Solutions to Practice Exercises

① $\sim((x>0 \text{ and } x<1) \text{ or } x=-1) = \sim(x>0 \text{ and } x<1) \text{ and } x\neq -1$
$= (x\leq 0 \text{ or } x\geq 1) \text{ and } x\neq -1$

② $\sim(x>0 \text{ and } (x<1 \text{ or } x=-1)) = x\leq 0 \text{ or } \sim(x<1 \text{ or } x=-1)$
$= x\leq 0 \text{ or } (x\geq 1 \text{ and } x\neq -1)$
$= x\leq 0 \text{ or } x\geq 1$

③ $\sim(\forall \triangle ABC, \angle A+\angle B+\angle C = 180^\circ) = \exists \triangle ABC$ such that $\angle A+\angle B+\angle C \neq 180^\circ$
(There is a triangle ABC such that $\angle A+\angle B+\angle C\neq 180^\circ$.)

④ $\sim(\exists$ man such that man does not have wife$) = \forall$ man, man has a wife
(Every man has a wife.)

⑤ $\sim(\forall x . \exists y$ such that $x+y=0) = \exists x\ \forall y,\ x+y\neq 0$
(There is an x such that for every y, $x+y\neq 0$.)

⑥ $\sim(\exists\alpha\ \forall\beta\ \exists\gamma$ such that $|\alpha-\beta|<\gamma) = \forall\alpha\ \exists\beta\ \forall\gamma,\ |\alpha-\beta|\geq\gamma$.

⑦ $\sim(\text{If } (x>0) \text{ and } (y>0), \text{ then } x+y>0) = (x>0) \text{ and } (y>0) \text{ and } (x+y\leq 0)$

⑧ (a) If $\angle B\neq\angle C$ in $\triangle ABC$, then $AB\neq AC$ in $\triangle ABC$.

(b) If a function is not continuous, then it is not differentiable.

(c) If $\lim_{x\to 0}(f(x)+g(x))\neq a+b$, then $\lim_{x\to 0} f(x)\neq a$ or $\lim_{x\to 0} g(x)\neq b$.

(d) If $x\neq \frac{-b+\sqrt{b^2-4c}}{2}$ and $x\neq\frac{-b-\sqrt{b^2-4c}}{2}$, then $x^2+bx+c\neq 0$.

⑨ (a) $(\{x,y,z\}\cup\{w,z\})\setminus\{u,v,w\} = \{w,x,y,z\}\setminus\{u,v,w\} = \{x,y,z\}$.

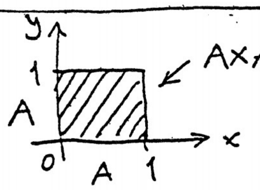
(b) $\{1,2\}\times\{3,4\}\times\{5\} = \{(1,3,5),(1,4,5),(2,3,5),(2,4,5)\}$.

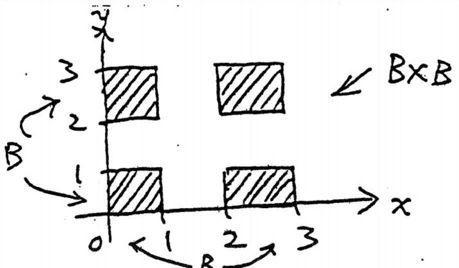
(c) $\mathbb{Z}\cap[0,10]\cap\{n^2+1 : n\in\mathbb{N}\} = \{0,1,2,\cdots,10\}\cap\{2,5,10,\cdots\} = \{2,5,10\}$.

(d) $\{n\in\mathbb{N} : 5<n<9\}\setminus\{2m : m\in\mathbb{N}\} = \{6,7,8\}\setminus\{2,4,6,8,10,\cdots\} = \{7\}$.

(e) $([0,2]\setminus[1,3])\cup([1,3]\setminus[0,2]) = [0,1)\cup(2,3]$.

⑩ (i)





(ii) $A=B$ (Reason: For every $a\in A$, $b\in B$, we have $(a,b)\in A\times B = B\times A$.
By the definition of Cartesian product, this means $a\in B$, $b\in A$. So $A\subseteq B$ and $B\subseteq A$.)

(11) (a) If $x \in A \cup B$, then $x \in A$ or $x \in B$, which implies $x \in A$ or $x \in C$ (because $B \subseteq C$ and $x \in B$ will yield $x \in C$). So $x \in A \cup C$.
So every element of $A \cup B$ is also an element of $A \cup C$. Therefore, $A \cup B \subseteq A \cup C$.

(b) If $x \in (X \setminus Y) \setminus Z$, then $x \in X \setminus Y$ and $x \notin Z$. So $x \in X$ and $x \notin Y$ and $x \notin Z$. Then $x \in X$ and $x \notin Z$ and $x \notin Y$. Hence, $x \in X \setminus Z$ and $x \notin Y$. Therefore, $x \in (X \setminus Z) \setminus Y$. We get $(X \setminus Y) \setminus Z \subseteq (X \setminus Z) \setminus Y$.

Interchanging Y, Z everywhere in the last paragraph, we also get $(X \setminus Z) \setminus Y \subseteq (X \setminus Y) \setminus Z$.
Therefore, $(X \setminus Y) \setminus Z = (X \setminus Z) \setminus Y$.

---

(12) (i) False. For example, $A = \mathbb{R} \setminus \mathbb{Q}$, $B = \mathbb{Q} = C$, then $(A \cup B) \cap C = \mathbb{R} \cap \mathbb{Q} = \mathbb{Q}$
$A \cup (B \cap C) = (\mathbb{R} \setminus \mathbb{Q}) \cup \mathbb{Q} = \mathbb{R}$

(ii) False. For example, $A = \mathbb{R} = B$, $C = \mathbb{Q}$, then $A \cup B = \mathbb{R} = A \cup C$, but $B \neq C$.

(iii) True. (<u>Reason</u>: For every $x \in A \setminus (B \cup C)$, we have $x \in A$ and $x \notin B \cup C$. Now $\boxed{x \notin B \cup C = \sim(x \in B \cup C) = \sim((x \in B) \text{ or } (x \in C)) = x \notin B \text{ and } x \notin C}$. So $x \in A \setminus B$ and $x \in A \setminus C$. We get $x \in (A \setminus B) \cap (A \setminus C)$. $\therefore$ $A \setminus (B \cup C) \subseteq (A \setminus B) \cap (A \setminus C)$.
Next we reverse steps. For every $x \in (A \setminus B) \cap (A \setminus C)$, we have $x \in A \setminus B$ and $x \in A \setminus C$. So $x \in A$ and $x \notin B$ and $x \notin C$. By the box above, we get $x \in A$ and $x \notin B \cup C$. So $x \in A \setminus (B \cup C)$. $\therefore$ $(A \setminus B) \cap (A \setminus C) \subseteq A \setminus (B \cup C)$.)

---

(13) (i) For every $x \in A \cup C$, we have $x \in A$ or $x \in C$. If $x \in A$, then $A \subseteq B$ implies $x \in B$. If $x \in C$, then $C \subseteq D$ implies $x \in D$. So $x \in B$ or $x \in D$, which implies $x \in B \cup D$.

(ii) False. For example, let $A = \{0\}$, $C = \{1\}$, $B = \{0,1\} = D$, then $A \cup C = \{0,1\} = B \cup D$.

(iii) Yes. (<u>Reason</u>: Since $(\frac{1}{n}, 2)_{\mathbb{Q}} \subseteq [\frac{1}{n}, 2)_{\mathbb{Q}}$ for each n, so as in part (i),

$$\bigcup_{n=1}^{\infty} (\tfrac{1}{n}, 2)_{\mathbb{Q}} = (1,2)_{\mathbb{Q}} \cup (\tfrac{1}{2}, 2)_{\mathbb{Q}} \cup (\tfrac{1}{3}, 2)_{\mathbb{Q}} \cup \cdots \subseteq [1,2)_{\mathbb{Q}} \cup [\tfrac{1}{2}, 2)_{\mathbb{Q}} \cup [\tfrac{1}{3}, 2)_{\mathbb{Q}} \cup \cdots = \bigcup_{n=1}^{\infty} [\tfrac{1}{n}, 2)_{\mathbb{Q}}.$$

For the reverse inclusion, since $[\frac{1}{n}, 2)_{\mathbb{Q}} \subseteq (\frac{1}{n+1}, 2)_{\mathbb{Q}}$ for each n, we have

$$\bigcup_{n=1}^{\infty} [\tfrac{1}{n}, 2)_{\mathbb{Q}} = [1,2)_{\mathbb{Q}} \cup [\tfrac{1}{2}, 2)_{\mathbb{Q}} \cup [\tfrac{1}{3}, 2)_{\mathbb{Q}} \cup \cdots \subseteq (\tfrac{1}{2}, 2)_{\mathbb{Q}} \cup (\tfrac{1}{3}, 2)_{\mathbb{Q}} \cup (\tfrac{1}{4}, 2)_{\mathbb{Q}} \cup \cdots = \bigcup_{n=1}^{\infty} (\tfrac{1}{n}, 2)_{\mathbb{Q}})$$

because $(\frac{1}{2}, 2)_{\mathbb{Q}} = (1,2)_{\mathbb{Q}} \cup (\frac{1}{2}, 2)_{\mathbb{Q}}$.

$\boxed{\text{Actually, } \bigcup_{n=1}^{\infty} [\tfrac{1}{n}, 2)_{\mathbb{Q}} = (0,2)_{\mathbb{Q}} = \bigcup_{n=1}^{\infty} (\tfrac{1}{n}, 2)_{\mathbb{Q}} \text{ but this is less rigorous.}}$

---

(14) f is not injective because $f(1) = 0 = f(2)$. f is not surjective because $f(\mathbb{R}) = \{0,1\} \neq \mathbb{R}$. g is injective because $g(x) = g(y) \Leftrightarrow 1 - 2x = 1 - 2y$ implies $x = y$.
g is surjective because for every $y \in \mathbb{R}$, $y = g(\frac{1-y}{2})$ and so $g(\mathbb{R}) = \mathbb{R}$.
$f \circ g : \mathbb{R} \to \mathbb{R}$ is given by $(f \circ g)(x) = f(g(x)) = f(1-2x) = \begin{cases} 0 & \text{if } 1-2x > 0 \\ 1 & \text{if } 1-2x \le 0 \end{cases}$
$= \begin{cases} 0 & \text{if } \frac{1}{2} > x \\ 1 & \text{if } \frac{1}{2} \le x \end{cases}$. $g \circ f : \mathbb{R} \to \mathbb{R}$ is given by $(g \circ f)(x) = g(f(x)) = \begin{cases} 1 = g(0) & \text{if } x > 0 \\ -1 = g(1) & \text{if } x \le 0 \end{cases}$.

---

(15) (i) To show f is injective, let $f(x)=f(y)$. Then $x=(g\circ f)(x)=g(f(x))=g(f(y))$
$=(g\circ f)(y)=y$. Next we will show f is surjective. For every $b\in B$, since
$b=(f\circ g)(b)=f(g(b))$, we see that $b\in f(A)$. $\therefore$ $f(A)=B$.

(ii) To show $h\circ f$ is injective, let $(h\circ f)(x)=(h\circ f)(y)$. Then $h(f(x))=h(f(y))$.
Since h is injective, we get $f(x)=f(y)$. Since f is injective, we get $x=y$.
Next we will show $h\circ f$ is surjective. For every $c\in C$, since h is surjective,
$C=h(B)$, which implies $c=h(b)$ for some $b\in B$. Since f is surjective,
$B=f(A)$, which implies $b=f(a)$ for some $a\in A$. Then $c=h(b)=h(f(a))$
$=(h\circ f)(a)\in(h\circ f)(A)$. $\therefore$ $(h\circ f)(A)=C$.

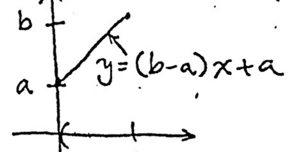
---

(16) For the 'at most once' case, to show f is injective, let $f(x_0)=f(y_0)$. Using
the choice $b=f(x_0)$, we see that the line $y=b$ intersects the graph of f at the
point $(x_0,f(x_0))$ and at the point $(y_0,f(y_0))$. Since the intersection is at most
one point, we have $(x_0,f(x_0))=(y_0,f(y_0))$, which implies $x_0=y_0$.

For the 'at least once' case, we can conclude f is surjective. (Reason:
For every $b\in B$, the line $y=b$ intersects the graph of f at least once.
This implies there is a point (a,b) on the graph of f. Then $b=f(a)\in f(A)$.
$\therefore$ $f(A)=B$.)

(Comments: Combining the two cases, we see that if for every $b\in B$, the horizontal
line $y=b$ intersects the graph of f exactly once, then f is a bijection. This
"horizontal line test" is useful to check bijections by inspecting the graphs.)

---

(17) 

The function $f:(0,1)\to(a,b)$ defined by $f(x)=(b-a)x+a$ is
a bijection. (This is clear from the graph: As x varies from
a to b, $f(x)$ takes each of the values between a and b
exactly once.) Since $(0,1)$ is uncountable, by the bijection theorem
we see that (a,b) is uncountable. Since $(a,b)\subseteq[a,b]$, by the countable
subset theorem, $[a,b]$ is uncountable.

---

(18) Let $S=\{(0,y): y\in\mathbb{R}\setminus\mathbb{Q}\}$. The function $f:\mathbb{R}\setminus\mathbb{Q}\to S$ defined by
$f(y)=(0,y)$ is a bijection. Since $\mathbb{R}\setminus\mathbb{Q}$ is uncountable, by the remarks,
S is uncountable. Since $S\subseteq\mathbb{Q}\times(\mathbb{R}\setminus\mathbb{Q})$, by the countable subset theorem,
$\mathbb{Q}\times(\mathbb{R}\setminus\mathbb{Q})$ is uncountable.

---

(19) For $n,m\in\mathbb{Z}$, $\frac{1}{2^n}+\frac{1}{3^m}\in\mathbb{Q}$. So $A\subseteq\mathbb{Q}$. Since $\mathbb{Q}$ is countable, by the
countable subset theorem, A is countable.

(20) For $x \in \mathbb{N}$, let $B_x = \{x + \sqrt{2}y : y \in \mathbb{N}\}$. The function $f: \mathbb{N} \to B_x$ defined by $f(y) = x + \sqrt{2}y$ is a bijection. So B_x is countable. Now $B = \bigcup_{x \in \mathbb{N}} B_x$, $\mathbb{N}$ is countable, each B_x is countable for $x \in \mathbb{N}$, so by the countable union theorem, B is countable.

---

(21) Let $S = \{L_m : L_m$ is the line with equation $y = mx$, $m \in \mathbb{R}\}$. The function $f: \mathbb{R} \to S$ defined by $f(m) = L_m$ is a bijection. Since $\mathbb{R}$ is uncountable, by the remarks, S is uncountable. Since $S \subseteq C$, by the countable subset theorem, C is uncountable.
C contains vertical line, not in S.

---

(22) For $r \in \mathbb{Q}$, $D_r = \{x \in \mathbb{R} \mid x^5 + x + 2 = r\}$ has at most 5 elements, so D_r is countable. Now $D = \bigcup_{r \in \mathbb{Q}} D_r$, $\mathbb{Q}$ is countable and each D_r is countable for $r \in \mathbb{Q}$, so by the countable union theorem, D is countable.

---

(23) Let $\mathbb{Q}^+$ be the positive rational numbers. Since $\mathbb{Q}^+ \subseteq \mathbb{Q}$ and $\mathbb{Q}$ is countable, by the countable subset theorem, $\mathbb{Q}^+$ is countable. Now the function $f: \mathbb{Q} \times \mathbb{Q} \times \mathbb{Q}^+ \to E$ defined by letting $f(x, y, r)$ be the circle centered at (x, y) and radius r is a bijection. Since $\mathbb{Q}$ and $\mathbb{Q}^+$ are countable, by the product theorem, $\mathbb{Q} \times \mathbb{Q} \times \mathbb{Q}^+$ is countable. By the bijection theorem, E is countable.

---

(24) Suppose $x^4 + ax - 5 = 0$ has a rational root r. (If $r = 0$, then $r^4 + ar - 5 \neq 0$.) We get $r \neq 0$ and $r^4 + ar - 5 = 0 \Rightarrow a = \frac{5 - r^4}{r} \in \mathbb{Q}$. So $F \subseteq \mathbb{Q}$. Therefore, F is countable.

---

(25) Since X is nonempty, let $a_0 \in X$. Consider the subset $G' = \{a_0^3 + b^3 : b \in Y\}$ of G. The function $f: Y \to G'$ defined by $f(b) = a_0^3 + b^3$ is a bijection (From $w = a_0^3 + b^3 \Leftrightarrow b = \sqrt[3]{w - a_0^3}$, we see $g: G' \to Y$ defined by $g(w) = \sqrt[3]{w - a_0^3}$ is the inverse of f.) Since Y is uncountable, so G' is uncountable. Since $G' \subseteq G$, so G is also uncountable.

---

(26) We will show $Y \setminus X$ is uncountable first. Suppose $Y \setminus X$ is countable. Since X is countable and $X \cap Y \subseteq X$, we get $X \cap Y$ countable by the countable subset theorem. Then $Y = (Y \setminus X) \cup (X \cap Y)$ is countable by the countable union theorem, a contradiction. $\therefore$ $Y \setminus X$ is uncountable. Since $Y \setminus X \subseteq (X \setminus Y) \cup (Y \setminus X)$, $H = (X \setminus Y) \cup (Y \setminus X)$ is uncountable by the countable subset theorem.

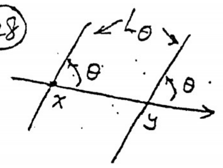
Solutions to Presentation Exercises (Week 6)

(27) Solution 1. For $k=0,1,2,\cdots$, let S_k be the set of all subsets of $\mathbb{N}$ having exactly k elements. Then $S_0=\{\emptyset\}$ has one element and so S_0 is countable. For $k\in\mathbb{N}$, the function $f_k: S_k \to \underbrace{\mathbb{N}\times\mathbb{N}\times\cdots\times\mathbb{N}}_{k\ \mathbb{N}'s}$ defined by $f(\{n_1,n_2,\cdots,n_k\})=(n_1,n_2,\cdots,n_k)$ is an injective function. ($n_1,n_2,\cdots,n_k$ in increasing order.) Since $\mathbb{N}\times\mathbb{N}\times\cdots\times\mathbb{N}$ is countable by the product theorem, we can use the bijection theorem to conclude that S_k is countable. Then $F=S_0\cup\left(\bigcup_{k=1}^{\infty}S_k\right)$ is countable by the countable union theorem.

Solution 2 Define $g: F\to\mathbb{N}\cup\{0\}$ by assigning to each finite subset S of $\mathbb{N}$ the nonnegative integer n having base 2 representation $n=(\cdots d_3d_2d_1)_2$, where $d_j=1$ if and only if $j\in S$. (For example, $S=\{1,2,4\}\to n=(1011)_2=8+2+1=11$.) Note g has the inverse $g^{-1}:\mathbb{N}\cup\{0\}\to F$ by assigning $n=(\cdots d_3d_2d_1)_2$ the subset $\{j: d_j=1\}$. It follows g is a bijection. As $\mathbb{N}\cup\{0\}$ is countable, so F is countable.

Solution 3 Let $F_0=\{\emptyset\}$. Let F_k be the set of all subsets of $\{1,2,\cdots,k\}$ for $k\in\mathbb{N}$. By math induction, we can check that F_k has 2^k elements. (This was discussed in the power set examples.) Clearly, $F_k\subseteq F$ because every element of F_k is a finite subset of $\mathbb{N}$. So $\bigcup_{k=0}^{\infty}F_k\subseteq F$ by the definitions of union and subset. For every $S\in F$, S is a finite subset of $\mathbb{N}$. If $S\neq\emptyset$, then S has a maximum element k. Then $S\in F_k$. Hence $F\subseteq\bigcup_{k=0}^{\infty}F_k$.
$\therefore$ $F=\bigcup_{k=0}^{\infty}F_k$ is a countable union of finite sets, hence countable by the countable union theorem.

---

(28) 

For $\theta\in(0,\pi)$, let L_θ be the pair of lines through x and y respectively making an angle θ with the axis from x to y. Let $T=\{L_\theta:\theta\in(0,\pi)\}$. The function $f:(0,1)\to T$ defined by $f(x)=L_{\pi x}$ has the inverse $g:T\to(0,1)$ given by $g(L_\theta)=\frac{\theta}{\pi}$. So f is a bijection. Since $(0,1)$ is uncountable, T is uncountable.
Next observe that for every $z\in S$, there are at most two θ's such that z is on one of the lines of L_θ, namely when $\overleftrightarrow{xz}$ or $\overleftrightarrow{yz}$ is one of the lines of L_θ. So $V=\{L_\theta: L_\theta \text{ contains some } z\in S\}=\underbrace{\bigcup_{z\in S}}_{\text{countable}}\underbrace{\{L_\theta: L_\theta \text{ contains } z\}}_{\text{at most 2 elements}}$ is countable.
Since T is uncountable and V is countable, so $T\setminus V$ is uncountable. In particular, taking two distinct L_θ's in $T\setminus V$, the parallelogram determined by them is in $\mathbb{R}^2\setminus S$ and has x,y as opposite vertices.

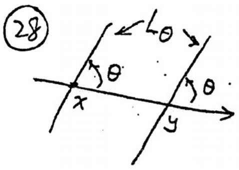
---

(27) Solution 1

For $k=0,1,2,\cdots$, let S_k be the set of all subsets of $\mathbb{N}$ having exactly k elements. Then $S_0=\{\emptyset\}$ has one element and so S_0 is countable. For $k\in\mathbb{N}$, the function $f_k: S_k \to \underbrace{\mathbb{N}\times\mathbb{N}\times\cdots\times\mathbb{N}}_{k\ \mathbb{N}'s}$ defined by $f(\{n_1,n_2,\cdots,n_k\})=(n_1,n_2,\cdots,n_k)$ is an injective function. ($\uparrow$ increasing order) Since $\mathbb{N}\times\mathbb{N}\times\cdots\times\mathbb{N}$ is countable by the product theorem, we can use the bijection theorem to conclude that S_k is countable. Then $F=S_0\cup\left(\bigcup_{k=1}^{\infty}S_k\right)$ is countable by the countable union theorem.

Solution 2 Define $g: F\to\mathbb{N}\cup\{0\}$ by assigning to each finite subset S of $\mathbb{N}$ the nonnegative integer n having base 2 representation $n=(\cdots d_3d_2d_1)_2$, where $d_j=1$ if and only if $j\in S$. (For example, $S=\{1,2,4\}\to n=(1011)_2=8+2+1=11$.) Note g has the inverse $g^{-1}:\mathbb{N}\cup\{0\}\to F$ by assigning $n=(\ldots d_3d_2d_1)_2$ the subset $\{j: d_j=1\}$. It follows g is a bijection. As $\mathbb{N}\cup\{0\}$ is countable, so F is countable.

---

(28) 

For $\theta\in(0,\pi)$, let L_θ be the pair of lines through x and y respectively making an angle θ with the axis from x to y. Let $T=\{L_\theta:\theta\in(0,\pi)\}$. The function $f:(0,1)\to T$ defined by $f(x)=L_{\pi x}$ has the inverse $g:T\to(0,1)$ given by $g(L_\theta)=\frac{\theta}{\pi}$. So f is a bijection. Since $(0,1)$ is uncountable, T is uncountable.

Next observe that for every $z\in S$, there are at most two θ's such that z is on one of the lines of L_θ, namely when $\overleftrightarrow{xz}$ or $\overleftrightarrow{yz}$ is one of the lines of L_θ. So $V=\{L_\theta: L_\theta \text{ contains some } z\in S\}=\underbrace{\bigcup_{z\in S}}_{\text{countable}}\underbrace{\{L_\theta: L_\theta \text{ contains } z\}}_{\text{at most 2 elements}}$ is countable.

Since T is uncountable and V is countable, so $T\setminus V$ is uncountable. In particular, taking two distinct L_θ's in $T\setminus V$, the parallelogram determined by them is in $\mathbb{R}^2\setminus S$ and has x,y as opposite vertices.

---

(29) For $x\in[0,1]$, let $x\doteq(0.a_1a_2a_3\cdots)_3$. Observe that $(\frac13,\frac23)=\{x: a_1=1\}$ where we take $\frac13=(0.022\cdots)_3$. So $K_1=[0,1]\setminus(\frac13,\frac23)=\{x: a_1\neq1\}$.

Also, $(\frac19,\frac29)=\{x: a_1=0, a_2=1\}$ where we take $\frac19=(0.0022\cdots)_3$ and $(\frac79,\frac89)=\{x: a_1=2, a_2=1\}$ where we take $\frac79=(0.2022\cdots)_3$.

So $K_2=K_1\setminus\left((\frac19,\frac29)\cup(\frac79,\frac89)\right)=\{x: a_1\neq1, a_2\neq1\}$. Similarly, we will get $K_n=\{x: a_1\neq1, a_2\neq1,\cdots,a_n\neq1\}$. Therefore $K=\{x: \text{all } a_i\neq1\}$ $=\{x: \text{all } a_i=0 \text{ or } 2\}$. Define $f:\{0,1\}\times\{0,1\}\times\cdots\to K$ by $f((b_1,b_2,\cdots))=x$ where $a_i=2b_i$ for $i=1,2,3,\cdots$. This function has the inverse $g:K\to\{0,1\}\times\{0,1\}\times\cdots$ defined by $g(x)=(b_1,b_2,\cdots)$, where $b_i=\frac{a_i}{2}$ for $i=1,2,3,\cdots$. So f is a bijection. Since $\{0,1\}\times\{0,1\}\times\cdots$ is uncountable, K is uncountable.

Remarks In the above solution, when we wrote K_1, K_2, K_n, K as sets of x with $a_i\neq1$, we mean "x has at least one base 3 representation, where the a_i's $\neq1$."