(1) **(a)** Find the distance (in terms of $\mathbf{n}$, $\mathbf{r}_0$ and $\mathbf{r}_1$ only) from the point $\mathbf{r}_1$ to the plane $(\mathbf{r}-\mathbf{r}_0)\cdot\mathbf{n} = 0$.

(b) A rigid body rotates about an axis through point O with angular velocity $\boldsymbol{\omega}$.

(i) Find the linear velocity $\mathbf{v}$ of a point P of the body with position vector $\mathbf{r}$. ?

(ii) Show that the vector $-\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r})$ is directed away from the axis of rotation and lies on the plane containing the vector $\boldsymbol{\omega}$ and $\mathbf{r}$. ?

a).

$$D = \frac{(\vec{r}_1 \cdot \vec{n}) - (\vec{r}_0 \cdot \vec{n})}{|r_1|}$$ ✓

b).

(2) **(a)** Can the function $f(x,y) = \dfrac{\sin x \sin^3 y}{1-\cos(x^2+y^2)}$ be defined at $(0,0)$ in such a way that it becomes continuous there? If so, how?

?!

(b) Let $f(x,y) = \begin{cases} \dfrac{x^3}{x^2+y^2}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$

Calculate each of the following partial derivatives or explain why it does not exist:

(i) $f_x(0,0)$, (ii) $f_y(0,0)$, (iii) $f_{yx}(0,0)$, (iv) $f_{xy}(0,0)$ and (v) $f_{xx}(0,0)$.

Is the function $f(x,y)$ differentiable at $(0,0)$? Explain.

a).

$$\lim_{(x,y)\to(0,0)} \frac{\sin x \sin^3 y}{1-\cos(x^2+y^2)}$$

$\sin^2 y \quad \sin x \sin y$

$\sin A \sin B$

$$\lim_{r\to 0} \frac{\sin(r\cos\theta)\sin^3(r\sin\theta)}{1-\cos(r^2)}$$

$$= \lim_{r\to 0} \frac{\sin(r\cos\theta)\, 3\sin^2(r\sin\theta)\cos(r\sin\theta)\sin\theta + \sin^3(r\sin\theta)\cos(r\cos\theta)(-\sin\theta)}{-2r\sin(r^2)}$$

$$= \sin^2(r\sin\theta)$$

$$(1-\cos^2(y))\frac{1}{2}(\cos(x-y)+\cos(x+y))$$

(2) **(a)** Can the function $f(x,y) = \dfrac{\sin x \sin^3 y}{1-\cos(x^2+y^2)}$ be defined at $(0,0)$ in such a way that it becomes continuous there? If so, how?

(b) Let $f(x,y) = \begin{cases} \dfrac{x^3}{x^2+y^2}, & \text{if} \quad (x,y) \neq (0,0), \\ 0, & \text{if} \quad (x,y) = (0,0). \end{cases}$

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Is the function $f(x,y)$ differentiable at $(0,0)$? Explain.

b).
$$f_x = \lim_{h\to 0^+} \frac{f(h,0)-f(0,0)}{h}$$

$$= \lim_{h\to 0^+} \frac{\frac{h^3}{h^2} - 0}{h}$$

$$= 1$$

$$f_y(0,0) = \lim_{h\to 0^+} \frac{f(0,h)-f(0,0)}{h}$$

$$= \lim_{h\to 0^+} \frac{0-0}{h}$$

$= \infty$ does not exist.

f_{yx} does not exist since f_y does not exist.

f_{xy} same, by mixed partial theorem.

$f_{xx} = \frac{d}{dx} 1 = 0.$

$$\frac{x^3}{x^2+y^2}$$

$x=0 \Rightarrow 0$

$y=0 \Rightarrow 0$

$x=y \Rightarrow \frac{y^3}{2y^2} \Rightarrow 0$

$y=x^{\frac{3}{2}} \Rightarrow \frac{x^3}{x^2+x^3} \Rightarrow 1$

does not exist

(3) **(a)** Show that the curve $\mathbf{r}(t) = t\cos t\,\mathbf{i} + t\sin t\,\mathbf{j} + t\,\mathbf{k}$, $t \geqslant 0$, lies on the surface of the form $z = f(x, y)$. Find $f(x, y)$. Describe (or sketch) the curve. ??

(b) Find a vector equation of the line tangent to the graph of

$$\mathbf{r}(t) = t^2\,\mathbf{i} - \frac{1}{t+1}\,\mathbf{j} + (4 - t^2)\,\mathbf{k}$$

at the point $(4, 1, 0)$ on the curve. Find also the arc length of the curve $\mathbf{r}(t)$ from point $(4, 1, 0)$ to point $(0, -1, 4)$.

3a). let $g(x,y,z) = t\cos t\, x + t\sin t\, y + tz$

$$\frac{g(x,y,z)}{t} = \cos t\, x + \sin t\, y + z$$

$$\frac{g(x,y,z)}{t} - z = \cos t\, x + \sin t\, y$$

b). $r'(t) = 2t\, i + (t+1)^{-2} j - 2t\, k$

$$r'(2) = 4i + \frac{1}{9} j - 4k.$$

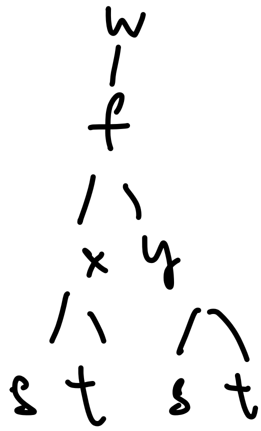
$$|r'(t)| = \sqrt{4t^2 + \frac{1}{(t+1)^4} + 4t^2}$$

$$= \sqrt{8t^2 + \frac{1}{t^4 + 4t^3 + 6t^2 + 4t + 1}}$$

$$= \sqrt{\frac{8t^6 + 24t^5 + 48t^4 + 32t^3 + 8t^2 + 1}{t^4 + 4t^3 + 6t^2 + 4t + 1}}$$

(4) **(a)** Find the equation of the level curve of the function $z = g(x, y) = xf(xy)$ at the point (x_0, y_0), where both f and g are differentiable. Show that $\nabla g(x_0, y_0)$ is normal to the tangent line to the level curve at (x_0, y_0). ??

(b) If $w = f(x, y)$ (assume f is differentiable) and $x = s^2 + t^2$, $y = s^2 - t^2$, use the chain rule to find (i) w_s, (ii) w_{st} and (iii) w_{stt}.



$$W_s = \frac{\partial w}{\partial f}\frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial f}\frac{\partial f}{\partial y}\frac{\partial y}{\partial s}$$

$$W_s = \frac{\partial w}{\partial f}(2s)\left(\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\right)$$

$$W_{st} = 2s\left(\frac{d}{dt}W_f\,(f_x + f_y) + \left(\frac{d}{dt}f_x + \frac{d}{dt}f_y\right)W_f\right)$$

$$= 2s\left(W_{ff}\, f_x\, X_t + W_{ff}\, f_y\, y_t\,(f_x + f_y) + (f_{xx} X_t + f_{xy} y_t + f_{yx} X_t + f_{yy} y_t)W_f\right)$$

(5) Find the point(s) on the surface $z^2 = -\frac{1}{2}x^2 + 2y^2 + xy$ that are closest to the point $\left(-\frac{1}{2}, -3, 0\right)$

(a) by reducing the problem to an unconstrained problem in two variables, and

(b) using the method of Lagrange multipliers.

√a).