Problems (Due Nov. 1 at 11:59 pm)

① Given $x_n \neq -1$ for all $n \in \mathbb{N}$. If $\lim\limits_{n \to \infty} x_n = 0$, then show that $\lim\limits_{n \to \infty} \dfrac{x_n}{1+x_n} = 0$ by checking the definition of limit.

② Let $a_1 = 9$ and $a_{n+1} = \dfrac{\sqrt{a_n} + 2a_n}{3}$ for $n = 1, 2, 3, \cdots$. Prove that $a_1, a_2, a_3, \cdots$ converges and find its limit.

③ Let $w_1, w_2, w_3, \cdots$ be a sequence such that for $k = 1, 2, 3, \cdots$, we have $|w_{k+1} - w_k| < \dfrac{1}{2^k}$. Then prove that $w_1, w_2, w_3, \cdots$ is a Cauchy sequence.

④ Show that for every $t \in \mathbb{R}$, there is a strictly increasing sequence of irrational numbers $t_1, t_2, t_3, \cdots$ converging to t.