## Math 2033 (Homework 5) L1 FALL 2021 Problems (Due Nov 29 at 11:59 pm) 1) If file > R is differentiable and lim f(x) exists, then prove that f'is continuous at 0. (2) If f: [a,b] > Ris continuous, fox) > 0 for all x and $\int_{a}^{b} f(x) dx = 0$ , then prove that f(x) = 0 for all xe[a,b]. (3) If f,g: [0,1] > R are Kiemann integrable, 25) then show that the function h(x) = min(f(x), g(x))is Riemann integrable on [0, 1].

(1) Let f: [0,1] > [-1,1] be Riemann integrable.

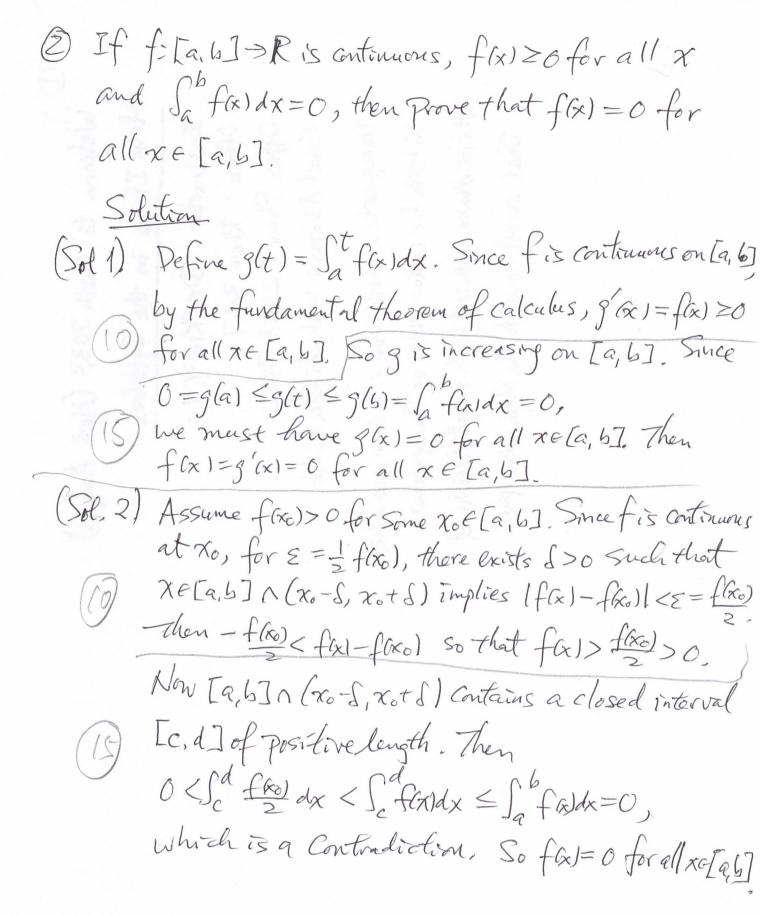
Using the integral criterion, Prove that

g(x) = { f(x) if 0 < x < 1}

o if x = 0 or 1

is also Riemann integrable on Lo, 1]

1) If fire-Risdifferentiable and lim f (x) exists, then prove that f'is continuous at O. Solution Since lim f(x) = lim f(x) exists, by l Hopital's rule, we have  $\lim_{x \to 0} \frac{f(x) - f(0)}{x \to 0} = \lim_{x \to 0} \frac{f(x)}{x \to 0} = \lim_{x \to 0} f(x)$  exists in R. Therefore,  $f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x \to 0} = \lim_{x \to 0} f(x)$ , by definition of f'(0) i.e.  $f'(0) = \lim_{x \to 0} f'(0) =$ 



3) If f,g:[0,1]=R are Riemann integrable, then show that the function h(x) = min(f(x), g(x)) is Riemann integrable on [0,1],

## Solution

Note max(f,g)+min(f,g)=f+g and max(f,g)-min(f,g)=H-g/.

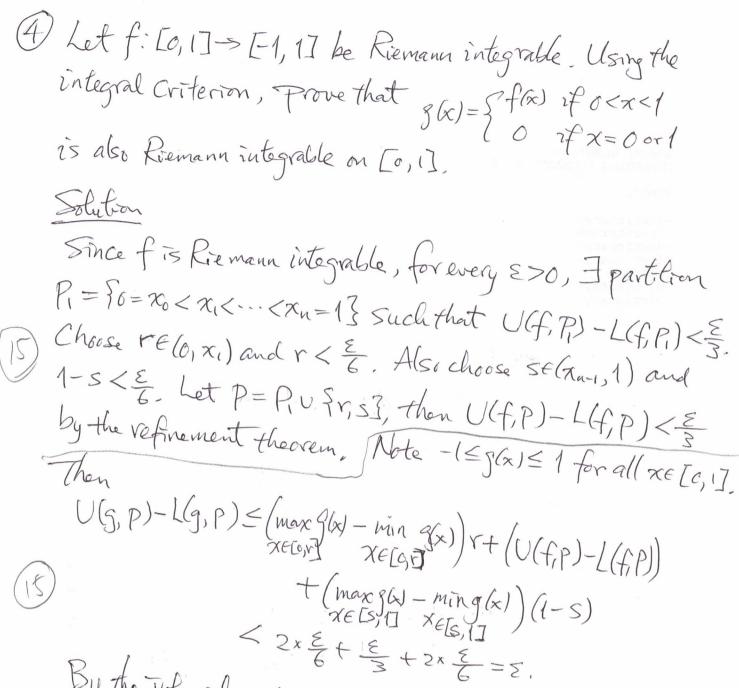
Subtracting, then dividing by 2, we have

(12) A = min (f,g) = ftg-1f-g1

If f, g are integrable, then f+g, f-g are integrable.

(3) Since 1x1 is Continuous, so 1f-gl is integrable.

Therefore h = f+g-|f-g| is integrable.



< 2x \(\xi \) + 2x \(\xi \) = \(\xi \). By the Vilegral Criterion, 3 is Riemann integrable.