

Math 3322 Matrix Computation

Topics Covered

① system of linear equations

$$\begin{array}{ccc} \nearrow & \uparrow & \nwarrow \\ A\vec{x} = \vec{b} \\ \uparrow & & \uparrow \\ \mathbb{R}^{n \times n} & \mathbb{R}^n & \mathbb{R}^n \end{array}$$

n unknown in n linear equations

Gaussian elimination problem

② Least squares problem

$$\begin{array}{ccc} \nearrow & \uparrow & \nwarrow \\ A\vec{x} \approx \vec{b} \\ \uparrow & & \uparrow \\ \mathbb{R}^{m \times n} & \mathbb{R}^n & \mathbb{R}^m \end{array}$$

$m \geq n$

$$\min_{\vec{x}} \|A\vec{x} - \vec{b}\|_2^2$$

minimize the errors.

③ Eigenvalue / Eigenvector

$$\begin{array}{ccc} \nearrow & \uparrow & \nwarrow \\ A\vec{v} = \lambda\vec{v} & \leftarrow \mathbb{C}^n \\ \uparrow & & \uparrow \\ \mathbb{R}^{n \times n} & \text{vector (can be} & \\ & \mathbb{C}^n & \text{imaginary)} \end{array}$$

Given A , eigenvector
find (λ, \vec{v})
eigenvalue.

④ Singular Value and Singular Vector problem
 Given A , want to find $\vec{u} \in \mathbb{R}^m$, $\vec{v} \in \mathbb{R}^n$, $\sigma \in \mathbb{R}$

$$A \in \mathbb{R}^{m \times n}$$

$$\begin{cases} A\vec{v} = \sigma\vec{u} \\ A^T\vec{u} = \sigma\vec{v} \end{cases}$$

Computational Methods:

• Matrix Factorization: LU , QR , $\overset{\text{least square}}{\downarrow}$ eigenvalue decomposition

Singular value decomposition
 \rightarrow singular value.

- Iterative Methods. (gradually find results)
- Randomized Numerical Algorithm

Applications:

- Image / Signal Processing, Computer vision.
- Data analysis / Machine Learning

Why these problems are important?

- Example 1: Adjacency matrix and Google's page rank.

We can use