MATH 2033 HW-3 Due Sept 27.

1. Prove that for any $a, b \in \mathbb{Q}$, there holds

$$||a| - |b|| \leq |a - b|.$$

(This is another version of the triangle inequality.)

2. Solve the inequalities:

(i). $|x - 3| \leq 6$;

(ii). $|3x - 7| \geq 4$,

(iii). $|\sin x| \leq \frac{1}{2}$.

3. Suppose $(a_n)_{n\in\mathbb{N}}$ and $(b_n)_{n\in\mathbb{N}}$ are two Cauchy sequences in $\mathbb{Q}$. Define

$$c_n = a_n + b_n \quad \text{for all } n \in \mathbb{N}.$$

Prove by definition that $(c_n)_{n\in\mathbb{N}}$ is also a Cauchy sequence in $\mathbb{Q}$.

4. Let

$$a_n = \frac{1}{n^2} + \frac{1}{n}, \quad n \in \mathbb{N}.$$

Verify by definition that $(a_n)_{n\in\mathbb{N}}$ is a Cauchy sequence in $\mathbb{Q}$.

5. Suppose $(a_n)_{n\in\mathbb{N}}$ is a sequence which converges in $\mathbb{Q}$ to $\frac{1}{100}$. Prove that there exists $N > 0$ such that $a_n > \frac{1}{200}$ for every $n \geq N$.

6. Let

$$a_n = (-1)^n.$$

Is $\{a_n\}_{n\in\mathbb{N}}$ a Cauchy sequence in $\mathbb{Q}$? Why or why not?