15. Remark: 1. set $\vec{r}(t) = 5\cos(\omega t)i + 5\sin(\omega t)j$

$$\frac{2\pi}{\omega} = 2. \left(\text{Period} = \frac{2\pi}{\omega}\right).$$

18. Remark: Given $\frac{dz}{dt}$, can chain rule and change as $\frac{dx}{dt}$

19. Remark: For $\vec{a}$, it is $\frac{d\vec{v}}{dt}$. $= \frac{d}{dt}\frac{d\vec{r}}{dt}$

$\therefore$ chain rule is required.

e.g.

$$\mathbf{r} = 3u\mathbf{i} + 3u^2\mathbf{j} + 2u^3\mathbf{k}$$
$$\mathbf{v} = \frac{du}{dt}(3\mathbf{i} + 6u\mathbf{j} + 6u^2\mathbf{k})$$
$$\mathbf{a} = \frac{d^2u}{dt^2}(3\mathbf{i} + 6u\mathbf{j} + 6u^2\mathbf{k}) + \left(\frac{du}{dt}\right)^2(6\mathbf{j} + 12u\mathbf{k}).$$

24. Technique: $\frac{d}{dt}|\vec{r}|^2$ first.

Remark: $|\vec{r}|$ is speed!

25. Technique: same as Q24, start with $\frac{d}{dt}|\vec{r} - \vec{r_0}|$,

Remark: $\frac{d}{dt}\vec{r_0} = \vec{0}$., $\vec{v} = \frac{d\vec{r}}{dt}$

26. Remark: $|\vec{r}| > 0$ means that it is moving away from the origin.

EXERCISES 11.1

In Exercises 1–14, find the velocity, speed, and acceleration at time t of the particle whose position is $\mathbf{r}(t)$. Describe the path of the particle.

1. $\mathbf{r} = \mathbf{i} + t\mathbf{j}$

2. $\mathbf{r} = t^2\mathbf{i} + \mathbf{k}$

3. $\mathbf{r} = t^2\mathbf{j} + t\mathbf{k}$

4. $\mathbf{r} = \mathbf{i} + t\mathbf{j} + t\mathbf{k}$

5. $\mathbf{r} = t^2\mathbf{i} - t^2\mathbf{j} + \mathbf{k}$

6. $\mathbf{r} = t\mathbf{i} + t^2\mathbf{j} + t^2\mathbf{k}$

7. $\mathbf{r} = a\cos t\,\mathbf{i} + a\sin t\,\mathbf{j} + ct\mathbf{k}$

8. $\mathbf{r} = a\cos\omega t\,\mathbf{i} + b\mathbf{j} + a\sin\omega t\,\mathbf{k}$

9. $\mathbf{r} = 3\cos t\,\mathbf{i} + 4\cos t\,\mathbf{j} + 5\sin t\,\mathbf{k}$

10. $\mathbf{r} = 3\cos t\,\mathbf{i} + 4\sin t\,\mathbf{j} + t\mathbf{k}$

11. $\mathbf{r} = ae^t\mathbf{i} + be^t\mathbf{j} + ce^t\mathbf{k}$

12. $\mathbf{r} = at\cos\omega t\,\mathbf{i} + at\sin\omega t\,\mathbf{j} + b\ln t\,\mathbf{k}$

13. $\mathbf{r} = e^{-t}\cos(e^t)\mathbf{i} + e^{-t}\sin(e^t)\mathbf{j} - e^t\mathbf{k}$

14. $\mathbf{r} = a\cos t\,\sin t\,\mathbf{i} + a\sin^2 t\,\mathbf{j} + a\cos t\,\mathbf{k}$

15. A particle moves around the circle $x^2 + y^2 = 25$ at constant speed, making one revolution in 2 s. Find its acceleration when it is at $(3, 4)$.

16. A particle moves to the right along the curve $y = 3/x$. If its speed is 10 when it passes through the point $\left(2, \frac{3}{2}\right)$, what is its velocity at that time?

17. A point P moves along the curve of intersection of the cylinder $z = x^2$ and the plane $x + y = 2$ in the direction of increasing y with constant speed $v = 3$. Find the velocity of P when it is at $(1, 1, 1)$.

18. An object moves along the curve $y = x^2$, $z = x^3$, with constant vertical speed $dz/dt = 3$. Find the velocity and acceleration of the object when it is at the point $(2, 4, 8)$.

19. A particle moves along the curve $\mathbf{r} = 3u\mathbf{i} + 3u^2\mathbf{j} + 2u^3\mathbf{k}$ in the direction corresponding to increasing u and with a constant speed of 6. Find the velocity and acceleration of the particle when it is at the point $(3, 3, 2)$.

20. A particle moves along the curve of intersection of the cylinders $y = -x^2$ and $z = x^2$ in the direction in which x increases. (All distances are in centimetres.) At the instant when the particle is at the point $(1, -1, 1)$, its speed is 9 cm/s, and that speed is increasing at a rate of 3 cm/s^2. Find the velocity and acceleration of the particle at that instant.

21. Show that if the dot product of the velocity and acceleration of a moving particle is positive (or negative), then the speed of the particle is increasing (or decreasing).

22. Verify the formula for the derivative of a dot product given in Theorem 1(c).

23. Verify the formula for the derivative of a 3×3 determinant in the second remark following Theorem 1. Use this formula to verify the formula for the derivative of the cross product in Theorem 1.

24. If the position and velocity vectors of a moving particle are always perpendicular, show that the path of the particle lies on a sphere.

25. Generalize Exercise 24 to the case where the velocity of the particle is always perpendicular to the line joining the particle to a fixed point P_0.

26. What can be said about the motion of a particle at a time when its position and velocity satisfy $\mathbf{r} \bullet \mathbf{v} > 0$? What can be said when $\mathbf{r} \bullet \mathbf{v} < 0$?

In Exercises 27–32, assume that the vector functions encountered have continuous derivatives of all required orders.

27. Show that $\dfrac{d}{dt}\left(\dfrac{d\mathbf{u}}{dt} \times \dfrac{d^2\mathbf{u}}{dt^2}\right) = \dfrac{d\mathbf{u}}{dt} \times \dfrac{d^3\mathbf{u}}{dt^3}$.

28. Write the Product Rule for $\dfrac{d}{dt}\Big(\mathbf{u} \bullet (\mathbf{v} \times \mathbf{w})\Big)$.

29. Write the Product Rule for $\dfrac{d}{dt}\Big(\mathbf{u} \times (\mathbf{v} \times \mathbf{w})\Big)$.

30. Expand and simplify: $\dfrac{d}{dt}\left(\mathbf{u} \times \left(\dfrac{d\mathbf{u}}{dt} \times \dfrac{d^2\mathbf{u}}{dt^2}\right)\right)$.

31. Expand and simplify: $\dfrac{d}{dt}\Big((\mathbf{u} + \mathbf{u}'') \bullet (\mathbf{u} \times \mathbf{u}')\Big)$.

32. Expand and simplify: $\dfrac{d}{dt}\Big((\mathbf{u} \times \mathbf{u}') \bullet (\mathbf{u}' \times \mathbf{u}'')\Big)$.

33. If at all times t the position and velocity vectors of a moving particle satisfy $\mathbf{v}(t) = 2\mathbf{r}(t)$, and if $\mathbf{r}(0) = \mathbf{r}_0$, find $\mathbf{r}(t)$ and the acceleration $\mathbf{a}(t)$. What is the path of motion?

34. Verify that $\mathbf{r} = \mathbf{r}_0\cos(\omega t) + (\mathbf{v}_0/\omega)\sin(\omega t)$ satisfies the initial-value problem

$$\frac{d^2\mathbf{r}}{dt^2} = -\omega^2\mathbf{r}, \qquad \mathbf{r}'(0) = \mathbf{v}_0, \qquad \mathbf{r}(0) = \mathbf{r}_0.$$

(It is the unique solution.) Describe the path $\mathbf{r}(t)$. What is the path if $\mathbf{r}_0$ is perpendicular to $\mathbf{v}_0$?

35. **(Free fall with air resistance)** A projectile falling under gravity and slowed by air resistance proportional to its speed has position satisfying

$$\frac{d^2\mathbf{r}}{dt^2} = -g\mathbf{k} - c\frac{d\mathbf{r}}{dt},$$

where c is a positive constant. If $\mathbf{r} = \mathbf{r}_0$ and $d\mathbf{r}/dt = \mathbf{v}_0$ at time $t = 0$, find $\mathbf{r}(t)$. (*Hint:* Let $\mathbf{w} = e^{ct}(d\mathbf{r}/dt)$.) Show that the solution approaches that of the projectile problem given in this section as $c \to 0$.

In Exercises 1–14, find the velocity, speed, and acceleration at time t of the particle whose position is $\mathbf{r}(t)$. Describe the path of the particle.

1. $\mathbf{r} = \mathbf{i} + t\mathbf{j}$
2. $\mathbf{r} = t^2\mathbf{i} + \mathbf{k}$
3. $\mathbf{r} = t^2\mathbf{j} + t\mathbf{k}$
4. $\mathbf{r} = \mathbf{i} + t\mathbf{j} + t\mathbf{k}$

$r' = \langle 0, 1, 0 \rangle$

speed $= 1$

$r'' = \langle 0, 0, 0 \rangle$

Path: $x = 1$

2. $\langle 2t, 0, 0 \rangle$

speed $= 2|t|$

$r'' = \langle 2, 0, 0 \rangle$

3. $r' = \langle 0, 2t, 1 \rangle$

speed $= \sqrt{4t^2+1}$

$r'' = \langle 0, 2, 0 \rangle$

4. $r' = \langle 0, 1, 1 \rangle$

$|r'| = \sqrt{2}$

$r'' = \langle 0, 0, 0 \rangle$

5. $\mathbf{r} = t^2\mathbf{i} - t^2\mathbf{j} + \mathbf{k}$
6. $\mathbf{r} = t\mathbf{i} + t^2\mathbf{j} + t^2\mathbf{k}$
7. $\mathbf{r} = a\cos t\,\mathbf{i} + a\sin t\,\mathbf{j} + ct\mathbf{k}$

5. $\langle 2t, -2t, 1 \rangle$

speed $= \sqrt{8t^2+1}$

$r'' \langle 2, -2, 0 \rangle$

6. $\langle 1, 2t, 2t \rangle$

speed $= \sqrt{8t^2+1}$

$r'' = \langle 0, 2, 2 \rangle$

7. $\langle -a\sin t, a\cos t, c \rangle$

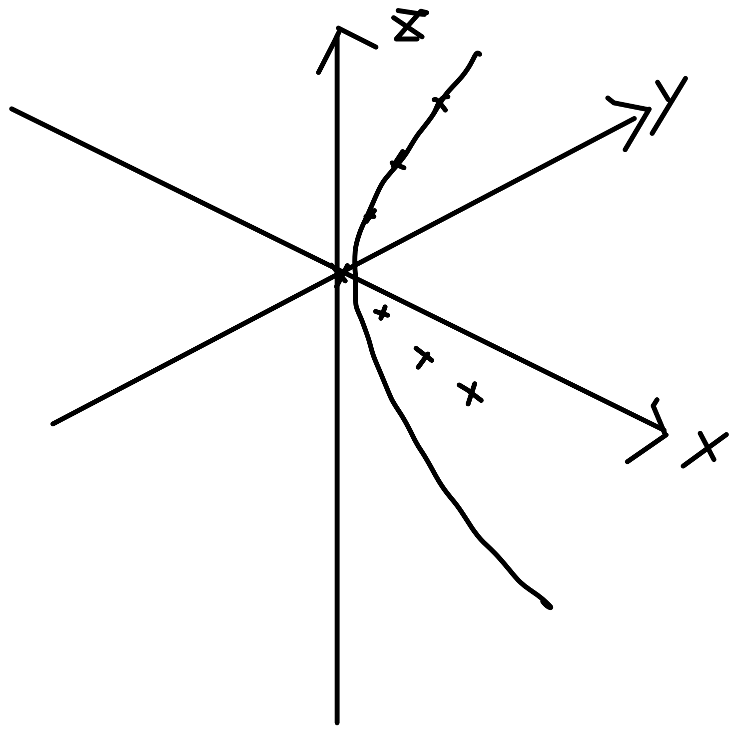
speed $\sqrt{a^2+c^2}$

Path: helix

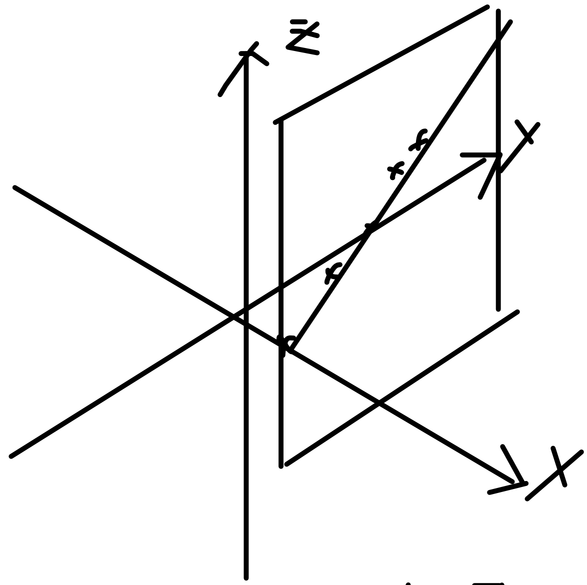
$r'' \langle -a\cos t, -a\sin t, 0 \rangle$

Made with Goodnotes

3. $\vec{r} = t^2\vec{j} + t\vec{k}$

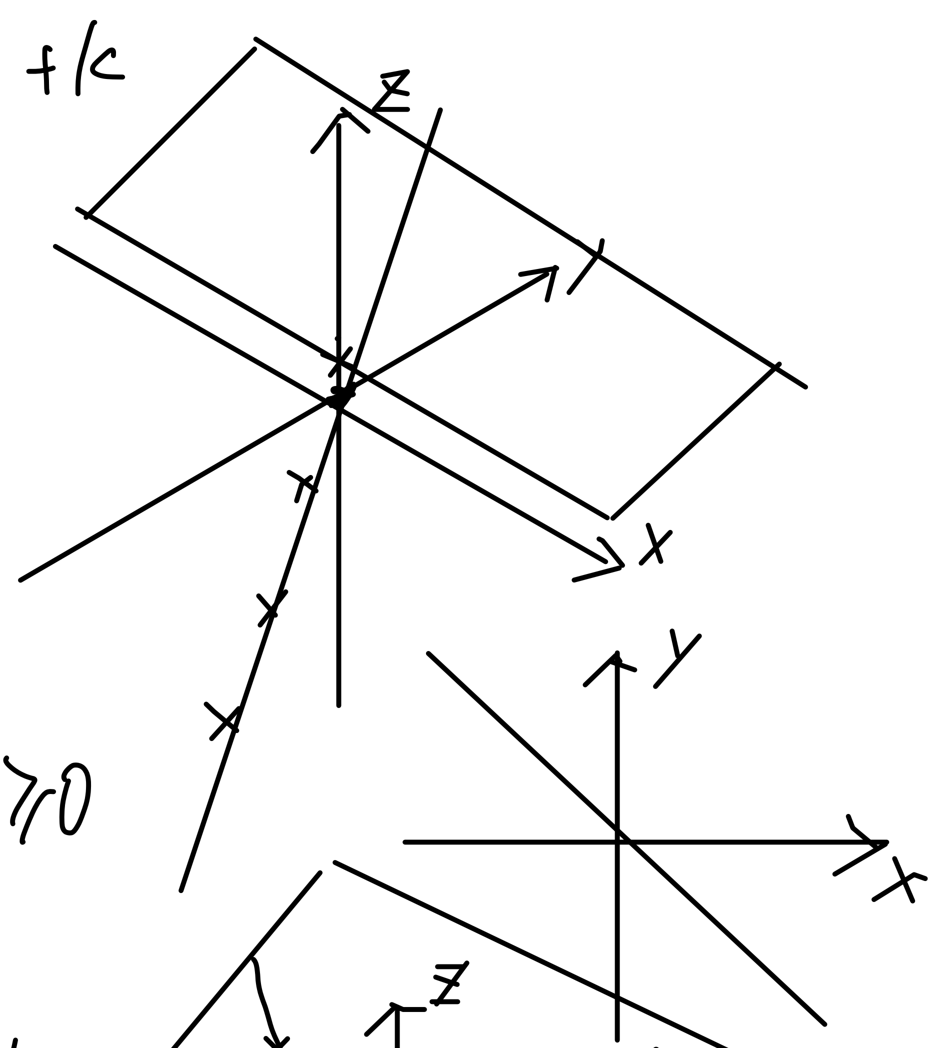


4. $\vec{r} = i + tj + tk$



$y = z,\ x = 1$

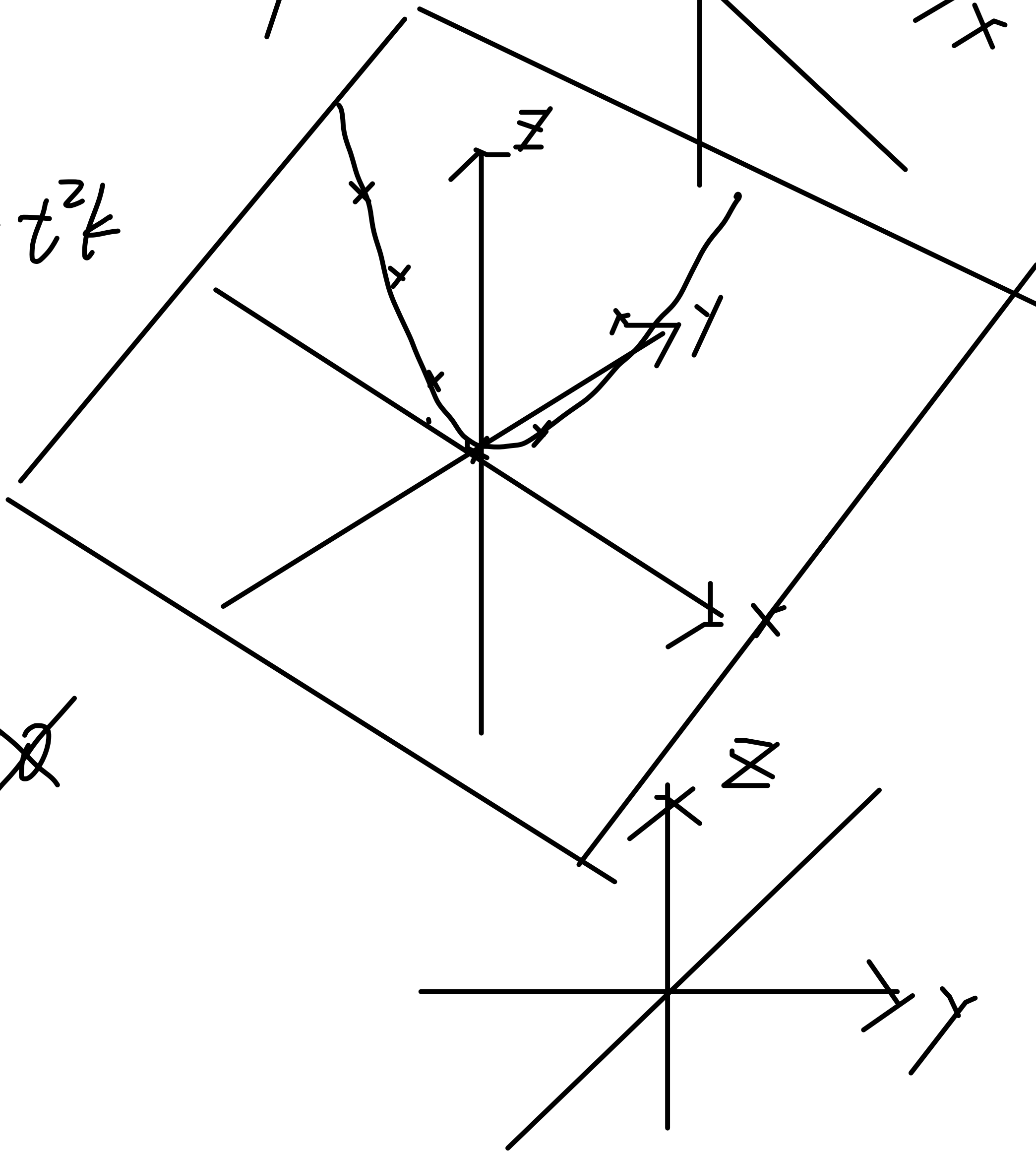
5. $\vec{r} = t^2 i - t^2 j + k$



$x + y = 0, z = 1$

$x = -y, x \geq 0$

6. $ti + t^2 j + t^2 k$



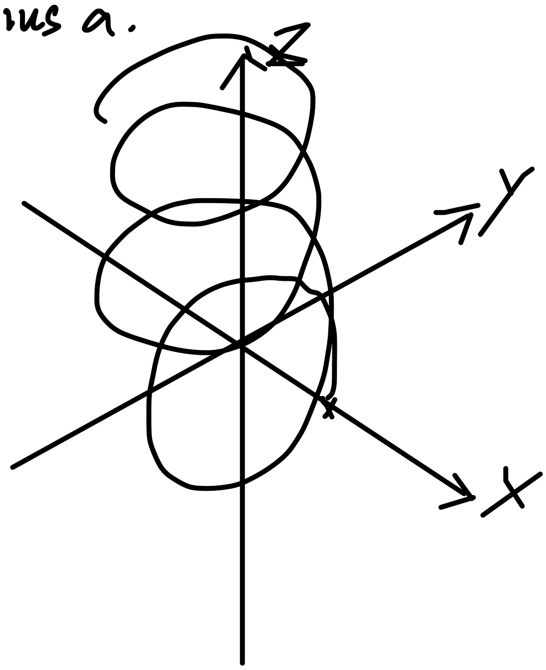
$z = y \geq 0$

$z = y = x^2 \cancel{\geq 0}$

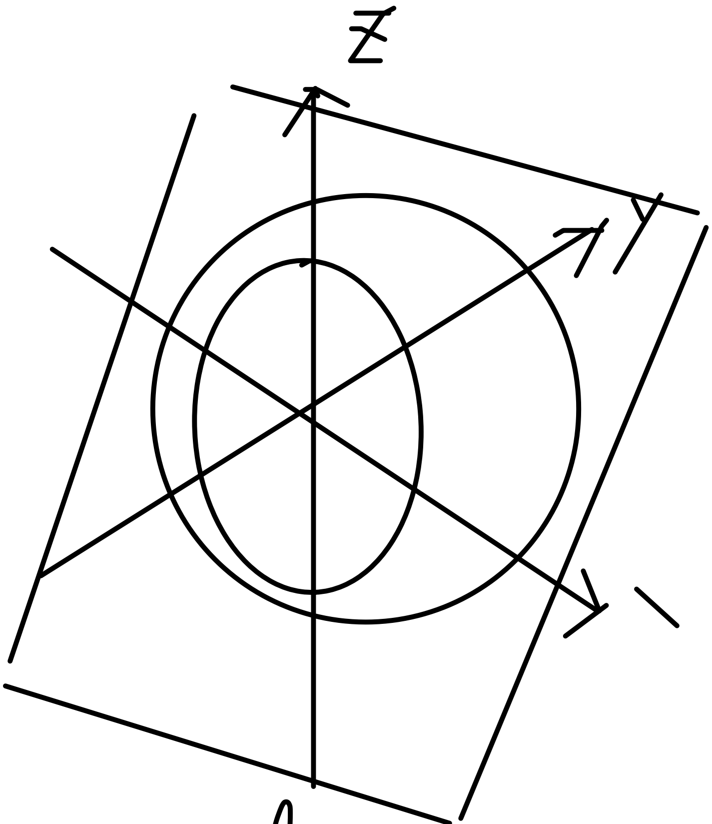
7. $\mathbf{r} = a\cos t\,\mathbf{i} + a\sin t\,\mathbf{j} + ct\,\mathbf{k}$ helix, radius a.

8. $\mathbf{r} = a\cos \omega t\,\mathbf{i} + b\mathbf{j} + a\sin \omega t\,\mathbf{k}$

9. $\mathbf{r} = 3\cos t\,\mathbf{i} + 4\cos t\,\mathbf{j} + 5\sin t\,\mathbf{k}$



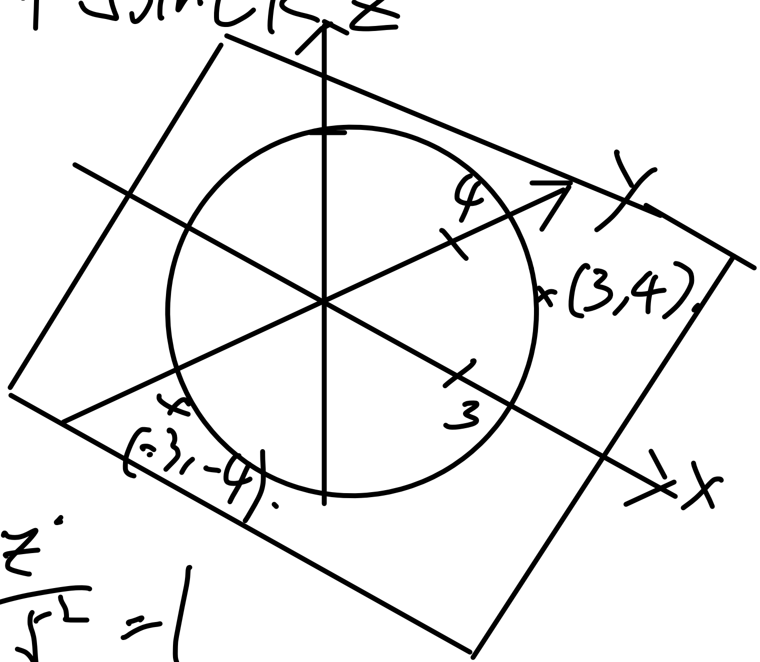
8.



Circle, centre 0, b, 0, radius a

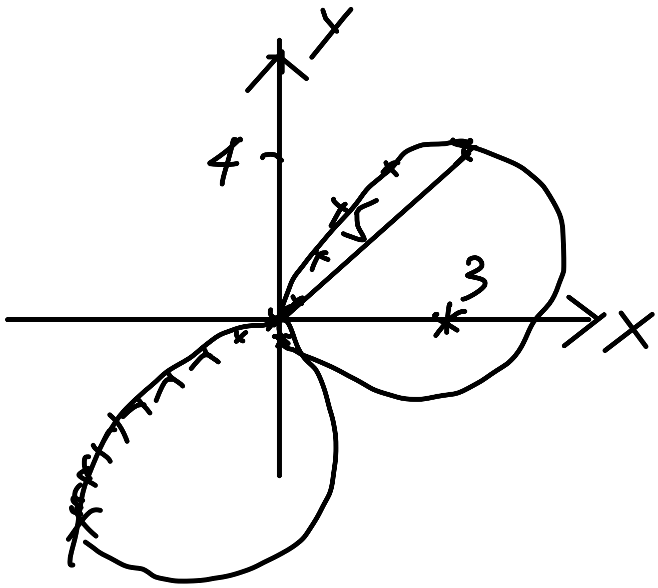
$x^2 + z^2 = a^2, \quad y = b$

9. $r = 3\cos t\, i + 4\cos t\, j + 5\sin t\, k$



ellipse,

$$\frac{x^2}{3^2} + \frac{y^2}{4^2} + \frac{z^2}{5^2} = 1$$



$$x^2 + y^2 + z^2 = 25$$

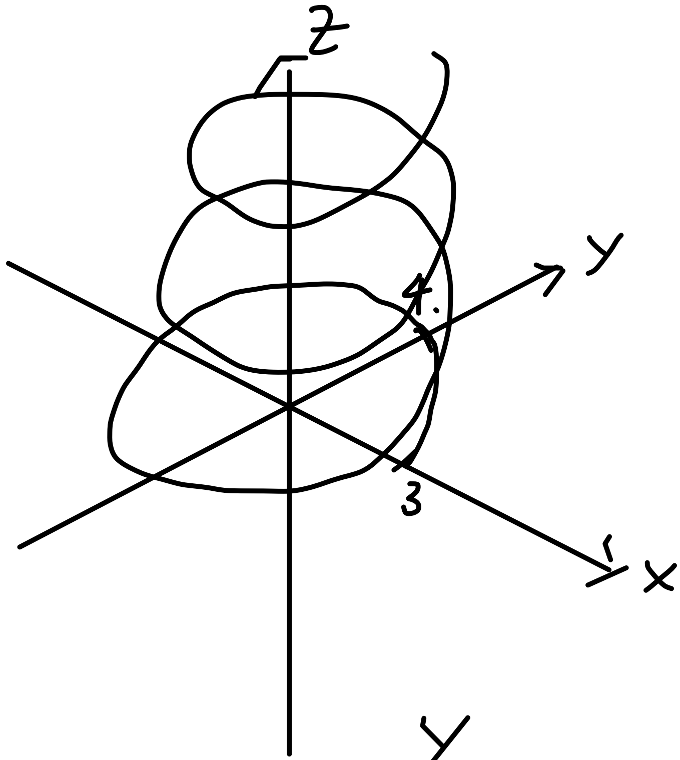
$$y = \frac{4}{3}x$$

$$3y = 4x.$$

10. $\mathbf{r} = 3\cos t\,\mathbf{i} + 4\sin t\,\mathbf{j} + t\,\mathbf{k}$

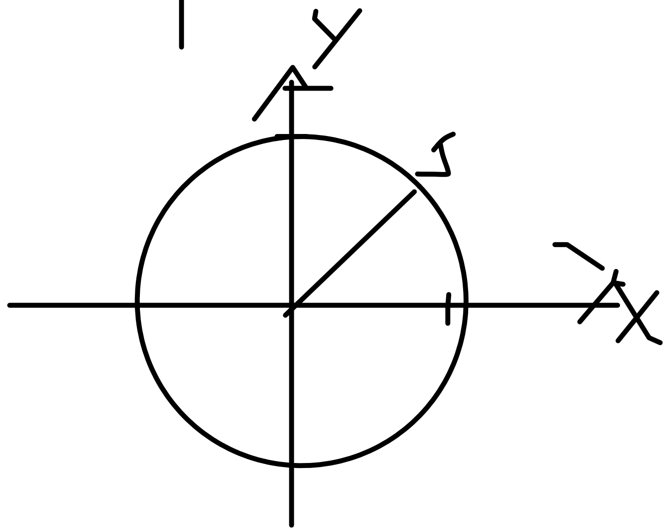
11. $\mathbf{r} = ae^t\mathbf{i} + be^t\mathbf{j} + ce^t\mathbf{k}$

10. circular helix, radius 5. centre (0,0,0).

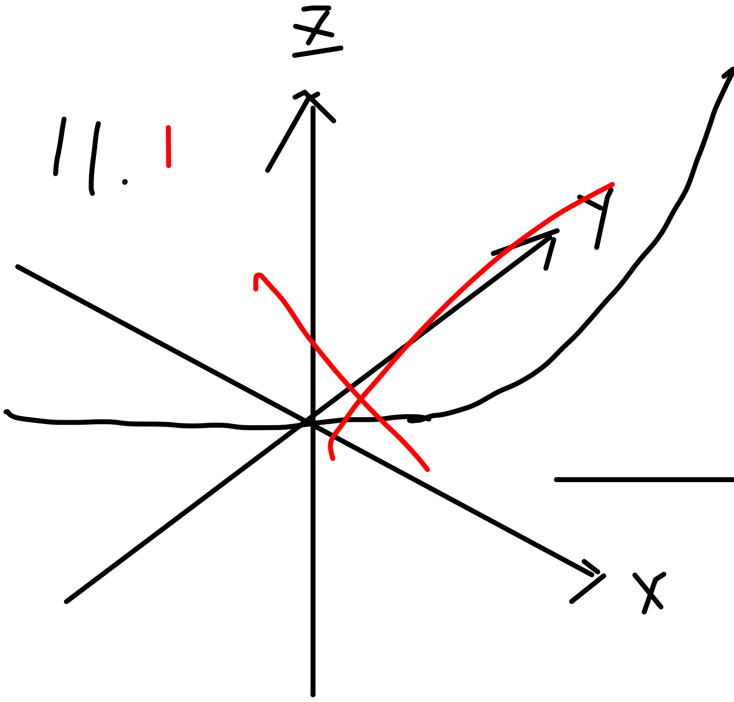


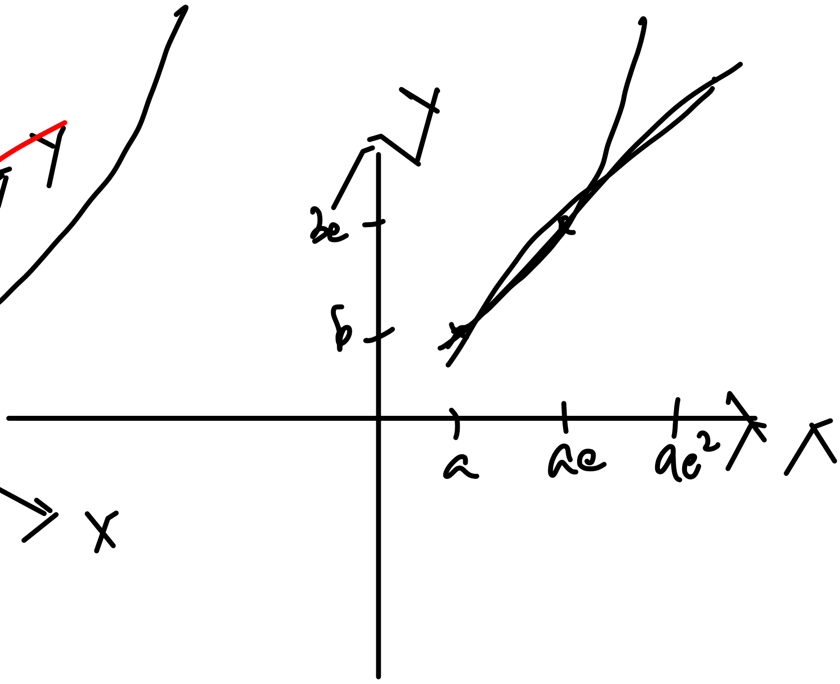
$x = ae^t$

$y = be^t$



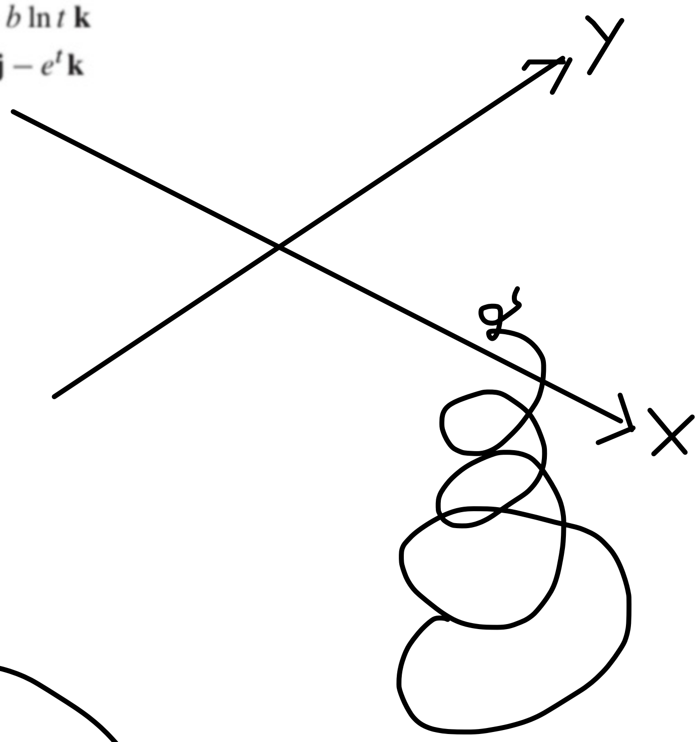
11. 1

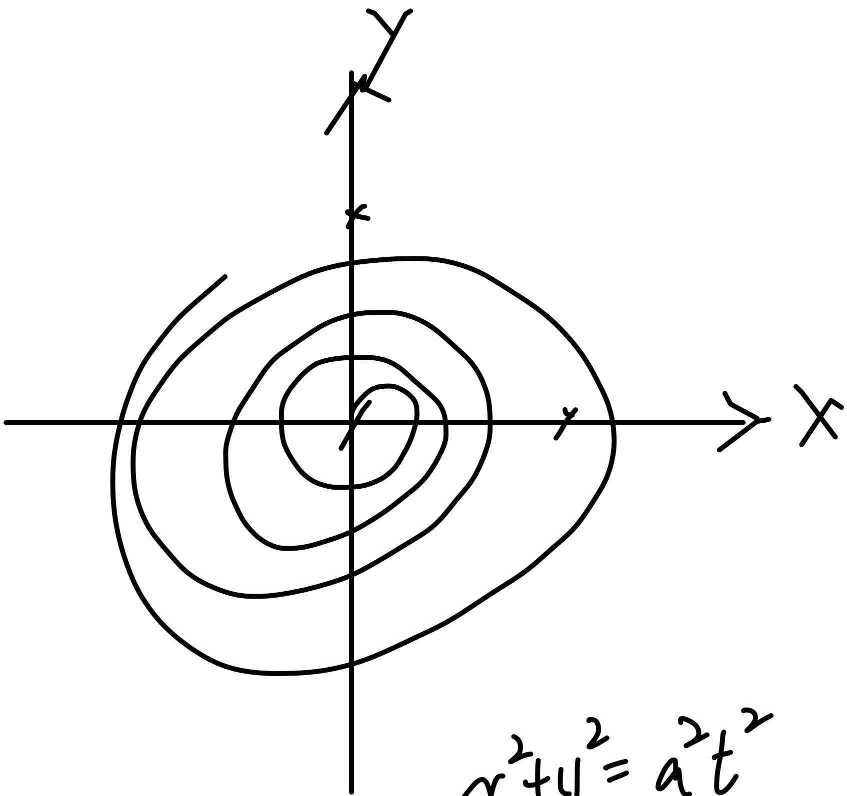




12. $\mathbf{r} = at\cos\omega t\,\mathbf{i} + at\sin\omega t\,\mathbf{j} + b\ln t\,\mathbf{k}$

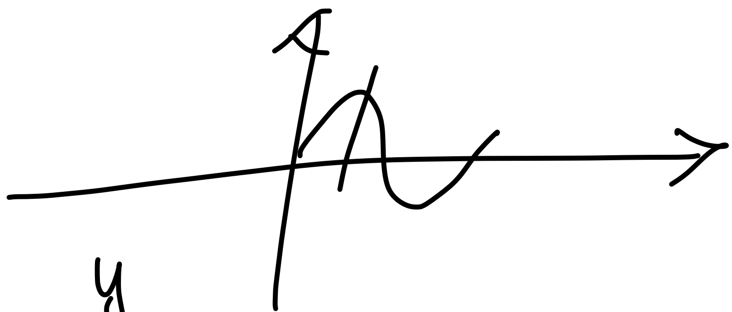
13. $\mathbf{r} = e^{-t}\cos(e^{t})\mathbf{i} + e^{-t}\sin(e^{t})\mathbf{j} - e^{t}\mathbf{k}$





t	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin t$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos t$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

$$x^2+y^2=a^2t^2$$



$$x = at\cos\omega t$$
$$y = at\sin\omega t$$

$$\frac{x}{\cos\omega t} = \frac{y}{\sin\omega t}$$
$$x\sin\omega t = y\cos\omega t$$

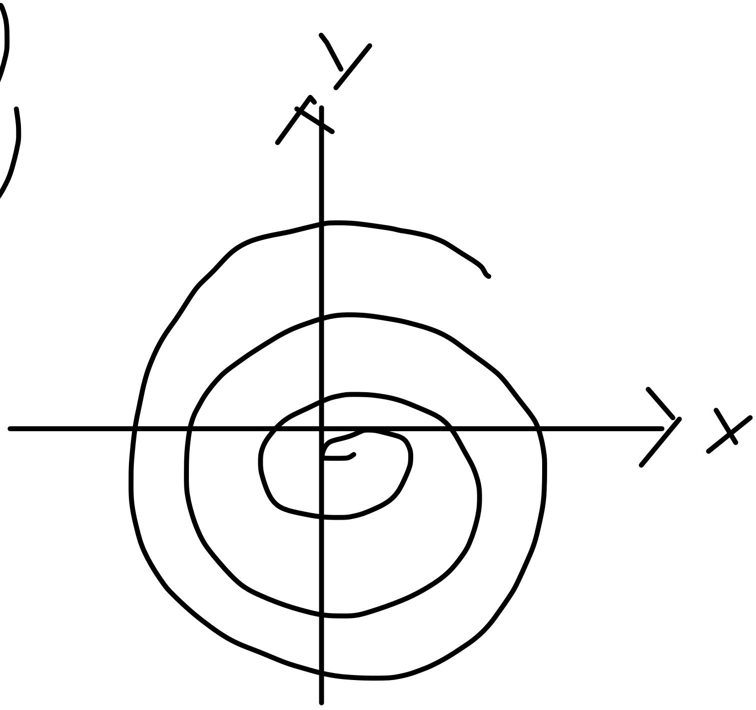
$$z = b\ln t$$
$$\frac{z}{b} = \ln t$$

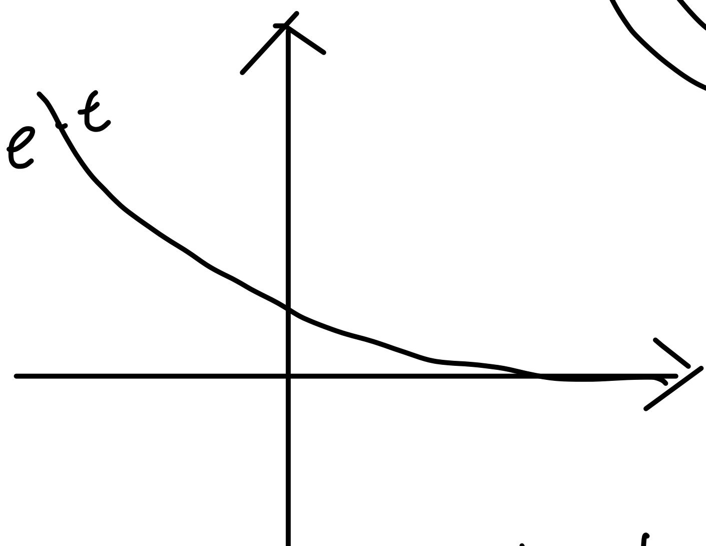
$$t = e^{z/b}$$

13. $\mathbf{r} = e^{-t}\cos(e^{t})\mathbf{i} + e^{-t}\sin(e^{t})\mathbf{j} - e^{t}\mathbf{k}$

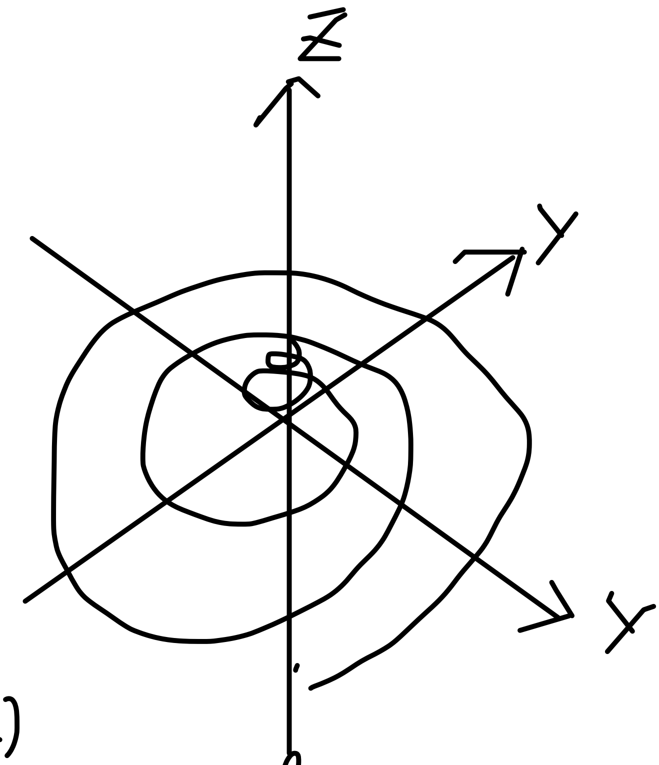
$x = e^{-t}\cos(e^{t})$

$y = e^{-t}\sin(e^{t})$





$x^2 + y^2 = \frac{1}{z^2}$



$x^2 + y^2 = e^{-2t}$

$e^{-2\ln(-z)}$

spiral.

$z = -e^{t}$,

$= e^{\ln(-z)^{-2}}$

$-z = e^{t}$

$= z^{-2}$

$\frac{1}{z^2}$

$\ln(-z) = t$

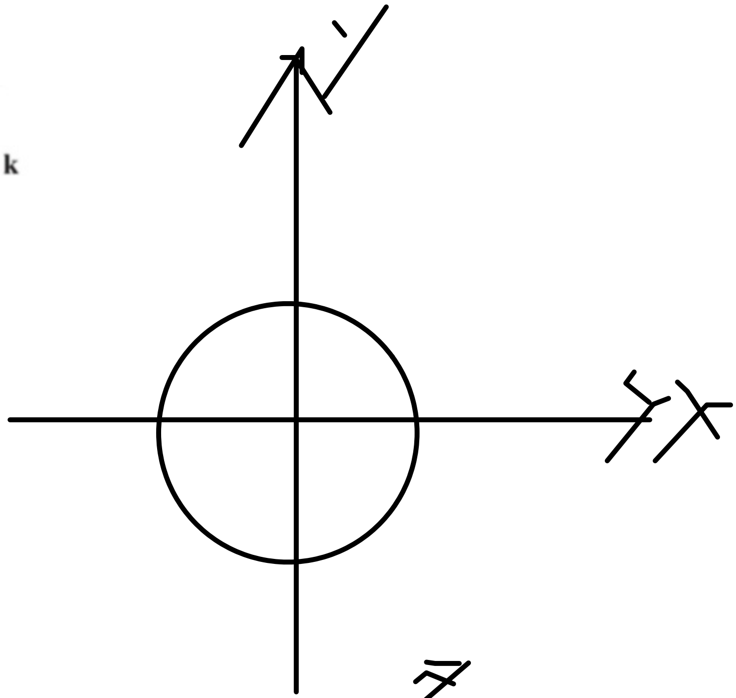
13. $\mathbf{r} = e^{-t}\cos(e^t)\mathbf{i} + e^{-t}\sin(e^t)\mathbf{j} - e^t\mathbf{k}$

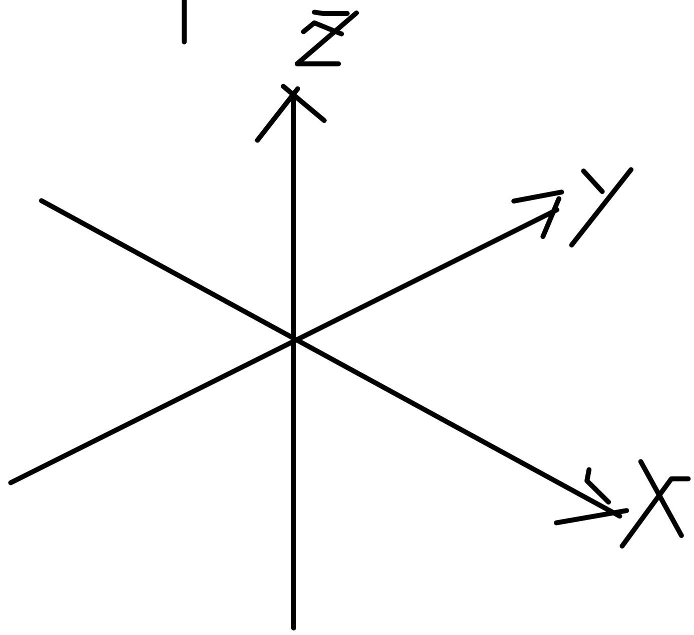
14. $\mathbf{r} = a\cos t\sin t\,\mathbf{i} + a\sin^2 t\,\mathbf{j} + a\cos t\,\mathbf{k}$

14.

$x = a\cos t\sin t$

$y = a\sin^2 t$





15. A particle moves around the circle $x^2 + y^2 = 25$ at constant speed, making one revolution in 2 s. Find its acceleration when it is at $(3, 4)$.

$$y^2 = 25 - x^2$$

$$r = x\,i + \sqrt{25-x^2}\,j$$

$$\vec{V} = \frac{dr}{dt} = \frac{dr}{dx} \cdot \frac{dx}{dt}$$

$$= \left(i + \tfrac{1}{2}(25-x^2)^{-\frac{1}{2}}(-2x)\,j\right)\frac{dx}{dt}$$

$$= \left(i - \frac{2x}{\sqrt{25-x^2}}\,j\right)\frac{dx}{dt}$$

$$|\vec{V}| = 5$$

$$\sqrt{1+\left(\frac{2x}{\sqrt{25-x^2}}\right)^2}\left|\frac{dx}{dt}\right| = 5$$

$$\sqrt{1+\frac{4x^2}{25-x^2}}\left|\frac{dx}{dt}\right| = 5$$

$$\sqrt{\frac{25-x^2+4x^2}{25-x^2}}\left|\frac{dx}{dt}\right| = 5$$

$$\frac{5}{\sqrt{\frac{28}{24}}}$$

$$\frac{dx}{dt} = \frac{5}{\sqrt{\frac{25+3x^2}{25-x^2}}}$$

$$\left.\frac{dx}{dt}\right|_{x=1} = \frac{5}{\sqrt{\frac{25+3}{25-1}}}$$

15. A particle moves around the circle $x^2 + y^2 = 25$ at constant speed, making one revolution in 2 s. Find its acceleration when it is at $(3, 4)$.

$$\frac{2\pi}{\omega} = 2$$

CHAPTER 11. VECTOR FUNCTIONS AND CURVES

Section 11.1 Vector Functions of One Variable (page 629)

1. Position: $\mathbf{r} = \mathbf{i} + t\mathbf{j}$
Velocity: $\mathbf{v} = \mathbf{j}$
Speed: $v = 1$
Acceleration : $\mathbf{a} = \mathbf{0}$
Path: the line $x = 1$ in the xy-plane.

2. Position: $\mathbf{r} = t^2\mathbf{i} + \mathbf{k}$
Velocity: $\mathbf{v} = 2t\mathbf{i}$
Speed: $v = 2|t|$
Acceleration : $\mathbf{a} = 2\mathbf{i}$
Path: the line $z = 1$, $y = 0$.

3. Position: $\mathbf{r} = t^2\mathbf{j} + t\mathbf{k}$
Velocity: $\mathbf{v} = 2t\mathbf{j} + \mathbf{k}$
Speed: $v = \sqrt{4t^2 + 1}$
Acceleration : $\mathbf{a} = 2\mathbf{j}$
Path: the parabola $y = z^2$ in the plane $x = 0$.

4. Position: $\mathbf{r} = \mathbf{i} + t\mathbf{j} + t\mathbf{k}$
Velocity: $\mathbf{v} = \mathbf{j} + \mathbf{k}$
Speed: $v = \sqrt{2}$
Acceleration : $\mathbf{a} = \mathbf{0}$
Path: the straight line $x = 1$, $y = z$.

5. Position: $\mathbf{r} = t^2\mathbf{i} - t^2\mathbf{j} + \mathbf{k}$
Velocity: $\mathbf{v} = 2t\mathbf{i} - 2t\mathbf{j}$
Speed: $v = 2\sqrt{2}t$
Acceleration: $\mathbf{a} = 2\mathbf{i} - 2\mathbf{j}$
Path: the half-line $x = -y \ge 0$, $z = 1$.

6. Position: $\mathbf{r} = t\mathbf{i} + t^2\mathbf{j} + t^2\mathbf{k}$
Velocity: $\mathbf{v} = \mathbf{i} + 2t\mathbf{j} + 2t\mathbf{k}$
Speed: $v = \sqrt{1 + 8t^2}$
Acceleration: $\mathbf{a} = 2\mathbf{j} + 2\mathbf{k}$
Path: the parabola $y = z = x^2$.

7. Position: $\mathbf{r} = a\cos t\mathbf{i} + a\sin t\mathbf{j} + ct\mathbf{k}$
Velocity: $\mathbf{v} = -a\sin t\mathbf{i} + a\cos t\mathbf{j} + c\mathbf{k}$
Speed: $v = \sqrt{a^2 + c^2}$
Acceleration: $\mathbf{a} = -a\cos t\mathbf{i} - a\sin t\mathbf{j}$
Path: a circular helix.

8. Position: $\mathbf{r} = a\cos\omega t\mathbf{i} + b\mathbf{j} + a\sin\omega t\mathbf{k}$
Velocity: $\mathbf{v} = -a\omega\sin\omega t\mathbf{i} + a\omega\cos\omega t\mathbf{k}$
Speed: $v = |a\omega|$
Acceleration: $\mathbf{a} = -a\omega^2\cos\omega t\mathbf{i} - a\omega^2\sin\omega t\mathbf{k}$
Path: the circle $x^2 + z^2 = a^2$, $y = b$.

9. Position: $\mathbf{r} = 3\cos t\mathbf{i} + 4\cos t\mathbf{j} + 5\sin t\mathbf{k}$
Velocity: $\mathbf{v} = -3\sin t\mathbf{i} - 4\sin t\mathbf{j} + 5\cos t\mathbf{k}$
Speed: $v = \sqrt{9\sin^2 t + 16\sin^2 t + 25\cos^2 t} = 5$
Acceleration : $\mathbf{a} = -3\cos t\mathbf{i} - 4\cos t\mathbf{j} - 5\sin t\mathbf{k} = -\mathbf{r}$
Path: the circle of intersection of the sphere $x^2 + y^2 + z^2 = 25$ and the plane $4x = 3y$.

10. Position: $\mathbf{r} = 3\cos t\mathbf{i} + 4\sin t\mathbf{j} + t\mathbf{k}$
Velocity: $\mathbf{v} = -3\sin t\mathbf{i} + 4\cos t\mathbf{j} + \mathbf{k}$
Speed: $v = \sqrt{9\sin^2 t + 16\cos^2 t + 1} = \sqrt{10 + 7\cos^2 t}$
Acceleration : $\mathbf{a} = -3\cos t\mathbf{i} - 4\sin t\mathbf{j} = t\mathbf{k} - \mathbf{r}$
Path: a helix (spiral) wound around the elliptic cylinder $(x^2/9) + (y^2/16) = 1$.

11. Position: $\mathbf{r} = ae^t\mathbf{i} + be^t\mathbf{j} + ce^t\mathbf{k}$
Velocity and acceleration: $\mathbf{v} = \mathbf{a} = \mathbf{r}$
Speed: $v = e^t\sqrt{a^2 + b^2 + c^2}$
Path: the half-line $\dfrac{x}{a} = \dfrac{y}{b} = \dfrac{z}{c} > 0$.

12. Position: $\mathbf{r} = at\cos\omega t\mathbf{i} + at\sin\omega t\mathbf{j} + b\ln t\mathbf{k}$
Velocity: $\mathbf{v} = a(\cos\omega t - \omega t\sin\omega t)\mathbf{i}$
$\qquad + a(\sin\omega t + \omega t\cos\omega t)\mathbf{j} + (b/t)\mathbf{k}$
Speed: $v = \sqrt{a^2(1 + \omega^2 t^2) + (b^2/t^2)}$
Acceleration: $\mathbf{a} = -a\omega(2\sin\omega t + \omega\cos\omega t)\mathbf{i}$
$\qquad + a\omega(2\cos\omega t - \omega\sin\omega t)\mathbf{j} - (b/t^2)\mathbf{k}$
Path: a spiral on the surface $x^2 + y^2 = a^2 e^{z/b}$.

13. Position: $\mathbf{r} = e^{-t}\cos(e^t)\mathbf{i} + e^{-t}\sin(e^t)\mathbf{j} - e^t\mathbf{k}$
Velocity: $\mathbf{v} = -\Big(e^{-t}\cos(e^t) + \sin(e^t)\Big)\mathbf{i}$
$\qquad - \Big(e^{-t}\sin(e^t) - \cos(e^t)\Big)\mathbf{j} - e^t\mathbf{k}$
Speed: $v = \sqrt{1 + e^{-2t} + e^{2t}}$
Acceleration: $\mathbf{a} = \Big((e^{-t} - e^t)\cos(e^t) + \sin(e^t)\Big)\mathbf{i}$
$\qquad + \Big((e^{-t} - e^t)\sin(e^t) - \cos(e^t)\Big)\mathbf{j} - e^t\mathbf{k}$
Path: a spiral on the surface $z\sqrt{x^2 + y^2} = -1$.

14. Position: $\mathbf{r} = a\cos t\sin t\mathbf{i} + a\sin^2 t\mathbf{j} + a\cos t\mathbf{k}$
$\qquad = \dfrac{a}{2}\sin 2t\mathbf{i} + \dfrac{a}{2}\Big(1 - \cos 2t\Big)\mathbf{j} + a\cos t\mathbf{k}$
Velocity: $\mathbf{v} = a\cos 2t\mathbf{i} + a\sin 2t\mathbf{j} - a\sin t\mathbf{k}$
Speed: $v = a\sqrt{1 + \sin^2 t}$
Acceleration: $\mathbf{a} = -2a\sin 2t\mathbf{i} + 2a\cos 2t\mathbf{j} - a\cos t\mathbf{k}$
Path: the path lies on the sphere $x^2 + y^2 + z^2 = a^2$, on the surface defined in terms of spherical polar coordinates by $\phi = \theta$, on the circular cylinder $x^2 + y^2 = ay$, and on the parabolic cylinder $ay + z^2 = a^2$. Any two of these surfaces serve to pin down the shape of the path.

15. The position of the particle is given by

$$\mathbf{r} = 5\cos(\omega t)\mathbf{i} + 5\sin(\omega t)\mathbf{j},$$

where $\omega = \pi$ to ensure that $\mathbf{r}$ has period $2\pi/\omega = 2$ s. Thus

$$\mathbf{a} = \frac{d^2\mathbf{r}}{dt^2} = -\omega^2\mathbf{r} = -\pi^2\mathbf{r}.$$

At $(3, 4)$, the acceleration is $-3\pi^2\mathbf{i} - 4\pi^2\mathbf{j}$.

8. $\mathbf{r} = a\cos\omega t\,\mathbf{i} + b\mathbf{j} + a\sin\omega t\,\mathbf{k}$

9. $\mathbf{r} = 3\cos t\,\mathbf{i} + 4\cos t\,\mathbf{j} + 5\sin t\,\mathbf{k}$

10. $\mathbf{r} = 3\cos t\,\mathbf{i} + 4\sin t\,\mathbf{j} + t\mathbf{k}$

11. $\mathbf{r} = ae^t\mathbf{i} + be^t\mathbf{j} + ce^t\mathbf{k}$

12. $\mathbf{r} = at\cos\omega t\,\mathbf{i} + at\sin\omega t\,\mathbf{j} + b\ln t\,\mathbf{k}$

13. $\mathbf{r} = e^{-t}\cos(e^t)\mathbf{i} + e^{-t}\sin(e^t)\mathbf{j} - e^t\mathbf{k}$

14. $\mathbf{r} = a\cos t\sin t\,\mathbf{i} + a\sin^2 t\,\mathbf{j} + a\cos t\,\mathbf{k}$

15. A particle moves around the circle $x^2 + y^2 = 25$ at constant speed, making one revolution in 2 s. Find its acceleration when it is at $(3, 4)$.

16. A particle moves to the right along the curve $y = 3/x$. If its speed is 10 when it passes through the point $(2, \frac{3}{2})$, what is its velocity at that time?

8. $r' \langle -a\omega\sin\omega t,\, 0,\, a\omega\cos\omega t\rangle$

speed $|a\omega|$

$r'' \langle -a\omega^2\cos\omega t,\, 0,\, -a\omega^2\sin\omega t\rangle$

9. $r' \langle -3\sin t,\, -4\sin t,\, 5\cos t\rangle$

$|r'|\ \sqrt{9\sin^2 t + 16\sin^2 t + 25\cos^2 t}$

$= 5.$

$r'' \langle -3\cos t,\, -4\cos t,\, -5\sin t\rangle = -r$

10. $r' \langle -3\sin t,\, 4\cos t,\, 1\rangle$ $\quad r'' \langle -3\cos t, -4\sin t, 0\rangle$

$|r'| = \sqrt{9\sin^2 t + 16\cos^2 t + 1}$

$= \sqrt{9 + 7\cos^2 t + 1} = \sqrt{10 + 7\cos^2 t}$

11. $\mathbf{r} = ae^t\mathbf{i} + be^t\mathbf{j} + ce^t\mathbf{k}$

12. $\mathbf{r} = at\cos\omega t\,\mathbf{i} + at\sin\omega t\,\mathbf{j} + b\ln t\,\mathbf{k}$

13. $\mathbf{r} = e^{-t}\cos(e^t)\mathbf{i} + e^{-t}\sin(e^t)\mathbf{j} - e^t\mathbf{k}$

11. $r' \langle ae^t, be^t, ce^t \rangle$

$|r'| = \sqrt{a^2+b^2+c^2}\, e^t$

$r'' = \langle ae^t, be^t, ce^t \rangle$

12. $r' \langle -awt\sin wt,\ awt\cos wt, \frac{b}{t} \rangle$

$|r'| = \sqrt{a^2w^2t^2 + \frac{b^2}{t^2}}$

$|r'| = t^{\frac{1}{2}} \sqrt{a^2w^2 + b^2t^2}$

$r'' = \langle -aw^2t\cos wt, -aw^2t\sin wt, -\frac{b}{t^2} \rangle$

13. $r' \langle -e^{-t}\cos(e^t) - \quad \sin(e^t)), 0, -e^t \rangle$

$|r'| = \sqrt{(-e^{-t}\cos(e^t) - \sin(e^t))^2 + e^{2t}}$

$= \sqrt{e^{-2t}\cos^2(e^t) + 2e^{-t}\cos(e^t)\sin(e^t) + \sin^2(e^t) + e^{2t}}$

14. $\mathbf{r} = a\cos t \sin t\,\mathbf{i} + a\sin^2 t\,\mathbf{j} + a\cos t\,\mathbf{k}$

15. A particle moves around the circle $x^2 + y^2 = 25$ at constant speed, making one revolution in 2 s. Find its acceleration when it is at $(3, 4)$.

16. A particle moves to the right along the curve $y = 3/x$. If its speed is 10 when it passes through the point $(2, \frac{3}{2})$, what is its velocity at that time?

14. $r' = \langle \alpha(-\sin^2 t + \cos^2 t),\ 2\alpha \sin t \cos t,\ -\alpha \sin t \rangle$

$|r'| = \sqrt{\alpha^2(\cos^2 t - \sin^2 t)^2 + 4\alpha^2 \sin^2 t \cos^2 t + \alpha^2 \sin^2 t}$

$= \alpha\sqrt{\cos^4 t - 2\sin^2 t\cos^2 t + \sin^4 t + 4\sin^2 t\cos^2 t + \sin^2 t}$

$= \alpha\sqrt{(\sin^2 t + \cos^2 t)^2 + \sin^2 t}$

$r'' = \langle \alpha(-2\sin t\cos t - 2\cos t\sin t,\ 2\alpha(-\sin^2 t + \cos^2 t),$

$-\alpha\cos t \rangle$

$= \langle -4\alpha\sin t\cos t,\ 2\alpha(\cos^2 t - \sin^2 t),\ -\alpha\cos t \rangle$

Made with Goodnotes

15. A particle moves around the circle $x^2 + y^2 = 25$ at constant speed, making one revolution in 2 s. Find its acceleration when it is at $(3, 4)$.

16. A particle moves to the right along the curve $y = 3/x$. If its speed is 10 when it passes through the point $(2, \frac{3}{2})$, what is its velocity at that time?

16.

$$\vec{r} = x i + \frac{3}{x} j$$

$$\frac{dr}{dt} = V = \left(i - \frac{3}{x^2} j\right) \frac{dx}{dt}$$

$$\left|\frac{dr}{dt}\right| = \sqrt{1 + \left(\frac{3}{x^2}\right)^2} \left|\frac{dx}{dt}\right|$$

$$10 = \sqrt{1 + \frac{9}{x^4}} \left|\frac{dx}{dt}\right|$$

$$\frac{dx}{dt} = \frac{10}{\sqrt{1 + \frac{9}{x^4}}}$$

When $x = 2$, $\frac{dx}{dt} = \frac{10}{\sqrt{1 + \frac{9}{16}}} = \frac{10}{\sqrt{\frac{25}{16}}}$

$$= \frac{10}{\frac{5}{4}} = 10 \times \frac{4}{5} = 8.$$

$$\left.\frac{dr}{dt}\right|_{x=2} = \left(i - \frac{3}{4} j\right)(8)$$

$$\vec{v} = 8i - 6j.$$

17. A point P moves along the curve of intersection of the cylinder $z = x^2$ and the plane $x + y = 2$ in the direction of increasing y with constant speed $v = 3$. Find the velocity of P when it is at $(1, 1, 1)$.

$$x+y=2$$
$$x=2-y.$$

$$z=x^2$$
$$z=(2-y)^2$$
$$z=4-\cancel{4}y+y^2$$

Let $\vec{r} = \langle 2-y, y, 4-\cancel{4}y+y^2 \rangle$

$$\vec{v} = \frac{dr}{dt} = \langle -1, 1, -\cancel{4}+2y \rangle \frac{dy}{dt}$$

$$\left|\frac{dr}{dt}\right| = \sqrt{1+1+(-\cancel{4}+2y)^2}\ \left|\cancel{\frac{dy}{dt}}\right|$$

$$\left|\frac{dr}{dt}\right| = \sqrt{2+16-16y+4y^2}\ \left|\frac{dy}{dt}\right|$$

$$\frac{3}{\sqrt{4y^2-16y+18}} = \frac{dy}{dt}$$

When $P=(1,1,1)$, $\frac{dy}{dt} = \frac{3}{\sqrt{4-16+18}} = \frac{3}{\sqrt{6}}$

$$\vec{v} = \langle -1, 1, -4+2(1) \rangle \frac{3}{\sqrt{6}}$$
$$= \langle -1, 1, -2 \rangle \frac{3}{\sqrt{6}}.$$

18. An object moves along the curve $y = x^2$, $z = x^3$, with constant vertical speed $dz/dt = 3$. Find the velocity and acceleration of the object when it is at the point $(2, 4, 8)$.

$x = z^{\frac{1}{3}}$ $y = z^{\frac{2}{3}}$

$\vec{r} = \langle z^{\frac{1}{3}}, z^{\frac{2}{3}}, z \rangle$

$\frac{dr}{dt} = \langle \frac{1}{3} z^{-\frac{2}{3}}, \frac{2}{3} z^{-\frac{1}{3}}, 1 \rangle \frac{dz}{dt}$

$v = \frac{dr}{dt} = \langle \frac{1}{3} z^{-\frac{2}{3}}, \frac{2}{3} z^{-\frac{1}{3}}, 1 \rangle (3)$

When $z = 8$,

$\vec{v} = \langle \frac{1}{3}(8)^{-\frac{2}{3}}, \frac{2}{3}(8)^{-\frac{1}{3}}, 1 \rangle (3)$

$= \langle \frac{1}{3}(2)^{-2}, \frac{2}{3}(2)^{-1}, 1 \rangle 3$

$= \langle \frac{1}{4}, 1, 3 \rangle$

$\frac{dv}{dt} = \langle -\frac{2}{9} z^{-\frac{5}{3}}, -\frac{2}{9} z^{-\frac{4}{3}}, 0 \rangle (3)$

When $z = 8$,

$\vec{a} = \langle -\frac{2}{9}(2)^{-5}, -\frac{2}{9}(2)^{-4}, 0 \rangle (3)$

$= \langle -\frac{2}{3}(2)^{-5}, -\frac{2}{3}(2)^{-4}, 0 \rangle$

$= \langle -\frac{1}{48}, -\frac{1}{24}, 0 \rangle$

19. A particle moves along the curve $\mathbf{r} = 3u\mathbf{i} + 3u^2\mathbf{j} + 2u^3\mathbf{k}$ in the direction corresponding to increasing u and with a constant speed of 6. Find the velocity and acceleration of the particle when it is at the point (3, 3, 2).

$$\vec{v} = \frac{dr}{dt} = (3i + 6uj + 6u^2k)\frac{du}{dt}$$

$$\frac{d}{dt}\frac{dr}{dt} = (0i + 6j + 12uk)\frac{du}{dt}\frac{du}{dt}$$

$$\left|\frac{dr}{dt}\right| = \sqrt{9 + 36u^2 + 36u^4}\ \frac{du}{dt}$$

$$\frac{6}{\sqrt{9+36u^2+36u^4}} = \frac{du}{dt}$$

when $u=1$,

$$\frac{du}{dt} = \frac{6}{\sqrt{9+36+36}} = \frac{6}{\sqrt{81}} = \frac{2}{3}.$$

$$\vec{V} = (3i + 6j + 6k)\frac{2}{3}$$

$$\vec{V} = (2i + 4j + 4k)$$

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20. A particle moves along the curve of intersection of the cylinders $y = -x^2$ and $z = x^2$ in the direction in which x increases. (All distances are in centimetres.) At the instant when the particle is at the point $(1, -1, 1)$, its speed is 9 cm/s, and that speed is increasing at a rate of 3 cm/s^2. Find the velocity and acceleration of the particle at that instant.

$$\vec{r} = \langle x, -x^2, x^2 \rangle$$

$$\vec{v} = \frac{dr}{dt} = \langle 1, -2x, 2x \rangle \frac{dx}{dt}$$

$$\left|\frac{dr}{dt}\right| = \sqrt{1 + 4x^2 + 4x^2}\left|\frac{dx}{dt}\right|$$

$$\left|\frac{dr}{dt}\right| = \sqrt{8x^2 + 1}\,\frac{dx}{dt}$$

$$\frac{9}{\sqrt{8(1)^2+1}} = \frac{dx}{dt}$$

$$\frac{dx}{dt} = \cancel{1}\ 3.$$

$$\vec{a} = \langle 1, -2, 2 \rangle$$

$$\vec{v} = \langle 3, -6, 6 \rangle$$

要d翻原本式.

$$\vec{a} = \frac{d}{dt}\frac{dr}{dt} = \left(\frac{dx}{dt}\right)^2 \langle \cancel{x}\,0, -2, 2 \rangle + \langle 1, -2x, 2x \rangle \frac{d^2x}{dt^2}$$

$$\vec{a} = 9\langle 0, -2, 2 \rangle + \frac{d^2x}{dt^2}\langle 1, -2x, 2x \rangle$$

$$3 \quad \cancel{|\vec{a}|} = 9\sqrt{8} \cdot \cancel{0} + \left|\frac{d^2x}{dt^2}\right|\sqrt{1+8x^2} \qquad \cancel{1}\ 4\sqrt{8}$$

$$\frac{3}{\sqrt{8}} \quad \frac{1}{3\sqrt{8}} = \frac{d^2x}{dt^2}\sqrt{1+8x^2} \qquad \frac{d^2x}{dt^2} = 0 \cdot \frac{1}{\sqrt{8}}$$

21. Show that if the dot product of the velocity and acceleration of a moving particle is positive (or negative), then the speed of the particle is increasing (or decreasing).

$V > 0$

$(x_0, y_0, z_0 > 0)$

$\frac{d}{dt}|\vec{V}| = \frac{\vec{V}\cdot\vec{V}'}{|\vec{V}|}$ If $\vec{v}' > 0$, $\vec{V} > 0$,

$\vec{v}\cdot\vec{V} > 0.$

$|\vec{v}| > 0.$

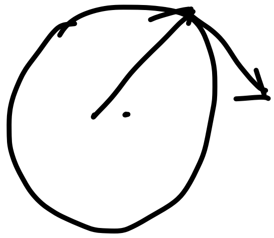
$\therefore \frac{d}{dt}|\vec{v}| > 0.$

And it is increasing

24. If the position and velocity vectors of a moving particle are always perpendicular, show that the path of the particle lies on a sphere.

$$\vec{r} \cdot \vec{v} = 0$$

$$\vec{r} = (x, y, z) \qquad \vec{v} = (u, v, w)$$



$$\sqrt{u^2 + v^2 + w^2}$$

$$|\vec{v}| = c$$

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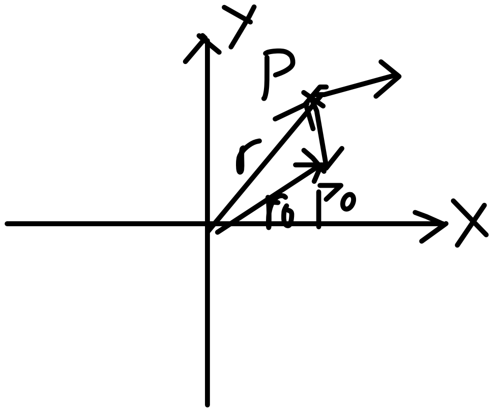
25. Generalize Exercise 24 to the case where the velocity of the particle is always perpendicular to the line joining the particle to a fixed point P_0.



$$\frac{d}{dt}|r-r_0|^2$$

$$=\frac{d}{dt}(r-r_0)\cdot(r-r_0)$$

$$=2(r-r_0)\frac{dr}{dt}$$

$$r+\overrightarrow{PP_0}=\overrightarrow{r_0}$$

$$=$$

26. What can be said about the motion of a particle at a time when its position and velocity satisfy $\mathbf{r} \bullet \mathbf{v} > 0$? What can be said when $\mathbf{r} \bullet \mathbf{v} < 0$?

$$r \cdot r = |r|^2$$

$$2 r \cdot v = \frac{d}{dt}|r|^2$$

$$r \cdot v = \frac{d}{dt}\frac{|r|^2}{2}$$

Speed is increasing.

In Exercises 27–32, assume that the vector functions encountered have continuous derivatives of all required orders.

27. Show that $\frac{d}{dt}\left(\frac{d\mathbf{u}}{dt} \times \frac{d^2\mathbf{u}}{dt^2}\right) = \frac{d\mathbf{u}}{dt} \times \frac{d^3\mathbf{u}}{dt^3}$.

$$\frac{du}{dt} \times \frac{d^3u}{dt^3} + \frac{d^2u}{dt^2} \times \frac{d^2u}{dt^2}$$

$$= \frac{du}{dt} \times \frac{d^3u}{dt^3}$$

28. Write the Product Rule for $\frac{d}{dt}\left(\mathbf{u} \bullet (\mathbf{v} \times \mathbf{w})\right)$.

$$u \cdot \frac{d}{dt}(v \times w) + u' \cdot (v \times w)$$

$$= u \cdot (v' \times w + v \times w') + u' \cdot (v \times w)$$

$$= u \cdot (v' \times w) + u \cdot (v \times w') + u' \cdot (v \times w)$$

29. Write the Product Rule for $\frac{d}{dt}\left(\mathbf{u} \times (\mathbf{v} \times \mathbf{w})\right)$.

$$u \times \frac{d}{dt}(v \times w) + u' \times (v \times w)$$

$$= u \times (v' \times w + v \times w') + u' \times (v \times w)$$

$$= u \times (v' \times w) + u \times (v \times w') + u' \times (v \times w)$$

30. Expand and simplify: $\frac{d}{dt}\left(\mathbf{u} \times \left(\frac{d\mathbf{u}}{dt} \times \frac{d^2\mathbf{u}}{dt^2}\right)\right)$.

31. Expand and simplify: $\frac{d}{dt}\left((\mathbf{u} + \mathbf{u}'') \bullet (\mathbf{u} \times \mathbf{u}')\right)$.

32. Expand and simplify: $\frac{d}{dt}\left((\mathbf{u} \times \mathbf{u}') \bullet (\mathbf{u}' \times \mathbf{u}'')\right)$.

30. $$u \times \frac{d}{dt}\left(\frac{du}{dt} \times \frac{d^2u}{dt^2}\right) + u' \times \left(\frac{du}{dt} \times \frac{d^2u}{dt^2}\right)$$

$$= u \times \left(\frac{d^2u}{dt^2} \times \frac{d^2u}{dt^2} + \frac{du}{dt} \times \frac{d^3u}{dt^3}\right) + u' \times \left(\frac{du}{dt} \times \frac{d^2u}{dt^2}\right)$$

$$= u \times \left(\frac{du}{dt} \times \frac{d^3u}{dt^3}\right) + u' \times \left(\frac{du}{dt} \times \frac{d^2u}{dt^2}\right)$$

31. Expand and simplify: $\frac{d}{dt}\left((\mathbf{u}+\mathbf{u}'')\bullet(\mathbf{u}\times\mathbf{u}')\right)$.

32. Expand and simplify: $\frac{d}{dt}\left((\mathbf{u}\times\mathbf{u}')\bullet(\mathbf{u}'\times\mathbf{u}'')\right)$.

31. $\frac{d}{dt}\left((u+u'')\cdot(u\times u')\right)$

$= (u+u'')\cdot\frac{d}{dt}(u\times u') + \frac{d}{dt}(u+u'')\cdot(u\times u')$

$= (u+u'')\cdot(u'\times u'+u\times u'') + (u'+u''')\cdot(u\times u')$

$= (u+u'')\cdot(u\times u'') + (u'+u''')\cdot(u\times u')$

$= \underbrace{u\cdot(u\times u'') + u''\cdot(u\times u'') + u'\cdot(u\times u')}_{0} + u'''\cdot(u\times u')$

$= u'''\cdot(u\times u')$

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32. Expand and simplify: $\dfrac{d}{dt}\Big((\mathbf{u}\times\mathbf{u}')\bullet(\mathbf{u}'\times\mathbf{u}'')\Big)$.

33. If at all times t the position and velocity vectors of a moving particle satisfy $\mathbf{v}(t) = 2\mathbf{r}(t)$, and if $\mathbf{r}(0) = \mathbf{r}_0$, find $\mathbf{r}(t)$ and the acceleration $\mathbf{a}(t)$. What is the path of motion?

32. $(u\times u')\cdot\frac{d}{dt}(u'\times u'') + \frac{d}{dt}(u\times u')\cdot(u'\times u'')$

$= (u\times u')\cdot(u'\times u''') + (u\times u'')\cdot(u'\times u'')$

33. $\int_0^t \vec{r}(t) = \vec{v}(t) + C$

$\int_0^t \vec{r}(t) = 2\vec{r}(t) + C$

$=$

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CHAPTER 11. VECTOR FUNCTIONS AND CURVES

Section 11.1 Vector Functions of One Variable (page 629)

1. Position: $\mathbf{r} = \mathbf{i} + t\mathbf{j}$
Velocity: $\mathbf{v} = \mathbf{j}$
Speed: $v = 1$
Acceleration : $\mathbf{a} = \mathbf{0}$
Path: the line $x = 1$ in the xy-plane.

2. Position: $\mathbf{r} = t^2\mathbf{i} + \mathbf{k}$
Velocity: $\mathbf{v} = 2t\mathbf{i}$
Speed: $v = 2|t|$
Acceleration : $\mathbf{a} = 2\mathbf{i}$
Path: the line $z = 1$, $y = 0$.

3. Position: $\mathbf{r} = t^2\mathbf{j} + t\mathbf{k}$
Velocity: $\mathbf{v} = 2t\mathbf{j} + \mathbf{k}$
Speed: $v = \sqrt{4t^2 + 1}$
Acceleration : $\mathbf{a} = 2\mathbf{j}$
Path: the parabola $y = z^2$ in the plane $x = 0$.

4. Position: $\mathbf{r} = \mathbf{i} + t\mathbf{j} + t\mathbf{k}$
Velocity: $\mathbf{v} = \mathbf{j} + \mathbf{k}$
Speed: $v = \sqrt{2}$
Acceleration : $\mathbf{a} = \mathbf{0}$
Path: the straight line $x = 1$, $y = z$.

5. Position: $\mathbf{r} = t^2\mathbf{i} - t^2\mathbf{j} + \mathbf{k}$
Velocity: $\mathbf{v} = 2t\mathbf{i} - 2t\mathbf{j}$
Speed: $v = 2\sqrt{2}t$
Acceleration: $\mathbf{a} = 2\mathbf{i} - 2\mathbf{j}$
Path: the half-line $x = -y \ge 0$, $z = 1$.

6. Position: $\mathbf{r} = t\mathbf{i} + t^2\mathbf{j} + t^2\mathbf{k}$
Velocity: $\mathbf{v} = \mathbf{i} + 2t\mathbf{j} + 2t\mathbf{k}$
Speed: $v = \sqrt{1 + 8t^2}$
Acceleration: $\mathbf{a} = 2\mathbf{j} + 2\mathbf{k}$
Path: the parabola $y = z = x^2$.

7. Position: $\mathbf{r} = a\cos t\mathbf{i} + a\sin t\mathbf{j} + ct\mathbf{k}$
Velocity: $\mathbf{v} = -a\sin t\mathbf{i} + a\cos t\mathbf{j} + c\mathbf{k}$
Speed: $v = \sqrt{a^2 + c^2}$
Acceleration: $\mathbf{a} = -a\cos t\mathbf{i} - a\sin t\mathbf{j}$
Path: a circular helix.

8. Position: $\mathbf{r} = a\cos\omega t\mathbf{i} + b\mathbf{j} + a\sin\omega t\mathbf{k}$
Velocity: $\mathbf{v} = -a\omega\sin\omega t\mathbf{i} + a\omega\cos\omega t\mathbf{k}$
Speed: $v = |a\omega|$
Acceleration: $\mathbf{a} = -a\omega^2\cos\omega t\mathbf{i} - a\omega^2\sin\omega t\mathbf{k}$
Path: the circle $x^2 + z^2 = a^2$, $y = b$.

9. Position: $\mathbf{r} = 3\cos t\mathbf{i} + 4\cos t\mathbf{j} + 5\sin t\mathbf{k}$
Velocity: $\mathbf{v} = -3\sin t\mathbf{i} - 4\sin t\mathbf{j} + 5\cos t\mathbf{k}$
Speed: $v = \sqrt{9\sin^2 t + 16\sin^2 t + 25\cos^2 t} = 5$
Acceleration : $\mathbf{a} = -3\cos t\mathbf{i} - 4\cos t\mathbf{j} - 5\sin t\mathbf{k} = -\mathbf{r}$
Path: the circle of intersection of the sphere $x^2 + y^2 + z^2 = 25$ and the plane $4x = 3y$.

10. Position: $\mathbf{r} = 3\cos t\mathbf{i} + 4\sin t\mathbf{j} + t\mathbf{k}$
Velocity: $\mathbf{v} = -3\sin t\mathbf{i} + 4\cos t\mathbf{j} + \mathbf{k}$
Speed: $v = \sqrt{9\sin^2 t + 16\cos^2 t + 1} = \sqrt{10 + 7\cos^2 t}$
Acceleration : $\mathbf{a} = -3\cos t\mathbf{i} - 4\sin t\mathbf{j} = t\mathbf{k} - \mathbf{r}$
Path: a helix (spiral) wound around the elliptic cylinder $(x^2/9) + (y^2/16) = 1$.

11. Position: $\mathbf{r} = ae^t\mathbf{i} + be^t\mathbf{j} + ce^t\mathbf{k}$
Velocity and acceleration: $\mathbf{v} = \mathbf{a} = \mathbf{r}$
Speed: $v = e^t\sqrt{a^2 + b^2 + c^2}$
Path: the half-line $\dfrac{x}{a} = \dfrac{y}{b} = \dfrac{z}{c} > 0$.

12. Position: $\mathbf{r} = at\cos\omega t\mathbf{i} + at\sin\omega t\mathbf{j} + b\ln t\mathbf{k}$
Velocity: $\mathbf{v} = a(\cos\omega t - \omega t\sin\omega t)\mathbf{i}$
$\quad + a(\sin\omega t + \omega t\cos\omega t)\mathbf{j} + (b/t)\mathbf{k}$
Speed: $v = \sqrt{a^2(1 + \omega^2 t^2) + (b^2/t^2)}$
Acceleration: $\mathbf{a} = -a\omega(2\sin\omega t + \omega\cos\omega t)\mathbf{i}$
$\quad + a\omega(2\cos\omega t - \omega\sin\omega t)\mathbf{j} - (b/t^2)\mathbf{k}$
Path: a spiral on the surface $x^2 + y^2 = a^2 e^{z/b}$.

13. Position: $\mathbf{r} = e^{-t}\cos(e^t)\mathbf{i} + e^{-t}\sin(e^t)\mathbf{j} - e^t\mathbf{k}$
Velocity: $\mathbf{v} = -\Big(e^{-t}\cos(e^t) + \sin(e^t)\Big)\mathbf{i}$
$\quad - \Big(e^{-t}\sin(e^t) - \cos(e^t)\Big)\mathbf{j} - e^t\mathbf{k}$
Speed: $v = \sqrt{1 + e^{-2t} + e^{2t}}$
Acceleration: $\mathbf{a} = \Big((e^{-t} - e^t)\cos(e^t) + \sin(e^t)\Big)\mathbf{i}$
$\quad + \Big((e^{-t} - e^t)\sin(e^t) - \cos(e^t)\Big)\mathbf{j} - e^t\mathbf{k}$
Path: a spiral on the surface $z\sqrt{x^2 + y^2} = -1$.

14. Position: $\mathbf{r} = a\cos t\sin t\mathbf{i} + a\sin^2 t\mathbf{j} + a\cos t\mathbf{k}$
$\quad = \dfrac{a}{2}\sin 2t\mathbf{i} + \dfrac{a}{2}\Big(1 - \cos 2t\Big)\mathbf{j} + a\cos t\mathbf{k}$
Velocity: $\mathbf{v} = a\cos 2t\mathbf{i} + a\sin 2t\mathbf{j} - a\sin t\mathbf{k}$
Speed: $v = a\sqrt{1 + \sin^2 t}$
Acceleration: $\mathbf{a} = -2a\sin 2t\mathbf{i} + 2a\cos 2t\mathbf{j} - a\cos t\mathbf{k}$
Path: the path lies on the sphere $x^2 + y^2 + z^2 = a^2$, on the surface defined in terms of spherical polar coordinates by $\phi = \theta$, on the circular cylinder $x^2 + y^2 = ay$, and on the parabolic cylinder $ay + z^2 = a^2$. Any two of these surfaces serve to pin down the shape of the path.

15. The position of the particle is given by

$$\mathbf{r} = 5\cos(\omega t)\mathbf{i} + 5\sin(\omega t)\mathbf{j},$$

where $\omega = \pi$ to ensure that $\mathbf{r}$ has period $2\pi/\omega = 2$ s. Thus

$$\mathbf{a} = \frac{d^2\mathbf{r}}{dt^2} = -\omega^2\mathbf{r} = -\pi^2\mathbf{r}.$$

At $(3, 4)$, the acceleration is $-3\pi^2\mathbf{i} - 4\pi^2\mathbf{j}$.