Tutorial Notes

Method 2. for $1^2+2^2+\dots+n^2$.

$$n^2 = n(n+1)-n$$

$$n(n+1) = \frac{1}{3}\left[n(n+1)(n+2)-(n-1)n(n+1)\right]$$

$$1\times 2+2\times 3+\dots+n(n+1) = \frac{1\times 2\times 3-0}{3}+\dots+\frac{1}{3}\left[(n-1)n(n+1)-(n-2)(n-1)n\right]$$

$$+\frac{1}{3}\left[n(n+1)(n+2)-(n-1)n(n+1)\right]$$

$$= \cancel{\frac{1}{3}(n+1)}\ \frac{1}{3}n(n+1)(n+2)$$

$$1^2+2^2+\dots+n^2 = 1\times 2+2\times 3+\dots+n(n+1)-\left[1+2+\dots+n\right]$$

$$= \frac{1}{3}n(n+1)(n+2)-\frac{n(n+1)}{2}$$

$$= \frac{n(n+1)(2n+1)}{6}$$

$\boxed{\text{main idea : change } n(n+1) \text{ to a difference } \frac{n(n+1)(n+2)}{3} - \frac{(n-1)n(n+1)}{3}}$

ex 4. $(1+x)^n \geq 1+nx \qquad n>1 \qquad x>-1$

$n=2$ ✓

assume $n=k$ ✓ $\qquad (1+x)^k \geq 1+kx$

when $n=k+1 \qquad (1+x)^{k+1} \geq (1+kx)(1+x) = 1+(k+1)x+kx^2$

$$\geq 1+(k+1)x$$

ex 5. $n! < \left(\frac{n+1}{2}\right)^n. \qquad n>1$

$n=2$ ✓

assume $n=k$. ✓. $\qquad k! < \left(\frac{k+1}{2}\right)^k.$

when $n=k+1 \qquad (k+1)! = k!\times(1+k) < \frac{(k+1)^{k+1}}{2^k}$

we need to prove $\frac{(k+1)^{k+1}}{2^k} < \frac{(k+2)^{k+1}}{2^{k+1}}$

$$\Leftrightarrow 2 < \left(\frac{k+2}{k+1}\right)^{k+1} = \left(1+\frac{1}{k+1}\right)^{k+1}$$

use the result of ex 4. $\quad x=\frac{1}{k+1} \qquad n=k+1.$

high order derivative.

$(e^x)^{(n)} = e^x \qquad (a^x)^{(n)} = a^x(\ln a)^n$

$(\sin x)^{(n)} = \sin(x+\frac{n\pi}{2}) \qquad (\cos x)^{(n)} = \cos(x+\frac{n\pi}{2})$

$(\ln x)^{(n)} = (-1)^{n-1}(n-1)!\frac{1}{x^n}$

ex 6. $(x^{n-1}e^{\frac{1}{x}})^{(n)} = \frac{(-1)^n}{x^{n+1}}e^{\frac{1}{x}}$

$n=1,2,$ ✓.

assume $n=k-1, k$ ✓.

when $n=k+1$

$$(x^k e^{\frac{1}{x}})^{(k+1)} = [(x^k e^{\frac{1}{x}})']^{(k)}$$

$$= (kx^{k-1}e^{\frac{1}{x}} - x^{k-2}e^{\frac{1}{x}})^{(k)}$$

$$= k(x^{k-1}e^{\frac{1}{x}})^{(k)} - (x^{k-2}e^{\frac{1}{x}})^{(k)}$$

$$= k(x^{k-1}e^{\frac{1}{x}})^{(k)} - \cancel{[(x^{k-2}e^{\frac{1}{x}})']^{(k-1)}}[(x^{k-2}e^{\frac{1}{x}})^{(k-1)}]'$$

use the assumption $n=k$ and $n=k-1$

$$= k\cdot\frac{(-1)^k}{x^{k+1}}e^{\frac{1}{x}} - [\frac{(-1)^{k-1}}{x^k}e^{\frac{1}{x}}]'$$

$$= \frac{k(-1)^k}{x^{k+1}}e^{\frac{1}{x}} - [\frac{(-1)^{k-1}}{x^{k+1}}(-k)e^{\frac{1}{x}} - \frac{(-1)^{k-1}}{x^{k+2}}e^{\frac{1}{x}}]$$

$$= \frac{k(-1)^k}{x^{k+1}}e^{\frac{1}{x}} - \frac{k(-1)^k}{x^{k+1}}e^{\frac{1}{x}} + \frac{(-1)^{k-1}}{x^{k+2}}e^{\frac{1}{x}}$$

$$= \frac{(-1)^{k-1}}{x^{k+2}}e^{\frac{1}{x}} = \frac{(-1)^{k+1}}{x^{k+2}}e^{\frac{1}{x}}$$