MATH 2023 – Multivariable Calculus

Lecture #18 Worksheet April 16, 2019

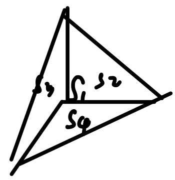
Problem 1. Find the surface integral

$$\iint_S z dS$$

where S is the surface of the solid bounded by the cylinder $x^2 + y^2 = 1$, the disk $x^2 + y^2 \leq 1$ and under the plane $z = x + 1$.

Problem 2. Find $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} = y\mathbf{i} + (z-y)\mathbf{j} + x\mathbf{k}$ and S is the surface of the tetrahedron bounded by the coordinate planes and the plane $x + y + z = 1$.

$z = 1 - x - y$



S_1: $\vec{n} = \langle 1, 1, 1 \rangle$

$$\iint z + x \, dA$$

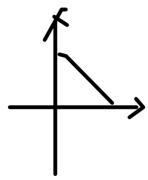
$$\int_0^1 \int_0^{1-y} 1 - y \, dx\,dy$$

$$= \int_0^1 (1-y)^2 \, dy$$

$$= \int_0^1 1 - 2y + y^2 \, dy$$

$$= \left[y - y^2 + \frac{y^3}{3} \right]_0^1$$

$$= \frac{1}{3}$$



S_2: $\vec{n} = \langle -1, 0, 0 \rangle$

$$\int_0^1 \int_0^{1-y} y \, dz\,dy$$

$$\int_0^1 y(1-y) \, dy$$

$$= \int_0^1 y - y^2 \, dy$$

$$= \left[\frac{y^2}{2} - \frac{y^3}{3} \right]_0^1$$

$$= \frac{1}{6}.$$

S_3: $\vec{n} = \langle 0, -1, 0 \rangle$

$$\iint y - z$$

$$\int_0^1 \int_0^{1-z} 1 - x - 2z \, dx\,dz$$

Problem 3. Let $\mathbf{G} = \dfrac{\mathbf{r}}{|\mathbf{r}|^3}$ be the gravitational field, where $\mathbf{r} = \langle x, y, z \rangle$.
Show that the flux of $\mathbf{G}$ across a sphere S with center at the origin is indepndent of the radius of S.