MATH 2023 – Multivariable Calculus

Lecture #09 Worksheet ♯ March 7, 2019

Problem 1. Find the point on the sphere $x^2 + y^2 + z^2 = 4$ that are closest and farthest from the point $(3, 1, -1)$.

$$D = (x-3)^2 + (y-1)^2 + (z+1)^2$$

Subject to $x^2+y^2+z^2=4$.

$$\nabla D = \langle 2(x-3), 2(y-1), 2(z+1) \rangle$$

$$\nabla g = \langle 2x, 2y, 2z \rangle$$

$$2x-6 = 2\lambda x$$

$$2y-2 = 2\lambda y$$

$$2z+2 = 2\lambda z$$

$$x^2+y^2+z^2=4.$$

$$\frac{2x-6}{2x} = \frac{2y-2}{2y} = \frac{2z+2}{2z}$$

$$4xy - 12y = 4xy - 4x$$

$$3y = x$$

$$y = \frac{x}{3}$$

$$4xz + 4x = 4xz - 12z$$

$$x = -3z$$

$$x^2 + \left(\frac{x}{3}\right)^2 + \left(\frac{x}{-3}\right)^2 = 4$$

$$\frac{11}{9}x^2 = 4$$

$$x^2 = \frac{36}{11}$$

$$x = \pm\frac{6}{\sqrt{11}}$$

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For $\left(\frac{6}{\sqrt{11}}, \frac{2}{\sqrt{11}}, -\frac{2}{\sqrt{11}}\right)$, $D = 1.31662479$ ← closest

For $\left(\frac{-6}{\sqrt{11}}, \frac{-2}{\sqrt{11}}, \frac{2}{\sqrt{11}}\right)$, $D = 5.31662479$. ← Furthest

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Problem 2. Find the maximum value of the function $f(x, y, z) = x + 2y + 3z$ on the curve of intersection of the plane $x - y + z = 1$ and the cylinder $x^2 + y^2 = 1$ using

- Lagrange multipliers

- parametric curve

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Problem 2. Find the maximum value of the function $f(x,y,z) = x + 2y + 3z$ on the curve of intersection of the plane $x - y + z = 1$ and the cylinder $x^2 + y^2 = 1$ using

- Lagrange multipliers

Max.

$f\left(\frac{-4}{\sqrt{17}}, \frac{1}{\sqrt{17}}, \frac{\sqrt{17}+5}{\sqrt{17}}\right) = 6.15256315$

-6.15256315

$\nabla f = \langle 1, 2, 3 \rangle$

$f($

$\nabla g = \langle 1, -1, 1 \rangle$

$\nabla h = \langle 2x, 2y, 0 \rangle$

$1 = \lambda + \mu(2x)$

$2 = -\lambda + \mu(2y)$ $\quad 3 = \lambda$

careless!

$3 = -\lambda + \mu(0) \leftarrow \lambda = -3$

$1 = -3 + \mu(2x)$ $\quad$ | $\quad$ $\cancel{2 = 3 + 2\mu y}$

$2 = \mu x$ $\quad$ | $\quad$ $\cancel{-\frac{1}{2} = \mu y}$

$\cancel{\frac{2}{x} = -\frac{1}{2y}}$

$\cancel{4y = -x}$

$\cancel{x = -4y}$

$16y^2 + y^2 = 1$

$17y^2 = 1$

$y = \pm\frac{1}{\sqrt{17}}$

$x = \mp\frac{4}{\sqrt{17}}$

$z = 1 - x + y$

$= 1 + \frac{4}{\sqrt{17}} + \frac{1}{\sqrt{17}}$ or $1 - \frac{4}{\sqrt{17}} - \frac{1}{\sqrt{17}}$

$= \frac{\sqrt{17}+5}{\sqrt{17}}$ or $\frac{\sqrt{17}-5}{\sqrt{17}}$

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Problem 2. Find the maximum value of the function $f(x, y, z) = x + 2y + 3z$ on the curve of intersection of the plane $x - y + z = 1$ and the cylinder $x^2 + y^2 = 1$ using

- Lagrange multipliers

$x = \cos t$

$y = \sin t$

$z : 1 - \cos t + \sin t.$

$f(t) = \cos t + 2\sin t + 3 - 3\cos t + 3\sin t.$

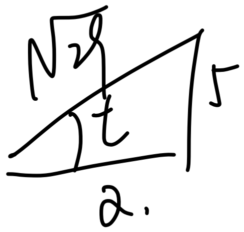
$f(t). = 3 - 2\cos t + 5\sin t$

$f'(t) = 2\sin t + 5\cos t.$

solving $2\sin t + 5\cos t = 0$

$\tan t = \frac{-5}{2}$

$-\tan(t) = \frac{5}{2}$



$=$

$x = \frac{2}{\sqrt{29}},\ y =$

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Problem 3. Prove the **AM-GM** inequality

$$\sqrt[n]{x_1 \cdots x_n} \leq \frac{x_1 + \cdots x_n}{n}$$

by finding the maximum value of

$$f(x_1, ..., x_n) = x_1 x_2 \cdots x_n$$

subject to the condition $x_1 + x_2 + \cdots + x_n = S$ where S is a constant. Find the conditions in which equality holds.

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