MATH 2023 – Multivariable Calculus

Lecture #22 Worksheet ⋆ May 7, 2019

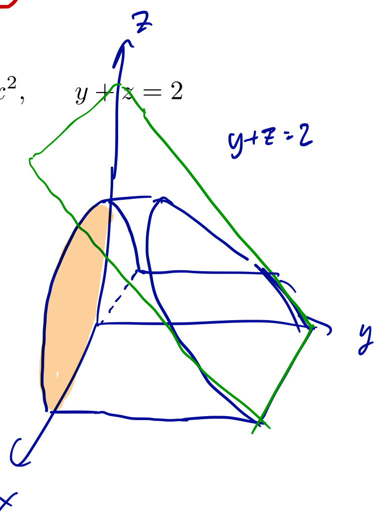
Problem 1. Let

$$\mathbf{F}(x,y,z) = \langle xy, y^2 + e^{xz^2}, \sin(xy)\rangle$$

and S be the surface bounded by

$$y = 0, \qquad z = 0, \qquad z = 1 - x^2, \qquad y + z = 2$$

Find $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

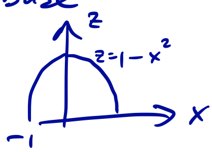


$$= \iiint_E \nabla \cdot \vec{F} \, dV$$

$$= \iiint_E y + 2y + 0 \, dV$$

$$= 3\iiint_E y \, dV$$

use xz as base



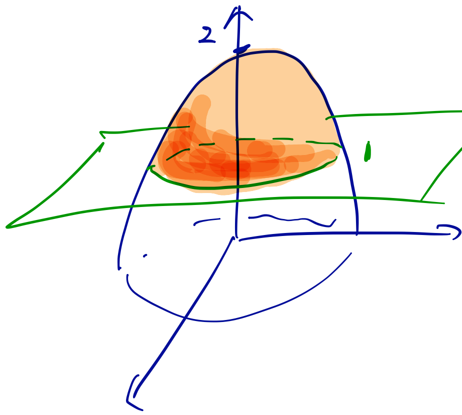
$$\int_{-1}^{1} \int_{0}^{1-x^2} \left(\int_0^{2-z} 3y \, dy \right) dz \, dx = \cdots$$

Problem 2. Let

$$\mathbf{F}(x,y,z) = \langle z\tan^{-1}(y^{2023}), z^3\ln(x^2+z^{777}), z\rangle$$

$z = 2 - x^2 - y^2$

Find the flux of $\mathbf{F}$ across the part of the paraboloid $x^2+y^2+z=2$ that lies above the plane $z=1$ and oriented upward.



$$\nabla\cdot\vec{F} = 0 + 0 + 1$$

$$\iiint_E \nabla\cdot\vec{F}\,dV = \underbrace{\iint \vec{F}\cdot d\vec{S}}_{\text{Question}} + \iint \vec{F}\cdot d\vec{S}$$

$\vec{n}$ $\vec{n}$

$$\iiint_E \nabla\cdot\vec{F}\,dV = \int_0^{2\pi}\int_0^1\int_1^{2-r^2} dz\, r\,dr\,d\theta = 2\pi\int_0^1 (1-r^2)r\,dr$$

$$= 2\pi\left(\frac{1}{2}-\frac{1}{4}\right) = \pi.$$

$$\iint \vec{F}\cdot d\vec{S} = \iint -1\,dS = -\pi$$

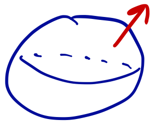
$\leftarrow z=1$

$\vec{n} = \langle 0,0,-1\rangle$

$$\therefore \iint \vec{F}\cdot d\vec{S} = \iiint \nabla\cdot\vec{F}\,dV - \iint \vec{F}\cdot d\vec{S} = \pi - (-\pi)$$

$$= 2\pi$$

Problem 3. Find $\iint_S (2x+2y+z^2)\,dS$ where S is the unit sphere $x^2+y^2+z^2=1$.



$\vec{n} = \langle x, y, z \rangle$

$$\iint_S \underset{\langle 2,2,z\rangle}{\vec{F}} \cdot \overset{\langle x,y,z\rangle}{\vec{n}}\, dS = \iint_S \vec{F} \cdot d\vec{S}$$

Div Thm: $= \iiint_E 1\, dV = \frac{4}{3}\pi$ //