Useful Facts

① $\max(r,s) + \min(r,s) = r+s$
$\max(r,s) - \min(r,s) = |r-s|$

$\Rightarrow \boxed{\max(r,s) = \frac{r+s+|r-s|}{2},\ \min(r,s) = \frac{r+s-|r-s|}{2}}$

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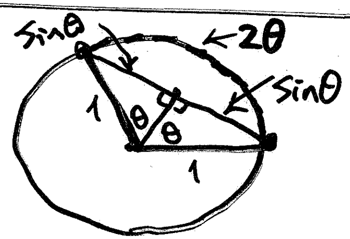
② $\boxed{|\sin\theta| \le |\theta|}$

For $|\theta| \le 1$, look at $\rightarrow$



$|\sin\theta| = \sin|\theta| \le |\theta|$

$2\sin\theta \le 2\theta,\ \theta \in [0, \frac{\pi}{2}]$

For $|\theta| > 1$, $|\sin\theta| \le 1 < |\theta|$.

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③ $\boxed{|\sin a - \sin b| \le |a-b|}$

Recall $\sin(x+y) = \sin x \cos y + \cos x \sin y$
$\sin(x-y) = \sin x \cos y - \cos x \sin y$

Set $x+y = a$, $x-y = b$ get $x = \frac{a+b}{2}$, $y = \frac{a-b}{2}$

$\sin a - \sin b = 2\cos x \sin y = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$

$|\sin a - \sin b| \le 2\cdot 1\cdot \left|\sin\left(\frac{a-b}{2}\right)\right| \le 2\left|\frac{a-b}{2}\right| = |a-b|$.

④ For $0 < r < 1$ and $a \ge b > 0$

let $t = \frac{b}{a} \in (0,1]$, then

$$\left.\begin{array}{l} t^r \ge t \\ (1-t)^r \ge (1-t) \end{array}\right\} \Rightarrow \begin{array}{l} t^r + (1-t)^r \ge 1 \\ (1-t)^r \ge 1 - t^r \end{array}$$

Multiply by a^r on both sides $\uparrow$, we get

$$a^r(1-t)^r \ge a^r(1-t^r)$$

$$\begin{array}{ccc} a^r(1-t)^r & \ge & a^r(1-t^r) \\ \| & & \| \\ (a-at)^r & & a^r - (at)^r \\ \| & & \| \\ (a-b)^r & & a^r - b^r \end{array}$$

For $r = \frac{1}{n}$, $n = 2,3,4,\dots$, we get $a,b > 0$

$\Rightarrow \boxed{\sqrt[n]{|a-b|} \ge |\sqrt[n]{a} - \sqrt[n]{b}|}$