MATH 2023 – Multivariable Calculus

Problem 1. Let

$$\mathbf{F}(x, y, z) = \langle xy, y^2 + e^{xz^2}, \sin(xy) \rangle$$

and S be the surface bounded by

$$y = 0, \qquad z = 0, \qquad z = 1 - x^2, \qquad y + z = 2$$

Find $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

Problem 2. Let

$$\mathbf{F}(x, y, z) = \langle z \tan^{-1}(y^{2023}), z^3 \ln(x^2 + y^{777}), z \rangle$$

Find the flux of $\mathbf{F}$ across the part of the paraboloid $x^2 + y^2 + z = 2$ that lies above the plane $z = 1$ and oriented upward.

Problem 3. Find $\iint_S (2x+2y+z^2)dS$ where S is the unit sphere $x^2 + y^2 + z^2 = 1$.