MATH 2023 – Multivariable Calculus

Lecture #02 Worksheet ♠ February 12, 2019

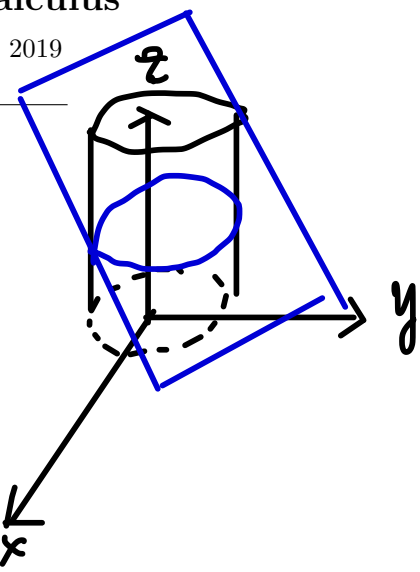
Problem 1. (a) Find the curve of intersection of

$$\begin{cases} x^2 + y^2 = 1 \\ y + z = 2 \end{cases}$$

plane: $\vec{n} = \langle 0, 1, 1 \rangle$

guess

$\langle \cos t, \sin t, 2 - \sin t \rangle$

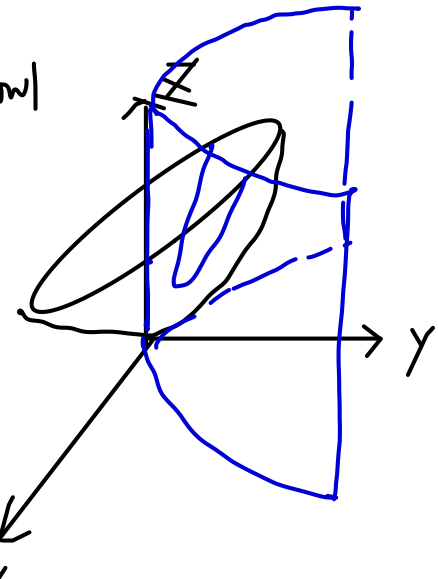


(b) Find the curve of intersection of

elliptic bowl

$$\begin{cases} z = 4x^2 + y^2 \\ y = x^2 \end{cases}$$

Sub $x = t$

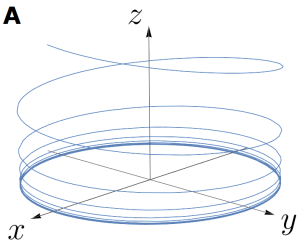


$\vec{r}(t) = \langle t, t^2, 4t^2 + t^4 \rangle$

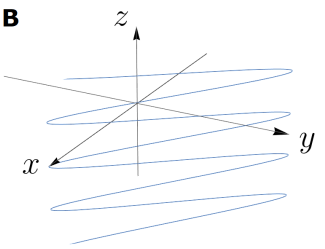
top view = ↓ circle, height = cosine curve

Problem 2. Identify the pictures with the corresponding vector functions $\mathbf{r}(t)$.

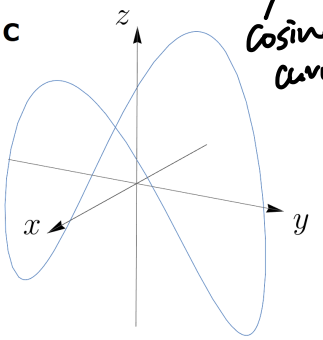
A

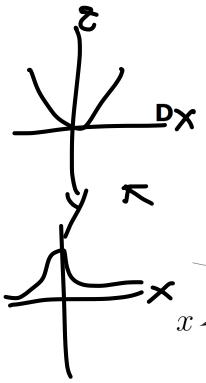


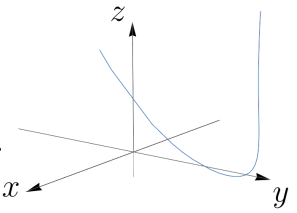
B



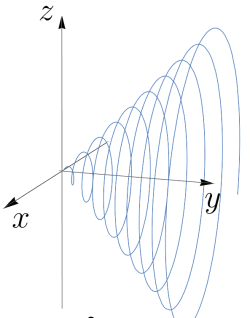
C



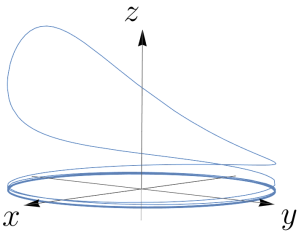




E



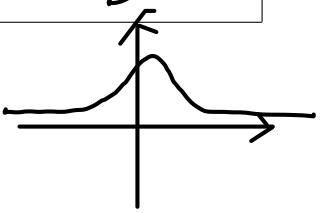
F



x,z form a circle of radius t, increasing circles

$\mathbf{r}_1(t) = \langle t\cos t, t, t\sin t\rangle, (t \geq 0)$	$\mathbf{r}_2(t) = \langle \cos t, \sin t, \frac{1}{1+t^2}\rangle$	$\mathbf{r}_3(t) = \langle t, \frac{1}{1+t^2}, t^2\rangle$
E	F	D
$\mathbf{r}_4(t) = \langle \cos 8t, \sin 8t, e^{-t}\rangle, (t \geq 0)$	$\mathbf{r}_5(t) = \langle \cos t, \sin t, \cos 2t\rangle$	$\mathbf{r}_6(t) = \langle \cos^2 t, \sin^2 t, t\rangle$
A	C	B

$\vec{r_2}(t) = \langle x,y\rangle$ = circle in $\mathbb{R}^2$, $\frac{1}{1+t^2}$ =



$\vec{r_3}(t) = \langle x,z\rangle$ = parabola

$\vec{r_6}(t)$, $\cos^2 t + \sin^2 t = 1$, $\vec{r}(t)$ lies on plane $x+y=1$

Problem 3. Consider the paths of two particles given by

$$\mathbf{r}_1(t) = (\sin t)\mathbf{i} + (\cos t)\mathbf{j} + 5t\mathbf{k}$$
$$\mathbf{r}_2(t) = \langle t, -1 + t, 5\pi - t\rangle$$

(a) Find the velocity, speed, and acceleration of each particle at time t.

$\vec{r}_1(t) = \langle \cos t, -\sin t, 5\rangle$ speed $= \sqrt{26}$

$\vec{a}_1(t) = \vec{r}_1''(t) = \langle -\sin t, -\cos t, 0\rangle$

$\vec{v}_2(t) = \langle 1, 1, -1\rangle$ speed $= \sqrt{3}$ $\vec{a}_2(t) = \langle 0, 0, 0\rangle$

(b) Do the two particles collide?

$\Leftrightarrow$ does $\vec{r}_1(t) = \vec{r}_2(t)$ has solution?

$z = 5t = 5\pi - t$ $\quad t = \frac{5\pi}{6}$

但 x: $\sin\frac{5\pi}{6} \neq \frac{5\pi}{6}$ No sol.

轨迹

(c) Do the trajectories of two particles intersect each other?

(c) does $\vec{r}_1(t) = \vec{r}_2(s)$ have solution

(guess) $\begin{cases} t = \pi \\ s = 0 \end{cases}$