# MATH 2023 – Multivariable Calculus

Lecture #06 Worksheet ♢ February 26, 2019

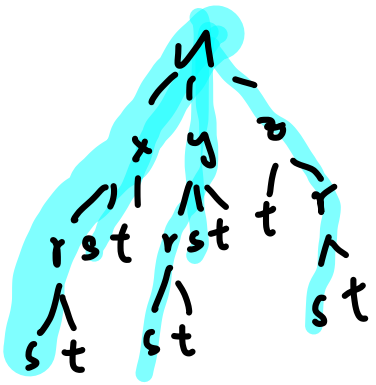
---

**Problem 1.** Let $u = x^4y + y^2z$ where

$$x = rse^t$$
$$y = s^2e^{-tr}$$
$$z = rt$$
$$r = st^2$$

Find $\frac{\partial u}{\partial s}$ in terms of $s, t$

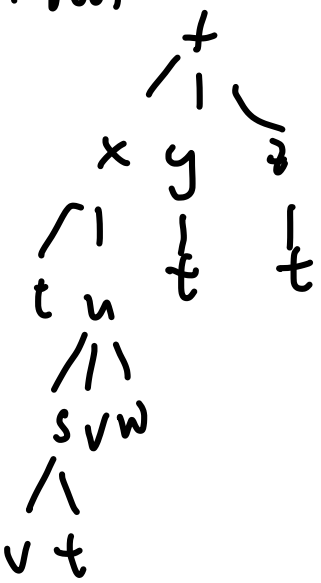
$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial r}\frac{\partial r}{\partial s} + \frac{\partial u}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r}\frac{\partial r}{\partial s} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial r}\frac{\partial r}{\partial s}$$



Let $f(x, y, z)$ be a function, where we have the dependence of variables:

$$x(t, u), y(t), u(s, v, w), s(v, t), z(t)$$

Find $\frac{\partial f}{\partial s}$ and $\frac{\partial f}{\partial t}$.





Made with Goodnotes

**Problem 1.** Let $u = x^4y + y^2z$ where

$$x = rse^t$$
$$y = s^2e^{-tr}$$
$$z = rt$$
$$r = st^2$$

Find $\frac{\partial u}{\partial s}$ in terms of $s, t$


$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x}\frac{dx}{dr}\frac{dr}{ds} \text{ ①}$$

$$+ \frac{\partial u}{\partial x}\frac{\partial x}{\partial s} \text{ ②} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r}\frac{\partial r}{\partial s} \text{ ③}$$

$$+ \frac{\partial u}{\partial y}\frac{\partial y}{\partial s} \text{ ④} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial r}\frac{\partial r}{\partial s} \text{ ⑤}$$

① : $4x^3y \cdot se^t t^2 = 4st^2 se^t s^2 e^{-tst^2} se^t t^2$

$= 4s^5 t^4 e^{2t} e^{-st^3}$

Made with Goodnotes

**Problem 2.** Let $u(r, \theta)$ be a function in polar coordinates. Express the Laplace equation

$$u_{xx} + u_{yy} = 0$$

in terms of $r$ and $\theta$.



Made with Goodnotes