MATH3322: Quiz 1

Student Name:
Student Number:

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1. (20') Let

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & -1 & 4 & 5 \\ 1 & 2 & 0 & -2 & 3 \end{pmatrix}, \tag{1}$$

$$\mathbf{b} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}. \tag{2}$$

Find 1) $\mathbf{A}^T\mathbf{b}$; 2) $\left\|\mathbf{A}^T\mathbf{b}\right\|_1$; 3) $\left\|\mathbf{A}^T\mathbf{b}\right\|_2$; 4)$\left\|\mathbf{A}^T\mathbf{b}\right\|_\infty$.

2. (20') Let

$$\mathbf{B} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 2 & 1 \end{pmatrix}, \tag{3}$$

Find 1)$\|\mathbf{B}\|_\infty$; 2) $\|\mathbf{B}\|_1$; $\|\mathbf{B}\|_2$; $\|\mathbf{B}\|_F$.

3. (20') Let $\mathbf{x} \in \mathbb{R}^n$. Show:
1)$\|\mathbf{x}\|_\infty \leq \|\mathbf{x}\|_2 \leq \sqrt{n}\,\|\mathbf{x}\|_\infty$.
2)$\|\mathbf{x}\|_\infty \leq \|\mathbf{x}\|_1 \leq n\,\|\mathbf{x}\|_\infty$.

4. (20') Show that $\|\mathbf{x}\|_2 := (\sum_{i=1}^n |x_i|^2)^{\frac{1}{2}}$ defines a norm on $\mathbb{R}^n$.

5. (20') Let

$$\mathbf{C} = \begin{pmatrix} 1 & 0 & 4 & 2 \\ 2 & -1 & 1 & 0 \\ 3 & 2 & 0 & -1 \\ 4 & -3 & -1 & 2 \end{pmatrix}, \tag{4}$$

Find the matrix $\mathbf{L}$ and $\mathbf{U}$ of the LU decomposition of $\mathbf{C}$. Show your computation process.

1. (20') Let

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & -1 & 4 & 5 \\ 1 & 2 & 0 & -2 & 3 \end{pmatrix}, \tag{1}$$

$$\mathbf{b} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}. \tag{2}$$

Find 1) $\mathbf{A}^T\mathbf{b}$; 2) $\|\mathbf{A}^T\mathbf{b}\|_1$; 3) $\|\mathbf{A}^T\mathbf{b}\|_2$; 4)$\|\mathbf{A}^T\mathbf{b}\|_\infty$.

1). $A^Tb = \begin{pmatrix} 2 & 1 \\ 1 & 2 \\ -1 & 0 \\ 4 & -2 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} -5 \\ -1 \\ 3 \\ -14 \\ -12 \end{pmatrix}$

$\|A^Tb\|_1 = 5+1+3+14+12 = 35$

$\|A^Tb\|_2 = 25 + 1 + 9 + 196 + 144$

$= 375$

$= \sqrt{375}$ program.

$\|A^Tb\|_\infty = 14$

2. (20') Let

$$\mathbf{B} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 2 & 1 \end{pmatrix}, \tag{3}$$

Find 1)$\|\mathbf{B}\|_\infty$; 2) $\|\mathbf{B}\|_1$; $\|\mathbf{B}\|_2$; $\|\mathbf{B}\|_F$.

1). $\|B\|_\infty = \max_{1 \le i \le 3} \|a^{(i)}\|_1$

$= 4$

$\|B\|_1 = \max_{1 \le j \le 3} \|a^{(j)}\|_1$

$= 3$

$\|B\|_F = \sqrt{1+1+1+4+1}$

$= 3$

$\|B\|_2 = (\text{max eigenvalue of } B^T B)^{\frac{1}{2}}$

$$B^T B = \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 2 & 2 \\ 2 & 5 & 2 \\ 2 & 2 & 2 \end{pmatrix}$$

$|B^TB - \lambda I| = 0$ & find determinant.

$$\begin{vmatrix} 2-\lambda & 2 & 2 \\ 2 & 5-\lambda & 2 \\ 2 & 2 & 2-\lambda \end{vmatrix}$$

$$(2-\lambda)((5-\lambda)(2-\lambda) - 4)$$

$$- 2((4-2\lambda) - 4)$$

$$+ 2(4 - 10 + 2\lambda)$$

$$-\lambda^3 + \lambda^2 + 2\lambda$$

$$2 \quad 0 \quad -1$$

5. (20') Let

$$\mathbf{C} = \begin{pmatrix} 1 & 0 & 4 & 2 \\ 2 & -1 & 1 & 0 \\ 3 & 2 & 0 & -1 \\ 4 & -3 & -1 & 2 \end{pmatrix}, \tag{4}$$

Find the matrix $\mathbf{L}$ and $\mathbf{U}$ of the LU decomposition of $\mathbf{C}$. Show your computation process.

$A(2, 2:n) = A(2, 2:n) - A(2,1) A(1, 2:n)$

$A(3:n, 2) = (A(3:n, 2) - A(3:n, 1:2) A(1:2, 3)) / A(2,2)$

At step 1,

$A(1, 1:4) = A(1, 1:4) - A(1, 1:0) A(1:0, 1:4)$

$A(2:4, 1) = (A(2:4, 1) - A(2:4, 1:0) A(1:0, 1)) / A(1,1)$

4x4 matti

$$\begin{pmatrix} 9 & -5 & 4 & -3 \\ -7 & -5 & -6 & -4 \\ 7 & -2 & 2 & -6 \\ -4 & -6 & -5 & -4 \end{pmatrix}$$

(6)-

$$\begin{pmatrix} A & B & C & D \\ E & G & & \\ F & H & & \\ A & I & & \end{pmatrix}$$

$$= \begin{pmatrix} 9 & -5 & 4 & -3 \\ -\frac{7}{9} & -5 & -6 & -4 \\ \frac{7}{9} & -2 & 2 & -6 \\ -4/9 & -6 & -5 & -4 \end{pmatrix}$$

$$= \begin{pmatrix} 9 & -5 & 4 & -3 \\ -\frac{7}{9} & -\frac{80}{9} & -\frac{26}{9} & -\frac{19}{3} \\ \frac{7}{9} & -\frac{17}{80} & \frac{-69}{40} & -\frac{121}{80} \\ -4/9 & \frac{1}{8} & -\frac{10}{23} & -\frac{25}{46} \end{pmatrix}$$

Program 1 & 2

計 eigenvalue

Program 3

3x3 Matrix Multiply

Program 4

4x4 Matrix

Program 1

平方根化簡

Program 2