1. Find the minimal distance between the point $(1,1,0)$ and points on the sphere $x^2+y^2+z^2-2x-4y=4$.

$D = \sqrt{d} = \sqrt{(x-1)^2+(y-1)^2+z^2}$ — (x,y,z)

subject to $x^2+y^2+z^2-2x-4y=4$.

$\nabla D = \langle 2(x-1), 2(y-1), 2z \rangle$

$\nabla g = \langle 2x-2, 2y-4, 2z \rangle$

$$\begin{cases} 2x-2 = \lambda(2x-2) \\ 2y-2 = (2y-4)\lambda \\ 2z = \lambda(2z) \\ x^2+y^2+z^2-2x-4y=4 \end{cases}$$

$\lambda = 1$ or $2x-2=0, \lambda \neq 1$

$2y-2 = 2y-4$

$2 = 4$

(reject)

$x = 1$

$z = 0$

$1+y^2-2-4y=4$

$y^2-4y=5$

$y^2-4y-5=0$

$(y-5)(y+1)=0$

$y=5$ or $y=-1$

y -5

y $+1$

$(1, -1, 0)$

$(1, 5, 0)$

$$D = \sqrt{0^2 + (-1-1)^2 + 0^2} = 2.$$

$$D = \sqrt{0^2 + (5-1)^2 + 0^2} = 4.$$

2. Find a vector equation for the intersection of the two planes $x+y+z=1$ and $x+2y+z=3$.

$$\begin{cases} x+y+z=1 \\ x+2y+z=3 \end{cases}$$

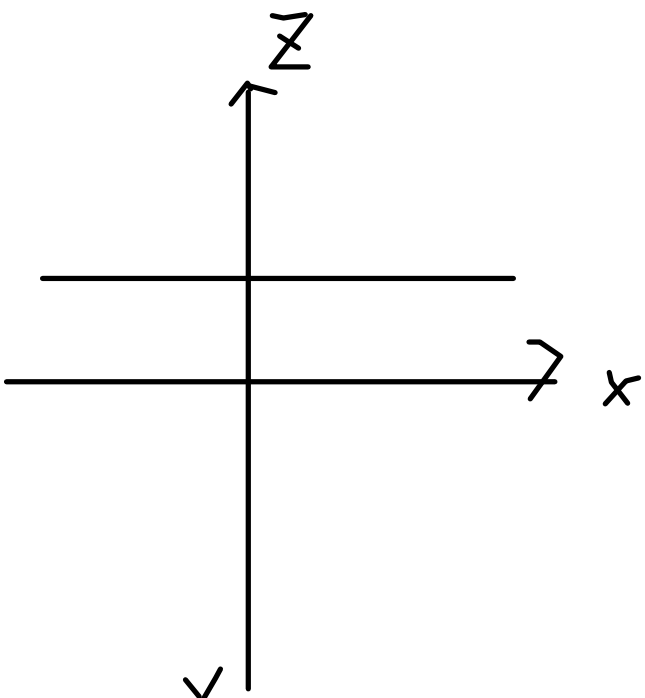
$$y=2.$$

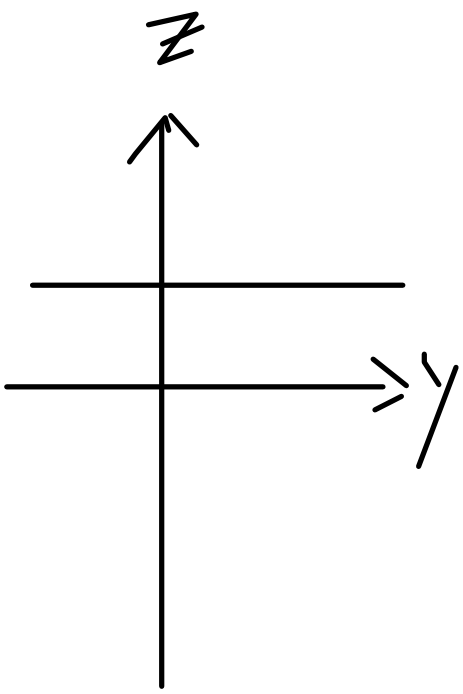
$$\vec{r}(t) = 2j$$

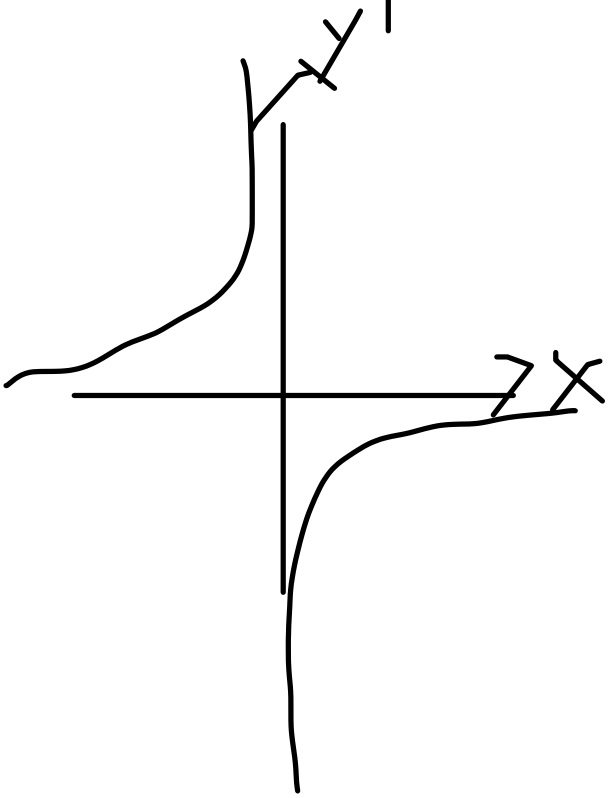
3. Sketch the surface $z = 1 + xy$.

When $x=0$, $z=1$

When $x \to \infty$, $y \to 0$, $z=$

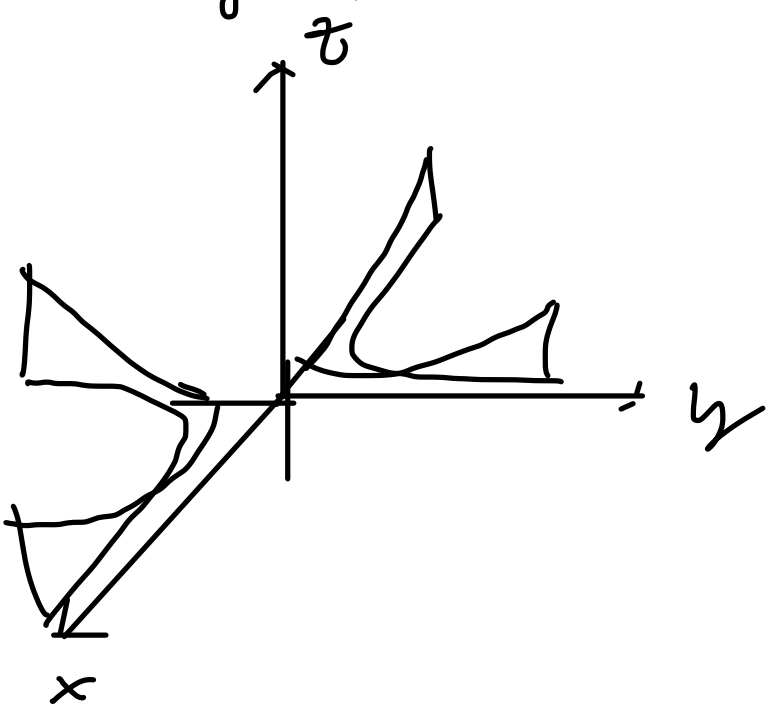






$xy = -1$

$y = -\frac{1}{x}$.



4. Let $f(x,y) = \ln(x^2+y^2-2)$, $g(x) = \sqrt{1-x^2}$. Find the domain of the composite function $g \circ f$.

$$\sqrt{1-(\ln(x^2+y^2-2))^2}$$

$$1-\ln(x^2+y^2-2)^2 > 0$$

$$\ln(x^2+y^2-2)^2 > 1$$

$$x^2+y^2-2 > 0$$

$$-1 > \ln(x^2+y^2-2) > 1$$

$$e^{-1} > x^2+y^2-2 > e$$

$$x^2+y^2-2 > 0$$

$$x^2+y^2 > 2$$

$$1-x^2 > 0$$

$$1 > x^2$$

$$x^2 < 1$$

$$-1 < x < 1$$

$$\begin{cases} x^2+y^2 > 2 \\ x^2 < 1 \end{cases}$$

$$y > 1 \text{ or } y < -1$$

$$y^2 > 1$$

5. Find the limit $\lim_{(x,y)\to(0,0)} \frac{x^2y + xy^2 + x^2 + y^2 + 2xy}{1 - \cos\sqrt{x^2+y^2}}$ if it exists.

Let $x = r\cos\theta$, $y = r\sin\theta$

$$\lim_{r\to 0} \frac{r^2\cos^2\theta\, r\sin\theta + r^2\sin^2\theta\, r\cos\theta + r^2 + 2r^2\sin\theta\cos\theta}{1 - \cos r}$$

$$= \lim_{r\to 0} \frac{r^3\cos\theta(\sin\theta\cos\theta + \sin^2\theta) + r^2 + 2r^2\sin\theta\cos\theta}{1 - \cos r}$$

$$\vdots$$

$$\frac{3r^2\cos\theta(\sin\theta\cos\theta + \sin^2\theta) + 2r + 4r\sin\theta\cos\theta}{\sin r}$$

$$\frac{6r\cos\theta(\sin\theta\cos\theta + \sin^2\theta) + 2 + 4\sin\theta\cos\theta}{\cos r}$$

$$= 2.$$