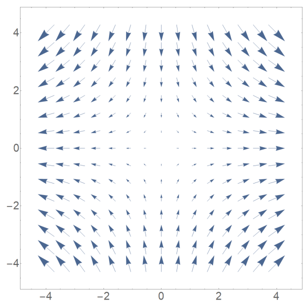
MATH 2023 – Multivariable Calculus

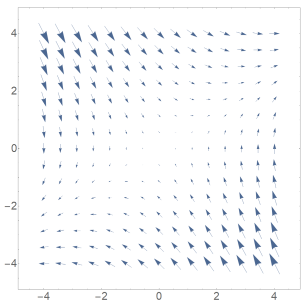
Lecture #13 Worksheet ♭ March 26, 2019

Problem 1. Identify vector fields $\mathbf{F}(x, y)$:

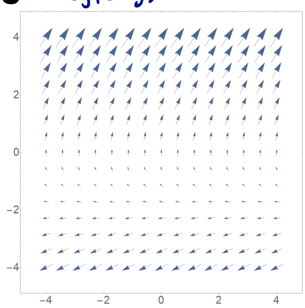
A



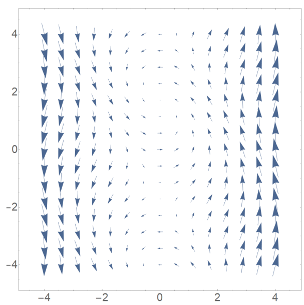
B



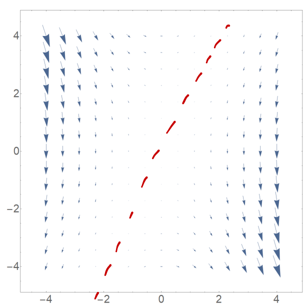
C $\langle y+2, y\rangle$



D

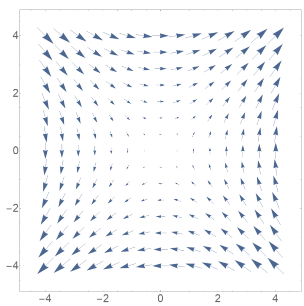


E



$y=2x$: no vectors

F



when $y=2x$ $\langle 0,0\rangle$

$\langle \cos(x+y), x\rangle$	$\langle y, y+2\rangle$	$\langle x, -y\rangle$	$\langle y, x\rangle$	$\langle y, x-y\rangle$	$\langle y^2-2xy, 3xy-6x^2\rangle$
D	C	A	F	B	E

$x=0$: $\langle \cos y, 0\rangle$

$x=1$ $\langle \cos(1+y), 1\rangle$

should be same arrow for any x

$\langle x,y\rangle$

$\langle x,-y\rangle$

$\langle -y,x\rangle$

$\langle y,x\rangle$

$\langle y,x\rangle - \langle 0,y\rangle$

$x=y$ $\langle y,0\rangle$

$y=0$ $\langle 0,x\rangle$

1

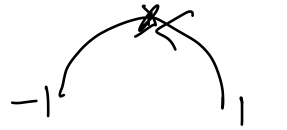
$$r'(t) = -\sin t\, i + \cos t\, j$$

$$|r'(t)| = 1 \qquad x = \cos t,\quad y = \sin t \qquad 0 < t < \pi$$

Problem 2. (a) Evaluate the line integral

$$\int_C (2 + x^2 y)\,ds$$

where C is the upper half unit circle going counterclockwise.



$$\int_0^{\pi} 2 + \cos^2 t \sin t \, dt$$

$$= 2\pi + \int_0^{\pi} \cos^2 t \sin t \, dt$$

Let $u = \cos t$, $du = -\sin t\,dt$, $dt = -\frac{1}{\sin t}\,du$

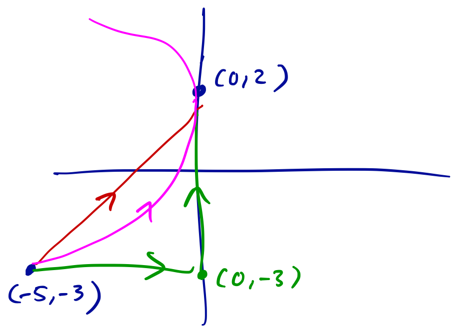
$$= 2\pi - \int_0^{\pi} u^2 du = 2\pi - \left[\frac{u^3}{3}\right]_0^{\pi} = 2\pi - \frac{\pi^3}{3}$$

(b) Evaluate the line integral

$$\int_C y^2 dx + x dy$$

where C is a curve from $(-5, -3)$ to $(0, 2)$

- (1) Along a straight line
- (2) Along the x and y direction passing through $(0, -3)$
- Along the curve $x = 4 - y^2$



$$\begin{cases} x = -5 + 5t \\ y = -3 + 5t \end{cases} \quad 0 \le t \le 1$$

$$x'(t) = 5, \quad y'(t) = 5$$

(1) $$\int_0^1 (-3+5t)5 + (-5+5t)5 \, dt = \cdots$$

(2) $$= \int_{C_1 \rightarrow} + \int_{C_2 \uparrow} = \int_{C_1} y^2 dx + \int_{C_2} x\,dy$$

$$= \int_{-5}^{0} (-3)^2 dx + \int_{-3}^{2} 0\, dy$$

$$= 45$$

(3) $$\begin{cases} x = 4 - t^2 \\ y = t \end{cases}, \quad -3 \le t \le 2$$

$$x'(t) = -2t$$
$$y'(t) = 1$$

$$\hookrightarrow \int_{-3}^{2} t^2(-2t)\,dt + \int_{-3}^{2} (4 - t^2)\,dt = \cdots$$