

MATH 2023 – Multivariable Calculus

Lecture #03 Worksheet ♡ February 14, 2019

Problem 1. Find the arc length parametrization of the following curve from the point $(1, 0)$.

$\swarrow t=0$ $\mathbf{r}(t) = \left(\frac{2}{t^2+1} - 1 \right) \mathbf{i} + \frac{2t}{t^2+1} \mathbf{j} + \cancel{0\mathbf{k}}$

What can you conclude about the curve?

$$\begin{aligned} \textcircled{1} \quad \vec{r}'(t) &= \left\langle -\frac{2(2t)}{(t^2+1)^2}, \frac{(t^2+1)2 - 2t(2t)}{(t^2+1)^2} \right\rangle \\ &= \left\langle -\frac{4t}{(t^2+1)^2}, \frac{2-2t^2}{(t^2+1)^2} \right\rangle \end{aligned}$$

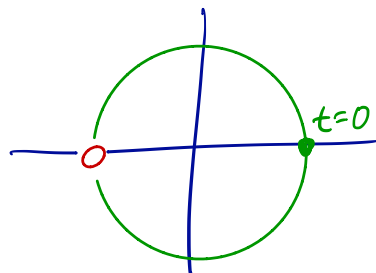
$$\begin{aligned} \textcircled{2} \quad |\vec{r}'(t)| &= \sqrt{\frac{16t^2 + (4 - 8t^2 + 4t^4)}{(t^2+1)^4}} = \sqrt{\frac{4 + 8t^2 + 4t^4}{(t^2+1)^4}} \\ &= \frac{2 + 2t^2}{(t^2+1)^2} = \frac{2}{t^2+1} \quad // \end{aligned}$$

$$\textcircled{3} \quad s = \int_0^t |\vec{r}'(\tau)| d\tau = \int_0^t \frac{2}{\tau^2+1} d\tau = 2 \tan^{-1} t \quad //$$

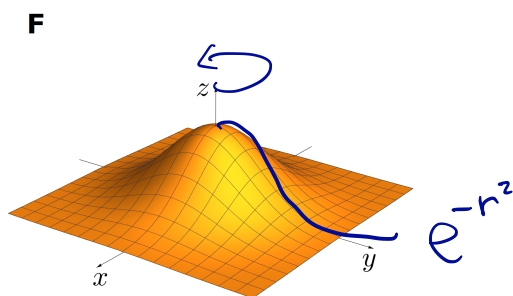
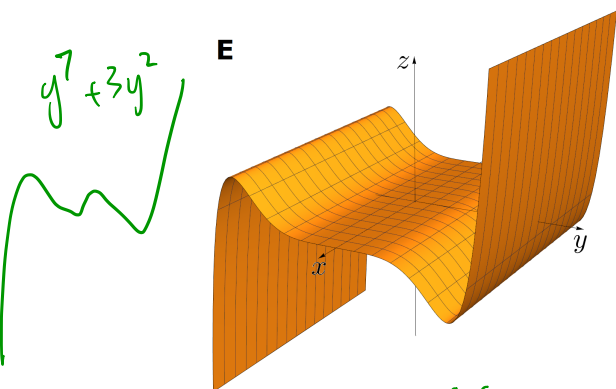
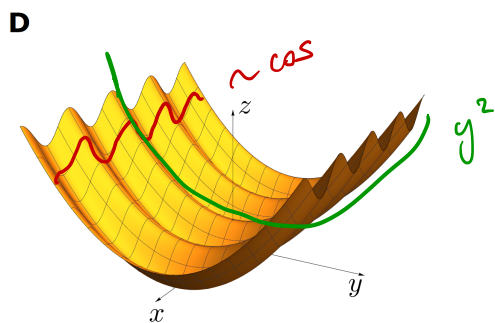
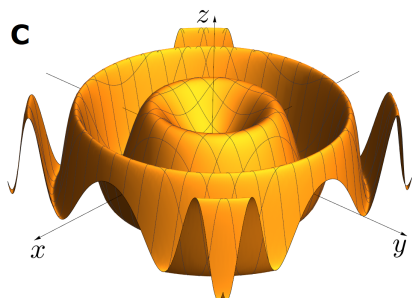
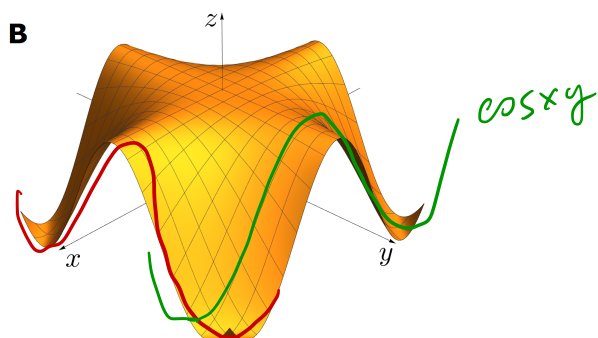
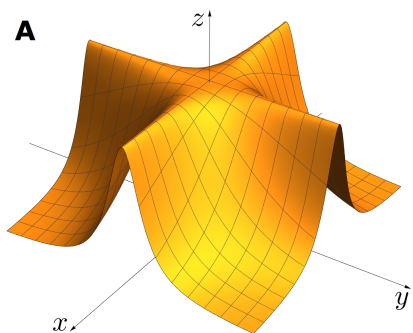
$$\textcircled{4} \quad t = \tan \frac{s}{2} \qquad \tan^2 + 1 = \frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$$

$$\begin{aligned} \textcircled{5} \quad \vec{r}(s) &= \left\langle 2\cos^2 \frac{s}{2} - 1, 2 \tan \frac{s}{2} \cos \frac{s}{2} \right\rangle \\ &\quad \text{(double angle formula)} \\ &= \langle \cos s, \sin s \rangle \end{aligned}$$

$\vec{r}(t)$ is a circle
(without the point $(-1, 0)$)



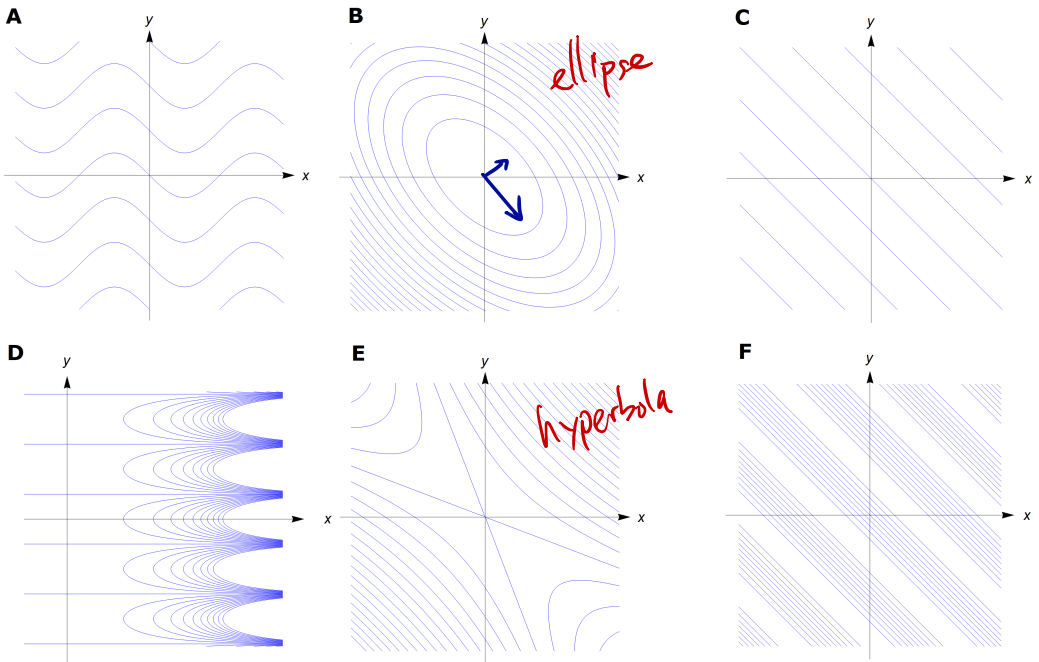
Problem 2. Identify the graphs with the corresponding functions $z = f(x, y)$.



rotate
a sin
curve

$\cos(x^2) + y^2$	$\sin(x^2 + y^2)$	$e^{-x^2 - y^2}$	$\cos(xy)$	$y^7 + 3y^2$	$\frac{1}{1+x^2y^2}$
D	C	F	B	E	A

Problem 3. Identify the level sets with the corresponding functions $z = f(x, y)$.



periodic in y

plane: $x+y=k$

$x + y$	$\sin(x + y)$	$\sin x + y$	$x^2 + xy + y^2$	$x^2 + 3xy + y^2$	$e^x \cos y$
C	F	A	B	E	D

$$e^x \cos y = 0 \\ \Rightarrow y = 2n\pi \pm \frac{\pi}{2}$$

$$\sin(x+y)=k \\ x+y=k'$$

$$\sin x + y = k \\ y = -\sin x + k$$

$$\begin{aligned} & \underline{x^2 + 3xy + \frac{9y^2}{4}} - \frac{9y^2}{4} + y^2 \\ &= \left(x + \frac{3y}{2}\right)^2 - \frac{5y^2}{4} \Rightarrow \text{hyperbola} \\ & \underline{x^2 + xy + y^2} + \underline{\frac{y^2}{4}} - \frac{y^2}{4} \\ &= \left(x + \frac{y}{2}\right)^2 + \frac{3y^2}{4} \\ & \text{ellipse.} \end{aligned}$$

Bonus Problem. Plot the graph and the level sets of the following function

$$f(x, y) = (x^2 + y^2 - 1)^3 - x^2 y^3.$$

