MATH2033 Mathematical Analysis (2021 Spring)
Assignment 5

Problem 1
(a) Using the definition of integrability or integral criterion, prove that $f(x) = |x - 1|$ is integrable on $[0,2]$.

(b) Using the definition of integrability or integral criterion, prove that the function $f : [0,1] \to \mathbb{R}$ defined by
$$f(x) = \begin{cases} x & if \ x \in \mathbb{Q} \\ -x & if \ x \in \mathbb{R} \backslash \mathbb{Q} \end{cases}$$
is not integrable on $[0,1]$.

Problem 2
We let f, g, h be three bounded functions on $[a, b]$ such that $f(x) \leq g(x) \leq h(x)$ for all $x \in [a, b]$. Suppose that f, h are integrable on $[a, b]$ and $\int_a^b f(x)dx = \int_a^b h(x)dx$.

(a) Show that g is integrable on $[a, b]$.

(b) Show that $\int_a^b g(x)dx = \int_a^b f(x)dx$.

Problem 3
(a) We let $f, g : [a, b] \to \mathbb{R}$ be two bounded Riemann integrable function on $[a, b]$, show that the function $h(x) = \min(f(x), g(x))$ is also Riemann integrable on $[a, b]$.

(b) We let $f : [a, b] \to \mathbb{R}$ be a bounded function on $[a, b]$.

    (i)   Suppose that f^2 is Riemann integrable, is it true that f is Riemann integrable? Explain your answer.

    (ii)   Suppose that f^3 is Riemann integrable, is it true that f is Riemann integrable? Explain your answer.

(*Note: If your answer is yes, please give a proof. If your answer is no, please provide a counter-example.)