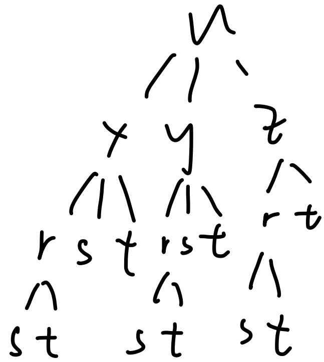
MATH 2023 – Multivariable Calculus

Lecture #06 Worksheet ♢ February 26, 2019

Problem 1. Let $u = x^4 y + y^2 z$ where

$$x = rse^t$$
$$y = s^2 e^{-tr}$$
$$z = rt$$
$$r = st^2$$



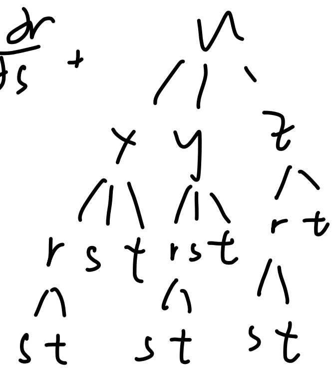
Find $\frac{\partial u}{\partial s}$ in terms of s, t

Let $f(x, y, t)$ be a function, where we have the dependence of variables:

$$x(t, u), y(t), u(s, v, w), s(v, t)$$

Find $\frac{\partial f}{\partial s}$ and $\frac{\partial f}{\partial t}$.

$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial u}{\partial x}\frac{\partial x}{\partial r}\frac{\partial r}{\partial s} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r}\frac{\partial r}{\partial s} + \frac{\partial z}{\partial r}\frac{\partial r}{\partial s}$$



Problem 1. Let $u = x^4y + y^2z$ where

$$x = rse^t$$
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Find $\frac{\partial u}{\partial s}$ in terms of s, t

$$= 4x^3y(re^t) + 4x^3y(se^t)(t^2) + (x^4+2yz)(2se^{-tr}) + (x^4+2yz)(-s^2te^{-tr})(t^2) + (t)(t^2)$$

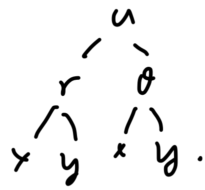
$$= 4x^3y(st^2e^t + se^tt^2) + (x^4+2yz)(2se^{-st^3} - s^2t^3e^{-st^3}) + t^3$$

Problem 2. Let $u(r,\theta)$ be a function in polar coordinates. Express the Laplace equation

$$u_{xx} + u_{yy} = 0$$

in terms of r and θ.

$x = r\cos\theta$
$y = r\sin\theta$
$r = \sqrt{x^2+y^2}$
$\theta = \tan^{-1}\left(\frac{y}{x}\right)$



$$u_x = u_r r_x + u_\theta \theta_x$$

$$u_x = u_r\left(\frac{x}{\sqrt{x^2+y^2}}\right) + u_\theta\left(\frac{x^2}{x^2+y^2}\right)\left(-\frac{y}{x^2}\right)$$

$$u_x = u_r \frac{x}{\sqrt{x^2+y^2}} - u_\theta\left(\frac{y}{x^2+y^2}\right)$$

$$u_{xx} = u_{rx}\left(\frac{x}{\sqrt{x^2+y^2}}\right) + u_r \frac{(x^2+y^2)^{\frac{1}{2}} - \frac{x^2}{(x^2+y^2)^{\frac{1}{2}}}}{(x^2+y^2)} - u_{\theta x} + \frac{2xy}{(x^2+y^2)^2}$$

$$u_{xx} = (u_{rr} r_x + u_{r\theta}\theta_x)\left(\frac{x}{\sqrt{x^2+y^2}}\right) + u_r \frac{y^2}{(x^2+y^2)^{\frac{3}{2}}}$$

$$-(u_{\theta r} r_x + u_{\theta\theta}\theta_x)\left(\frac{y}{x^2+y^2}\right) + \frac{2xy}{(x^2+y^2)^2} u_\theta$$