MATH 2023 – Multivariable Calculus

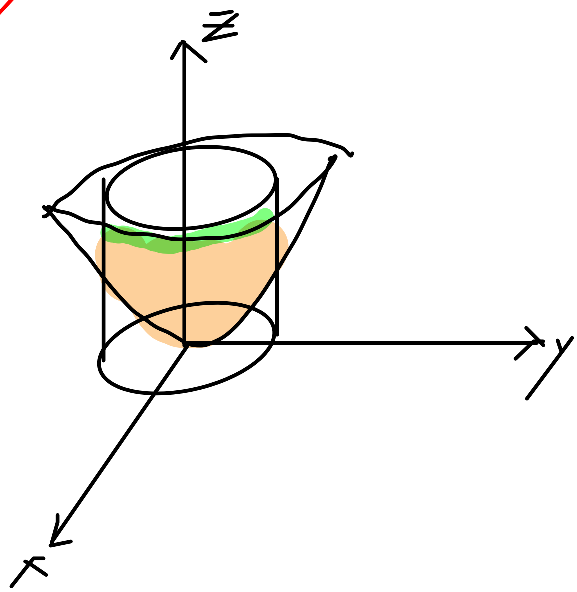
Lecture #19 Worksheet   April 25, 2019

Problem 1. Let $\mathbf{F} = \langle x^2z^2, y^2z^2, xyz\rangle$. Let S be the part of paraboloid $z = x^2+y^2$ inside the cylinder $x^2+y^2 = 4$, oriented downward. Find $\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$ by

(a) Changing to a line integral

(b) Evaluate on a different surface.

$\oint_C \vec{F} \cdot d\vec{r}$



$\vec{r} = \langle 2\cos\theta, 2\sin\theta, 4\rangle$

$\times$

$\langle 2\sin\theta, 2\cos\theta, 4\rangle$

$$\int_0^{2\pi} x^2z^2(-2\sin\theta)\,d\theta + y^2z^2(2\cos\theta)\,d\theta$$

$$= \int_0^{2\pi} -4\cos^2\theta \cdot 16 \cdot 2\sin\theta + 4\sin^2\theta \cdot 16 \cdot 2\cos\theta \, d\theta = 0$$

let $u = \cos\theta$,

$du = -\sin\theta\, d\theta$.

Problem 2. Let C be a simple closed curve that lies in the plane $x + y + z = 1$. Show that the line integral

$$\oint_C zdx - 2xdy + 3ydz$$

depends only on the area of the region enclosed by C and not on the shape of C or its location in the plane.

$\vec{F} = \langle z, -2x, 3y \rangle$

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z & -2x & 3y \end{vmatrix}$$

$\langle 3, 1, -2 \rangle$

$\vec{n} \langle 1, 1, 1 \rangle$

$\iint_S 2 \cdot dS$ Area

Problem 3. Evaluate

$$\oint_C (y + \sin x)dx + (z^2 + \cos y)dy + x^3 dz$$

where C is the curve $\mathbf{r}(t) = \langle \sin t, \cos t, \sin 2t \rangle$

$$\vec{F} = \langle y + \sin x, \; z^2 + \cos y, \; x^3 \rangle$$

Problem 4. If $\mathbf{a}$ is a constant vector, and $\mathbf{r} = \langle x, y, z\rangle$ is the divergence vector field, show that

$$\iint_S 2\mathbf{a} \cdot d\mathbf{S} = \oint_C (\mathbf{a} \times \mathbf{r}) \cdot d\mathbf{r}$$

where the assumptions on S and C are as in Stokes' Theorem.