MATH 2033 HW-7 Due Nov 1.

1. Find the accumulation points of the following sets in $\mathbb{R}$. NO justification is needed.

   i. $S = (0, 1)$;

   ii. $S = \{(-1)^n + \frac{1}{n} \mid n \in \mathbb{N}\}$;

   iii. $S = \mathbb{Q}$;

   iv. $S = \mathbb{R}$;

2.

   i. Show, by example, that an infinite intersection of open sets in $\mathbb{R}$ is not necessarily open.

   ii. Show, by example, that an infinite union of closed sets in $\mathbb{R}$ is not necessarily closed.

   iii. Show that $\emptyset$ and $\mathbb{R}$ are the only two subsets of $\mathbb{R}$ that are both open and closed in $\mathbb{R}$.

   iv. Show that a subset of $\mathbb{R}$ is closed if and only if it contains all its accumulation points.

   v. Suppose S is a bounded and closed nonempty subset of $\mathbb{R}$. Prove that $\sup S \in S$.