MATH 2023 – Multivariable Calculus

Lecture #07 Worksheet ◇ February 28, 2019

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e^{-x-5} $e^{-5}\cdot e^{-x}$

Problem 1. Let $z = f(x, y) = e^{-x-y}$.

(a) Find ∇f at the point $P = (\ln 2, \ln 3)$

(b) Find the directional derivative $D_{\mathbf{u}}f$ where $\mathbf{u}$ is the unit vector parallel to $\mathbf{v} = \mathbf{i} + 2\mathbf{j}$

(c) Find the unit direction such that $|D_{\mathbf{u}}f|$ is maximum.

$$\nabla f = \langle -e^{-x-y}, -e^{-x-y} \rangle$$

$$\nabla f(\ln 2, \ln 3) = \langle -e^{-\ln 2-\ln 3}, -e^{-\ln 2-\ln 3} \rangle$$

$$= \langle -e^{\ln\left(\frac{1}{2}\cdot\frac{1}{3}\right)}, -e^{\ln\left(\frac{1}{2}\cdot\frac{1}{3}\right)} \rangle$$

$$= \langle -\frac{1}{6}, -\frac{1}{6} \rangle$$

b). $\hat{v} = \frac{1}{\sqrt{5}} i + \frac{2}{\sqrt{5}} j.$

$$\nabla f \cdot \hat{v} = -\frac{1}{6}\cdot\frac{1}{\sqrt{5}} - \frac{2}{\sqrt{5}}\cdot\frac{1}{6} = -\frac{1}{2\sqrt{5}}$$

c) $-\nabla f \langle \frac{1}{6}, \frac{1}{6} \rangle$

$$-\hat{\nabla f} = \left\langle \frac{\frac{1}{6}}{\sqrt{\frac{1}{18}}}, \frac{\frac{1}{6}}{\sqrt{\frac{1}{18}}} \right\rangle$$

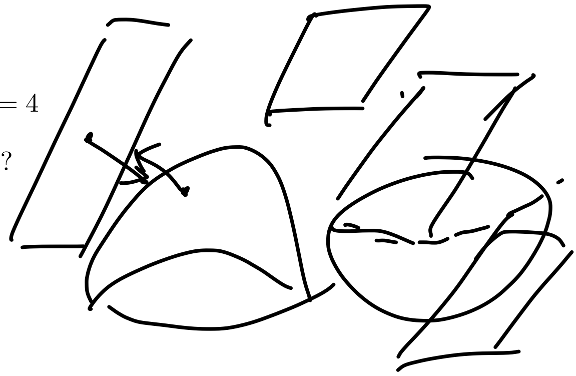
$$= \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$$

Made with Goodnotes

Problem 2. At what point on the surface

$$x^2 + 2y^2 + 3z^2 = 4$$

is the tangent plane parallel to $x + 2y + 3z = 4$?



$\vec{n} = \langle 1, 2, 3 \rangle$

$\nabla g = \langle 2x, 4y, 6z \rangle$

$x = y = z$.

$x^2 + 2x^2 + 3x^2 = 4$

$6x^2 = 4$

$\cancel{x^2 = \frac{2}{3}}$

$\cancel{x = \pm \frac{\sqrt{2}}{\sqrt{3}}}$

$\left(\pm\sqrt{\frac{2}{3}}, \pm\sqrt{\frac{2}{3}}, \pm\sqrt{\frac{2}{3}} \right)$

Problem 3. Two surfaces are **orthogonal** at a point of intersection if their normal lines are perpendicular at that point.

(a) Show that two surfaces $F(x, y, z) = 0, G(x, y, z) = 0$ are orthogonal at a point P where $\nabla F \neq 0, \nabla G \neq 0$ if and only if

$$F_x G_x + F_y G_y + F_z G_z = 0 \qquad \text{at } P$$?

(b) Given $r > 0$. Show that the surfaces $z^2 = x^2 + y^2$ and $x^2 + y^2 + z^2 = r^2$ intersects orthogonally everywhere.

(c) Explain (b) without using calculus.

a).