# MATH 2023 – Multivariable Calculus

Lecture #21 Worksheet ~~~~ April 30, 2019

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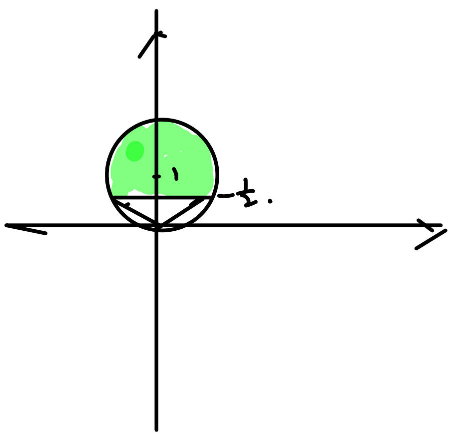
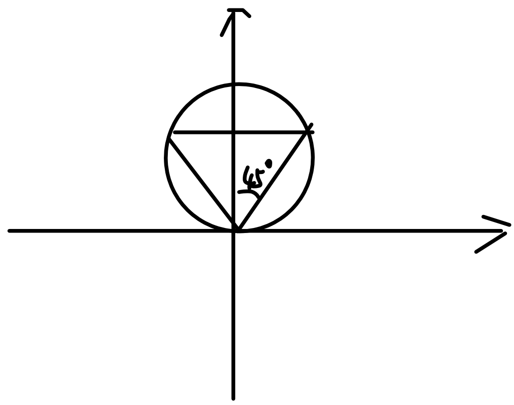
**Problem 1.** Find the volume of the solid lying above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$.

$x^2+y^2+x^2+y^2 = \sqrt{x^2+y^2}$

$2z^2 - z = 0$



$\iiint_E 1 \, dV$



**Problem 2.** Find the volume of the sphere inside the sphere $x^2 + y^2 + z^2 = 4$, under $z = \sqrt{x^2 + y^2}$ and above the $xy$ plane.

**Problem 3.** Evaluate the integrals:

$$\int_0^{\pi/4} \int_0^{2\pi} \int_0^{\sec\phi} \rho^2 \sin\phi d\rho\theta d\phi$$

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xydzdydx$$

$$\int_0^{2\pi} \int_{\pi/6}^{\pi/2} \int_{a/\sin\phi}^{2a} \rho^4 \sin^3\phi d\rho d\phi d\theta$$

**Problem 4.** Find the volume of the region bounded by the surface

$$\sqrt{x} + \sqrt{y} + \sqrt{z} = 1$$

and the coordinate planes.

**Problem 5.** If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are constant vectors, $\mathbf{r} = \langle x, y, z \rangle$ is the position vector, and $E$ is the region bounded by

$$0 \le \mathbf{a} \cdot \mathbf{r} \le \alpha, \qquad 0 \le \mathbf{b} \cdot \mathbf{r} \le \beta, \qquad 0 \le \mathbf{c} \cdot \mathbf{r} \le \gamma$$

show that

$$\iiint_E (\mathbf{a} \cdot \mathbf{r})(\mathbf{b} \cdot \mathbf{r})(\mathbf{c} \cdot \mathbf{r}) dV = \frac{(\alpha\beta\gamma)^2}{8|\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|}$$