MATH 2033 HW-1 Due Sept 13.

1. Use ONLY the empty set $\emptyset$ to construct three different nonempty sets.

2. Let $A = \{5, c, Q\}$. List out all the subsets of A, and find the power set $\wp(A)$.

3. Let A, B, C be subsets of some universal set X. Prove the following identities:

(a) $(A \cap B)^c = A^c \cup B^c$;

(b) $A \triangle B = \emptyset$ if and only if $A = B$;

(c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

4. Suppose $A, B \neq \emptyset$ (i.e., both A and B are nonempty sets).

(a) Prove that $A \times B = B \times A$ if and only if $A = B$.

(b) Where in your proof do you use the condition $A, B \neq \emptyset$?

(c) Does the same conclusion hold if we allow $A = \emptyset$ or $B = \emptyset$?

5. Suppose A, B, C are nonempty sets and $f : A \to B$ and $g : B \to C$ are functions.

(a) Prove that if $g \circ f : A \to C$ is injective, then f is injective. Can we say anything about g?

(b) Prove that if $g \circ f : A \to C$ is surjective, then g is surjective. Can we say anything about f?

6. Let $f : A \to B$ be a function. Denote I_A as the identity function $A \to A$, and I_B is the identity function $B \to B$, that is, $I_A(a) = a$ for every $a \in A$, and $I_B(b) = b$ for every $b \in B$. Suppose there exist two functions $g, h : B \to A$ such that $f \circ g = I_B$ and $h \circ f = I_A$. Show that f is a bijection and that $g = h = f^{-1}$.

7. Construct a set X and an equivalence relation on X which is different from what we have seen so far. Verify what you have defined is really an equivalence relation.