Lecture 25

09-05-2019

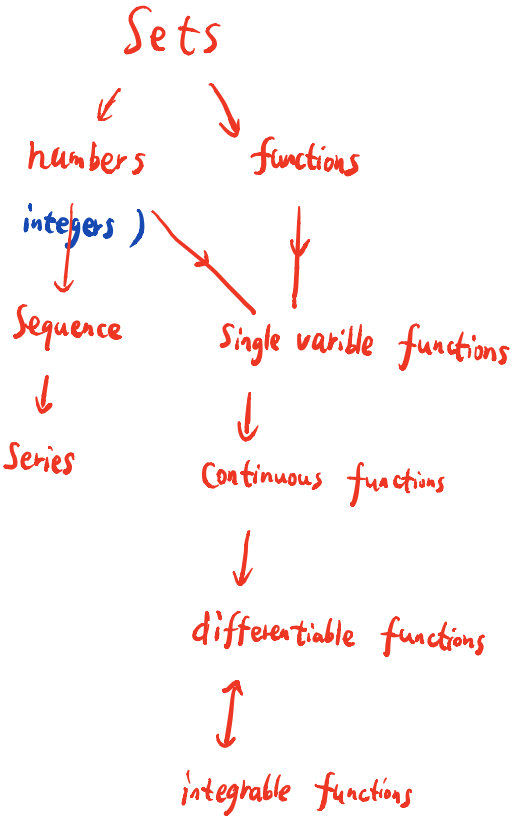
Review of Mathematical analysis

—— an introductory course aims to provide a rigorious mathematical foundations to single variable Calculas.

Content:

1. Basics of logic
2. Basics of Set theory
3. Basics of counting (emergence of positive integers)
4. Theory of real numbers
5. Theory of limit { sequence, function
6. Theory of continuity
7. Theory of differentiation
8. Theory of Riemann integral
9. Fundamental theorem of Calculas

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$



Check list for the Final Exam

For each concept X, (X = limit of sequence, continuity, ...)

① What is the definition of X

② typical examples of X

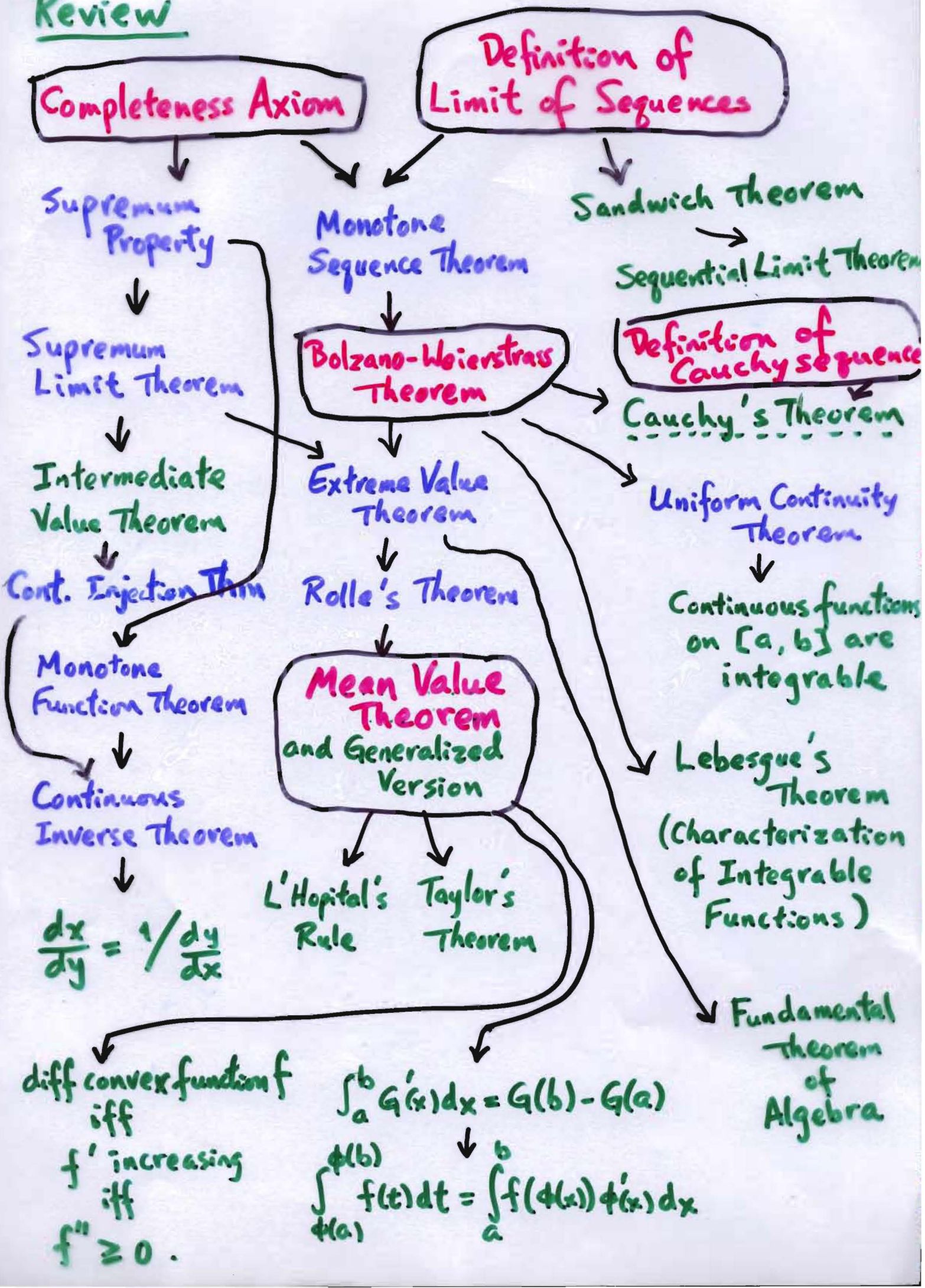
③ What are the important properties of X (computational rules and theorems about X)

The key ideas in the proofs?

Final Exam: focus on these basic concepts, basic properties, and the basic ideas in the proofs.

Deeper questions:
- Why X is defined in that way?
- Why X is needed, and how X paved the mathematical foundation of Calculas?
- how to develop a rigorous theory?

Review


Completeness Axiom
Definition of Limit of Sequences
Supremum Property
Monotone Sequence Theorem
Sandwich Theorem
Sequential Limit Theorem
Supremum Limit Theorem
Bolzano-Weierstrass Theorem
Definition of Cauchy sequence
Cauchy's Theorem
Intermediate Value Theorem
Extreme Value Theorem
Uniform Continuity Theorem
Cont. Injection Thm
Rolle's Theorem
Continuous functions on [a, b] are integrable
Monotone Function Theorem
Mean Value Theorem and Generalized Version
Lebesgue's Theorem (Characterization of Integrable Functions)
Continuous Inverse Theorem
L'Hopital's Rule
Taylor's Theorem
$\frac{dx}{dy} = 1/\frac{dy}{dx}$
Fundamental theorem of Algebra
diff conver function f iff f' increasing iff $f'' \geq 0$.
$\int_a^b G'(x)dx = G(b) - G(a)$
$\int_{\phi(a)}^{\phi(b)} f(t)dt = \int_a^b f(\phi(x))\phi'(x)dx$

Need your help : SFQ survey