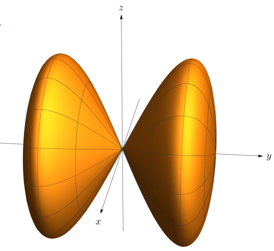
MATH 2023 – Multivariable Calculus

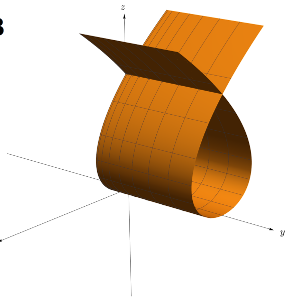
Lecture #17 Worksheet April 11, 2019

Problem 1. Identify the following surfaces with their parametric equations.

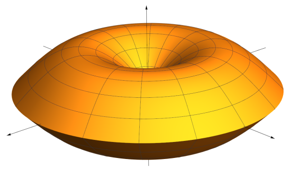
A



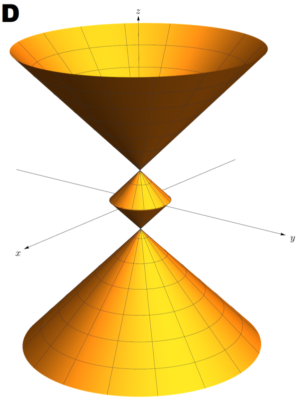
B



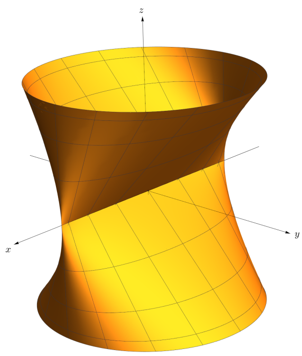
C



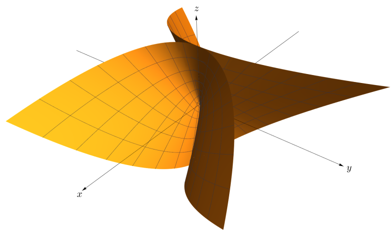
D



E



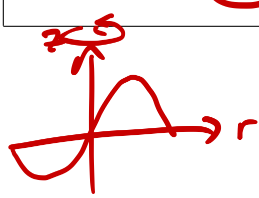
F



$(r, y) = (\sin 2v, \sin v)$

rotate around y-axis.

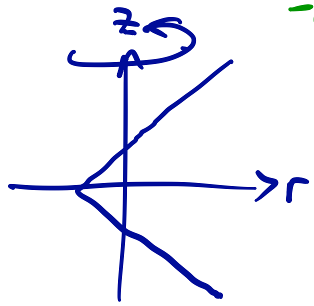
$\langle (1-\lvert u\rvert)\cos v, (1-\lvert u\rvert)\sin v, u\rangle$	$\langle \cos u \sin 2v, \overset{y}{\sin v}, \sin u \overset{r}{\sin 2v}\rangle$	$\langle uv^2, u^2v, u^2-v^2\rangle$
D	A	F
$z = \sin r$ $\langle u\cos v, u\sin v, \sin u\rangle$, $-\pi \le u \le \pi$	$\langle \sin u, \cos u \sin v, \sin v\rangle$	$\langle u^3-u, v^2, u^2\rangle$
C	E	B



Problem 1. (cont'd)

$$\langle \underbrace{(1-|u|)}_{r}\cos v,\ \underbrace{(1-|u|)}_{r}\sin v,\ \underbrace{u}_{z} \rangle$$

rotate about z -axis.



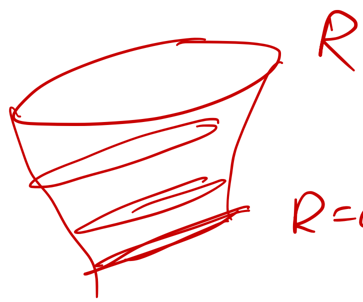
rotating $r = 1 - |z|$

D

$$\langle \sin u,\ \cos u \underbrace{\sin v}_{},\ \underbrace{\sin v}_{} \rangle$$

fix v. const.

$$\langle \sin u,\ \cos u\, R,\ R \rangle$$



E

$$x^2 + \frac{y^2}{R^2} = 1$$

$$\langle uv^2,\ u^2 v,\ u^2 - v^2 \rangle$$

fix v $\longrightarrow$ looks like parabola.

$v = 0 \Longrightarrow \langle 0, 0, u^2 \rangle$

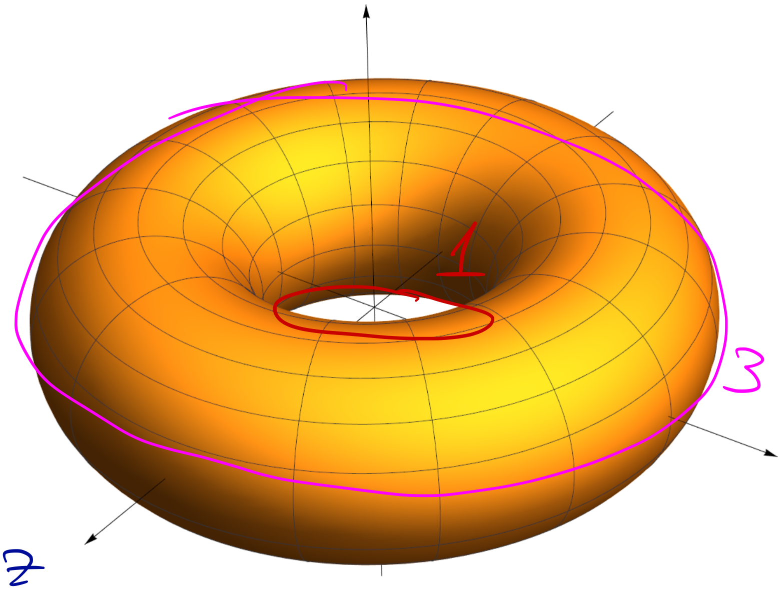
F

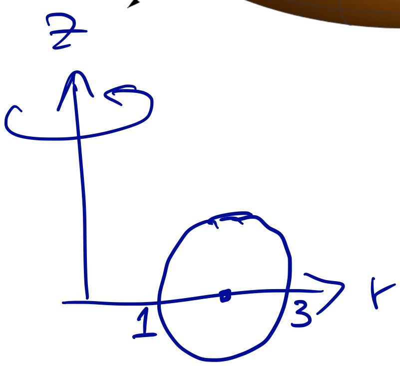
$$\langle \underbrace{u^3 - u}_{x},\ v^2,\ \underbrace{u^2}_{z} \rangle$$

elliptic curve. B

$x = z^{3/2} - z^{1/2} = z^{1/2}(z-1) \Rightarrow x^2 = z(z-1)^2$ γ

Problem 2. Find the parametric equation for the torus with inner radius 1 and outer radius 3 around the z-axis.





$$(r-2)^2 + z^2 = 1.$$

parametric curve ?

$$\begin{cases} r = 2 + \cos\phi \\ z = \sin\phi \end{cases}$$

$$\vec{r}(\theta, \phi) = \langle \underbrace{(2+\cos\phi)}_{r}\cos\theta, \underbrace{(2+\cos\phi)}_{r}\sin\theta, \sin\phi \rangle$$

θ: polar on x-y

ϕ: circle on r-z

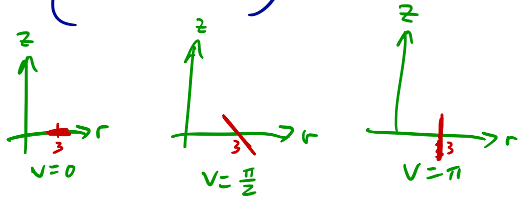
Problem 3. Describe the following surfaces.

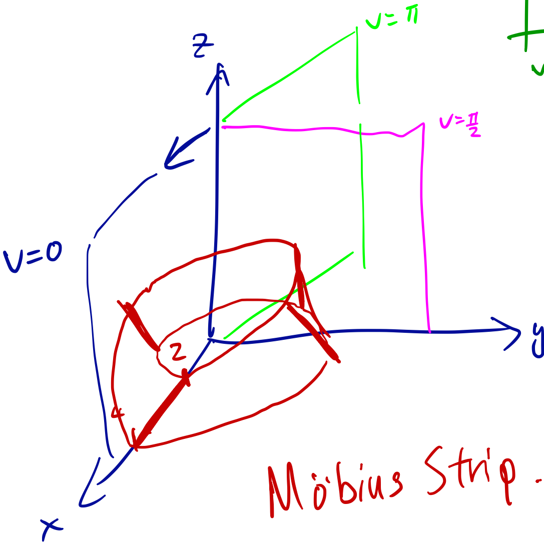
$$\begin{cases} x = (3 - u\cos\frac{v}{2})\cos v \\ y = (3 - u\cos\frac{v}{2})\sin v \quad , \qquad -1 \le u \le 1, \qquad 0 \le v \le 2\pi \\ z = u\sin\frac{v}{2} \end{cases}$$

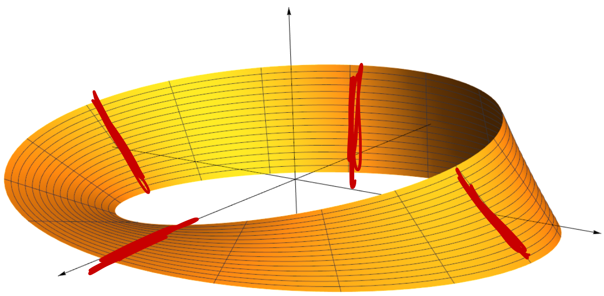
rotate around z-axis.

$\begin{pmatrix} 3 \\ 0 \end{pmatrix} + u \begin{pmatrix} -\cos\frac{v}{2} \\ \sin\frac{v}{2} \end{pmatrix}$

$\begin{cases} r = 3 - u\cos\frac{v}{2} \\ z = u\sin\frac{v}{2} \end{cases}$ (v = angle)







Möbius Strip.

v=0: ⟨3-u, 0⟩

Problem 3. (cont'd)

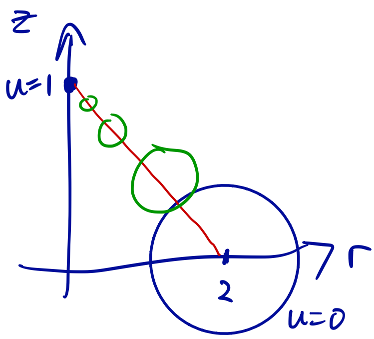
$$\begin{cases} x = (1-u)(2+\cos v)\cos(6\pi u) \\ y = (1-u)(2+\cos v)\sin(6\pi u) \\ z = 4u + (1-u)\sin v \end{cases}, \qquad 0 \le u \le 1, \qquad 0 \le v \le 2\pi$$

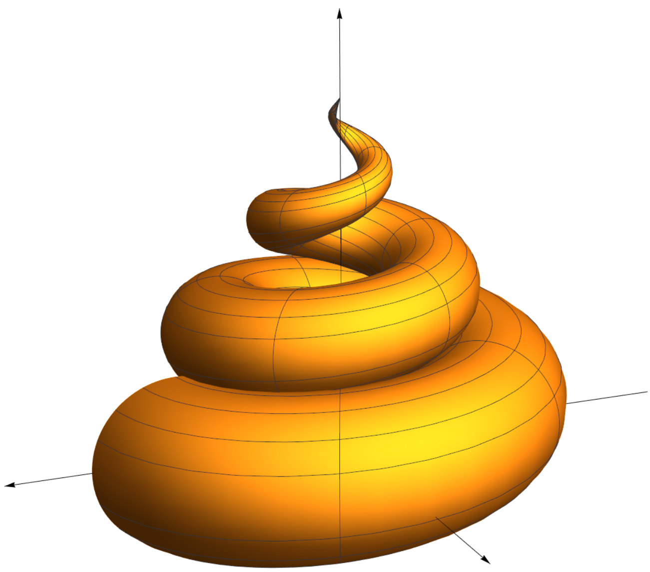
rotate around z-axis.

$$\begin{cases} r = (1-u)(2+\cos v) = 2-2u+(1-u)\cos v. \\ z = 4u + (1-u)\sin v. \end{cases}$$

$$(r-(2-2u))^2 + (z-4u)^2 = (1-u)^2$$

circle of radius $1-u$ centered at $(2-2u, 4u)$





Problem 4. Find the surface area of the part of the sphere $x^2 + y^2 + z^2 = 4z$ that lies inside the paraboloid $z = x^2 + y^2$.