

# MATH2023 Multivariable Calculus 2013

From the textbook Calculus of Several Variables (5th) by R. Adams, Addison Wesley.

## Homework 7

(Total: 12 questions)

### Ex. 14.5

- 4 Evaluate the triple integral  $\iiint_R x \, dV$ , where  $R$  is the tetrahedron bounded by the coordinate planes and the plane  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ .

Be alert for simplifications and auspicious orders of iteration.

- 11 Evaluate the triple integral  $\iiint_R \frac{1}{(x+y+z)^3} \, dV$ , where  $R$  is the region bounded by the six planes  $z = 1$ ,  $z = 2$ ,  $y = 0$ ,  $y = z$ ,  $x = 0$ , and  $x = y + z$ .

Be alert for simplifications and auspicious orders of iteration.

- 16 Sketch the region  $R$  in the first octant of 3-space that has finite volume and is bounded by the surfaces  $x = 0$ ,  $z = 0$ ,  $x + y = 1$ , and  $z = y^2$ . Write six different iterations of the triple integral of  $f(x, y, z)$  over  $R$ .

- 19 Express the iterated integral as a triple integral and sketch the region over which it is taken. Reiterate the integral so that the outermost integral is with respect to  $x$  and the innermost is with respect to  $z$ .

$$\int_0^1 \int_z^1 \int_0^{x-z} f(x, y, z) \, dy \, dx \, dz.$$

- 27 Evaluate the iterated integral by reiterating it in a different order. (You will need to make a good sketch of the region.)

$$\int_0^1 \int_z^1 \int_0^x e^{x^3} \, dy \, dx \, dz.$$

### Ex. 14.6

- 19 Find the volume of the region above the  $xy$ -plane, inside the cone  $z = 2a - \sqrt{x^2 + y^2}$  and inside the cylinder  $x^2 + y^2 = 2ay$ .

- 25 Find  $\iiint_B (x^2 + y^2) \, dV$ , where  $B$  is the ball given by  $x^2 + y^2 + z^2 \leq a^2$ .

- 30 Evaluate  $\iiint_R (x^2 + y^2) \, dV$  over the region  $R$ , where  $R$  is the region which lies above the cone  $z = c\sqrt{x^2 + y^2}$  and inside the sphere  $x^2 + y^2 + z^2 = a^2$ .

### Ex. 14.7

- 2 Use double integral to calculate the area of the part of the plane  $5z = 3x - 4y$  inside the elliptic cylinder  $x^2 + 4y^2 = 4$ .

- 6 Use double integral to calculate the area of the paraboloid  $z = 1 - x^2 - y^2$  in the first octant.

- 10 Show that the parts of the surfaces  $z = 2xy$  and  $z = x^2 + y^2$  that lie in the same vertical cylinder have the same area.

- Qu Find the volume bounded by the surface with equation  $x^{2/3} + y^{2/3} + z^{2/3} = a^{2/3}$ .

\* Only hand in the underlined ones, the others are recommended exercises.