MATH 2023 – Multivariable Calculus

Problem 1. Let

$$f(x,y) = \begin{cases} \dfrac{3x^2y}{x^2 + y^2}, & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

Is this function continuous on $\mathbb{R}^2$?

$$\frac{3r^3\cos^2\theta \sin\theta}{r^2}$$

$$= 3\cos^2\theta \sin\theta$$

depends on θ, no

Problem 2. Let
$$f(x, y) = x^{y^{x^y}}.$$
Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$.

Problem 3. Find $f(x, y)$ such that

$$\begin{cases} \dfrac{\partial f}{\partial x} = 4x - y \\[2mm] \dfrac{\partial f}{\partial y} = -x + 6y^2 \end{cases}$$

$$\int \frac{\partial f}{\partial x}\, \partial x = 2x^2 - yx + C(y)$$

$$\int \frac{\partial f}{\partial y}\, \partial y = -xy + 2y^3 + C(x)$$

$$F(x,y) = 2x^2 - xy + 2y^3$$