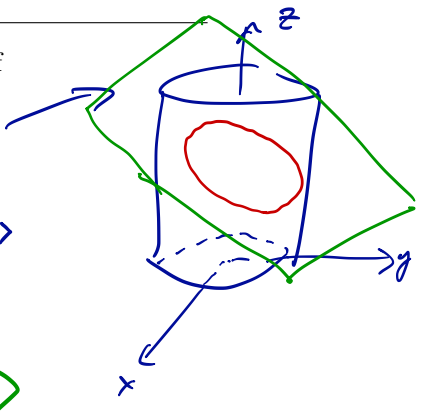
MATH 2023 – Multivariable Calculus

Lecture #02 Worksheet ♠ February 12, 2019

Problem 1. (a) Find the curve of intersection of

$$\begin{cases} x^2 + y^2 = 1 \\ y + z = 2 \end{cases}$$

plane: $\vec{n} = \langle 0, 1, 1 \rangle$

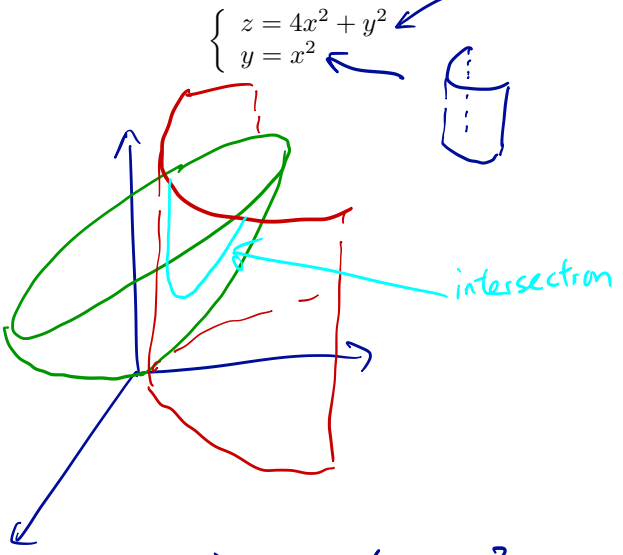


$\vec{r}(t) = \langle \cos t, \sin t, 2 - \sin t \rangle$

(b) Find the curve of intersection of

$$\begin{cases} z = 4x^2 + y^2 \\ y = x^2 \end{cases}$$

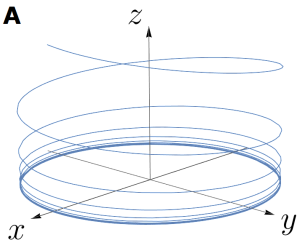
elliptic bowl



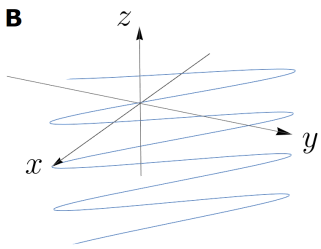
$\vec{r}(t) = \langle t, t^2, 4t^2 + t^4 \rangle$

Problem 2. Identify the pictures with the corresponding vector functions $\mathbf{r}(t)$.

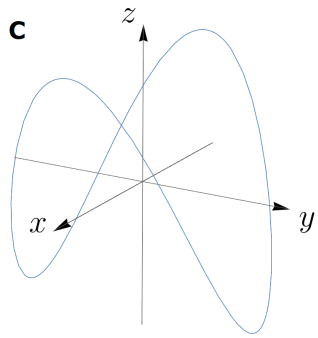
A

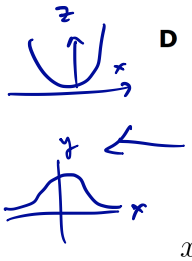


B

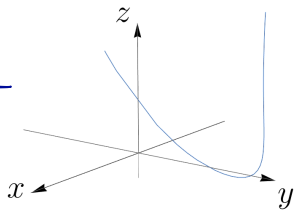


C

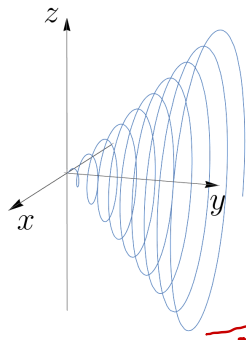




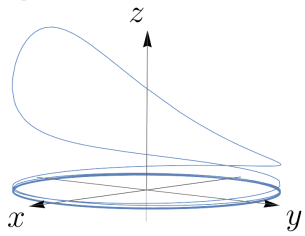
D



E



F



increasing "circles" in x-z

circle in x-y

parabola

$\mathbf{r}_1(t) = \langle t\cos t, t, t\sin t\rangle, (t \geq 0)$	$\mathbf{r}_2(t) = \langle \cos t, \sin t, \frac{1}{1+t^2}\rangle$	$\mathbf{r}_3(t) = \langle t, \frac{1}{1+t^2}, t^2\rangle$
E	F	D
$\mathbf{r}_4(t) = \langle \cos 8t, \sin 8t, e^{-t}\rangle, (t \geq 0)$	$\mathbf{r}_5(t) = \langle \cos t, \sin t, \cos 2t\rangle$	$\mathbf{r}_6(t) = \langle \cos^2 t, \sin^2 t, t\rangle$
A	C	B

$x+y=1$

Problem 3. Consider the paths of two particles given by

$$\mathbf{r}_1(t) = (\sin t)\mathbf{i} + (\cos t)\mathbf{j} + 5t\mathbf{k}$$
$$\mathbf{r}_2(t) = \langle t, -1 + t, 5\pi - t\rangle$$

(a) Find the velocity, speed, and acceleration of each particle at time t.

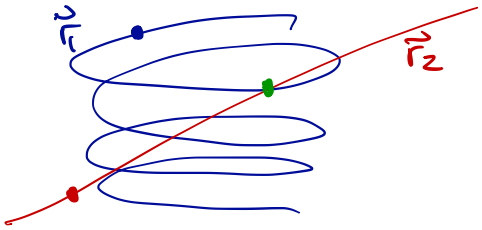
$\vec{v}_1(t) = \vec{r}_1'(t) = \langle \cos t, -\sin t, 5\rangle$ speed $= \sqrt{26}$

$\vec{a}_1(t) = \vec{r}_1''(t) = \langle -\sin t, -\cos t, 0\rangle$

$\vec{v}_2(t) = \vec{r}_2'(t) = \langle 1, 1, -1\rangle$, speed $= \sqrt{3}$

$\vec{a}_2(t) = \vec{r}_2''(t) = \langle 0, 0, 0\rangle$

(b) Do the two particles collide?



$\Longleftrightarrow$ does $\vec{r}_1(t) = \vec{r}_2(t)$ have solution?

z: $5t = 5\pi - t \Rightarrow 6t = 5\pi \Rightarrow t = \frac{5\pi}{6}$

x: $\sin\frac{5\pi}{6} \neq \frac{5\pi}{6}$ No solution!

(c) Do the trajectories of two particles intersect each other?

$\Longleftrightarrow$ does $\vec{r}_1(t) = \vec{r}_2(s)$ have solution?

Yes: $\begin{cases} t = \pi \\ s = 0 \end{cases}$ is a solution.