

MATH 2023 – Multivariable Calculus

Lecture #02 Worksheet ♠ February 12, 2019

Problem 1. (a) Find the curve of intersection of

$$\begin{cases} x^2 + y^2 = 1 \\ y + z = 2 \end{cases}$$

$$x = \cos \theta, y = \sin \theta, z = 2 - \sin \theta$$

$$\vec{r}(\theta) = \langle \cos \theta, \sin \theta, 2 - \sin \theta \rangle$$

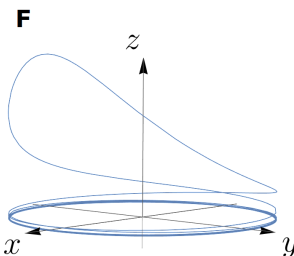
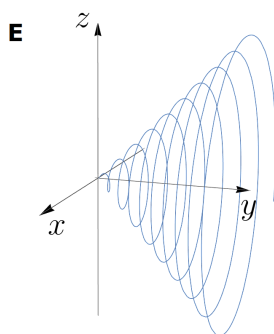
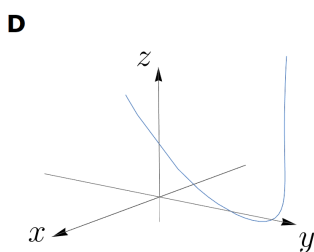
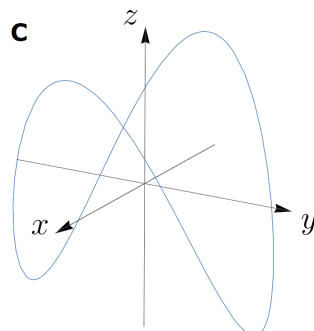
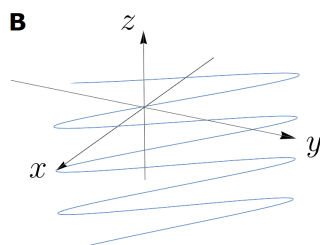
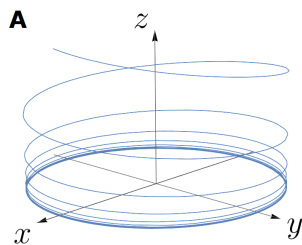
(b) Find the curve of intersection of

$$\begin{cases} z = 4x^2 + y^2 \\ y = x^2 \end{cases} \quad (4x^2 + x^4)!$$

$$z = 5x^2, y = x^2, x = t.$$

$$\vec{r}(t) = \langle t, t^2, 5t^2 \rangle$$

Problem 2. Identify the pictures with the corresponding vector functions $\mathbf{r}(t)$.



$\mathbf{r}_1(t) = \langle t \cos t, t, t \sin t \rangle, (t \geq 0)$	$\mathbf{r}_2(t) = \langle \cos t, \sin t, \frac{1}{1+t^2} \rangle$	$\mathbf{r}_3(t) = \langle t, \frac{1}{1+t^2}, t^2 \rangle$
E	F	D
$\mathbf{r}_4(t) = \langle \cos 8t, \sin 8t, e^{-t} \rangle, (t \geq 0)$	$\mathbf{r}_5(t) = \langle \cos t, \sin t, \cos 2t \rangle$	$\mathbf{r}_6(t) = \langle \cos^2 t, \sin^2 t, t \rangle$
A	C	B

Problem 3. Consider the paths of two particles given by

$$\mathbf{r}_1(t) = (\sin t)\mathbf{i} + (\cos t)\mathbf{j} + 5t\mathbf{k}$$

$$\mathbf{r}_2(t) = \langle t, -1 + t, 5\pi - t \rangle$$

(a) Find the velocity, speed, and acceleration of each particle at time t .

(b) Do the two particles collide?

(c) Do the trajectories of two particles intersect each other?

$$r_1(t) = \sin(t)\mathbf{i} + \cos(t)\mathbf{j} + 5t\mathbf{k}$$

$$r_2(t) = t\mathbf{i} - 1 + t\mathbf{j} + (5\pi - t)\mathbf{k}$$

$$\sin t = s$$

$$\cos t = -1 + s$$

$$5t = 5\pi - s$$

$$s = 5\pi - 5t$$

$$\sin t = 5\pi - 5t$$

$$\text{When } t = \pi,$$

$$\cos t = -1 + 5\pi - 5t$$

$-1 = -1 + 5\pi - 5\pi$, Yes, they will collide when $t = \pi$,

