MATH 2023 – Multivariable Calculus

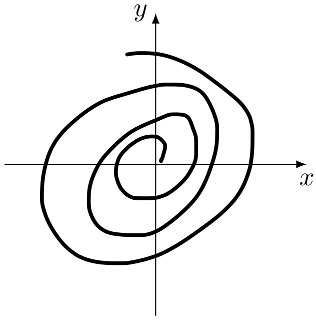
Lecture #11 Worksheet ♮ March 14, 2019

Problem 1. Sketch the following curves in polar coordinates :

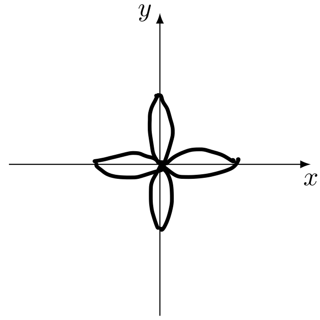
(a) $r = \ln\theta$

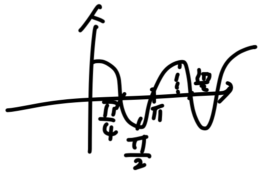
ln 1 = 0

x ln 0

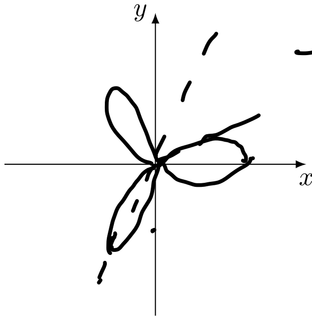


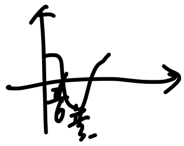
(b) $r = \cos 2\theta$



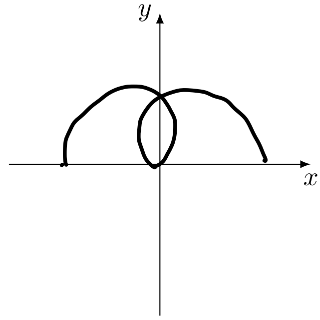


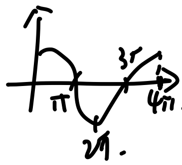
(c) $r = \cos 3\theta$



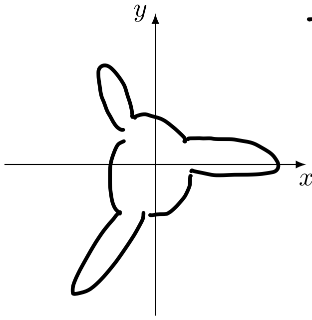


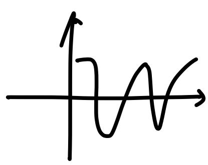
(d) $r = \cos\dfrac{\theta}{2}$



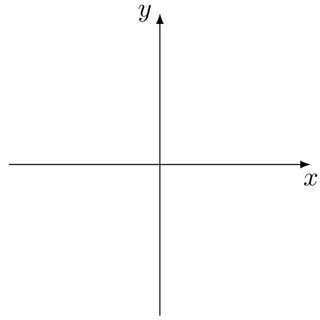


(e) $r = 2 + \cos 3\theta$



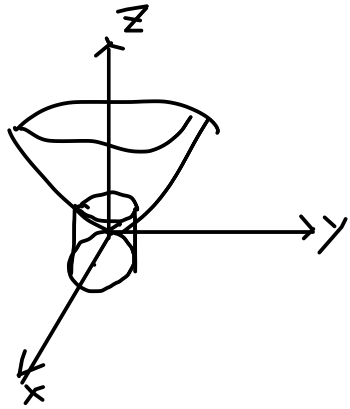


(f) $r = 1 + \sin\theta$



Problem 2. Find the volume:

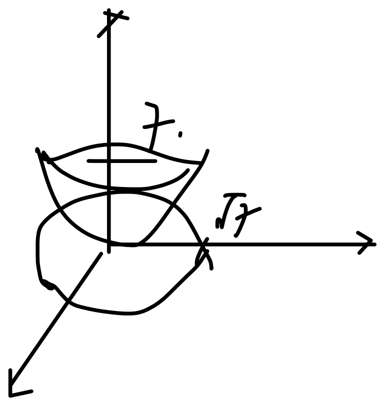
(a) Under $z = x^2 + y^2$ and inside the cyliner $x^2 + y^2 = 2x$



(b) Under the sphere $x^2 + y^2 + z^2 = 9$ and inside the cylinder $x^2 + y^2 = 4$

$$\int_0^{2\pi}\int_0^{2} \sqrt{9-x^2-y^2}\, dA$$

$$\int_0^{2\pi}\int_0^{2} \sqrt{9-r^2}\, r\, dr\, d\theta$$



(c) Bounded by $z = x^2 + y^2$ and $z = 7$

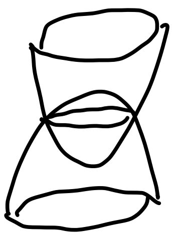
r^2

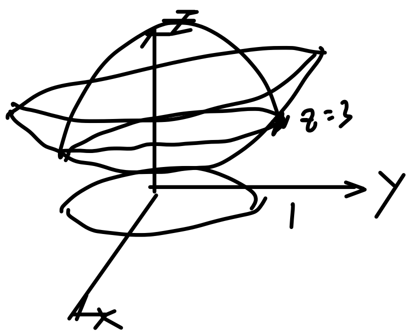
$$\int_0^{2\pi}\int_0^{\sqrt{7}} r^3\, dr\, d\theta$$

$z = 3r^2 \quad z = 4 - r^2$

(d) Bounded by $z = 3x^2 + 3y^2$ and $z = 4 - x^2 - y^2$

$$\int_0^{2\pi}\int_0^{1}$$

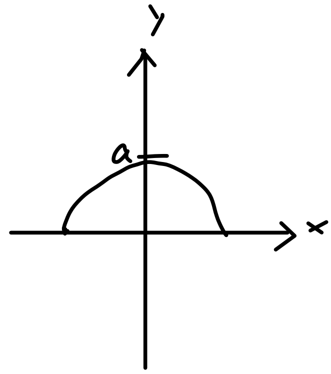




Problem 3. Convert the following integral into polar coordinates:

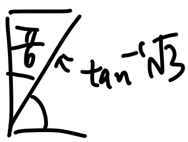
(a) $\int_0^a \int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} dxdy$

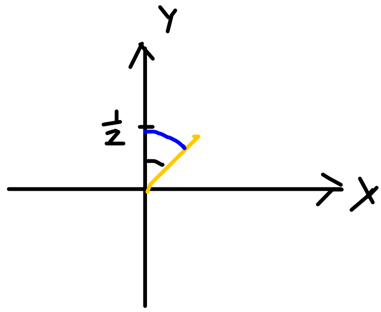
$$\int_0^{\pi}\int_0^{a} r\,dr\,d\theta$$



(b) $\int_0^{\frac{1}{2}} \int_{\sqrt{3}y}^{\sqrt{1-y^2}} dxdy$

$$\int_0^{\frac{\pi}{6}}\int_0^{\frac{1}{2}} r\,dr\,d\theta$$





(c) $\int_{\frac{1}{\sqrt{2}}}^{1} \int_{\sqrt{1-x^2}}^{x} dxdy + \int_1^{\sqrt{2}} \int_0^{x} dxdy + \int_{\sqrt{2}}^{2} \int_0^{\sqrt{4-x^2}} dxdy$