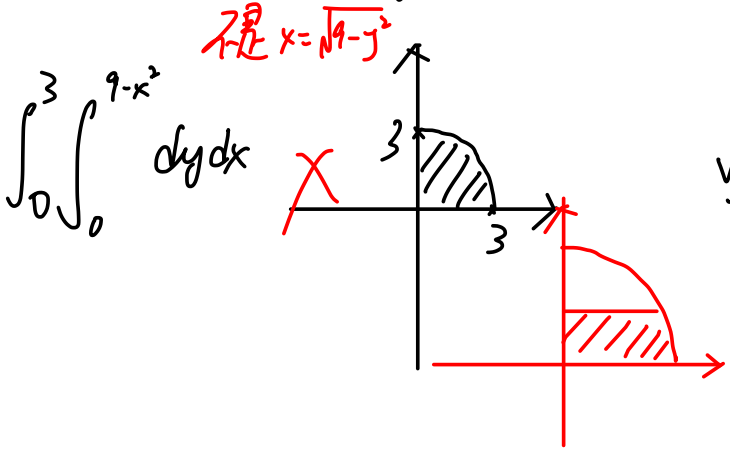
MATH 2023 – Multivariable Calculus

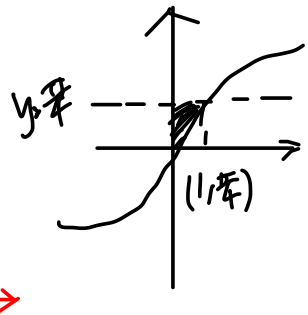
Lecture #10 Worksheet ♮ March 12, 2019

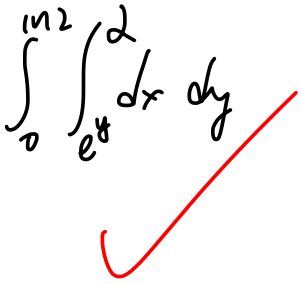
Problem 1. Sketch the region of integration, and interchange the order:

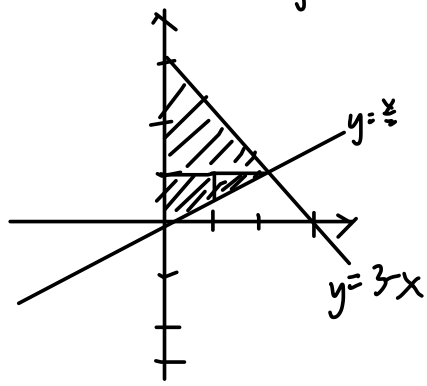
(a) $\int_0^3 \int_0^{\sqrt{9-y}} dx dy$, (b) $\int_0^1 \int_{\tan^{-1} x}^{\pi/4} dy dx$

(c) $\int_1^2 \int_0^{\ln x} dy dx$, (d) $\int_0^1 \int_0^{2y} dx dy + \int_1^3 \int_0^{3-y} dx dy$









Problem 2. Set up the two types of integrations of the following regions D bounded by:

(a) $y = 2x^2$ and $y = 1 + x^2$

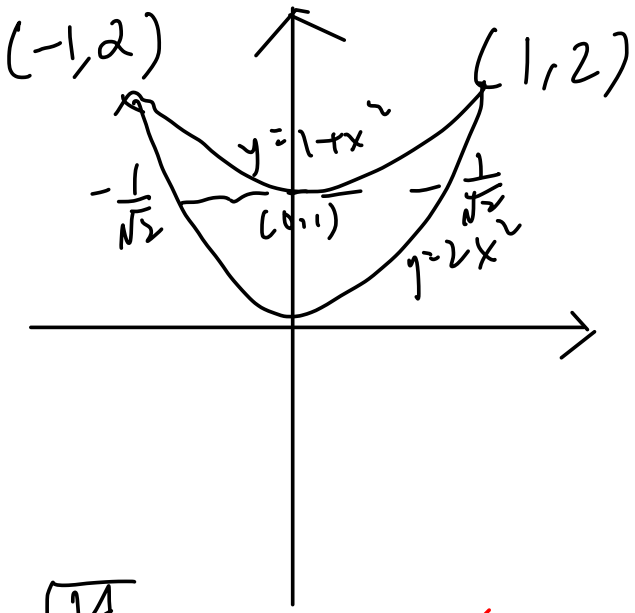
(b) $y = 2x$ and $y = x^2$

(c) $y = (x+1)^2, x = y - y^3, x = -1$ and $y = -1$

(d) $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$

Problem 2. Set up the two types of integrations of the following regions D bounded by:

(a) $y = 2x^2$ and $y = 1 + x^2$ (b) $y = 2x$ and $y = x^2$



a).

Type I:

$$\int_0^1 \int_{-\sqrt{\frac{y}{2}}}^{\sqrt{\frac{y}{2}}} dx\,dy + \int_1^2 \int_{-\sqrt{\frac{y}{2}}}^{-\sqrt{y-1}} dx\,dy +$$

$$\int_1^2 \int_{\sqrt{y-1}}^{\sqrt{\frac{y}{2}}} dx\,dy$$

Type II:

$$\int_{-1}^{1} \int_{2x^2}^{1+x^2} dy\,dx$$

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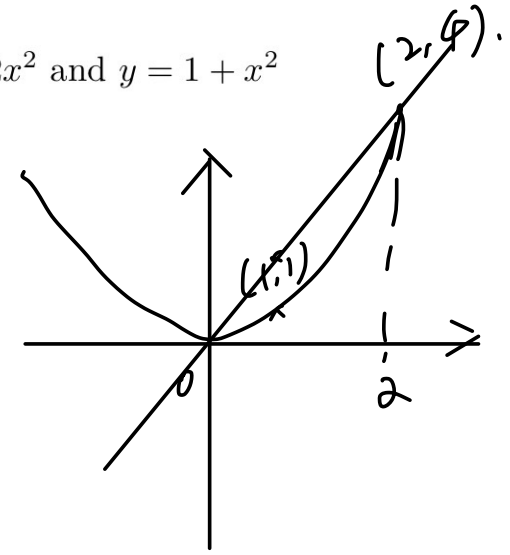


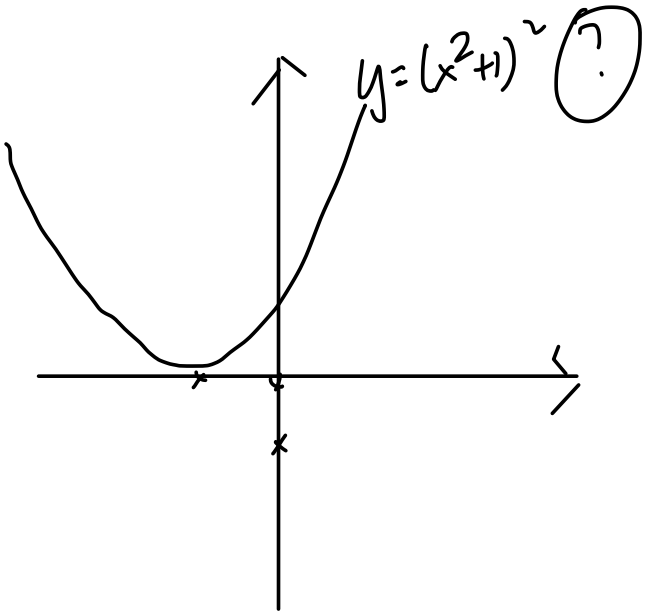
Type I:

$$\int_0^4 \int_{\frac{y}{2}}^{\sqrt{y}} dx\, dy$$

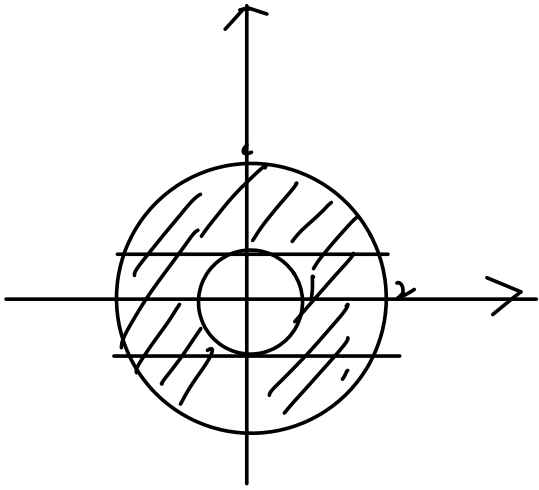
Type II:

$$\int_0^2 \int_{x^2}^{2x} dy\, dx$$

(c) $y = (x+1)^2, x = y - y^3, x = -1$ and $y = -1$



(d) $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$



$z = x+2$

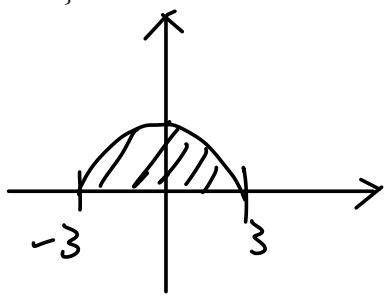
Problem 3. Use geometry or symmetry to evaluate the following integrals:

(a) $\iint_D (x+2)dA, \qquad D = \{(x,y) : 0 \le y \le \sqrt{9-x^2}\}$

$$\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}} (x+2)\,dy\,dx$$

$$= \int_{-3}^{3} xy + 2y \Big|_0^{\sqrt{9-x^2}}\,dx$$

$$= \int_{-3}^{3} x\sqrt{9-x^2} + 2\sqrt{9-x^2}\,dx$$



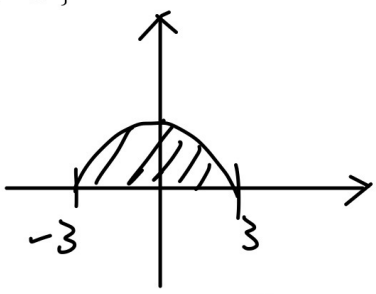
(b) $\iint_D (2 + x^2y^3 - y^2 \sin x)dA, \qquad D = \{(x,y) : |x| + |y| \le 1\}$

(c) $\iint_D (ax^3 + by^3 + \sqrt{a^2 - x^2})dA, \qquad D = [-a,a] \times [-b,b]$

$z = x+y$

Problem 3. Use geometry or symmetry to evaluate the following integrals:

(a) $\iint_D (x+2)dA, \qquad D = \{(x,y) : 0 \le y \le \sqrt{9-x^2}\}$



$$\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}} (x+2)\,dy\,dx$$

$$= \int_{-3}^{3} xy + 2y \Big|_0^{\sqrt{9-x^2}}\,dx$$

$$= \int_{-3}^{3} x\sqrt{9-x^2} + 2\sqrt{9-x^2}\,dx$$

$$= 0 + \int_{-3}^{3} 2\sqrt{9-x^2}\,dx$$

$$= 9$$