MATH 2033 HW-4 Due Oct 11.

1. Prove that an ordered field has the least upper bound property if and only if it has the greatest lower bound property.

2. Let A be a bounded nonempty subset of $\mathbb{R}$. Define

$$-A = \{-x \mid x \in A\}.$$

Prove that

$$\sup(-A) = -\inf A$$

and

$$\inf(-A) = -\sup A.$$

3. Suppose A and B are bounded sets in $\mathbb{R}$. Prove or disprove the following

i. $\sup(A \cup B) = \max\{\sup A, \sup B\}$.

ii. If $A + B = \{a + b \mid a \in A,\ b \in B\}$, then $\sup(A + B) = \sup A + \sup B$.

iii. If the elements of A and B are positive and $A \cdot B = \{ab \mid a \in A,\ b \in B\}$, then

$$\sup(A \cdot B) = \sup A \cdot \sup B.$$

iv. Formulate the analogous problems for the greatest lower bound.