

# MATH 2023 – Multivariable Calculus

Lecture #03 Worksheet    ♡    February 14, 2019

**Problem 1.** Find the arc length parametrization of the following curve from the point  $(1, 0)$ .

$$\mathbf{r}(t) = \left( \frac{2}{t^2+1} - 1 \right) \mathbf{i} + \frac{2t}{t^2+1} \mathbf{j}$$

What can you conclude about the curve?

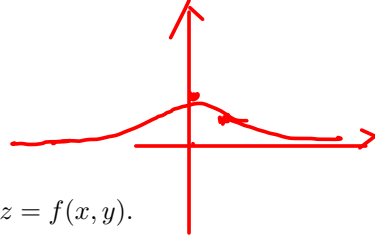
$$\mathbf{r}'(t) = \frac{-2(2t)}{(t^2+1)^2} \mathbf{i} + \frac{\overset{(2)}{(t^2+1)(2t)} - 2t(2t)}{(t^2+1)^2} \mathbf{j}$$

$$\mathbf{r}'(t) = \frac{-4t}{(t^2+1)^2} \mathbf{i} + \frac{2t^2+2t-4t^2}{(t^2+1)^2} \mathbf{j}$$

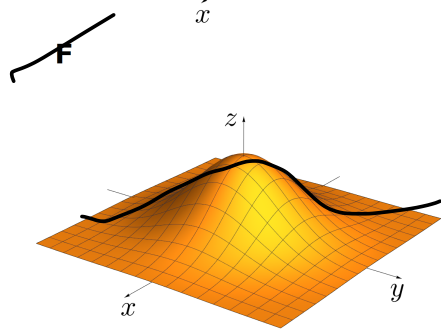
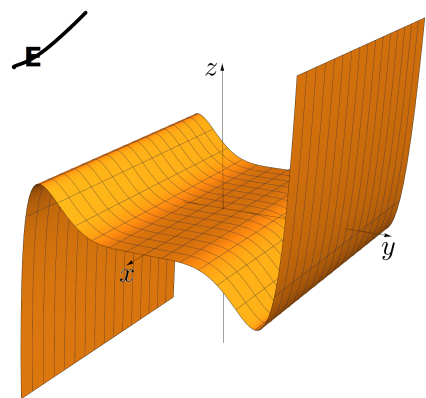
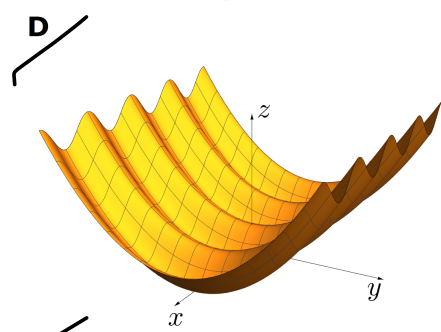
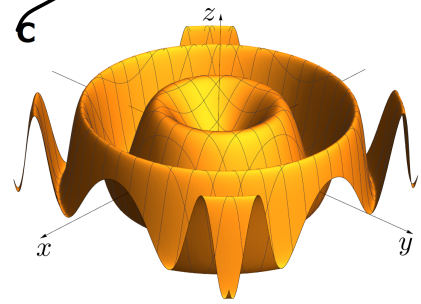
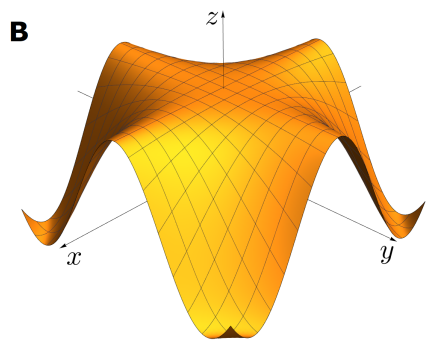
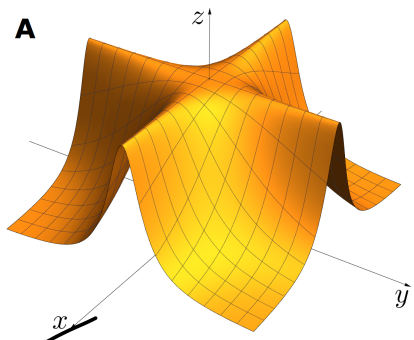
$$\mathbf{r}'(t) = \frac{-4t}{(t^2+1)^2} \mathbf{i} + \frac{2t-2t^2}{(t^2+1)^2} \mathbf{j}$$

$$\begin{aligned} |\mathbf{r}'(t)| &= \sqrt{\frac{(-4t)^2 + (2t-2t^2)^2}{(t^2+1)^4}} \\ &= \sqrt{\frac{16t^2 + 4t^2 - 8t^3 + 4t^4}{(t^2+1)^4}} \\ &= \sqrt{\frac{4t^4 - 8t^3 + 20t^2}{(t^2+1)^4}} \end{aligned}$$





**Problem 2.** Identify the graphs with the corresponding functions  $z = f(x, y)$ .



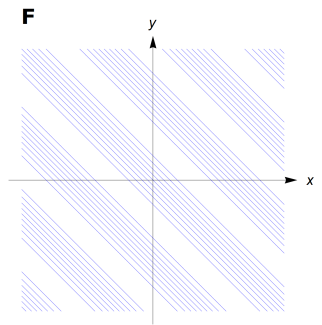
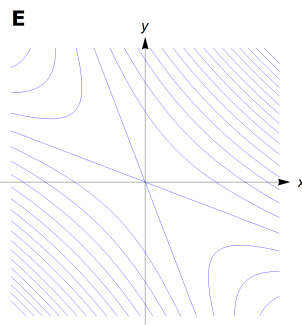
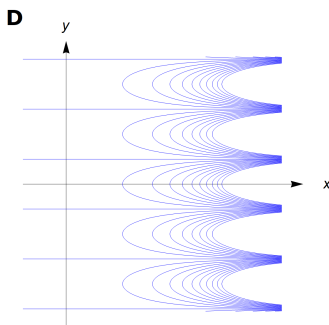
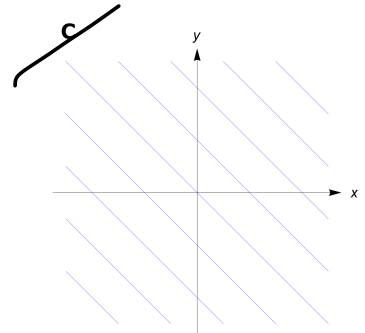
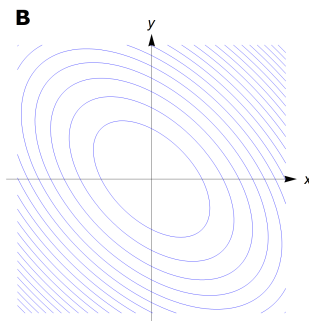
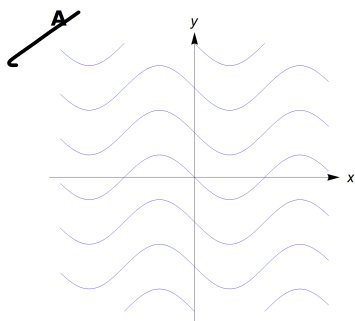
$\sin(r^2)$   $e^{-r^2}$

$\cos(x^2) + y^2$	$\sin(x^2 + y^2)$	$e^{-x^2 - y^2}$	$\cos(xy)$	$y^7 + 3y^2$	$\frac{1}{1+x^2y^2}$
D	C	<del>A</del>	B	E	F

A  $\frac{1}{1+v^2}$

↖ ↗  
?

**Problem 3.** Identify the level sets with the corresponding functions  $z = f(x, y)$ .



$x + y$	$\sin(x + y)$	$\sin x + y$	$x^2 + xy + y^2$	$x^2 + 3xy + y^2$	$e^x \cos y$
C	D <del>X</del>	A	B	F	E

$\sin(x+y) = k$   
 $x+y = k'$

$e^x \cos y = 0$   
 $e^x \cos y = 1$   
 $x=0$  or  $y=0$

**Bonus Problem.** Plot the graph and the level sets of the following function

$$f(x, y) = (x^2 + y^2 - 1)^3 - x^2 y^3.$$