基礎課程文憑 - 中六全日制 - × WeBWorK : 2019_SPRING_M ×

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Problem 1

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Homework-11: Problem 1

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(4 points) **(a)** Show that each of the vector fields $\vec{F} = 3y\,\vec{i} + 3x\,\vec{j}$, $\vec{G} = \frac{3y}{x^2+y^2}\vec{i} + \frac{-3x}{x^2+y^2}\vec{j}$, and $\vec{H} = \frac{3x}{\sqrt{x^2+y^2}}\vec{i} + \frac{3y}{\sqrt{x^2+y^2}}\vec{j}$ are gradient vector fields on some domain (not necessarily the whole plane) by finding a potential function for each.

For $\vec{F}$, a potential function is $f(x, y) =$ 3xy

For $\vec{G}$, a potential function is $g(x, y) =$

For $\vec{H}$, a potential function is $h(x, y) =$

$$f_x = \frac{3y}{x^2+y^2}$$

$$f_y = \frac{-3x}{x^2+y^2}$$

(b) Find the line integrals of $\vec{F}, \vec{G}, \vec{H}$ around the curve C given to be the unit circle in the xy-plane, centered at the origin, and traversed counterclockwise.

$\int_C \vec{F} \cdot d\vec{r} =$

$\int_C \vec{G} \cdot d\vec{r} =$

$\int_C \vec{H} \cdot d\vec{r} =$

(c) For which of the three vector fields can Green's Theorem be used to calculate the line integral in part (b)?

It may be used for ?

(Be sure that you are able to explain why or why not.)

Note: *You can earn partial credit on this problem.*

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$$f_x = \frac{3y}{x^2+y^2}$$

$$f_y = \frac{-3x}{x^2+y^2}$$

$$\frac{\sqrt{x^2+y^2}\,\cancel{d} - \cancel{x}\frac{x}{\sqrt{x^2+y^2}}}{\left(\sqrt{x^2+y^2}\right)^2}$$

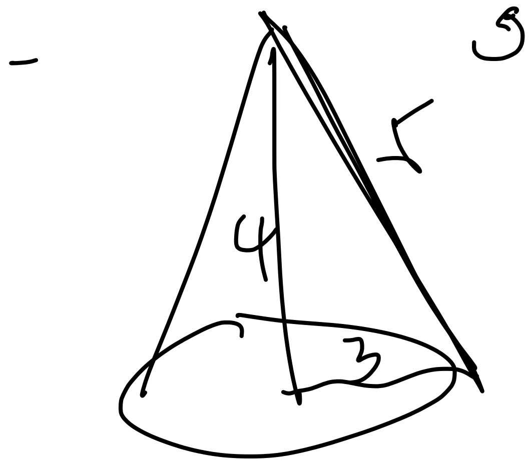
$$\frac{\cancel{3}\;-\frac{3}{2}}{\sqrt{x^2+y^2}}$$

$$3\left(-\frac{1}{2}\right)(x^2+y^2)^{-\frac{3}{2}}(2x)$$

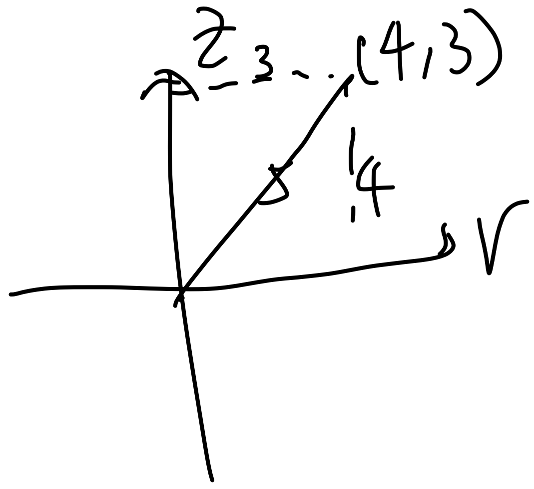
$$-6x$$

$$\frac{-3x}{\sqrt{x^2+y^2}} - \frac{3x}{\sqrt{x^2+y^2}}$$

$$\int \frac{3y}{x^2+y^2}\,dx$$

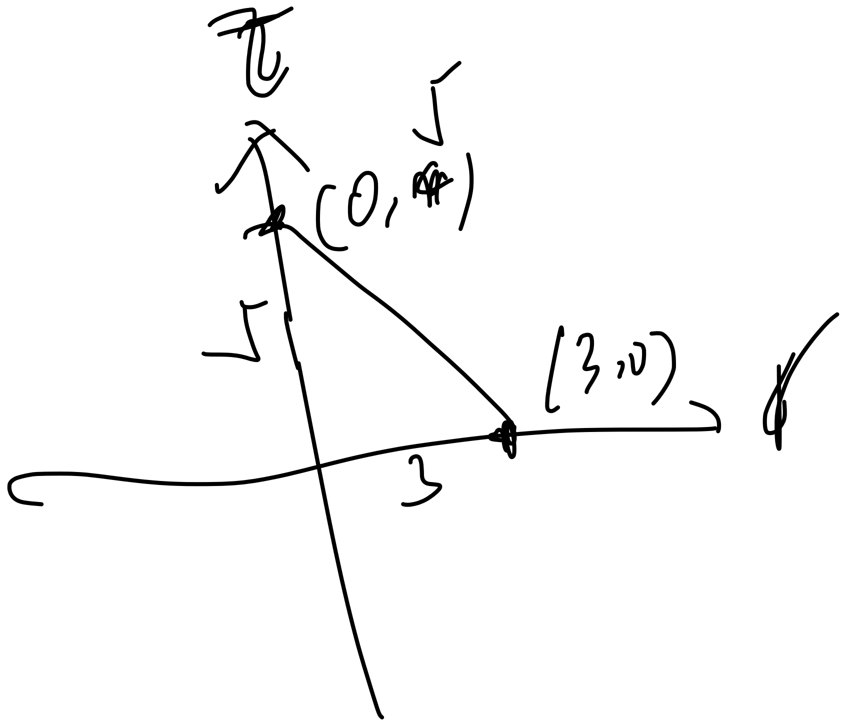


r along z.



$r\cos\theta$

$r\sin\theta$



$$\frac{z-5}{r} = \frac{-5}{3}$$

$$3z-15 = -5r$$

$$3z = -5r+15$$

$$z = \frac{-5r+15}{3}$$

$$\frac{z-4}{r} = \frac{-4}{3}$$

$$3(z-4) = -4r$$

$$3z-12 = -4r$$

$$3z = -4r-12$$

$$z = \frac{-4r-12}{3}$$

$$\frac{2x}{\sqrt{x^2+y^2}}$$ let $u = x^2+y^2$, $du = 2x\,dx$, $dx = \frac{1}{2x}du$.

$$\frac{3}{2}\left(u^{-\frac{1}{2}}\right)$$

$$\frac{3}{2}\left(2u^{\frac{1}{2}}\right)$$

$$= 3u^{\frac{1}{2}}.$$

$$3\sqrt{x^2+y^2} + g(y)$$

$$\frac{\partial g}{\partial y} = 3\left(\frac{1}{2}\right)\left(x^2+y^2\right)^{-\frac{1}{2}}(2y) + g'(y)$$

$$= \frac{3y}{\sqrt{x^2+y^2}}$$

Made with Goodnotes

(4 points) **(a)** Show that each of the vector fields $\vec{F} = 3y\,\vec{i} + 3x\,\vec{j}$, $\vec{G} = \frac{3y}{x^2+y^2}\,\vec{i} + \frac{-3x}{x^2+y^2}\,\vec{j}$, and $\vec{H} = \frac{3x}{\sqrt{x^2+y^2}}\,\vec{i} + \frac{3y}{\sqrt{x^2+y^2}}\,\vec{j}$ are gradient vector fields on some domain (not necessarily the whole plane) by finding a potential function for each.

For $\vec{F}$, a potential function is $f(x, y) =$ 3xy

For $\vec{G}$, a potential function is $g(x, y) =$ 3*arctan(x/y)

For $\vec{H}$, a potential function is $h(x, y) =$ 3sqrt(x**2+y**2)

(b) Find the line integrals of $\vec{F}, \vec{G}, \vec{H}$ around the curve C given to be the unit circle in the xy-plane, centered at the origin, and traversed counterclockwise.

$\int_C \vec{F} \cdot d\vec{r} =$

$\int_C \vec{G} \cdot d\vec{r} =$

$\int_C \vec{H} \cdot d\vec{r} =$

(c) For which of the three vector fields can Green's Theorem be used to calculate the line integral in part (b)?

It may be used for ?

(Be sure that you are able to explain why or why not.)

$$\int_C 3y\,dx + \int_C 3x\,dy.$$

$$= \int_0^{2\pi} -3\sin^2\theta\,d\theta + \int_0^{2\pi} 3\cos^2\theta\,d\theta$$

$$= \int_0^{2\pi} (-3 + 3\cos^2\theta)\,d\theta + \int_0^{2\pi} 3\cos^2\theta\,d\theta$$

$$= -6\pi + \int_0^{2\pi} 6\cos^2\theta\,d\theta$$

$$3\int_0^{2\pi} 1 + \cos 2\theta\,d\theta$$

$$3\left[2\pi + \frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

$3\tan^{-1}\left(\frac{x}{y}\right).$

$$\left\langle \frac{3y}{x^2+y^2}, \frac{-3x}{x^2+y^2} \right\rangle$$

$$\int_C \frac{3y}{x^2+y^2}\,dx + \int_C \frac{-3x}{x^2+y^2}\,dy$$

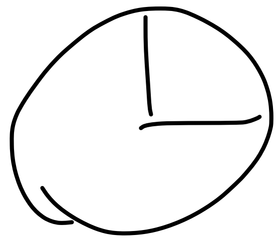
$$= \int_0^{2\pi} -3\sin^2\theta\,d\theta + \int_0^{2\pi} -3\cos^2\theta\,d\theta$$

$$= \int_0^{2\pi} -3(1-\cos^2\theta)\,d\theta$$

$$\int \frac{3x}{\sqrt{x^2+y^2}}\,dx + \frac{3y}{\sqrt{x^2+y^2}}\,dy$$

$$-3\cos\theta\sin\theta\,d\theta + 3\sin\theta\cos\theta\,d\theta$$

$x^2+y^2=1$

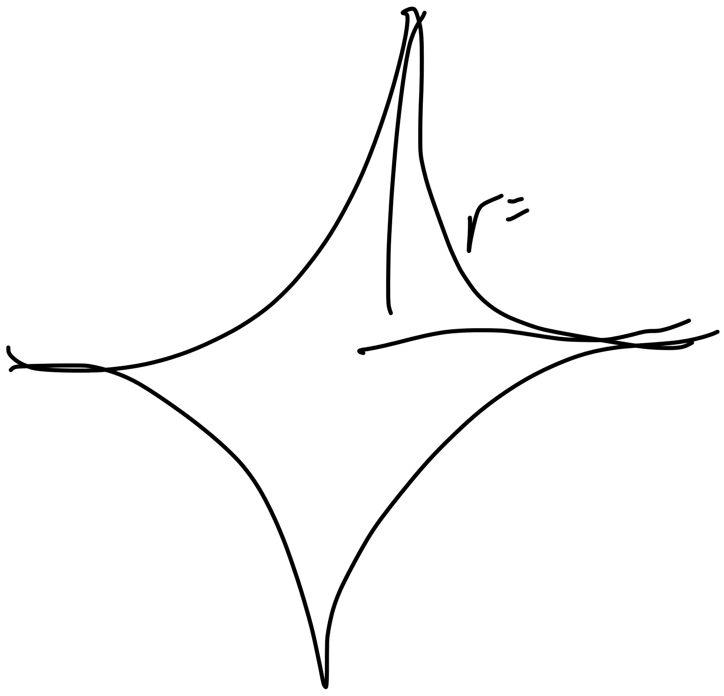


$dr\,d\theta$

$x=(\cos\theta)^3$

$y=(\sin\theta)^3$

$x^{\frac{2}{3}}+y^{\frac{2}{3}}=1$



$$\int_0^{\frac{\pi}{2}}\int_0^{\cos^3\theta+\sin^3\theta} dr\,d\theta$$

$$\int_0^{\frac{\pi}{2}} \cos^3\theta+\sin^3\theta \, d\theta$$