HKUST **MATH 102**

Midterm One Examination

Multivariable and Vector Calculus

30 Oct 2007

Answer ALL 5 questions

Time allowed – 120 minutes

Problem 1

Identify the following surfaces

(a) $\mathbf{r}\cdot\hat{\mathbf{u}} = 0$.

(b) $(\mathbf{r}-\mathbf{a})\cdot(\mathbf{r}-\mathbf{b}) = k$.

(c) $\|\mathbf{r}-(\mathbf{r}\cdot\hat{\mathbf{u}})\hat{\mathbf{u}}\| = k$. [Hint: What are the vectors $(\mathbf{r}\cdot\hat{\mathbf{u}})\hat{\mathbf{u}}$ and $\mathbf{r}-(\mathbf{r}\cdot\hat{\mathbf{u}})\hat{\mathbf{u}}$?]

Here k is fixed scalar, $\mathbf{a}$, $\mathbf{b}$ are fixed 3D vectors and $\hat{\mathbf{u}}$ is a fixed 3D unit vector and $\mathbf{r} = (x, y, z)$.

Problem 2

(a) Find the velocity, speed and acceleration at time t of the particle whose position is $\mathbf{r}(t)$. Describe the path of the particle.

$$\mathbf{r} = at\,\cos\omega t\,\mathbf{i} + at\,\sin\omega t\,\mathbf{j} + b\,\ln t\,\mathbf{k}$$

(b) Find the required parametrization of the first quadrant part of the circular arc $x^2+y^2=a^2$ in terms of arc length measured from $(0, a)$, oriented clockwise.

(c) Let C be the curve $x^{2/3}+y^{2/3}=a^{2/3}$ on the xy-plane, find the parametric equation of the curve C. Hence find the tangent line to the curve C at $(a, 0)$.

Problem 3

(a) Assume $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, $\mathbf{x}$ and $\mathbf{y}$ are three dimensional vectors and if

$$(\mathbf{a}\times\mathbf{b})\cdot(\mathbf{b}\times\mathbf{c})\times(\mathbf{c}\times\mathbf{a}) = [\mathbf{x}\cdot\mathbf{y}]^2.$$

Use suffix notation to find $\mathbf{x}$ and $\mathbf{y}$ in terms of the vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.

(b) Show that the line $x=-1+t$, $y=3+2t$, $z=-t$ and the plane $2x-2y-2z+3=0$ are parallel, and find the distance between them.

Problem 4

(a) Find the parametric equation of the curve of intersection C between the plane $z = 2y+3$ and the surface $z = x^2+y^2$. Find also the equation of the projection curve of the curve of intersection C onto the xz-plane.

(b) Evaluate $\displaystyle\lim_{(x,y)\to(0,0)} \frac{|x|+|y|}{\sqrt{x^2+y^2}}$.

Problem 5

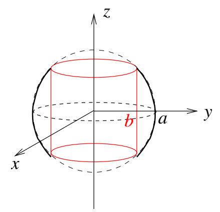
(a) Let $f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0). \end{cases}$

(i) Use the definition of partial derivative to show that $f_x(0,0)$ and $f_y(0,0)$ exist.

(ii) Is the function f continuous at $(0,0)$?

(b) Determine (sketch) the graph of the spherical-coordinate equation $\rho = 2\cos\phi$.

(c) A sphere of radius a is centered at the origin. A hole of radius b is drilled through the sphere, with the axis of the hole lying on the z-axis. Describe the solid region that remains (see Figure) in a (i) cylindrical coordinates; (ii) spherical coordinates.



Bonus question

Find the maximum and minimum distances between the point $(1,1,1)$ and a point on the curve of intersection of the cone $z=\sqrt{x^2+y^2}$ and the sphere $x^2+y^2+z^2=z$. (Note that the curve of intersection is not the origin).