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1. $PA = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 2 & -1 & 2 \\ -2 & 1 & 2 \\ -4 & 0 & -1 \end{pmatrix} = \begin{pmatrix} -4 & 0 & -1 \\ -2 & 1 & 2 \\ 2 & -1 & 2 \end{pmatrix}$

$PA = \begin{pmatrix} -4 & 0 & -1 \\ 1/2 & 1 & 2 \\ -1/2 & -1 & 2 \end{pmatrix}$

$PA = \begin{pmatrix} -4 & 0 & -1 \\ 1/2 & 1 & 5/2 \\ -1/2 & -1 & 2 \end{pmatrix}$

$PA = \begin{pmatrix} -4 & 0 & -1 \\ 1/2 & 1 & 5/2 \\ -1/2 & -1 & 4 \end{pmatrix}$

$\therefore$

$$\underbrace{\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}}_{P}\underbrace{\begin{pmatrix} 2 & -1 & 2 \\ -2 & 1 & 2 \\ -4 & 0 & -1 \end{pmatrix}}_{A} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ -1/2 & -1 & 1 \end{pmatrix}}_{L}\underbrace{\begin{pmatrix} -4 & 0 & -1 \\ 0 & 1 & 5/2 \\ 0 & 0 & 4 \end{pmatrix}}_{U}$$

2. Use $[L \mid I]$ $\quad L \in \mathbb{R}^{n \times n}$

$$
= \left[\begin{array}{cccc|cccc}
l_{11} & & & & 1 & & & \\
l_{21} & l_{22} & & & & 1 & & \\
\vdots & \vdots & \ddots & & & & \ddots & \\
l_{n1} & l_{n2} & \cdots & l_{nn} & & & & 1
\end{array}\right]
$$

$$
= \left[\begin{array}{cccc|cccc}
1 & & & & 1/l_{11} & & & \\
0 & l_{22} & & & -\frac{l_{21}}{l_{11}} & 1 & & \\
\vdots & \vdots & \ddots & & \vdots & & \ddots & \\
0 & l_{n2} & \cdots & l_{nn} & -\frac{l_{n1}}{l_{11}} & & & 1
\end{array}\right]
$$

$$
= \left[\begin{array}{ccccc|ccccc}
1 & & & & & 1/l_{11} & & & & \\
0 & 1 & & & & -\frac{l_{21}}{l_{11}} & 1/l_{22} & & & \\
\vdots & 0 & l_{33} & & & \vdots & \vdots & 1 & & \\
\vdots & \vdots & \vdots & \ddots & & & & & \ddots & \\
0 & 0 & l_{n3} & \cdots & l_{nn} & -\frac{l_{n1}}{l_{11}} & -\frac{l_{n2}}{l_{22}} & & & 1
\end{array}\right]
$$

$= \cdots$

$$
= \left[\begin{array}{cccc|cccc}
1 & & & & 1/l_{11} & & & \\
& 1 & & & -\frac{l_{21}}{l_{11}} & 1/l_{22} & & \\
& & \ddots & & \vdots & \vdots & \ddots & \\
& & & 1 & -\frac{l_{n1}}{l_{11}} & -\frac{l_{n2}}{l_{22}} & \cdots & 1/l_{nn}
\end{array}\right]
$$

$\Rightarrow L^{-1}$ is lower triangular matrix.

Algorithm for computing L^{-1}:

```
for j = 1 to n:
    for i = 1 to n:
        if i > j
            L^-1[i,j] = 0

        else if i = j
            L^-1[i,j] = 1
            L^-1[i,j] = L^-1[i,j] / L[i,j]
        else if i < j



            for k = j : n
                L^-1[k,i] = L^-1[k,i] - L[k,i] / L[i,i]
            end


    end
end
```

Computational cost:

$$\sum_{i=1}^{n}\left(1+\sum_{j=i+1}^{n}\left(\sum_{k=j}^{n}(2)+1\right)\right)$$

$$=\sum_{i=1}^{n}\left(1+\sum_{j=i+1}^{n}\left(2(n-j+1)+1\right)\right)$$

$$=\sum_{i=1}^{n}\left(1+\sum_{j=i+1}^{n}\left(-2j+2n+3\right)\right)$$

$$=\sum_{i=1}^{n}\left(1+(2n+3)(n-i)-2\left(\sum_{j=1}^{n}j-\sum_{j=1}^{i}j\right)\right)$$

$$=\sum_{i=1}^{n}\left(1+2n^2-2ni+3n-3i-2\left(\frac{n(n+1)}{2}-\frac{i(i+1)}{2}\right)\right)$$

$$=\sum_{i=1}^{n}\left(2n^2+3n+1-2ni-3i-n^2-n+i^2+i\right)$$

$$=(n^2+2n+1)n+\frac{n(n+1)(2n+1)}{6}-2(n+1)\frac{(n)(n+1)}{2}$$

$$=n^3+2n^2+n+\frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6}-n^3-2n^2-n$$

$$=\frac{1}{3}n^3+\frac{1}{2}n^2+\frac{1}{6}n=\frac{1}{3}n^3+O(n^2)$$