

Math 2033 (Mathematical Analysis)

Spring 2016

Midterm

Directions: This is a closed book exam. Every student must show work in every problem with full details legibly to receive marks. Answers alone are worth very little!!! Calculators are not allowed in this midterm.

Notations: \mathbb{R} denotes the set of all real numbers. \mathbb{Q} denotes the set of all rational numbers.

1. (10 marks) Let A be a nonempty bounded subset of \mathbb{R} such that $\inf A = 3$ and $\sup A = 6$. Let

$$B = \{y^2 + \sqrt{x^2 - 16} : x \in [5, \sqrt{80}) \setminus \mathbb{Q}, y \in A\}.$$

Prove that B is bounded. Determine (with proof) the infimum and supremum of B .

2. (13 marks) Prove that the sequence $\{x_n\}$ converges, where

$$x_1 = 35 \quad \text{and} \quad \text{for } n = 1, 2, 3, \dots, \quad x_{n+1} = \frac{120}{5 + x_n}$$

and find its limit. Show all details.

3. (12 marks) Let $S = \{r\sqrt{2} : r \in \mathbb{Q}\}$. Determine if S is a countable set or not. Prove that there exist infinitely many real numbers c such that the equation

$$2^x \sin y - 2^y \cos x = c$$

does not have any solution with $x, y \in S$.

4. (10 marks) Prove that

$$\lim_{n \rightarrow \infty} \left(\frac{4n\sqrt{n} - 3}{n\sqrt{n} + 5} + \frac{6n^2 - n + 3}{3n^2 - 1} \right) = 6$$

by checking the definition of limit of a sequence only.

(Do not use computation formulas, sandwich theorem or l'Hopital's rule! Otherwise, you will get zero mark for this problem.)

–End of Paper–