Grading Criteria for Quiz 1.

1. 1) $A^Tb = \begin{pmatrix} 2 & 1 \\ 1 & 2 \\ -1 & 0 \\ 4 & -2 \\ 5 & 3 \end{pmatrix}\begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} -5 \\ -1 \\ 3 \\ -14 \\ -12 \end{pmatrix}$ (each entry is 1')

2) $\|A^Tb\|_1 = 5+1+3+14+12 = 35$

3) $\|A^Tb\|_2 = \sqrt{25+1+9+14^2+12^2} = \sqrt{375} = 5\sqrt{15}$

4) $\|A^Tb\|_\infty = \max\{5, 1, 3, 14, 12\} = 14$

For 2), 3), 4), miscalculation : taking 1' off

wrong expression of definition : taking 5' off.

2. $\|B\|_\infty = \max\{2, 1, 4\} = 4$

$\|B\|_1 = \max\{2, 3, 2\} = 3$

$\|B\|_F = \sqrt{1+1+1+1+4+1} = 3$

miscalculation : taking 1' off

wrong expression of definition : taking 5' off.

$\|B\|_2$. $B^TB = \begin{pmatrix} 2 & 2 & 2 \\ 2 & 5 & 2 \\ 2 & 2 & 2 \end{pmatrix}$ —— 2'

$$|B^TB - \lambda I| = \left|\begin{pmatrix} 2-\lambda & 2 & 2 \\ 2 & 5-\lambda & 2 \\ 2 & 2 & 2-\lambda \end{pmatrix}\right| = (2-\lambda)\begin{vmatrix} 5-\lambda & 2 \\ 2 & 2-\lambda \end{vmatrix} - 2\cdot\begin{vmatrix} 2 & 2 \\ 2 & 2-\lambda \end{vmatrix} + 2\cdot\begin{vmatrix} 2 & 5-\lambda \\ 2 & 2 \end{vmatrix}$$

$$= -\lambda(\lambda^2 - 9\lambda + 12) = 0$$

$$\lambda_1 = 0,\ \lambda_{2,3} = \frac{9 \pm \sqrt{81-48}}{2} = \frac{9 \pm \sqrt{33}}{2} \quad -2'$$

$$\|B\|_2 = \sqrt{\frac{9+\sqrt{33}}{2}} \quad -1'$$

3. 1) $\|x\|_\infty = \max_{1 \le i \le n} |x_i|$ — 1'

$\|x\|_2 = \left(\sum_{i=1}^n |x_i|^2\right)^{1/2}$ — 1'

$\|x\|_\infty^2 = \max_{1 \le i \le n} |x_i|^2 \le \sum_{i=1}^n |x_i|^2 = \|x\|_2^2$ — 4'

$\|x\|_2^2 = \sum_{i=1}^n |x_i|^2 \le \sum_{i=1}^n \left(\max_{1 \le j \le n} |x_j|^2\right) = n \cdot \|x\|_\infty^2$ — 4'

$\Rightarrow \|x\|_\infty \le \|x\|_2 \le \sqrt{n} \cdot \|x\|_\infty$ #

2) $\|x\|_\infty = \max_{1 \le i \le n} |x_i|$ — 1'

$\|x\|_1 = \sum_{i=1}^n |x_i|$ — 1'

$\|x\|_\infty = \max_{1 \le i \le n} |x_i| \le \sum_{i=1}^n |x_i| = \|x\|_1$ — 4'

$\|x\|_1 = \sum_{i=1}^n |x_i| \le \sum_{i=1}^n \left(\max_{1 \le j \le n} |x_j|\right) = n \cdot \|x\|_\infty$ — 4'

$\Rightarrow \|x\|_\infty \le \|x\|_1 \le n \cdot \|x\|_\infty$ #

4. ① — 5'

② — 5'

③ — 10'

5. $\begin{pmatrix} 1 & 0 & 4 & 2 \\ 2 & -1 & 1 & 0 \\ 3 & 2 & 0 & -1 \\ 4 & -3 & -1 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 4 & 2 \\ 2 & -1 & -7 & -4 \\ 3 & 2 & -12 & -7 \\ 4 & -3 & -17 & -6 \end{pmatrix}$

$\rightarrow \begin{pmatrix} 1 & 0 & 4 & 2 \\ 2 & -1 & -7 & -4 \\ 3 & -2 & -26 & -15 \\ 4 & 3 & 4 & 6 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 4 & 2 \\ 2 & -1 & -7 & -4 \\ 3 & -2 & -26 & -15 \\ 4 & 3 & -\frac{2}{13} & \frac{48}{13} \end{pmatrix}$

$L = \begin{pmatrix} 1 & & \\ 2 & 1 & 0 \\ 3 & -2 & 1 \\ 4 & 3 & -\frac{2}{13} & 1 \end{pmatrix}$, $U = \begin{pmatrix} 1 & 0 & 4 & 2 \\ 0 & -1 & -7 & -4 \\ 0 & 0 & -26 & -15 \\ 0 & 0 & 0 & \frac{48}{13} \end{pmatrix}$

Basically, for L, each column is 2.5'
for U, each row is 2.5'.
And round up (i.e. 12.5 → 13).
Some students didn't do the reduction for fractions (i.e. $-\frac{2}{13}$ (✓) $-\frac{4}{26}$ (×)). 1' is taken off.