Math 2033 (Mathematical Analysis) **Spring 2020**

<u>Final Examination (Part A) – (Duration: 40 minutes)</u>

Directions: **For every problem of this exam, detailed written works supported by correct reasons must be shown legibly to receive credits**.

Notations: $\mathbb{R}$ is the set of all real numbers.

<u>Problems</u>

1. (15 marks) Using the continuous injection theorem or otherwise, prove that if $f : \mathbb{R} \to [0, +\infty)$ is a bijection, then f is not continuous on $\mathbb{R}$.

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2. (15 marks) Let $h : [0, 1] \to \mathbb{R}$ be continuous. If $h(0) = 0$ and h is differentiable on $(0, 1)$ with $h'(x)$ decreasing on $(0, 1)$, then prove that for $0 \leq a \leq b \leq a + b \leq 1$, we have $h(a + b) \leq h(a) + h(b)$.

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3. (20 marks) Define $g : (0, 4) \to \mathbb{R}$ by $g(x) = \dfrac{x + 1}{2\sqrt{x} - x}$. Prove that $\lim_{x \to 1} g(x) = 2$ by checking the ε-δ definition of limit of $g(x)$ as x tends to 1.

– End of Part A –