MATH 2023 – Multivariable Calculus

Lecture #01 Worksheet ♠ January 31, 2019

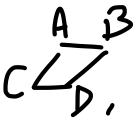
Problem 1. Let

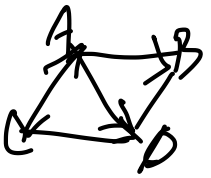
$$A = (1, 2, 3), \qquad B = (3, 4, 5), \qquad C = (1, 0, -1), \qquad D = (3, 2, 1)$$

be four points in $\mathbb{R}^3$.

(a) Show that $ABCD$ is a parallelogram

(b) Find the area of this parallelogram.





a). $\overrightarrow{AB} = \langle 2, 2, 2 \rangle$

$\overrightarrow{AC} = \langle 0, -2, -4 \rangle$

$\overrightarrow{AD} = \langle 2, 0, -2 \rangle$

$\overrightarrow{BC} = \langle -2, -4, -6 \rangle$

$\overrightarrow{BD} = \langle 0, -2, -4 \rangle$

$\overrightarrow{CD} = \langle 2, 2, 2 \rangle$

$AC = BD$

$AB = CD$.

b). $|\overrightarrow{AB} \times \overrightarrow{AC}|$

$$= \begin{vmatrix} i & j & k \\ 2 & 2 & 2 \\ 0 & -2 & -4 \end{vmatrix}$$

$$\begin{vmatrix} i & j & k \\ a & b & c \\ d & x & y \end{vmatrix}$$

$\langle -4, 8, -4 \rangle = \sqrt{96}$.

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Problem 2. Describe the four different relationships between the line L

$$L = \begin{cases} x = 1 + 4s \\ y = 2 + 5s \\ z = 3 + 6s \end{cases}$$

and the lines

$$\ell_1 = \begin{cases} x = 9 - 8t \\ y = 12 - 10t \\ z = 15 - 12t \end{cases}$$

$$\ell_2 = \begin{cases} x = 12t \\ y = 3 + 15t \\ z = 5 + 18t \end{cases}$$

$$\ell_3 = \begin{cases} x = -2 + 3t \\ y = 4 - 2t \\ z = -1 + 4t \end{cases}$$

$$\ell_4 = \begin{cases} x = -1 + t \\ y = t \\ z = 2 + t \end{cases}$$

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Problem 3. Find the angle between the planes and their line of intersection

$$\begin{cases} x+y+z=1 \\ x-2y+3z=1 \end{cases}$$

$\langle 1,1,1 \rangle \cdot \langle 1,-2,3 \rangle = |\langle 1,1,1 \rangle| \, |\langle 1,-2,3 \rangle| \cos\theta$

$2 = (\sqrt{3})(\sqrt{14})\cos\theta$

$\frac{2}{\sqrt{42}} = \cos\theta$

$\theta = 72.0247^\circ$

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Problem 4. Find the distance between the skew lines

$$L = \begin{cases} x = 1 + 4s \\ y = 2 + 5s \\ z = 3 + 6s \end{cases} \qquad \text{and} \qquad \ell_4 = \begin{cases} x = -1 + t \\ y = 0 + t \\ z = 2 + t \end{cases}$$

$$\frac{(r_2 - r_1) \cdot (v_1 \times v_2)}{(v_1 \times v_2)} \quad ? ?$$

$=$

Made with Goodnotes