

MATH 2023 – Multivariable Calculus

Lecture #01 Worksheet ♠ January 31, 2019

Problem 1. Let

$$A = (1, 2, 3), \quad B = (3, 4, 5), \quad C = (1, 0, -1), \quad D = (3, 2, 1)$$

be four points in \mathbb{R}^3 .

- (a) Show that $ABCD$ is a parallelogram
- (b) Find the area of this parallelogram.

Problem 2. Describe the four different relationships between the line L

$$L = \begin{cases} x = 1 + 4s \\ y = 2 + 5s \\ z = 3 + 6s \end{cases}$$

and the lines

$$\ell_1 = \begin{cases} x = 9 - 8t \\ y = 12 - 10t \\ z = 15 - 12t \end{cases}$$

$$\ell_2 = \begin{cases} x = 12t \\ y = 3 + 15t \\ z = 5 + 18t \end{cases}$$

$$\ell_3 = \begin{cases} x = -2 + 3t \\ y = 4 - 2t \\ z = -1 + 4t \end{cases}$$

$$\ell_4 = \begin{cases} x = -1 + t \\ y = t \\ z = 2 + t \end{cases}$$

Problem 3. Find the angle between the planes and their line of intersection

$$\begin{cases} x + y + z = 1 \\ x - 2y + 3z = 1 \end{cases}$$

Problem 4. Find the distance between the skew lines

$$L = \begin{cases} x = 1 + 4s \\ y = 2 + 5s \\ z = 3 + 6s \end{cases} \quad \text{and} \quad \ell_4 = \begin{cases} x = -1 + t \\ y = 0 + t \\ z = 2 + t \end{cases}$$