MATH 2023 – Multivariable Calculus

Lecture #03 Worksheet ♡ February 14, 2019

Problem 1. Find the arc length parametrization of the following curve from the point $(1, 0)$. ← $t=0$

$$\mathbf{r}(t) = \left(\frac{2}{t^2+1} - 1\right)\mathbf{i} + \frac{2t}{t^2+1}\mathbf{j} \quad + \cancel{\vec{c}}$$

What can you conclude about the curve?

① $$\vec{r}'(t) = \left\langle -\frac{2(2t)}{(t^2+1)^2}, \frac{(t^2+1)2 - 2t(2t)}{(t^2+1)^2} \right\rangle$$

$$= \left\langle -\frac{4t}{(t^2+1)^2}, \frac{2-2t^2}{(t^2+1)^2} \right\rangle$$

② $$|\vec{r}'(t)| = \sqrt{\frac{16t^2 + (4 - 8t^2 + 4t^4)}{(t^2+1)^4}} = \sqrt{\frac{4+8t^2+4t^4}{(t^2+1)^4}}$$

$$= \frac{2+2t^2}{(t^2+1)^2} = \frac{2}{t^2+1} \;//$$

③ $$s = \int_0^t |\vec{r}'(\tau)|\,d\tau = \int_0^t \frac{2}{\tau^2+1}\,d\tau = 2\tan^{-1} t \;//$$

④ $$t = \tan\frac{s}{2}$$

$$\tan^2 + 1 = \frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$$

⑤ $$\vec{r}(s) = \left\langle 2\cos^2\frac{s}{2} - 1, \; 2\cancel{\tan\frac{s}{2}\cos^2\frac{s}{2}} \right\rangle$$

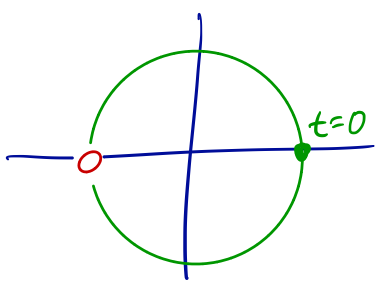
$$2\sin\frac{s}{2}\cos\frac{s}{2}$$

(double angle formula)

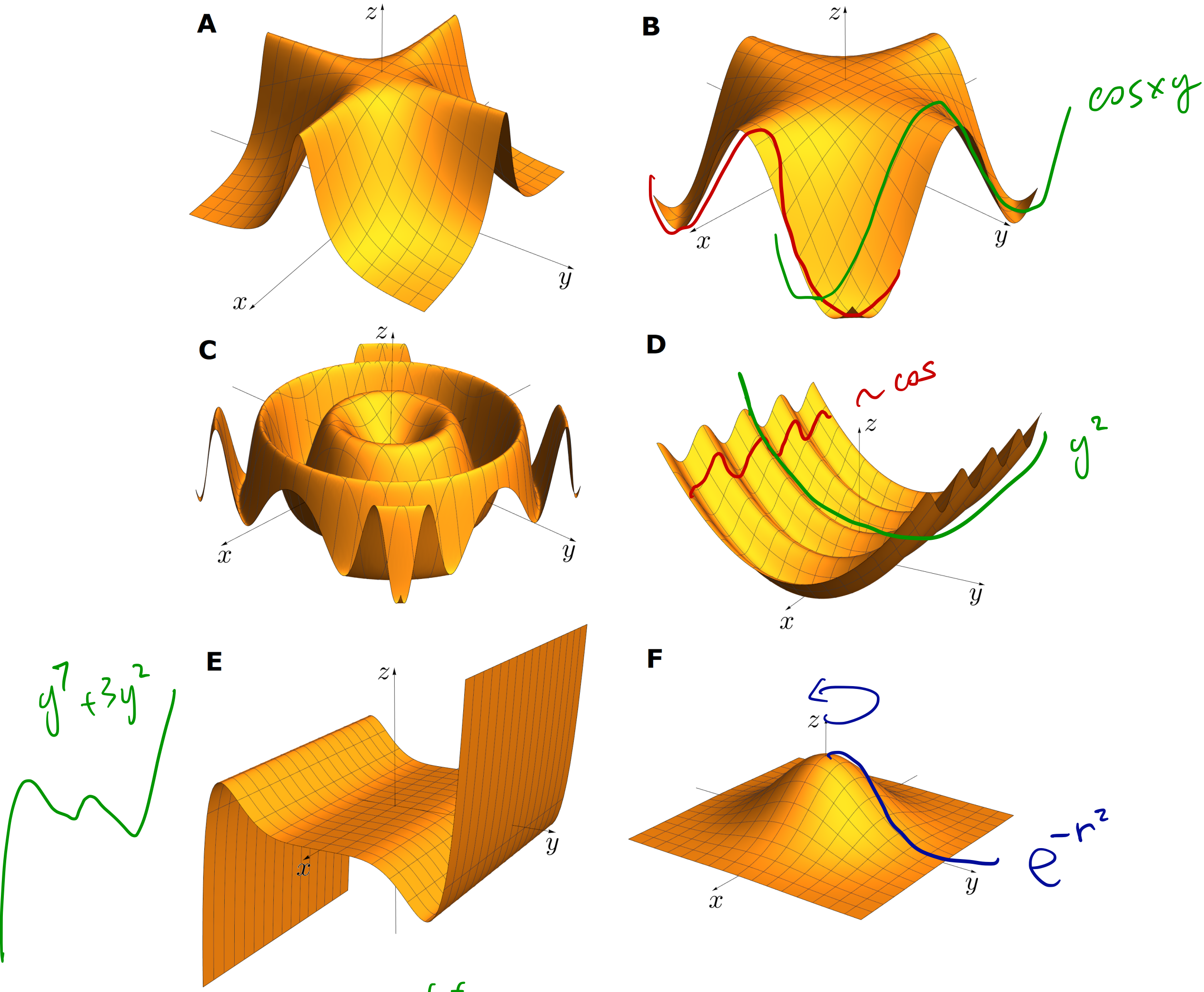
$$= \langle \cos s, \; \sin s \rangle$$

$\vec{r}(t)$ is a circle

(without the point $(-1,0)$)



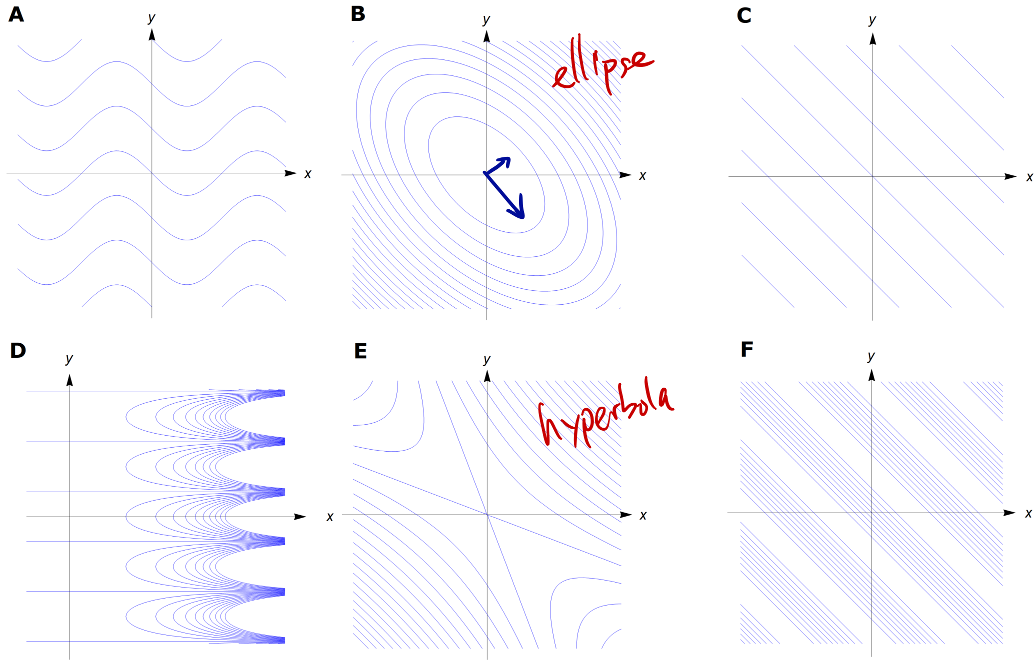
Problem 2. Identify the graphs with the corresponding functions $z = f(x, y)$.



rotate
a sin
curve

$\cos(x^2) + y^2$	$\sin(x^2 + y^2)$	$e^{-x^2-y^2}$	$\cos(xy)$	$y^7 + 3y^2$	$\frac{1}{1+x^2y^2}$
D	C	F	B	E	A

Problem 3. Identify the level sets with the corresponding functions $z = f(x, y)$.



periodic in y

plane : $x+y=k$

$e^x \cos y = 0$

$\Rightarrow y = 2\pi k \pm \frac{\pi}{2}$

$x+y$	$\sin(x+y)$	$\sin x + y$	x^2+xy+y^2 $=k$	$x^2+3xy+y^2$ $=k$	$e^x \cos y$
C	F	A	B	E	D

$\sin(x+y)=k$

$x+y = k'$

$\sin x + y = k$

$y = -\sin x + k$

$x^2+3xy+\frac{9y^2}{4} - \frac{9y^2}{4} + y^2$

$= (x+\frac{3}{2}y)^2 - \frac{5y^2}{4} \Longrightarrow$ hyperbola

$x^2+xy+y^2+\frac{y^2}{4}-\frac{y^2}{4}$

$= (x+\frac{y}{2})^2 + \frac{3y^2}{4}$

ellipse.

Bonus Problem. Plot the graph and the level sets of the following function

$$f(x, y) = (x^2 + y^2 - 1)^3 - x^2y^3.$$