

## MATH2033 Mathematical Analysis (2021 Spring)

### Assignment 5

**Submission deadline of Assignment 5: 11:59p.m. of 12<sup>th</sup> May, 2020 (Wed)**

Instruction: Please complete all required problems. Full details (including description of methods used and explanation, key formula and theorem used and final answer) must be shown clearly to receive full credits. Marks can be deducted for incomplete solution or unclear solution.

Please submit your completed work via the submission system in canvas before the deadline. Late assignment will not be accepted.

Your submission must (1) be hand-written (typed assignment will not be accepted), (2) in a single pdf. file (other file formats will not be accepted) and (3) contain your full name and student ID on the first page of the assignment.

#### Problem 1

- (a) Using the definition of integrability or integral criterion, prove that  $f(x) = |x - 1|$  is integrable on  $[0, 2]$ .
- (b) Using the definition of integrability or integral criterion, prove that the function  $f: [0, 1] \rightarrow \mathbb{R}$  defined by

$$f(x) = \begin{cases} x & \text{if } x \in \mathbb{Q} \\ -x & \text{if } x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}$$

is not integrable on  $[0, 1]$ .

#### Problem 2

We let  $f, g, h$  be three bounded functions on  $[a, b]$  such that  $f(x) \leq g(x) \leq h(x)$  for all  $x \in [a, b]$ . Suppose that  $f, h$  are integrable on  $[a, b]$  and  $\int_a^b f(x)dx = \int_a^b h(x)dx$ .

- (a) Show that  $g$  is integrable on  $[a, b]$ .
- (b) Show that  $\int_a^b g(x)dx = \int_a^b f(x)dx$ .

#### Problem 3

- (a) We let  $f, g: [a, b] \rightarrow \mathbb{R}$  be two bounded Riemann integrable function on  $[a, b]$ , show that the function  $h(x) = \min(f(x), g(x))$  is also Riemann integrable on  $[a, b]$ .
- (b) We let  $f: [a, b] \rightarrow \mathbb{R}$  be a bounded function on  $[a, b]$ .
- (i) Suppose that  $f^2$  is Riemann integrable, is it true that  $f$  is Riemann integrable? Explain your answer.
  - (ii) Suppose that  $f^3$  is Riemann integrable, is it true that  $f$  is Riemann integrable? Explain your answer.

(\*Note: If your answer is yes, please give a proof. If your answer is no, please provide a counter-example.)