MATH 2033 HW-2 Due Sept 20.

1. Let $A = \emptyset$. Find $\mathscr{P}(\mathscr{P}(\mathscr{P}(A)))$.

2. Suppose A and B are two sets and $f : A \to B$ is a function. Let $A_1, A_2 \subseteq A$ and $B_1, B_2 \subseteq B$. Prove all the following.

(i). $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$;

(ii). $f(A_1 \cap A_2) \subseteq f(A_1) \cap f(A_2)$;

(iii). $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$;

(iv). $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$.

Also, find an example to show that equality does not necessarily hold in (ii).

3. Check whether the following relations are equivalence relations. Prove it if your answer is yes. Point out ALL the violations of the definition of the equivalence relation if your answer is no.

(a) For the set of all subsets of $\mathbb{Z}$, let $A \sim B$ mean that $A \subsetneq B$ or $B \subsetneq A$.

(b) For the set of all people on earth, let $P1 \sim P2$ mean that $P1$ has the same biological parents as $P2$.

(c) In the set X, let $x \sim y$ hold for all $x, y \in X$. If your answer is yes, find out all the equivalence class(es) in this case.

4. Verify that the multiplication on $\mathbb{Q}$ is well-defined.

5. Verify the distributive law for $\mathbb{Q}$.

6. Let $n \geq 2$ be an integer. Verify that the addition and multiplication on $\mathbb{Z}_n$ are well-defined. What are the additive identity and multiplicative identity? Justify your answers.