Math 2033 (Homework 5) L1 FALL 2021

Problems (Due Nov 29 at 11:59 pm)

1. If $f: \mathbb{R} \to \mathbb{R}$ is differentiable and $\lim_{x\to 0} f'(x)$ exists, then prove that f' is continuous at 0.

2. If $f: [a,b] \to \mathbb{R}$ is continuous, $f(x) \geq 0$ for all x and $\int_a^b f(x)\,dx = 0$, then prove that $f(x) = 0$ for all $x \in [a,b]$.

3. If $f, g: [0,1] \to \mathbb{R}$ are Riemann integrable, then show that the function $h(x) = \min(f(x), g(x))$ is Riemann integrable on $[0,1]$.

4. Let $f: [0,1] \to [-1,1]$ be Riemann integrable. Using the integral criterion, Prove that

$$g(x) = \begin{cases} f(x) & \text{if } 0 < x < 1 \\ 0 & \text{if } x = 0 \text{ or } 1 \end{cases}$$

is also Riemann integrable on $[0,1]$