$$\int_0^{\frac{\pi}{4}} 30 \sin(2x)\cos^2(4x)\,dx$$

$$= 30\int_0^{\frac{\pi}{4}} \frac{1-\cos(4x)}{2}\cos^2(4x)\,dx$$

Let $u = \cos(4x)$, $du = -4\sin(4x)\,dx$,

$$dx = -\frac{1}{4\sin(4x)}\,du.$$

$$= 30\int_1^{-1} \left(\frac{1}{2}u^2 - \frac{1}{2}u^3\right)\left(-\frac{1}{4\sqrt{1-u^2}}\right)du$$

$$= -\frac{30}{8}\int_1^{-1} \frac{u^2}{\sqrt{1-u^2}} + \frac{30}{8}\int_1^{-1} \frac{u^3}{\sqrt{1-u^2}}\,du$$

$$I = \int_0^{\frac{\pi}{4}} 30 \sin(2x) \cos^2(4x)\, dx$$

$$\cos(4x) = \cos^2(2x) - \sin^2(2x)$$

$$\sin^2(2x) = 1 - \cos^2(2x)$$

$$\cos(4x) = \cos^2(2x) - (1 - \cos^2(2x))$$

$$\cos(4x) = 2\cos^2(2x) - 1$$

$$I = \int_0^{\frac{\pi}{4}} 30 \sin(2x) \left(2\cos^2(2x) - 1\right) dx$$

Let $u = \cos(2x)$, $du = -2\sin(2x)\,dx$,

$$dx = -\frac{1}{2\sin(2x)}\, du.$$

$$= -15 \int_1^{-1} \cancel{\sin(2x)} \left(2u^2 - 1\right) \frac{1}{\cancel{\sin(2x)}}\, du.$$

$$\vdots$$

$$\frac{1}{32}\int_0^{32}\left(\frac{y}{2}\right)^2\pi\,dy$$

要寫 x 或 terms of y，因為你係 dy 緊，求到係 πr^2，$r=\left(\frac{y}{2}\right)^{\frac{1}{2}}$.

$$=\frac{\pi}{32}\int_0^{32}\sqrt{\frac{y}{2}}\,dy.$$

Let $u=\frac{y}{2}$, $du=\frac{1}{2}dy$, $dy=2du$

$$=\frac{\pi}{16}\int_0^{16}\sqrt{u}\,du$$

$$=\frac{\pi}{16}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_0^{16}$$

$$=\frac{\pi}{16}\left(\frac{2}{3}\cdot 64\right)$$

$$=\frac{8}{3}\pi$$