Math 2033 (Mathematical Analysis) Spring 2014

Midterm

Directions: This is a closed book exam. Every student must show work in every problem with *full* details legibly to receive marks. *Answers alone are worth very little!!!*

Notations: $\mathbb{R}$ denotes the set of all real numbers. $\mathbb{Q}$ denotes the set of all rational numbers.

1. (14 marks) Prove that there exist infinitely many real numbers r such that the equation $10^{xy} + r - y^3 = xy$ does not have any solution with $x, y \in \mathbb{Q}$.

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2. (16 marks) Let A be a nonempty bounded subset of $\mathbb{R}$ such that $\inf A = 0$ and $\sup A = 3$. Let
$$B = \{x + 2^{xy} + y : x \in [1,2] \setminus \mathbb{Q},\ y \in A\}.$$
Prove that B is bounded. Determine (with proof) the infimum and supremum of B.

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3. (20 marks) Prove that the sequence $\{x_n\}$ converges, where
$$x_1 = 11 \quad \text{and} \quad \text{for } n = 1, 2, 3, \ldots,\ x_{n+1} = \frac{18}{x_n + 7}$$
and find its limit. Show all details.

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4. (20 marks) Prove that
$$\lim_{n\to\infty}\Big(\frac{6n^2 + n - 3}{1 + 2n^2} + \frac{n + 5\sqrt{n} + \sqrt[3]{n}}{6 + n}\Big) = 4$$
by checking the definition of limit of a sequence only.

(Do not use computation formulas, sandwich theorem or l'Hopital's rule! Otherwise, you will get zero mark for this problem.)

–End of Paper–