MATH2023 Multivariable Calculus 2013

From the textbook Calculus of Several Variables (5th) by R. Adams, Addison Wesley.

Homework 7

(*Total: 12 questions*)

Ex. 14.5

4 Evaluate the triple integral $\iiint_R x\,dV$, where R is the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

Be alert for simplifications and auspicious orders of iteration.

11 Evaluate the triple integral $\iiint_R \frac{1}{(x+y+z)^3}\,dV$, where R is the region bounded by the six planes $z=1$, $z=2$, $y=0$, $y=z$, $x=0$, and $x=y+z$.

Be alert for simplifications and auspicious orders of iteration.

16 Sketch the region R in the first octant of 3-space that has finite volume and is bounded by the surfaces $x=0$, $z=0$, $x+y=1$, and $z=y^2$. Write six different iterations of the triple integral of $f(x,y,z)$ over R.

19 Express the iterated integral as a triple integral and sketch the region over which it is taken. Reiterate the integral so that the outermost integral is with respect to x and the innermost is with respect to z.

$$\int_0^1\int_z^1\int_0^{x-z} f(x,y,z)\,dydxdz.$$

27 Evaluate the iterated integral by reiterating it in a different order. (You will need to make a good sketch of the region.)

$$\int_0^1\int_z^1\int_0^x e^{x^3}\,dydxdz.$$

Ex. 14.6

19 Find the volume of the region above the xy-plane, inside the cone $z=2a-\sqrt{x^2+y^2}$ and inside the cylinder $x^2+y^2=2ay$.

25 Find $\iiint_B (x^2+y^2)\,dV$, where B is the ball given by $x^2+y^2+z^2\leqslant a^2$.

30 Evaluate $\iiint_R (x^2+y^2)\,dV$ over the region R, where R is the region which lies above the cone $z=c\sqrt{x^2+y^2}$ and inside the sphere $x^2+y^2+z^2=a^2$.

Ex. 14.7

2 Use double integral to calculate the area of the part of the plane $5z=3x-4y$ inside the elliptic cylinder $x^2+4y^2=4$.

6 Use double integral to calculate the area of the paraboloid $z=1-x^2-y^2$ in the first octant.

10 Show that the parts of the surfaces $z=2xy$ and $z=x^2+y^2$ that lie in the same vertical cylinder have the same area.

Qu Find the volume bounded by the surface with equation $x^{2/3}+y^{2/3}+z^{2/3}=a^{2/3}$.

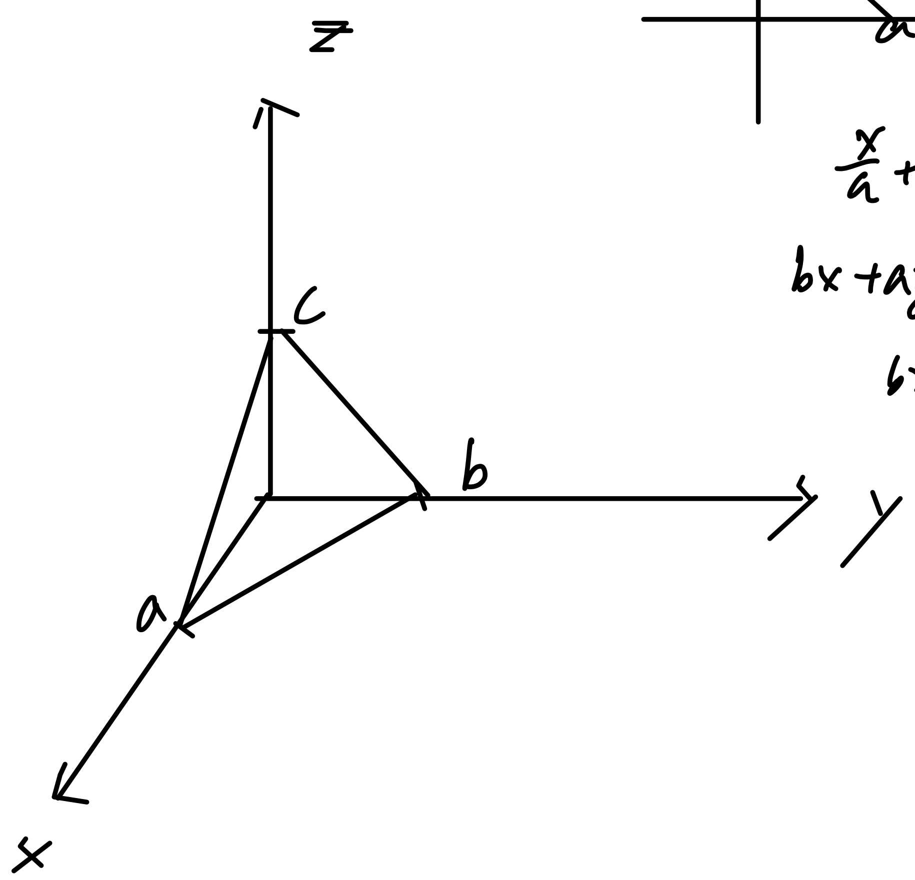
* **Only hand in the underlined ones, the others are recommended exercises.**

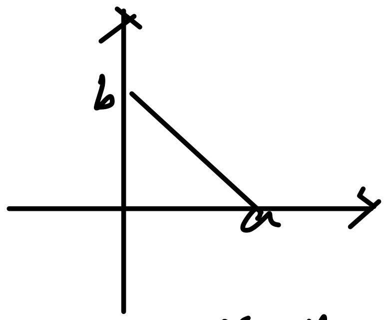
Ex. 14.5

4 Evaluate the triple integral $\iiint_R x\, dV$, where R is the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

Be alert for simplifications and auspicious orders of iteration.

$$\int_0^b \int_0^{\frac{a(b-y)}{b}} \int_0^{c(1-\frac{x}{a}-\frac{y}{b})} x \, dz\, dx\, dy$$





$$\frac{x}{a} + \frac{y}{b} = 1$$

$$bx + ay = ab$$

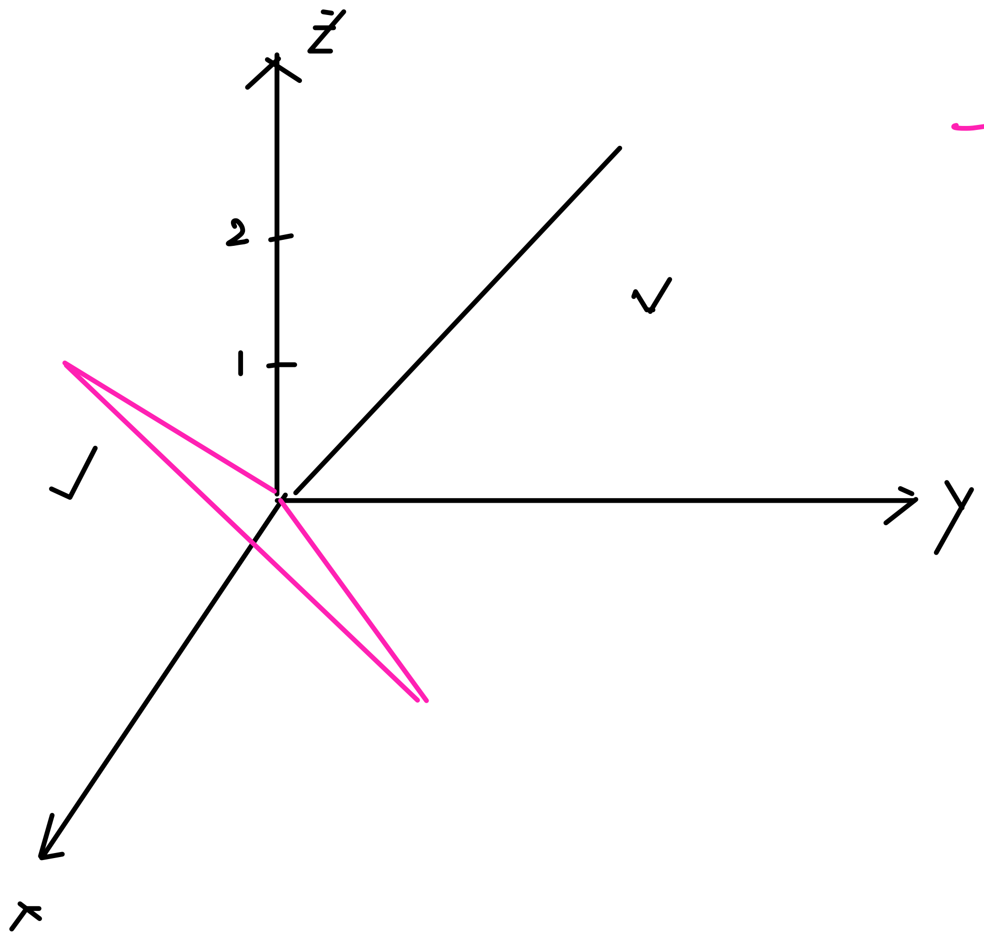
$$bx = ab - ay$$

$$x = \frac{ab - ay}{b}$$

$\underline{11}$ Evaluate the triple integral $\iiint_R \frac{1}{(x+y+z)^3}\, dV$, where R is the region bounded by the six planes $z = 1$, $z = 2$, $y = 0$, $y = z$, $x = 0$, and $x = y + z$.

Be alert for simplifications and auspicious orders of iteration.

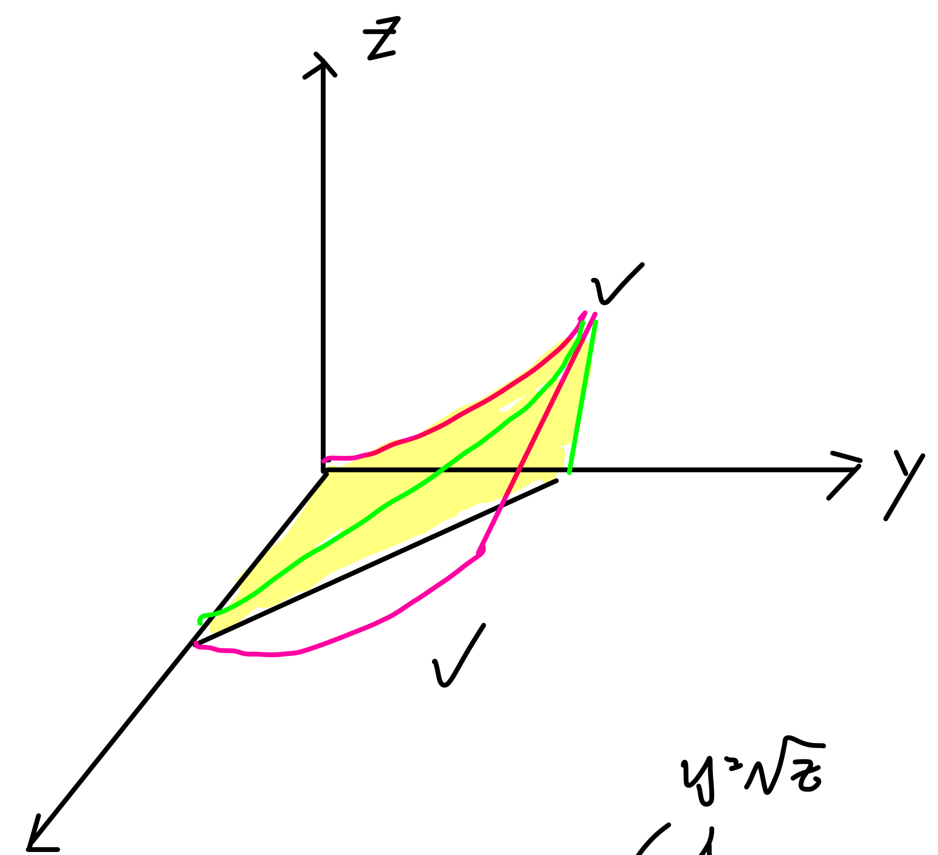
$z = x - y$



16 Sketch the region R in the first octant of 3-space that has finite volume and is bounded by the surfaces $x=0$, $z=0$, $x+y=1$, and $z=y^2$. Write six different iterations of the triple integral of $f(x,y,z)$ over R.

$y=\sqrt{z}$

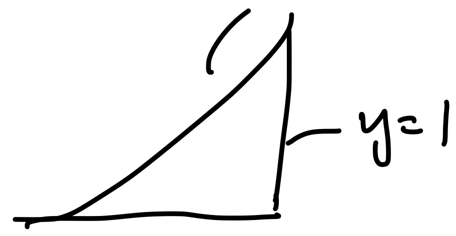
$x+\sqrt{z}=1$



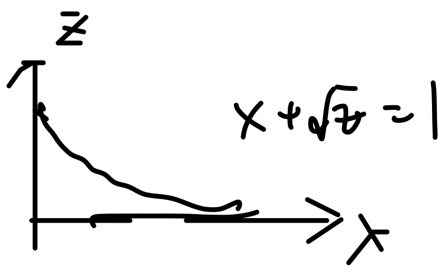
$$\int_0^1\int_0^{1-y}\int_0^{y^2} f\,dz\,dx\,dy$$

$$\int_0^1\int_0^{1-x}\int_0^{y^2} f\,dz\,dy\,dx$$

$$\int_0^1\int_{\sqrt{z}}^1\int_0^{1-y} f\,dx\,dy\,dz$$



$$\int_0^1\int_0^{y^2}\int_0^{1-y} f\,dx\,dz\,dy$$

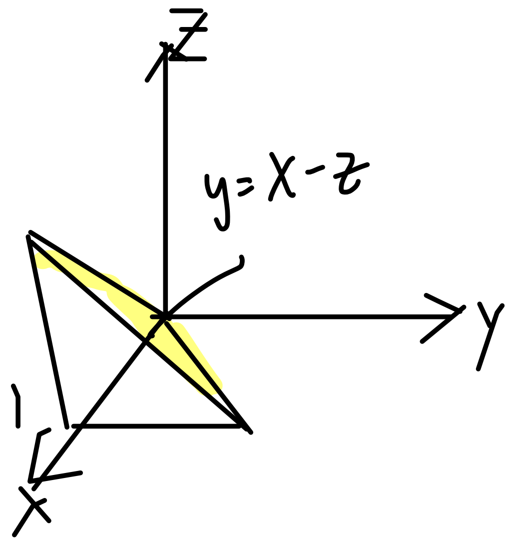


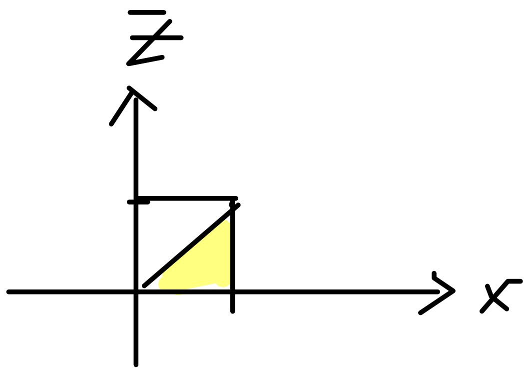
$$\int_0^1\int_0^{1-\sqrt{z}}\int_{\sqrt{z}}^{1-x} f\,dy\,dx\,dz$$

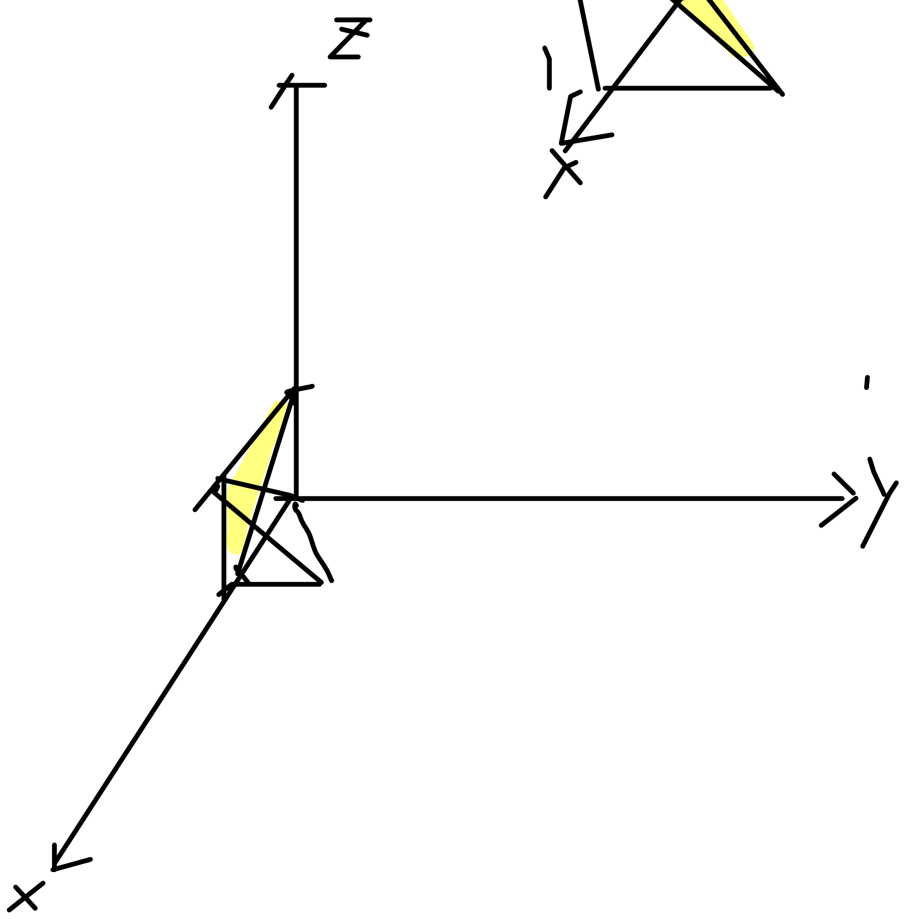
$$\int_0^1\int_0^{(1-x)^2}\int_{\sqrt{z}}^{1-x} f\,dy\,dz\,dx$$

19 Express the iterated integral as a triple integral and sketch the region over which it is taken. Reiterate the integral so that the outermost integral is with respect to x and the innermost is with respect to z.

$$\int_0^1 \int_z^1 \int_0^{x-z} f(x, y, z)\, dy dx dz.$$

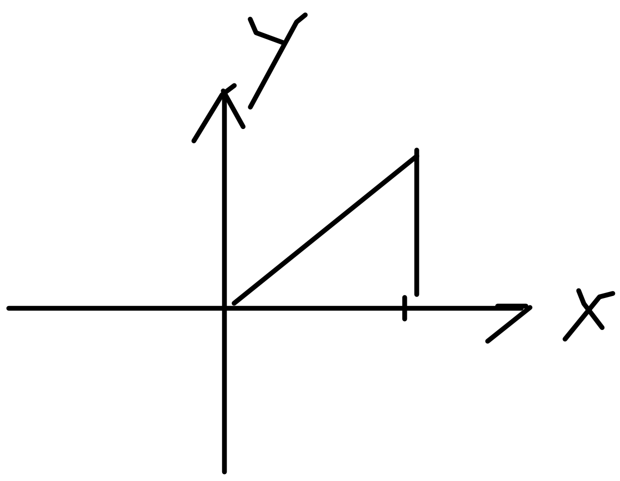






$y = x - z$

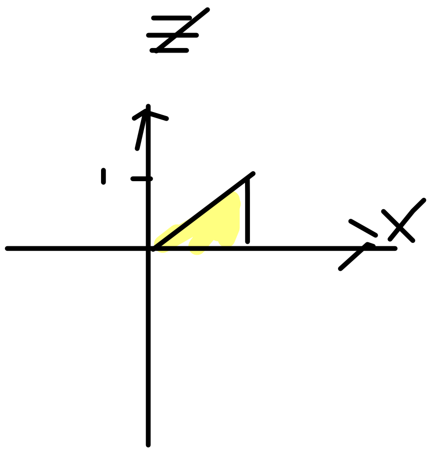
$z = x - y$

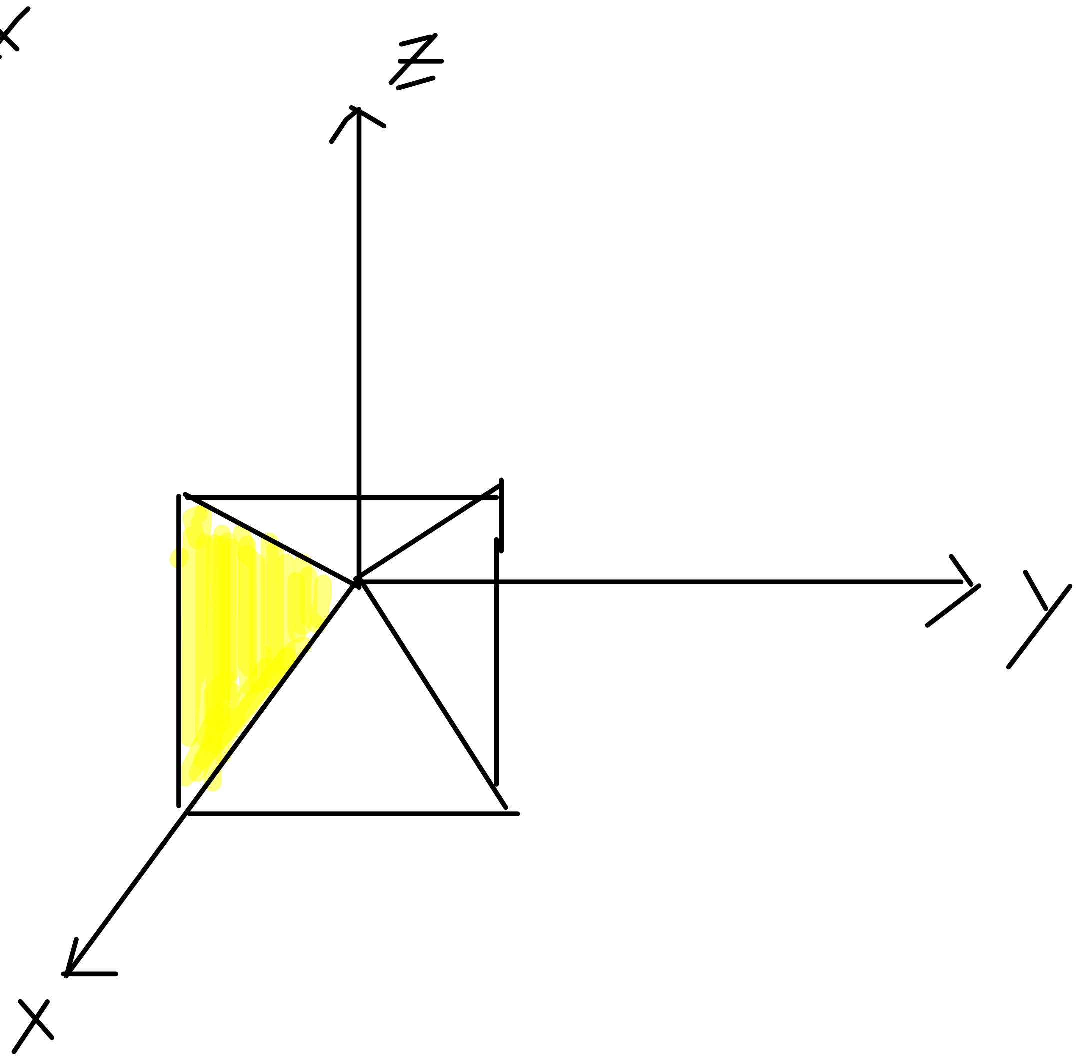


$$\int_0^1 \int_0^x \int_0^{x-y} f\, dz\, dy\, dx$$

27 Evaluate the iterated integral by reiterating it in a different order. (You will need to make a good sketch of the region.)

$$\int_0^1 \int_z^1 \int_0^x e^{x^3}\,dy\,dx\,dz.$$





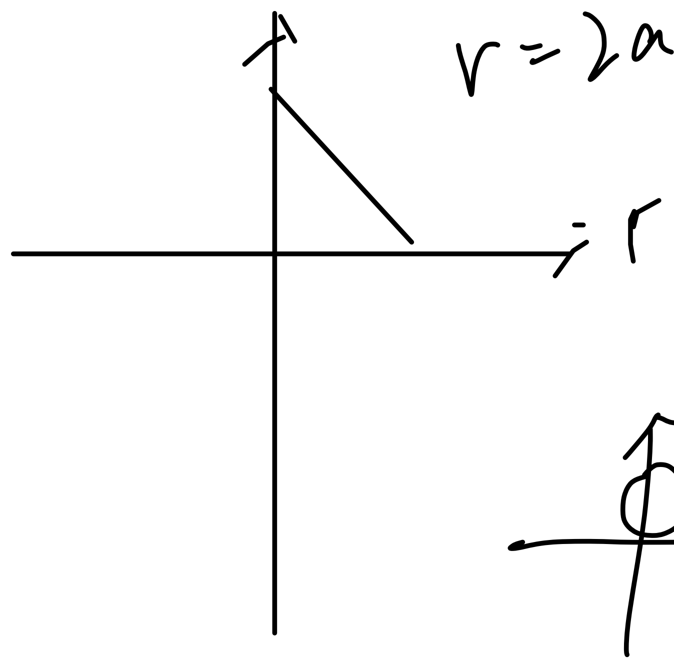
$$\int_0^1 \int_0^x \int_0^x e^{x^3}\,dz\,dy\,dx$$

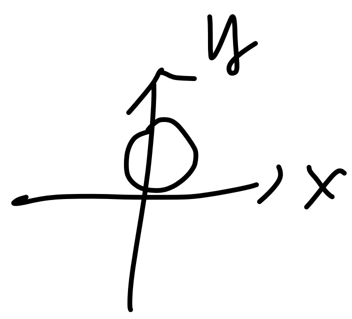
Ex. 14.6

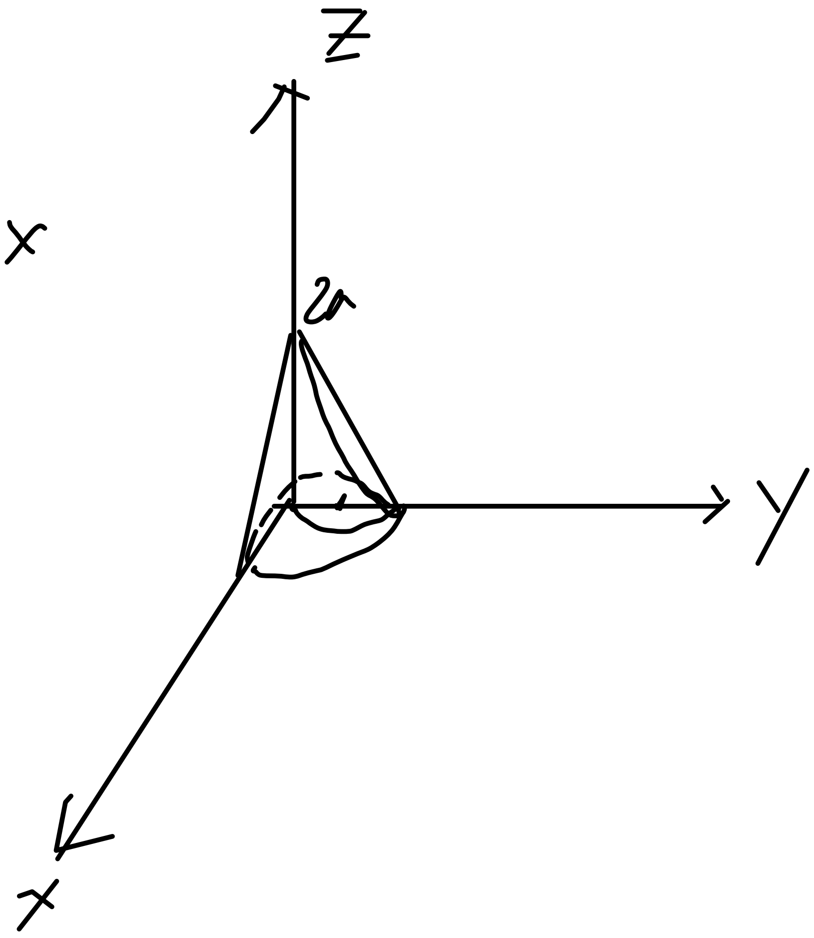
19 Find the volume of the region above the xy-plane, inside the cone $z = 2a - \sqrt{x^2+y^2}$ and inside the cylinder $x^2 + y^2 = 2ay$.

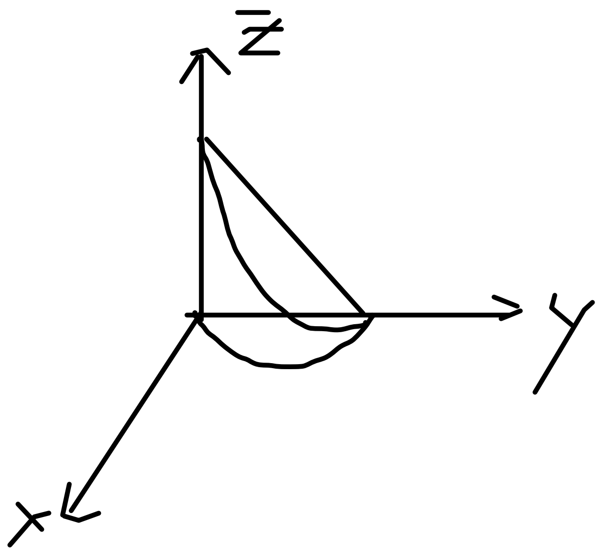
$r^2 = \cancel{z}\ 2ar\sin\theta$ $\qquad z = 2a - r$

$r = 2a\sin\theta$









$\alpha \cdot x \int_0^{\pi}\int_0^{2a\sin\theta} (2a - r)\, r\, dr\, d\theta$

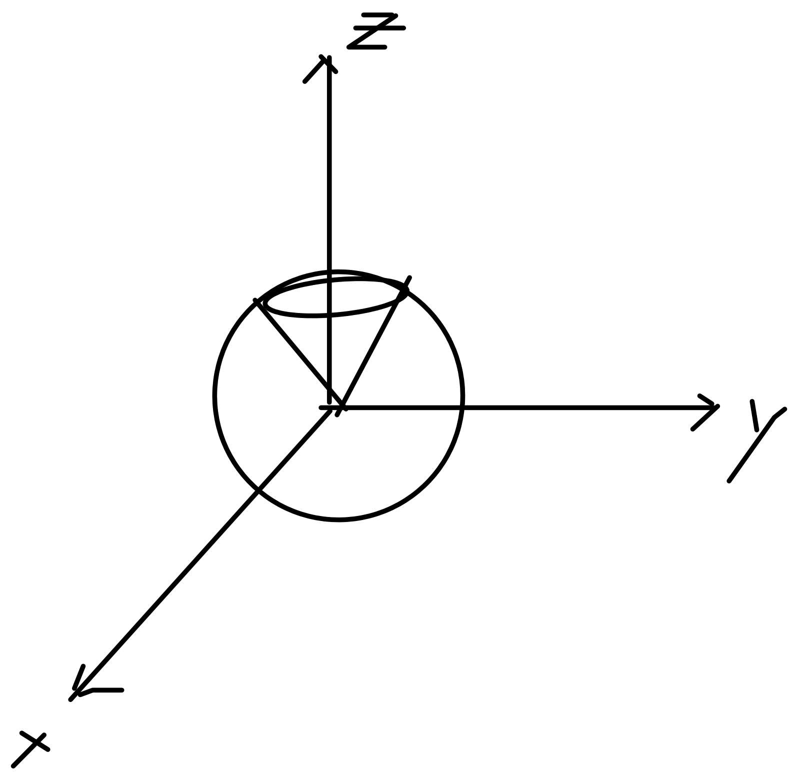
25 Find $\iiint_B (x^2+y^2)\,dV$, where B is the ball given by $x^2+y^2+z^2 \leqslant a^2$.

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{a} \rho^2 \sin\varphi \; \rho^2 \sin\varphi \; d\rho\, d\varphi\, d\theta$$

Made with Goodnotes

30 Evaluate $\iiint_R (x^2+y^2)\,dV$ over the region R, where R is the region which lies above the cone $z = c\sqrt{x^2+y^2}$ and inside the sphere $x^2+y^2+z^2=a^2$.

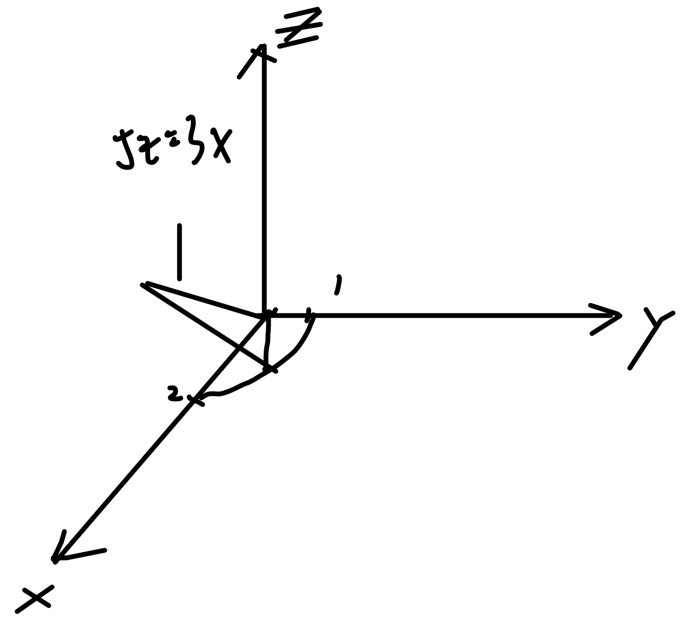
$$x^2+y^2+c^2(x^2+y^2)=a^2$$
$$(1+c^2)(x^2+y^2)=a^2$$
$$x^2+y^2=\frac{a^2}{1+c^2}$$



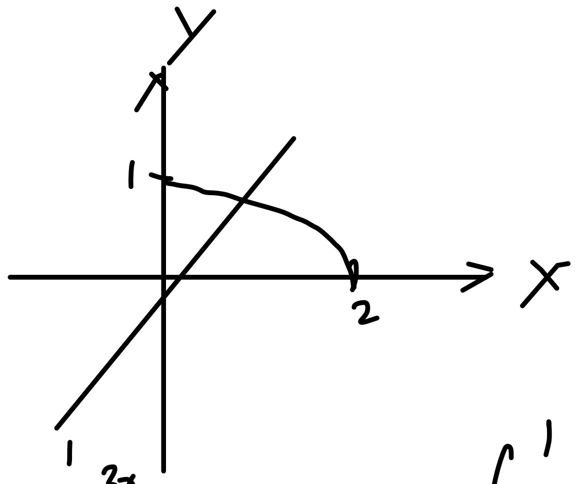
Ex. 14.7

2 Use double integral to calculate the area of the part of the plane $5z = 3x - 4y$ inside the elliptic cylinder $x^2 + 4y^2 = 4$.

$$\frac{x^2}{4} + y^2 = 1$$



$$3x - 4y = 0$$
$$3x = 4y$$
$$y = \frac{3x}{4}.$$



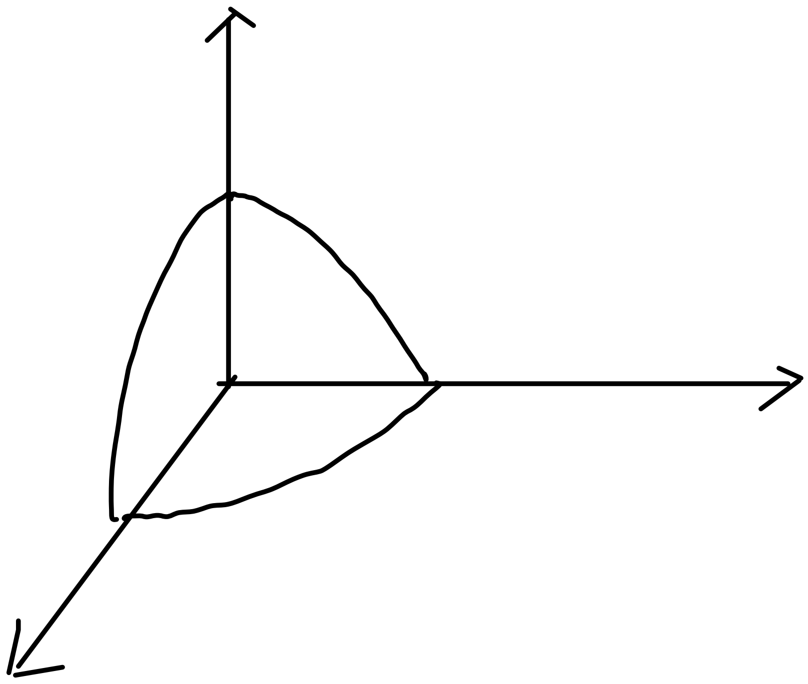
$y = \frac{3x}{4}.$

$$x^2 = 4 - 4y^2$$

$$\int_{-1}^{1} \int_{-\sqrt{4-4y^2}}^{\sqrt{4-4y^2}} \frac{3x-4y}{5} \, dx \, dy$$

6 Use double integral to calculate the area of the paraboloid $z = 1 - x^2 - y^2$ in the first octant.

$\iint_S 1 dS$



10 Show that the parts of the surfaces $z = 2xy$ and $z = x^2 + y^2$ that lie in the same vertical cylinder have the same area.

Qu Find the volume bounded by the surface with equation $x^{2/3} + y^{2/3} + z^{2/3} = a^{2/3}$.