$$\int_0^1 -3\sin(t^4)\cdot(4t^3)\,dt +$$

$$\cos(-t^3)\cdot(-3t^2)\,dt +$$

$$-(t^6)(2t)\,dt$$

$$= -12\int_0^1 t^3\sin(t^4)\,dt +$$

let $u = -t^3$, $du = -3t^2$, $dt = \frac{1}{-3}du$

$$-3\int_0^{-1} t^2\cos(-t^3)\,dt +$$

$$-2\int t^7\,dt$$

$$= -12\left(\tfrac{1}{4}\right)[\cos u]_0^1 - 3\left(\tfrac{-1}{3}\right)[\sin u]_0^{-1} - 2\left[\left(\tfrac{t^8}{8}\right)\right]_0^1$$

$$= -3\cos 1 + 3 + \sin(-1) - \tfrac{1}{4}$$

5 $\int_0^{2\pi} -8\cos(t)\sin(t)\,dt +$

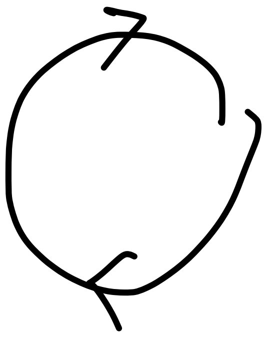
$\int_0^{2\pi} 8\sin(t)\cos(t)\,dt +$

$\int_0^{2\pi} 5 \cdot 4\,dt$

$=$

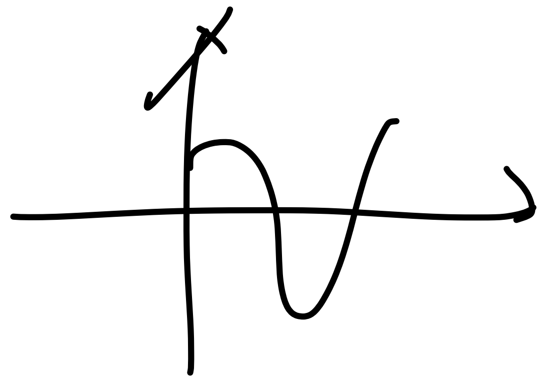
Made with Goodnotes

(6)



$$\iint y - x \, dA$$

$$\int_0^{2\pi}\int_0^{r} r\sin\theta - r\cos\theta\,(r\,dr\,d\theta)$$



$$\iint y\,dA - \iint x\,dA$$

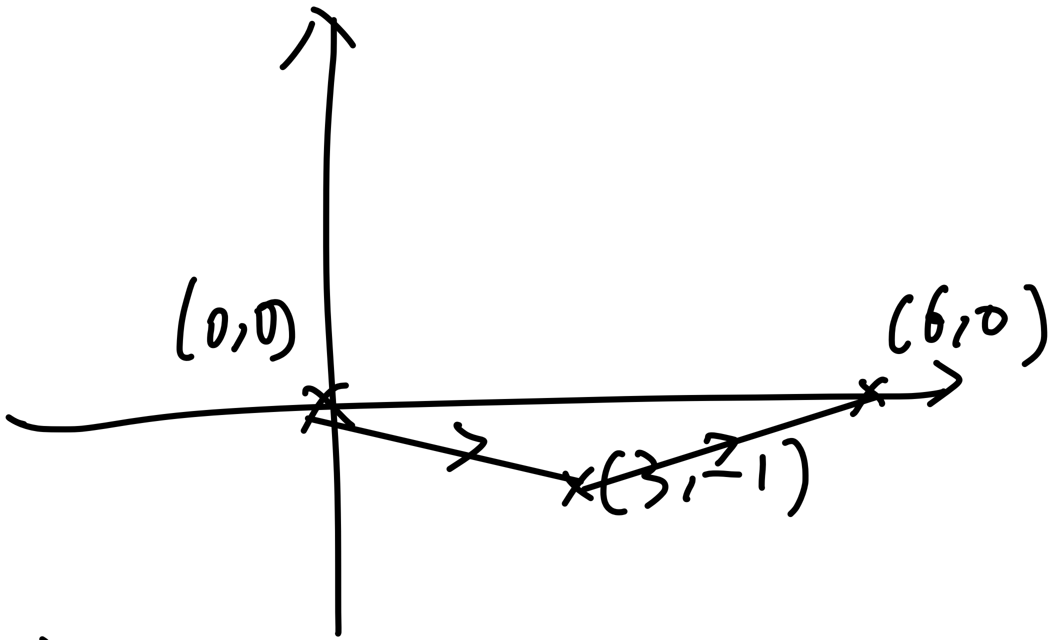
$$\int_0^{2\pi}\int_0^{R} r^2\sin\theta\,dr\,d\theta - \iint r^2\cos\theta\,dr\,d\theta$$

$$\left[\int \frac{R^3}{3}\sin\theta\right]_0^{2\pi} - \left[\frac{R^3}{3}\sin\theta\right]_0^{2\pi}$$

$$\left[-\frac{R^3}{3}\cos\theta\right]_0^{2\pi}$$

$$-\frac{R^3}{3} + \frac{R^3}{3}$$

7.



$\langle P, Q \rangle = \langle 1, 3 \rangle$

$C_2 = y = 0$

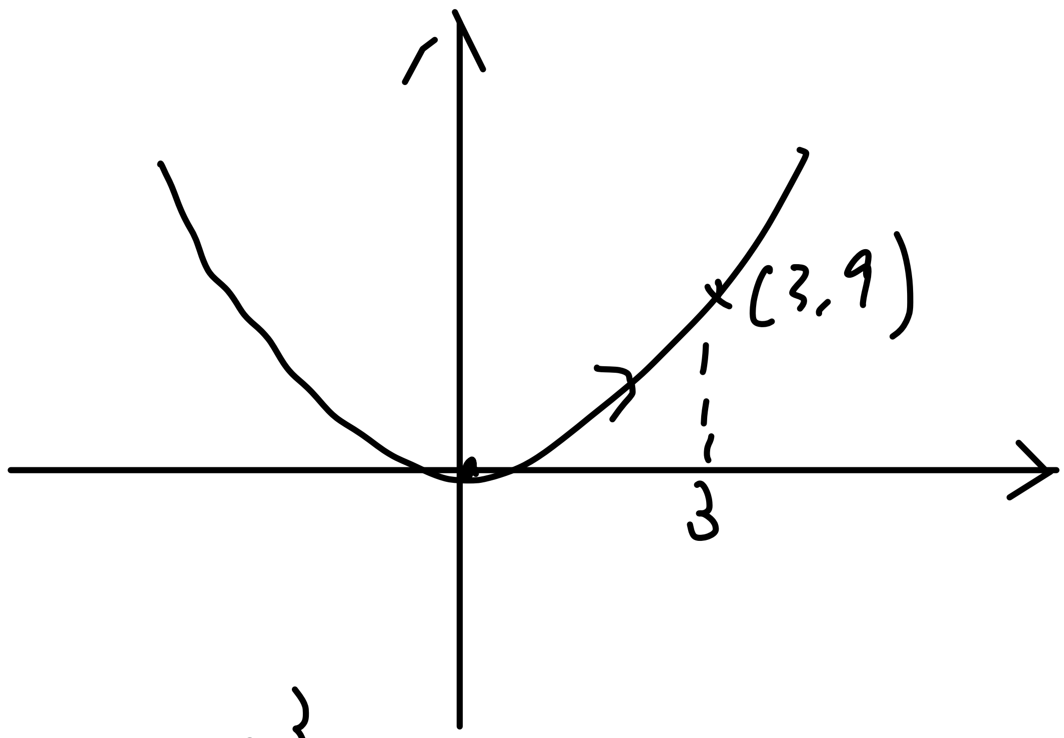
$x = t \quad 0 < t < 6$

$$\oint_{\Delta} \vec{F} \cdot d\vec{r} = \underbrace{\int_{C_1} \vec{F} \cdot d\vec{r}}_{\star} + \int_{C_2} \vec{F} \cdot d\vec{r}$$

$$= \star + \int_0^6 1 \, dt$$

$$\iint_{\Delta} \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \, dA = \star + 6$$

8.



$$\int_0^3 F_1\,dx + F_2\,dy$$

$$= \int_0^3 x(1)\,dt + \int_0^3 y(2t)\,dt$$

$$= \int_0^3 t\,dt + 2\int_0^3 t^3\,dt$$

$$= \left[\frac{t^2}{2}\right]_0^3 + 2\left[\frac{t^4}{4}\right]_0^3$$

$$=$$

$$\int_0^{3} x(6t)\,dt + \int_0^{3} y(18t)\,dt$$

$$= \int_0^{1} 18t^3\,dt + \int_0 18\cdot 9\,t^3\,dt$$

$$= (18 + 18\cdot 9)\left(\frac{t^4}{4}\right)_0^3$$

$$W = \int_0^{2} f_x (-3\sin t)\, dt + \int_0^{2} f_y (3\cos t) dt +$$

$$\int_0^{2\pi} f (4) dt$$

$=$

Q13.

$$\int_0^1 x^2 + y^2 \, dt + \int_0^1 10xy \, (2t) \, dt$$

$$= \int_0^1 t^2 + t^4 \, dt + 20 \int_0^1 t^4 \, dt$$

$$= \frac{1}{3} + 21 \left(\frac{1}{5} \right)$$

$=$

$$\int_0^1 x^2 + y^2 \, dt + \int_0^1 10xy \, dt$$

$$= \int_0^1 2t^2 \, dt + 10 \int_0^1 t^2 \, dt$$

$$= 12 \left[\frac{1}{3} \right]$$

14. $\frac{x^2+y^2-x(2x)}{(x^2+y^2)^2}$

$= \frac{y^2-x^2}{(x^2+y^2)^2}$

$\frac{(x^2+y^2)(-1)-(-y)(2y)}{(x^2+y^2)^2}$

$= -x^2-y^2+2y^2$

y^2-x^2

$\int_0^{2\pi} \frac{-y}{x^2+y^2}(-\sin t)\,dt + \int_0^{2\pi} \frac{x}{x^2+y^2}\cos t\,dt$

$= \int_0^{2\pi} \sin^2 t\,dt + \int_0^{2\pi} \cos^2 t\,dt$

$= \int \frac{1-\cos 2\theta}{2}\,dt$

$\int \sin^2 t\,dt$

$= \int_0^{2\pi} \frac{1}{2} - \frac{\cos 2\theta}{2}\,dt$

$= \pi + \left[\frac{\sin 2\theta}{4}\right]_0^{2\pi}$

π.

$\int_0^{2\pi} \frac{1}{2} + \frac{\cos 2\theta}{2}\,dt$

$= \pi \quad \left[\frac{\sin 2\theta}{4}\right]_0^{2\pi}$

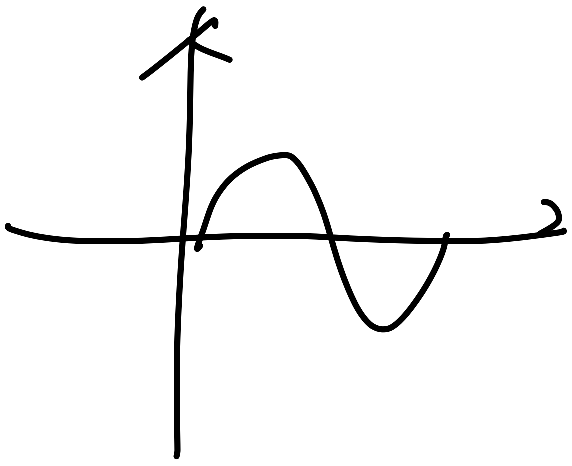
Made with Goodnotes

$3xz + 4xy + 4yz$

$f(\vec{r}(1,1,1)) - f(\vec{r}(0,0,0))$

$=$ 4

11 -



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16. $f \Rightarrow \frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2}$

$\vec{r}\left(\frac{\pi}{2}\right) = \langle 3, 3, 3 \rangle$

$\vec{r}(0) = \langle 1, 1, 1 \rangle$

---

17. P $\qquad \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$.

$-3x^2 \quad -7xy + 2y^2$

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -3x & -2y & 1 \end{vmatrix}$$

$\langle 0, 0, 0 \rangle$

$-\frac{3}{2}x^2 - y^2 + z$

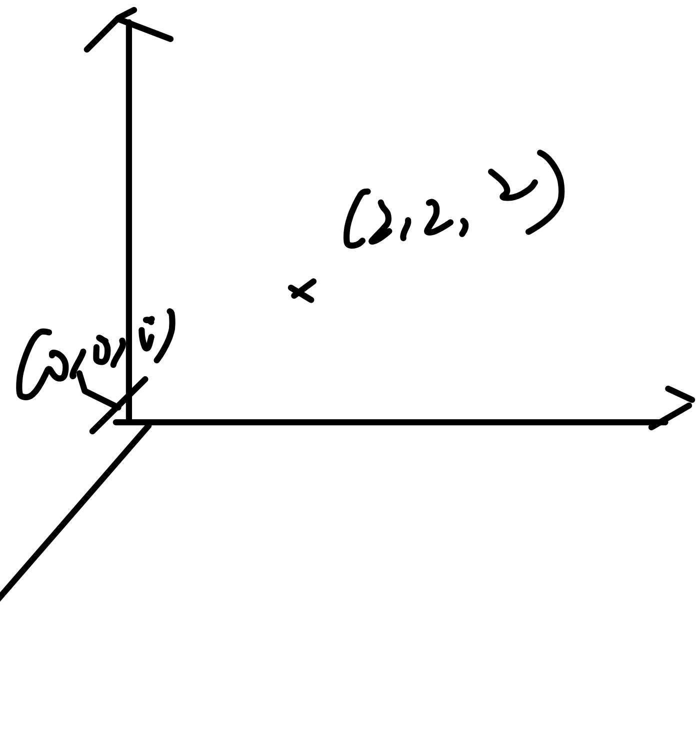
$-3\cos y \cdot \quad -3\cos y$

$-3x\sin y - 7y^2$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -3x^2 & -7y^2 & 2z^2 \end{vmatrix}$$

$-x^3 - \frac{7}{3}y^3 + \frac{2}{3}z^3$

$0, 0, 0$



$r = \langle t, t, t \rangle$

$\int F \cdot dr \qquad = f(2,2,2) + 2$

$\int_0^2 2t^3 e^{t^2} dt + \int_0^2 t e^{t^2} dt + \int_0^2 t e^{t^2} dt$

let $u = t^2$, $du = 2t\,dt$

late.

$\int_0^4 u e^u du + (e^4 - 1)$

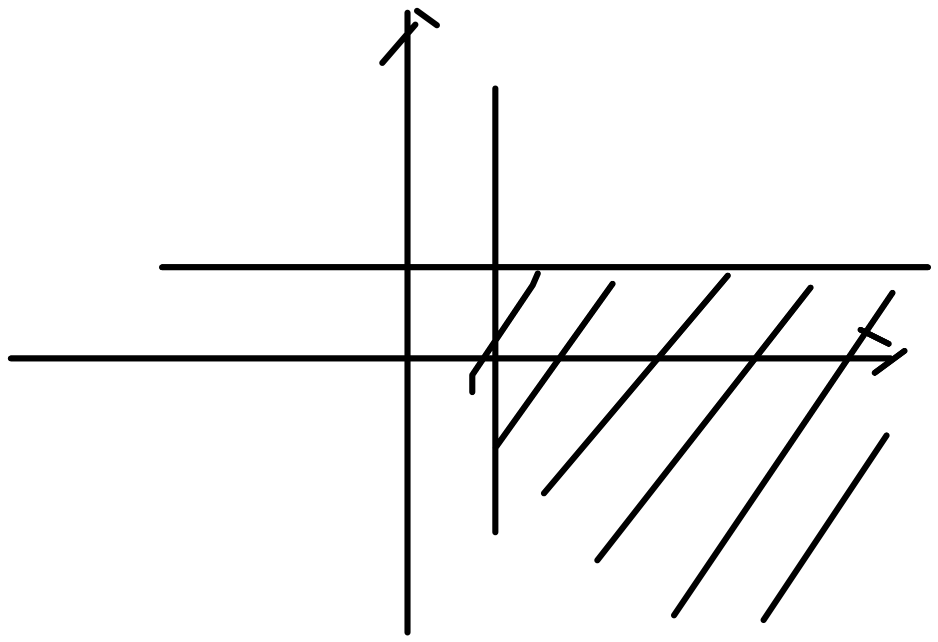
$u \quad v'$

$v = e^u$

$u' = 1$

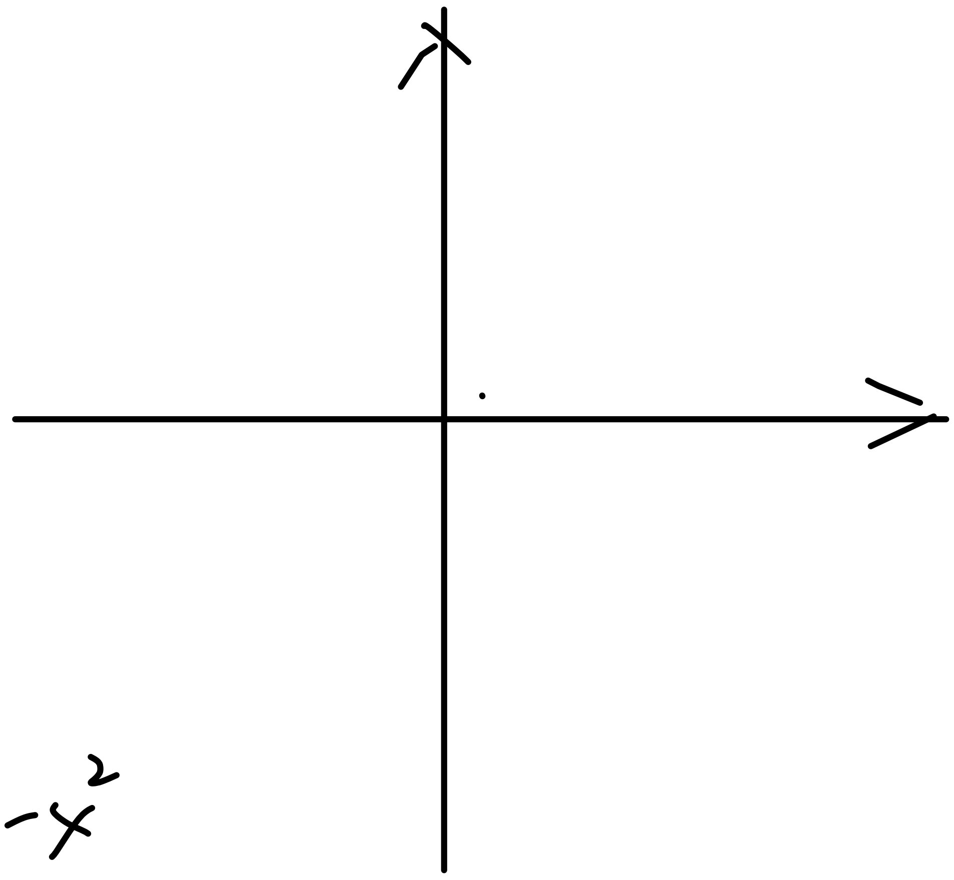
$[ue^u]_0^4 - \int_0^4 e^u du$

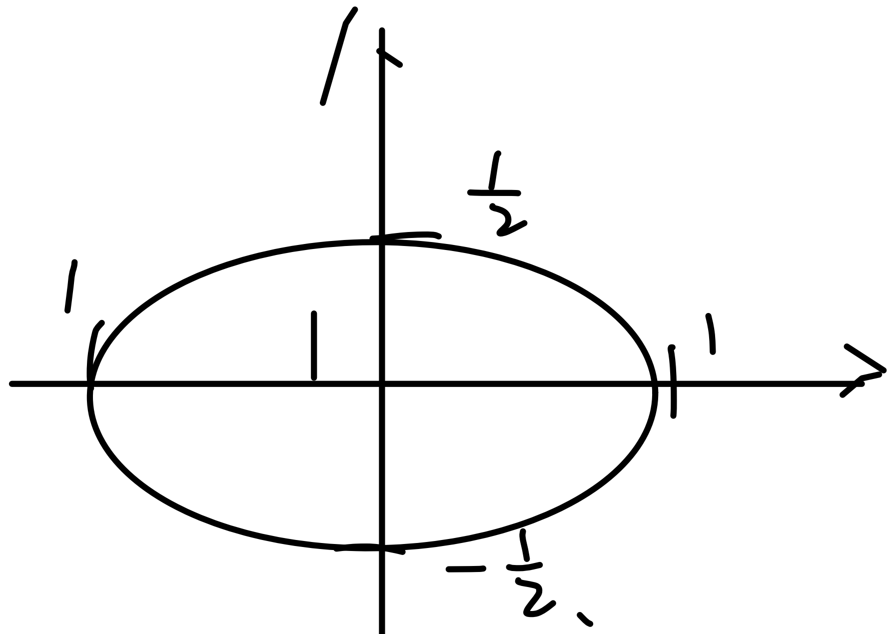
4.



$x^2 - 1 < y^2$

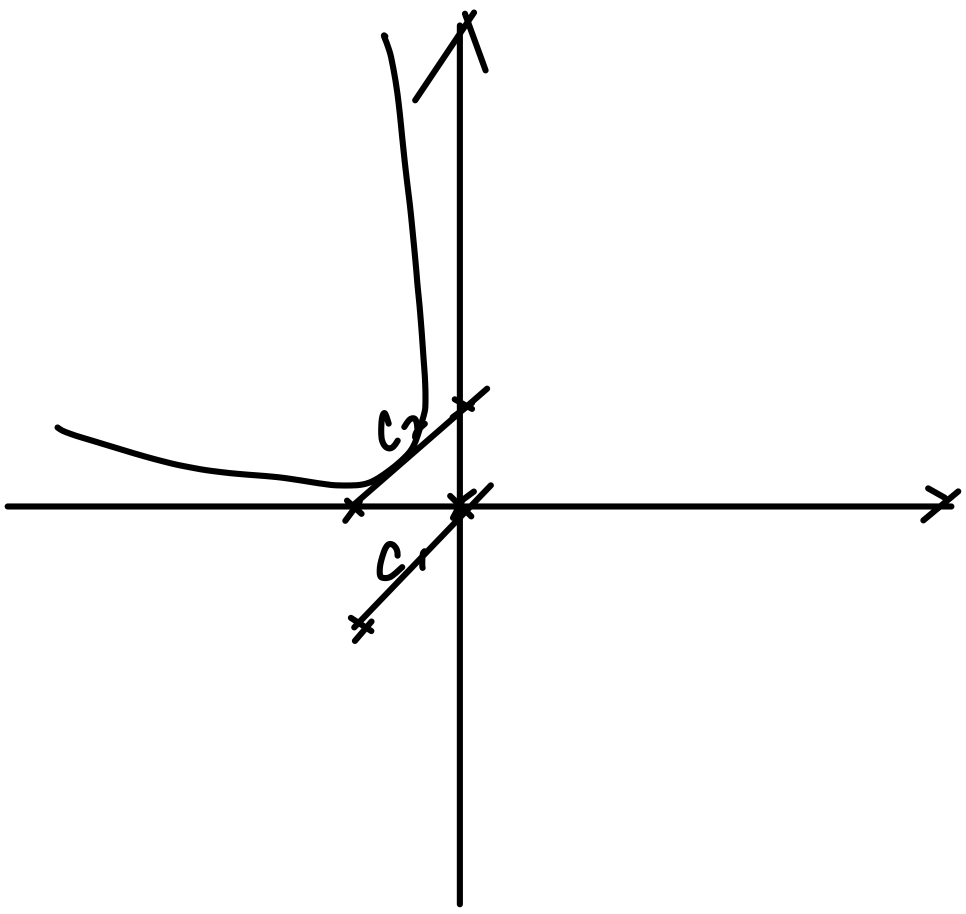
$y > \sqrt{x^2-1}$





$-y^2 < 1 - x^2$

$y^2 > x^2 - 1$



27.

$\frac{\partial a}{\partial x} -$ $\quad 2$

$y-x$

$\langle -y, x \rangle$

$\frac{x^2}{2} - \frac{y^2}{2}.$

$\frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z}$

$4z \quad 7y \quad 6x$

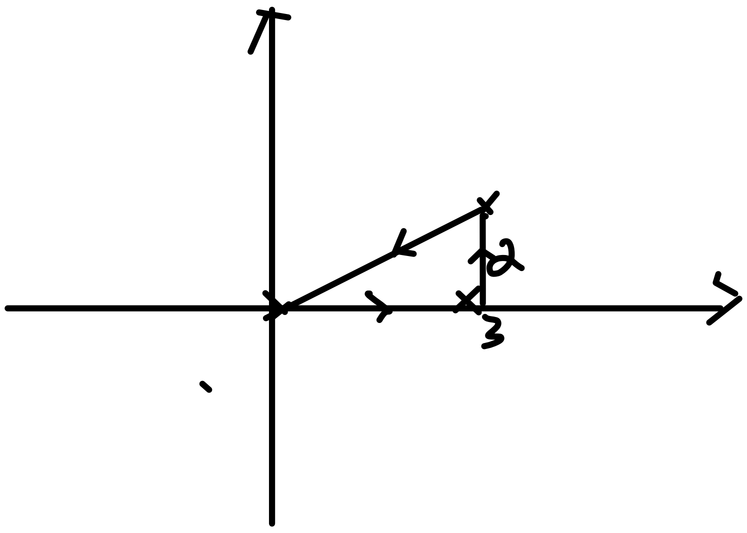
$\langle 0, 4+6, 0 \rangle$

$\frac{\partial}{\partial x}$ $\frac{\partial}{\partial y}$ $\frac{\partial}{\partial z}$

$3x^3$ $\sin z$ e^y

$e^y \cos z$

$4x^2y + 2y^3$



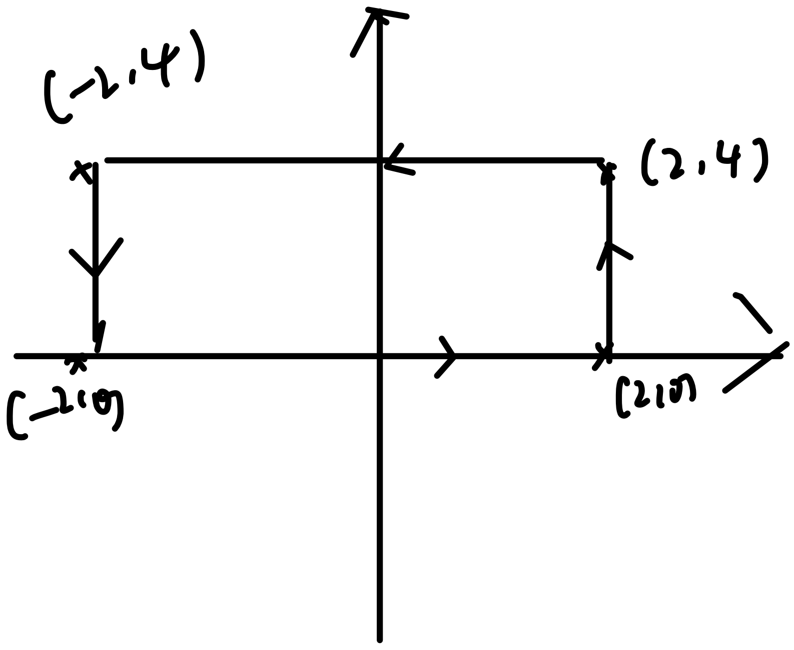
$$\iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \, dA$$

$7x^2e^y$

21.3

$7 + 14$

$-7(\triangle)$



$-3 \, dA$

$$\iint -4x \, dA$$

$$\int_0^5 \int_0^4 -4x \, dy \, dx$$

$$= \int_0^5 (-4x)_0^4 \, dx$$

$$= \int_0^5 -16x \, dx$$

$$= \left[\frac{-16x^2}{2}\right]_0^5$$

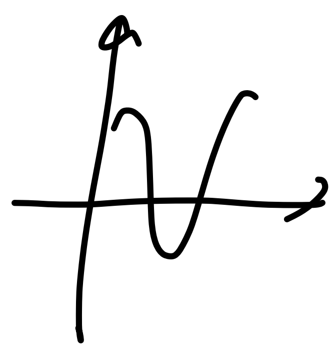
$$= -8(25)$$

$$\int_0^{2\pi} \int_0^1 (4y - 1) \, r \, dr \, d\theta$$

$$= \int_0^{2\pi} \int_0^1 4r^2 \sin\theta \, dr \, d\theta \quad - \int_0^{2\pi} \int_0^1 r \, dr \, d\theta$$

$$= \int_0^{2\pi} \left[4 \frac{r^3}{3} \sin\theta\right]_0^1 d\theta \quad - \quad \left[\frac{r^2}{2}\right]_0^1$$

$$= \int_0^{2\pi} \frac{4}{3} \sin\theta \, d\theta$$



$$\frac{1}{2} \cdot 2\pi$$

$$-\pi$$

$$\frac{4}{3}\left[-\cos\theta\right]_0^{2\pi}$$

$$= \frac{4}{3}(-1 + 1)$$

$x = a + m\cos\theta$
$y = b + m\sin\theta$

$$\iint 3 - 8 \, dA$$

$$-5(m^2\pi)$$

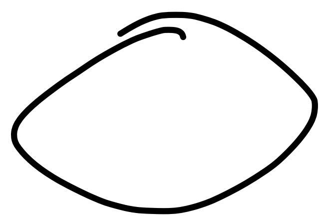
$2+3$

5

$18xy - 8xy$

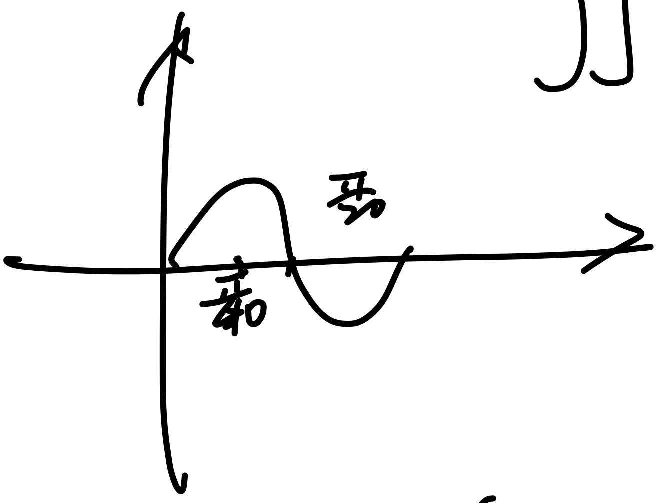
$10xy \, dA$

$$\int_0^2\int_0^2 10xy \, dx \, dy$$
$$= 10\int_0^2\int_0^2 xy \, dx \, dy$$
$$= 10\int_0^2 \left(\frac{x^2}{2}y\right)\Big|_0^2 \, dy$$
$$= 10\int_0^2 2y \, dy$$
$$= 20\left[\frac{y^2}{2}\right]_0^2$$
$$= 40$$



$\pi 4.13$

$$\iint 1\, dA$$



$$C(t) = \vec{r}(t) =$$

$$\langle 8\sin(20\theta)\cos\theta, \quad 8\sin(20\theta)\sin\theta \rangle$$

$$\int_0^{\frac{\pi}{20}} 8\sin(20\theta)\cos\theta \left(8\sin(20\theta)\cos\theta + 160\cos(20\theta)\sin\theta\right) d\theta \; -$$

$$\int_0^{\frac{\pi}{20}} 8\sin(20\theta)\sin\theta\left(-8\sin(20\theta)\sin\theta + 160\cos(20\theta)\cos\theta\right)$$

$$\int_0^{\frac{\pi}{20}} 8\sin^2(20\theta) \quad -16\sin^2(20\theta)\sin^2\theta\, d\theta$$

$$8\int \frac{1}{2} - \frac{\cos 40\theta}{2}\, d\theta \qquad \frac{1-\cos 2\theta}{2}$$

$$= 8\left[\frac{1}{2}\theta - \frac{\sin 40\theta}{80}\right]_0^{\frac{\pi}{20}}$$

$$= 8\left(\frac{1}{2}\cdot\frac{\pi}{20}\right)$$

$$\left(\frac{1}{2} - \frac{\cos 40\theta}{2}\right)\left(\frac{1}{2} - \frac{\cos 2\theta}{2}\right)$$

$$-16$$

$$= \left(\frac{1}{4}\cdot\frac{\pi}{} - \cancel{\frac{\cos 2\theta}{4}} - \cancel{\frac{\cos 40\theta}{4}} + \frac{\cos 40\theta \cos 2\theta}{4}\right)$$

$$= -16\left(\frac{1}{4}\right)\left(\frac{\pi}{20}\right)$$

$$\int_0^{\frac{\pi}{2}} \cos 40\theta \cos 2\theta \, d\theta$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \cos 42\theta + \cos 38\theta \, d\theta$$

$$= \frac{1}{2}\left[\frac{\sin 42\theta}{42} + \frac{\sin 38\theta}{38} \right]_0^{\frac{\pi}{20}}$$

$$\frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z}$$

$$\frac{1}{x} \quad y \quad \frac{1}{z}$$

$$\frac{1}{x}$$

$$\frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z}$$

$$-2y \quad 5x \quad 5z$$

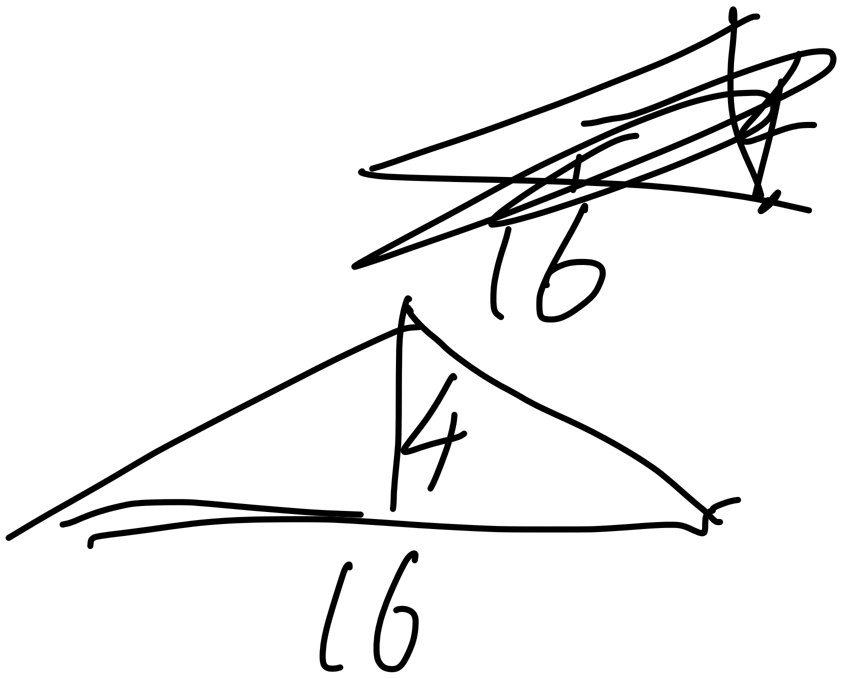
$$5+2$$

$2yz\,e^{2xyz} + 5z^2\cos(xz^2)$

$e^{2xyz} + 5\sin(xz^2)$

$7\left(\frac{16\pi}{6} \quad \frac{3}{2}\right)$

2



$\cdot\,32$

$$\int_0^4\int_0^3 -\cos x - \frac{\cancel{9}e^{\frac{y}{5}}}{5}\,dx\,dy$$

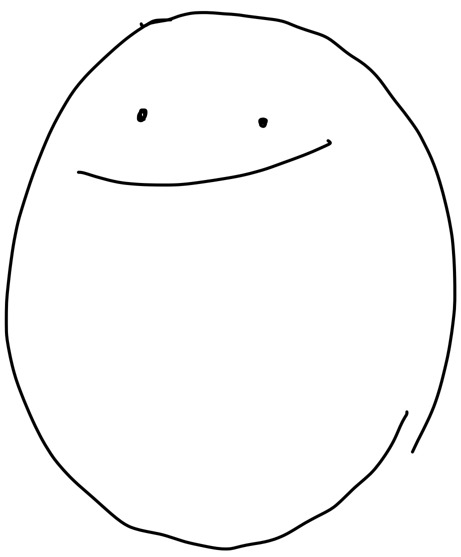
$$[\sin x]_0^3 - \frac{3}{5}e^{\frac{y}{5}}$$

$$-4\sin 3 - 3e^{\frac{4}{5}} + 3$$

$$\int_0^3\int_0^4 \cos x\,dy\,dx - \int_0^4\int_0^3 5e^{\frac{y}{5}}\,dx\,dy$$

$$= 4\sin 3 - 15\left[5e^{\frac{y}{5}}\right]_0^4$$

$$= 4\sin 3 - 75e^{\frac{4}{5}} + 75$$



$$\int_0^{2\pi}\int_0^4 (6y-7)\,r\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^4 6r^2\sin\theta - 7r\,dr\,d\theta$$

$$= \int_0^{2\pi}\left[6\frac{r^3}{3}\sin\theta\right]_0^4 d\theta - \int_0^{2\pi}\left[\frac{7r^2}{2}\right]_0^4 d\theta$$

$$= \int_0^{2\pi} 128\sin\theta\,d\theta - \int_0^{2\pi} 56\,d\theta$$

$$= 128\left[-\cos\theta\right]_0^{2\pi} - 56(2\pi)$$

$$= 128\,($$

F
F
F
T
F
T
T
F

$x(t) = 8\cos t$
$y(t) = 16\sin t$

$$\begin{array}{r} 16 \\ \times\ 8 \\ \hline 128 \\ \hline 2 \end{array}$$

$$\frac{1}{2}\int_0^{2\pi} -16\sin t\,(-8\sin t)\,dt + 8\cos t\,(16\cos t)\,dt$$

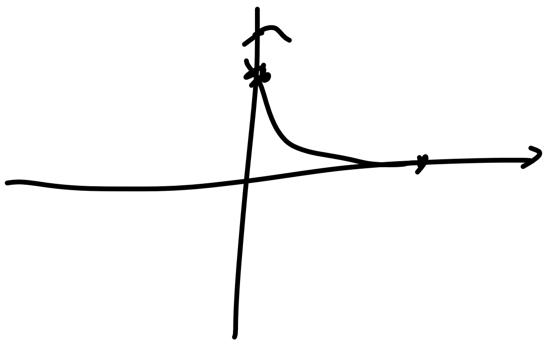
$$= 64\int_0^{2\pi} \sin^2 t + \cos^2 t\,dt$$

$$= 64\int_0^{2\pi} \frac{1}{2} - \frac{\cos 2\theta}{2} + \frac{1}{2} + \frac{\cos 2\theta}{2}\,dt$$

$$= 64\int_0^{2\pi} 1\,dt$$

$$= 64 \cdot 2\pi$$

$x(t) = 2\cos^3 t$
$y(t) = 2\sin^3 t$



$$2 \int_C -y\,dx + x\,dy.$$

$$= 2\int_0^{\frac{\pi}{2}} -2\sin^3 t\,(6\cos^2 t(-\sin t))\,dt + 2\cos^3 t\,(6\sin^2 t\cos t)\,dt$$

$$= 24\int_0^{\frac{\pi}{2}} \sin^4 t\cos^2 t + \sin^2 t\cos^4 t\,dt$$

$$\sin^2 t\cos^2 t(\sin^2 t + \cos^2 t)\,dt$$

$$= 24\int_0^{\frac{\pi}{2}} \sin^2 t\cos^2 t\,dt$$

$$= 24\int_0^{\frac{\pi}{2}} \left(\frac{1}{2} - \frac{\cos 2\theta}{2}\right)\left(\frac{1}{2} - \frac{\sin 2\theta}{2}\right) dt$$

$$= \underbrace{24\int_0^{\frac{\pi}{2}} \frac{1}{4}}_{\frac{\pi}{\frac{4}{3}}} - \cancel{\frac{\sin 2\theta}{4}} - \cancel{\frac{\cos 2\theta}{4}} + \cancel{\frac{\cos 2\theta \sin 2\theta}{4}}$$

let $u = \sin 2\theta$,
$du = 2\cos 2\theta\,d\theta$,
$d\theta = \frac{1}{2\cos 2\theta}\,du$

$6xy - \cancel{4xy}\ 10xy$

$$\int_0^1 \int_0^1 4xy \, dx \, dy$$

$$= \int_0^1 \left[\frac{x^2}{2} y\right]_0^1 dy$$

$$= \int_0^1 \frac{1}{2} y \, dy$$

$$\left[\frac{y^2}{4}\right]_0^1$$

$$\frac{17-13}{4}$$

$$\frac{16xy - 2xy}{14}$$

$$\frac{1+3}{4}$$

$$\vec{r}(u,v) = \langle -4u^2, 2v^2, -4u^2+2v^2 \rangle$$

$$\vec{n} = \vec{r}_u \times \vec{r}_v$$

$$\begin{array}{ll} & \qquad\quad i \qquad\quad j \qquad\quad k \\ r_u = & \left| \begin{array}{l} \langle -8u, \; 0, \; -8u \rangle \\ \langle \; 0, \; 4v, \; 4v \rangle \end{array} \right| \\ r_v = & \end{array}$$

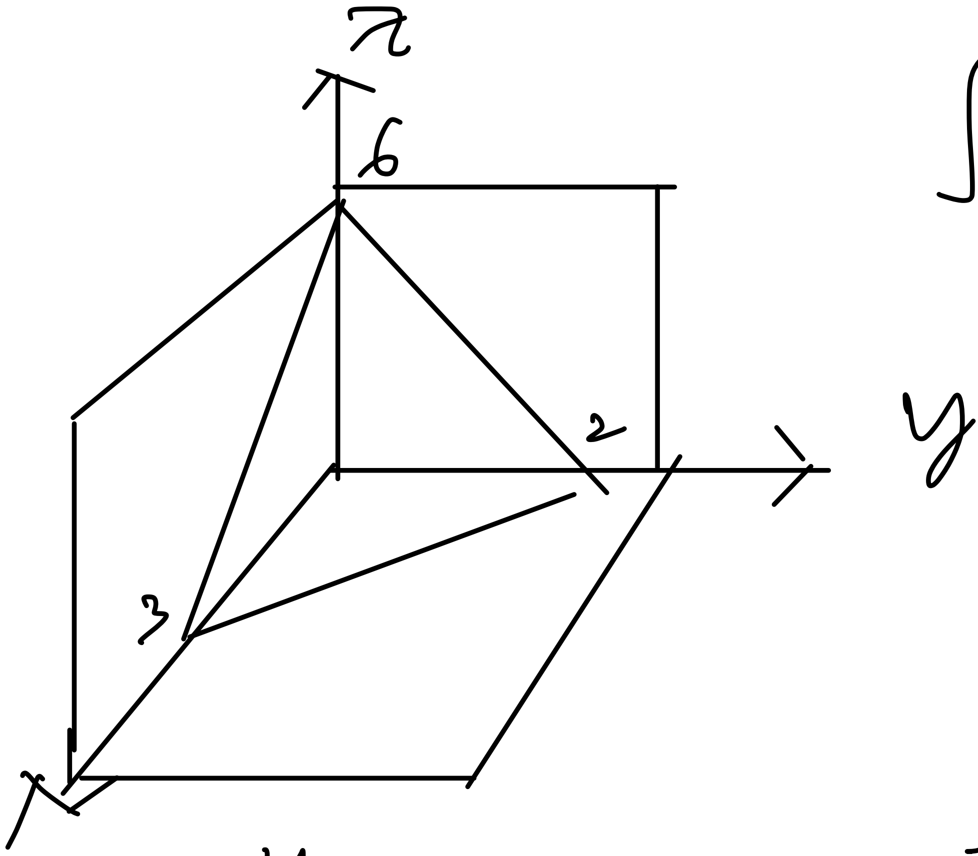
$$= \langle 32uv, \; 32uv, \; -32uv \rangle$$

$$\vec{n} = \langle -64, -64, 64 \rangle$$

$$\vec{r}(-2,1) = (-16, \; 2, \; -14)$$

$$-64x - 64y + 64z = 1024 - 128 - 896$$

$$x + y - z \qquad = 0$$



$$\int_0^2\int_0^3\int_0^{6-2x-3y} x+2y \; dz\, dx\, dy$$

$$\int_0^2\int_0^3 (x+2y)(6-2x-3y)\,dx\,dy$$

$$=\int_0^2\int_0^3 6x-2x^2-3xy+12y-4xy-6y^2\,dx\,dy$$

$$=\int_0^2\int_0^3 -2x^2+6x-6y^2+12y-7xy\,dx\,dy$$

$$=\int_0^2\left[-\frac{2x^3}{3}+3x^2-6y^2x+12xy-\frac{7}{2}x^2y\right]_0^3 dy$$

$$=\int_0^2 -\frac{2\cdot 27}{3}+27-18y^2+36y-\frac{63}{2}y$$

$$=\int_0^2 9-18y^2+\frac{9}{2}y\,dy$$

$$=\left[9y-6y^3+\frac{9}{4}y^2\right]_0^2$$

$$=18-48+9$$

$$=-30+9$$

$$=-27.$$

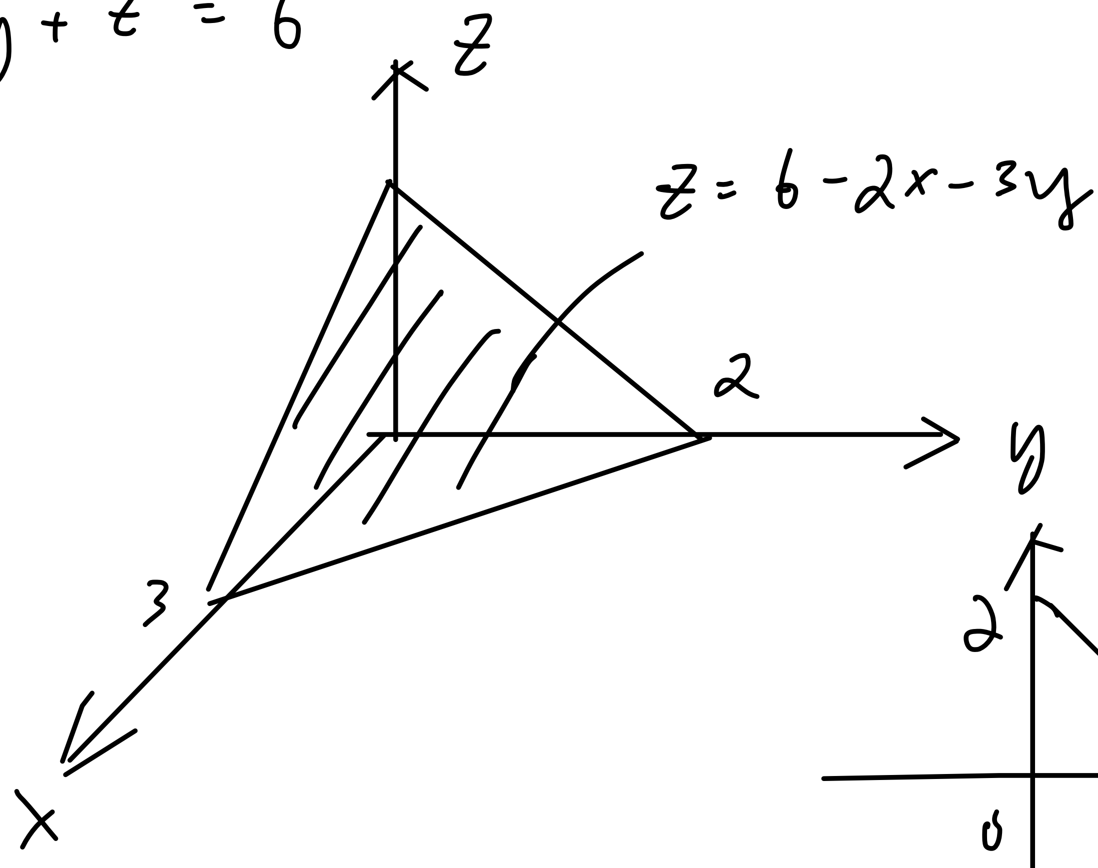
$$\int_0^{6-2x-3y}\int_0^2\int_0^3 (x+2y)\,dx\,dy\,dz$$

$$=\int_0^{6-2x-3y}\int_0^2\left[\frac{x^2}{2}+2xy\right]_0^3 dy\,dz$$

$$=\int_0^{6-2x-3y}\int_0^2 \frac{9}{2}+6y\,dy\,dz$$

$$=\int_0^{6-2x-3y} 9+\left[3y^2\right]_0^2$$

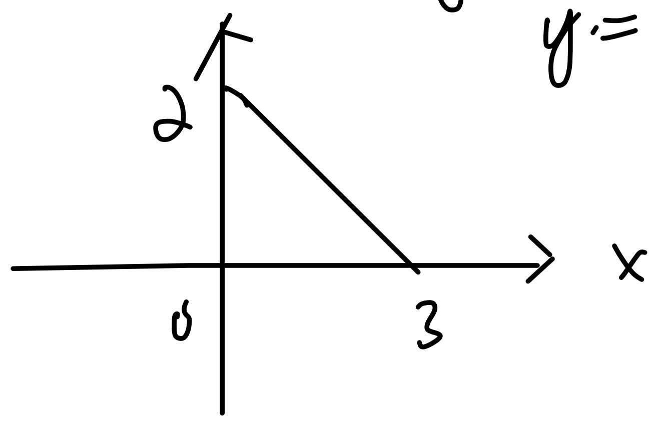
$2x + 3y + z = 6$



$\frac{y}{2} + \frac{x}{3} = 1$

$3y + 2x = 6$

$y = \frac{6-2x}{3}$



$$\int_0^{\frac{6-2x}{3}} \int_0^{3} \int_0^{6-2x-3y} x+2y \; dz \, dx \, dy$$

$$= \underbrace{(x+2y)(6-2x-3y)}$$

$$\left[6x - 2x^2 + 12y - 7xy - 6y^2\right]_0^{3} dx$$

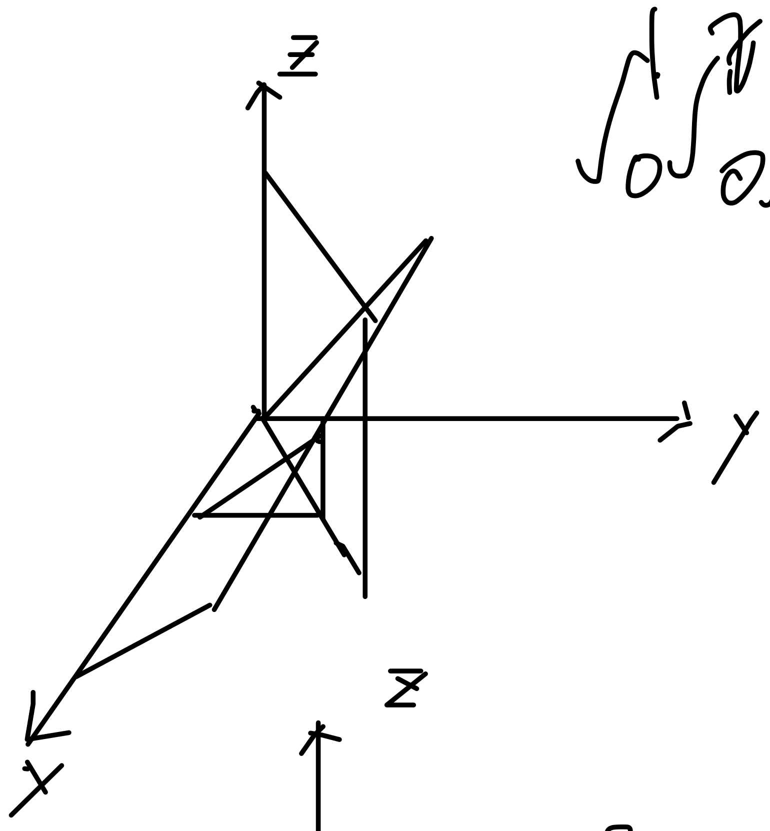
$$= \left[6\frac{x^2}{2} - \frac{2x^3}{3} + 12xy - 7\frac{x^2}{2}y - 6xy^2\right]_0^3$$

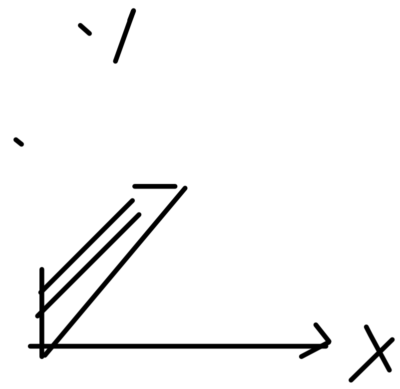
$$= \left[9y + 18y^2 - \frac{63}{4}y^2 - 6y^3\right]_0$$

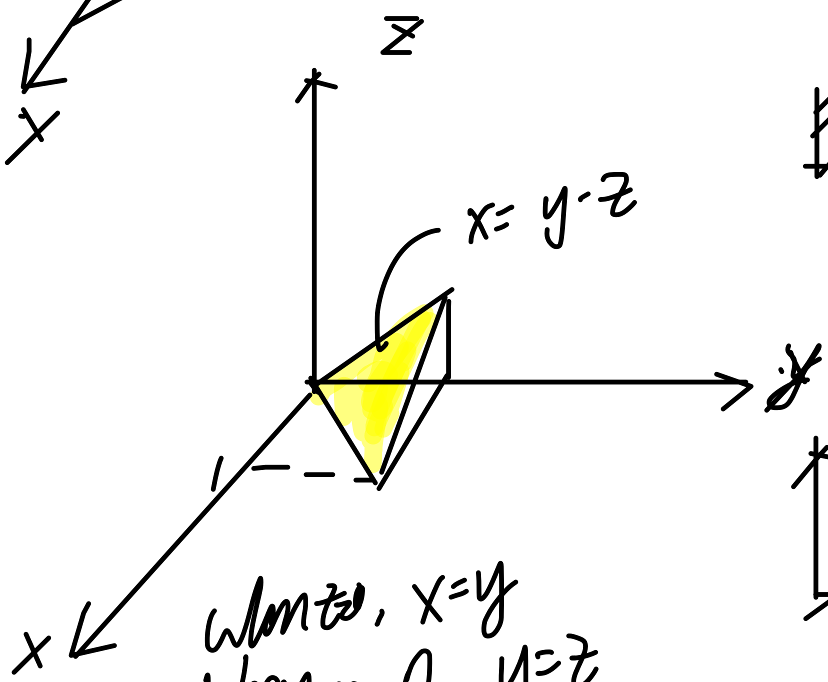
$$= -21$$

Made with Goodnotes

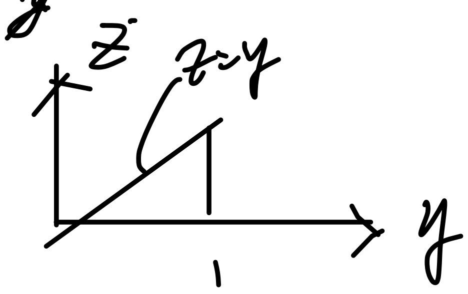
$$\int_0^1\int_0^z\int_0^{y-z} dx\,dy\,dz$$



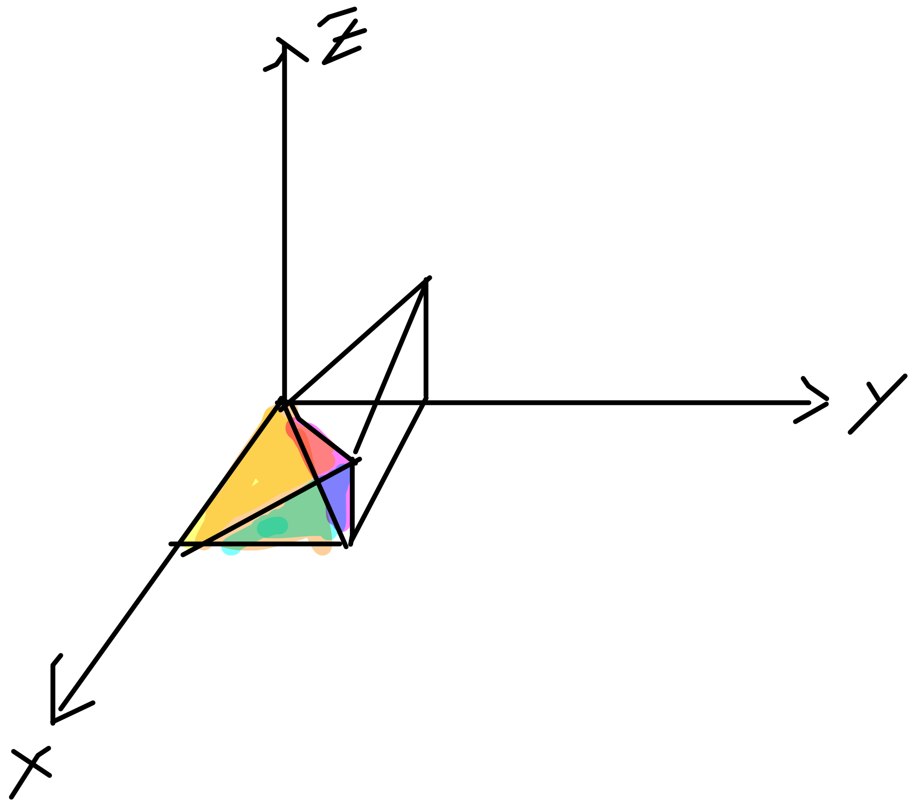


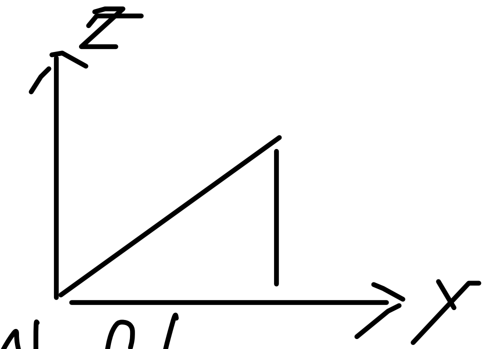


when z=0, x=y
when x=0, y=z

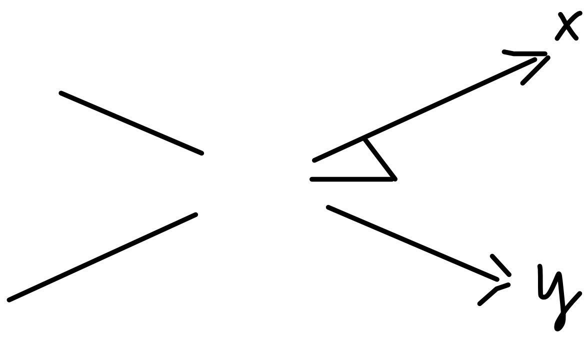


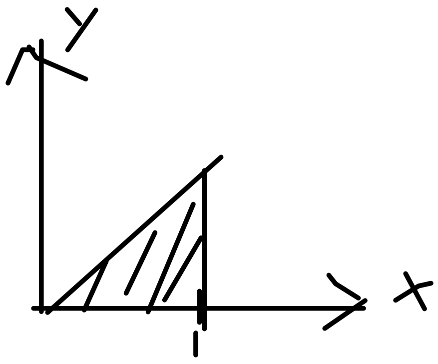
$x-y+z=0$

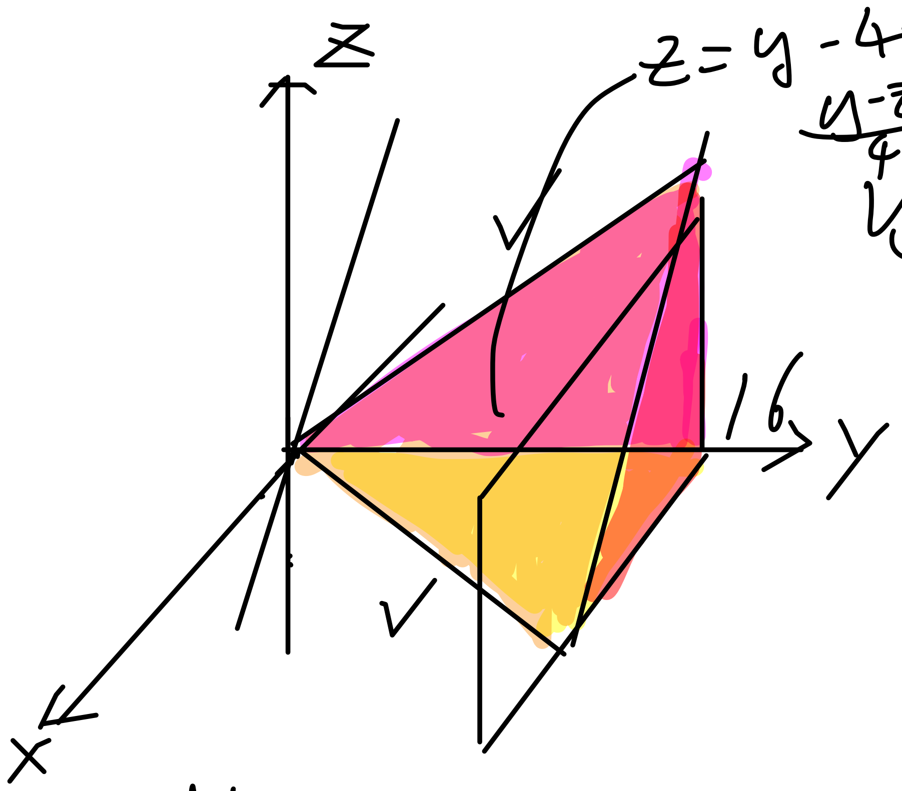




$$\int_0^1\int_z^1\int_y^1 dy\,dz$$







$z = y - 4x$

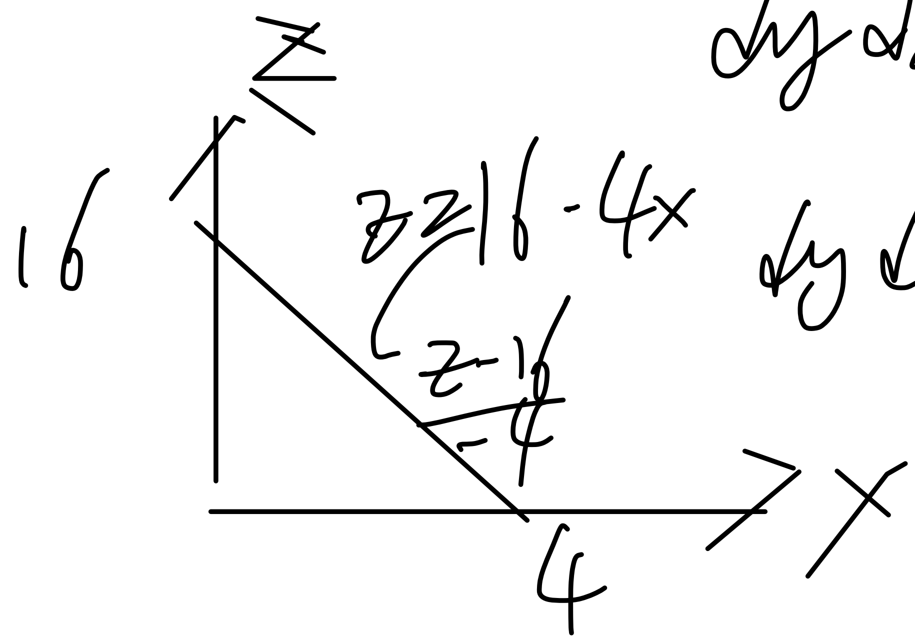
$\frac{y-z}{4} = x$

$y = z + 4x$

$$\int_0^{16}\int_0^{y}\int_0^{\frac{y-z}{4}} dx\,dz\,dy$$

$$\int_0^{4}\int_0^{16-4x}\int_{z+4x}^{16}$$

$dy\,dz\,dx$



$dy\,dx\,dz$

$$\int_0^{16}\int_0^{\frac{16-z}{4}}\int_{z+4x}^{16}$$

$x = \rho\sin\varphi\cos\theta$

$y = \rho\sin\varphi\sin\theta$

$z = \rho\cos\varphi$

$$\int_0^{2\pi}\int_0^{\pi}\int_1^{2} \frac{e^{-\rho^2}}{\rho}\,\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta$$

$$\int_0^{2\pi}\int_0^{\pi}\int_1^{2} \rho e^{-\rho^2}\sin\varphi\,d\rho\,d\varphi\,d\theta$$

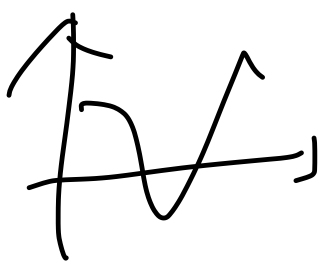
let $u = -\rho^2$, $du = -2\rho\,d\rho$.

$d\rho = -\frac{1}{2\rho}\,du$.

$$-\frac{1}{2}\int_0^{2\pi}\int_0^{\pi}\int_{-1}^{-4} e^{u}\sin\varphi\,du\,d\varphi\,d\theta$$

$$-\pi\left(\int_0^{\pi} (e^{-4}-e^{-1})\sin\varphi\,d\varphi\right)$$

$$(e^{-1}-e^{-4})\pi\left(\left[-\cos\varphi\right]_0^{\pi}\right)$$



2π $\quad -(-1)-(-1) \quad 1+1$

$$\int_{\partial V} F \cdot \vec{n}\, ds = \iiint_E \nabla \cdot F \; dV$$

$$\int_0^9 \int_0^{10} \int_0^4 (2x-1)\, dz\, dy\, dx$$

$$(4)(10)\int_0^9 (2x-1)\, dx$$

$$= 40\left(\left[x^2 - x\right]_0^9\right)$$

$$= 40 \cdot (81 - 9)$$

$$\nabla \cdot F = 3y^2 + 3x^2 + 3z^2$$

$$\iiint_E 3x^2 + 3y^2 + 3z^2 \, dV$$

$$\int_0^{2\pi} \int_0^{\pi} \int_0^{9} (3\rho^4) \quad \sin\varphi \, d\rho \, d\varphi \, d\theta$$

$$= 6\pi \int_0^{\pi} \left[\frac{\rho^5}{5} \sin\varphi \right]_0^9 d\varphi$$

$$= 6\pi \int_0^{\pi} \sin\varphi \left(\frac{9^5}{5} \right) d\varphi$$

$$= 6\pi \quad \cancel{\left[-\cos\varphi \right]_0^{\pi}} \; 2$$

$$12\pi \cdot \frac{9^5}{5}$$

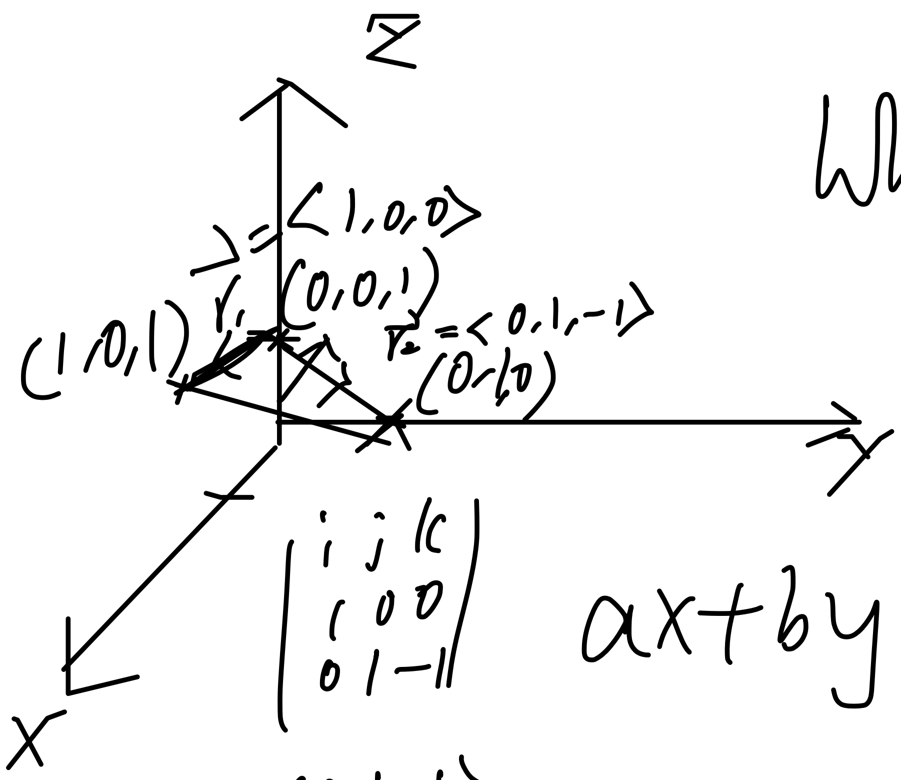
	1st	2nd	3rd	4th
b	250	150	100	50
c	150	60	60 30,	30
d	240	140	140	
e	300			
f	280	180		
g	150	50	50	30
h	180	80	40	40

when $y=0$, $x=z=1$

$ax+cz=1$

When $z=0$, $y=1$, $x=0$

$b=0$



$$\begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix} = \langle 0,1,1\rangle$$

$ax+by+cz=0$

$$\iint_S \nabla\times \vec{F}\, dS = \int_C \vec{F}\cdot d\vec{r}$$

when $y=0$, $z=1$

$y+z=1$

When $x=0$, $y+z=1$

When

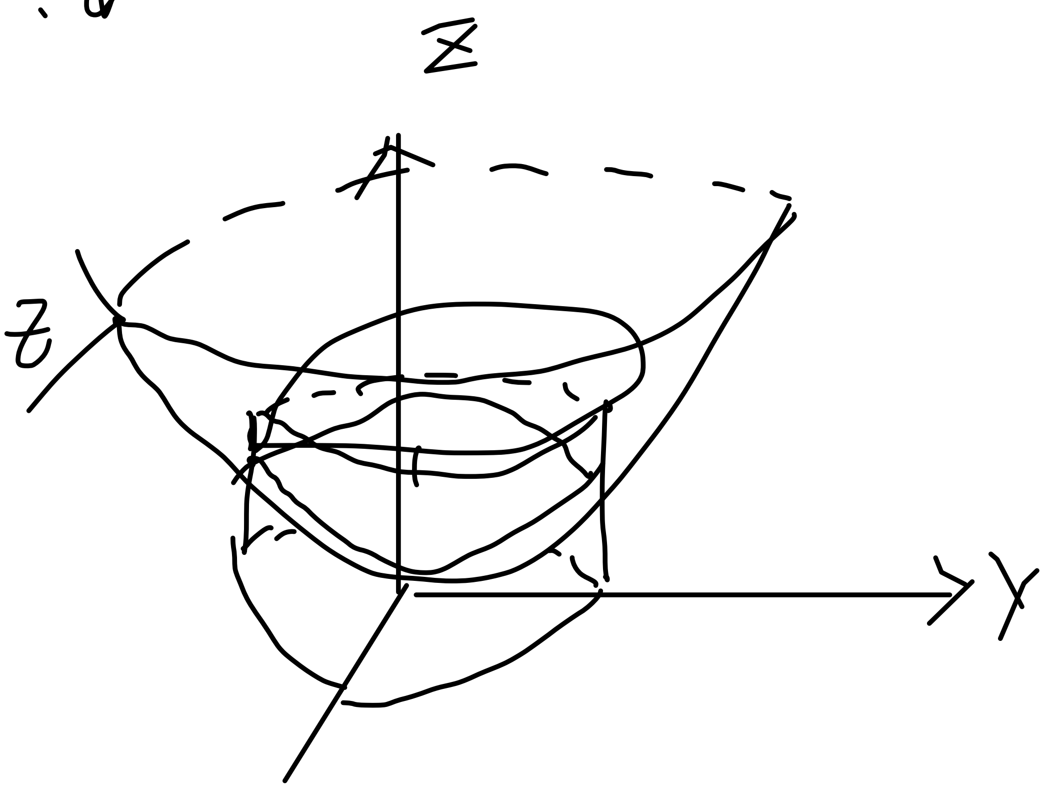
$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -5yz & 5xz & 17(x^2+y^2) \end{vmatrix}$$

$$\langle 5x, -5y - 34x, 5z + 5z \rangle$$

$$\iint \nabla \times F \cdot dS = \int_C \vec{F} \cdot d\vec{r}$$

$$\langle \cos\theta, \sin\theta, 1 \rangle$$

$$\langle -5yz, 5xz, 17(x^2+y^2)z \rangle$$



$$\int_0^{2\pi} -5\sin\theta(-\sin\theta)\,d\theta +$$

$$\int_0^{2\pi} 5\cos\theta(\cos\theta)\,d\theta +$$

$$\int_0^{2\pi} 17(0)\,d\theta$$

$$\int_0^{2\pi} 5\,d\theta$$

$$\iint_S F \cdot dS$$

$\langle 0, 1, 0\rangle, \langle 1, 0, -4\rangle$

$$\vec{n} = \langle \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ 1 & 0 & -4 \end{vmatrix} \quad -4, 0, -1\rangle$$

$$-4x - z = -4$$

$$-z = -4 + 4x$$

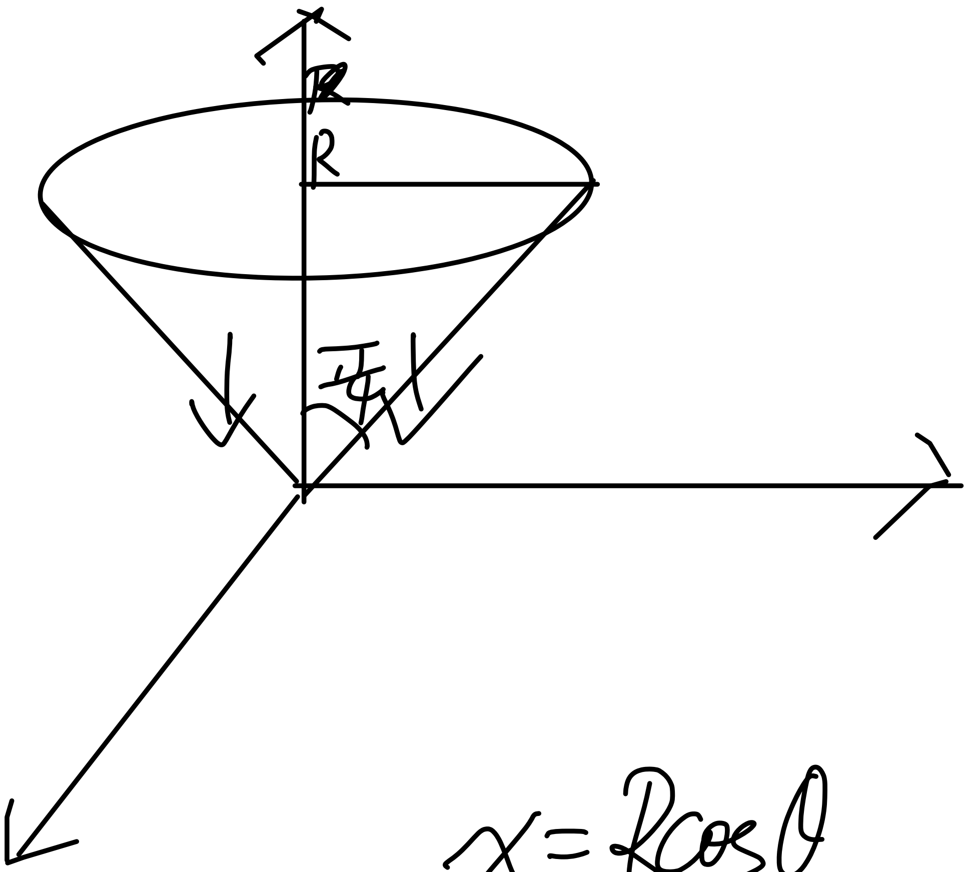
$$z = 4 - 4x$$

$\sqrt{17}$

$$\vec{n} = \langle 4, 0, 1\rangle = \langle \frac{4}{\sqrt{17}}, 0, \frac{1}{\sqrt{17}}\rangle$$

$$\vec{F} = \langle 1, -5, -1\rangle$$

$$\left(\frac{4}{\sqrt{17}} - \frac{1}{\sqrt{17}}\right) \cdot (\sqrt{17}) \quad 4 - 1 \quad \iint 3\,d$$



$$x = R\cos\theta$$

$$y = R\sin\theta$$

$$z = R$$

$$\frac{1}{3}(R)$$

$$d\vec{A}$$

$$\int\int\int_0^4 \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

$$\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^4 \frac{\rho^3}{3} \sin\varphi \, d\varphi \, d\theta$$

$\vec F \cdot \vec r$

$\iint_S \vec r \cdot d\vec S$ $\quad \rho^2 \cos^2\varphi \, \vec k$

$\vec r(\rho, \varphi, \theta) =$

$\iint_S (\nabla \times \vec F) \, dS =$ $\quad dz$

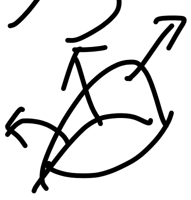
$\iiint_E$

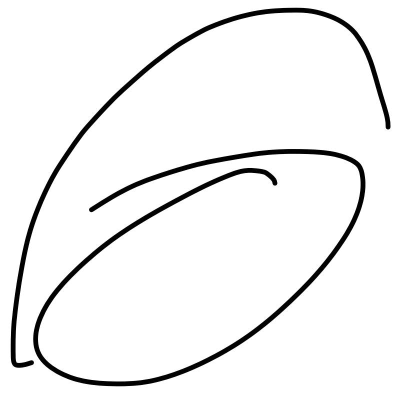
$\int_C \vec F \cdot d\vec r$

$\rho\cos\varphi$

$\cancel{\frac{\rho^3}{3}}$ $\quad \frac{\varphi^3}{3}$

$\iint_S F \cdot \vec n \, dS$





$\langle 0, 0, z^2 \rangle \cdot \langle 2x, 2y, 2z \rangle$

$\iint_S 2z^3$ $\quad dS$

d

$$\iint_S \vec{F} \cdot d\vec{S}$$

$$\vec{r}(r,\theta) = \langle r\cos\theta,\ r\sin\theta,\ r\rangle$$

$$\begin{vmatrix} & \vec{i} & \vec{j} & \vec{k} \\ \vec{S}_r(r,\theta) = & \langle \cos\theta, & \sin\theta, & 1\rangle \\ \vec{S}_\theta(r,\theta) = & \langle -r\sin\theta, & r\cos\theta, & 0\rangle \end{vmatrix}$$

$$\left(\langle -r\cos\theta,\ r\sin\theta,\ r\rangle\right)$$

$$\int_0^{2\pi}\int_0^{R} 9r^4\cos^2\theta\sin^2\theta\, dr\, d\theta$$

$$\int_0^{2\pi} \frac{9R^5}{5}\left(\cos^2\theta - \cos^4\theta\right) d\theta$$

$$= \int_0^{2\pi} \left(\frac{1}{2}+\frac{\cos 2\theta}{2}\right) - \left(\frac{1}{2}+\frac{\cos 2\theta}{2}\right)^2 d\theta$$

$$\left(\frac{9R^5}{5}\right)\left(\pi + \cancel{\frac{\sin 2\theta}{4}} - \left(\frac{1}{4} + \cancel{\frac{\cos 2\theta}{2}} + \cancel{\frac{\cos^2 2\theta}{4}}\right)\right)$$

$$\left(\frac{\pi}{2}\right) \qquad \frac{\pi}{2}$$

$$\begin{array}{ccc} & i & j & k \\ \Phi_u = & \langle -2, & -4u, & 0 \rangle \\ \Phi_v = & \langle 0, & -5, & -4v \rangle \end{array}$$

$$\langle 16uv, -8v, 10 \rangle$$

$$\begin{array}{cc} u & v \\ 3 & -3 \end{array}$$

$$\Downarrow$$

$$\langle -144, 24, 10 \rangle$$

$$-144x + \cancel{72}\overset{24}{}y + 10z =$$

$$\left| \begin{array}{l} r_\theta = \langle -2\sin\varphi\sin\theta,\ 2\cos\theta\sin\varphi,\ 0\rangle \\ r_\varphi = \langle 2\cos\theta\cos\varphi,\ 2\sin\theta\cos\varphi,\ -2\sin\varphi\rangle \end{array} \right|$$

$$\langle -4\cos\theta\sin^2\varphi,\ -4\sin^2\varphi\sin\theta,\ -4\sin\varphi\cos\varphi$$

$$\sin^2\theta - 4\cos^2\theta$$

$$\sin\varphi\cos\varphi\rangle$$

$$-4\sin\varphi\cos\varphi\rangle$$

$$\nabla f = \langle 2x, 2y, 2z\rangle$$

$$2\rho\cos\theta\sin\varphi$$

$r_\theta = \langle -(3+\cos\varphi)\sin\theta,\ (3+\cos\varphi)\cos\theta,\ 0\rangle$

$r_\varphi = \langle -\cos\theta\sin\varphi,\ -\sin\theta\sin\varphi,\ \cos\varphi\rangle$

$r_\theta \times r_\varphi = \left|\langle (3+\cos\varphi)\cos\theta\cos\varphi,\ (3+\cos\varphi)\sin\theta\cos\varphi,\ (3+\cos\varphi)\sin\varphi\rangle\right|$

$$\sqrt{(3+\cos\varphi)^2\cos^2\theta\cos^2\varphi + (3+\cos\varphi)^2\sin^2\theta\cos^2\varphi + (3+\cos\varphi)^2\sin^2\varphi}$$

$$= \sqrt{(3+\cos\varphi)^2(\cos^2\theta\cos^2\varphi + \sin^2\theta\cos^2\varphi + \sin^2\varphi)}$$

$$= (3+\cos\varphi)$$

$$\int_0^{2\pi}\int_0^{2\pi} 3+\cos\varphi \, d\theta \, d\varphi$$

$$\int_0^{2\pi}\int_0^{2\pi} 3+\cos\varphi \quad d\theta\, d\varphi$$

$$2\pi \int_0^{2\pi} (3+\cos\varphi)\, d\varphi$$

$$= 2\pi \left[3\varphi + \sin\varphi \right]_0^{2\pi}$$

$$= 2\pi \cdot 6\pi$$

Made with Goodnotes

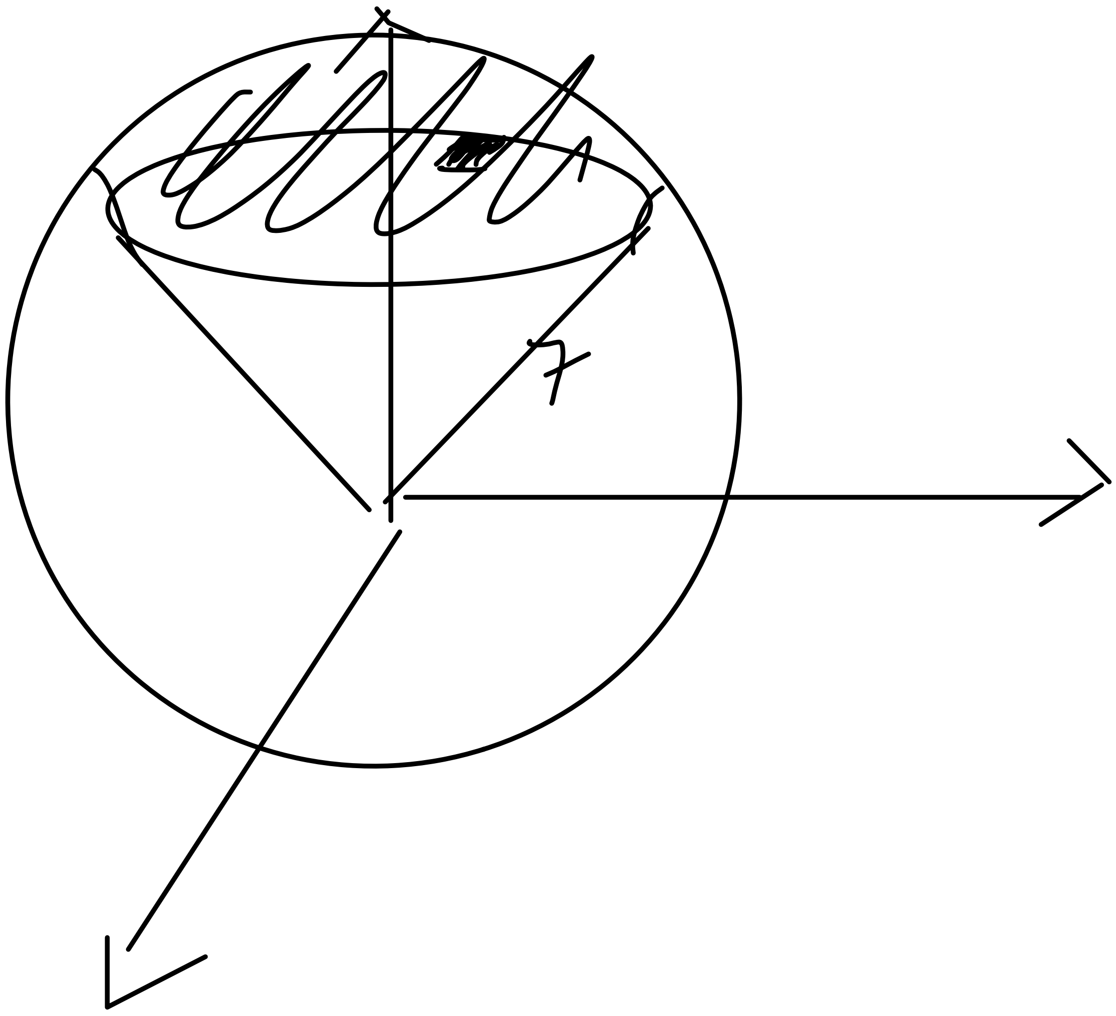
$$\int_{-2}^{2}\int_{-1}^{1} |r_u \times r_v| \, du\,dv$$

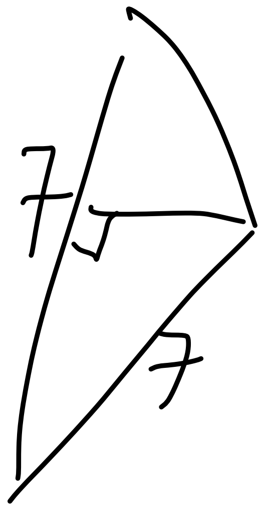
$i \quad j \quad k$

$3r_1 u_i \quad 3r_1 u_j \quad 3r_1 u_k$

9

$z = r^2$





$$\vec{r}(\theta, \varphi) = \langle 7\sin\varphi\cos\theta,$$
$$7\sin\varphi\sin\theta,$$
$$7\cos\varphi \rangle$$

$0 \le \theta \le 2\pi, \; 0 \le \varphi \le \frac{\pi}{4}.$

$$\vec{r}(\theta,\varphi) = \langle 7\sin\varphi\cos\theta,$$
$$7\sin\varphi\sin\theta,$$
$$7\cos\varphi\rangle$$

$0 \leq \theta \leq 2\pi,\ 0 \leq \varphi \leq \frac{\pi}{4}$

$$\begin{array}{cccc} & i & j & k \\ r_\theta = & \langle -7\sin\varphi\sin\theta, & 7\sin\varphi\cos\theta, & 0\rangle \\ r_\varphi = & \langle 7\cos\varphi\cos\theta, & 7\cos\varphi\sin\theta, & -7\sin\varphi\rangle \end{array}$$

$$r_\theta \times r_\varphi = \langle -49\sin^2\varphi\cos\theta,\ -49\sin^2\varphi\sin\theta,$$
$$-49\sin\varphi\cos\varphi\sin^2\theta - 49\sin\varphi\cos\varphi\cos^2\theta$$

$$|\quad| = \sqrt{(-49)^2 \quad \sin^4\varphi + \sin^2\varphi\cos^2\varphi}$$

$$\sqrt{49^2\sin^2\varphi} = 49\sin\varphi$$

$$\int_0^{\frac{\pi}{4}}\int_0^{2\pi} 49\sin\varphi\, d\theta\, d\varphi$$

$$= 49 \cdot 2\pi \quad [-\cos\varphi]_0^{\frac{\pi}{4}}$$

$$= 49 \cdot 2\pi \cdot \left(\frac{\sqrt{2}}{2} + 1\right)$$

Made with Goodnotes

$r_s = \langle t, 1, 1 \rangle$

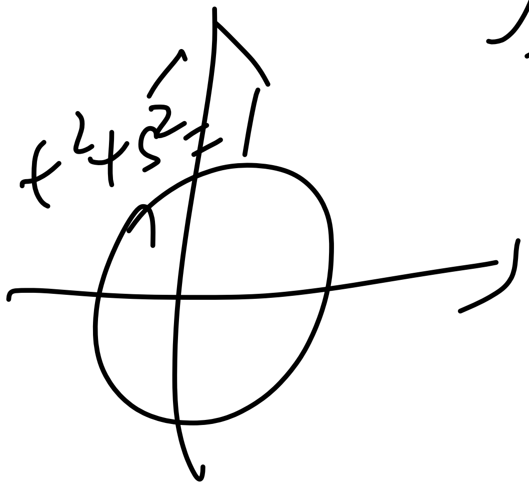
$r_t = \langle s, 1, -1 \rangle$

$\langle -2, s+t, t-s \rangle$

$4 + s^2 + 2st + t^2 + t^2 - 2st + s^2$

$\sqrt{4 + 2(s^2+t^2)}$

$$\int_{-1}^{1} \int_{-\sqrt{1-t^2}}^{\sqrt{1-t^2}} \sqrt{4+2(s^2+t^2)} \, ds\, dt$$



πr^2

(2π)

$$\int_0^{2\pi} \int_0^1 \sqrt{4+2r^2} \, r\, dr\, d\theta$$

Let $u = 4+2r^2$, $du = 2r\,dr$,

$dr = \frac{1}{2r} du$.

$$\frac{1}{2}\int_4^6 \sqrt{u}\, du = \frac{1}{2}\left[\frac{2}{3} u^{\frac{3}{2}}\right]_4^6$$

$= \frac{1}{3}(6)^{\frac{3}{2}} - \frac{1}{2}\left(\frac{2}{3}\right)\cdot(8)$

$\vec{r}(r,\theta) = \langle r^2\cos\theta\sin\theta, r(\cos\theta+\sin\theta), r(\cos\theta-\sin\theta) \rangle$

$s = r\cos\theta$
$t = r\sin\theta$

$\vec{r}_r = \langle 2r\cos\theta\sin\theta, \cos\theta+\sin\theta, \cos\theta-\sin\theta \rangle$

$\vec{r}_\theta = \langle$

$t^2+2st+s^2+t^2-2st+s^2$

$|r_s \times r_t|$

$2(t^2+s^2)$

r^2+4

$2r^2$

$$\int_0^{2\pi}\int_0^1 \sqrt{2r^2+4}\; r\,dr\,d\theta$$

let $u = 2r^2+4$, $du = 4r\,d\theta$,
$d\theta = \frac{1}{4r}du$

$$\frac{1}{4}\cdot 2\pi \int_4^6 \sqrt{u}\,du$$

$$\frac{\pi}{2}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_4^6$$

Made with Goodnotes

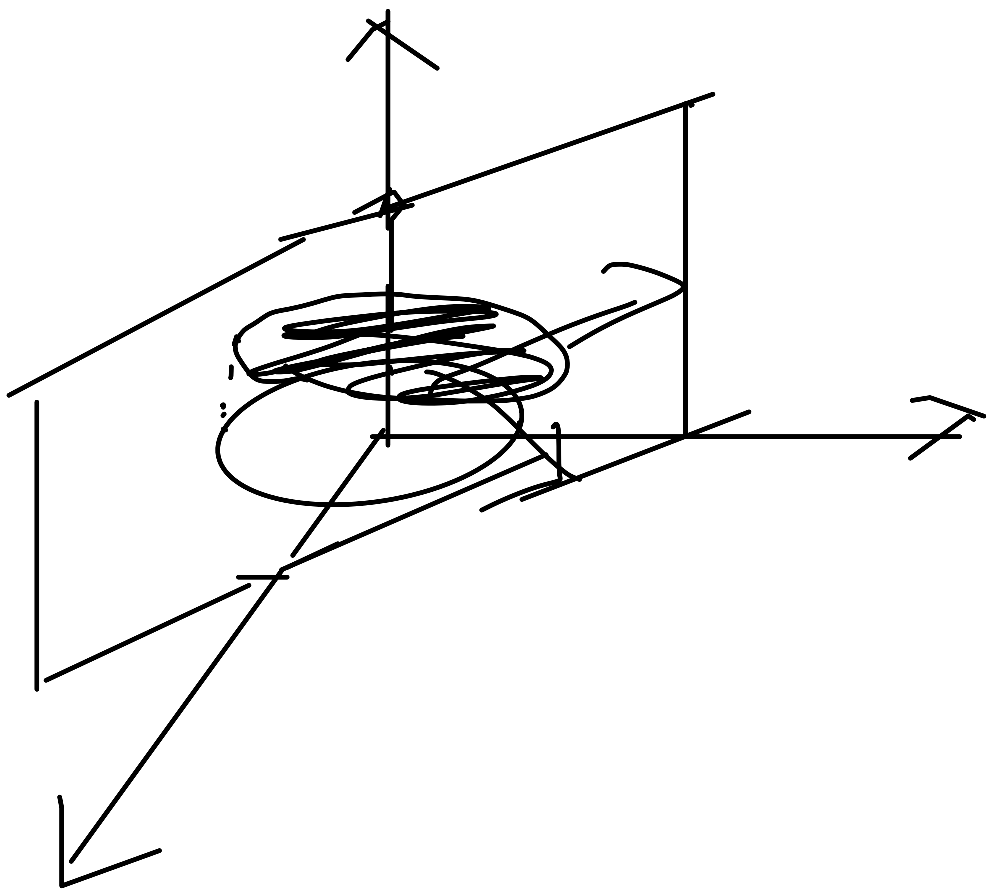
$z = 10 - 3x - 2y$

$z_x = -3$

$z_y = -2$

$\langle -3, -2, -1 \rangle$

$\sqrt{14}$. π

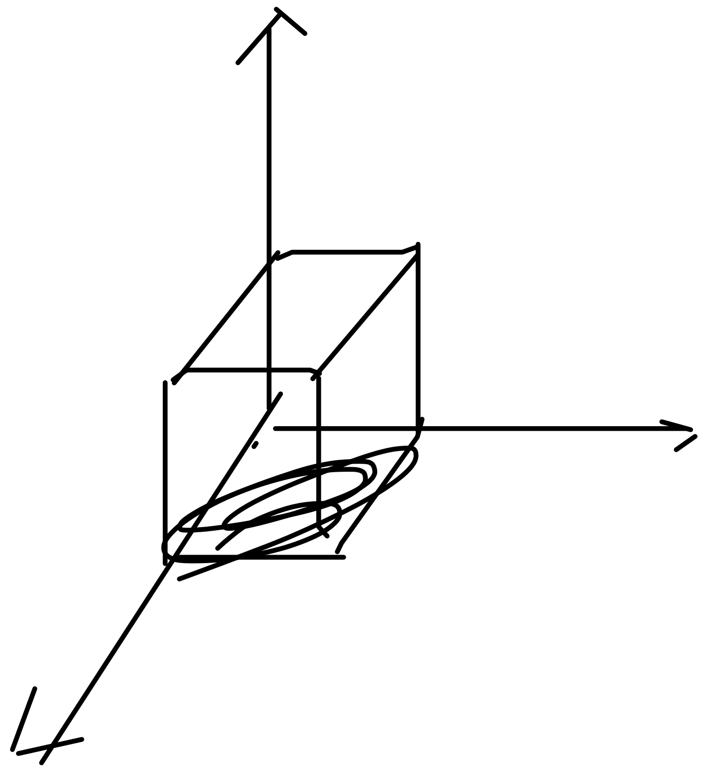


$x = 9\cos\theta$
$y = 10\sin\theta$

$$\iint_S F \cdot dS =$$

$$\iiint_E \nabla \cdot F \, dV$$

$$0$$

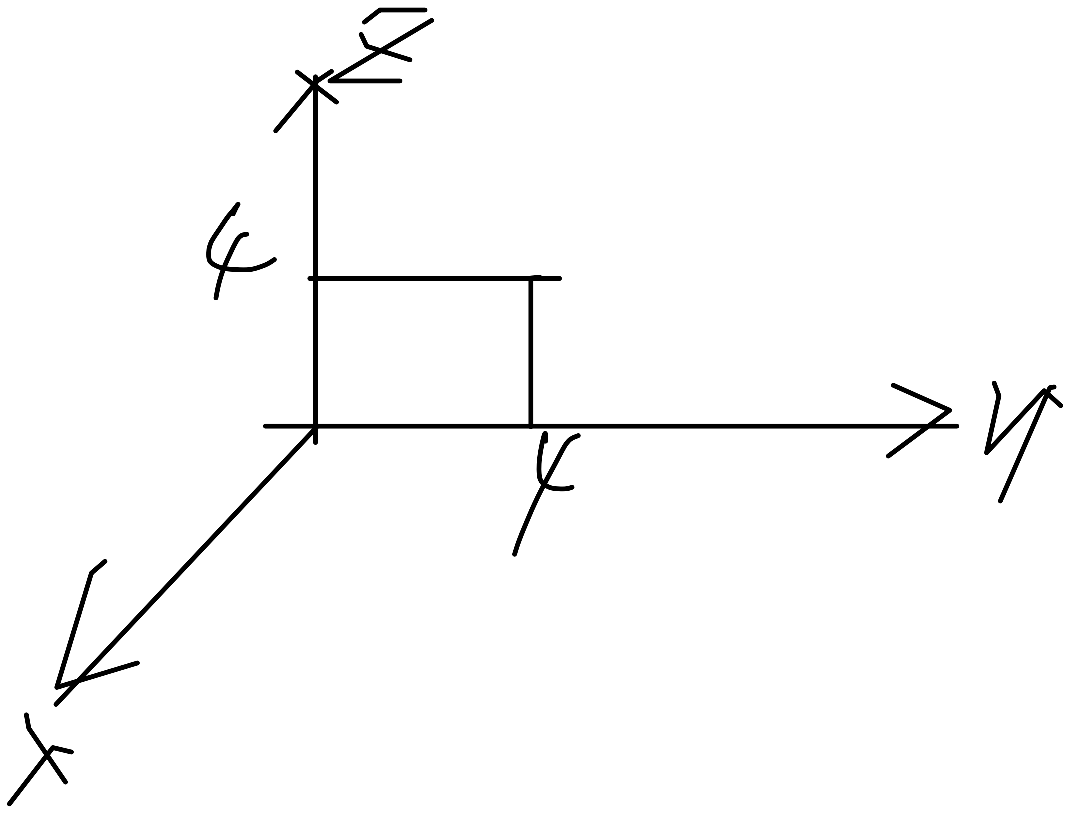


$$\iiint_{E} \nabla \cdot F \, dV = \iint dS + \iint dS$$

$$0 \qquad \alpha.$$

$$\iint \langle 1, -1, -1 \rangle \cdot \langle 0, 0, -1 \rangle \, dS$$

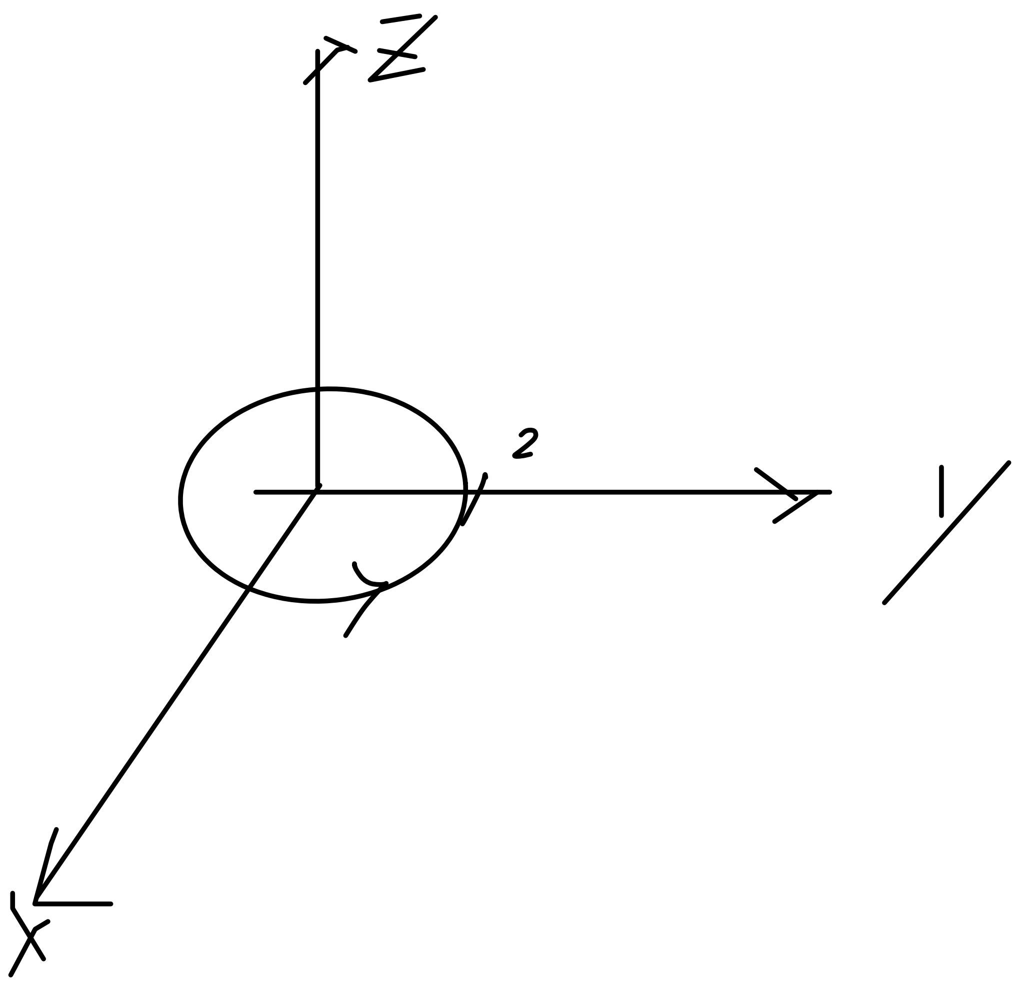
$$= \int_0^3 \int_0^3 1 \, dx\, dy$$



$$\iint \vec{F} \cdot dS$$

$$\iint \langle 1, -1, 5 \rangle \cdot \langle 1, 0, 0 \rangle \, dA$$

$$= 1$$



$\langle 0, 0, x^2+y^2 \rangle \langle 0, 0, 1 \rangle$

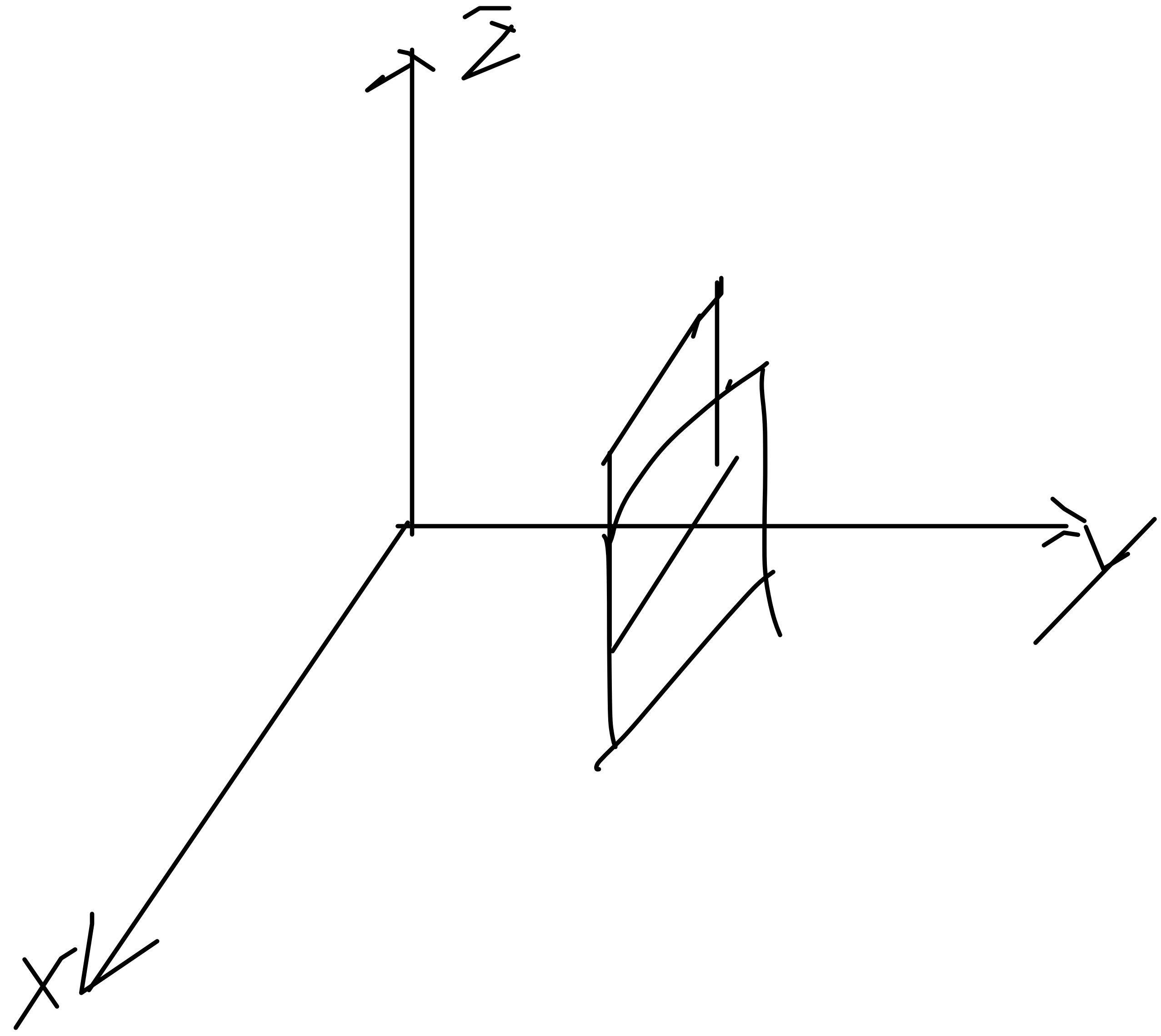
$\int_0^{2\pi} \int_0^{2} r^2 \, r \, dr \, d\theta$ // x^2+y^2

$= \frac{2^4}{4} \cdot 2\pi$

$\frac{1}{4} \cdot 2\pi$

$4 \cdot 2\pi$

8π



$\langle 8, 2x^2, -3 \rangle \ \langle 0, 1, 0 \rangle$

$$\int_{-2}^{2}\int_{0}^{4} 2x^2 \, dz\, dx$$

$4(2x^2) \qquad 8\left[\frac{x^3}{3}\right]_{-2}^{2} \qquad \frac{16}{3}\cdot 8$

$= 8\left(\frac{8}{3}+\frac{8}{3}\right)$

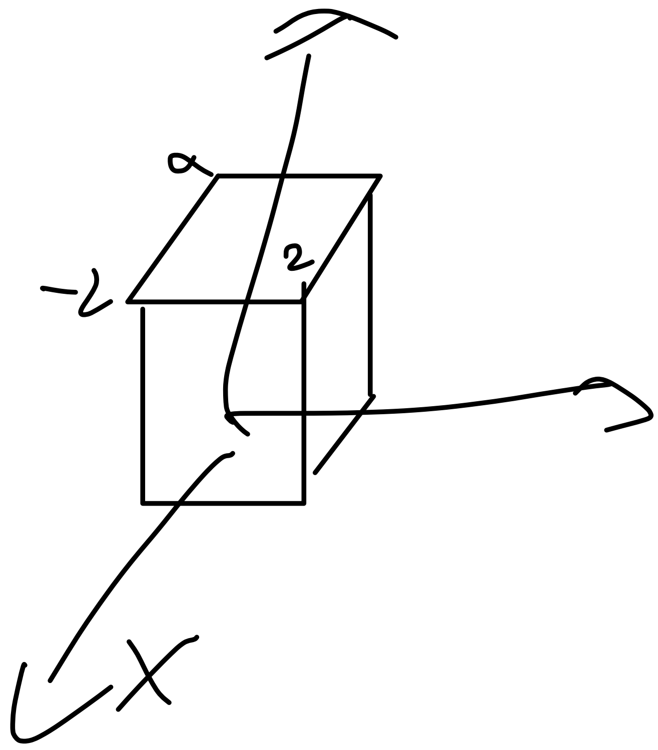
$\langle -1, 3, -3 \rangle \quad \sqrt{19}$

$\langle 0, 1, 0 \rangle \times \langle 1, 0, -2 \rangle$

$\sqrt{5} \quad 1 \quad \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ 1 & 0 & -2 \end{vmatrix}$

$\langle \frac{-2}{\sqrt{5}}, 0, \frac{-1}{\sqrt{5}} \rangle$

$\left(\frac{2}{\sqrt{5}} + \frac{3}{\sqrt{5}} \right) \sqrt{5}$



$\langle 1,0,0 \rangle$

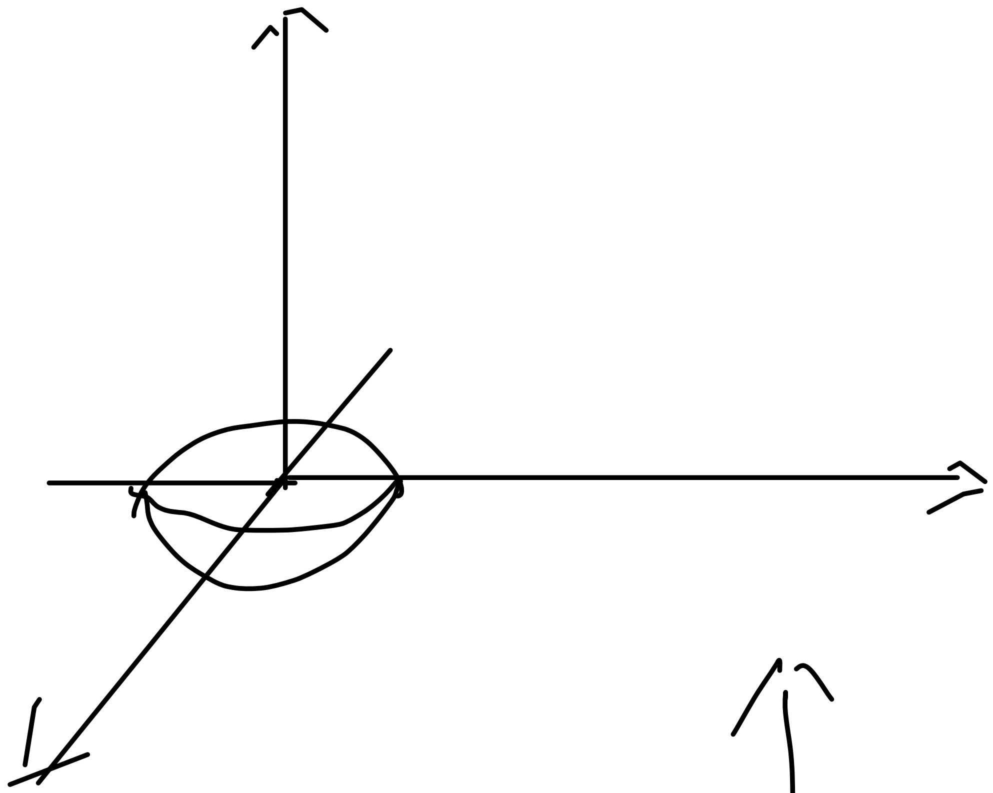
$\cancel{-\frac{1}{4}}$ $-5 \cdot 4$

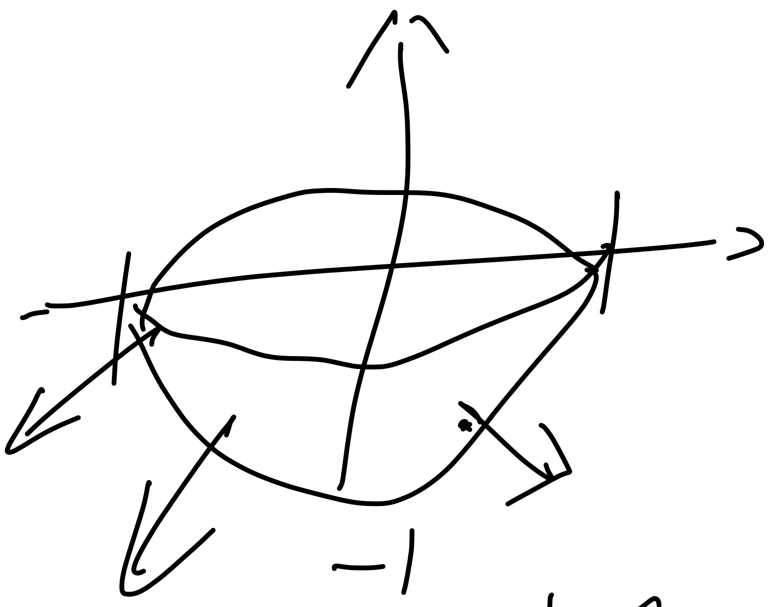
$F \cdot \hat{n}\, dS$

$\langle -x, -5, -1 \rangle \langle 1, 0, 0 \rangle$

$-5 \iint dS$

$= -5 \cdot 16.$





$-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

$-\frac{1}{\sqrt{3}}a - \frac{1}{\sqrt{3}}b \frac{1}{\sqrt{3}}c > 0$

$\langle -1, -1, -1 \rangle$ $\quad -1 < x < 1$

$\iint F \cdot \hat{n} \, dS$ $\quad -1 < y < 1$

2π

πr $\quad \cancel{z < 0}$

$-1 < z < 0$

$\langle a, b, c \rangle \langle x, y, z \rangle$

$-1 < x < 1$

$-a < ax < a$

$x + y + z$

$\cancel{ax > 0}$

$by > 0$

$cz > 0$

$ax + by + cz > 0$

$-a - b - c > 0$

$-a > b + c$

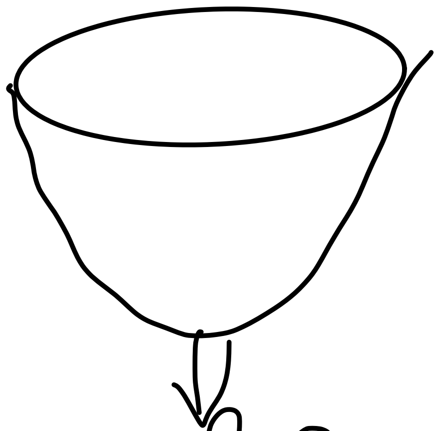
$a < -b - c$

$by > 0 \quad -b < by < b$

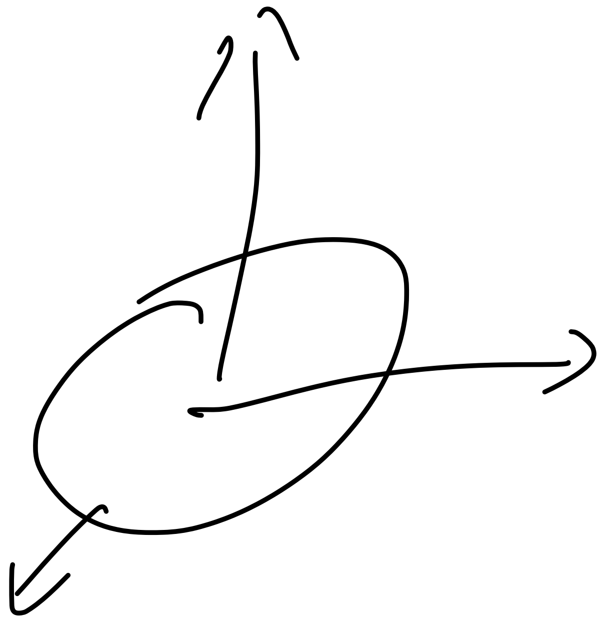
$b = 1$

$by + \cancel{cz} > 0$

$by > -1$

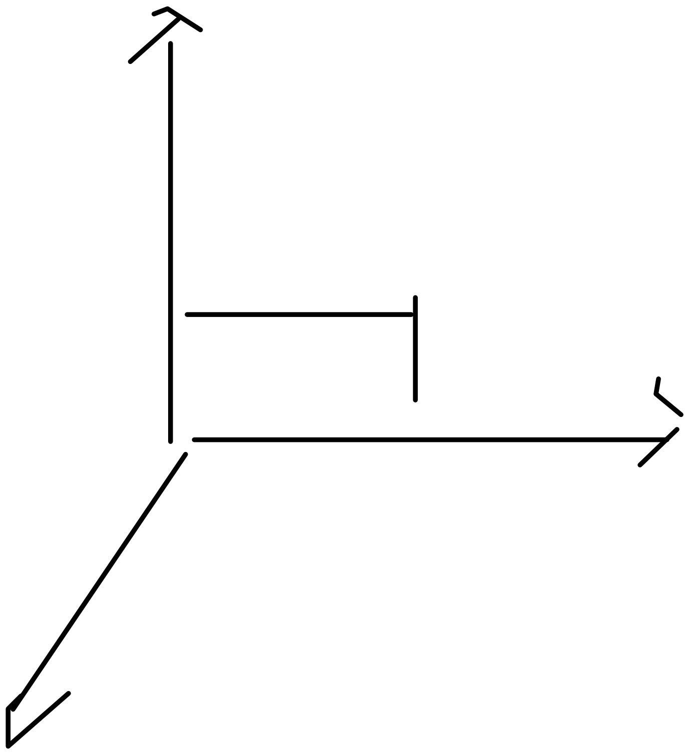


$\langle 0,0,-1 \rangle$



$cz>0$

$-1(z)>0$



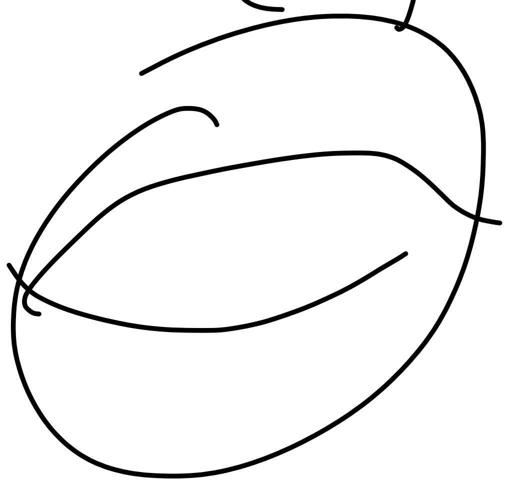
$\nabla \cdot \vec{F} \, dV$

$\vec{F}$

$\iint_D -(x+6) \, dy \, dz$

$\frac{1}{(x^2+y^2+z^2)^{\frac{1}{2}}} \, dS$

$\frac{x^2+y^2+z^2}{(x^2+y^2+z^2)^{3/2}}$



$\iint x^{\frac{1}{2}}+y^{\frac{1}{2}}+z^{\frac{1}{2}} \, dS$

~~dxdy~~

$\vec{F} \cdot \vec{n}$

$\frac{\langle x,y,z \rangle}{(x^2+y^2+z^2)^{3/2}} \cdot \langle x,y,z \rangle$

$x^{2-3/2}$

$\frac{x^2}{x^{3/2}} + y^2$

Made with Goodnotes

$$\nabla \cdot \frac{\langle x, y, z\rangle}{\langle x, y, z\rangle^{3/2}} = \frac{\partial}{\partial x} \frac{x}{(x^2+y^2+z^2)^{3/2}}$$

$$(x^2+y^2+z^2)^{3/2} \qquad = \frac{(x^2+y^2+z^2)^{3/2} - x(3/2)(x^2+y^2+z^2)^{1/2}(2x)}{(x^2+y^2+z^2)^3}$$

$$= \cancel{3x^2(x^2+y^2+z^2)}$$

$$\iiint_E \nabla \cdot F \, dV = \iint_S F \cdot d\vec{S}$$

$$\frac{\partial}{\partial x} \frac{x}{\sqrt{x^2+y^2+z^2}^{\,3}} \qquad \cancel{(x^2+y^2+z^2)}$$

$$\iiint \frac{3(x^2+y^2+z^2)^{3/2} - 3(x^2+y^2+z^2)^2}{(x^2+y^2+z^2)^3} \, dV$$

$$\frac{\partial}{\partial y} \frac{(x^2+y^2+z^2)^{3/2} - y(3/2)}{(x^2+y^2+z^2)^3}$$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{3} \frac{3\rho^{3/2} - \cancel{3\rho^2}}{\cancel{\rho^2}} \qquad 3\rho^{\frac{1}{2}}(1-\rho^{\frac{1}{2}}) \, \rho \sin\varphi \, d\rho \, d\varphi \, d\theta$$

$$3\rho^{\frac{1}{2}}(1-\rho^{\frac{1}{2}})\sin\varphi$$

2π

$$(3\rho^{\frac{1}{2}} - 3\rho)\sin\varphi \, d\rho \, d\varphi \, d\theta$$

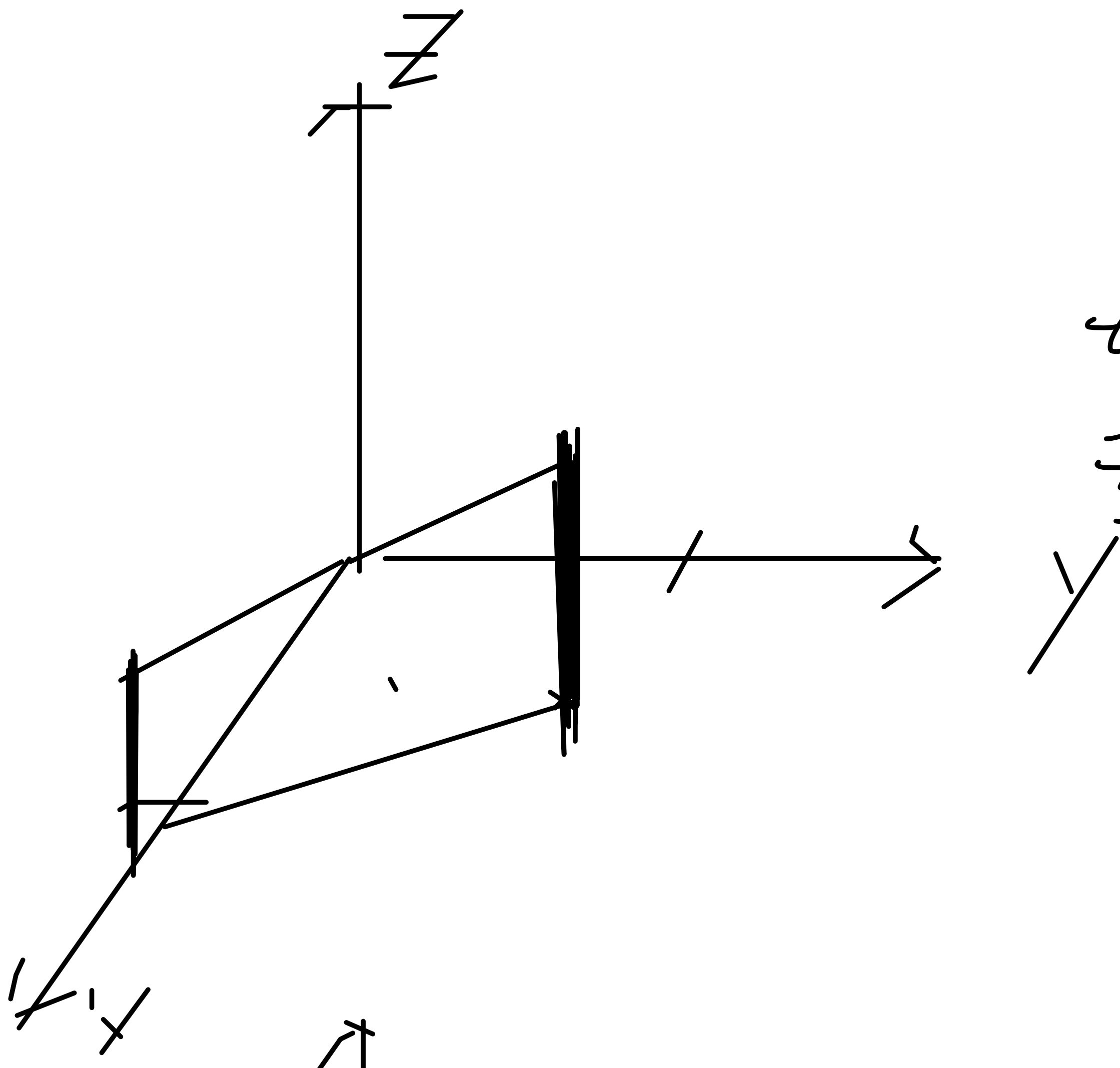
$$\left[2\rho^{3/2} - \frac{3\rho^2}{2}\right]_0^3 \left(-\cos\varphi\right)_0^{\pi}$$

$$\left(2\sqrt{27} - \frac{27}{2}\right)\alpha.$$

$$\iiint \nabla \cdot F \, dV$$

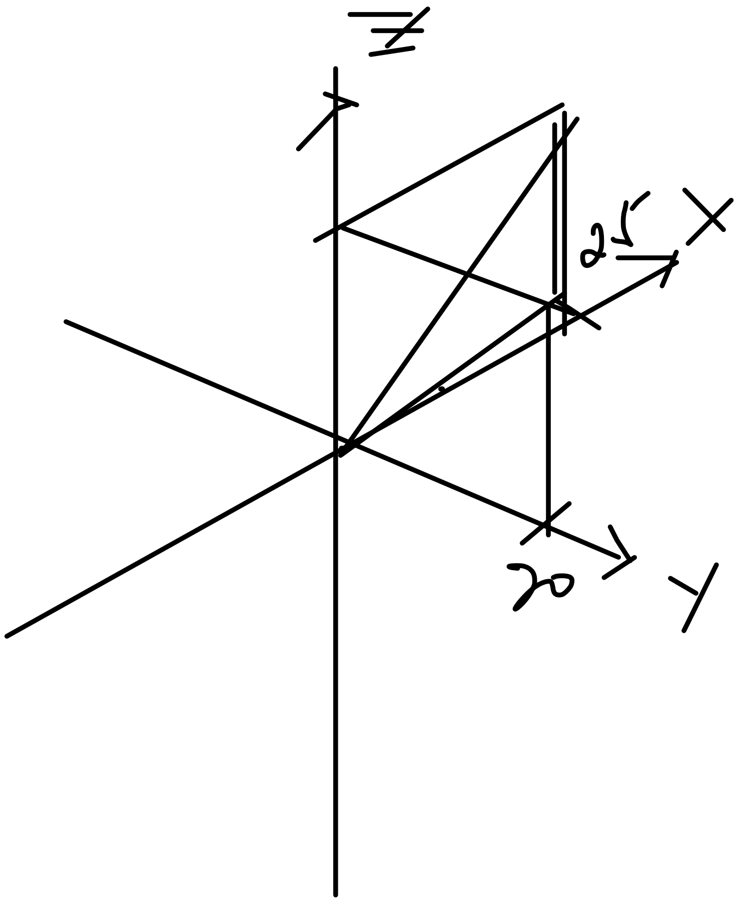
$$\frac{\partial}{\partial x} F + \frac{\partial}{\partial y} F + \frac{\partial}{\partial z} F$$

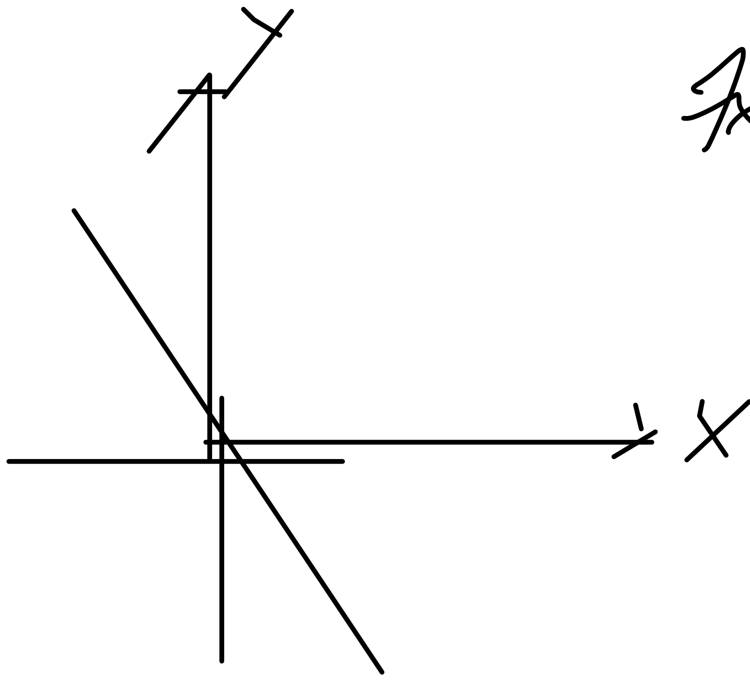
$$\frac{\langle x, y, z \rangle}{\sqrt{x^2+y^2+z^2}^{\,3}}$$



$$z - 5y = 7x$$

$$\frac{z-5y}{7} = x$$





$7x+5y=0$

$5y=-7x$

$y=-\frac{7}{5}x$

$z_x = 7$

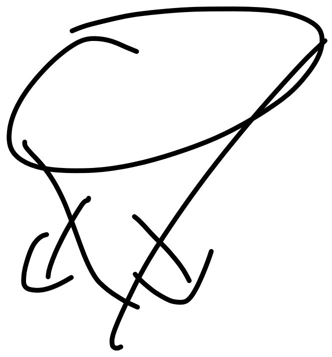
$z_y = 5$

$\langle 7, 5, -1 \rangle$

$\langle -7, -5, 1 \rangle$

$\iint 1 \, dS$

$z = r$



$$\iint \vec{F} \cdot \hat{n}\, dS$$

$$\iint \quad i \quad j \quad k$$

$$r_r = \langle \cos\theta, \sin\theta, 1 \rangle$$

$$r_\theta = \langle -r\sin\theta, r\cos\theta, 0 \rangle$$

$$= \left\langle \frac{r\cos\theta}{\sqrt{2}\,r}, r\sin\theta, \frac{-r}{\sqrt{2}\,r} \right\rangle$$

$$r^2 + r^2$$

$6r^5\cos^2\theta\sin^2\theta$

$$\int_0^{2\pi}\int_0^R 6r^5\cos^2\theta\sin^2\theta \quad r\,dr\,d\theta$$

$$= \quad \frac{r^7}{7} \quad \frac{6R^7}{7}\cos^2\theta\sin^2\theta$$

$$z = \sqrt{x^2+y^2}$$

$$z_x = \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}(2x)$$

$$= \frac{x}{\sqrt{x^2+y^2}} \quad z_y = \frac{y}{\sqrt{x^2+y^2}}$$

$$\left\langle \frac{-x}{\sqrt{x^2+y^2}}, \frac{-y}{\sqrt{x^2+y^2}}, 1 \right\rangle$$

$$\left\langle \frac{-r\cos\theta}{r}, -\frac{r\sin\theta}{r}, 1 \right\rangle$$

$$\int_0^{2\pi}\int_0^{R} 9\, r^6 \sin^2\theta \cos^2\theta \, dr \, d\theta$$

$$= \left(\frac{\sin 2\theta}{2}\right)^2$$

$$\frac{1-\cos 4\theta}{2} r^6 \, dr \, d\theta$$

$$-\frac{9}{4}. \qquad \left[\theta - \frac{\sin 4\theta}{4}\right]_0^{2\pi} \qquad \left(\frac{R^7}{7}\right)$$

$$x = 4$$

$x = 4\cos\theta\sin\varphi$

$y = 4\sin\theta\sin\varphi$

$z = 4\cos\varphi$

$$\int_{\textcircled{a}} F \cdot \underset{(r_\theta \times r_\varphi)}{\hat{n}}\, dS$$

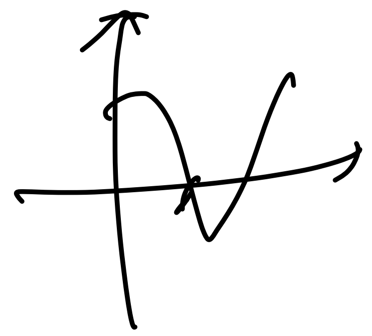
$16\cos^2\varphi \quad \cancel{4\cos\varphi} \quad ; \quad d\varphi\, d\theta \qquad k$

$r_\theta = \langle -4\sin\theta\sin\varphi,\ 4\cos\theta\sin\varphi,\ 0\rangle$

$r_\varphi = \langle 4\cos\theta\cos\varphi,\ 4\sin\theta\cos\varphi,\ -4\sin\varphi\rangle$

$r_\theta \times r_\varphi = \langle -16\cos\theta\sin^2\varphi,\ -16\sin\theta\sin^2\varphi,$

$-16\sin\varphi\cos\varphi \rangle \quad \cancel{X}$



$16\cos^2\varphi\ (-16)\sin\varphi\cos\varphi$

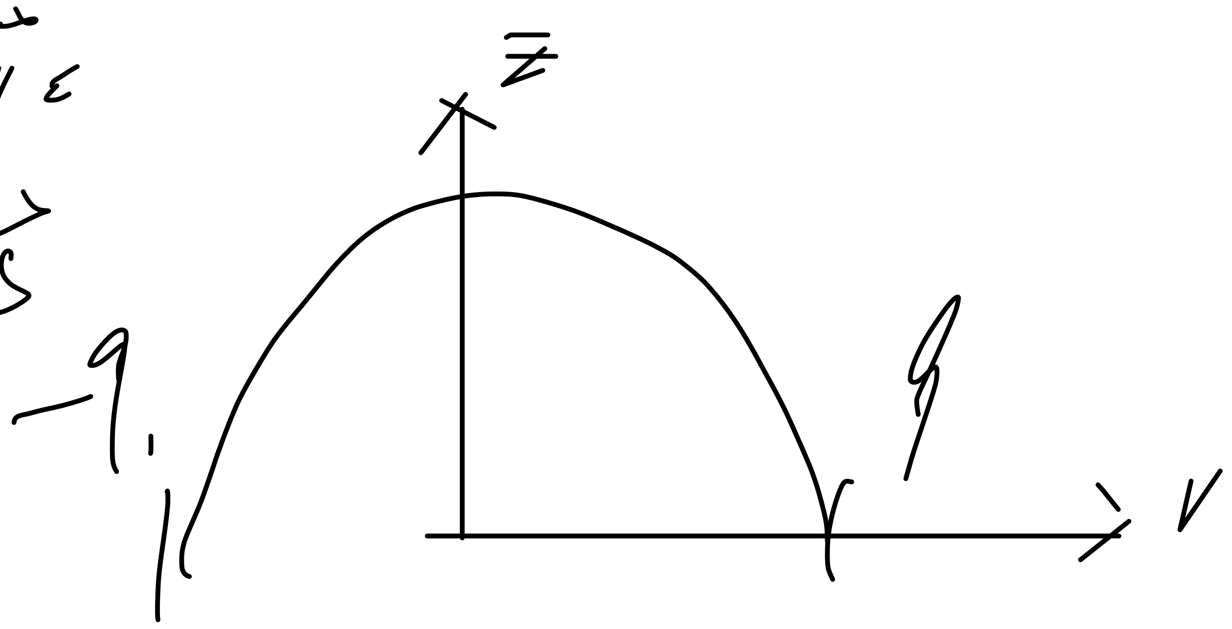
$$\int_0^{\frac{\pi}{2}} 16^2 \sin\varphi\cos^3\varphi\ d\varphi \quad \times 2\pi.$$

let $u = \cos\varphi$, $du = -\sin\varphi$

$= 16^2 \int_0^1 u^3 \qquad u \qquad \left(\frac{1}{4}\right) 16^2 \cdot 2\pi.$

$\int_C \vec{F}\cdot d\vec{r} =$

$\iint_S (\nabla\times\vec{F})\cdot d\vec{S}$



$z_x = -2x$

$z_y = -2y$

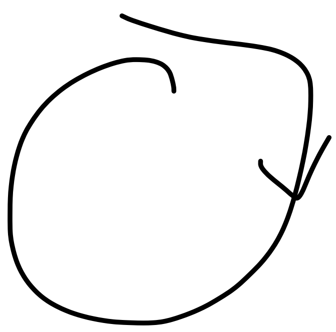
$\langle 2x, 2y, 1\rangle$

$2x^2+2y^2+7x^2+7y^2$

$\iint 9x^2+9y^2\, dx\,dy.$

$\int_0^{2\pi}\int_0^9 9r^2 \; r\,dr\,d\theta$

$\frac{r^4}{4}$ $\left(-\frac{9^3}{3}\cdot 9\cdot 2\pi\right)$



∇. (7).

$-2\pi \cdot 1 \cdot 1^2$

$\iint_S \nabla \times F \, dA$

$= \int_C \vec{F} \cdot d\vec{r}$

$\int_0^{2\pi} -6y(-4\sin t)\,dt +$

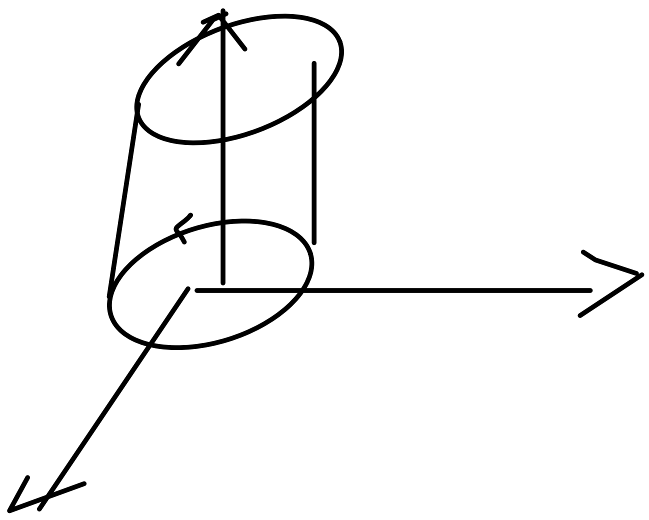
$6x(-4\cos t)\,dt +$

$7z(0)\,dt$

$= 24\int_0^{2\pi} -4\sin^2 t - 4\cos^2 t \, dt$

$24(-4)\ 2\pi.$

T
T
T
F
T
F
F
F.



F.

$\langle x, y, 0 \rangle$ $\quad r\cos\theta, r\sin\theta, 0$

$\cancel{r\cos\theta}$

$x(r\sin\theta,$

$-r^2\cos\theta\sin\theta - r^2\cos\theta\sin\theta$

$$\iint F \cdot \hat{n}\, dS$$

$6x^2y^2z^2$

$$\begin{vmatrix} & i & j & k \\ r_r = & \langle \cos\theta, & \sin\theta, & 1 \rangle \\ r_\theta = & \langle -r\sin\theta, & r\cos\theta, & 0 \rangle \end{vmatrix}$$

$\langle r\cos\theta, -r\sin\theta, r \rangle$

$\langle +r\cos\theta, +r\sin\theta, -r \rangle$

$r\cos\theta$

$F \cdot d\vec{A}$

$-6r$

$$-\frac{9\pi R^7}{7 \cdot 4}$$

$r_s = \langle 2s, 2, 0 \rangle$

$r_t = \langle 0, 2t, 4 \rangle$

$r_s \times r_t = \langle 8, -8s, 4st \rangle$

$8z - 8s(4x)$

$= 8(4t) - 8s(4s^2)$

$= \int_2^4 \int_0^4 32t - 32s^3 \, ds\, dt$

$= \int_2^4 \left(32ts - 8s^4\right)\Big|_0^4 dt$

$= \int_2^4 128t - 2048 \, dt$

$= 128\left[\frac{t^2}{2}\right]_2^4 - 2048(2)$

$= 128\left(\frac{4^2}{2} - \frac{2^2}{2}\right)$

Made with Goodnotes

$$r_s = \langle e^s, 0, 8 \rangle$$
$$r_t = \langle 0, -3\sin(3t), 0 \rangle$$

$$r_s \times r_t = \langle 24\sin(3t), 0, -3e^s\sin(3t) \rangle$$

$$\int_0^{\frac{\pi}{6}} \int_0^4 24\sin(3t)(e^s) \, ds \, dt$$

$$(e^4 - 1)\left(24 \left[-\frac{1}{3}\cos(3t) \right]_0^{\frac{\pi}{6}} \right)$$

$$= (e^4 - 1)\left(24\left(-\frac{1}{3}(0) + \frac{1}{3} \right) \right)$$

$$= (e^4 - 1)(8)$$

$$r_s = \left| \begin{matrix} \langle 2, 2, 2s \rangle \\ r_t = \langle 5, -t, 2t \rangle \end{matrix} \right|$$

$$-20$$

$$= \langle 4t + 10s, 10s - 4t, \cancel{-10} -10 \rangle$$

$$-20t$$

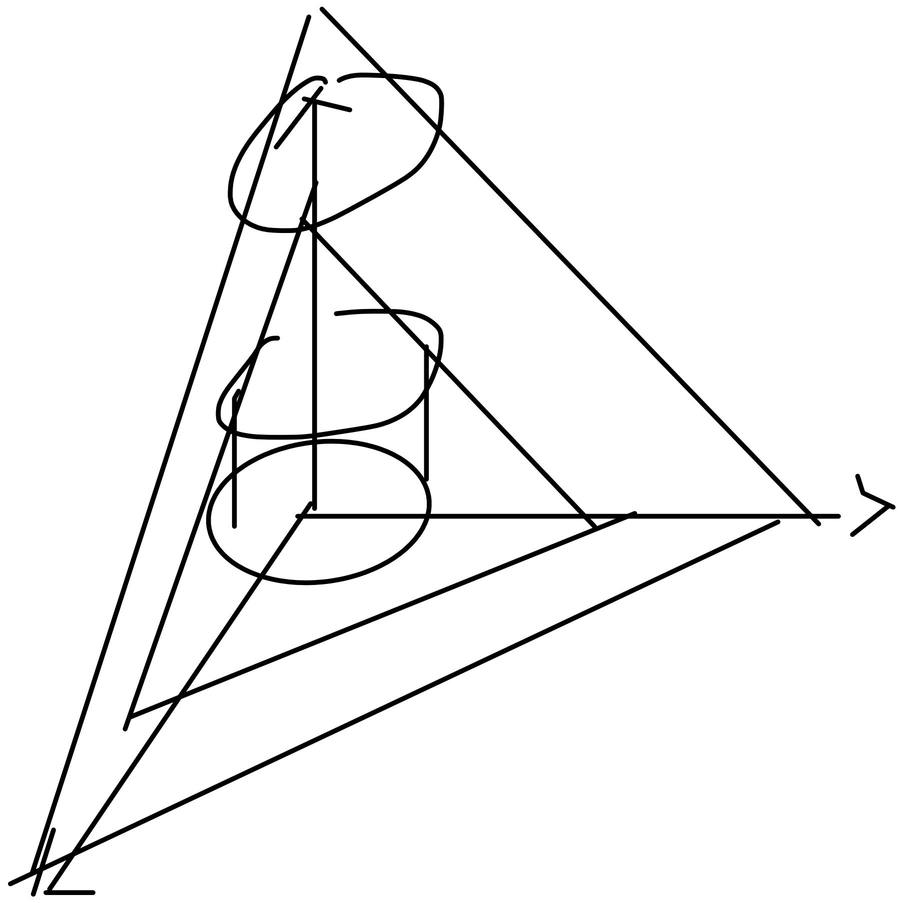
$$\int_0^3 \int_0^1 -20(s^2 + t^2)\, ds\, dt$$

$$-20 \int_0^3 \int_0^1 \frac{s^3}{3} + t^2 s \quad ds \quad dt$$

$$= \int_0^3 \frac{1}{3} + t^2 \, dt$$

$$= \left[\frac{1}{3}t + \frac{t^3}{3} \right]_0^3$$

$$= 1 + 9$$



$z = 1 - x - y$

$3 - r\cos\theta - r\sin\theta$

$z : 3 - x - y$

$F \cdot \hat{n}\, dS$

$$\int_0^{2\pi} \int_0^1 \int_{1-x-y}^{3-x-y} r\, dz\, dr\, d\theta$$

$$\iiint_E \nabla \cdot F \, dV = \iint_S F \cdot dS$$

$$3$$

$$\int_0^{2\pi}\int_0^1\int_{1-x-y}^{3-x-y} 3 \quad r\,dz\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^1 3r(3-x-y-1+x+y)\,dr\,d\theta$$

$$= 6\left[\frac{r^2}{2}\right]_0^1 (2).$$

$$6$$

$$(2\pi)(3)\left[6r\right]_0^1$$

$$\left[3r^2\right]_0^1$$

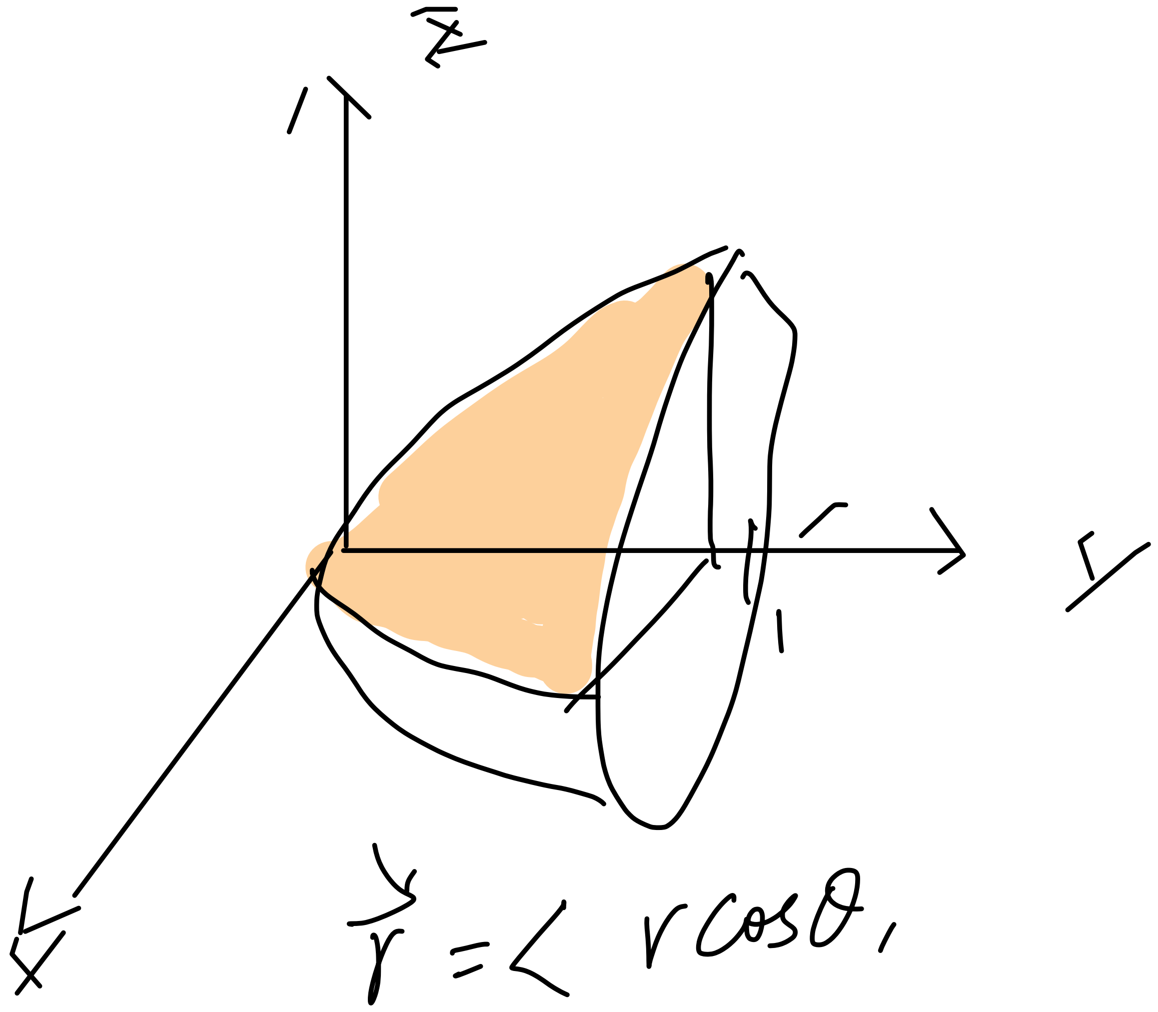
$$\iiint_E 3 \, dV$$

0 0

$$4^2 \times \pi \times 8 \times 3 - \iint$$

$$\iiint_E 6 \, dV$$

$$6 \, (5^2 \pi \times 6)$$



$\vec{r} = \langle r\cos\theta, \quad , r\sin\theta \rangle$

$r_\theta = \langle -r\sin\theta, 0, r\cos\theta \rangle$

$r_r = \langle \cos\theta, 2r, \sin\theta \rangle$

$r_\theta \times r_r = \langle -2r^2\cos\theta, r, -2r^2\sin\theta \rangle$

$\int_0^{\frac{\pi}{4}} \int_0^1 4(x+z)(-2r^2\cos\theta) + 3r + 4z(-2r^2\sin\theta)\, dr\, d\theta$

$4(x+z)(-2r^2\cos\theta) - 8zr^2\sin\theta + 3r\, dr\, d\theta$

$4x(-2r^2\cos\theta) - \cancel{8z}r^2\cos\theta - 8zr^2\sin\theta$

$= 4r\cos\theta(-2r^2\cos\theta) - 8(r^3\sin\theta\cos\theta) - 8(\sin^2\theta\, r^3)$

$-8r^3\cos^2\theta - 8r^3\sin^2\theta - 8r^3\sin\theta\cos\theta + 3r\, dr\, d\theta$

$-8r^3 + 3r - 8r^3\sin\theta\cos\theta$

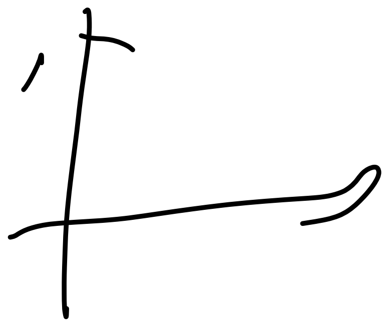
$\frac{-8}{4} + \frac{3}{2} - \frac{8}{4}\sin\theta\cos\theta$

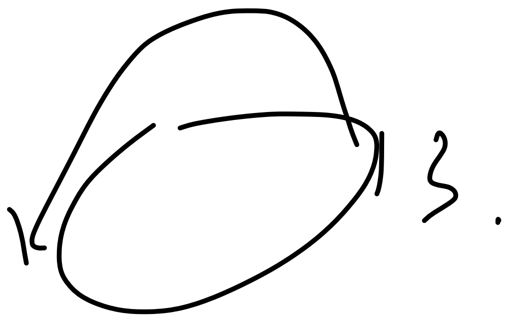
$= \int_0^{\frac{\pi}{4}} -\frac{1}{2} - 2\sin\theta\cos\theta\, d\theta$

$= \int_0^{\frac{\pi}{4}} \frac{1}{2} - 2\sin\theta\cos\theta \, d\theta$

$\frac{1}{2}\left(\frac{\pi}{4}\right) - 2\int_0^{\frac{\pi}{4}} \frac{\sin\theta\cos\theta}{4}$

$\frac{\pi}{8} - 1 \int_0^{1} du$





$$\iiint_E 2 \, dV = \underbrace{\iint_{S_{\ominus}} + \iint_{\ominus}}_{Q}.$$

$$2\left(\frac{4}{3}\right)(3^3)\left(\frac{\pi}{2}\right) \iint 2zC$$

$$\vec{r}(u,v) = \langle u\cos v,\ u\sin v,\ 0 \rangle$$

$$\begin{vmatrix} r_u = \langle \cos v, & \sin v, & 0 \rangle \\ r_v = \langle -u\sin v, & u\cos v, & 0 \rangle \end{vmatrix}$$

$$\langle 0,\ 0,\ u \rangle$$

$z_x = 0$

$z_y = -1$

$\langle 0, -1, 1 \rangle$

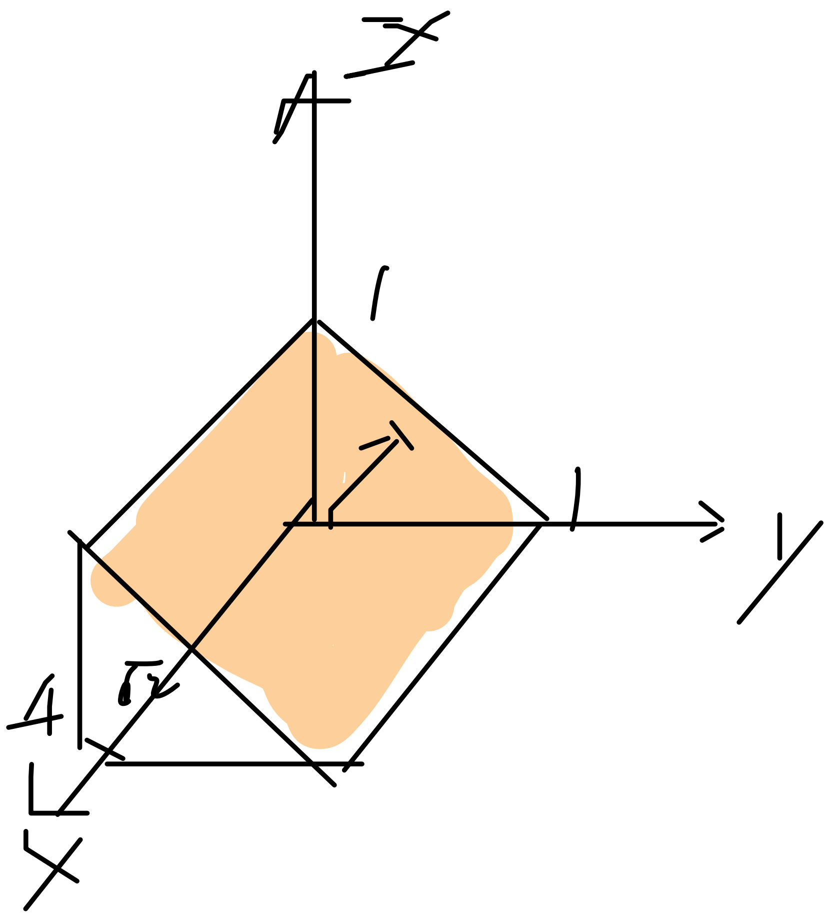
$\langle 0, 4x, 5y \rangle$

$\langle \cancel{0, 4, 5} \rangle$

$z_y = 1.$

$-4 + 5$

$\iint 1 \, dS$



$\int_0^1 \int_0^4 4x + 5y \, dx\, dy$

$= \left[4\left(\frac{x^2}{2}\right) + 5xy \right]_0^4$

$= \cancel{-4\left(\frac{16}{2}\right)} \; -32 + 20y$

$-32 + 20y$

$-32 + 10$

$\int_0^1 \left[-4\left(\frac{x^2}{2}\right) \right]_0^4 + 20y \; dy$

$= -4(8) \quad \int_0^1 -32 + 20y \, dy$

$-32 + 20\left(\frac{y^2}{2}\right) \quad -32 + 10$

$$\iiint_E 5z \cdot dV \, . - \iint_O \vec{F} \cdot d\vec{S}$$

$$5\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^{5} \rho\cos\varphi \; \rho^2\sin\varphi \; d\rho\, d\varphi\, d\theta$$

$$\rho^3 \cos\varphi \sin\varphi$$

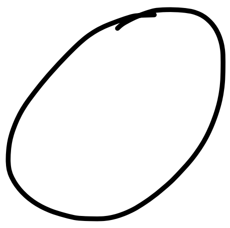
$$(5)(2\pi) \int_0^{\frac{\pi}{2}} \frac{5^4}{4} \cos\varphi \sin\varphi \, d\varphi$$

$$\frac{5^5}{4}(2\pi) \qquad \text{of } u = \sin\varphi$$

$$\int_0^1 u \, du$$

$$\frac{5^5}{4}(2\pi)\left(\frac{1}{2}\right) 5^5 \pi \quad - \iint_O$$

Made with Goodnotes



$$\vec{r}(u,v) = \langle u\cos v,\ u\sin v,\ 0\rangle$$

$$\vec{r}_u = \langle \cos v,\ \sin v,\ 0\rangle$$

$$r_v = \langle -u\sin v,\ u\cos v,\ 0\rangle$$

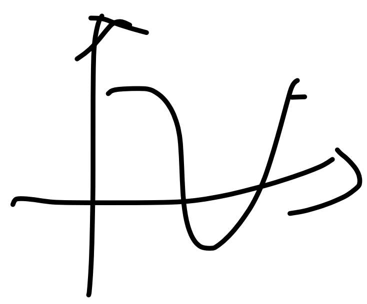
$$r_u \times r_v = \langle 0, 0, u\rangle$$

$$\int_0^{2\pi}\int_0^{5} 3yu$$

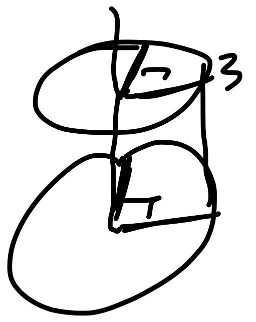
$$3u^2\sin v \, du\, dv$$

$$= \int_0^{2\pi} 5^3 \sin v \, dv$$

$$= 5^3 \left[-\cos v\right]_0^{2\pi}$$



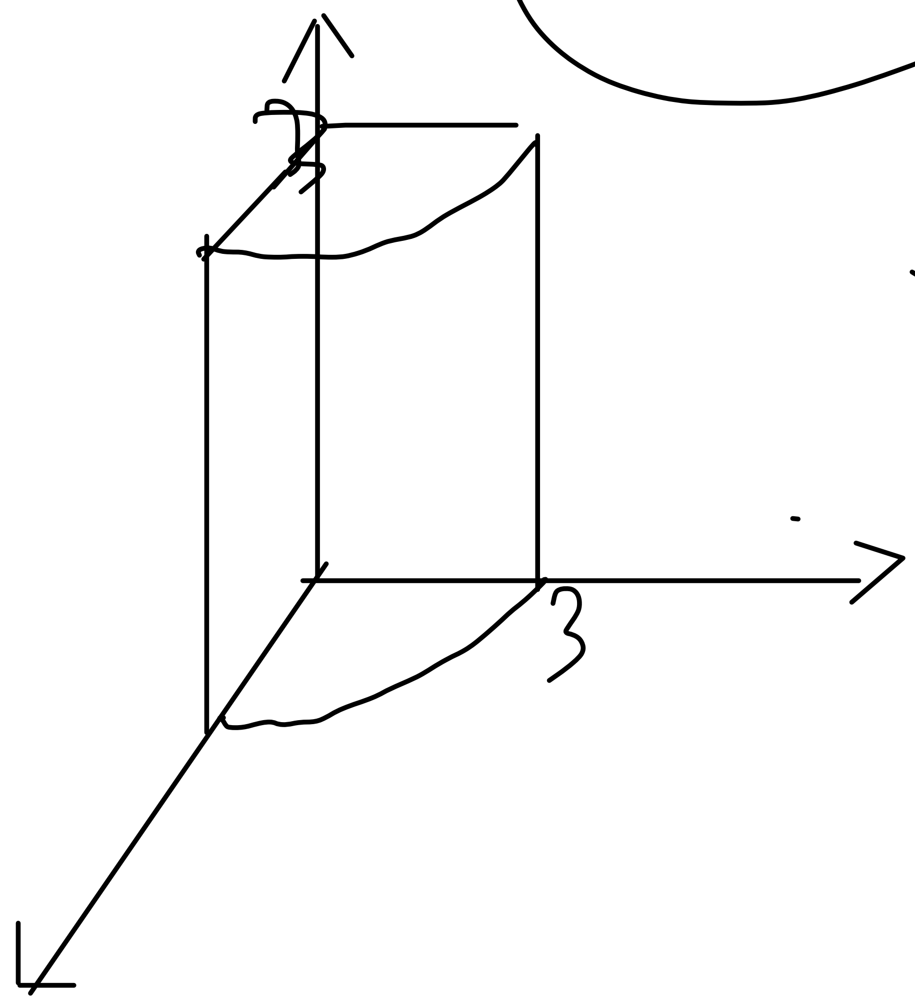
$$= 5^3\left(-1 - (-1)\right)$$



$$\iiint \quad 2+2+1$$

$$9. \quad \frac{3^2 \pi \times 3}{4}.$$

$x=0. \quad y=0$



$27 - 27.$

$$\iint -2x \, dS$$

$$\iint F \cdot \hat{n}$$

$\cancel{\langle 0, 0, 1 \rangle}$

$\langle -1, 0, 0 \rangle$

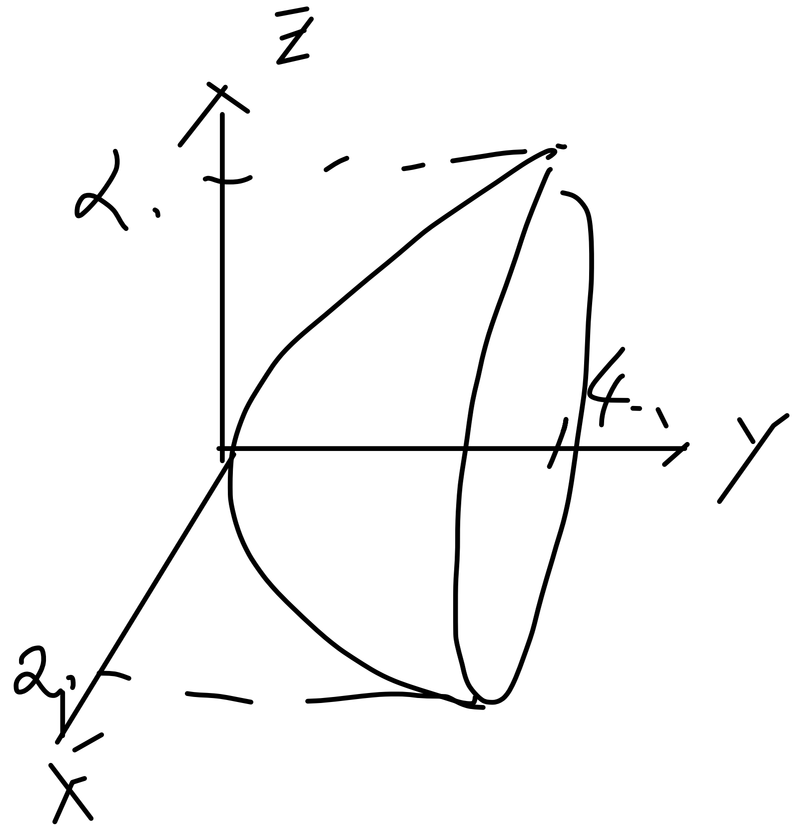
$\iint_{\square} -2x\,dS$

$F \cdot \langle 0, 1, 0 \rangle$

$\iint_S F \cdot$

$1\,dS$

$\langle r\cos\theta,\ r^2,\ r\sin\theta \rangle$ α.



$y \le 4$.

$\iiint_E (-4x)\,dV = \underbrace{\iint_S 0}_{0} + \iint_D$

$\int_0^{2\pi}\int_0^2\int_0^4 (-4x)\,dy\,r\,dr\,d\theta - \iint_D$

$\pi(4)^2$

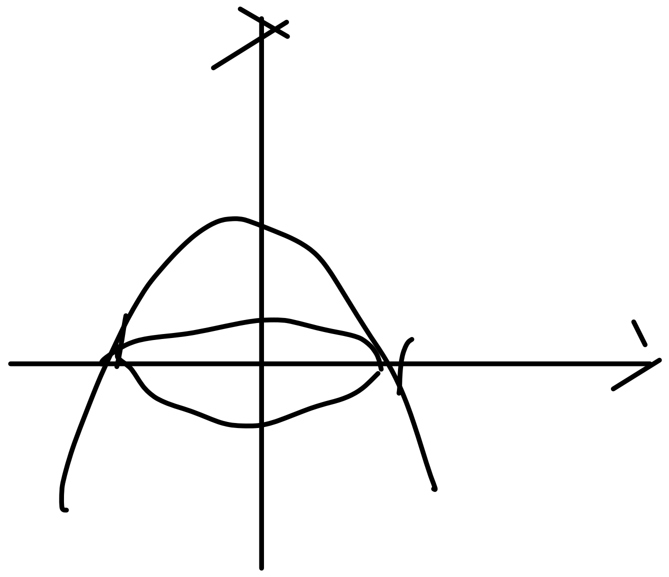
$= \int_0^{2\pi}\int_0^2\int_0^4 -4r^2\cos\theta\,dy\,dr\,d\theta$

$= \int_0^2 -16r^2\cos\theta\,dr$

$\int_0^{2\pi} -16\left(\frac{2^3}{3}\right)\cos\theta\,d\theta$

$[\sin\theta]_0^{2\pi}$

$=$

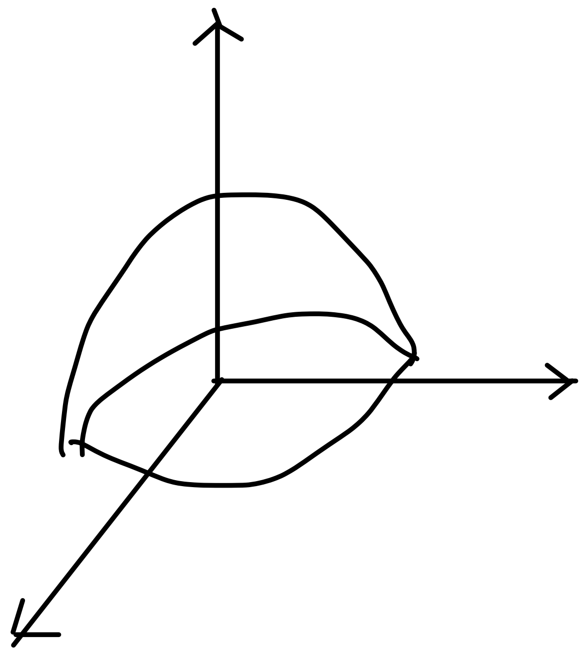


$\vec{F}\cdot$

$$\iiint_E 4\,dV + \iint_{S(0)} (0)\,dS$$

$$\vec{F}\cdot\vec{n}\,dS$$

$$0\,dS$$



$\Downarrow$

$$d\sin x = \cos x$$

$$d\cos x = -\sin x$$

$-\cot$.

$$\frac{4(6^3)(\frac{4}{3})\pi}{2}$$

2.

$$\iint \vec{F} \cdot dS.$$

$z_x = -2x$

$z_y = -2y$

$\vec{n} \langle 2x, 2y, 1 \rangle$

$\vec{F} \langle 2x, 2y, 0 \rangle$

$$\iint 4x^2 + 4y^2 \, dS$$

$$\int_0^{2\pi} \int_0^6 (4r^2) \quad r\,dr\,d\theta$$

$$= \quad \cancel{4} \quad \cancel{4} \quad (6^4) \cdot 2\pi.$$

$$\frac{\frac{4}{3}(6)^3 \, 4}{2}$$

$$\iint s\sqrt{1+u^2}\quad \sqrt{1+u^2}\,dA.$$

$$\left|\begin{matrix} r_u = \langle \cos v,\ \sin v,\ 0\rangle \\ r_v = \langle -u\sin v,\ u\cos v,\ 1\rangle \end{matrix}\right|$$

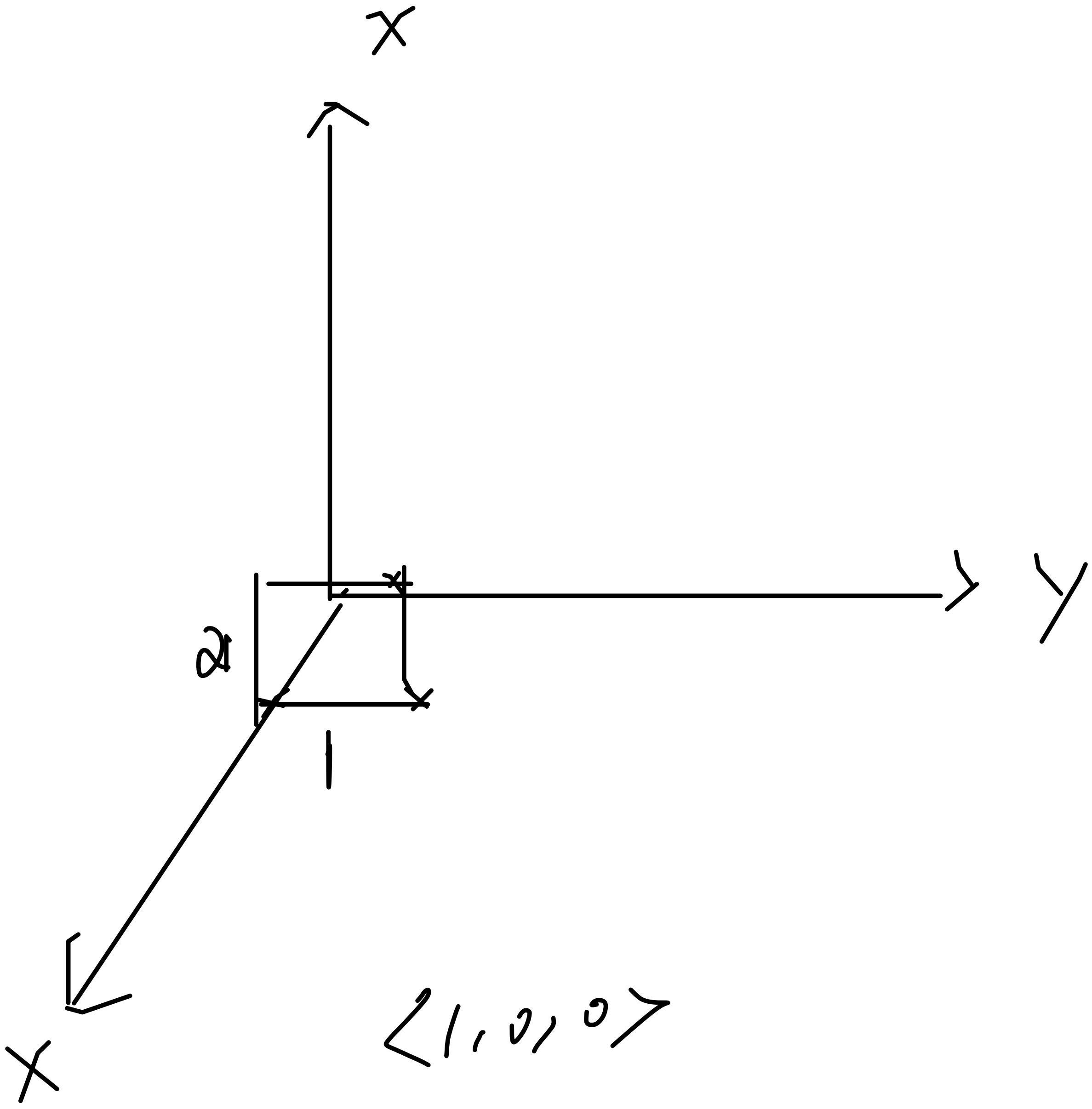
$$|\langle \sin v,\ -\cos v,\ u\rangle|$$

$$= \sqrt{1+u^2}$$

$$= \int_0^{5\pi}\int_0^{5} 1+u^2\ du\,dv$$

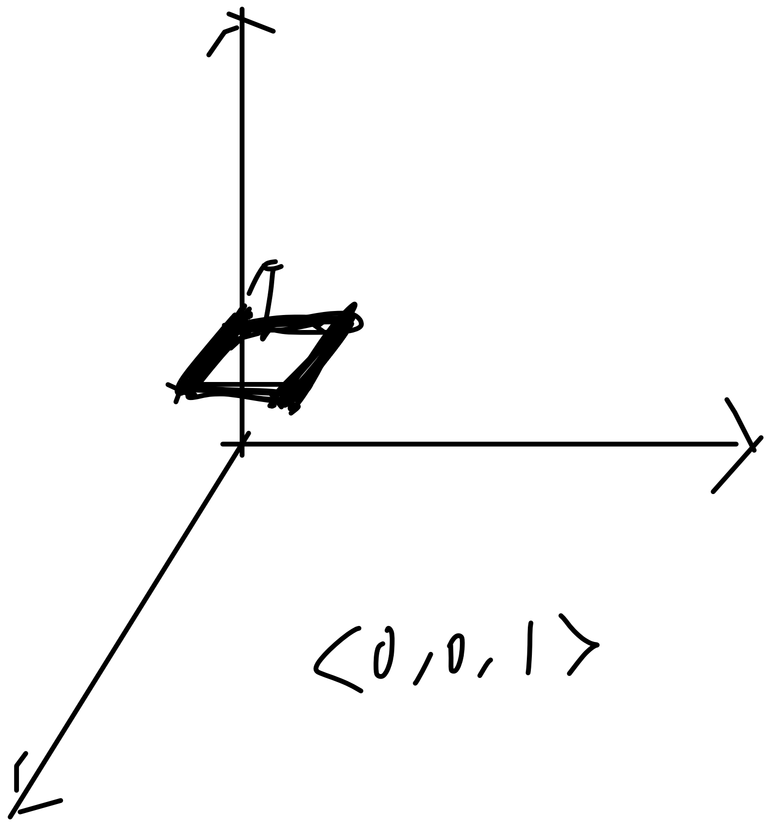
$$= (5\pi)\left(\left[u+\frac{u^3}{3}\right]_0^5\right)$$

$$= 5\pi\left(5+\frac{5^3}{3}\right)$$



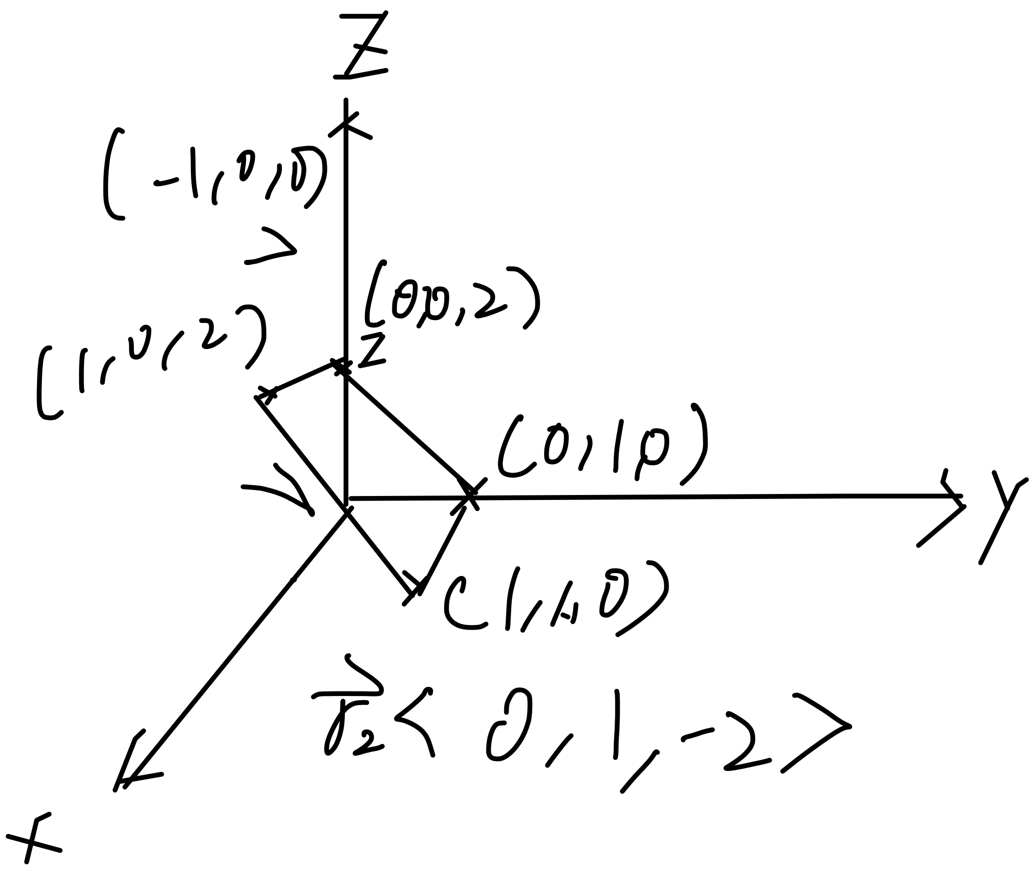
$\langle 1, 0, 0 \rangle$

$\iint 2 \, dS.$



$\langle 0,0,1\rangle$

∂ -



$\vec{r}_2 \langle 0, 1, -2 \rangle$

$$r_1 \times r_2 = \begin{vmatrix} i & j & k \\ -1 & 0 & 0 \\ 0 & 1 & -2 \end{vmatrix}$$

$$= \langle 0, -2, -1 \rangle$$

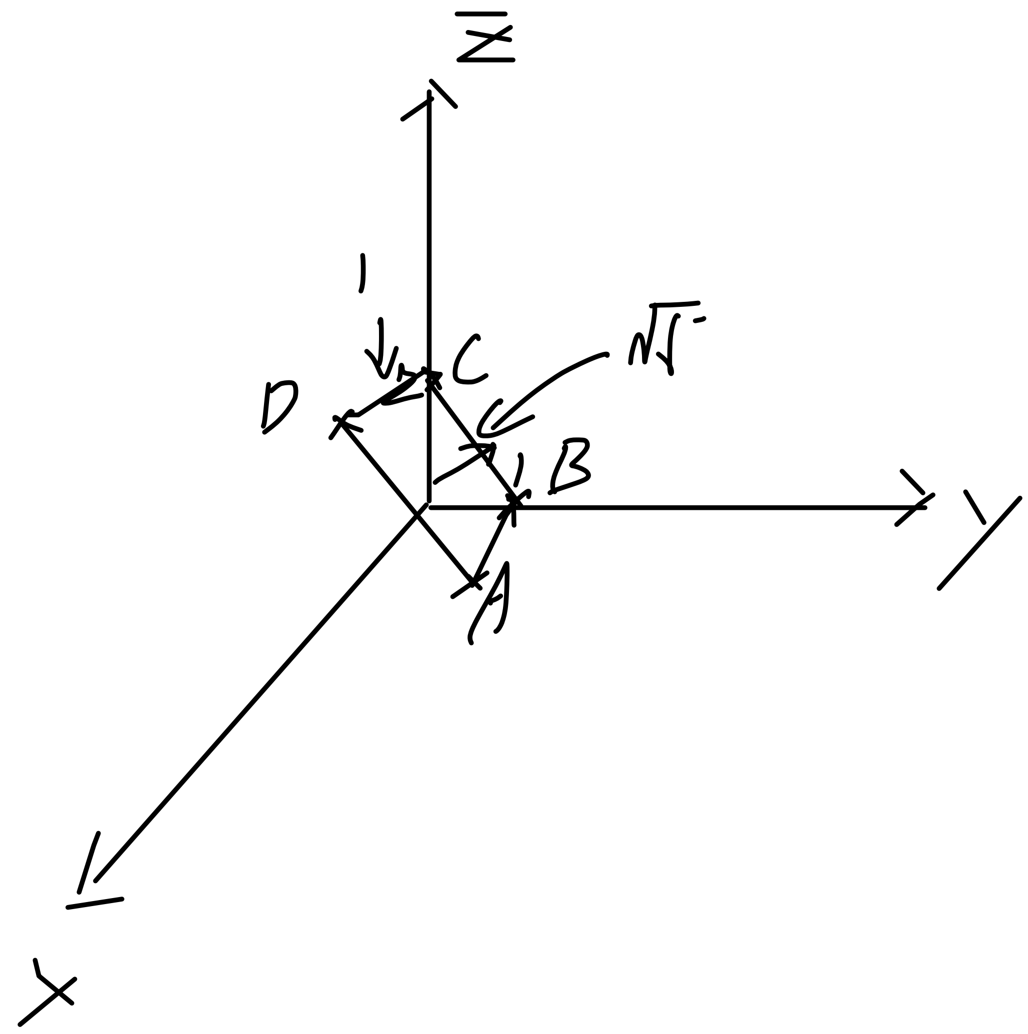
$$\langle 0, 2, 1 \rangle$$

$$\iint F \cdot \hat{n} \, dS$$

$$= \quad 5(-2) - 2$$

$$\iint -12 \, dS$$

$$12\sqrt{5}$$

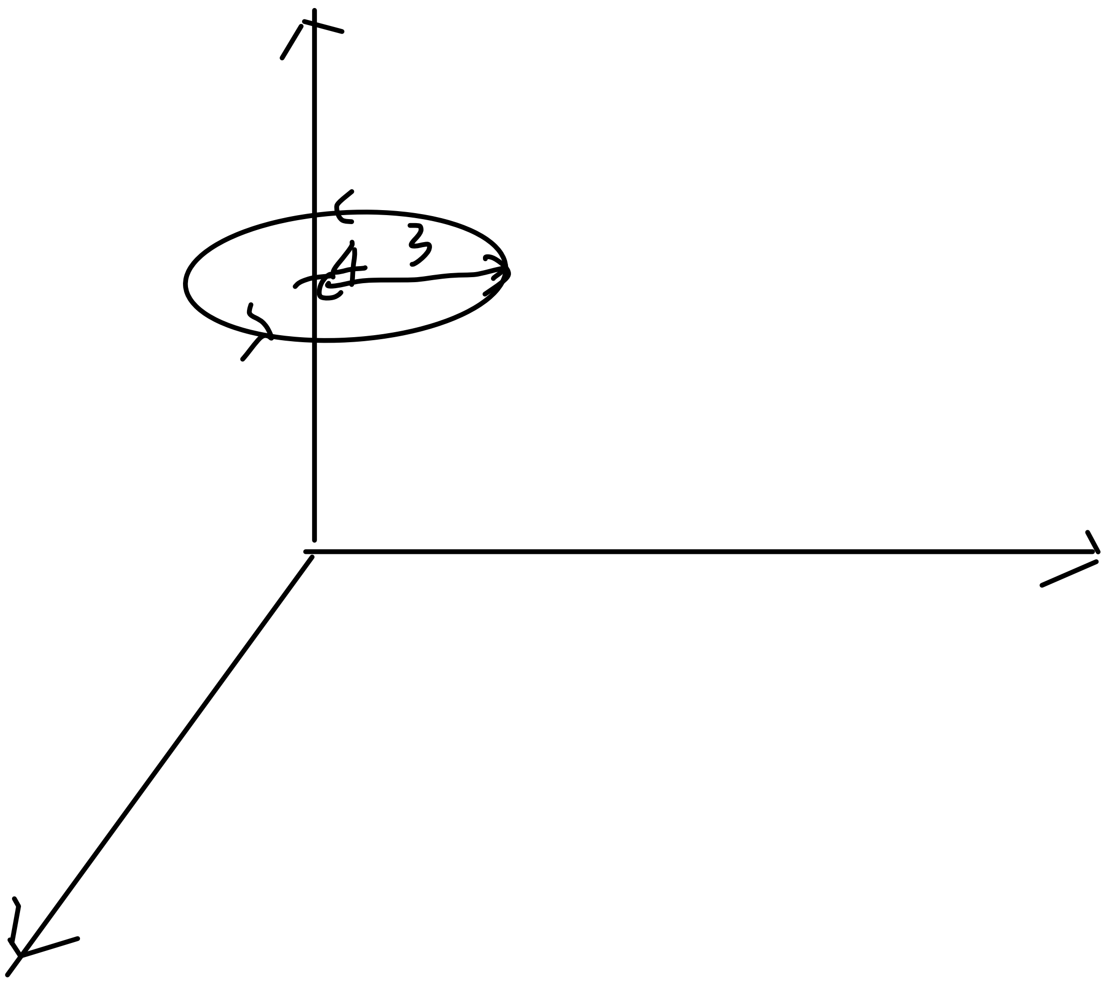


A $(1,1,0)$

B $(0,1,0)$

C $(0,0,2)$

D $(1,0,2)$



$$\iint \nabla \times F \cdot d\vec{S} = \oint_C F \cdot d\vec{r}$$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 7xz & 2x+5yz & 6x^2 \end{vmatrix}$$

$$\langle -5y,\ 7x-12x,\ 2 \rangle$$

$$\langle -5y,\ -5x,\ 2 \rangle$$

$\langle 0, 0, 1 \rangle$

$\iint 2 \, dS$

$\vec{r} = \langle r\cos\theta, r\sin\theta, 4 \rangle$

$$\begin{vmatrix} i & j & k \\ \vec{r}_r = \langle \cos\theta, & \sin\theta, & 0 \rangle \\ \vec{r}_\theta = \langle -r\sin\theta, & r\cos\theta, & 0 \rangle \end{vmatrix}$$

$\vec{r}_r \times \vec{r}_\theta = \langle 0, 0, r \rangle$

$$\int_0^{2\pi} \int_0^3 2 \quad r \, dr \, d\theta$$

$$= \quad [r^2]_0^3$$

9

$$\nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & x^2+z^2 & 2 \end{vmatrix}$$

$$\langle -2z, 0, 2x \rangle$$

$$\vec{n} = \langle 0, 0, 1 \rangle$$

$$\iint 2x \, dS$$

$$\int_0^{2\pi}\int_0^1 2r\cos\theta \quad r\,dr\,d\theta$$

$$= \int_0^{2\pi} \left[\frac{2r^3}{3} \cos\theta \right]_0^1 d\theta$$

$$= \int_0^{2\pi} \frac{2}{3} \cos\theta \, d\theta$$

$$= [\sin\theta]$$

$\nabla \times F = \langle 0, x^2+z^2, 2 \rangle$

$\iint 2 \, dS$

$2 \int \int_0 r \, dr \, d\theta$

$\langle 0, 1, 0 \rangle$

$\iint x^2+z^2 \, dS$

$\vec{r} \langle r\cos\theta, 0, r\sin\theta \rangle$

$\vec{r_r} = \langle \cos\theta, 0, \sin\theta \rangle$

$\vec{r_\theta} = \langle -r\sin\theta, 0, r\cos\theta \rangle$

$r_r \times r_\theta = \langle 0, r, 0 \rangle$

$\int_0^{2\pi} \int_0^1 (r^2) \, r \, dr \, d\theta$

$= \frac{1}{4} \cdot 2\pi.$

$$\iint \nabla \times F \cdot d\vec{S}$$

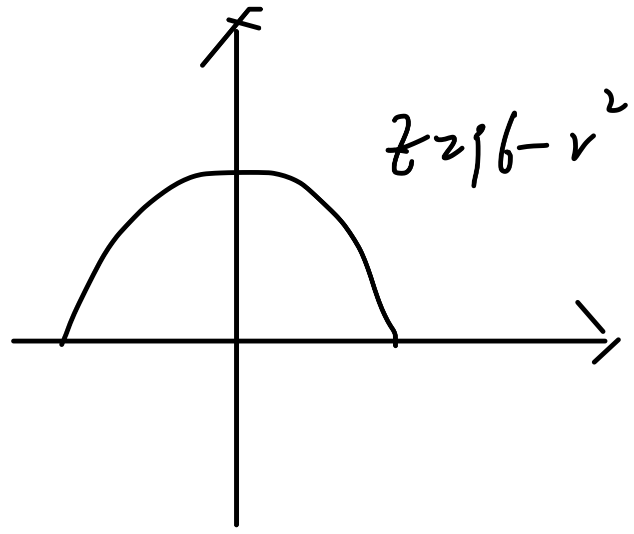
$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2y-5z & 2x+5z & xy \end{vmatrix}$$

$$\langle x-5,\ 5-y,\ 0 \rangle$$

$$\vec{n} = \langle 0,0,1 \rangle$$

$$\iint 0$$

$\langle 4\cos t \quad , 4\sin t \quad , 0 \rangle$



$$\int_0^{2\pi} 7y(-4\sin t)\,dt \; +$$

$$\int_0^{2\pi} -7x(4\cos t)\,dt$$

$$= \quad -28(4)(\sin^2 t + \cos^2 t)$$

$F = \langle 7y, -7x, 0 \rangle$

$\nabla \times F =$
$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 7y & -7x & 0 \end{vmatrix}$$

$\langle 0, 0, -7-7 \rangle$

$\langle 0, 0, -14 \rangle$

~~$r(t) =$~~

$f_x = \langle$

$\langle 2x, 2y, 1 \rangle$

$4x^2 + 4y^2 + 1$

$r\,dr\,d\theta$

$-14 \quad \left[\frac{r^2}{2}\right]_0^4 \qquad \theta \ldots 2\pi.$

when $t=0$, $\vec{r}(0) = (2,7,0)$

$t=1$, $\vec{r}(1) = (0,0,0)$

$2-2t$, $7-7t$

$$\vec{F} = \langle 4x-4y+5z,\ 4x-4y+5z,\ 0 \rangle$$

$$\vec{r}(t) = \langle 2-2t,\ 7-7t,\ 0 \rangle$$

$\cancel{\vec{F_3}\cdot d\vec{r}}$

$$\int_{C_3} \vec{F}\cdot d\vec{r} = \int_0^1 (4x-4y+5z)(-2)\,dt$$

$$+ \int_0^1 (4x-4y+5z)(-7)\,dt$$

$$\int_0^1 -9\left(4(2-2t) - 4(7-7t)\right)dt$$

$$= -9(4)\int_0^1 -5+5t\,dt$$

$$= \cancel{-9(4)}\ -36\left(-5+\frac{5}{2}\right)$$

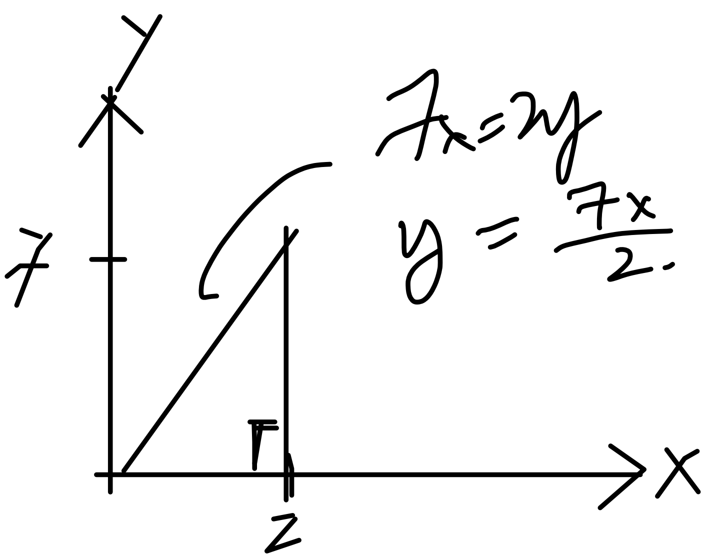
$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 4x-4y+\sqrt{z} & 4x-4y+\sqrt{z} & 0 \end{vmatrix}$$

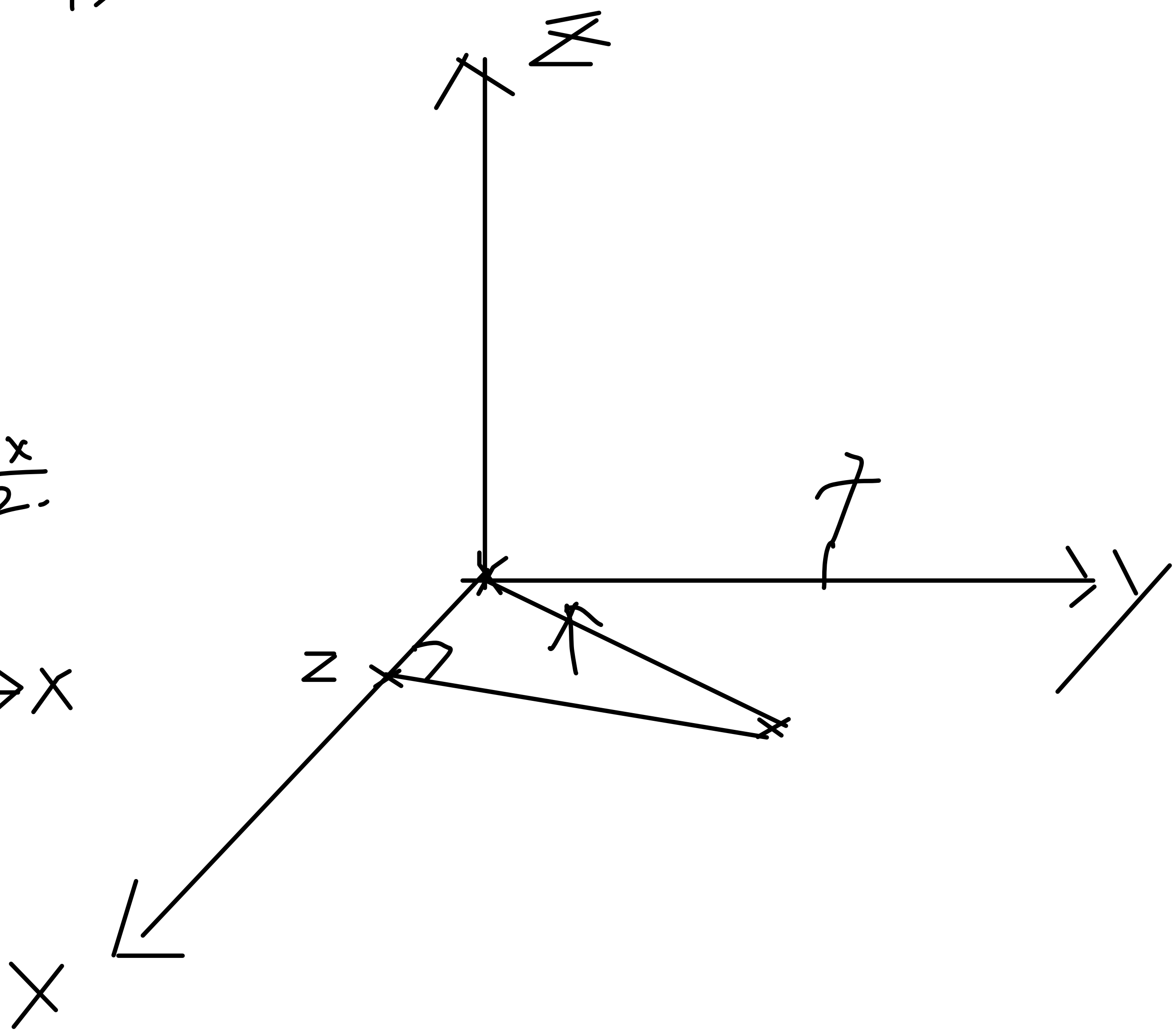
$\langle -5, 5, 4+4 \rangle$

$\langle -5, 5, 8 \rangle$

$7x = 2y$

$y = \frac{7x}{2}$





$$\int_0^{x} \int_0^{\frac{7x}{2}} 8 \, dy \, dx$$

$$\int_0^2 8\left(\frac{7x}{2}\right) dx$$

$$\int_0^2 28x \, dx$$

$$14[x^2]_0^2$$

$$14 \cdot 4$$

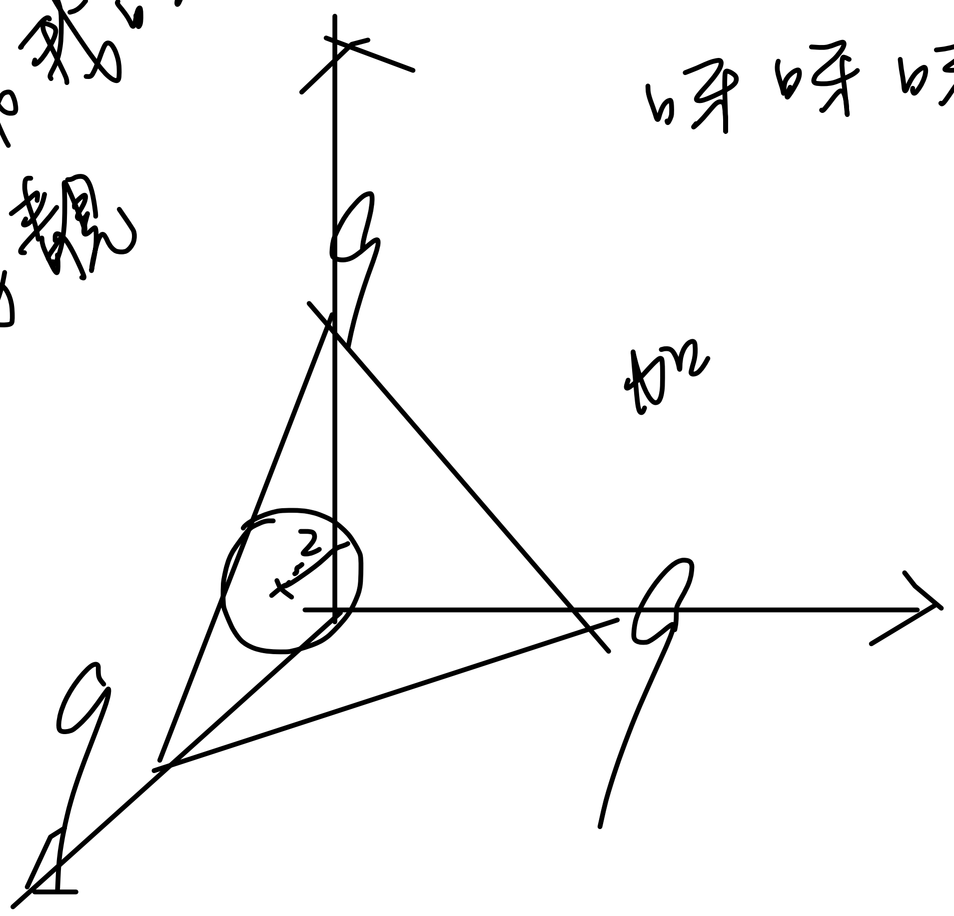
$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & -3z & 5y \end{vmatrix}$$

$$= \langle 8, 0, 0 \rangle$$

數学好难
人生好难
呼呼呼呼

$S(u,v) = \langle$

救命我的字
好靚



加2

好煩呀

MATH
MENTAL
ABUSE
TO
HUMAN

唔明呀 唔明呀
我到底計緊咩

$z = 9 - x - y$

$\langle 1, 1, 1 \rangle \cdot \langle 8, 0, 0 \rangle$

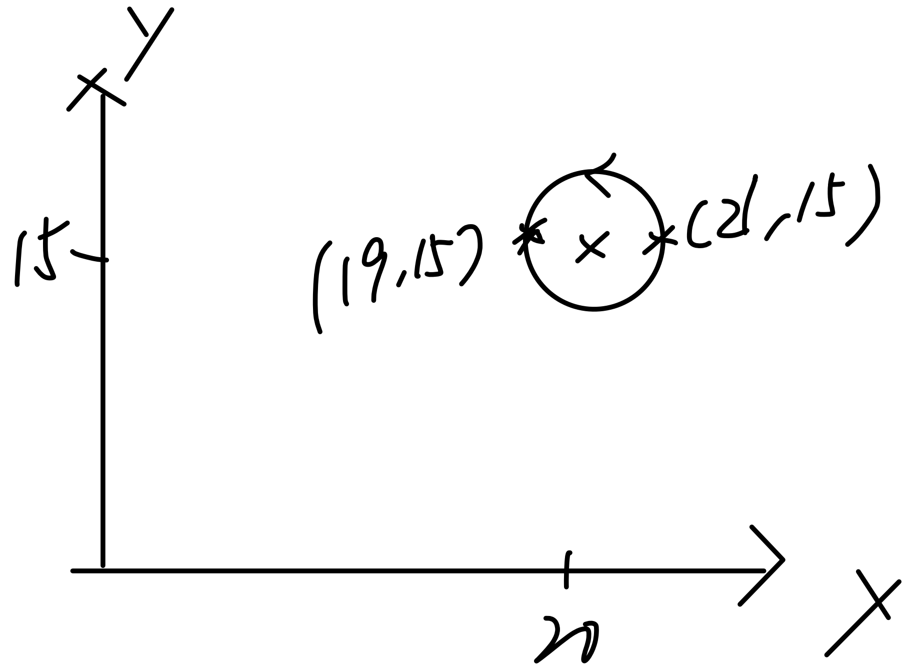
$\iint 8 \, dA$

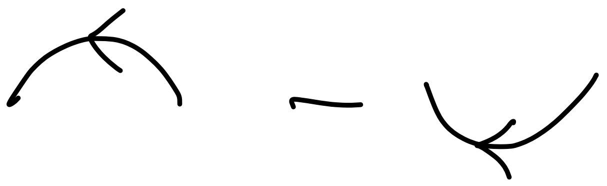
$8 \cdot \pi (2)^2$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 4z+4x^3 & 6y+6z+6\sin(y^3) & 4x+6y+6e^{z^3} \end{vmatrix}$$

$$\langle 6-6,\ 4-4,\ 0-0 \rangle$$

$$\iint \nabla \times F \, dS.$$





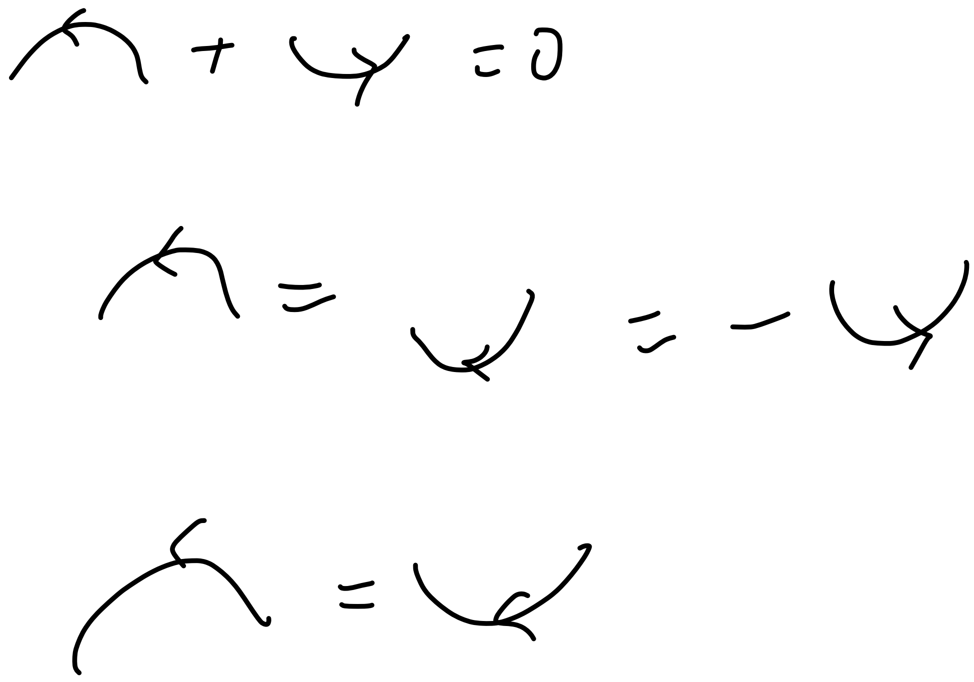
$= 0$

$\langle \cos t, \sin t \rangle \quad 0 < t < \pi$

$$\int_0^{\pi} 4z + 4x^3 (-\sin t)\, dt +$$

$$6y + 6z + 6\sin(y^3) \cos t \, dt$$

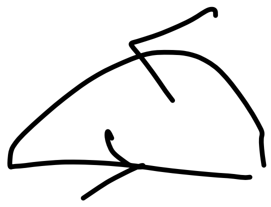
$\vec{r}(19,15) - \vec{r}(21,15) =$



$$\iint \nabla \times F \cdot dS$$

$\vec{r}(t) = \langle 19+2t, 0, 0 \rangle,$
$0 \le t \le 1.$

$\vec{r}'(t) = \langle 2, 0, 0 \rangle$



$(4t + 4x^3)(2)\,dt$

$$\int_0^1 4(19+2t)^3 (2)\,dt$$

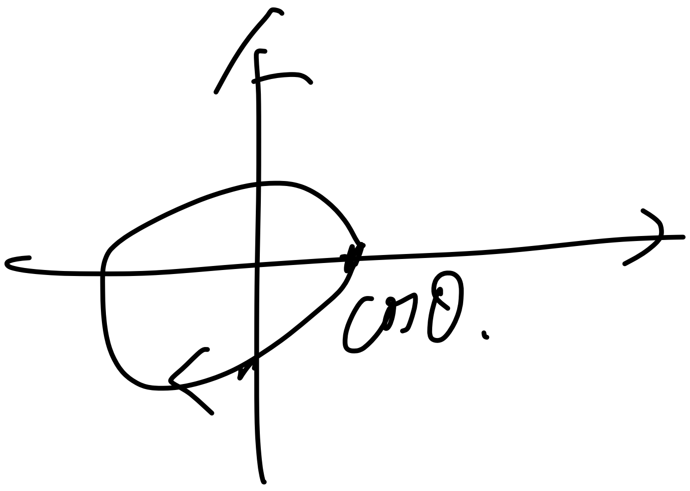
$$= 8\int_0^1 (19+2t)^3\,dt$$

Let $u = 19+2t$, $du = 2dt$, $dt = \frac{1}{2}du$

$21^4 - 19^4$

$$4\int_{19}^{21} u^3\,du$$

$$\left[\frac{u^4}{4}\right]_{19}^{21} \quad 4\left(\frac{21^4}{4} - \frac{19^4}{4}\right)$$



$\vec{r} = \langle 3\cos t, -3\sin t, 5\rangle$

$0 \le t \le 2\pi.$

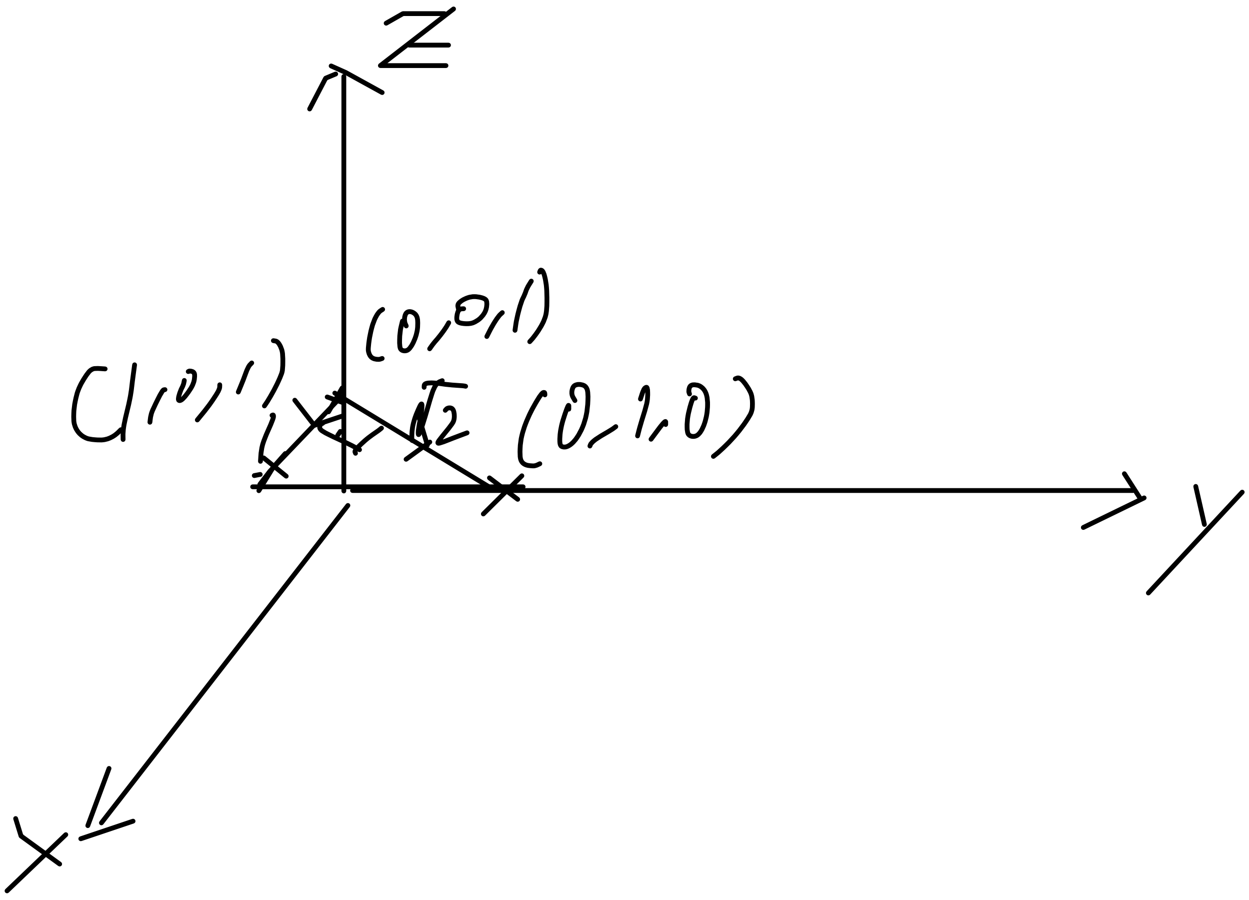
$$\iint_S \nabla\times F\cdot d\vec{A} = \int_C \vec{F}\cdot d\vec{r}$$

$$\int_0^{2\pi} -6y(-3\sin t)\,dt + 6x(-3\cos t)\,dt + \cancel{z(0)\,d\vec{t}}$$

$$= \int_0^{2\pi} 18(-3\sin^2 t - 3\cos^2 t)\,dt$$

$$= \cancel{18}$$

$$-54\cdot 2\pi.$$



$$\vec{r_1} = \langle 1, 0, 0 \rangle$$

$$\vec{r_2} = \langle 0, 1, -1 \rangle$$

$$\overrightarrow{r_1 \times r_2} = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= \left\langle 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

$$\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x & 2y & 2x+2z \end{vmatrix}$$

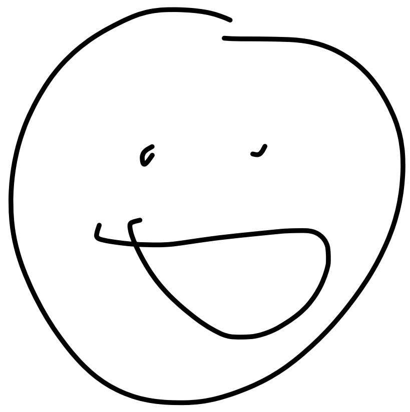
$$= \langle 0, -2, 0 \rangle \qquad \left(\frac{\sqrt{2}}{2}\right)(-2)$$

哇～好難!

究竟點解世界要有數學～

唉～唔識就溫返以前

咁我要由幼稚園開始了



計唔到呀計唔到呀

點解 計唔到

屌那媽

頂硬上

$$\begin{vmatrix} \Psi_r = & \langle \cos\theta, & \sin\theta, & 0\rangle \\ \Psi_\theta = & \langle -r\sin\theta, & r\cos\theta, & 1\rangle \end{vmatrix}$$

(with i, j, k above the first row)

$$\Psi_r \times \Psi_\theta = \langle \sin\theta, -\cos\theta, r\rangle$$

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3z & 9x & 5y \end{vmatrix}$$

$$= \langle 5, 3, 9\rangle$$

$$\int_0^{\frac{\pi}{2}}\int_0^1 (5\sin\theta - 3\cos\theta + 9r)\,dr\,d\theta$$

$$= \int_0^{\frac{\pi}{2}} 5\sin\theta - 3\cos\theta + \frac{9}{2} \quad d\theta$$

$$= 5\left[-\cos\theta\right]_0^{\frac{\pi}{2}} - 3\left[\sin\theta\right]_0^{\frac{\pi}{2}} + \frac{9}{2}\left(\frac{\pi}{2}\right)$$

$$= 5 - 3 + \frac{9\pi}{4}$$

$\vec{r}(\theta) = \langle \cos\theta, \sin\theta, \theta \rangle$

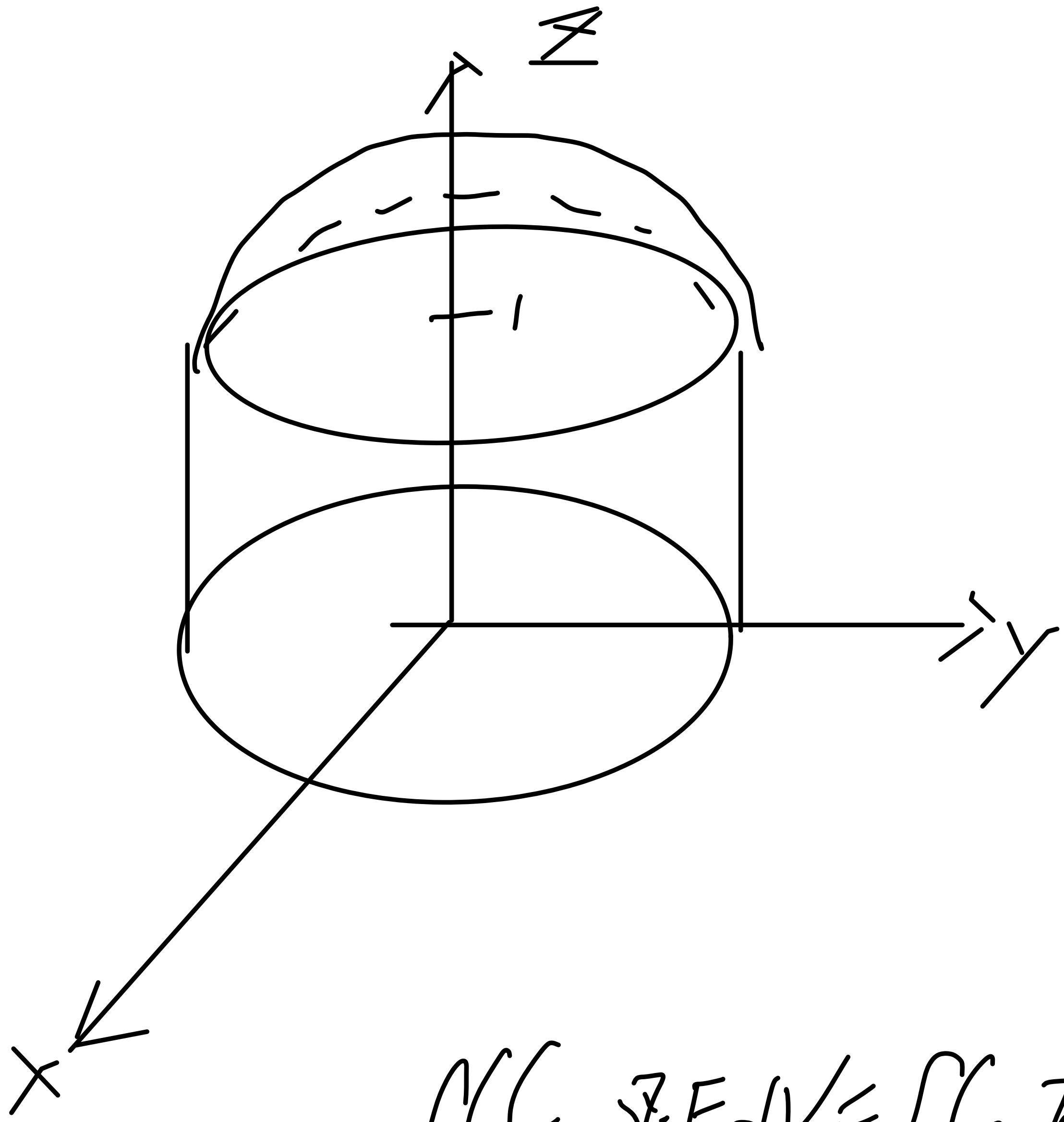
$0 \leq \theta \leq \frac{\pi}{2}$

$\int$

$$\int_0^{\frac{\pi}{2}} 3z(-\sin\theta \, d\theta) + 9x \cos\theta \, d\theta +$$

$$+ y \, d\theta$$

$$= -3\theta \sin\theta + 9\cos^2\theta + 5\sin\theta \, d\theta$$



$$\iiint_E \nabla\cdot F\,dV = \iint_S F\cdot dS$$

$$\iint \nabla\times F\, d\vec{S} = \iiint_E \nabla\cdot(\nabla\times F)\,dV.$$

$$\nabla\times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ zx+z^2y+3y & z^3yx+7x & z^4x^2 \end{vmatrix}$$

$$\langle -3z^2yx,\ x+2zy-2z^4x,\ z^3y+7-z^2-3\rangle$$

$$\iiint_E -3z^2y+2z+3zy-2z\,dV.$$

$$\iiint_E -3z^2y + 2z + 3zy - 2z \, dV.$$

$$\iint_M \nabla \times F \cdot dS = \int_C F \cdot d\vec{r}$$

$$\vec{r} = \langle 8\cos t, 8\sin t \rangle$$

$$\int zx + z^2y + 3y(-8\sin t\,dt) + z^3yx + 7x(8\cos t\,dt)$$

$$\int 3(8\sin t)(-8\sin t\,dt) + 7(8\cos t)(8\cos t\,dt)$$

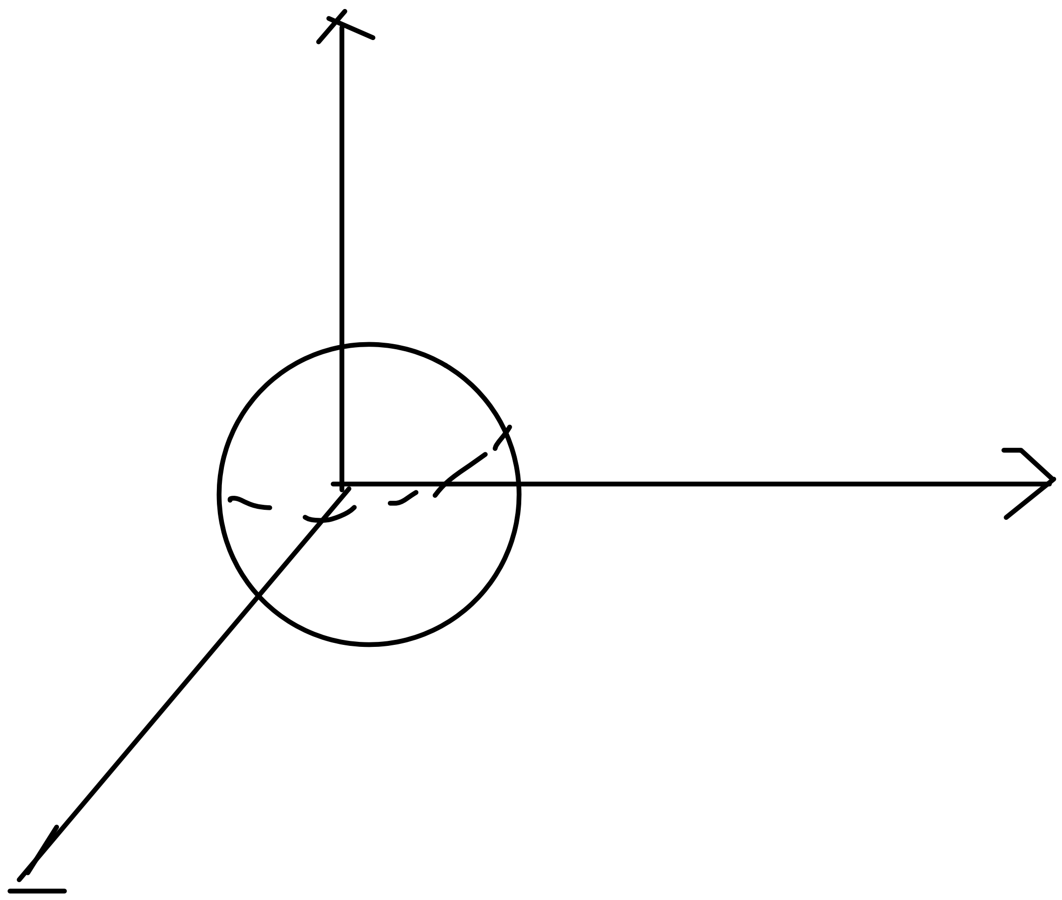
$$= \int_0^{2\pi} -192\sin^2 t + 448\cos^2 t\,dt$$

$$\frac{1}{2} + \frac{\cos 2\theta}{2}$$

$$= \int_0^{2\pi} -192\sin^2 t - 192\cos^2 t + 640\,\cancel{\cos^2} t\,dt$$

$$= -192(2\pi) + 640\left[\frac{1}{2}\theta + \frac{\sin 2\theta}{4}\right]_0^{2\pi}$$

$$-384\pi + 640(\pi).$$



$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 9z & 2x & 9y \end{vmatrix}$$

$$= \langle 9, 9, 2 \rangle$$

$$\begin{array}{lc} & \begin{array}{ccc} i & j & k \end{array} \\ r_r = & \left| \begin{array}{ccc} \langle \cos\theta, & \sin\theta, & 0 \rangle \\ \langle -r\sin\theta, & r\cos\theta, & 1 \rangle \end{array} \right| \\ r_\theta = & \end{array}$$

$$\langle \sin\theta, -\cos\theta, r \rangle$$

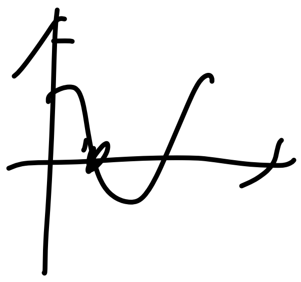
$$\sqrt{r^2+1}$$

$$\int_0^{\frac{\pi}{2}}\int_0^1 9\sin\theta - 9\cos\theta + 2r \, dr \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} 9\sin\theta - 9\cos\theta + 1 \, d\theta$$

$$= \left[-9\cos\theta - 9\sin\theta + \theta\right]_0^{\frac{\pi}{2}}$$

$$= +9(1) - 9(1) + \frac{\pi}{2}$$



$$\vec{r}(\theta) = \langle \cos\theta, \sin\theta, \theta \rangle$$

$$r'(\theta) = \langle -\sin\theta, \cos\theta, 1 \rangle$$

$$9z(-\sin\theta)\,d\theta +$$

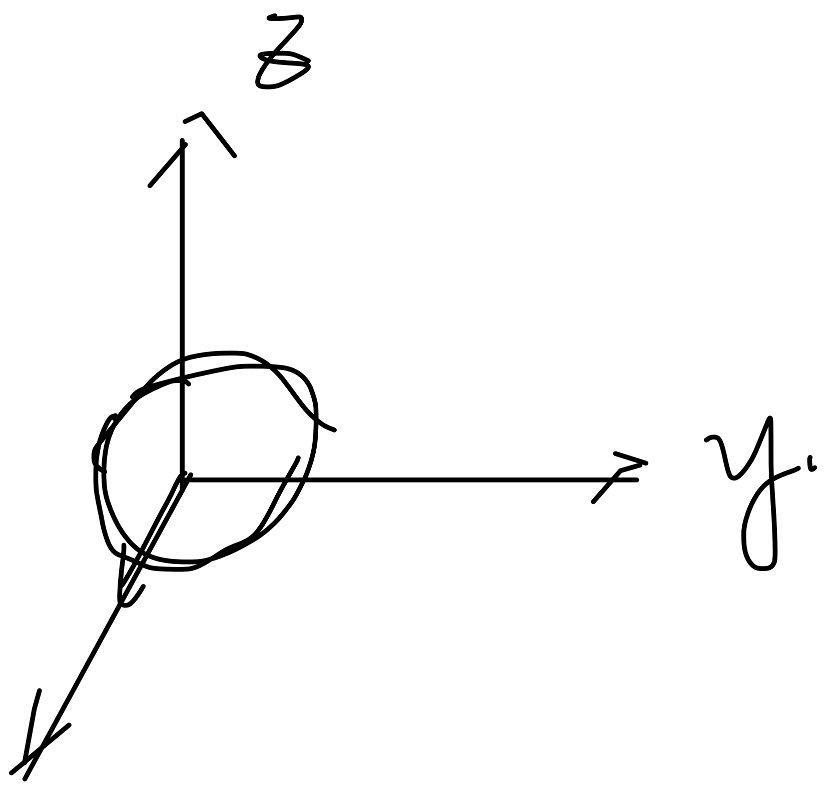
$$2x(\cos\theta)\,d\theta +$$

$$9y(d\theta)$$

$$\cancel{-9\sin\theta - 2\sin\theta\cos\theta + 9\cos\theta\,d\theta}$$

$$9\theta(-\sin\theta)$$

Made with Goodnotes



$$\langle 0 \quad 2\cos\theta \quad 2\sin\theta \rangle$$

$$y^2 \cos\theta \, d\theta$$

$$\int_0^{2\pi} \cos^3 d\theta$$

$$(1-\sin^2\theta)\cos\theta \, d\theta$$

let $u = \sin\theta$, $du = \cos\theta \, d\theta$

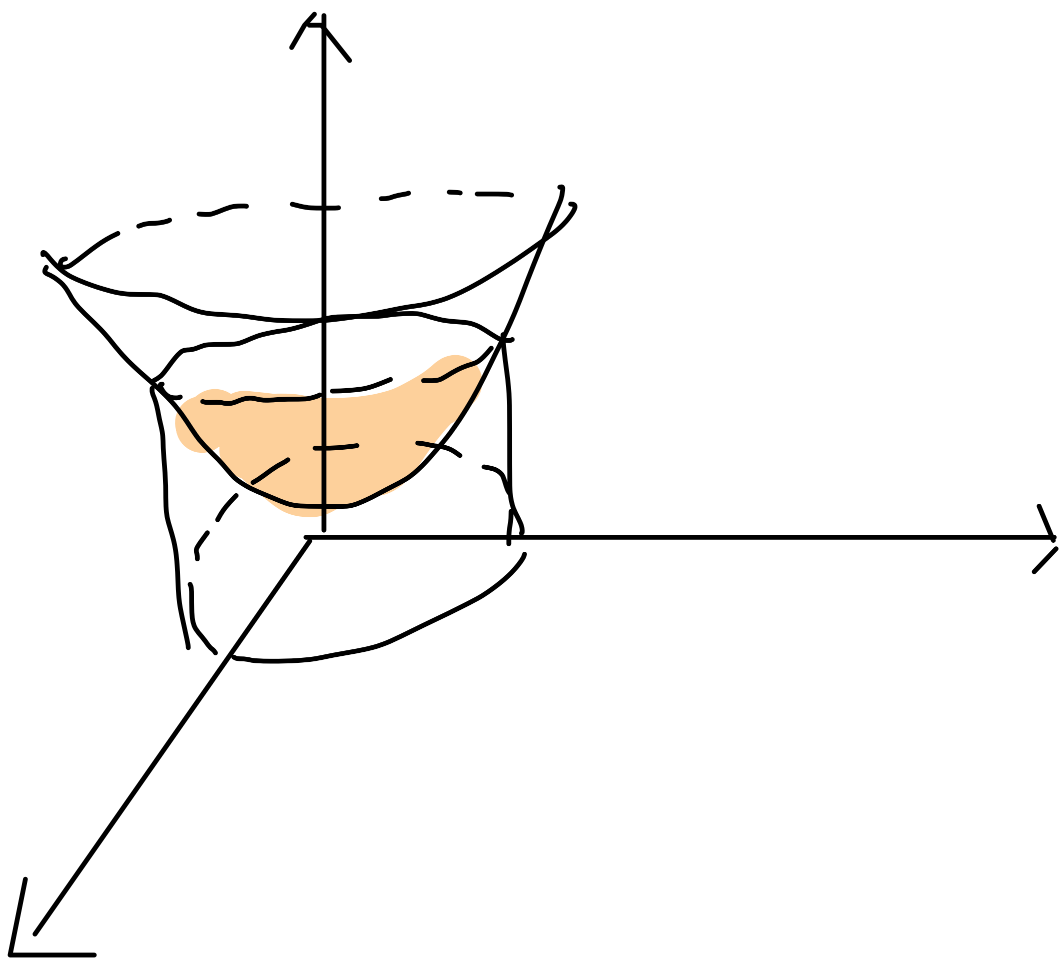
$$\int$$

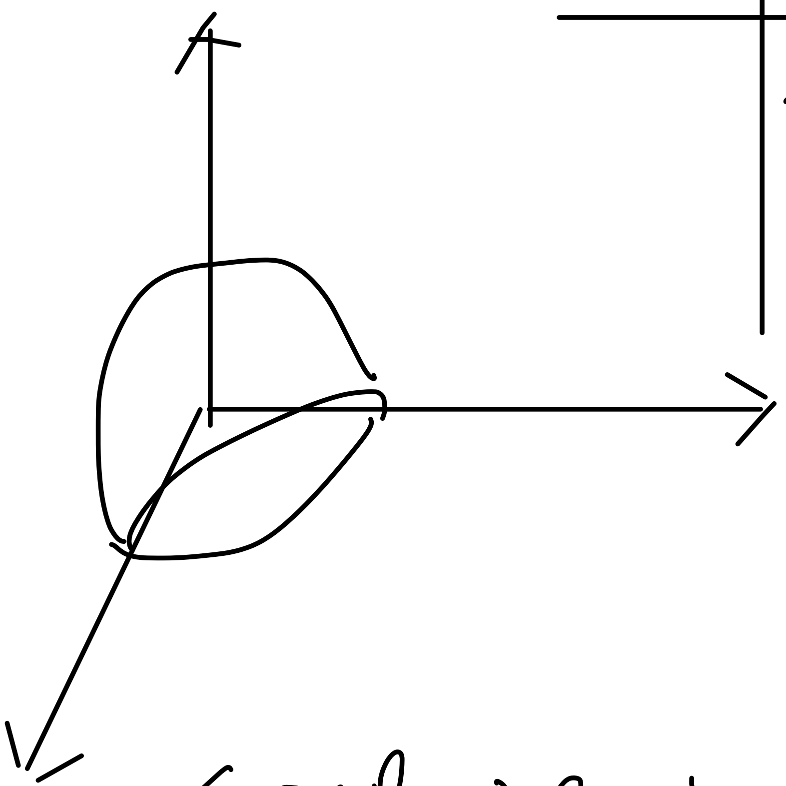
$\vec{r}(\theta) = \langle$

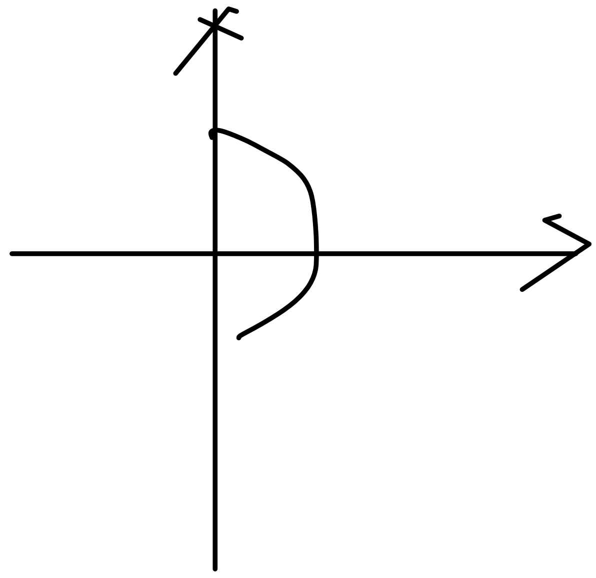
$$\int_0^{2\pi} -19yz(-\sin\theta)d\theta +$$

$$19xz(\cos\theta)d\theta +$$

$$8\cancel{(x^2+y^2)z}\ \theta(d\theta)$$





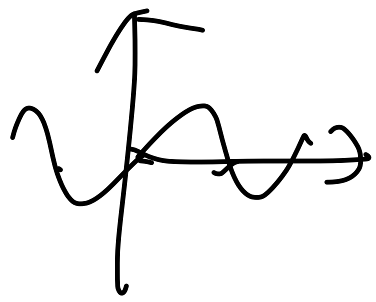


$\langle \cos\theta, \sin\theta, 0 \rangle$

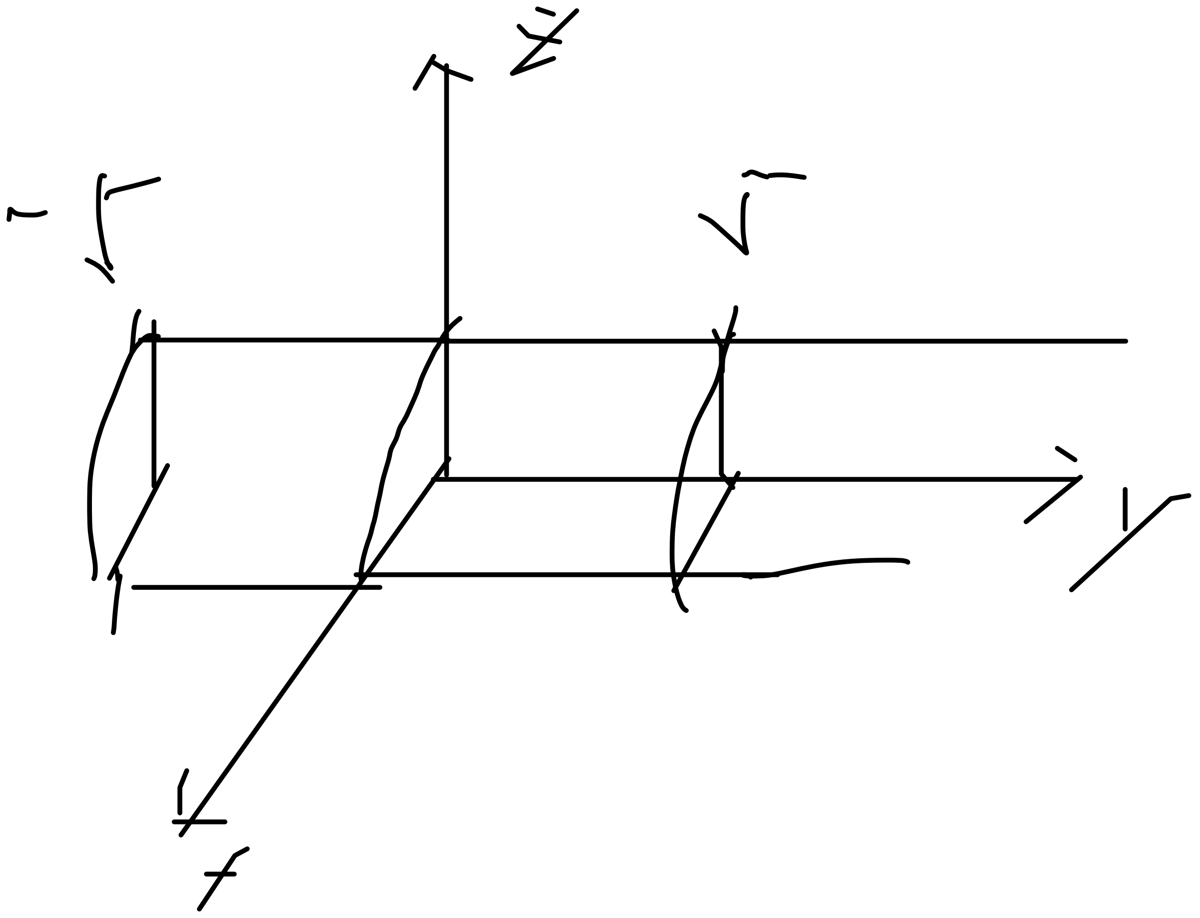
$xy(-\sin\theta)\,d\theta +$

$yz(\cos\theta)\,d\theta$

$\int_{\cancel{0}-\pi}^{\cancel{2}\pi} -\cos\theta\sin^2\theta\,d\theta +$



ut $u = \sin\theta$, $du = \sin\theta$



$$\vec{r}(u,v) = \langle u, v, 25 - u^2 \rangle$$

$$0 \le u \le 5$$

$$-5 \le v \le 5$$

$$\begin{array}{ll} & \quad i \quad\; j \quad\;\; k \\ \begin{array}{l} r_u = \\ r_v = \end{array} & \begin{vmatrix} \langle 1, & 0, & -2u \rangle \\ \langle 0, & 1, & 0 \rangle \end{vmatrix} \end{array}$$

$$r_u \times r_v = \langle 2u, 0, 1 \rangle$$

$$\iint xy \cdot (2u) + xz \quad du\,dv$$

$$= \int_{-5}^{5} \int_{0}^{5} 2u^2 v + 25u - u^3 \quad du\,dv$$

$$= \int_{-5}^{5} \left[\frac{2u^3}{3} v + \frac{25u^2}{2} - \frac{u^4}{4} \right]_0^5 du\,dv$$

$$\int_{-5}^{5} \left[\frac{2u^3}{3} v + \frac{25u^2}{2} - \frac{u^4}{4} \right]_0^5 du\,dv$$

$$= \int_{-5}^{5} \frac{2(5)^3}{3} v + \frac{25(5)^2}{2} - \frac{(5)^4}{4} \, dv$$

$$= \frac{5^4}{4}(10)$$

$$\frac{2(5)^3}{3} \; \frac{v^2}{2} \; \cancel{+ \frac{(5)^2}{2}}$$

$$5^2$$

$$\nabla \times \vec{F}: \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & yz & xz \end{vmatrix}$$

$$\langle -y, -z, -x \rangle$$

$$r_u \times r_v = \langle 2u, 0, 1 \rangle$$

$$r = \langle u, v, 25 - u^2 \rangle$$

$$\iint_S -yu - zv - x(25-u^2)\, du\, dv$$

$$= \iint_S -vu - (25-u^2)v - u(25-u^2)\, du\, dv$$

$$= \iint_S -uv - 25v + u^2 v - 25u + u^3\, du\, dv$$

$$= \int_{-5}^{5} \left[-\frac{u^2}{2}v - 25uv + \frac{u^3}{3}v - \frac{25u^2}{2} + \frac{u^4}{4} \right]_0^5 dv$$

$$= \int_{-5}^{5} -\frac{25}{2}v - 125v + \frac{125}{3}v - \frac{625}{2} + \frac{625}{4}\, dv$$

$$= \int_{-5}^{5} \frac{-575}{6}v - \frac{625}{4}\, dv$$

$$= \frac{-575}{6}\left[\frac{v^2}{2}\right]_{-5}^{5} - \frac{625}{4}(10)$$

$$= \frac{-575}{6}(0) - \frac{625}{4}(10)$$

$$\langle -y, -z, -x \rangle$$

$$r_u \times r_v = \langle 2u, 0, 1 \rangle$$

$$\iint -y(2u) - x \, dA.$$

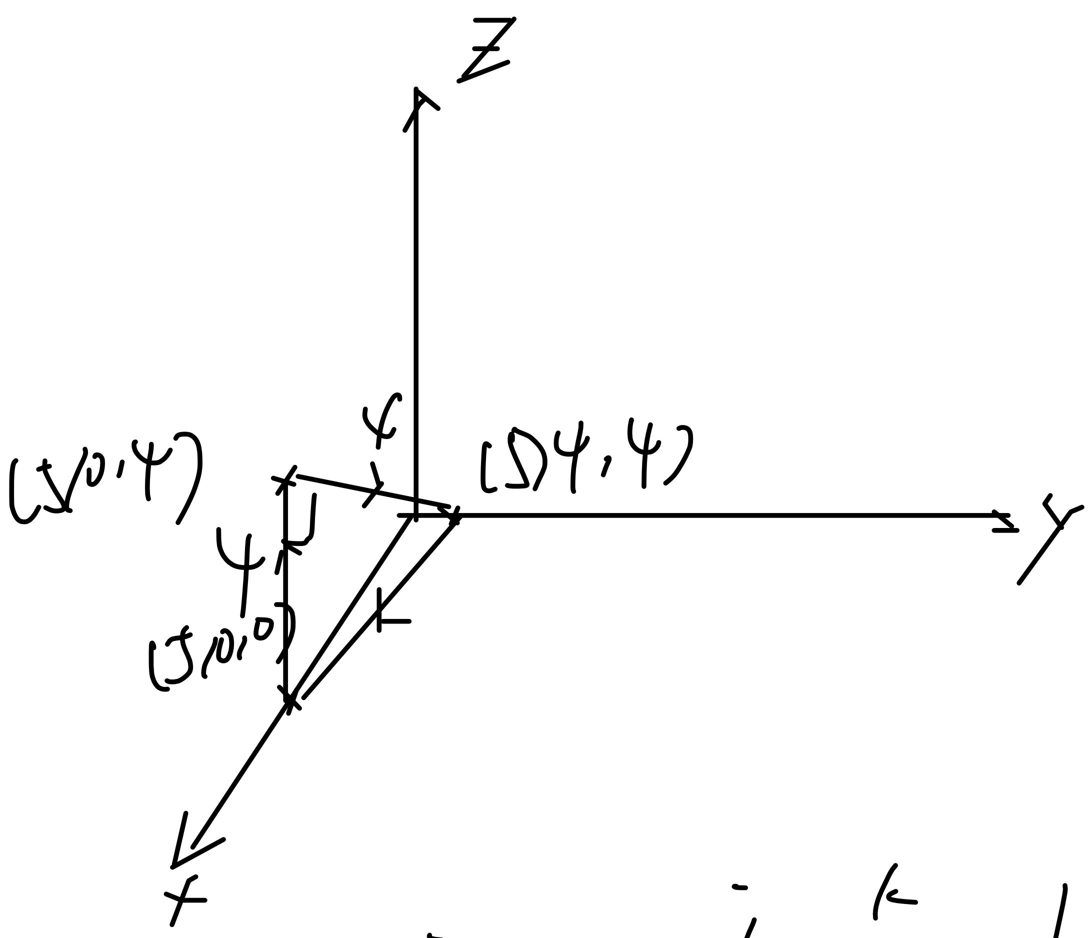
$$= \int_{-5}^{5}\int_{0}^{5} -2xy - x \, dx \, dy$$

$$= \int_{-5}^{5} -\left[x^2 y\right]_0^5 - \left[\frac{x^2}{2}\right]_0^5 \, dy$$

$$= \int_{-5}^{5} -25y - \frac{25}{2} \, dy$$

$$= -\frac{25}{2}\left[y^2\right]_{-5}^{5} - \frac{25}{2}(10)$$

$$= -\frac{25}{2}(25-25) - 25(5)$$



$$\nabla \times F: \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2y & 6z & 2x \end{vmatrix}$$

$$\langle -6, -2, -2 \rangle$$

$$\vec{n} = \langle -1, 0, 0 \rangle$$

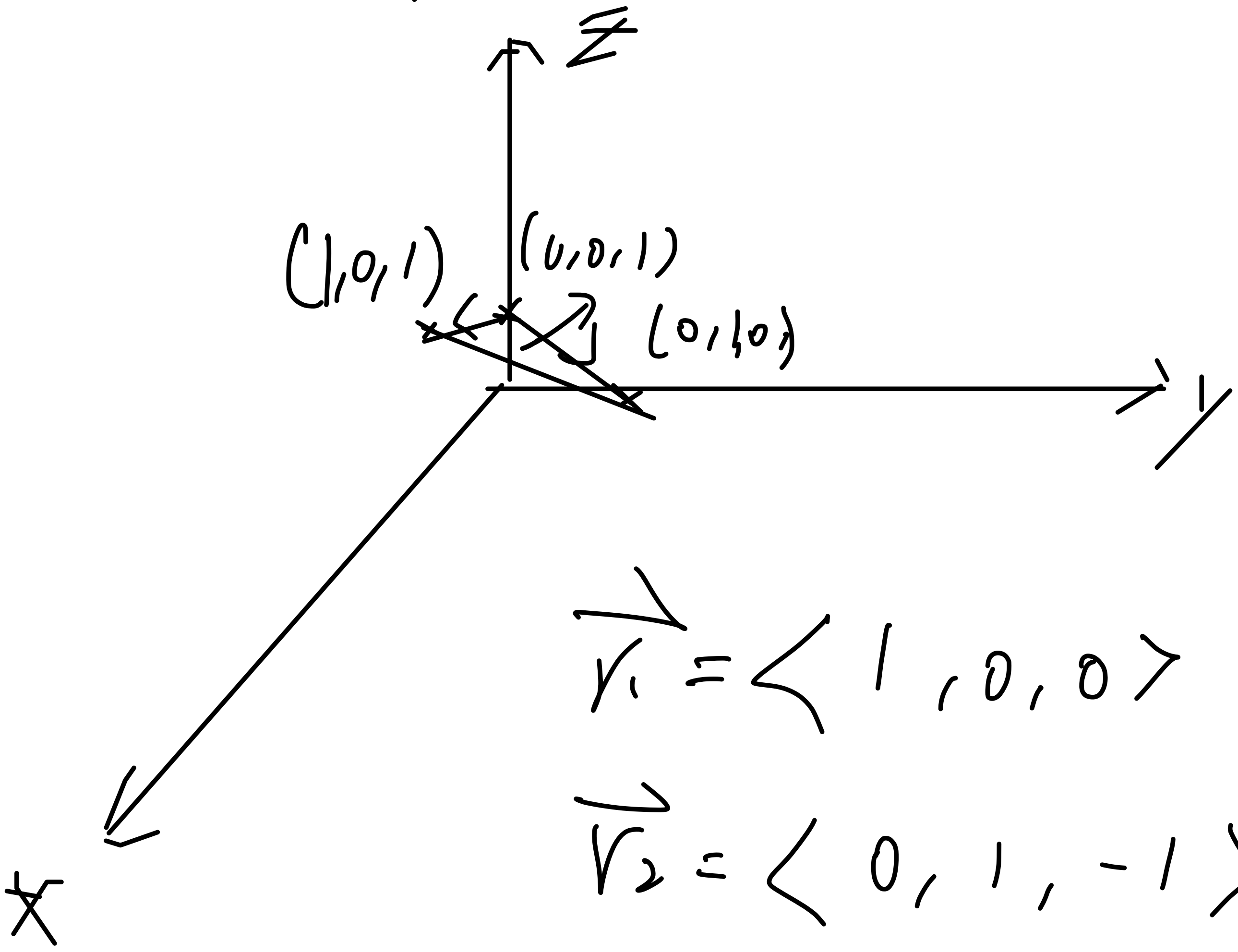
$$6 \iint_S dS. \quad \left(\frac{4 \times 4}{2}\right).$$

$$\iint -\frac{3}{\sqrt{2}} \, dA.$$

$$-\frac{2}{\sqrt{2}}$$

$\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x & 2y & 2x+2z \end{vmatrix}$

$\langle 0, -2, 0 \rangle$



$\vec{r}_1 = \langle 1, 0, 0 \rangle$

$\vec{r}_2 = \langle 0, 1, -1 \rangle$

$r_1 \times r_2 = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix}$

$F \cdot \hat{n} dS.$

$= \langle 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$\int$

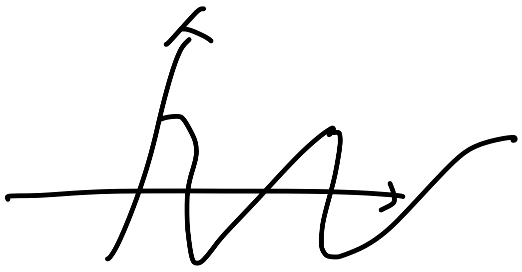
$-\frac{2}{\sqrt{2}} (\sqrt{2})$

$$\vec{r}(\theta) = \langle 4\cos\theta, 4\sin\theta, 0 \rangle$$

$$zx + z^2y + 4y \,(-4\sin\theta)\,d\theta +$$

$$z^3yx + 7x\,(4\cos\theta)\,d\theta$$

$$= \int_0^{2\pi} \sin^2\theta\, d\theta \cdot \quad 7\cos^2\theta\, d\theta$$



$$= \int_0^{2\pi} (-4) + 11\cos^2\theta\, d\theta$$

$$= (-4)(2\pi) + 11 \underbrace{\left[\frac{1}{2} + \frac{\cos 2\theta}{2} \right]_0^{2\pi}}_{\phi}$$

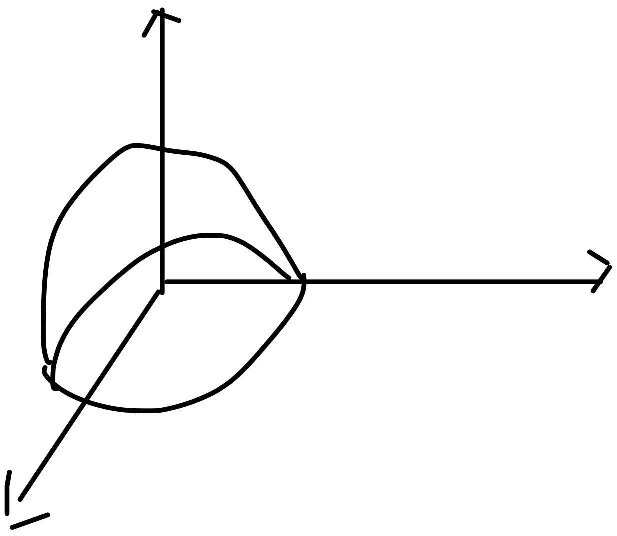
$$= -8\pi + 11(\pi)$$

$$\left[\frac{1}{2}\theta + \frac{\sin 2\theta}{4} \right]_0^{2\pi}$$

$\vec{r}(\theta) = \langle \cos\theta, \sin\theta, 1 \rangle$

$+ 7\sin\theta(\sin\theta)\,d\theta +$

7



$\vec{r}(\theta) = \langle 3\cos\theta, 3\sin\theta, 0 \rangle$

$3\cos\theta(-3\sin\theta) + 3\sin\theta(\cos\theta)$

$\nabla \cdot F$

$36a^2 - 3a - 3 = 0$

$a = \frac{1}{3}$ or $a = -\frac{1}{4}$.

$36(a^2 - \frac{1}{12}a) - 3$

$36(a^2 - \frac{1}{12}a + (\frac{1}{24})^2$

$a - \frac{1}{24}$

$$\frac{\partial}{\partial x}\left(\frac{-y}{(x^2+y^2)^5}\right)$$

$$\frac{+y\,(5(x^2+y^2)^4(2x))}{(x^2+y^2)^{10}} + \frac{-x\,(5(x^2+y^2)^4)(2y)}{(x^2+y^2)^{10}}$$

$y \cdot 2x$

$y - y^2$

$-3(2)(x+y) + 14(x+y+z)$

$= -6(x+y) + 14(x+y+z)$

$= 8(x+y) + 14z$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & -3(x+y)^2 & 7(x+y+z)^2 \end{vmatrix}$$

$14(x+y+z), \ -14(x+y+z), \ -6(x+y)$

$$\begin{vmatrix} \hat{i} & \hat{j} & k \\ \partial x & \partial y & \partial z \\ -4yz & xz & 4xy \end{vmatrix}$$

$\langle 4x - x, -4y - 4y, z + 4z \rangle$

$\langle$

$$\begin{vmatrix} \partial x & \partial y & \partial z \\ 5x & 6y & 5z \end{vmatrix}$$

$\langle 0, 0,$

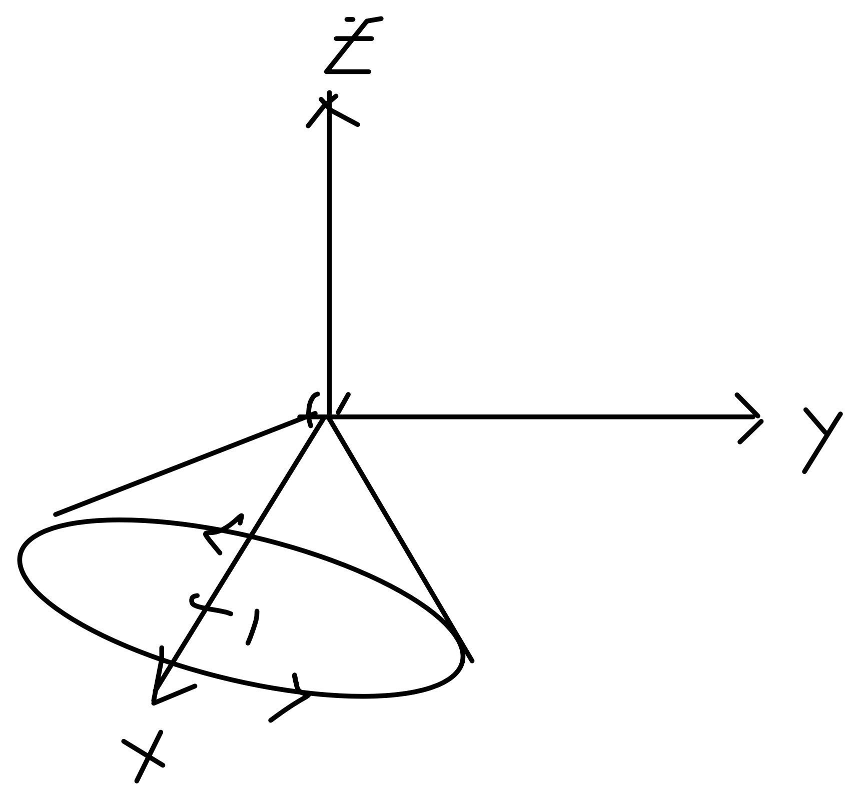
∇

$$\begin{vmatrix} \partial x & \partial y & \partial z \\ 4xy & 6y & 7z \end{vmatrix}$$

$= \langle 0, 0, -4x \rangle$

$\nabla \cdot (\nabla \times F) = 0.$

$\nabla \cdot F$



$$\iint_{S_1} \vec{F} \cdot d\vec{S} + \iint_{S_2} \vec{F} \cdot d\vec{S}$$

$$\vec{r}(r,\theta) = \langle r, r\cos\theta, r\sin\theta \rangle$$

$$\begin{array}{l} r_r = \\ r_\theta = \end{array} \begin{vmatrix} \langle 1, \cos\theta, \sin\theta \rangle \\ \langle 0, -r\sin\theta, r\cos\theta \rangle \end{vmatrix}$$

$$r_r \times r_\theta = \langle r, r\cos\theta, -r\sin\theta \rangle$$

$$\int_0^{2\pi}\int_0^1 2x^2(r) + 4z(r\cos\theta) + 5y(-r\sin\theta)\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^1 2r^3\cos^2\theta + 4r^2\sin\theta\cos\theta - 5r^2\sin\theta\cos\theta\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^1 2r^3\cos^2\theta - r^2\sin\theta\cos\theta\, dr\, d\theta$$

$$= \int_0^{2\pi} \left[\frac{r^4}{2}\cos^2\theta - \frac{r^3}{3}\sin\theta\cos\theta \right]_0^1 d\theta$$

$$= \int_0^{2\pi} \frac{1}{2}\cos^2\theta - \frac{1}{3}\sin\theta\cos\theta\, d\theta =$$

$$\int_0^{2\pi} \frac{1}{2}\cos^2\theta - \frac{1}{3}\sin\theta\cos\theta \, d\theta$$

$$\iint_{S_1} \vec{F} \cdot d\vec{S} = \int_C \vec{F} \cdot d\vec{r} \quad 0.$$

TSL

$$\vec{r}(\theta) = \langle 1, \cos\theta, \sin\theta \rangle$$

$$\int_0^{2\pi} 2x^2(0)\,d\theta + 4z(\cos\theta\, d\theta) + 5y(-\sin\theta)\,d\theta$$

$$= \int_0^{2\pi} 4\sin\theta\cos\theta - 5\sin\theta\cos\theta \quad d\theta$$

$$= -\int_0^{2\pi} \sin\theta\cos\theta \, d\theta$$

Let $u = \sin\theta$, $du = \cos\theta$

$$[-\cos\theta]_0^{\theta}$$

$$-1 - (-1)$$

$$\iint_D \vec{F} \cdot d\vec{S}$$

$$\vec{n} = \langle 1, 0, 0 \rangle$$

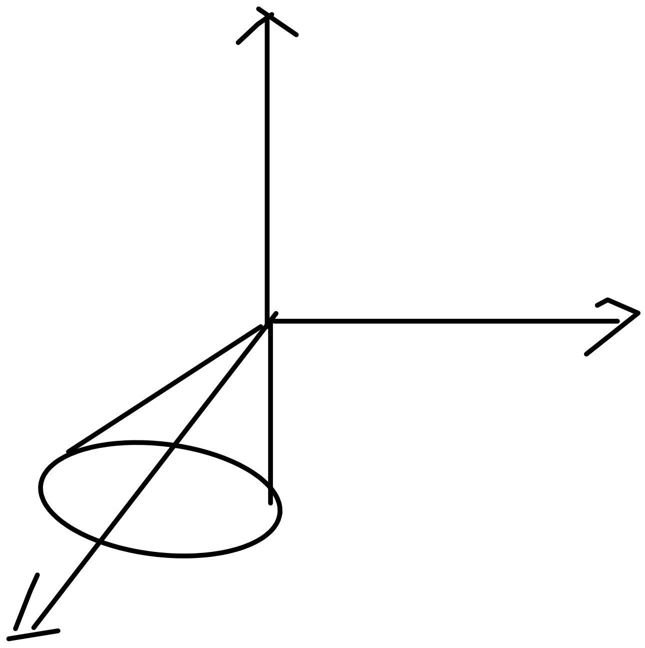
$$\iint_D 2x^2 \, d\vec{S} \qquad r = \langle 1, \cos\theta, \sin\theta \rangle$$

$$\iiint \nabla \cdot F\,dV = \iint_S \vec{F} \cdot d\vec{S}$$

$$\iiint 4x\,dV$$

$$\vec{r}(r,\theta) = \langle r,\ r\cos\theta,\ r\sin\theta \rangle$$

$$0 \le r \le 3,$$



$$\int_0^3\int_0^4 6x+8y+48 \, dx\, dy$$

$$= \int_0^3 \left[3x^2 + 8xy + 48x\right]_0^4 dy$$

$$= \int_0^3 48 + 32y + 192 \, dy$$

$$= \int_0^3 240 + 32y \, dy$$

$$= 240(3) + 16\left[y^2\right]_0^3$$

$$= 720 + 16(9)$$

$$\int_5^1 \int_1^{-2} 2x+5 \, dx\, dy$$

$$= \int_5^1 \left[x^2+5x\right]_1^{-2} dy$$

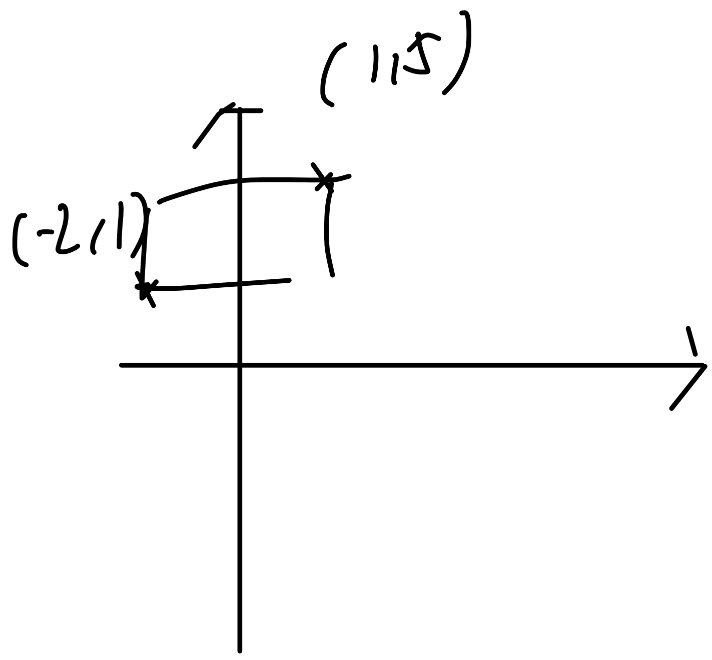
$$= \int_5^1 4-10-(1+5)\, dy$$

$$= \int_5^1 -12\, dy$$

$$= \left[-12y\right]_5^1$$

$$= -12(1)+60$$

$$= 48$$



$$\left[2xy+5y\right]_1^5$$

$$= 10x+25-2x-5$$

$$= \int_{-2}^1 8x+20\, dx$$

$$= \left[4x^2+20x\right]_{-2}^1$$

$$= 4+20-(16-40)$$

$$\int_0^2 \left[4x^3y^3 \right]_0^1 dy$$

$$= \int_1^2 4y^3 \, dy$$

$$= \left[y^4 \right]_0^2$$

$$\int_0^2 \int_0^2 81 - x^2 - 4y^2 \, dx \, dy$$

$$= \int_0^2 \left[81x - \frac{x^3}{3} - 4xy^2 \right]_0^2 dy$$

$$= \int_0^2 162 - \frac{8}{3} - 8y^2 \, dy$$

$$= \int_0^2 \frac{478}{3} - 8y^2 \, dy$$

$$= \left[\frac{478}{3}y - \frac{8y^3}{3} \right]_0^2$$

$$=$$

$$\int_4^3 \int_{-4}^{-3} 4x^2+5y^2 \; dx \, dy.$$

$$= \int_4^3 \left[\frac{4x^3}{3} + 5xy^2\right]_{-4}^{-3} dy$$

$$= \int_4^3 -36 - 15y^2 - \left(\frac{-256}{3} - 20y^2\right) dy$$

$$= \int_4^3 \frac{148}{3} + 5y^2 \, dy$$

$$= \left[\frac{148}{3}y + \frac{5y^3}{3}\right]_4^3$$

$$\int_0^1 \int_3^7 xye^{x+y}\,dy\,dx$$

let $u = x+y$, $\frac{du}{dy} = 1$ $dy = du$.

$$\int_0^1 \int_3^7 e^u \quad xe^x \cdot ye^y$$

$$\int_0^1 \int_3^7 xe^x \cdot ye^y\,dy\,dx$$

$$= \left(\int_0^1 \underset{u}{x}\underset{v'}{e^x}\,dx\right)\left(\int_3^7 ye^y\,dy\right)$$

$u' = 1$

$v = e^x$

$$= \left([xe^x]_0^1 - \int_0^1 e^x\,dx\right)\left([ye^y]_3^7 - \int_3^7 e^y\,dy\right)$$

$$= (e - (e-1))(7e^7 - 3e^3 - e^7 + e^3)$$

$$6e^7 - 2e^3$$

Made with Goodnotes

$$\int_3^4 \int_{-4}^{-3} 4x^2 + 5y^2 \, dx\, dy$$

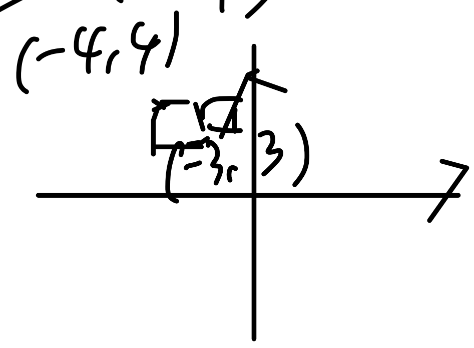
$$= \int_3^4 \left[\frac{4x^3}{3} + 5xy^2\right]_{-4}^{-3} dy$$

$$= \int_3^4 -36 - 15y^2 - \left(\frac{-256}{3} - 20y^2\right) dy$$

$$= \int_3^4 \frac{148}{3} + 5y^2 \, dy$$

$$= \frac{148}{3} + \frac{5}{3}\left[y^3\right]_3^4$$

$$= \frac{148}{3} + \frac{5}{2}(37)$$



$$\int_4^5 \int_4^5 \frac{1}{(x+y)^2}\, dy\, dx$$

$$\int_{-2}^{1} \int_{1}^{5} 2x+5 \quad dx\, dy$$

$$\int_{-2}^{1} \left[x^2+5x\right]_1^5 dy$$

$$= \int_{-2}^{1} 25+25-1-5\, dy$$

$$= \int_{-2}^{1} 44\, dy$$

$$= 44(\cancel{1+2}\ 3)$$

$$\begin{array}{r} 44 \\ \times\ 3 \\ \hline 132 \end{array}$$

$$\int_4^5\int_4^5 \frac{1}{(x+y)^2}\,dy\,dx \qquad \frac{1}{(y+3)^2}$$

Let $u = x+y$, $du = dy$

$$\int_4^5 \int_{x+4}^{x+5} u^{-2}\,du\,dx$$

$$= \int_4^5 \left[-u^{-1}\right]_{x+4}^{x+5} dx$$

$$= \int_4^5 -\frac{1}{(x+5)} + \frac{1}{(x+4)}\,dx$$

$$= \left[\ln(x+4) - \ln(x+5)\right]_4^5$$

$$= \ln 9 - \ln 10 - (\ln 8 - \ln 9)$$

$$= 2\ln 9 - \ln 10 - \ln 8$$

$$\int_0^{\frac{\pi}{3}}\int_0^{\frac{\pi}{4}} x\cos(2x+y)\,dy\,dx$$

Let $u = 2x+y$, $du = dy$

$$\int_0^{\frac{\pi}{3}}\int_0^{\frac{\pi}{4}} x\cos u \,du\,dx$$

$$= \int_0^{\frac{\pi}{3}} x\left[\sin u\right]_0^{\frac{\pi}{4}}\,dx$$

$$= \int_0^{\frac{\pi}{3}} \frac{\sqrt{2}}{2}x\,dx$$

$$= \frac{\sqrt{2}}{2}\left[\frac{x^2}{2}\right]_0^{\frac{\pi}{3}}$$

$$= \frac{\sqrt{2}}{2}\left(\frac{\pi^2}{18}\right) = \frac{\sqrt{2}\,\pi^2}{36}$$

$2x+y$

Made with Goodnotes

$$\int_0^{\frac{\pi}{3}}\int_0^{\frac{\pi}{4}} x\cos(2x+y)\ dy\,dx$$

$=$ let $u = 2x+y$,

$$\int_0^{\frac{\pi}{3}}\int_{2x}^{2x+\frac{\pi}{4}} x\cos u\ du\ dx$$

$$= \int_0^{\frac{\pi}{3}} x\left[\sin u\right]_{2x}^{2x+\frac{\pi}{4}}\ dx$$

$$= \int_0^{\frac{\pi}{3}} x\left(\sin\left(2x+\tfrac{\pi}{4}\right) - \sin(2x)\right)dx$$

$$= \int_0^{\frac{\pi}{3}} x\sin\left(2x+\frac{\pi}{4}\right) - x\sin(2x)\ dx$$

$=$

$$\int_4^6 \int_2^3 x \, dy \, dx + \int_2^3 \int_4^6 \ln y \quad dx \, dy$$

$$= \left[\frac{x^2}{2}\right]_4^6 + \int_2^3 2\ln y \, dy$$

$$= 10 + 2\left([y \ln y]_2^3 - \int_2^3 dy \right)$$

$$= 10 + 2\left(3\ln 3 - 2\ln 2 - 1 \right)$$

$u = \ln y$
$v' = 1$

$v = y$
$u' = \frac{1}{y}$

Made with Goodnotes

$$\int_{-4}^{4}\int_{-3}^{3} 4x^2+5y^2 \, dy\,dx$$

$$= \int_{-4}^{4} \left[4x^2y + \frac{5y^3}{3} \right]_{-3}^{3} dx$$

$$= \int_{-4}^{4} 12x^2 + 45 - (-12x^2 - 45) \, dx$$

$$= \int_{-4}^{4} 24x^2 + 90 \, dx$$

$$= \left[8x^3 + 90x \right]_{-4}^{4}$$

$$= 8(4)^3 + 90(4) - (8(-4)^3 - 360)$$

$$=$$

Made with Goodnotes

$$\int_0^{\frac{\pi}{6}} \int_0^3 y \cos x \, dy \, dx \qquad + \quad \pi$$

$$= \int_0^{\frac{\pi}{6}} \left[\frac{y^2}{2} \cos x \right]_0^3 dx$$

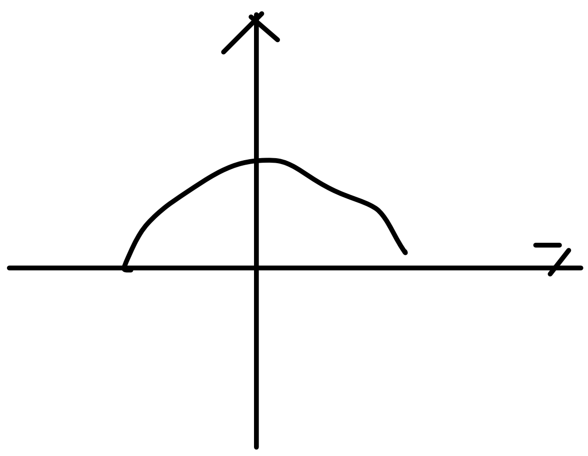
$$= \int_0^{\frac{\pi}{6}} \frac{9}{2} \cos x \, dx$$

$$= \frac{9}{2} \left[\sin x \right]_0^{\frac{\pi}{6}}$$

$$= \frac{9}{2} \left(\frac{1}{2} \right) + \pi.$$

$$z = \sqrt{49 - r^2}$$

$$\frac{\frac{4}{3} \pi (7)^3}{2.}$$

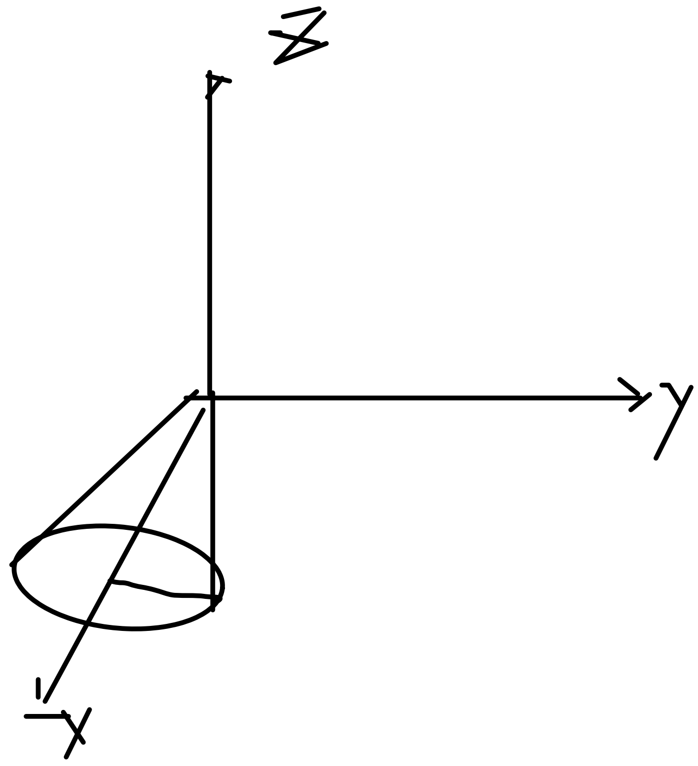


$$\iiint_E 4x \, dV$$

$\langle \quad r \quad , r\cos\theta , \; r\sin\theta \; \rangle$

$0 \leq \theta \leq 2\pi$

$0 \leq r \leq 1$



$$\int_0^{2\pi}\int_0^1\int_0^1 4xr \, dx \quad dr\, d\theta$$

$$\int_0^{2\pi}\int_0^1\int_0^1 4r^2 \, dx\, dr\, d\theta$$

$=$ $\qquad 4r^2$

$$\left[\frac{4r^3}{3}\right]_0^1$$

$$\frac{4}{3}.$$

$\langle \; x ,$

$2r \, dr$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{1} \rho^2 + 4 \left(\rho^2 \sin\varphi\right) \, d\rho \, d\varphi \, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{1} \rho^4 \sin\varphi + 4\rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{1}{5}\sin\varphi + \frac{4}{3}\sin\varphi \, d\varphi \, d\theta$$

$$= 2\pi \cdot \frac{23}{15}\left[-\cos\varphi\right]_0^{\pi}$$

$$=$$

Made with Goodnotes

$\nabla \cdot F = 3x^2 + 3y^2 + 3z^2$

$$\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_2^3 3\rho^2 \ \rho^2 \sin\varphi \ d\rho \, d\varphi \, d\theta$$

$$= 2\pi \cdot \int_0^{\frac{\pi}{4}} 3\left[\frac{\rho^5}{5}\sin\varphi\right]_2^3 d\varphi$$

$$= 2\pi \cdot 3\left(\frac{211}{5}\right)\left[-\cos\varphi\right]_0^{\frac{\pi}{4}}$$

$$= 2\pi\left(\frac{633}{5}\right)\left(-\frac{\sqrt{2}}{2}+1\right)$$

$$\int_0^4 \int_0^8 \int_0^2 2yz \ . dx \, dy \, dz$$

$$= 2 \cdot 2 \int_0^4 \int_0^8 yz \ dy \, dz$$

$$= \int_0^4 32z \ dz$$

$$32 \cdot 8 \cdot 4.$$

Made with Goodnotes

政府 A 政策 $\longrightarrow$ 项目 A $\longrightarrow$ 医疗人手不足

$\uparrow$

Recommend

B $\longrightarrow$ B $\longrightarrow$

$\uparrow$

Recommend

公 $\longrightarrow$ 私

由

$$\frac{-y(2x)}{(x^2+y^2)^2}$$

$$\frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z}$$

$$\frac{-y}{x^2+y^2} \quad \frac{x}{x^2+y^2} \quad 0$$

$-x^2-y^2+2y^2$

y^2-x^2

y^2-x^2

x^2-y^2

$$\langle 0, 0, \frac{x^2+y^2-2x^2}{(x^2+y^2)^2} - \frac{(x^2+y^2)(-1)+2y^2}{(x^2+y^2)^2}$$

$2(y^2-x^2)$

政策: 拨款不足，对病人、私家医生都缺乏吸引力

私营

原因: 公营薪金不足，流向私营

加人工

$3x^2+3y^2+2z$

$$\int_0^8 (3r^2+2z)\cdot r\,dz\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^6\int_0^8 3r^3+2zr\,dz\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^6 24r^3+64r\,dr\,d\theta$$

$$= \left[6r^4+32r^2\right]_0^6 dr \cdot 2\pi$$

$$6\cdot 6^4+32\cdot 6^2$$

$$\iiint 2x-1 \, dx\, dy\, dz$$

(5) (2).

$$\int_0^8 2x-1 \, dx$$

$$= [x^2]_0^8 - 8$$

$$= 64-8$$

$$= 56$$

$$\frac{\partial}{\partial x}\frac{-y}{(x^2+y^2)} + \frac{\partial}{\partial y}\frac{x}{(x^2+y^2)}$$

$$= \frac{-2xy}{(x^2+y^2)^2} +$$

$$\nabla f = \langle ay + 2axy, \quad ax + ax^2 + 3y^2 \rangle$$

$$\nabla \cdot \nabla f = 2ay + 6y$$

$$2a$$

$$\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_2^4 3\rho^2\,\rho^2\sin\varphi\;d\rho\,d\varphi\,d\theta$$

$$= \int_0^{2\pi}\int_0^{\frac{\pi}{4}} \left[\frac{3\rho^5}{5}\sin\varphi\right]_2^4 d\varphi\,d\theta$$

$$= \int_0^{2\pi}\int_0^{\frac{\pi}{4}} \frac{3\cdot 4^5}{5}\sin\varphi - \frac{3(2)^5}{5}\sin\varphi\;d\varphi\,d\theta$$

$$= 2\pi\left(\frac{3\cdot 4^5}{5}\left[-\cos\varphi\right]_0^{\frac{\pi}{4}} + \frac{3(2)^5}{5}\left[\cos\varphi\right]_0^{\frac{\pi}{4}}\right)$$

$$= 2\pi\left(\frac{3\cdot 4^5}{5}\left(-\left(\frac{\sqrt{2}}{2}\right)+1\right) + \frac{3\cdot 2^5}{5}\left(\frac{\sqrt{2}}{2}-1\right)\right)$$

$$\int_0^3\int_0^2\int_0^3 2x + 6y + 4z\;dx\,dy\,dz$$

$$= \int_0^3\int_0^2 \left[x^2\right]_0^3 + 18y + 12z\;dy\,dz$$

$$= \int_0^3\int_0^2 9 + 18y + 12z\;dy\,dz$$

$$= \int_0^3 18 + 18\left(\frac{4}{2}\right) + 24z\;dz$$

$$= 18(3) + 18(2)(3) + 24\left(\frac{3^2}{2}\right)$$

$3y^2 + 3x^2 + 3z^2$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{3} 3\rho^2 \ \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{3^6}{5} \sin\varphi \, d\varphi \, d\theta$$

$$= \int_0^{2\pi} \frac{3^6}{5} \left[-\cos\varphi \right]_0^{\pi} \, d\theta$$

$$= \frac{3^6}{5} (1) \cdot 2\pi$$