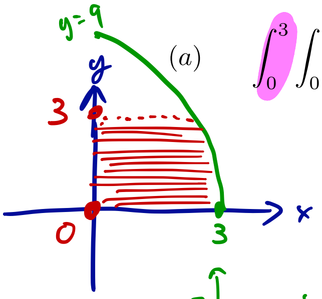
MATH 2023 – Multivariable Calculus

Lecture #10 Worksheet ♮ March 12, 2019

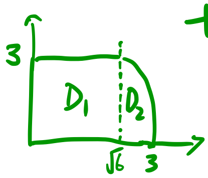
Problem 1. Sketch the region of integration, and interchange the order:

(a) $\displaystyle\int_0^3 \int_0^{\sqrt{9-y}} dxdy,$



$x=\sqrt{9-y}$

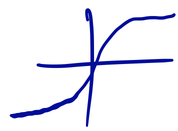
$y=9-x^2$

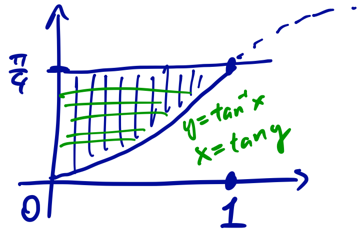


$$\int_0^{\sqrt6}\int_0^3 dy\,dx + \int_{\sqrt6}^3\int_0^{9-x^2} dy\,dx$$

(b) $\displaystyle\int_0^1 \int_{\tan^{-1} x}^{\pi/4} dydx$

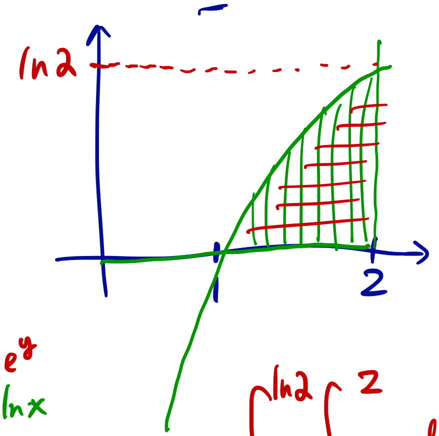
$y=\tan^{-1}x$





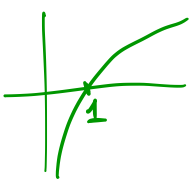
$$=\int_0^{\frac{\pi}{4}}\int_0^{\tan y} \quad dy$$

(c) $\displaystyle\int_1^2 \int_0^{\ln x} dydx,$



$x=e^y$

$y=\ln x$

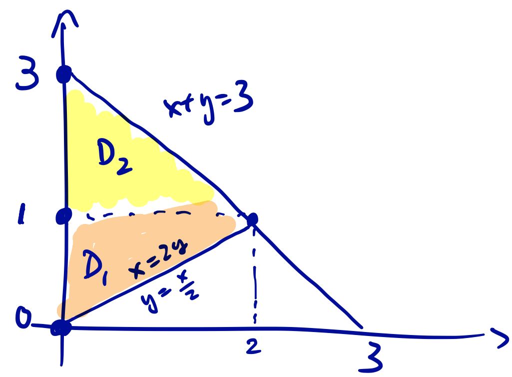


$$\int_0^{\ln 2}\int_{e^y}^2 dx\,dy$$

(d) $\displaystyle\underbrace{\int_0^1 \int_0^{2y} dxdy}_{D_1} + \underbrace{\int_1^3 \int_0^{3-y} dxdy}_{D_2}$

$x=3-y$

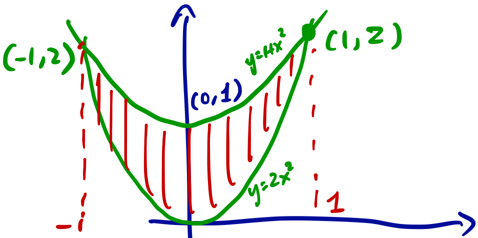
$x+y=3$



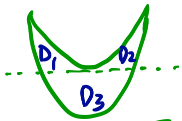
$$\int_0^2\int_{\frac{x}{2}}^{3-x} dy\,dx$$

Problem 2. Set up the two types of integrations of the following regions D bounded by:

(a) $\quad y = 2x^2$ and $y = 1 + x^2$



$$\int_{-1}^{1}\int_{2x^2}^{1+x^2} dy\,dx$$

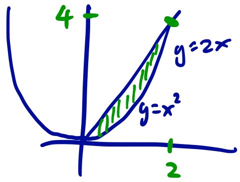


D_1: $\displaystyle\int_{1}^{2}\int_{-\sqrt{\frac{y}{2}}}^{-\sqrt{y-1}} dx\,dy$ $+$

D_3: $\displaystyle\int_{1}^{2}\int_{\sqrt{y-1}}^{\sqrt{\frac{y}{2}}} dx\,dy$ $+$

D_3: $\displaystyle\int_{0}^{1}\int_{-\sqrt{\frac{y}{2}}}^{\sqrt{\frac{y}{2}}} dx\,dy$

(b) $\quad y = 2x$ and $y = x^2$

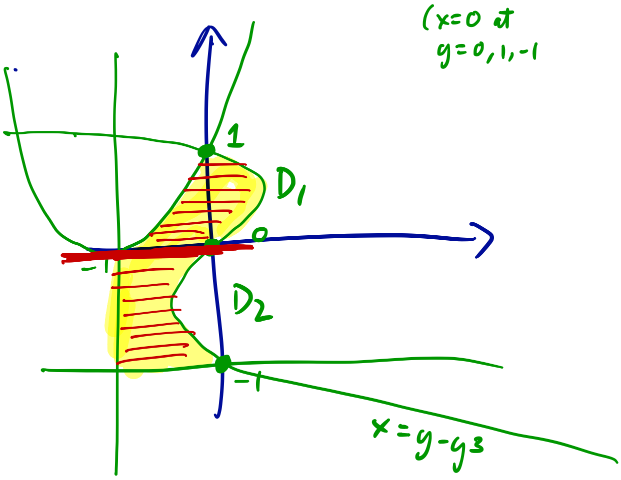


$$\int_{0}^{2}\int_{x^2}^{2x} dy\,dx$$

or $\displaystyle\int_{0}^{4}\int_{\frac{y}{2}}^{\sqrt{y}} dx\,dy$

(c) $\quad y = (x+1)^2, x = y - y^3, x = -1$ and $y = -1$

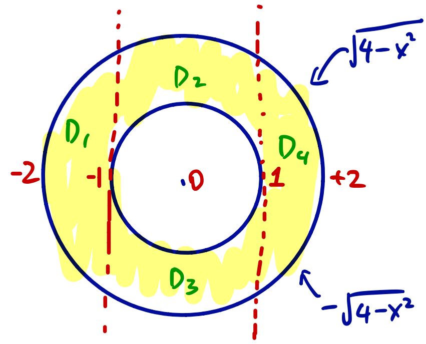
(x=0 at
y=0,1,-1



Find type II

$$\underbrace{\int_{0}^{1}\int_{\sqrt{y}-1}^{y-y^3} dx\,dy}_{D_1} + \underbrace{\int_{-1}^{0}\int_{-1}^{y-y^3} dx\,dy}_{D_2}$$

(d) $\quad x^2 + y^2 = 1$ and $x^2 + y^2 = 4$



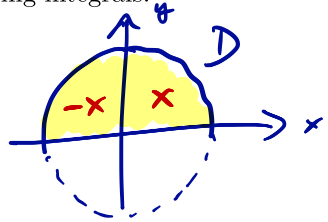
D_4: $\displaystyle\int_{1}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} dy\,dx$

D_3: $\displaystyle\int_{-1}^{1}\int_{-\sqrt{4-x^2}}^{-\sqrt{1-x^2}} dy\,dx$

D_2, D_1.

Problem 3. Use geometry or symmetry to evaluate the following integrals:

(a) $\iint_D (x+2)dA, \qquad D=\{(x,y): 0\le y\le \sqrt{9-x^2}\}$

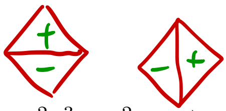


odd!

$y=\sqrt{9-x^2}$

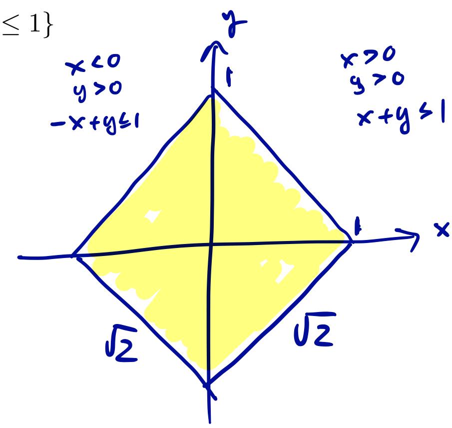
$x^2+y^2=9$

$$= \iint_D 2\,dA = 2\,A(D) = 2\cdot\frac{9\pi}{2} = 9\pi$$



(b) $\iint_D (2+x^2y^3-y^2\sin x)dA, \qquad D=\{(x,y): |x|+|y|\le 1\}$

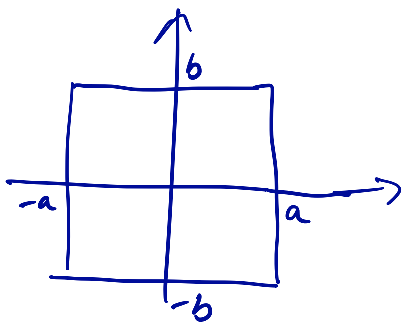
odd odd



$$=2\iint_D dA = 2\,A(D)$$
$$= 2\times 2$$
$$= 4$$

(c) $\iint_D (ax^3+by^3+\sqrt{a^2-x^2})dA, \qquad D=[-a,a]\times[-b,b]$

odd odd



$$\int_{-b}^{b}\int_{-a}^{a}\sqrt{a^2-x^2}\,dx\,dy$$

$$=\left(\int_{-b}^{b}dy\right)\left(\int_{-a}^{a}\sqrt{a^2-x^2}\,dx\right)$$

$=2b$

$x=a\sin u$ $\qquad \sqrt{a^2-x^2}=a\cos u$

$dx=a\cos u\,du$ $\qquad x=a \Rightarrow u=\frac{\pi}{2}$

$$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} a^2\cos^2u\,du = a^2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\cancel{\cos 2u}^{\,0}+1}{2}\,du = a^2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1}{2}\,du = \frac{\pi a^2}{2}$$

$=\pi a^2 b.$