Math 2033 (Homework 4) L1 Fall 2021

Problems (Due Nov 15 at 11:59 pm)

① Prove that the function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=\frac{2+3x}{x^2+4}$ is continuous at $x=2$ by checking the ε-δ definition of a function continuous at a point.

② Prove that there does not exist any continuous function $f:\mathbb{R}\to\mathbb{R}$ such that $f(f(x))+x=0$ for every $x\in\mathbb{R}$.

③ Let $f:\mathbb{R}\to\mathbb{R}$ be continuous such that $|f(x)-f(y)|\le\frac{1}{2}|x-y|$ for every $x,y\in\mathbb{R}$.

(a) Let $w\in\mathbb{R}$. Define $x_1=w$ and $x_{n+1}=f(x_n)$ for $n\in\mathbb{N}$. Show that $x_1, x_2, x_3, \ldots$ is a Cauchy sequence.

~~(b) Show that there is $x\in\mathbb{R}$ such that $f(x)=x$.~~

~~No need to give a solution!~~