MATH 2023 – Multivariable Calculus

Lecture #14 Worksheet April 2, 2019

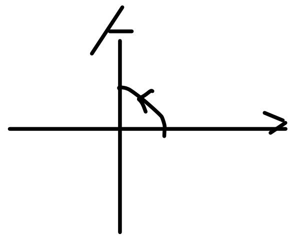
Problem 1. Find the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ for

$$\mathbf{F}(x, y) = x^2\mathbf{i} - xy\mathbf{j}$$

along the quarter circle in the first quadrant with counterclockwise orientation.

$\langle \cos\theta, \sin\theta \rangle$

$0 \le \theta \le \frac{\pi}{2}.$



$$\int_0^{\frac{\pi}{2}} x^2(-\sin\theta)\,d\theta - xy(\cos\theta)\,d\theta$$

$$= \int_0^{\frac{\pi}{2}} -2\cos^2\theta \sin\theta \quad d\theta$$

Problem 2. Let

$$\mathbf{F}(x, y) = \langle 3 + 2xy, x^2 - 3y^2 \rangle$$

(a) Show that this is a conservative vector field

(b) Hence find $f(x, y)$ such that $\nabla f = \mathbf{F}$.

(c) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ for

$$\mathbf{r}(t) = \langle e^t \sin t, e^t \cos t \rangle, \qquad 0 \leq t \leq 2\pi.$$

a). $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 2x - 2x = 0.$

b). $f(x,y) = 3x + x^2 y - y^3$

c). $f(\vec{r}(2\pi)) - f(\vec{r}(0))$

$= f((0, e^{2\pi}) - f((0,1))$

$= -e^{6\pi} + 1$

Problem 3. Let $\mathbf{F}(x, y) = \langle y^2, 2xy + e^{3z}, 3ye^{3z} \rangle$

(a) Show that this is a conservative vector field.

(b) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ for $C : \mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ from $t = 0$ to $t = 1$.

a). $\nabla \times F = \begin{vmatrix} \vec{i} & \vec{j} & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & 2xy + e^{3z} & 3ye^{3z} \end{vmatrix}$

$= \langle 3e^{3z} - 3e^{3z}, 0, 2y - 2y \rangle$

$= \langle 0, 0, 0 \rangle$ $\quad f(x, y, z) = y^2 x + ye^{3z}$

b). $f(\vec{r}(1)) - f(\vec{r}(0))$

$= f(1, 1, 1) - f(0, 0, 0)$

$= 1 + e^3 - 0$

$=$

Problem 4. Let $\mathbf{F}(x,y) = \langle \cos(x+2y), 2\cos(x+2y)\rangle$. Find curves C_1 and C_2 that are not closed, such that

$$\int_{C_1} \mathbf{F}\cdot d\mathbf{r} = 0, \qquad \text{and} \qquad \int_{C_2} \mathbf{F}\cdot d\mathbf{r} = 1$$

$r_1(t) = \langle a_1, b_1 \rangle$

$\cos(x+2y)\, a_1' + 2\cos(x+2y)\, b_1' = 0$

$a_1' = -2b_1'$

$\langle -2t, t \rangle$

$r_2(t) = \langle t, t \rangle \qquad 0 \leq t \leq \frac{\pi}{2}$

$\int_c \cos(x+2y)\, a_2' + 2\cos(x+2y)\, b_2' \, dt = 1$

$\int_c \cos(a_2+2b_2)(a_2' + 2b_2')\, dt = 1$

let $u = a_2 + 2b_2$, $du = a_2' + 2b_2' dt$

$[\sin(u)]_0^{\frac{\pi}{4}}$

$\sin d - \sin c = 1$

$\sin d = 1 + \sin c$