2. (10 points) .
Let $f \in C([a,b]), a < b$. Assume that f is differentiable in (a,b), and

$$f'(a^+) = \lim_{x \to 0+} \frac{f(a+x) - f(a)}{x} > 0$$
$$f'(b^-) = \lim_{x \to 0-} \frac{f(b+x) - f(b)}{x} < 0.$$

2.1 Show that the maximum of f is not achieved at $x = a$ and $x = b$.
2.2 Show that $\exists c \in (a,b)$ s.t

$$f'(c) = 0.$$

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Solution:
2.1 Since $f'(a^+) > 0$, there exists $x_1 \in (a,b]$ such that $\frac{f(x_1)-f(a)}{x_1-a} > 0$, which implies $f(x_1) > f(a)$. So f cannot attain the maximum at a. Since $f'(b^-) < 0$, there exists $x_2 \in [a,b)$ such that $\frac{f(x_2)-f(b)}{x_2-b} < 0$, which implies $f(x_2) > f(b)$. So f cannot attain the maximum at b.

4 2.2 Since f is continuous, f must attain its maximum at some point $c \in (a,b)$. Therefore, $f'(c) = 0$.