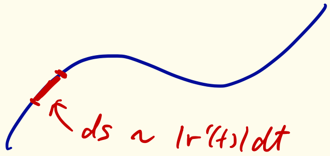
Review

$f(x,y,z)$ scalar , $\vec{F}(x,y,z)$ vector field.
" $\langle P, Q, R\rangle$

$$\int_C f(x,y,z)\,ds = \int_C f(\vec{r}(t))\,|\vec{r}\,'(t)|\,dt$$



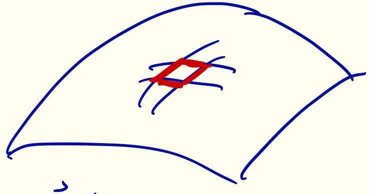
WORK DONE

$$\int \vec{F}\cdot\vec{T}\,ds = \int_C \vec{F}\cdot d\vec{r} = \int_C P\,\underbrace{dx}_{x'(t)dt} + Q\,dy + R\,dz$$

unit tangent $= \dfrac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|}$

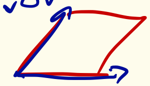
$$\iint_S f(x,y,z)\,dS = \iint_S f(\vec{r}(u,v))\,|\vec{r}_u\times\vec{r}_v|\,du\,dv$$

$d\vec{A}$



FLUX

$$\iint_S \vec{F}\cdot\vec{n}\,dS = \iint_S \vec{F}\cdot d\vec{S} = \iint_S \vec{F}\cdot(\vec{r}_u\times\vec{r}_v)\,du\,dv.$$



unit normal $= \dfrac{\vec{r}_u\times\vec{r}_v}{|\vec{r}_u\times\vec{r}_v|}$

☆ $\vec{n}$: outward pointing

: graph : upward.

$z = z(x,y)$

$\vec{n} = \langle -z_x, -z_y, 1\rangle$

$$\int_a^b F'(t)\,dt = F(b) - F(a)$$

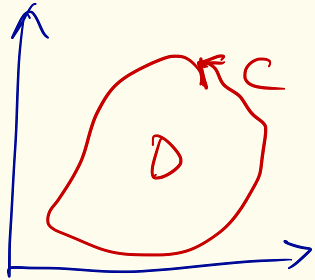
$$\int_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$$

Green's Thm

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \oint_C \vec{F} \cdot d\vec{r}$$

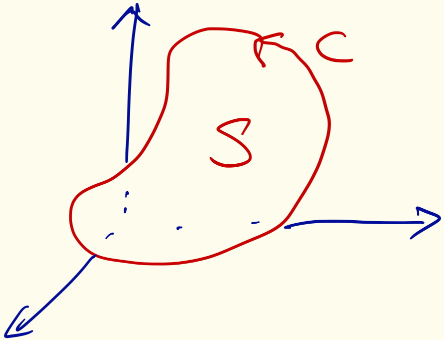
$$\parallel$$

$$(\nabla \times \vec{F}) \cdot \vec{k}$$



Stokes' Thm

$$\iint_S (\nabla \times \vec{F}) \cdot \vec{n}\,dS = \oint_C \vec{F} \cdot d\vec{r}$$

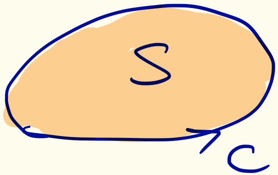


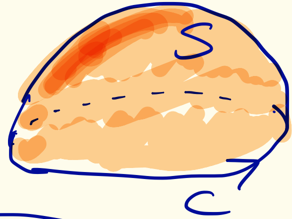
Stokes' Thm

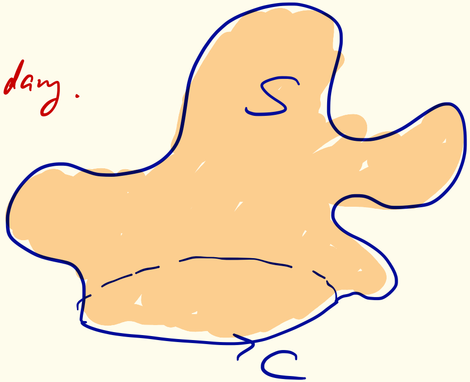
$$\oint_{\partial S} \vec{F} \cdot d\vec{r} = \iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$$

($\vec{F}$: cont. partial derivatives in $\mathbb{R}^3$)

positively oriented boundary.







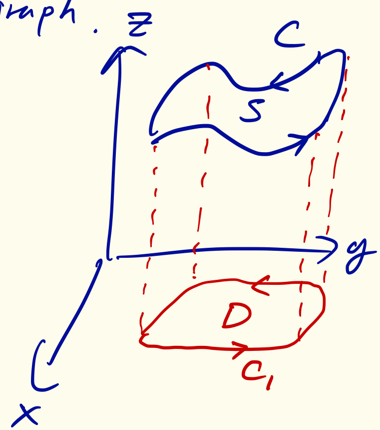
$$\boxed{\partial S = C}$$

Special Case Green's Thm: $\vec{F} = \langle P(x,y), Q(x,y), 0 \rangle$

$$\iint_S (\nabla \times \vec{F}) \cdot d\vec{S} = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \cancel{\vec{k} \cdot \vec{k}} \, dA$$

Proof Similar to Green's Thm:

Let's prove in special case when S is a graph.



$$S: z = g(x,y)$$

$$\vec{F} = \langle P, Q, R \rangle$$

$$C_1 = \langle x(t), y(t) \rangle$$

$$C = \langle x(t), y(t), g(x(t), y(t)) \rangle$$

$$\underline{\text{RHS}} = \iint_S \nabla \times \vec{F} \cdot d\vec{S} = \iint_D (\nabla \times \vec{F}) \cdot \langle -z_x, -z_y, 1 \rangle \, dA$$

$\uparrow$ graph!

$$= \iint_D -(R_y - Q_z) z_x - (P_z - R_x) z_y + (Q_x - P_y) \, dA.$$

$$\underline{\text{LHS}} = \oint_C \vec{F} \cdot d\vec{r} = \int_a^b \left(P \cdot \frac{dx}{dt} + Q \frac{dy}{dt} + R \frac{dz}{dt} \right) dt.$$

$$\frac{dz}{dt} = \frac{\partial g}{\partial x}\frac{dx}{dt} + \frac{\partial g}{\partial y}\frac{dy}{dt}$$

$$\oint_C \vec{F}\cdot d\vec{r} = \int_a^b \left((P + R g_x)\frac{dx}{dt} + (Q + R g_y)\frac{dy}{dt} \right) dt$$

$$= \int_a^b \underbrace{(P + R g_x)}_{P}\, dx + \underbrace{(Q + R g_y)}_{Q}\, dy .$$

Green's Thm

$$= \iint_D \left(\frac{\partial Q}{dx} - \frac{\partial P}{\partial y} \right) dA$$

$$\left(\frac{\partial Q}{\partial x} = Q_x + Q_z \frac{\partial z}{\partial x} \right)$$

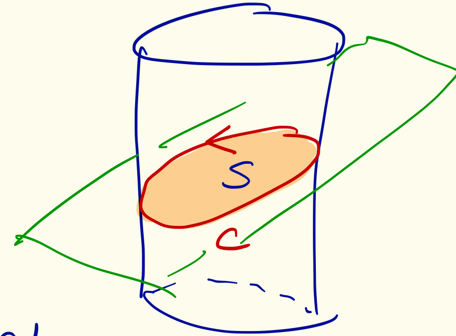
$$= \iint_D \left(\underset{Q_z g_x}{Q_x} + R_x g_y + \cancel{R g_{yx}} - \underset{-R_z g_y}{P_y} - R_y g_x - \cancel{R g_{xy}} \right) dA$$

$= RHS$ //

<u>Ex</u> Find $\oint_C \vec{F} \cdot d\vec{r}$, $\vec{F} = \langle -y^2, x, z^2 \rangle$

C : intersection of $y+z=2$ and $x^2+y^2=1$

counterclockwise.



$$\nabla \times \vec{F} : \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_x & \partial_y & \partial_z \\ -y^2 & x & z^2 \end{vmatrix} = \langle 0, 0, 1+2y \rangle$$

$$= \iint_S \nabla \times \vec{F} \cdot d\vec{S} = \iint_D 1+2y \, dA = \int_0^{2\pi} \int_0^1 (1+2r\sin\theta)\, r\, dr\, d\theta$$

$x^2+y^2 \le 1$

$= \pi$ //

S : $y+z=2$

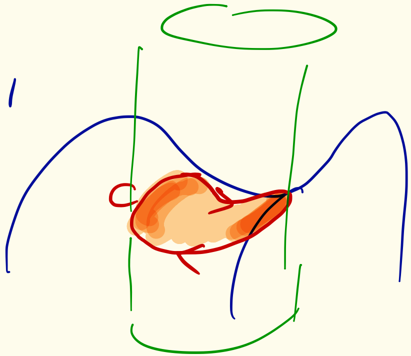
$z=2-y$

$\vec{n} = \langle 0, 1, 1 \rangle$

Ex $\vec{F} = \langle x^2 y,\ \frac{1}{3}x^3,\ xy \rangle$

C : intersection of $z = y^2 - x^2$, $x^2+y^2=1$

clockwise.



By Stokes':

$$= \iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$$

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_x & \partial_y & \partial_z \\ x^2 y & \frac{1}{3}x^3 & xy \end{vmatrix} = \langle x, -y, 0 \rangle$$

S : $z = y^2 - x^2$

$\vec{n} = \langle 2x, -2y, 1 \rangle$

$$= \iint_D 2x^2 + 2y^2 \, dA$$

$x^2+y^2 \le 1$

$$= \int_0^{2\pi} \int_0^1 (2r^2)\, r\,dr\,d\theta$$

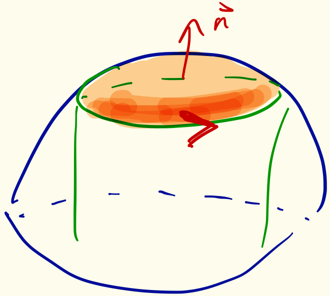
$$= \pi$$

Made with Goodnotes

Ex $\vec{F} = \langle xz, yz, xy \rangle$

S : $x^2+y^2+z^2=4$ above xy plane, inside $x^2+y^2=1$

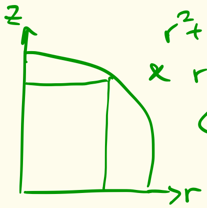
Find $\iint_S \nabla \times \vec{F} \cdot d\vec{S}$



By Stokes':

$$\oint_C \vec{F} \cdot d\vec{r}$$

$\leftarrow$ circle at $z=\sqrt{3}$ $(\cos t, \sin t, \sqrt{3})$



$r^2+z^2=4$

$\times$ $r^2=1$

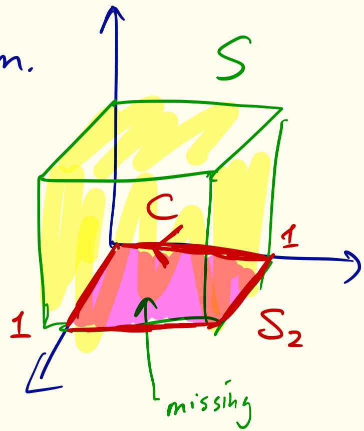
$\Leftrightarrow z^2=3$

$z=\sqrt{3}$

$$= \oint_C \langle \sqrt{3}\cos t, \sqrt{3}\sin t, 3\cos t \sin t \rangle \cdot \langle -\sin t, \cos t, 0 \rangle \, dt = 0$$

Ex $\vec{F} = \langle xyz, xy, x^2yz \rangle$

S : length 1 square box with open bottom.



Find $\iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$

Using Stokes': $\oint_C \vec{F} \cdot d\vec{r} = \oint \langle 0, xy, 0 \rangle \cdot d\vec{r}$

$\leftarrow z = 0$

along $x = 1$, $\oint \langle 0, y, 0 \rangle \cdot \langle 0, 1, 0 \rangle \, dy = \frac{y^2}{2}\Big|_0^1 = \frac{1}{2}$

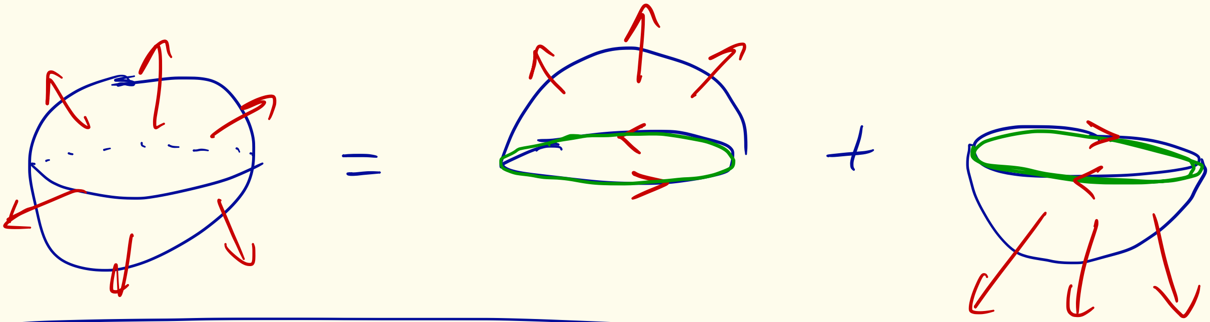
along all other segments: 0.

$$= \iint_{S_2} \nabla \times \vec{F} \cdot d\vec{S}$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_x & \partial_y & \partial_z \\ xyz & xy & x^2yz \end{vmatrix} = \langle x^2 z, \; xy - 2xyz, \; y - xz \rangle$$

$$= \iint \langle 0, xy, y \rangle \cdot \vec{k} = \int_0^1\int_0^1 y \, dx\, dy = \frac{1}{2}$$

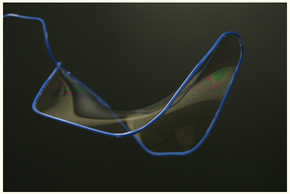
Rem $\iint_S (\nabla \times \vec{F}) \cdot d\vec{S} = 0$ if S is closed surface (no boundary!)



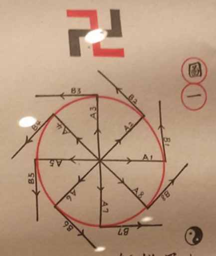
Rem **Thm D** $\nabla \times \vec{F} = 0$ + simply connected

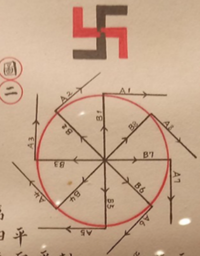
$\Rightarrow \vec{F}$ is conservative.

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_S (\nabla \times \vec{F}) \cdot d\vec{S} = 0$$ for any simple closed curve.



Made with Goodnotes

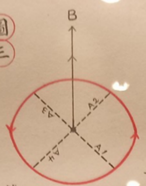


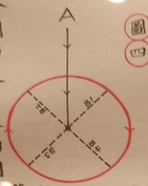


太極為易卦之核心則盡人皆知矣惟卐卍亦為易卦之核心則罕有述及之見於卍字文選者亦僅見卍字分配八卦而已其鎮化寶以先後天八卦中立卍字易之核心於斯可見矣先聖創立紅卍字會其殆以易之核心為中心者耶其宣化之道亦本之羲黃聖道者耶是以其對天人一貫之道以卍字之哲理解析亦无異於易理其端倪已備見於訓文既詳且盡蘊以加之矣惟卍字更能普及當今科學之定理則先聖猶未詳宣於經文因此撰集先賢有關卍字之科學解析以為好道者共擘幾之

圖半徑與周切線之互易

設有圓平面A_1A_2而至A_8乃代表平面上一切之方向俱為半徑而B_1B_2而至B_8乃代表圓周上種〻之切線皆垂直於半徑如圖一所示依此種〻切線之方向另作圓平面如圖二示之於是B_1B_2而至B_8俱為半徑而圓周切線乃為A_1A_2而至A_8若逆矢向則圖一為右旋圖二為左旋矣





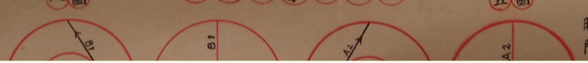
圖平面與中心垂直線之互易

圖三$A_1A_2A_3A_4$俱為圓平面之半徑B為垂直於圓中心之線圖四之為圓平面而$B_1B_2B_3B_4$為之半徑A為垂直於圓中心之線凡平面上一切方向可以xy兩軸交直角者代表之蓋任何向量依其與x軸所成角度可析為正弦餘弦兩向量而並行於y與x凡一圓平面以對徑為軸而旋一周則包舉一空間故空間三度得以圓平面代表之命圖三圓平面A代表空間則垂直於中之B軸代表第四度時間向量此與空間中任何向量均垂直但在空間中僅能以一點代表之蓋時間向量在空間中為合一也若必欲作線則垂直於種〻方向者必畫種〻線如圖一之所示B_1B_2而至B_8之為圓周切線即代表第四度而符合於圖三之B軸斯乃表之以斯篤克定理 *Stokes's theorem.* 其式為 $\int curl\, nA\, df = \int A_s\, ds$ 此式所示為一旋轉之向量之垂直分向之積分等於此向量之分向沿曲線之還繞面積者之線積分又以圓平面代表空間而垂直於圓中之線代表第四度則亦符於哥斯定理 *Gauss's theorem.* 其式為 $\int A_n\, df = \int div A\, d\tau$ 其式所示為經由一閉合面之向量川流等於積分向量之散佈取諸全部容積包舉於面者也此乃哥斯首得之故曰哥斯定理圖一A_1A_2至A_8為圓半徑代表空間一切向量B_1B_2至B_8為圓周切線代表第四度時間一線圖二B_1B_2至B_8為圓半徑代表空間一切向量A_1A_2至A_8為圓周切線代表第四度一線謂之時間然而圖一之A_1A_2至A_8亦即圖二之A_1A_2至A_8圖一之B_1B_2至B_8亦即圖二之B_1B_2至B_8故此兩空間者為交錯量易言之B為空間者於A為時間線而A為空間者於B為時間線此即$S_xS_yS_z$為空間三度則$T_xT_yT_z$合一為第四度反之$T_xT_yT_z$為空間三度則$S_xS_yS_z$合一為第四度圖三垂直於圓中之B線即圖一圓周之B_1等種〻線圖四垂直於圓中之A線即圖二圓周之A_1等種〻線故圖三之B線即圖四之圓平面而圖四A線即圖三之圓平面故AB兩線為交錯量也

卐卍兩象之解析

圖八

圖五



新空 有A_1 之若 形成 其義

卍卐 可用 中心 互交 表圖 命此 時間

by 遊自在