Math 2033 (Homework 2) L1 Fall 2021

Problems (Due Oct. 11 at 11:59 pm)

① Prove that there is a bijection from $[0,1]$ to $(0,1]$.

② Determine if the set A of all intersection points in $\mathbb{R}^2$ of the family of lines $\{y=mx : m\in\mathbb{Z}\}$ with the family of circles $\{x^2+y^2=r^2 : r\in\mathbb{Q}\}$ is countable or uncountable. Here A is the set of all points in $\mathbb{R}^2$ that are on at least one of the lines $y=mx$ $(m\in\mathbb{Z})$ and at least one of the circles $x^2+y^2=r^2$ $(r\in\mathbb{Q})$.

③ Prove that there exist infinitely many positive real numbers r such that the equation $2^x+3^y+5^z=r$ has no solution $(x,y,z)\in\mathbb{Q}\times\mathbb{Q}\times\mathbb{Q}$.

④ Let T be a nonempty subset of the interval $(0,1)$. If every finite subset $\{x_1,x_2,\dots,x_n\}$ of T (with no two of $x_1,x_2,\cdots,x_n$ equal) has the property that $x_1^2+x_2^2+\dots+x_n^2<1$, then prove that T is a countable set.