SHEN Yinan, yshenay@connect.ust.hk. 4381.

0. Prove $\sin(\frac{1}{x})$ is continuous at every point in $(0,1)$.

$\varepsilon - \delta.$

$\uparrow$

$f_2 \xrightarrow{\varepsilon} \delta_1.$

Define $f_1(x) = \frac{1}{x}$. $x \in (0,1)$.

Define $f_2(y) = \sin y$ $y \in (1, +\infty)$

Define $f(x) = f_2(f_1(x)) = \sin\frac{1}{x}$. $x \in (0,1)$

$f = f_2 \circ f_1$

f_1 is continuous at every point in $(0,1)$

f_2 is continuous at every point in $(1, +\infty)$.

$\Rightarrow$ f is continuous at every point in $(0,1)$

for f is $\because$ the composite function

Some problems in Problem Set 1.

1. Problem 4: Prove that $\sqrt[3]{3}$ is an irrational number.

Proof: By contradiction, suppose $\sqrt[3]{3}$ is a rational number:

$\times$

$\longrightarrow$ $\sqrt[3]{3} = \frac{p}{q}$, $p, q \in \mathbb{Z}_+$. $(p,q)=1$. $(p,q)=1$

$\longrightarrow$ $3 = \frac{p^3}{q^3}$. $\Rightarrow$ $p^3 = 3q^3$. $p^3 = 3q^3$ $3 \mid p^3$

$\Rightarrow 3 \mid p$. (for 3 is a prime number) 3: primenumber

$3 = 3p_1$. $\Rightarrow$ $9p_1^3 = q^3$ $\Rightarrow 3 \mid p$.

$\Rightarrow 3 \mid q$. $\Rightarrow$ $(p,q)=1$. $(3 \mid p, 3 \mid q)$ $\sqrt{2}$:

The assumption is false. irrational

$\Rightarrow$ $\sqrt[3]{3}$ is an irrational number. $\nearrow$ number

$a \mid b$ means $b \div a = ? \ldots 0$.

$a \mid b$

$b \div a = ? \ldots 0$.

$a \equiv b \pmod{c}$

$c \mid a-b$

$(a-b) \div c = ? \ldots 0.$

2. Problem 5: Prove that there does not exist integers a and b such that $21a+30b=1$.

Proof: By contradiction, suppose there exist integers a and b such that $21a+30b=1$.

$3 \mid 21$
$3 \mid 30$

We find that $3 \mid 21$, $3 \mid 30$. $\Rightarrow 3 \mid 21a+30b$
$\Rightarrow 3 \mid 1$. ×

$5a+25b=3$.
5, a^2 b^2

$\Rightarrow$ the assumption is false.

$1 \div 3 = ? \ldots a$

3. Problem 6. Let a and b be two real numbers. Prove that if $a, b > 0$, then $\frac{2}{a}+\frac{2}{b} \neq \frac{4}{a+b}$.

Proof: By contradiction, suppose there exist real numbers $a, b > 0$ such that $\frac{2}{a}+\frac{2}{b}=\frac{4}{a+b}$.

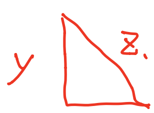
$a^2+b^2 \geq 2ab$,
$((a-b)^2 \geq 0)$.

$\Rightarrow \frac{a+b}{ab} = \frac{2}{a+b} \Rightarrow (a+b)^2 = 2ab \Rightarrow a^2+b^2=0$.

$\Rightarrow a=0, \ b=0$.

$\Rightarrow$ The assumption is false.

4. Problem 8. We let x, y, z be three positive integers satisfying $x^2+y^2=z^2$. Show that if x and y are relatively prime, then one of them is odd and another one is even.



x
$\rightarrow 3^2+4^2=5^2$.
$12^2+5^2=13^2$

$2 \mid (x, y)$

Proof: $(x,y)=1$. $\Rightarrow$. x and y are both even is impossible.

Without loss of generality x is odd, $\boxed{x=2k+1}$.

$\Rightarrow x^2 = 4k^2+4k+1 \Rightarrow x^2 \equiv 1 \pmod 4$.
if y is odd, $y^2 \equiv 1 \pmod 4$.
$x^2+y^2 \equiv 2 \pmod 4$.

$$\boxed{\begin{array}{l} x \equiv a \pmod b \\ \text{means} \quad x \div b = ? \ldots a \\ \qquad a = 0, 1, \ldots, b-1 \end{array}}$$

$\begin{cases} 5 \equiv 2 \pmod 3 \\ 18 \equiv 3 \pmod 5 \end{cases}$

$\times$ $\Rightarrow$ $z^2 = x^2 + y^2$ (even) z is even. $4 \mid z^2$ $\Rightarrow$ $z^2 \equiv 0 \pmod 4$, $x^2 + y^2 \equiv 2 \pmod 4$

$\Rightarrow$ y is not odd

$\Rightarrow$ y is even.

5. Problem 10: $n = d_r d_{r-1} \cdots d_0 \Leftrightarrow \sum_{i=0}^{r} d_i$

Answer: $n = d_r d_{r-1} \cdots d_1 d_0 = 10^r d_r + 10^{r-1} d_{r-1} + \dots + 10 d_1 + d_0$

$n = 123$ $d_2 = 1$ $d_1 = 2$ $d_0 = 3$. Good why? $d_0 \neq 10 d_1$

$18 \div 9 = 2 \cdots 0$

$(1+8) \div 9 = 1 \cdots 0$.

$\longrightarrow 10 = 9+1$

$\longrightarrow 100 = 99+1$

$\vdots$

$10^r \equiv ? \pmod 9$.

$\longrightarrow 10^r = 9 + 10 \cdot 9 + \dots + 10^{r-1} \cdot 9 + 1$.

$\left(= 9 \times \frac{10^r - 1}{10 - 1} + 1\right)$ $10^r \equiv 1 \pmod 9$.

$\longrightarrow 12222 \div 9 = 1358$.

$$\begin{array}{r} 1358 \\ 9\,\overline{)12222} \\ \underline{9} \\ 32 \\ \underline{27} \\ 52 \\ \underline{45} \\ 72 \end{array}$$

$0 \equiv n \equiv d_r + d_{r-1} + \dots + d_1 + d_0 \pmod 9$. ✓

$10^r d_r = (9 + 9 \cdot 10 + \dots + 9 \cdot 10^{r-1}) d_r + d_r \equiv d_r \pmod 9$

$10^{r-1} d_{r-1} \equiv d_{r-1} \pmod 9$.

$10 d_1 \equiv d_1 \pmod 9$.

$\longrightarrow (1+2+2+2+2) \div 9 = 1$.

3. $\times$. $x^2 \equiv ? \pmod 3$.

5, 7.

$x =$	1	2	3	4
	1	1	0	1

Number Theory.

4.

数论.

初等数论.

$x = 3k+1$ $\quad$ $x = 3k+2$ $\quad$ $x = 3k$.

$x^2 = 9k^2 + 6k + 1$ $\quad$ $x^2 = 9k^2 + 12k + 4$ $\quad$ $x^2 \equiv 0 \pmod 3$

$x^2 \equiv 1 \pmod 3$ $\quad$ $x^2 \equiv 1 \pmod 3$

$x^2 \equiv 2 \pmod 3$ $\times$.

$\longrightarrow x^2 = 9y + 12z + 2$. $\leftarrow$ impossible.

mod 3 $\quad$ mod 3. $\to x^2 \equiv 2 \pmod 3$

Problem 7: x: rational

y: irrational

Prove: $x+y$, xy irrational.

By contradiction, $\underline{x+y \text{ is rational}}$

$x+y = \frac{m_1}{n_1}$ $m_1, n_1 \in \mathbb{Z}$, $n_1 \neq 0$,

$x = \frac{m_2}{n_2}$, $m_2, n_2 \in \mathbb{Z}$, $n_2 \neq 0$

$y = \frac{m_1}{n_1} - \frac{m_2}{n_2} = \frac{m_1 n_2 - m_2 n_1}{n_1 n_2}$ $\in \mathbb{Z}$

$\in \mathbb{Z}$,

$n_1 n_2 \neq 0$.

$\Rightarrow$ y: rational.

$x+y$ is irrational.