Math 2033 (Mathematical Analysis) **Spring 2016**

Homework # 2 – Due Thursday, March 31, 2016 at noon

Please submit your homework to my office in Room 3471 (by slipping it under the door if I am not in the office). *Be sure to write your name (as shown on your student ID card) and your tutorial section number on the homework! Show work. Answers are worth very little.* **Take pictures of your homework and submit the original.**

1. Let
$$x_1 = 1, \quad \text{and} \quad x_{n+1} = \frac{1}{3}\sqrt{5x_n^2 + 11x_n} \quad \text{for } n = 1, 2, 3, \ldots.$$
Determine if the sequence $x_1, x_2, x_3, \ldots$ converges or not with proof. In case of convergence, also determine the limit of the sequence.

2. Let
$$x_1 = 1 \qquad \text{and} \qquad x_{n+1} = \frac{7}{4} + \frac{1}{2x_n} \quad \text{for } n = 1, 2, 3, \ldots.$$
Determine if the sequence $x_1, x_2, x_3, \ldots$ converges or not with proof. In case of convergence, also determine the limit of the sequence.

3. Prove that
$$\lim_{n\to\infty} \Big(\frac{n + \sqrt{n}}{4n - 3} + \frac{7n^2}{4n^2 + 1} \Big) = 2$$
by checking the definition of limit of sequences only.

4. Let $w_n \in [2, 4]$ for every positive integer n and $\lim\limits_{n\to\infty} w_n = 3$. Prove that
$$\lim_{n\to\infty} \Big(\sqrt{\frac{w_n}{2w_n + 6}} + \frac{2w_n + n}{2n + w_n} \Big) = 1$$
by checking the definition of limit of sequences only.

5. Let
$$x_1 = 2, \quad x_2 = 4 \quad \text{and} \quad x_{n+2} = \sqrt{10x_n - 9} \quad \text{for } n = 1, 2, 3, \ldots.$$
Determine if the sequence $x_1, x_2, x_3, \ldots$ converges or not with proof. In case of convergence, also determine the limit of the sequence.