EXERCISES 10.4

1. A single equation involving the coordinates (x, y, z) need not always represent a two-dimensional "surface" in $\mathbb{R}^3$. For example, $x^2 + y^2 + z^2 = 0$ represents the single point $(0, 0, 0)$, which has dimension zero. Give examples of single equations in x, y, and z that represent
   (a) a (one-dimensional) straight line,
   (b) the whole of $\mathbb{R}^3$,
   (c) no points at all (i.e., the empty set).

In Exercises 2–9, find equations of the planes satisfying the given conditions.

2. Passing through $(0, 2, -3)$ and normal to the vector $4\mathbf{i} - \mathbf{j} - 2\mathbf{k}$
3. Passing through the origin and having normal $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$
4. Passing through $(1, 2, 3)$ and parallel to the plane $3x + y - 2z = 15$
5. Passing through the three points $(1, 1, 0)$, $(2, 0, 2)$, and $(0, 3, 3)$
6. Passing through the three points $(-2, 0, 0)$, $(0, 3, 0)$, and $(0, 0, 4)$
7. Passing through $(1, 1, 1)$ and $(2, 0, 3)$ and perpendicular to the plane $x + 2y - 3z = 0$
8. Passing through the line of intersection of the planes $2x + 3y - z = 0$ and $x - 4y + 2z = -5$, and passing through the point $(-2, 0, -1)$
9. Passing through the line $x + y = 2$, $y - z = 3$, and perpendicular to the plane $2x + 3y + 4z = 5$
10. Under what geometric condition will three distinct points in $\mathbb{R}^3$ not determine a unique plane passing through them? How can this condition be expressed algebraically in terms of the position vectors, $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$, of the three points?
11. Give a condition on the position vectors of four points that guarantees that the four points are *coplanar*, that is, all lie on one plane.

Describe geometrically the one-parameter families of planes in Exercises 12–14. (λ is a real parameter.)

12. $x + y + z = \lambda$.
13. $x + \lambda y + \lambda z = \lambda$.
14. $\lambda x + \sqrt{1 - \lambda^2}\, y = 1$.

In Exercises 15–19, find equations of the line specified in vector and scalar parametric forms and in standard form.

15. Through the point $(1, 2, 3)$ and parallel to $2\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$
16. Through $(-1, 0, 1)$ and perpendicular to the plane $2x - y + 7z = 12$
17. Through the origin and parallel to the line of intersection of the planes $x + 2y - z = 2$ and $2x - y + 4z = 5$
18. Through $(2, -1, -1)$ and parallel to each of the two planes $x + y = 0$ and $x - y + 2z = 0$
19. Through $(1, 2, -1)$ and making equal angles with the positive directions of the coordinate axes

In Exercises 20–22, find the equations of the given line in standard form.

20. $\mathbf{r} = (1 - 2t)\mathbf{i} + (4 + 3t)\mathbf{j} + (9 - 4t)\mathbf{k}$.

21. $\begin{cases} x = 4 - 5t \\ y = 3t \\ z = 7 \end{cases}$

22. $\begin{cases} x - 2y + 3z = 0 \\ 2x + 3y - 4z = 4 \end{cases}$

23. If $P_1 = (x_1, y_1, z_1)$ and $P_2 = (x_2, y_2, z_2)$, show that the equations

$$\begin{cases} x = x_1 + t(x_2 - x_1) \\ y = y_1 + t(y_2 - y_1) \\ z = z_1 + t(z_2 - z_1) \end{cases}$$

represent a line through P_1 and P_2.

24. What points on the line in Exercise 23 correspond to the parameter values $t = -1$, $t = 1/2$, and $t = 2$? Describe their locations.
25. Under what conditions on the position vectors of four distinct points P_1, P_2, P_3, and P_4 will the straight line through P_1 and P_2 intersect the straight line through P_3 and P_4 at a unique point?

Find the required distances in Exercises 26–29.

26. From the origin to the plane $x + 2y + 3z = 4$
27. From $(1, 2, 0)$ to the plane $3x - 4y - 5z = 2$
28. From the origin to the line $x + y + z = 0$, $2x - y - 5z = 1$
29. Between the lines

$$\begin{cases} x + 2y = 3 \\ y + 2z = 3 \end{cases} \quad \text{and} \quad \begin{cases} x + y + z = 6 \\ x - 2z = -5 \end{cases}$$

30. Show that the line $x - 2 = \dfrac{y + 3}{2} = \dfrac{z - 1}{4}$ is parallel to the plane $2y - z = 1$. What is the distance between the line and the plane?

In Exercises 31–32, describe the one-parameter families of straight lines represented by the given equations. (λ is a real parameter.)

31. $(1 - \lambda)(x - x_0) = \lambda(y - y_0)$, $z = z_0$.
32. $\dfrac{x - x_0}{\sqrt{1 - \lambda^2}} = \dfrac{y - y_0}{\lambda} = z - z_0$.
33. Why does the factored second-degree equation

$$(A_1x + B_1y + C_1z - D_1)(A_2x + B_2y + C_2z - D_2) = 0$$

represent a pair of planes rather than a single straight line?

1. A single equation involving the coordinates (x, y, z) need not always represent a two-dimensional "surface" in $\mathbb{R}^3$. For example, $x^2 + y^2 + z^2 = 0$ represents the single point $(0, 0, 0)$, which has dimension zero. Give examples of single equations in x, y, and z that represent

(a) a (one-dimensional) straight line,

(b) the whole of $\mathbb{R}^3$,

(c) no points at all (i.e., the empty set). $x^2 + y^2 + z^2 < 0$

In Exercises 2–9, find equations of the planes satisfying the given conditions.

2. Passing through $(0, 2, -3)$ and normal to the vector $4\mathbf{i} - \mathbf{j} - 2\mathbf{k}$

3. Passing through the origin and having normal $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$

4. Passing through $(1, 2, 3)$ and parallel to the plane $3x + y - 2z = 15$

2. $4(x-0) - (y-2) - 2(z+3) = 0$

$4x - y + 2 - 2z - 6 = 0$

$4x - y - 2z = 4$

3. $x - y + 2z = 0$

4. $3(x-1) + (y-2) - 2(z-3) = 0$

$3x - 3 + y - 2 - 2z + 6 = 0$

$3x + y - 2z = -1$

5. Passing through the three points $(1, 1, 0)$, $(2, 0, 2)$, and $(0, 3, 3)$

A = $(1, 1, 0)$, B = $(2, 0, 2)$, C = $(0, 3, 3)$

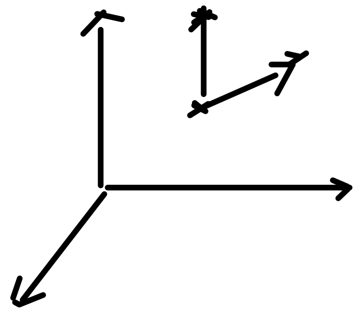
6. Passing through the three points $(-2, 0, 0)$, $(0, 3, 0)$, and $(0, 0, 4)$

A = $(-2, 0, 0)$, B = $(0, 3, 0)$, C = $(0, 0, 4)$

7. Passing through $(1, 1, 1)$ and $(2, 0, 3)$ and perpendicular to the plane $x + 2y - 3z = 0$

8. Passing through the line of intersection of the planes $2x + 3y - z = 0$ and $x - 4y + 2z = -5$, and passing through

5. $\vec{AB} = \langle 1, -1, 2 \rangle$



$\vec{AC} = \langle -1, 2, 3 \rangle$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{vmatrix} = \langle -7, -5, 1 \rangle$$

Equation: $-7(x-1) - 5(y-1) + z = 0$

$$-7x + 7 - 5y + 5 + z = 0$$

$$-7x - 5y + z = -12$$

$$7x + 5y - z = 12$$

6. $\vec{AB} = \langle 2, 3, 0 \rangle$ $\quad \vec{AC} = \langle 2, 0, 4 \rangle$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 2 & 3 & 0 \\ 2 & 0 & 4 \end{vmatrix} = \langle 12, -8, -6 \rangle$$

$$= 2\langle 6, -4, -3 \rangle$$

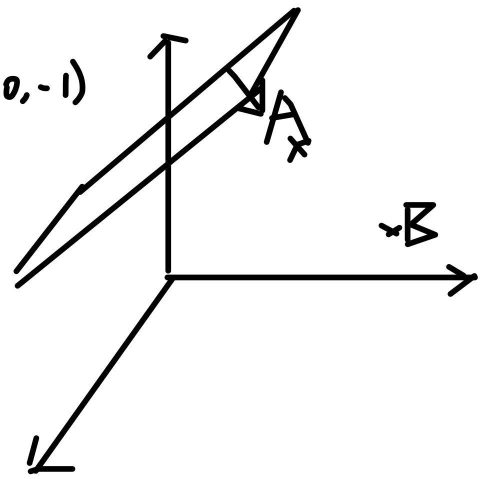
Equation: $6(x+2) - 4y - 3z = 0$

$$6x - 4y - 3z = -12$$

A B

7. Passing through $(1, 1, 1)$ and $(2, 0, 3)$ and perpendicular to the plane $x + 2y - 3z = 0$

8. Passing through the line of intersection of the planes $2x + 3y - z = 0$ and $x - 4y + 2z = -5$, and passing through $(-2, 0, -1)$



$\overrightarrow{AB} = \langle 1, -1, 2 \rangle$

$\vec{v} = \langle 1, 2, -3 \rangle$

$$\vec{n} = \overrightarrow{AB} \times \vec{v} = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ 1 & 2 & -3 \end{vmatrix}$$

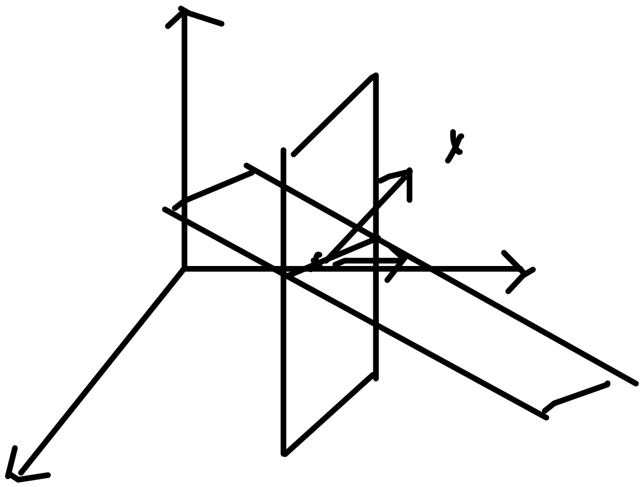
$$= \langle -1, 5, 3 \rangle$$

Equation: $-(x-1) + 5(y-1) + 3(z-1) = 0$

$$-x + 1 + 5y - 5 + 3z - 3 = 0$$

$$-x + 5y + 3z = 7$$

$$x - 5y - 3z = -7$$



8. Passing through the line of intersection of the planes $2x + 3y - z = 0$ and $x - 4y + 2z = -5$, and passing through $(-2, 0, -1)$

Sub $y = 0$, $\quad 2x - z = 0 \qquad\qquad 2x - z = 0$

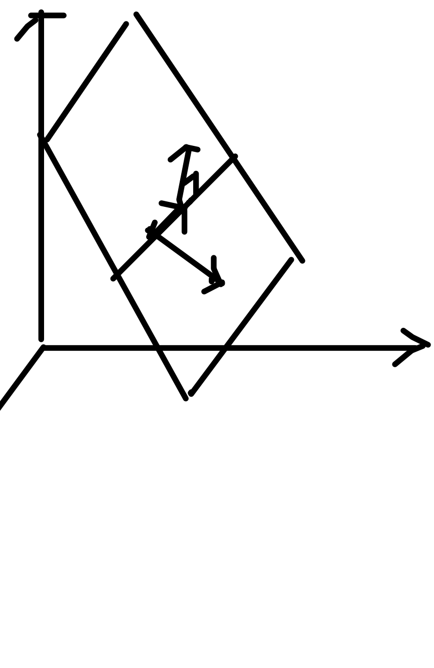
$\qquad\qquad x + 2z = -5 \qquad\qquad 2x + 4z = -10$

$$-5z = 10$$

$$z = -2, \; x = -1$$

vector of the line: $\langle 2, 3, -1\rangle \times \langle 1, -4, 2\rangle$

$$= \begin{vmatrix} \hat{i} & \hat{j} & k \\ 2 & 3 & -1 \\ 1 & -4 & 2 \end{vmatrix} = \langle 2, -5, -11\rangle$$



$$\vec{V_2} = \langle -2, 0, -1\rangle - \langle -1, 0, -2\rangle$$

$$= \langle -1, 0, 1\rangle$$

$$\vec{n} = \langle 2, -5, -11\rangle \times \langle -1, 0, 1\rangle$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & k \\ 2 & -5 & -11 \\ -1 & 0 & 1 \end{vmatrix} = \langle -5, 9, -5\rangle$$

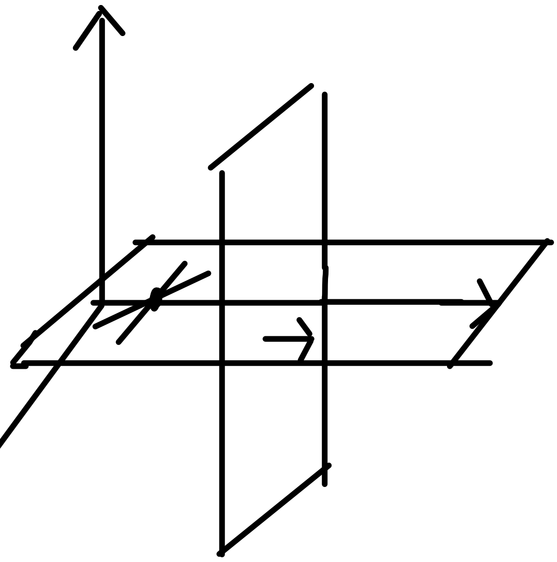
equation: $-5(x+2) + 9y - 5(z+1) = 0$

$$-5x - 10 + 9y - 5z - 5 = 0$$

$$-5x + 9y - 5z = 15$$

$$5x - 9y + 5z = -15$$

9. Passing through the line $x + y = 2$, $y - z = 3$, and perpendicular to the plane $2x + 3y + 4z = 5$



Sub $y=2, x=0, z=-1$

$\vec{n_1} = \langle 2, 3, 4 \rangle$

$(1, 1, -2)$ is also on the plane

$\vec{v} = \langle -1, 1, 1 \rangle$

$$\vec{n} = \vec{v} \times \vec{n_1} = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ -1 & 1 & 1 \end{vmatrix} = \langle -1, -6, 5 \rangle$$

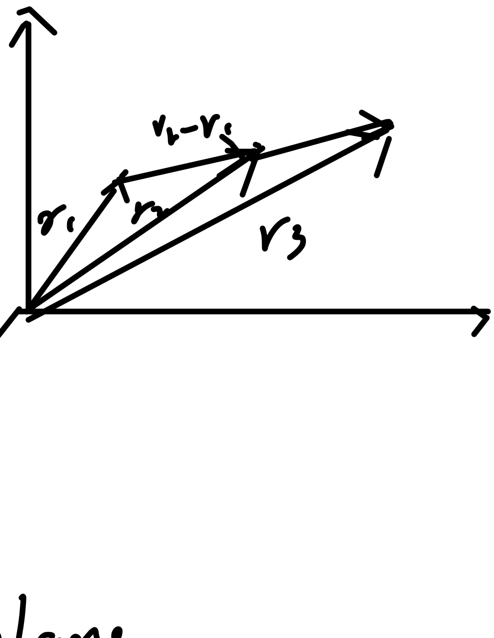
Equation: $-(x-1) - 6(y-1) + 5(z+2) = 0$

$$-x + 1 - 6y + 6 + 5z + 10 = 0$$

$$-x - 6y + 5z + 17 = 0$$

$$x + 6y - 5z = 17$$

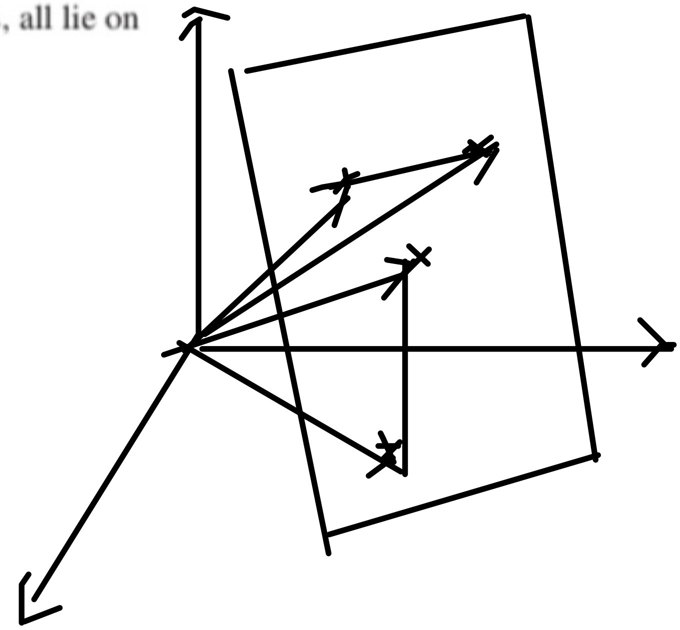
10. Under what geometric condition will three distinct points in $\mathbb{R}^3$ not determine a unique plane passing through them? How can this condition be expressed algebraically in terms of the position vectors, $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$, of the three points?



if $r_3 - r_1 = \lambda(r_2 - r_1)$,

then it does not uniquely determine a plane.

11. Give a condition on the position vectors of four points that guarantees that the four points are *coplanar*, that is, all lie on one plane.



cf $(r_2 - r_1) \times (r_4 - r_3) =$

$(r_3 - r_1) \times (r_4 - r_2)$

Describe geometrically the one-parameter families of planes in Exercises 12–14. (λ is a real parameter.)

12. $x + y + z = \lambda$.

13. $x + \lambda y + \lambda z = \lambda$.

14. $\lambda x + \sqrt{1-\lambda^2}\, y = 1$.

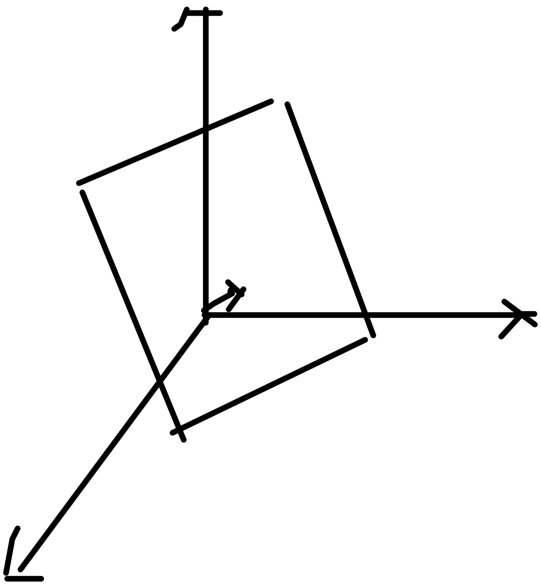
In Exercises 15–19, find equations of the line specified in vector and scalar parametric forms and in standard form.

15. Through the point $(1, 2, 3)$ and parallel to $2\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$

16. Through $(-1, 0, 1)$ and perpendicular to the plane $2x - y + 7z = 12$

17. Through the origin and parallel to the line of intersection of the planes $x + 2y - z = 2$ and $2x - y + 4z = 5$

18. Through $(2, -1, -1)$ and parallel to each of the two planes $x + y = 0$ and $x - y + 2z = 0$

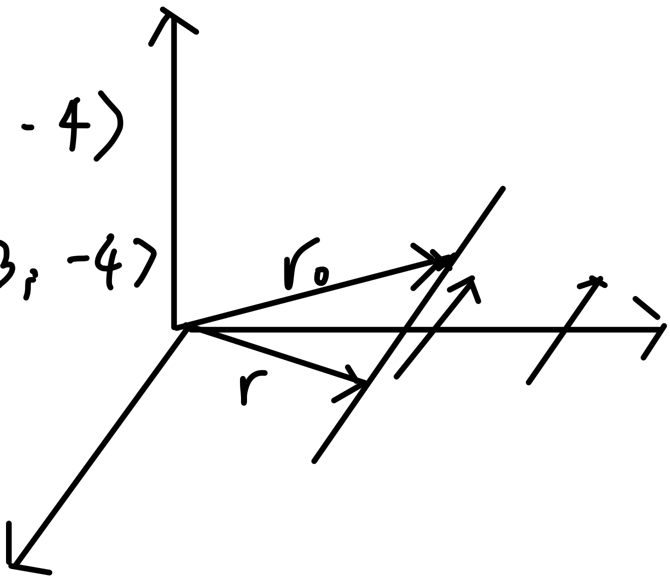


12. $\vec{n} = \langle 1, 1, 1 \rangle$, shift from origin by λ unit

15.

$(r - r_0) = t\langle 2, -3, -4 \rangle$

$r - \langle 1, 2, 3 \rangle = t\langle 2, -3, -4 \rangle$



vector: $r = \langle 1 + 2t, 2 - 3t, 3 - 4t \rangle$

scalar: $x = 1 + 2t$, $y = 2 - 3t$, $z = 3 - 4t$

standard: $\dfrac{x-1}{2} = \dfrac{y-2}{-3} = \dfrac{z-3}{-4}$

In Exercises 15–19, find equations of the line specified in vector and scalar parametric forms and in standard form.

15. Through the point $(1, 2, 3)$ and parallel to $2\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$

16. Through $(-1, 0, 1)$ and perpendicular to the plane $2x - y + 7z = 12$

17. Through the origin and parallel to the line of intersection of the planes $x + 2y - z = 2$ and $2x - y + 4z = 5$

18. Through $(2, -1, -1)$ and parallel to each of the two planes $x + y = 0$ and $x - y + 2z = 0$

16. $\vec{v} = \langle 2, -1, 7 \rangle$

$\vec{r} - \langle -1, 0, 1 \rangle = t \langle 2, -1, 7 \rangle$

vector: $\vec{r} = \langle -1+2t, -t, 1+7t \rangle$

scalar: $x = -1+2t, \; y = -t, \; z = 1+7t$

standard: $\dfrac{x+1}{2} = -y = \dfrac{z-1}{7}$

17. $\vec{v} = \langle 1, 2, -1 \rangle \times \langle 2, -1, 4 \rangle = \begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ 2 & -1 & 4 \end{vmatrix}$

$= \langle 7, -6, -5 \rangle$

$r - \langle 0, 0, 0 \rangle = t \langle 7, -6, -5 \rangle$

$r = \langle 7t, -6t, -5t \rangle$

$x = 7t, \; y = -6t, \; z = -5t$

$\dfrac{x}{7} = \dfrac{y}{-6} = \dfrac{z}{-5}$

Made with Goodnotes

18. Through $(2, -1, -1)$ and parallel to each of the two planes $x + y = 0$ and $x - y + 2z = 0$

$$\vec{v} = \langle 1, 1, 0 \rangle \times \langle 1, -1, 2 \rangle = \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 1 & -1 & 2 \end{vmatrix} =$$

$$\langle 2, -2, -2 \rangle = 2\langle 1, -1, -1 \rangle$$

$$\vec{r} - \langle 2, -1, -1 \rangle = t\langle 1, -1, -1 \rangle$$

$$\vec{v} = \langle 2+t, -1-t, -1-t \rangle$$

$$x = 2+t, \quad y = -1-t, \quad z = -1-t$$

$$x - 2 = \cancel{\frac{y+1}{-1}} = \frac{z+1}{-1}$$

19. Through (1, 2, −1) and making equal angles with the positive directions of the coordinate axes

In Exercises 20–22, find the equations of the given line in standard form.

20. $\mathbf{r} = (1-2t)\mathbf{i} + (4+3t)\mathbf{j} + (9-4t)\mathbf{k}$.

21. $\begin{cases} x = 4-5t \\ y = 3t \\ z = 7 \end{cases}$ **22.** $\begin{cases} x - 2y + 3z = 0 \\ 2x + 3y - 4z = 4 \end{cases}$

$x = 1-2t$, $y = 4+3t$, $z = 9-4t$

20. $\dfrac{x-1}{-2} = \dfrac{y-4}{3} = \dfrac{z-9}{-4}$

21. $\dfrac{x-4}{-5} = \dfrac{y}{3}$, $z = 7$

22. $\vec{v} = \langle 1, -2, 3\rangle \times \langle 2, 3, -4\rangle = \begin{vmatrix} i & j & k \\ 1 & -2 & 3 \\ 2 & 3 & -4 \end{vmatrix}$

$= \langle -1, 10, 7\rangle$

Sub $x=1$, $\begin{cases} -2y + 3z = -1 \\ 3y - 4z = 2 \end{cases}$ $\begin{bmatrix} -2 & 3 & -1 \\ 3 & -4 & 2 \end{bmatrix}$ $\frac{3}{2}$

$\begin{bmatrix} -2 & 3 & -1 \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \sim \begin{bmatrix} -2 & 3 & -1 \\ 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} -2 & 0 & -4 \\ 0 & 1 & 1 \end{bmatrix} \sim$

$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix}$ $y = 2, z = 1$

$$\vec{r} - \langle 1, 2, 1 \rangle = t\langle -1, 10, 7 \rangle$$

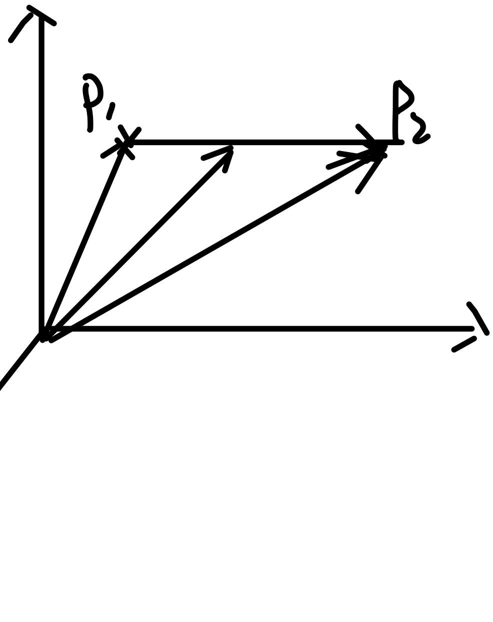
$$\vec{r} = \langle 1 - t, 2 + 10t, 1 + 7t \rangle$$

$$\frac{x-1}{-1} = \frac{y-2}{10} = \frac{z-1}{7}$$

23. If $P_1 = (x_1, y_1, z_1)$ and $P_2 = (x_2, y_2, z_2)$, show that the equations

$$\begin{cases} x = x_1 + t(x_2 - x_1) \\ y = y_1 + t(y_2 - y_1) \\ z = z_1 + t(z_2 - z_1) \end{cases}$$

represent a line through P_1 and P_2.



Let $\vec{r}$ = position vector of a point on the line.

$\overrightarrow{P_1P_2} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$

Let $\vec{r_1}$ = position vector of P_1

$$\vec{r} - \vec{r_1} = t\,\overrightarrow{P_1P_2}$$

$$\vec{r} - \langle x_1, y_1, z_1 \rangle = t \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

$$\vec{r} = \langle x_1 + t(x_2 - x_1), y_1 + t(y_2 - y_1), z_1 + t(z_2 - z_1) \rangle$$

Made with Goodnotes

24. What points on the line in Exercise 23 correspond to the parameter values $t = -1$, $t = 1/2$, and $t = 2$? Describe their locations.

$$x_1 - (x_2 - x_1)$$
$$= x_1 - x_2 + x_1$$
$$= 2x_1 - x_2$$

point: $(2x_1 - x_2, 2y_1 - y_2, 2z_1 - z_2)$

$$x_1 - \frac{1}{2}(x_2 - x_1) = x_1 - \frac{1}{2}x_2 + \frac{1}{2}x_1 = \frac{3}{2}x_1 - \frac{1}{2}x_2$$

point $\left(\frac{3}{2}x_1 - \frac{1}{2}x_2, \frac{3}{2}y_1 - \frac{1}{2}y_2, \frac{3}{2}z_1 - \frac{1}{2}z_2\right)$

$$x_1 - 2(x_2 - x_1) = x_1 - 2x_2 + 2x_1 = 3x_1 - 2x_2$$

point $(3x_1 - 2x_2, 3y_1 - 2y_2, 3z_1 - 2z_2)$

25. Under what conditions on the position vectors of four distinct points P_1, P_2, P_3, and P_4 will the straight line through P_1 and P_2 intersect the straight line through P_3 and P_4 at a unique point?

Find the required distances in Exercises 26–29.

26. From the origin to the plane $x + 2y + 3z = 4$

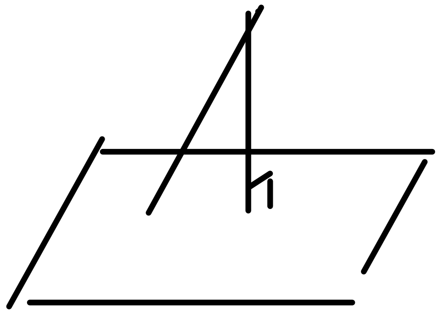
27. From $(1, 2, 0)$ to the plane $3x - 4y - 5z = 2$

28. From the origin to the line $x + y + z = 0$, $2x - y - 5z = 1$

29. Between the lines

$$\begin{cases} x + 2y = 3 \\ y + 2z = 3 \end{cases} \quad \text{and} \quad \begin{cases} x + y + z = 6 \\ x - 2z = -5 \end{cases}$$

26 $\left|\dfrac{-4}{\sqrt{1^2+2^2+3^2}}\right| = \dfrac{4}{\sqrt{14}}$



27. $s = \left|\dfrac{3(1)-4(2)-5(0)-2}{\sqrt{3^2+4^2+5^2}}\right| = \left|\dfrac{3-8-2}{\sqrt{50}}\right| = \dfrac{7}{\sqrt{50}}$

28. $\vec{v} = \langle 1, 1, 1\rangle \times \langle 2, -1, -5\rangle$

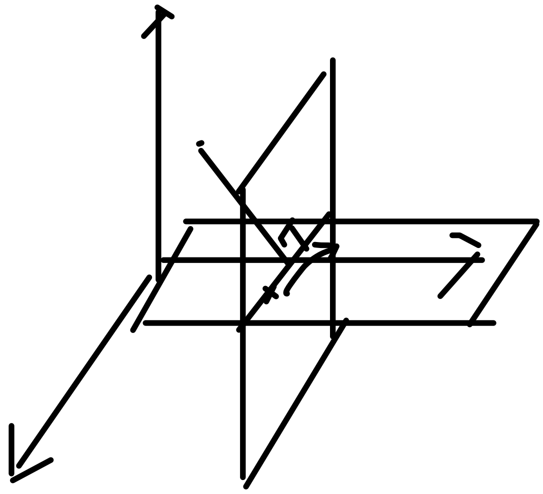
$= \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2 & -1 & -5 \end{vmatrix} = \langle -4, 7, -3\rangle$

Made with Goodnotes

28. From the origin to the line $x + y + z = 0$, $2x - y - 5z = 1$

29. Between the lines

$$\begin{cases} x + 2y = 3 \\ y + 2z = 3 \end{cases} \quad \text{and} \quad \begin{cases} x + y + z = 6 \\ x - 2z = -5 \end{cases}$$



$\langle 1,1,1\rangle \times \langle 2,-1,-5\rangle$

$= \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2 & -1 & -5 \end{vmatrix}$

$= \langle -4, 7, -3 \rangle$

$y=0,\quad x+z=0$

$2x-5z=1$

$2x+2z=0$

$2x-5z=1$

$-3z=1$

$z=-\frac{1}{3}$

$x=\frac{1}{3}$.

$\langle -\frac{1}{3}, 0, \frac{1}{3} \rangle$

$\begin{vmatrix} i & j & k \\ -4 & 7 & -3 \\ -\frac{1}{3} & 0 & \frac{1}{3} \end{vmatrix} = \langle \frac{7}{3}, \frac{7}{3}, -\frac{7}{3} \rangle$

$= \frac{7}{3}\langle 1, 1, -1 \rangle$

$-\frac{4}{3} - 1$

$$\frac{\frac{7}{3}\sqrt{3}}{\sqrt{16+49+9}} = \frac{\frac{7}{3}\sqrt{3}}{\sqrt{74}} = \frac{7\sqrt{3}}{3\sqrt{74}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{21}{3\sqrt{222}}$$

$= \frac{7}{\sqrt{222}}$

$$\begin{array}{r} 58 \\ +16 \\ \hline 74 \end{array} \qquad \begin{array}{r} 74 \\ \times 3 \\ \hline 222 \end{array}$$

29. Between the lines

$$\begin{cases} x + 2y = 3 \\ y + 2z = 3 \end{cases} \quad \text{and} \quad \begin{cases} x + y + z = 6 \\ x - 2z = -5 \end{cases}$$

1 3

$$V_0 \quad \begin{vmatrix} i & j & k \\ 1 & 2 & 0 \\ 0 & 1 & 2 \end{vmatrix} = \langle 4, -2, 1 \rangle$$

$$V_1 \quad \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & 0 & -2 \end{vmatrix} = \langle -2, 3, -1 \rangle$$

$$r_0 = \langle -3, 3, 0 \rangle \quad r_1 = \langle 1, 2, 3 \rangle$$

$$\frac{|(r_1 - r_0) \cdot (V_0 \times V_1)|}{|V_0 \times V_1|}$$

$$= \frac{\langle 4, -1, 3 \rangle \cdot \langle -1, 2, 8 \rangle}{\sqrt{69}}$$

$$= \frac{-4 - 2 + 24}{\sqrt{69}} = \frac{18}{\sqrt{69}}$$

30. Show that the line $x - 2 = \dfrac{y+3}{2} = \dfrac{z-1}{4}$ is parallel to the plane $2y - z = 1$. What is the distance between the line and the plane?

$x - 2 = t$

$y + 3 = 2t$

$z - 1 = 4t$

direction vector of the line = $\vec{v} = \langle 1, 2, 4 \rangle$

normal vector of the plane = $\langle 0, 2, -1 \rangle$
$\vec{n}$

$\vec{v} \cdot \vec{n} = 4 - 4 = 0$

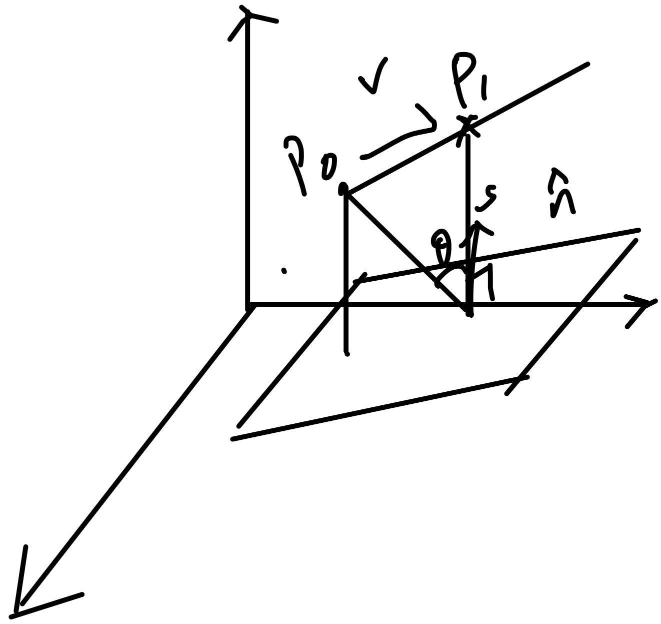
$\therefore$ the line is normal to normal vector of the plane

$\therefore$ the line $\parallel$ plane.

a point on line
$(2, -1, 1)$

distance :

$\vec{v} = \langle 1, 2, 4 \rangle$



In Exercises 31–32, describe the one-parameter families of straight lines represented by the given equations. (λ is a real parameter.)

31. $(1-\lambda)(x-x_0) = \lambda(y-y_0),\ z = z_0.$

32. $\dfrac{x-x_0}{\sqrt{1-\lambda^2}} = \dfrac{y-y_0}{\lambda} = z - z_0.$

33. Why does the factored second-degree equation

$$(A_1x + B_1y + C_1z - D_1)(A_2x + B_2y + C_2z - D_2) = 0$$

represent a pair of planes rather than a single straight line?

31. $x - x_0 - \lambda x + \lambda x_0 = \lambda y - \lambda y_0$

33. $A = 0$ / $B = 0$.