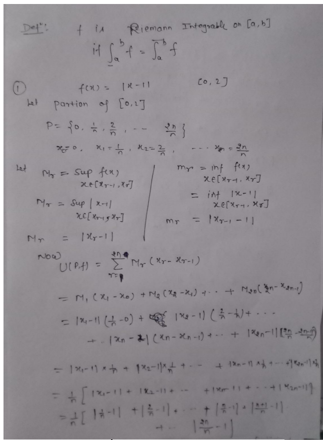
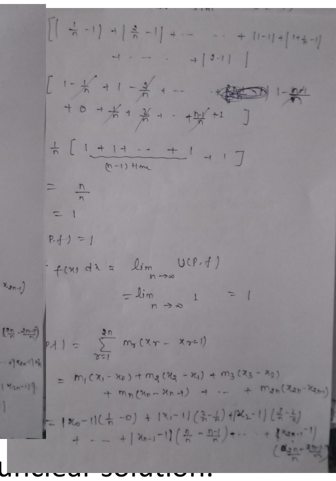
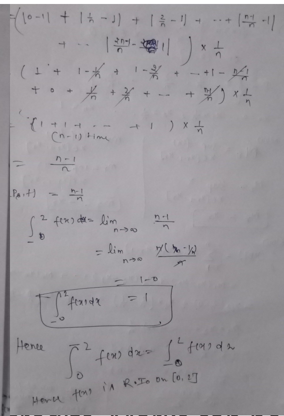
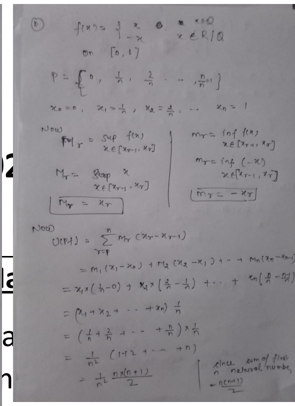
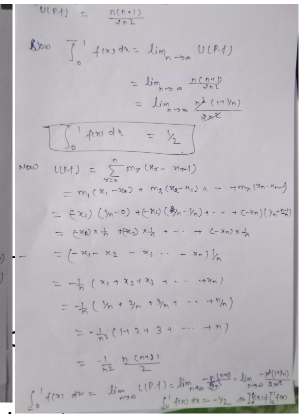










credits. Marks can be deducted for incomplete solution or u...

Please submit your completed work via the submission system in canvas before the deadline. Late assignment will not be accepted.

Your submission must (1) be hand-written (typed assignment will not be accepted), (2) in a single pdf. file (other file formats will not be accepted) and (3) contain your full name and student ID on the first page of the assignment.

Problem 1

(a) Using the definition of integrability or integral criterion, prove that $f(x) = |x-1|$ is integrable on $[0,2]$.

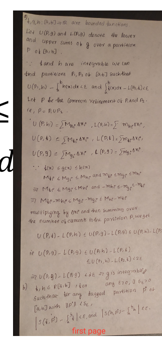
(b) Using the definition of integrability or integral criterion, prove that the function $f:[0,1] \to \mathbb{R}$ defined by

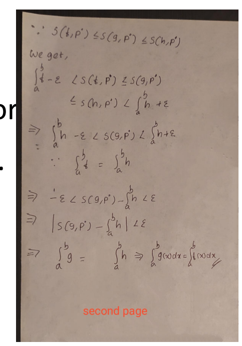
$$f(x) = \begin{cases} x & if \;\; x \in \mathbb{Q} \\ -x & if \;\; x \in \mathbb{R}\backslash\mathbb{Q} \end{cases}$$

is not integrable on $[0,1]$.

Problem 2

We let f, g, h be three bounded functions on $[a,b]$ such that $f(x) \leq$ ... $)$ for all $x \in [a,b]$. Suppose that f, h are integrable on $[a,b]$ and $\int_a^b f(x)d$ ... $)dx$.





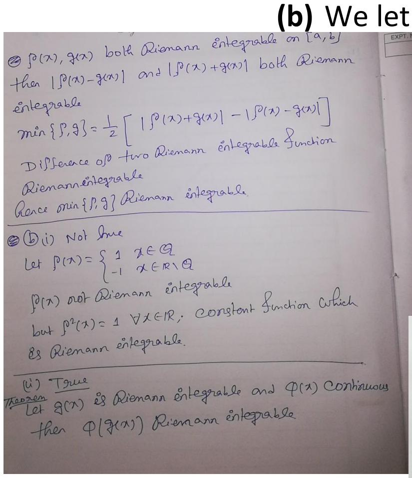
(a) Show that g is integrable on $[a,b]$.

(b) Show that $\int_a^b g(x)dx = \int_a^b f(x)dx$.

Problem 3

(a) We let $f, g: [a,b] \to \mathbb{R}$ be two bounded Riemann integrable function on $[a,b]$, show that the function $h(x) = \min(f(x), g(x))$ is also Riemann integrable on $[a,b]$.

(b) We let $f: [a,b] \to \mathbb{R}$ be a bounded function on $[a,b]$.



$f(x), g(x)$ both Riemann integrable on $[a,b]$ then $|f(x)-g(x)|$ and $|f(x)+g(x)|$ both Riemann integrable

$\min\{f,g\} = \frac{1}{2}\left[|f(x)+g(x)| - |f(x)-g(x)| \right]$

Difference of two Riemann integrable function Riemann integrable

Hence $\min\{f,g\}$ Riemann integrable

(b)(i) Not true

Let $f(x) = \begin{cases} 1 & x \in \mathbb{Q} \\ -1 & x \in \mathbb{R}\backslash\mathbb{Q} \end{cases}$

$f(x)$ not Riemann integrable

but $f^2(x) = 1 \;\; \forall x \in \mathbb{R}$, constant function which is Riemann integrable.

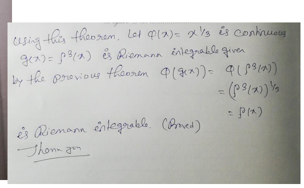
(ii) True

Theorem: Let $g(x)$ is Riemann integrable and $\varphi(x)$ continuous then $\varphi(g(x))$ Riemann integrable

Suppose that f^2 is Riemann integrable, is it true that f is Riemann integrable? Explain your answer.

Suppose that f^3 is Riemann integrable, is it true that f is Riemann integrable? Explain your answer.

: If your answer is yes, please give a proof. If your answer is no, please ...e a counter-example.)



Using this theorem. Let $\varphi(x) = x^{1/3}$ is continuous $g(x) = f^3(x)$ is Riemann integrable given by the previous theorem $\varphi(g(x)) = \varphi(f^3(x)) = (f^3(x))^{1/3} = f(x)$

is Riemann integrable. (Proved)

Thank you