Note: Every metric on $\mathbb{R}$ throughout this part is the usual one defined by the absolute value.

1. (20 points). Let $A = [0,1) \cup \{2\}$. Just answer **True** or **False** to the following questions. You do NOT need to justify your answers.

(a) A is open.

(b) A is compact.

(c) Both 1 and 2 are accumulation points of A.

(d) Both 1 and 2 are boundary points of A.

(e) The closure of A is $[0,2]$.

2. (10 points). Find ALL the real numbers x so that $\sum_{n=1}^{\infty} \frac{x^n}{n^7}$ converges.

3. (10 points). Suppose that both $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are uniformly continuous functions on $\mathbb{R}$.

(i) Prove $f + g$ is uniformly continuous on $\mathbb{R}$.

(ii) Is fg (f multiplies g) always uniformly continuous on $\mathbb{R}$? Prove it if your answer is yes, or disprove it by showing a counterexample.

4. (10 points). For each $n \in \mathbb{N}$, let

$$f_n(x) = \frac{1 - x^n}{1 + x^n}.$$

(i) Find the pointwise limit f of $(f_n)_{n\in\mathbb{N}}$ on $[0,2]$.

Hint to (i): The limit f is piece-wisely defined by

$$f(x) = \begin{cases} ? & x \in [0,1) \\ ? & x = 1 \\ ? & x \in (1,2]. \end{cases}$$

Answer these question marks. NO explanation is needed.

(ii) Does $(f_n)_{n\in\mathbb{N}}$ uniformly converge to the limit f on $[0,2]$? **Explain** your answer.