2. (a) $\langle f, g\rangle = \int_a^b f(t)g(t)\,dt + \int_a^b f'(t)g'(t)\,dt$

$= \langle g, f\rangle$

$\langle f, f\rangle = \int_a^b (f(t))^2\,dt + \int_a^b (f'(t))^2\,dt \geq 0$

and $\langle f, f\rangle = 0 \iff f = 0.$

$\langle \alpha f_1 + \beta f_2, g\rangle = \int_a^b (\alpha f_1(t) + \beta f_2(t)) \cdot g(t)\,dt + \int_a^b (\alpha f_1'(t) + \beta f_2'(t)) \cdot g'(t)\,dt$

$= \alpha\langle f_1, g\rangle + \beta\langle f_2, g\rangle.$

(b). $\|f\| \geq 0$ and $\|f\| = 0 \iff f = 0$

$\|\alpha f\| = \int_a^b |\alpha f(t)|dt + \int_a^b |\alpha f'(t)|dt$

$= |\alpha| \cdot \int_a^b |f(t)|\,dt + |\alpha| \cdot \int_a^b |f'(t)|\,dt$

$= |\alpha| \cdot \|f\|$

$\|f+g\| = \int_a^b |f(t)+g(t)|dt + \int_a^b |f'(t)+g'(t)|dt$

$\leq \int_a^b |f(t)|\,dt + \int_a^b |f'(t)|\,dt + \int_a^b |g(t)|dt + \int_a^b |g'(t)|dt$

$= \|f\| + \|g\|$

3. • $k := \sum_{i=1}^{d} k_i$

$k(x,y) = \sum_{i=1}^{d} k_i(x,y)$

$= k(y,x).$

• $\begin{bmatrix} k(y_1,y_1) & \cdots & k(y_1,y_m) \\ \vdots & & \\ k(y_m,y_1) & \cdots & k(y_m,y_m) \end{bmatrix} = \sum_{i=1}^{d} \begin{bmatrix} k_i(y_1,y_1) & \cdots & k_i(y_1,y_m) \\ \vdots & & \\ k_i(y_m,y_1) & \cdots & k_i(y_m,y_m) \end{bmatrix} \succeq 0$