MATH 2023 – Multivariable Calculus

Lecture #17 Worksheet 🖐 April 11, 2019

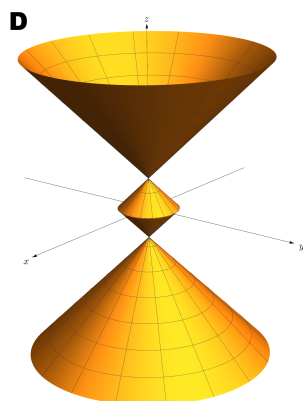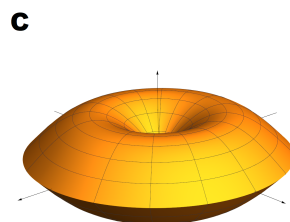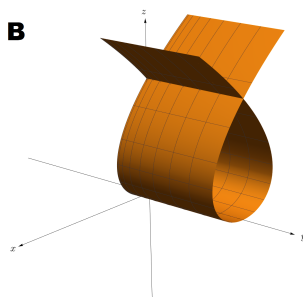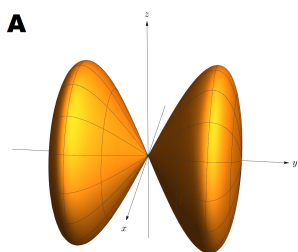**Problem 1.** Identify the following surfaces with their parametric equations.

A

B

C

D

E

F
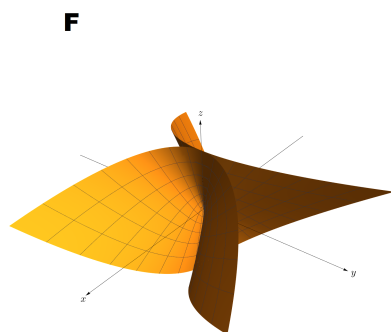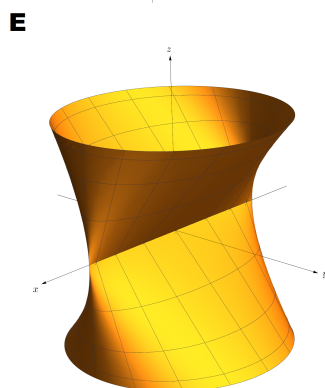
$\langle (1-|u|)\cos v, (1-|u|)\sin v, u\rangle$	$\langle \cos u \sin 2v, \sin v, \sin u \sin 2v\rangle$	$\langle uv^2, u^2v, u^2 - v^2\rangle$
$\langle u\cos v, u\sin v, \sin u\rangle, \quad -\pi \le u \le \pi$	$\langle \sin u, \cos u \sin v, \sin v\rangle$	$\langle u^3 - u, v^2, u^2\rangle$

Problem 2. Find the parametric equation for the torus with inner radius 1 and outer radius 3 around the z-axis.
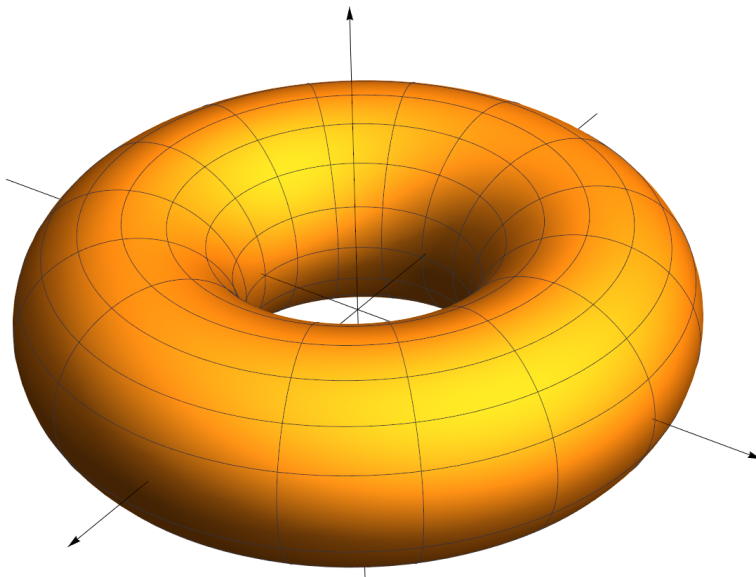
Problem 3. Describe the following surfaces.

$$\begin{cases} x & = (2 - u \cos \dfrac{v}{2}) \cos v \\[2mm] y & = (2 - u \cos \dfrac{v}{2}) \sin v \\[2mm] z & = u \sin \dfrac{v}{2} \end{cases} \quad , \qquad -\dfrac{1}{2} \le u \le \dfrac{1}{2}, 0 \le v \le 2\pi$$
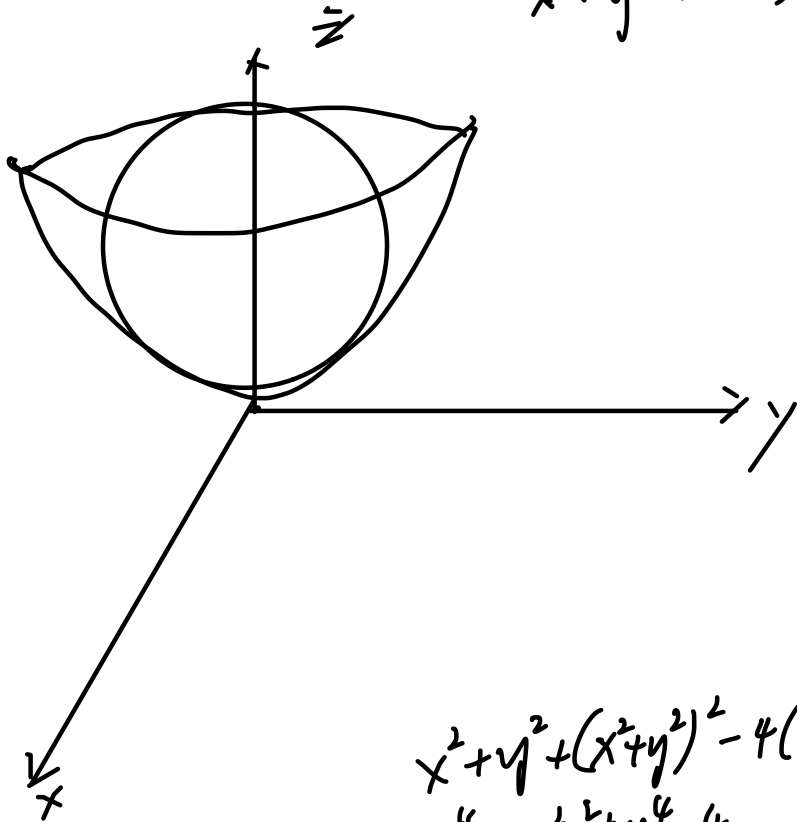
$$\begin{cases} x & = (1 - u)(2 + \cos v) \cos(6\pi u) \\ y & = (1 - u)(2 + \cos v) \sin(6\pi u) \\ z & = 4u + (1 - u) \sin v \rangle \end{cases} \qquad 0 \le u \le 1, 0 \le v \le 2\pi$$

$$x^2 + y^2 + z^2 - 4z + 4 - 4 = 0$$
$$(z-1)^2 = 4$$

Problem 4. Find the surface area of the part of the sphere $x^2 + y^2 + z^2 = 4z$ that lies inside the paraboloid $z = x^2 + y^2$.

$$x^2 + y^2 + (z-2)^2 = 4$$

$$x^2 + y^2 + (x^2 + y^2)^2 - 4(x^2 + y^2) = 0$$
$$x^2 + y^2 + x^4 + 2x^2y^2 + y^4 - 4 \qquad = 0$$
$$x^2 - 4y^2$$
$$x^4 + 2x^2y^2 + y^4 - 3x^2 - 3y^2 = 0$$

Problem 4. Find the surface area of the part of the sphere $x^2 + y^2 + z^2 = 4z$ that lies inside the paraboloid $z = x^2 + y^2$.

$x^2 + y^2 + (x^2 + y^2)^2 = 4(x^2 + y^2)$

$(x^2 + y^2)^2 = 3(x^2 + y^2)$