Math 2033 Comments on HW 1

March 23, 2016

In general

1. State clearly what the ”variable” comes from.
2. Be careful the definition of the supremum.

   Just supposing $\sup M \leq a$ does not yield any contradiction: (Consider $M = (0, 1)$, $a = 1$.)

   One Should show a is an upper bound of M (which implies $\sup M \leq a$) and then suppose $\sup M < a$ (which is same as $\sup M \neq a$ since $\sup M \leq a$) in order to yield a contradiction (Recall YOUR GOAL is to show $\sup M = a$.)
3. When Attempting to use limit supremum/infimum theorem to conclude $\sup M = a$ $\left(\inf M = b \text{ resp.}\right)$, make sure you have done the follwoing two steps:
   1. Show $\underline{a \text{ is an upper bound of } M}$ $\left(\underline{b \text{ is a lower bound of } M} \text{ resp.}\right)$
   2. Construct a sequence <u>lying inside M</u> that is <u>approaching to a</u> (<u>approaching to b</u> resp.).
4. Be careful with the Set notations:
   $\in$ and $\subseteq$ are NOT the SAME.
   Also, DO NOT make up intersection and union of two sets, namely, $A \cap B$ and $A \cup B$ respectively.
5. Do Not just list out special case(s), you need to show your arguments hold for everything that you want to show/prove.

   Listing Special case is NOT a proof!!!
6. We use the terminology: supremum of a set, but NOT suprmeum of a number/element.
7. Note that $\sup A$ is NOT NECESSARY lying in A. For example, $A = (0, 1)$, in which $\sup A = 1 \notin A$.

Problem 1

Many students forget to explain why the map is a bijection.

Problem 2

The presentation is not so clear and have some mistakes. Please learn from Professor Li.

Problem 3

1. Many student confuses the concepts ”finite” and ”countable”.
2. Students did not consider $(0, +\infty)\backslash S$, that is to prove $(0, +\infty)\backslash S$ is uncountable.

   Note that $g(x, y, z) = 2^x + 3^y + 5^z$ is not that special, we can use other map f as long as the $f : \mathbb{R} \to (0, +\infty)$, the same argument works.
   Note that $S = \{f(x, y, z) : (x, y, z) \in \mathbb{Q} \times \mathbb{Q} \times \mathbb{Q}\} = \{r \in (0, +\infty) : f(x, y, z) = r$ have a solution $(x, y, z) \in \mathbb{Q} \times \mathbb{Q} \times \mathbb{Q}\}$.
   This implies $\{r \in (0, +\infty) : f(x, y, z) = r$ have no solution $(x, y, z) \in \mathbb{Q} \times \mathbb{Q} \times \mathbb{Q}\} = (0, +\infty)/S$, is uncountable, and thus infinite.

   Hence, there exist infinitely many positive real numbers r such that the equation $f(x, y, z) = r$ has no solution $(x, y, z) \in \mathbb{Q} \times \mathbb{Q} \times \mathbb{Q}$.

Problem 4

1. DO NOT assume T or $T \cap \left[\frac{1}{k+1}, \frac{1}{k}\right)$ is finite/countable since it is what you need to show.

   Write $T = \{a_1, a_2, \cdots, a_n\}$ already means that T is finite.

   Write $T = \{a_1, a_2, \cdots\}$ already means that T is countable.
2. One Should use correct Theorem/definition for a countable Set.

Problem 5

1. Some students define sequences $\{x_n\}$, $\{y_n\}$ and $\{w_n\}$ in which $x_n \to 5$ and $y_n \to \sqrt{2}$ and $w_n \to 10\sqrt{2} - \frac{1}{5}$, however the sequence they constructed are NOT in the set concerned. So that they cannot apply supremum/infimum limit theorem.

2. Some students mentioned the fact that $10\sqrt{2} - \frac{1}{5}$ is an upper bounded of E without giving any reason OR forget to mention $10\sqrt{2} - \frac{1}{5}$ is an upper bounded of E.

3. In order to use Limit Supremum/infimum Theorem, One should show $10\sqrt{2} - \frac{1}{5}$ is an upper bound of E by constructing a sequence lying in E that approaching to $10\sqrt{2} - \frac{1}{5}$.

4. The number that we considered is

$$x(y + \sqrt{2}) - \frac{1}{x}$$

Remember two x are the same. If you do something like simply taking $\sup E = \sup D\left(\sqrt{2} + \sqrt{2}\right) - \frac{1}{\sup D}$ directly without some inequality kind of explanation, it works this time but it fails for the set $F = \left\{x(y + \sqrt{2}) + \frac{1}{x} : x \in D,\ y \in [0, \sqrt{2}) \cap \mathbb{Q}\right\}$.

Problem 6

Similar mistakes as in Problem 5.