MATH2023 Multivariable Calculus 2013

From the textbook Calculus of Several Variables (5th) by R. Adams, Addison Wesley.

Homework 8

(Total: 12 questions)

Ex. 15.1

7 Sketch the plane vector field $\mathbf{F}(x,y) = \nabla \ln(x^2+y^2)$ and determine its field lines.

9 Describe the streamlines of the velocity fields $\mathbf{v}(x,y,z) = y\,\mathbf{i} - y\,\mathbf{j} - y\,\mathbf{k}$.

16 Describe the streamlines of the velocity fields $\mathbf{v}(x,y) = x\,\mathbf{i} + (x+y)\,\mathbf{j}$. (Hint: let $y = xv(x)$.)

Ex. 15.3

2 Let C be the conical helix with parametric equations $x = t\cos t$, $y = t\sin t$, $z = t$, $(0 \leqslant t \leqslant 2\pi)$. Find $\int_C z\,ds$.

8 Find $\int_C \sqrt{1+4x^2z^2}\,ds$, where C is the curve of intersection of the surfaces $x^2+z^2=1$ and $y = x^2$.

15 Find $\int_C \frac{ds}{(2y^2+1)^{3/2}}$, where C is the parabola $z^2 = x^2+y^2$, $x+z=1$.

Ex. 15.4

3 Evaluate the line integral of the tangential component of the vector field

$$\mathbf{F}(x,y,z) = y\,\mathbf{i} + z\,\mathbf{j} - x\,\mathbf{k}$$

along the straight line from $(0,0,0)$ to $(1,1,1)$.

5 Evaluate the line integral of the tangential component of the vector field $\mathbf{F}(x,y,z) = yz\,\mathbf{i} + xz\,\mathbf{j} + xy\,\mathbf{k}$ from $(-1,0,0)$ to $(1,0,0)$ along either direction of the curve of intersection of the cylinder $x^2+y^2=1$ and the plane $z=y$.

11 Determine the values of A and B for which the vector field

$$\mathbf{F} = Ax\ln z\,\mathbf{i} + By^2z\,\mathbf{j} + \left(\frac{x^2}{z} + y^3\right)\mathbf{k}$$

is conservative. If C is the straight line from $(1,1,1)$ to $(2,1,2)$, find

$$\int_C 2x\ln z\,dx + 2y^2z\,dy + y^3\,dz.$$

13 If C is the intersection of $z = \ln(1+x)$ and $y = x$ from $(0,0,0)$ to $(1,1,\ln 2)$, evaluate

$$\int_C (2x\sin(\pi y) - e^z)\,dx + (\pi x^2\cos(\pi y) - 3e^z)\,dy - xe^z\,dz.$$

14 Is each of the following sets a domain? a connected domain? a simply connected domain?

(a) the set of points (x,y) in the plane such that $x>0$ and $y \geqslant 0$

(b) the set of points (x,y) in the plane such that $x=0$ and $y \geqslant 0$

(c) the set of points (x,y) in the plane such that $x \neq 0$ and $y > 0$

(d) the set of points (x,y,z) in 3-space such that $x^2 > 1$

(e) the set of points (x,y,z) in 3-space such that $x^2+y^2 > 1$

(f) the set of points (x,y,z) in 3-space such that $x^2+y^2+z^2 > 1$

22 Evaluate $\frac{1}{2\pi}\oint_C \frac{-y\,dx + x\,dy}{x^2+y^2}$

(a) counterclockwise around the circle $x^2+y^2=a^2$,

(b) clockwise around the square with vertices $(-1,-1)$, $(-1,1)$, $(1,1)$, and $(1,-1)$,

(c) counterclockwise around the boundary of the region $1 \leqslant x^2+y^2 \leqslant 2$, $y \geqslant 0$.

Homework 9

(Total: 9 questions)

Ex. 15.2

5 Determine whether the vector field

$$\mathbf{F}(x,y,z) = (2xy - z^2)\,\mathbf{i} + (2yz + x^2)\,\mathbf{j} - (2zx - y^2)\,\mathbf{k}$$

is conservative and find a potential if it is conservative.

7 Find the three-dimensional vector field with potential $\phi(\mathbf{r}) = \frac{1}{\|\mathbf{r} - \mathbf{r}_0\|^2}$.

9 Show that the vector field

$$\mathbf{F}(x,y,z) = \frac{2x}{z}\,\mathbf{i} + \frac{2y}{z}\,\mathbf{j} - \frac{x^2+y^2}{z^2}\,\mathbf{k}$$

is conservative, and find its potential. Describe the equipotential surfaces. Find the field lines of $\mathbf{F}$.

Ex. 15.5

10 Find the area of the part of the cylinder $x^2+z^2=a^2$ that lies inside the cylinder $y^2+z^2=a^2$.

14 Find $\iint_S y\,dS$, where S is the part of the cone $z=\sqrt{2(x^2+y^2)}$ that lies below the plane $z=1+y$.

Ex. 15.6

1 Find the flux of $\mathbf{F}=x\,\mathbf{i}+z\,\mathbf{j}$ out of the tetrahedron bounded by the coordinate planes and the plane $x+2y+3z=6$.

6 Find the flux of $\mathbf{F}=x\,\mathbf{i}+x\,\mathbf{j}+\mathbf{k}$ upward through the part of the surface $z=x^2-y^2$ lying inside the cylinder $x^2+y^2=a^2$.

10 Find the flux of $\mathbf{F}=2x\,\mathbf{i}+y\,\mathbf{j}+z\,\mathbf{k}$ upward through the surface $\mathbf{r}=u^2v\,\mathbf{i}+uv^2\,\mathbf{j}+v^3\,\mathbf{k}$, $(0\leqslant u\leqslant 1,\ 0\leqslant v\leqslant 1)$.

15 Define the flux of a *plane* vector field across a piecewise smooth *curve*. Find the flux of $\mathbf{F}=x\,\mathbf{i}+y\,\mathbf{j}$ outward across

(a) the circle $x^2+y^2=a^2$.

(b) the boundary of the square $-1\leqslant x,\ y\leqslant 1$.

Homework 10

(Total: 9 questions)

Ex. 16.4

4 Use the Divergence Theorem to calculate the flux of the vector field

$$\mathbf{F}=x^3\,\mathbf{i}+3yz^2\,\mathbf{j}+(3y^2z+x^2)\,\mathbf{k}$$

out of the sphere S with equation $x^2+y^2+z^2=a^2$, where $a>0$.

8 Evaluate the flux of $\mathbf{F}=x^2\,\mathbf{i}+y^2\,\mathbf{j}+z^2\,\mathbf{k}$ outward across the boundary of the solid cylinder $x^2+y^2\leqslant 2y$, $0\leqslant z\leqslant 4$.

11 A conical domain with vertex $(0,0,b)$ and axis along the z-axis has as base a disk of radius a in the xy-plane. Find the flux of

$$\mathbf{F}=(x+y^2)\,\mathbf{i}+(3x^2y+y^3-x^3)\,\mathbf{j}+(z+1)\,\mathbf{k}$$

upward through the conical part of the surface of the domain.

23 If $\mathbf{F}$ is a smooth vector field on D, show that

$$\iiint_D \phi\,\nabla\cdot\mathbf{F}\,dV+\iiint_D \nabla\phi\cdot\mathbf{F}\,dV=\oiint_S \phi\,\mathbf{F}\cdot\hat{\mathbf{n}}\,dS.$$

24 If $\nabla^2\phi=0$ in D and $\phi(x,y,z)=0$ on S, show that $\phi(x,y,z)=0$ in D.

Ex. 16.5

2 Evaluate $\oint_C y\,dx-x\,dy+z^2\,dz$ around the curve C of intersection of the cylinders $z=y^2$ and $x^2+y^2=4$, oriented counterclockwise as seen from a point high on the z-axis.

3 Evaluate $\iint_S \nabla\times\mathbf{F}\cdot\hat{\mathbf{n}}\,dS$, where S is the hemisphere $x^2+y^2+z^2=a^2$, $z\geqslant 0$ with outward normal, and $\mathbf{F}=3y\,\mathbf{i}-2xz\,\mathbf{j}+(x^2-y^2)\,\mathbf{k}$.

8 Evaluate $\oint_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}=ye^x\,\mathbf{i}+(x+e^x)\,\mathbf{j}+z^2\,\mathbf{k}$ and C is the curve

$$\mathbf{r}=(1+\cos t)\,\mathbf{i}+(1+\sin t)\,\mathbf{j}+(1-\sin t-\cos t)\,\mathbf{k},$$

where $0\leqslant t\leqslant 2\pi$.

9 Let C_1 be a straight line joining $(-1,0,0)$ to $(1,0,0)$ and let C_2 be the semicircle $x^2+y^2=1$, $z=0$, $y\geqslant 0$. Let S be a smooth surface joining C_1 to C_2 having upward normal, and let

$$\mathbf{F}=(\alpha x^2-z)\,\mathbf{i}+(xy+y^3+z)\,\mathbf{j}+\beta y^2(z+1)\,\mathbf{k}.$$

Find the values of α and β for which $\mathrm{I}=\iint_S \mathbf{F}\cdot\hat{\mathbf{n}}\,dS$ is independent of the choice of S, and find the value of I for these values of α and β.

Ex. 15.3

2 Let C be the conical helix with parametric equations $x = t\cos t$, $y = t\sin t$, $z = t$, $(0 \leqslant t \leqslant 2\pi)$.
Find $\int_C z\,ds$.

8 Find $\int_C \sqrt{1+4x^2z^2}\,ds$, where C is the curve of intersection of the surfaces $x^2+z^2=1$ and $y=x^2$.

15 Find $\int_C \frac{ds}{(2y^2+1)^{3/2}}$, where C is the parabola $z^2 = x^2+y^2$, $x+z=1$.

2. $r'(t) = \langle \cos t - t\sin t,\ \sin t + t\cos t,\ 1 \rangle$

$|r'(t)| = \sqrt{(\cos t - t\sin t)^2 + (\sin t + t\cos t)^2 + 1}$

$= \sqrt{\cos^2 t - 2t\sin t\cos t + t^2\sin^2 t + \sin^2 t + 2t\cos t\sin t + t^2\cos^2 t + 1}$

$= \sqrt{t^2 + 2}$

$\int_0^{2\pi} t\sqrt{t^2+2}\,dt$

8 Find $\int_C \sqrt{1+4x^2z^2}\, ds$, where C is the curve of intersection of the surfaces $x^2+z^2=1$ and $y=x^2$.

15 Find $\int_C \frac{ds}{(2y^2+1)^{3/2}}$, where C is the parabola $z^2=x^2+y^2$, $x+z=1$.

8. $\langle \cos t, \cos^2 t, \sin t \rangle$

Ex. 15.4

3 Evaluate the line integral of the tangential component of the vector field

$$\mathbf{F}(x, y, z) = y\,\mathbf{i} + z\,\mathbf{j} - x\,\mathbf{k}$$

along the straight line from $(0, 0, 0)$ to $(1, 1, 1)$.

$$r(t) = \langle t, t, t \rangle$$

$$\int_0^1 t \, dt$$

5 Evaluate the line integral of the tangential component of the vector field $\mathbf{F}(x, y, z) = yz\,\mathbf{i} + xz\,\mathbf{j} + xy\,\mathbf{k}$ from $(-1, 0, 0)$ to $(1, 0, 0)$ along either direction of the curve of intersection of the cylinder $x^2 + y^2 = 1$ and the plane $z = y$.

$$\vec{r}(t) = \langle \cos t, \sin t, \sin t \rangle$$

$$\pi \le t \le 2\pi$$

$$\int_{\pi}^{2\pi} yz(-\sin t)\,dt + xz(\cos t)\,dt + xy(\cos t)\,dt$$

11 Determine the values of A and B for which the vector field

$$\mathbf{F} = Ax \ln z \, \mathbf{i} + By^2 z \, \mathbf{j} + \left(\frac{x^2}{z} + y^3 \right) \mathbf{k}$$

is conservative. If C is the straight line from $(1,1,1)$ to $(2,1,2)$, find

$$\int_C 2x \ln z \, dx + 2y^2 z \, dy + y^3 \, dz.$$

$\nabla \times F = 0.$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ Ax \ln z & By^2 z & \frac{x^2}{z} + y^3 \end{vmatrix}$$

$$\left\langle 3y^2 - By^2, \ \frac{Ax}{z} - \frac{2x}{z}, \ 0 \right\rangle$$

$B = 3, \ A = 2.$

13 If C is the intersection of $z = \ln(1+x)$ and $y = x$ from $(0,0,0)$ to $(1,1,\ln 2)$, evaluate

$$\int_C (2x\sin(\pi y) - e^z)\,dx + (\pi x^2\cos(\pi y) - 3e^z)\,dy - xe^z\,dz.$$

$\langle t, t, \ln(1+t) \rangle$

$0 \leq t \leq 1$

14 Is each of the following sets a domain? a connected domain? a simply connected domain?

(a) the set of points (x, y) in the plane such that $x > 0$ and $y \geqslant 0$

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(f) the set of points (x, y, z) in 3-space such that $x^2 + y^2 + z^2 > 1$

22 Evaluate $\dfrac{1}{2\pi} \oint_C \dfrac{-y\,dx + x\,dy}{x^2 + y^2}$

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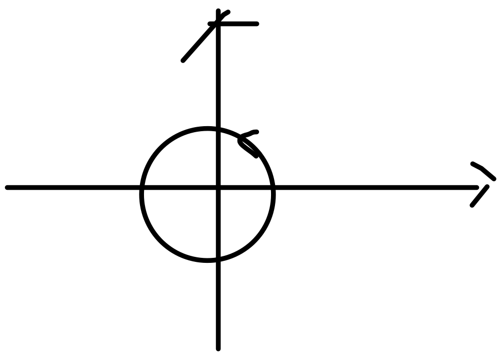
(b) clockwise around the square with vertices $(-1, -1)$, $(-1, 1)$, $(1, 1)$, and $(1, -1)$,

(c) counterclockwise around the boundary of the region $1 \leqslant x^2 + y^2 \leqslant 2$, $y \geqslant 0$.



Homework 9

(Total: 9 questions)

Ex. 15.2

5 Determine whether the vector field

$$\mathbf{F}(x,y,z) = (2xy - z^2)\,\mathbf{i} + (2yz + x^2)\,\mathbf{j} - (2zx - y^2)\,\mathbf{k}$$

is conservative and find a potential if it is conservative.

7 Find the three-dimensional vector field with potential $\phi(\mathbf{r}) = \dfrac{1}{\|\mathbf{r} - \mathbf{r}_0\|^2}$. ?

$x^2y \quad -z^2x \quad + \quad y^2z$

7.

9 Show that the vector field

$$\mathbf{F}(x,y,z) = \frac{2x}{z}\,\mathbf{i} + \frac{2y}{z}\,\mathbf{j} - \frac{x^2+y^2}{z^2}\,\mathbf{k}$$

is conservative, and find its potential. Describe the equipotential surfaces. Find the field lines of $\mathbf{F}$.

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10 Find the area of the part of the cylinder $x^2 + z^2 = a^2$ that lies inside the cylinder $y^2 + z^2 = a^2$.

14 Find $\iint_S y\, dS$, where S is the part of the cone $z = \sqrt{2(x^2 + y^2)}$ that lies below the plane $z = 1 + y$.

14.

1. 求出两者相交之面

2. $z = \sqrt{2(x^2+y^2)}$, 求 z_x , z_y.

$$\iint_S dS = \iint_S \sqrt{1+2x^2+2y^2}\, dA$$

3. change of variable,

$x = u$, $y = 1 + \sqrt{2}\, v$

$(u^2 + v^2 = 1)$

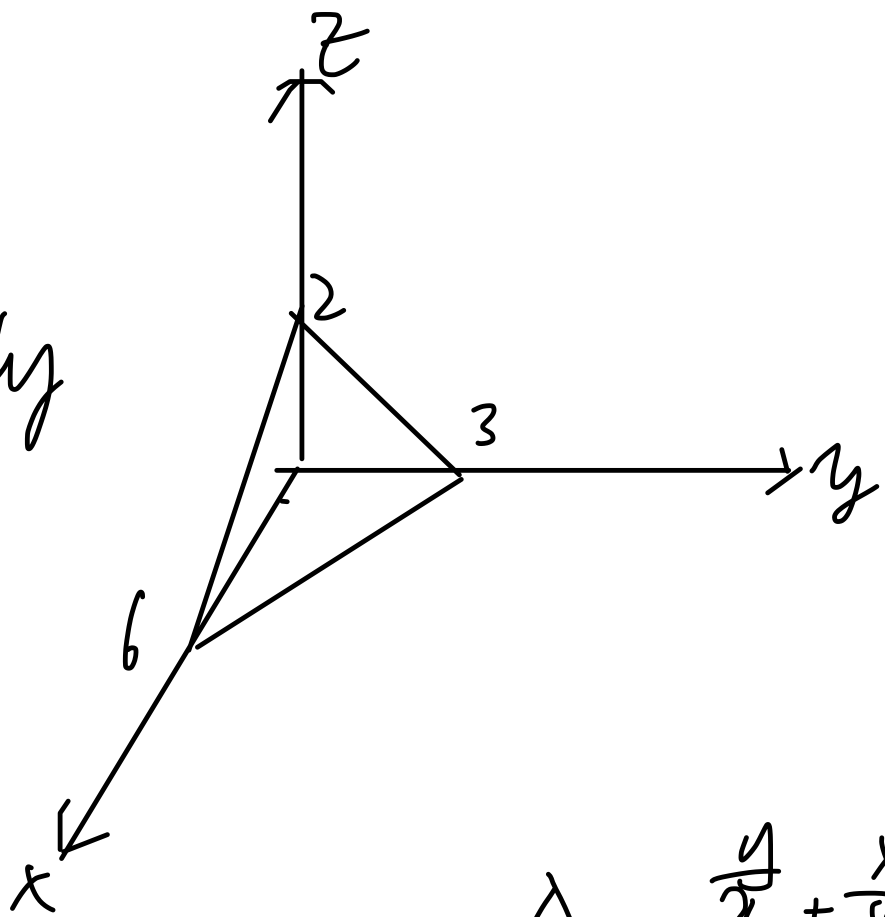
Made with Goodnotes

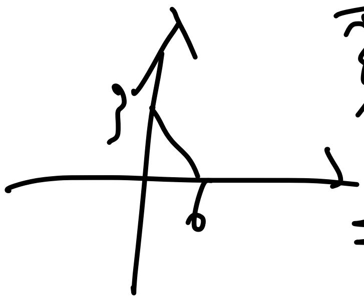
Ex. 15.6

1 Find the flux of $\mathbf{F} = x\,\mathbf{i} + z\,\mathbf{j}$ out of the tetrahedron bounded by the coordinate planes and the plane $x + 2y + 3z = 6$.

$$\iiint_E 1\,dV$$

$$= \int_0^3 \int_0^{\frac{6-x}{2}} \int_0^{\frac{6-x-2y}{3}} dz\,dx\,dy$$





$$\frac{y}{3} + \frac{x}{6} = 1$$

$$\cancel{y + 2x = 6}$$

$$2y + x = 6$$

$$= 6 - x$$

Ex. 15.6

1 Find the flux of $\mathbf{F} = x\,\mathbf{i} + z\,\mathbf{j}$ out of the tetrahedron bounded by the coordinate planes and the plane $x + 2y + 3z = 6$.

6 Find the flux of $\mathbf{F} = x\,\mathbf{i} + x\,\mathbf{j} + \mathbf{k}$ upward through the part of the surface $z = x^2 - y^2$ lying inside the cylinder $x^2 + y^2 = a^2$.

Q6. ① 求 $Z_x = 2x$

求 $Z_y = -2y$

② $dS = \sqrt{1 + Z_x^2 + Z_y^2}\, dA$

③ Let $g(x,y,z) = z - x^2 + y^2$ (level set in 3D)

$\vec{n} = \langle -2x, 2y, 1 \rangle$

$\hat{n} = \left\langle \dfrac{-2x,\ 2y,\ 1}{\sqrt{4x^2 + 4y^2 + 1}} \right\rangle$

④ $\iint_S F \cdot \hat{n}\, dS = \iint -2x^2 + 2xy + 1\, dA$

Made with Goodnotes

10 Find the flux of $\mathbf{F} = 2x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ upward through the surface $\mathbf{r} = u^2v\,\mathbf{i} + uv^2\,\mathbf{j} + v^3\,\mathbf{k}$, $(0 \leqslant u \leqslant 1, 0 \leqslant v \leqslant 1)$.

15 Define the flux of a *plane* vector field across a piecewise smooth *curve*. Find the flux of $\mathbf{F} = x\,\mathbf{i} + y\,\mathbf{j}$ outward across

(a) the circle $x^2 + y^2 = a^2$.

(b) the boundary of the square $-1 \leqslant x, y \leqslant 1$.

10. $r_u = \langle 2uv, v^2, 0 \rangle$

$r_v = \langle u^2, 2uv, 3v^2 \rangle$

$r_u \times r_v = \langle 3v^4, -6uv^3, 3u^2v^2 \rangle$

$$\int_0^1 \int_0^1 2x(3v^4) + y(-6uv^3) + z(3u^2v^2)\, du\, dv$$

$$= \int_0^1 \int_0^1 2u^2v(3v^4) + (uv^2)(-6uv^3) + (v^3)(3u^2v^2)\, du\, dv$$

Homework 10

(Total: 9 questions)

Ex. 16.4

4 Use the Divergence Theorem to calculate the flux of the vector field

$$\mathbf{F} = x^3\,\mathbf{i} + 3yz^2\,\mathbf{j} + (3y^2z + x^2)\,\mathbf{k}$$

out of the sphere S with equation $x^2 + y^2 + z^2 = a^2$, where $a > 0$.

8 Evaluate the flux of $\mathbf{F} = x^2\,\mathbf{i} + y^2\,\mathbf{j} + z^2\,\mathbf{k}$ outward across the boundary of the solid cylinder $x^2 + y^2 \leqslant 2y$, $0 \leqslant z \leqslant 4$.

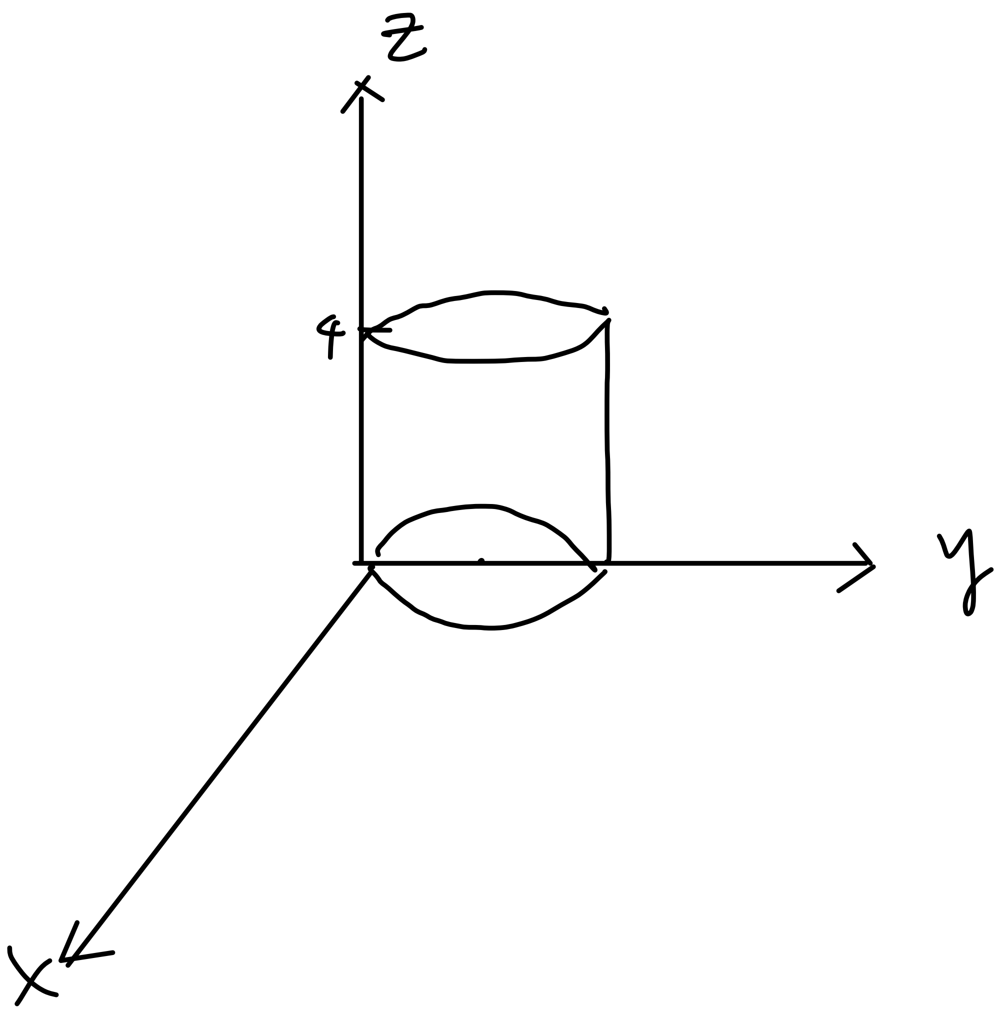
$$\nabla \cdot F = 3x^2 + 3z^2 + 3y^2$$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{a} 3\rho^2 (\rho^2 \sin\varphi)\, d\rho\, d\varphi\, d\theta$$

8 Evaluate the flux of $\mathbf{F} = x^2\,\mathbf{i} + y^2\,\mathbf{j} + z^2\,\mathbf{k}$ outward across the boundary of the solid cylinder $x^2 + y^2 \leqslant 2y$, $0 \leqslant z \leqslant 4$.

$x^2 + y^2 - 2y + 1 - 1 \leq 0$

$x^2 + (y-1)^2 \leq 1$



$r^2 = 2r\sin\theta$

$r = 2\sin\theta$

$\nabla F = 2x + 2y + 2z$

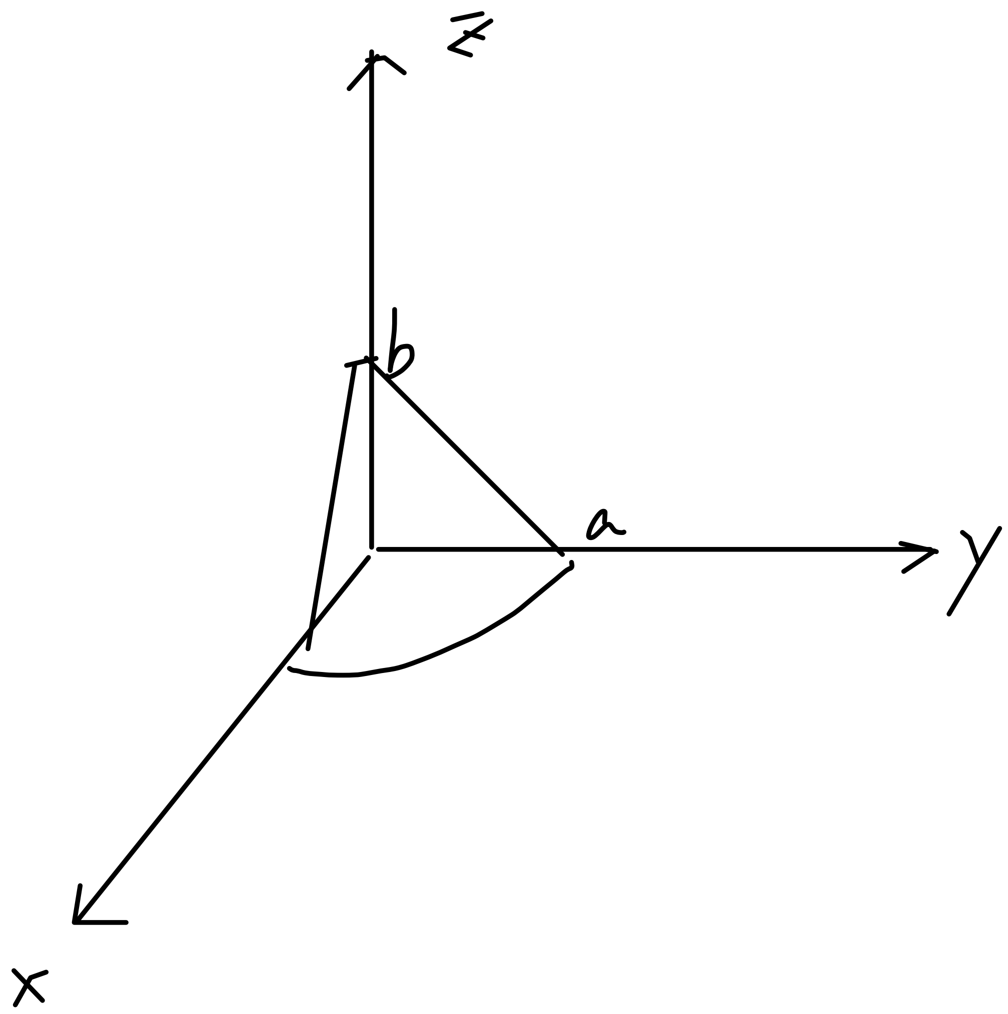
$$\int_0^{\pi} \int_0^{2\sin\theta} \int_0^{4} 2r\cos\theta + 2r\sin\theta + 2z \quad dz\, r\,dr\,d\theta$$

Made with Goodnotes

11 A conical domain with vertex $(0,0,b)$ and axis along the z-axis has as base a disk of radius a in the xy-plane. Find the flux of

$$\mathbf{F} = (x+y^2)\,\mathbf{i} + (3x^2y + y^3 - x^3)\,\mathbf{j} + (z+1)\,\mathbf{k}$$

upward through the conical part of the surface of the domain.



23 If $\mathbf{F}$ is a smooth vector field on D, show that

$$\iiint_D \phi \nabla \cdot \mathbf{F}\, dV + \iiint_D \nabla\phi \cdot \mathbf{F}\, dV = \oiint_S \phi \mathbf{F} \cdot \hat{\mathbf{n}}\, dS.$$

$$\phi \nabla \cdot F + \nabla \phi \cdot F = \nabla(\phi \cdot F)$$

24 If $\nabla^2\phi = 0$ in D and $\phi(x, y, z) = 0$ on S, show that $\phi(x, y, z) = 0$ in D.

$$\phi_{xx} + \phi_{yy} + \phi_{zz} = 0$$

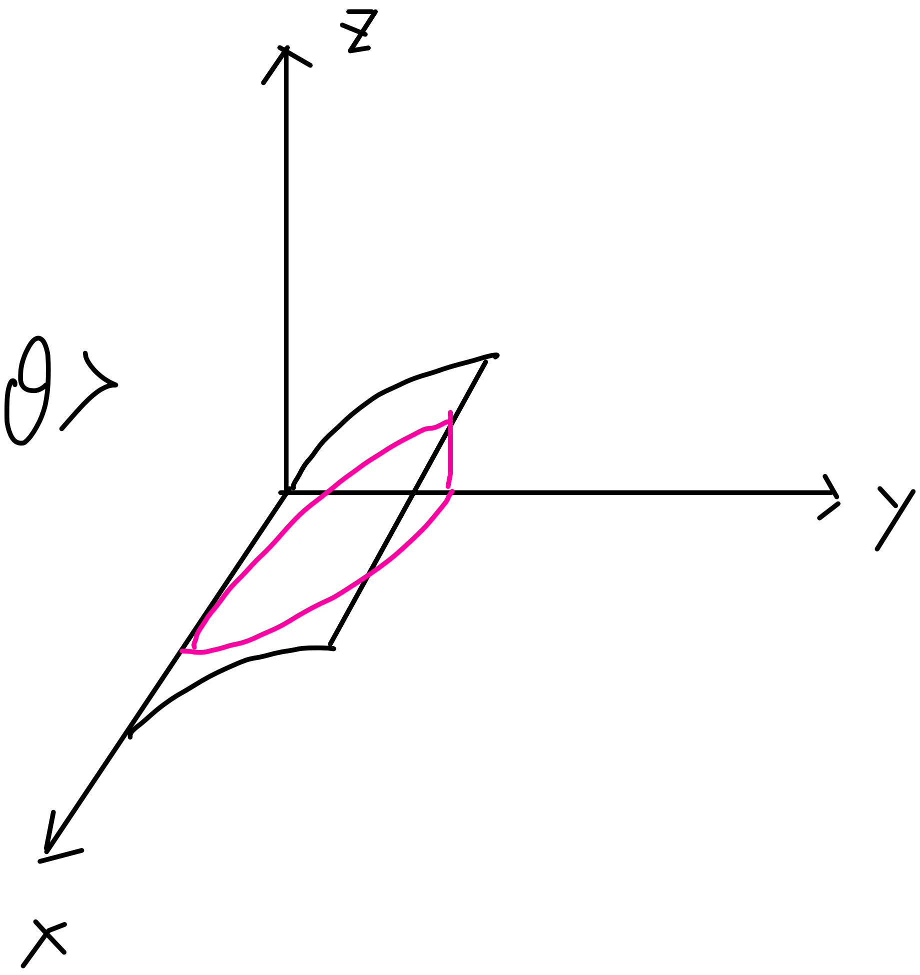
$$\phi = 0$$

Ex. 16.5

2 Evaluate $\oint_C y\,dx - x\,dy + z^2\,dz$ around the curve C of intersection of the cylinders $z = y^2$ and $x^2 + y^2 = 4$, oriented counterclockwise as seen from a point high on the z-axis.

$$x^2 + z = 4$$

$$z = 4 - x^2$$



$$\langle 2\cos\theta,\ 2\sin\theta,\ 4\sin^2\theta \rangle$$

$$\int_0^{2\pi} y(-2\sin\theta)\,d\theta - x(2\cos\theta)\,d\theta + z^2(8\sin\theta\cos\theta)\,d\theta$$

3 Evaluate $\iint_S \nabla \times \mathbf{F} \cdot \hat{\mathbf{n}} \, dS$, where S is the hemisphere $x^2 + y^2 + z^2 = a^2, z \geqslant 0$ with outward normal, and $\mathbf{F} = 3y\,\mathbf{i} - 2xz\,\mathbf{j} + (x^2 - y^2)\,\mathbf{k}$.

$r(u,v) = \langle a\sin u \cos v, a \sin u \sin v, a\cos u \rangle$ $\quad 0 \leq u \leq \frac{\pi}{2}$, $0 \leq v \leq 2\pi$

$|r_u \times r_v| = a^2 \sin u$

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3y & -2xz & x^2 - y^2 \end{vmatrix}$$

$$= \langle -2y + 2x, -2x, -2z - 3 \rangle$$

$$\hat{n} = \langle x, y, z \rangle$$

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① 計 $\nabla \times F$.

② $\iint_S \nabla \times F \cdot \hat{n} \, dS = \oint_C \vec{F} \cdot d\vec{r}$

8 Evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = ye^x\,\mathbf{i} + (x + e^x)\,\mathbf{j} + z^2\,\mathbf{k}$ and C is the curve

$$\mathbf{r} = (1 + \cos t)\,\mathbf{i} + (1 + \sin t)\,\mathbf{j} + (1 - \sin t - \cos t)\,\mathbf{k},$$

where $0 \leqslant t \leqslant 2\pi$.

$$ye^x(-\sin t)\,dt +$$

$$(x + e^x)(\cos t)\,dt +$$

$$z^2(-\cos t + \sin t)\,dt$$

$$= \int_0^{2\pi} (1+\sin t)\,e^{1+\cos t}(-\sin t) + (1+\cos t + e^{1+\cos t})\cos t + (1-\sin t - \cos t)^2(-\cos t + \sin t)\,dt$$

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Made with Goodnotes

9 Let C_1 be a straight line joining $(-1, 0, 0)$ to $(1, 0, 0)$ and let C_2 be the semicircle $x^2 + y^2 = 1$, $z = 0$, $y \geqslant 0$. Let S be a smooth surface joining C_1 to C_2 having upward normal, and let

$$\mathbf{F} = (\alpha x^2 - z)\,\mathbf{i} + (xy + y^3 + z)\,\mathbf{j} + \beta y^2(z+1)\,\mathbf{k}.$$

Find the values of α and β for which $\mathrm{I} = \iint_S \mathbf{F} \cdot \widehat{\mathbf{n}}\, dS$ is independent of the choice of S, and find the value of I for these values of α and β.