

1. In this exercise, we consider isometries from \mathbb{R} to itself in the usual metric.
 - i. Is $f(x) = x^3$ a bijection? A homeomorphism? An isometry?
 - ii. Is $f(x) = x + \sin x$ a bijection? A homeomorphism? An isometry?
 - iii. Find all isometries from \mathbb{R} to itself.

2. Let $X = (0, \infty)$ with the usual metric and determine whether the following functions are uniformly continuous on X :
 - i. $f(x) = \ln(x)$;
 - ii. $f(x) = \sqrt{x}$;

3.
 - (i) Show that a finite union of compact sets is compact.
 - (ii). Give an example of a countable union of compact sets that is not compact.
 - (iii). Show that a closed subset of a compact set is compact.
 - (iv). Show that a compact metric space is complete.

4. Let $(X, d), (X', d')$ be two metric spaces, and $f : X \rightarrow X'$ is a continuous function. Show that if $A \subseteq X$ is compact, then $f(A) \subseteq X'$ is compact.