MATH 2033 HW-8 Due Nov 8.

1. Prove that a subset of $\mathbb{R}$ is compact if and only if it is sequentially compact.

2. Let $a_i \geq 0$ for all $i = 1, 2, \cdots$.

(i) Suppose that $\sum_{k=1}^{\infty} a_k$ converges. Prove that $\sum_{k=1}^{\infty} a_k^2$ also converges.

(ii) Find an example that $\sum_{k=1}^{\infty} a_k^2$ converges but $\sum_{k=1}^{\infty} a_k$ diverges.

3. Prove that the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges.

4. Find all possible values of x so that the following series converge.

(i)
$$\sum_{n=1}^{\infty} \frac{x^n}{n},$$

(ii)
$$\sum_{n=1}^{\infty} \frac{x^n}{n^2},$$

(iii)
$$\sum_{n=1}^{\infty} n x^n.$$