MATH 2023 – Multivariable Calculus

Lecture #01 Worksheet ♠ January 31, 2019

Problem 1. Let

$$A = (1, 2, 3), \qquad B = (3, 4, 5), \qquad C = (1, 0, -1), \qquad D = (3, 2, 1)$$

be four points in $\mathbb{R}^3$.

(a) Show that $ABCD$ is a parallelogram

(b) Find the area of this parallelogram.

Problem 2. Describe the four different relationships between the line L

$$L = \begin{cases} x = 1 + 4s \\ y = 2 + 5s \\ z = 3 + 6s \end{cases}$$

and the lines

$$\ell_1 = \begin{cases} x = 9 - 8t \\ y = 12 - 10t \\ z = 15 - 12t \end{cases}$$

$$\ell_2 = \begin{cases} x = 12t \\ y = 3 + 15t \\ z = 5 + 18t \end{cases}$$

$$\ell_3 = \begin{cases} x = -2 + 3t \\ y = 4 - 2t \\ z = -1 + 4t \end{cases}$$

$$\ell_4 = \begin{cases} x = -1 + t \\ y = t \\ z = 2 + t \end{cases}$$

Problem 3. Find the angle between the planes and their line of intersection

$$\begin{cases} x + y + z = 1 \\ x - 2y + 3z = 1 \end{cases}$$

Problem 4. Find the distance between the skew lines

$$L = \begin{cases} x = 1 + 4s \\ y = 2 + 5s \\ z = 3 + 6s \end{cases} \quad \text{and} \quad \ell_4 = \begin{cases} x = -1 + t \\ y = 0 + t \\ z = 2 + t \end{cases}$$

$\vec{V}_{L1} = \langle 4, 5, 6 \rangle \quad \vec{V}_{\ell 4} = \langle 1, 1, 1 \rangle$

$\vec{n} = \langle 4, 5, 6 \rangle \times \langle 1, 1, 1 \rangle = \begin{vmatrix} i & j & k \\ 4 & 5 & 6 \\ 1 & 1 & 1 \end{vmatrix}$

$= \langle -1, 2, -1 \rangle$

Equation of plane: $-x + 2y - z = 0$

$$\left| \frac{-(-1) + 2(0) - (2)}{\sqrt{1+4+1}} \right| = \left| \frac{-1}{\sqrt{6}} \right| = \frac{1}{\sqrt{6}}$$