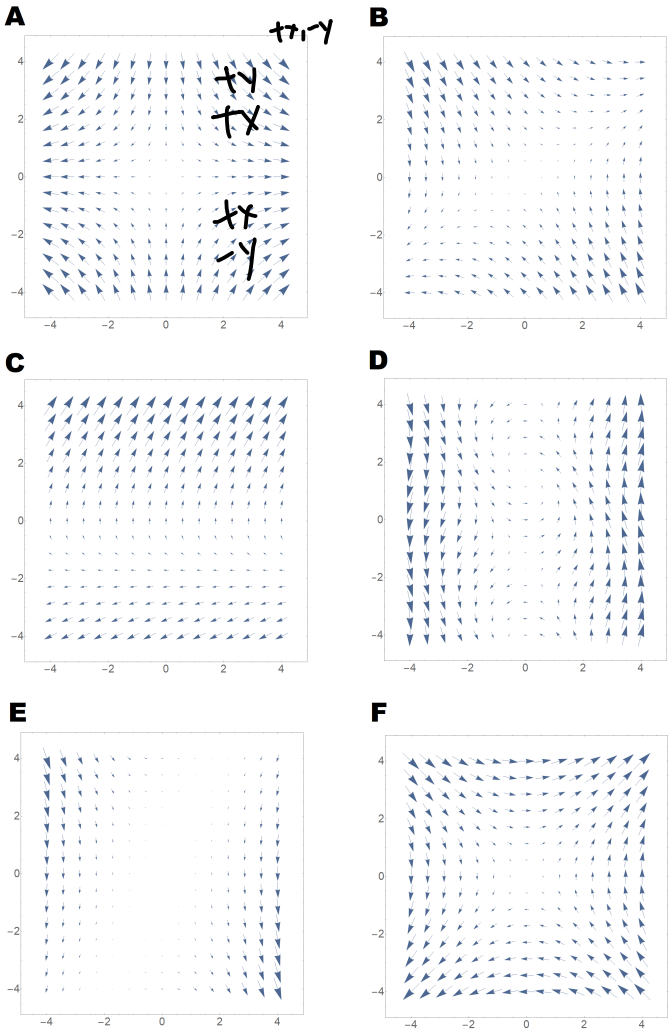
MATH 2023 – Multivariable Calculus

Lecture #13 Worksheet ♭ March 26, 2019

Problem 1. Identify vector fields $\mathbf{F}(x, y)$:



$\langle \cos(x+y), x \rangle$	$\langle y, y+2 \rangle$	$\langle x, -y \rangle$	$\langle y, x \rangle$	$\langle y, x-y \rangle$	$\langle y^2 - 2xy, 3xy - 6x^2 \rangle$
D	C	A	F	B	E

Problem 2. (a) Evaluate the line integral

$$\int_C (2 + x^2 y) ds$$

$\langle \cos\theta, \sin\theta \rangle$

$0 \le \theta \le \pi.$

where C is the upper half unit circle going counterclockwise.

$$\int_0^{\pi} (2 + \cos^2\theta \sin\theta) \, d\theta$$

(b) Evaluate the line integral

$$\int_C y^2 dx + x dy$$

where C is a curve from $(-5, -3)$ to $(0, 2)$

- Along a straight line
- Along the x and y direction passing through $(0, -3)$
- Along the curve $x = 4 - y^2$

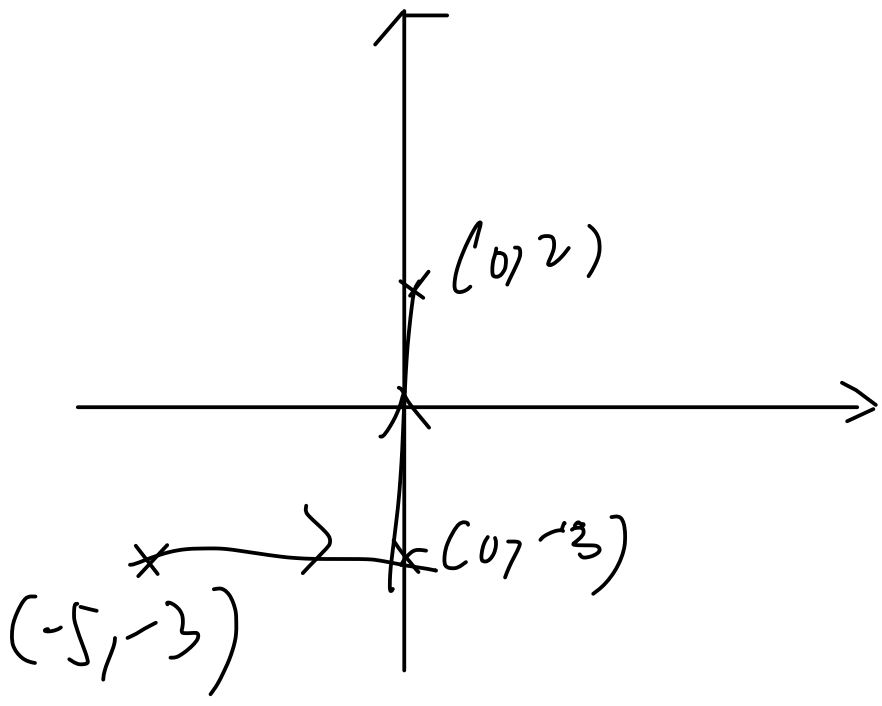
(1) $\vec{r}_1(t) = \langle -5+5t, \; -3+5t \rangle$

$0 \le t \le 1$

$y^2(5dt) + x(5dt)$

$\int_0^1 (-3+5t)^2 (5dt) + 5(-5+5t)\, dt$

2.



$r_2(t) = \langle -5+5t, -3 \rangle$ when $0 \le t \le 1$,

$r_2(t) = \langle 0, -3+5(t-1) \rangle$ when $1 \le t \le 2$

$y^2 dx + x dy$.

$-8+5t$

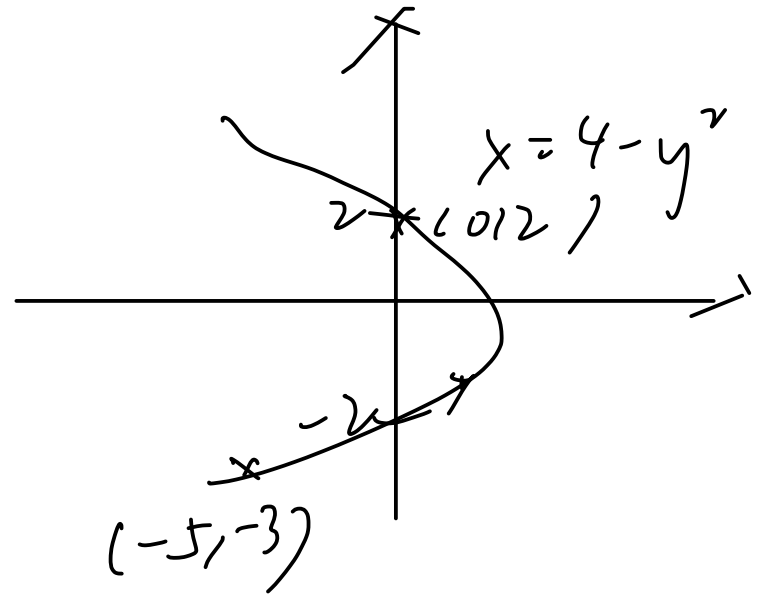
$$\int_0^1 y^2(5)\,dt + x(0)\,dt +$$

$$\int_1^2 0\,dt + x(5)\,dt$$

$$\int_0^1 9(5)\,dt + \int_1^2 0\,dt$$

$$= 45$$

3.



$\vec{\gamma_3}(t) = \langle 4 - t^2, t \rangle$

$y^2 dx + x dy$. $-3 \le t \le 2$

$$\int_{-3}^{2} y^2 (-2t)\, dt + \int_{-3}^{2} x\, dt$$

$$= \int_{-3}^{2} -2t^3\, dt + \int_{-3}^{2} 4 - t^2\, dt.$$