(3). prove. If $\lim\limits_{n\to\infty} x_n = a$. Then $\lim\limits_{n\to\infty} \frac{x_1+x_2+\cdots+x_n}{n} = a$

a can be real number or $+\infty$ or $-\infty$

(4). If $x_n > 0$. $\lim\limits_{n\to\infty} x_n = a$. ($a>0$ or $a=+\infty$)
Then $\lim\limits_{n\to\infty} \left(\frac{x_1^{-1}+x_2^{-1}+\cdots+x_n^{-1}}{n} \right)^{-1} = a$.

(5) If $x_n > 0$. $\lim\limits_{n\to\infty} x_n = a$. ($a \geq 0$ or. $a=+\infty$)

Then. $\lim\limits_{n\to\infty} \sqrt[n]{x_1 \cdots x_n} = a$.

(6). If $\lim\limits_{n\to\infty} \frac{x_{n+1}}{x_n} = a$. $x_n > 0$. ($a \geq 0$ or $a = +\infty$)
Then. $\lim\limits_{n\to\infty} \sqrt[n]{x_n} = a$.

(7). prove. $\lim\limits_{n\to\infty} \frac{n}{\sqrt[n]{n!}} = e$.

(8). prove $\frac{\log_a n}{n} = 0$ $(a>1)$

(9) find $\lim\limits_{n\to\infty} \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}$

(10) prove. $\lim\limits_{n\to\infty} \frac{a^n}{n!} = 0$ $(a>0)$

(11) If $\lim\limits_{n\to\infty} x_n = a$ $\lim\limits_{n\to\infty} y_n = b$. prove $\lim\limits_{n\to\infty} \frac{x_1y_n + x_2y_{n-1} + \cdots + x_ny_1}{n} = ab$.

(12). prove: $\frac{1}{2} \cdot \frac{3}{4} \cdot \cdots \frac{2n-1}{2n} < \frac{1}{\sqrt{2n+1}}$
then we have $\lim\limits_{n\to\infty} \frac{1}{2} \cdot \frac{3}{4} \cdots \frac{2n-1}{2n} = 0$

(13). find $\lim_{n\to\infty} \sqrt[n]{1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}}$

(14). find $\lim_{n\to\infty} \frac{1}{\sqrt[n]{n!}}$

(15) prove $\sin n$. does not converge.

(16). $\{x_n\}$. $\{y_n\}$ is bdd. prove $\sup\{x_n+y_n\} \le \sup\{x_n\}+\sup\{y_n\}$

(17). A. B are two set of non negative numbers.

prove $\sup_{x\in A} x \cdot \sup_{y\in B} y = \sup_{\substack{x\in A \\ y\in B}} xy$

(18). f. g. are two functions on R.

$f(x+y)+f(x-y) = 2f(x)g(x) \quad \forall x.\ y\in R.$

f(x) is bdd and f(x) ~~doesnot~~ does not always equal to 0

prove that $|g(y)|\le 1 \quad (\forall y\in R)$

(19). f is increasing on $[a, b]$. $\exists f$ $f(a)\ge a$ $f(b)\le b$.

prove. $\exists x_0 \in [a, b]$. $f(x_0)=x_0$

(20). f is increasing on $[0,1]$ $f(0)>0$ $f(1)<1$

prove $\exists x_0 \in [0,1]$ $f(x_0)=x_0^2$

(21). $\lim x_n = a \Leftrightarrow \forall p\in\mathbb{N}.$ $\lim_{n\to\infty} |x_{n+p}-x_n| = 0$

ture or false ?

(22). If $\lim\limits_{n\to\infty} \frac{x_1+x_2+\cdots+x_n}{n} = a.$ $a\in R.$

prove. $\lim\limits_{n\to\infty} \frac{x_n}{n} = 0.$

(23) $a_1, b_1 > 0,$ $a_{n+1} = \sqrt{a_n b_n}$ $b_{n+1} = \frac{a_n+b_n}{2}$

prove $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} b_n.$

(24). $a_{n+1} = a_n + a_n^{-1}$ $a_1 = 1$

prove ~~并且~~ (1). $\lim\limits_{n\to\infty} a_n = +\infty$

(2). $\sum\limits_{n=1}^{\infty} a_n^{-1} = +\infty$

(25). If $|x_{n+1} - x_n| \leq r\,|x_n - x_{n-1}|$ $0<r<1.$

prove $\{x_n\}$ converge.

(26). $x_0 = 1$ $x_{n+1} = \frac{x_n+2}{x_n+1}.$ prove $x_n \to \sqrt{2}$

(27). $x_1 = 1$ $x_{n+1} = \sqrt{2+x_n}$ find $\lim x_n.$

(28). $x_0 = 1$ $x_{n+1} = 1+\frac{1}{x_n}$ find $\lim x_n.$

(29). $x_1 = 0$ $x_{n+1} = \frac{x_n+3}{4}$ find $\lim x_n.$

(30). $0 \leq x_{m+n} \leq x_m + x_n$ $\forall\, m,n \in \mathbb{N}.$ prove $\frac{x_n}{n}$ converge.

① completeness axiom. ② monotone sequence theorem.
③ nested interval Theorem. ④ Weierstrass Theorem. ⑤ Cauchy theorem.

~~try to~~ start from one to prove other four theorems.