# MATH 2023 – Multivariable Calculus

Lecture #05 Worksheet ♣ February 21, 2019

---

**Problem 1.** Find $\dfrac{\partial}{\partial y}\left(\dfrac{\partial f}{\partial x}\right)$ where

$$f(x,y) = \frac{e^{2019x^2}}{\ln\sqrt{x^2+2023}} + \sin(xy)$$

$\frac{\partial}{\partial x}\ \frac{\partial f}{\partial y} \Rightarrow \cos(xy)(x)$

$\frac{\partial}{\partial x} \Rightarrow x(-\sin(xy)(y)) + \cos(xy)$

$= -xy\sin(xy) + \cos(xy)$

**Problem 2.** Consider the function

$$f(x,y) = \begin{cases} \dfrac{xy(x^2-y^2)}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

Show that $f$ is continuous, $f_x, f_y$ continuous, but

$$f_{xy}(0,0) \neq f_{yx}(0,0)$$

Why does this violate the Mixed Partial Theorem?

why?

$f$: $\dfrac{r^2\cos\theta\sin\theta(r^2\cos 4\theta - r^2\sin^2\theta)}{r^2}$

$\Rightarrow 0$.

$f_x$: 0

$f_y$: 0

**Problem 3.** (a) Show that

$$u(x,t) = \sin(x - at)$$

is a solution to the **wave equation**

$$u_{tt} = a^2 u_{xx}$$

$u_t = \cos(x-at)(-a)$

$u_{tt} = -a^2 \sin(x-at)$

$u_x = \cos(x-at)$

$u_{xx} = -\sin(x-at)$

(b) Show that

$$u(x,y,z) = e^{3x+4y}\sin 5z$$

$e^{3x} \cdot e^{4y} \sin 5z.$

is a solution to the **Laplace's equation**

$$u_{xx} + u_{yy} + u_{zz} = 0$$

$u_x = e^{4y}\sin 5z \; 3e^{3x}$

$u_{xx} = 9e^{3x}e^{4y}\sin 5z$

$u_{yy} = 16e^{4y}e^{3x}\sin 5z$

$u_{zz} = e^{4y}e^{3x}\cos 5z$

$u_{zz} = -25\sin 5z\, e^{4y}e^{3x}$

**Problem 4.** Let $z = f(x, y) = x^2 + 3xy - y^2$.

(a) Find the differential $dz$

(b) Find the tangent plane of $f(x, y)$ at $(2, 3)$

(c) Compare the values of $\Delta z$ and $dz$ when $x$ changes from 2 to 2.05 and $y$ changes from 3 to 2.96.