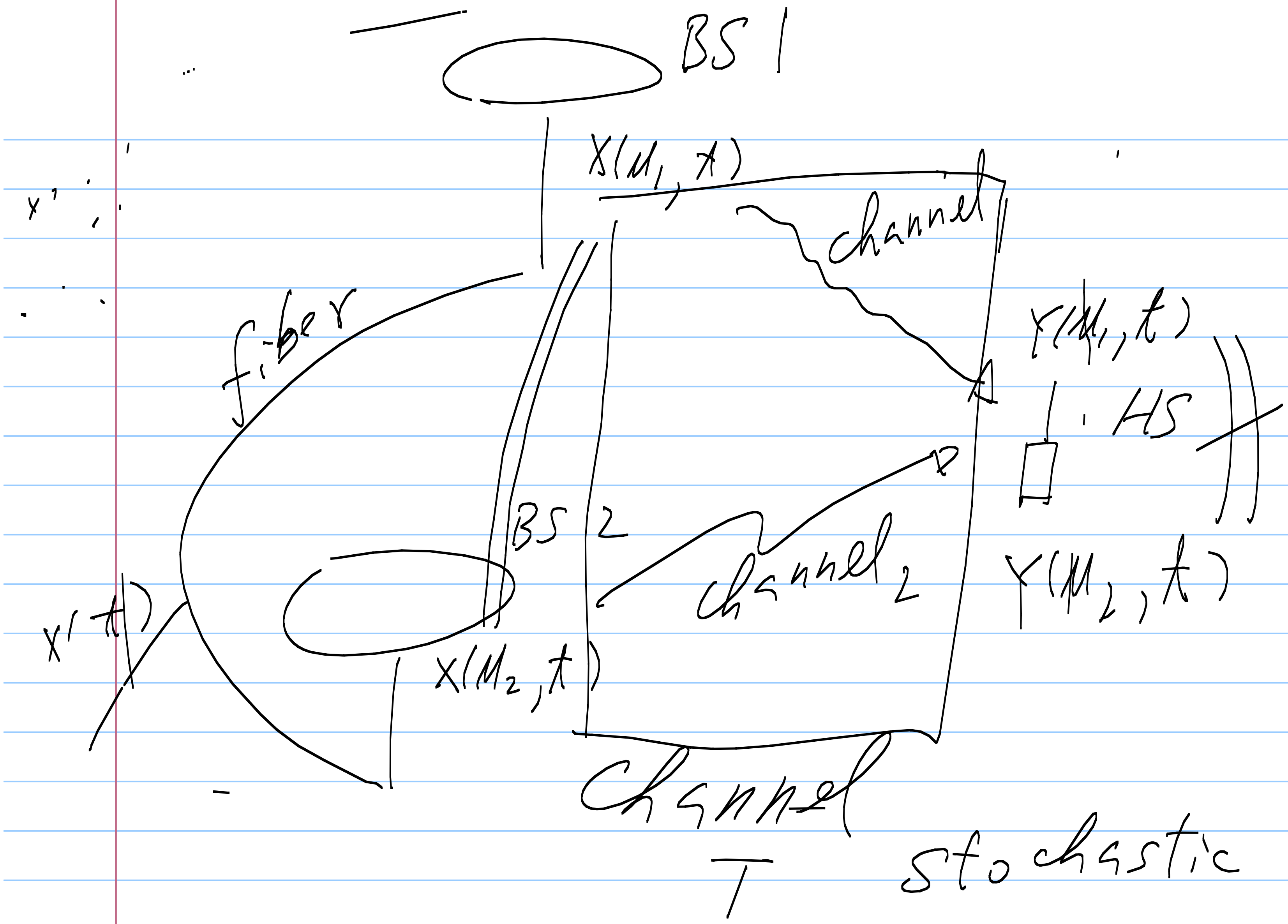
✓ linear

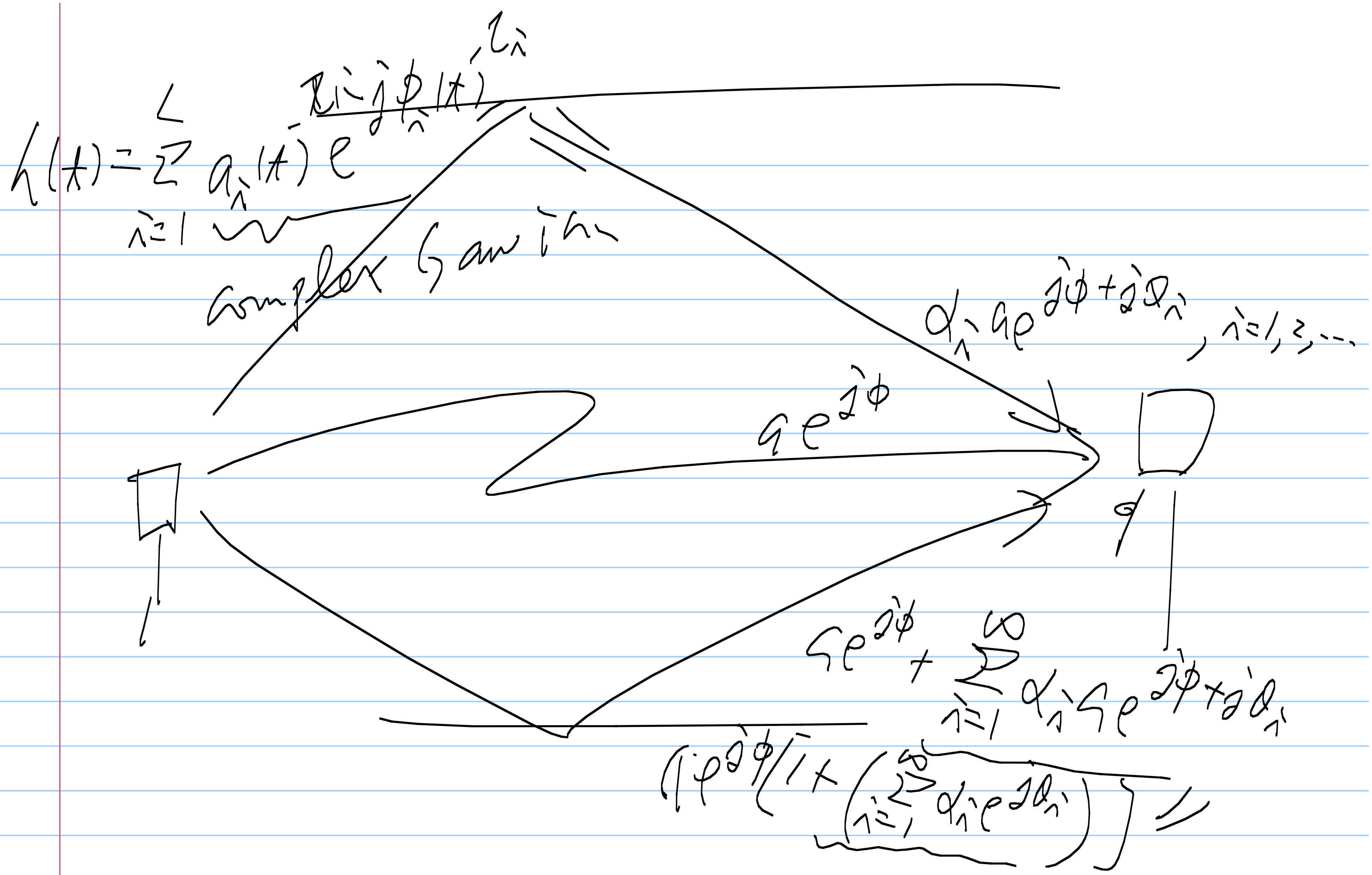
✗ stable

causal

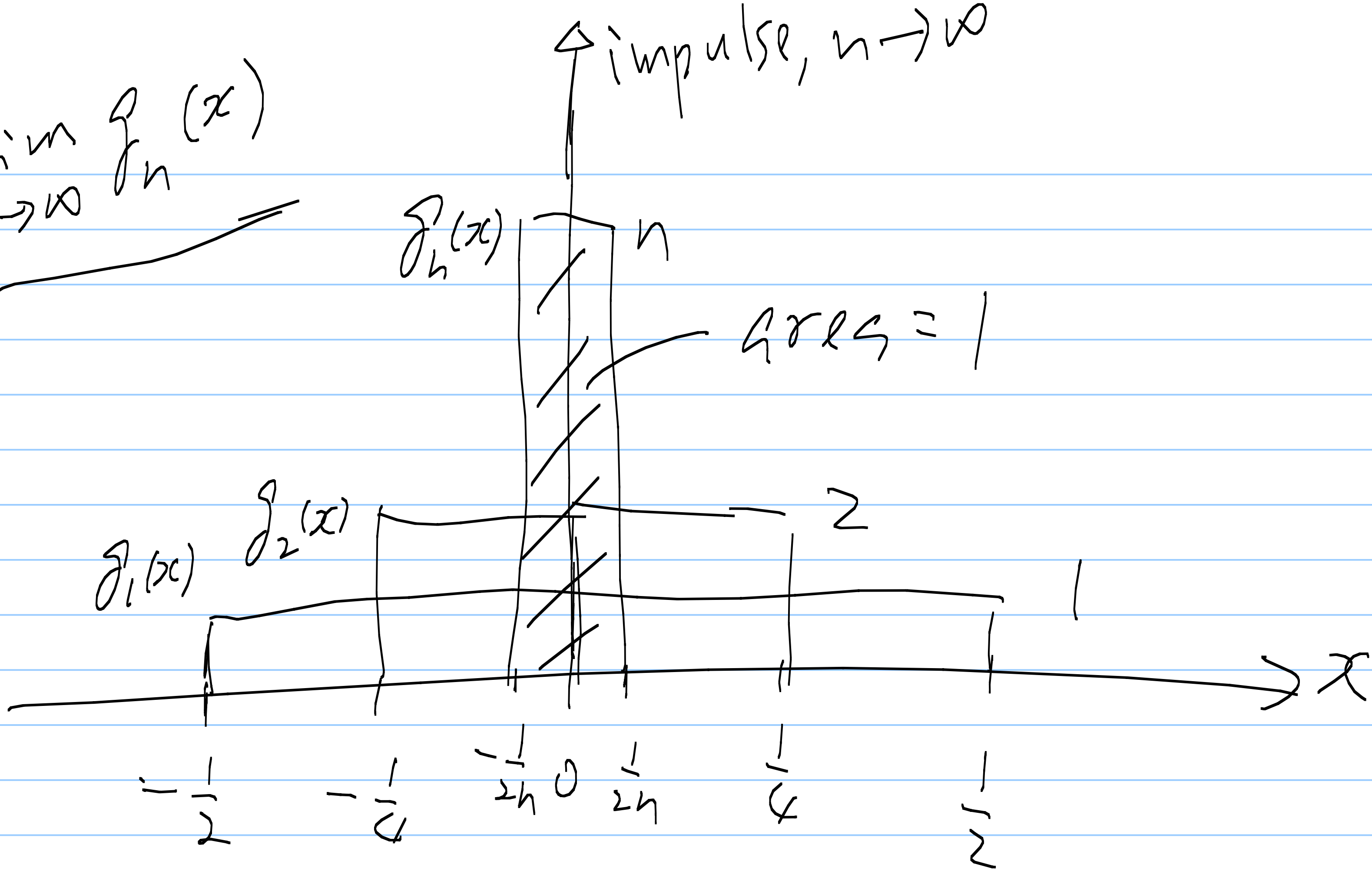
✓ time-invariant

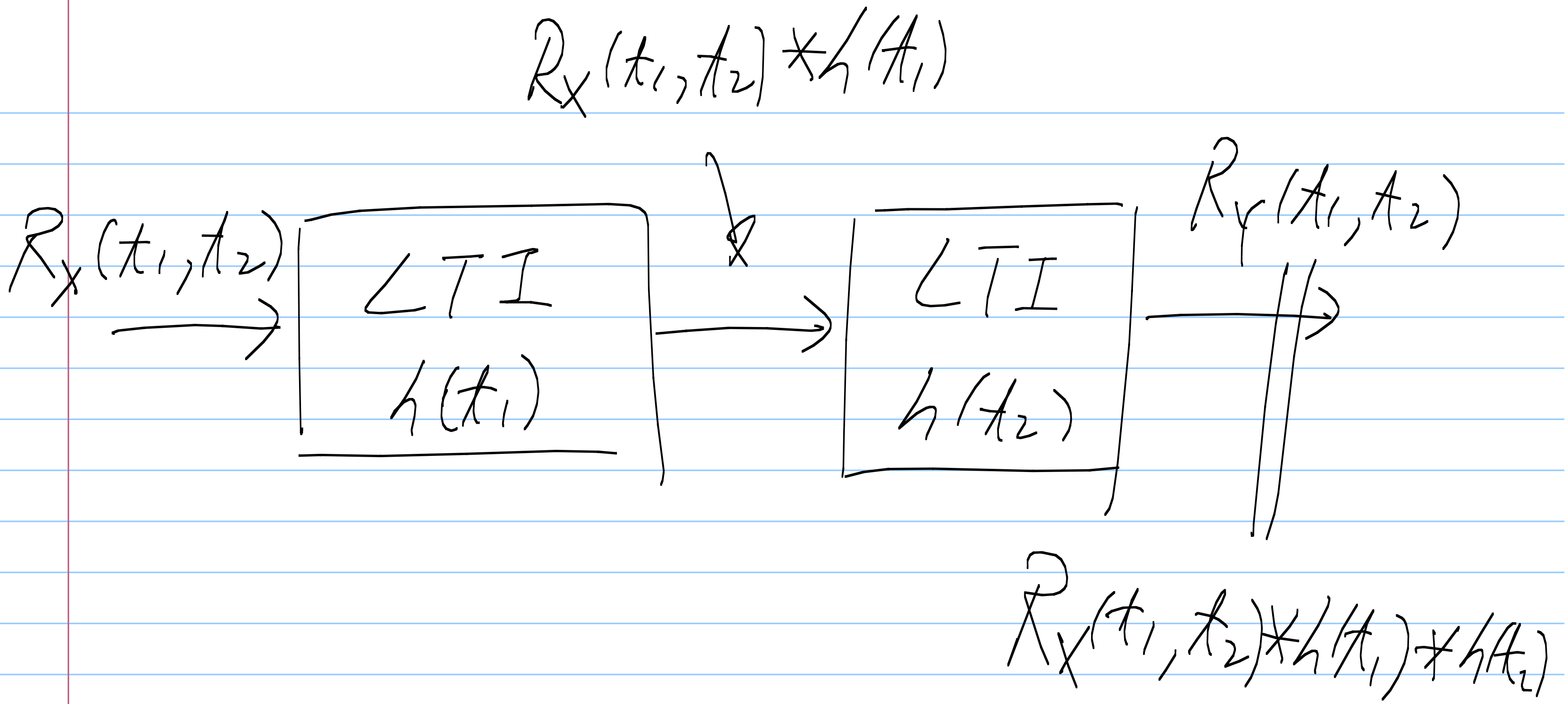
memoryless


BS 1
X(M1, t)
channel
fiber
Y(M1, t)
HS
BS 2
channel2
Y(M2, t)
x'(t)
X(M2, t)
Channel
T
stochastic


$h(t) = \sum_{n=1}^{L} a_n(t)\, e^{-j2\pi f_c \tau_n(t)}$
complex Gaussian
$\alpha_n a e^{j\phi + j\theta_n}$, $n = 1, 2, \ldots$
$a e^{j\phi}$
$a e^{j\phi} + \sum_{n=1}^{\infty} \alpha_n a e^{j\phi + j\theta_n}$
$|a e^{j\phi}| \left(1 + \sum_{n=1}^{\infty} \alpha_n e^{j\theta_n}\right)$

$$\delta(x) = \lim_{n\to\infty} \delta_n(x)$$




$R_X(t_1, t_2)$
LTI
$h(t_1)$
$R_X(t_1, t_2) * h(t_1)$
LTI
$h(t_2)$
$R_Y(t_1, t_2)$
$R_X(t_1, t_2) * h(t_1) * h(t_2)$

Binary Phase shift keyed (BPSK)

Transmitter $\xrightarrow{\sqrt{2P}\cos(2\pi f_c t)}$ AWGN $\rightarrow$ $r(\omega,t) = \begin{cases} s_0(t) + n(\omega,t) & \text{"0"} \\ s_1(t) + n(\omega,t) & \text{"1"} \end{cases}$

"0" $\longrightarrow s_0(t),\ t \in [0,T]$

$s_0(t) = \sqrt{2P}\cos(2\pi f_c t)$

"1" $\longrightarrow s_1(t),\ t \in [0,T]$

$s_1(t) = \sqrt{2P}\cos(2\pi f_c t + \pi)$

$T = 1$

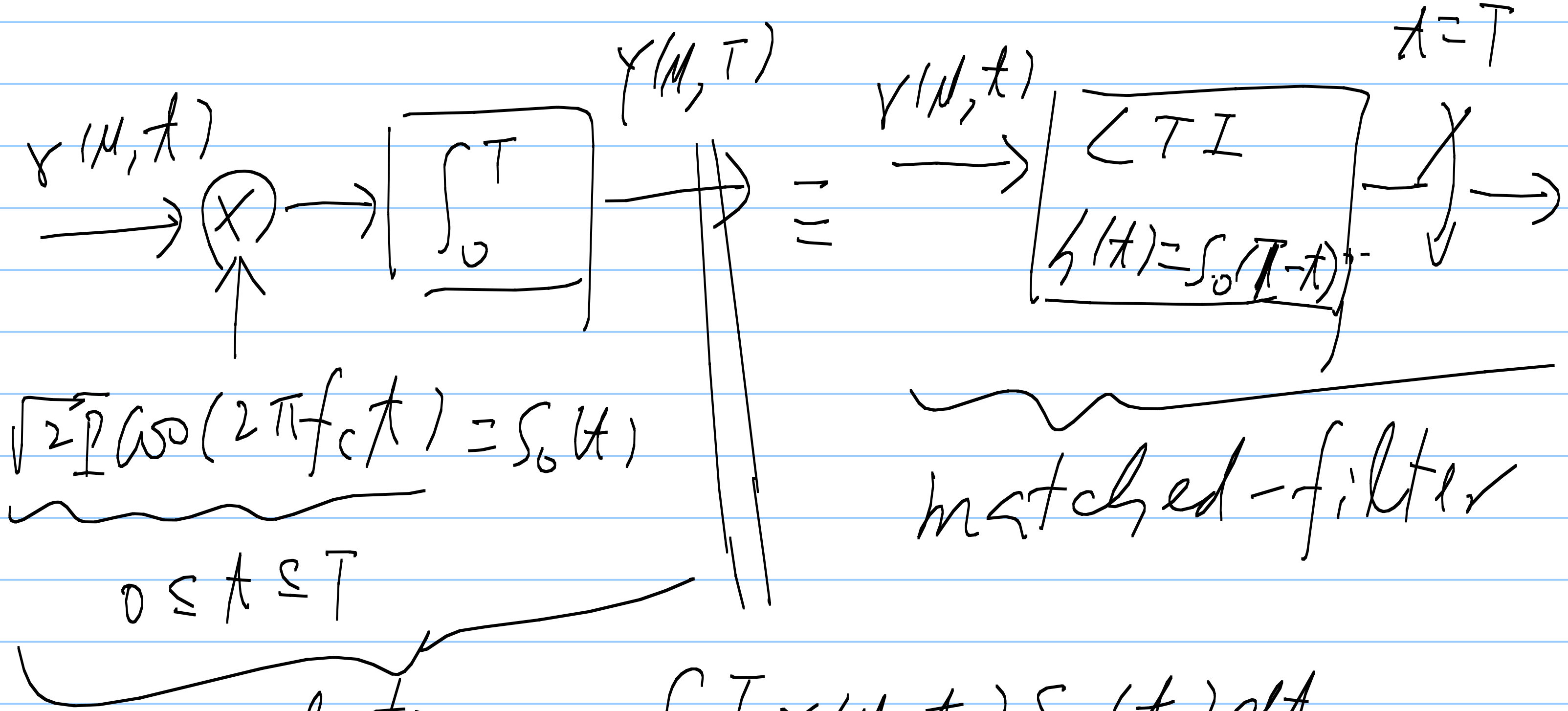
$r(\omega,t) \rightarrow$ LTI, $h(t) = s_0(T-t)$ $\rightarrow$ sample at $t = T$ $\rightarrow Y(\omega,T)$

decide "0" if $Y(\omega,T) > 0$

"1" if $Y(\omega,T) < 0$

$P_b \equiv$ bit error probability

$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right), \quad E_b = PT$$



correlation receiver

$$\int_0^T r(u,t)\, s_0(t)\, dt = r(u,t) * h(t)\Big|_{t=T}$$