Tutorial 4.

Completeness Axiom

$X \neq \phi$ $X \subseteq R$ If X is boundd above, Then $\sup X$ exists in R

$\Leftrightarrow$. $X, Y \neq \phi$ $X, Y \subseteq R$. If $\forall x \in X$ $y \in Y$ $x \leq y$. Then. $\exists$ $c \in R$.
for $\forall x, y$. ~~$x \leq y$~~, we have. $x \leq c \leq y$

Prove. $\Rightarrow$. just choose. $c = \sup X$. so $\forall x \in X$ $x \leq c$. by defination of sup.
$c \leq y$ for $\forall y \in Y$.

$\Leftarrow$ assume $X \neq \phi$ $X \subseteq R$. X is bounded above. we need to prove $\sup X$ exists

X is bdd above. so X has upper bound

$Y = \{ y \in R : \forall x \in X \ x \leq y \}$. Y is the set of all upper bounds

so. $\forall x \in X$ $\forall y \in Y$. we have $x \leq y$

$\exists$ $c \in R$. $x \leq c \leq y$.

so c is also upper bound $c \in Y$

by $c \leq y$ we know c is min element in Y

by defination of sup. $c = \sup X$

3 theorems will be used frequently

(1). $x \leq y \Leftrightarrow \forall \varepsilon > 0$. $x < y + \varepsilon$.

(2). If S has sup in R. then. ~~$\exists x \in S$~~ $\forall \varepsilon > 0$. $\exists$ $x \in S$ $\sup S - \varepsilon < x \leq \sup S$

(3). If S has inf in R then $\forall \varepsilon > 0$ $\exists$ $x \in S$ $\inf S + \varepsilon > x \geq \inf S$