Q2:

$$\underbrace{\min_{a \in \mathbb{R}^n} \sum_{i=1}^{N} (\langle a, x_i \rangle - y_i)^2 + \lambda \|a\|_2^2}$$

$$\shortparallel$$

$$(Xa - y)^T (Xa - y) + \lambda a^T a$$

Take derivative wrt a.

$$\Rightarrow \quad -2X^T(y - Xa) + 2\lambda a = 0$$

$$\Rightarrow \quad a = (X^T X + \lambda I)^{-1} \cdot X^T y.$$

Q3: (a)

$$\langle u, v \rangle^2 = \left(\sum_{i=1}^{n} u_i v_i\right)^2$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} u_i u_j v_i v_j$$

$$[\phi_1(u)]_{i,j} = u_i u_j$$

$$\langle u, v \rangle^3 = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} u_i u_j u_k \cdot v_i v_j v_k$$

$$[\phi_2(u)]_{i,j,k} = u_i u_j u_k.$$

$$\phi(u) = \left(\sqrt{2}\, \phi_1(u), \sqrt{5}\, \phi_2(u)\right) \in \mathbb{R}^{n \times n} \times \mathbb{R}^{n \times n \times n}.$$

(b) $f(x) = a_m x^m + \cdots + a_0 \qquad a_0, \cdots, a_m \geq 0.$

$$H = \mathbb{R}^0 \times \mathbb{R}^n \times \cdots \times \mathbb{R}^{\overbrace{n \times \cdots \times n}^{m}}$$

$$\phi(u) = \left(\sqrt{a_0}, \sqrt{a_1}\, \phi_1(u), \cdots, \sqrt{a_m}\, \phi_m(u)\right)$$

$$[\phi_k(u)]_{i_1 \cdots i_k} = u_{i_1} \cdots u_{i_k},$$