HW 3: Q1.

$$\int_0^\infty cxe^{-\frac{x}{2}}\,dx = 1$$

$u \quad v'$

$u' = 1$

$v = -2e^{-\frac{x}{2}}$

$$\left[xe^{-\frac{x}{2}}\right]_0^\infty + \int_0^\infty 2e^{-\frac{x}{2}}\,dx = \frac{1}{c}$$

$$2\left[-2e^{-\frac{x}{2}}\right]_0^\infty = \frac{1}{c}$$

$$2 = \frac{1}{2c}$$

$$1 = \frac{1}{4c}$$

$$4C = 1$$

$$C = \frac{1}{4}$$

i). $P(X \geq 5) = \int_5^\infty \frac{1}{4} x e^{-\frac{x}{2}}\,dx$

$$= \frac{1}{4}\left[xe^{-\frac{x}{2}}\right]_5^\infty + \frac{1}{4}\int_5^\infty 2e^{-\frac{x}{2}}\,dx$$

$$= \frac{1}{4}\left(-5e^{-\frac{5}{2}}\right) + \frac{1}{4}\left(-2e^{-\frac{5}{2}}\right)$$

$=$

$$\int_0^\infty cxe^{-\frac{x}{2}}dx = 1$$

$u' = 1$

$$\int_0^\infty xe^{-\frac{x}{2}}dx = \frac{1}{c}$$

$v = -2e^{-\frac{x}{2}}$

$u \; v'$

$$\left[-2xe^{-\frac{x}{2}}\right]_0^\infty + 2\int_0^\infty e^{-\frac{x}{2}}dx = \frac{1}{c}$$

$$\left[-2e^{-\frac{x}{2}}\right]_0^\infty = \frac{1}{2c}$$

$$0-(-2) = \frac{1}{2c}$$

$$1 = \frac{1}{4c}$$

$$c = \frac{1}{4}$$

$$\int_5^\infty$$

$$P(X>5) = \int_5^\infty \frac{1}{4} x e^{-\frac{x}{2}}\,dx$$

$u' = 1$

$v = -2e^{-\frac{x}{2}}$

$$= \frac{1}{4}\left[-2xe^{-\frac{x}{2}}\right]_5^\infty + \frac{1}{2}\int_5^\infty e^{-\frac{x}{2}}\,dx$$

$$= \frac{1}{4}\left(2(5)e^{-\frac{5}{2}}\right) + \frac{1}{2}\left[-2e^{-\frac{x}{2}}\right]_5^\infty$$

$$= \frac{5}{2}e^{-\frac{5}{2}} + e^{-\frac{5}{2}}$$

$0.2872.$

(i). $E(X) = \int_0^\infty \frac{1}{4} x^2 e^{-\frac{x}{2}} dx$

$= \frac{1}{4} \int_0^\infty \underset{u}{x^2} \underset{v'}{e^{-\frac{x}{2}}} dx$ $u' = 2x$

$v = -2e^{-\frac{x}{2}}$

$= \frac{1}{4} \left[-2x^2 e^{-\frac{x}{2}}\right]_0^\infty + \frac{1}{4}\int_0^\infty 4xe^{-\frac{x}{2}} dx$

$= \int_0^\infty \underset{u}{x} \underset{v'}{e^{-\frac{x}{2}}} dx$ $u' = 1$

$v = -2e^{-\frac{x}{2}}$

$= \left[-2xe^{-\frac{x}{2}}\right]_0^\infty + 2\int_0^\infty e^{-\frac{x}{2}} dx$

$= 2\left[-2e^{-\frac{x}{2}}\right]_0^\infty$

$= 2(0) - 2(-2)$

$= 4$

2.
$$\int_0^a 5(1-x)^4\,dx = 0.99$$

5 let $u = 1-x$, $du = -dx$

$$-5\int_1^{1-a} u^4\,du = 0.99$$

$$-5\left[\frac{u^5}{5}\right]_1^{1-a} = 0.99$$

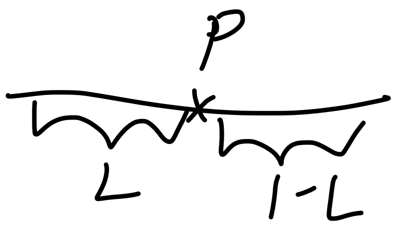
$$-5\left(\frac{(1-a)^5}{5} - \frac{1}{5}\right) = 0.99$$

$$1 - (1-a)^5 = 0.99$$

$$(1-a)^5 = 0.01$$

$$a = 0.6018$$

3.



$$L = f_P(x) = \begin{cases} 1 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$X(L) = \begin{cases} \frac{L}{1-L} & \text{if} \quad L < \frac{1}{2} \\ \frac{1-L}{L} & \text{if} \quad L > \frac{1}{2} \end{cases}$$

$$E(X) = \int_0^{\frac{1}{2}} \frac{L}{1-L}\, dL + \int_{\frac{1}{2}}^{1} \frac{1-L}{L}\, dL$$

let $u = 1-L,\ du = -dL$

$$= -\int_1^{\frac{1}{2}} \frac{1-u}{u}\, du + \int_{\frac{1}{2}}^{1} \frac{1}{L} - 1\, dL$$

$$= \int_{\frac{1}{2}}^{1} \frac{1}{u} - 1\, du + \int_{\frac{1}{2}}^{1} \frac{1}{L} - 1\, dL$$

$$= \left[\ln u - u\right]_{\frac{1}{2}}^{1} + \left[\ln L - L\right]_{\frac{1}{2}}^{1}$$

$$= -1 - \ln\tfrac{1}{2} \cancel{+ \tfrac{1}{2}} + - \ln\tfrac{1}{2} \cancel{+ \tfrac{1}{2}} = -2\ln\tfrac{1}{2} - 1$$

$$= 0.3862$$

$$X = \begin{cases} \dfrac{L}{1-L} & L < \frac{1}{2} \\ \dfrac{1-L}{L} & L > \frac{1}{2} \end{cases}$$

$$E(X) =$$

$$P(X<\tfrac{1}{4}) = P\left(\frac{L}{1-L} < \frac{1}{4}\right)P(L<\tfrac{1}{2}) + P\left(\frac{1-L}{L}<\frac{1}{4}\right)P(L>\tfrac{1}{2})$$

$$= 0.5\, P(4L<1-L) + P(4-4L<L)\tfrac{1}{2}$$

$$= 0.5\, P(L<\tfrac{1}{5}) + P(L>\tfrac{4}{5})\tfrac{1}{2}$$

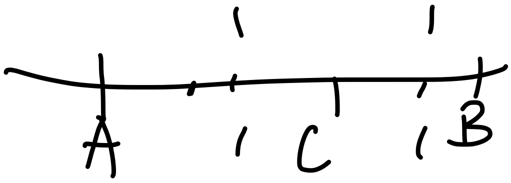
$$= \tfrac{1}{2}(\tfrac{1}{5}) + \tfrac{1}{5}(\tfrac{1}{2}) = \tfrac{1}{5}$$

No need cond. prob.

$$P\left(\frac{L}{1-L}<\frac{1}{4}\right) + P\left(\frac{1-L}{L}<\frac{1}{4}\right).$$

$p(x < \frac{1}{4}) =$

Q4.



$$y \begin{cases} (25-L) & L<25 \\ |50-L| & 25<L<75 \\ 100-L & L>75 \end{cases}$$

$$\int_0^{25} \frac{25-L}{100}\,dL + \int_{25}^{50} \frac{50-L}{100}\,dL +$$

$$\int_{50}^{75} \frac{L-50}{100}\,dL + \int_{75}^{100} \frac{100-L}{100}\,dL$$

$$= \frac{1}{100}\left[25L - \frac{L^2}{2}\right]_0^{25} + \frac{1}{100}\left[50L - \frac{L^2}{2}\right]_{25}^{50} +$$

$$\frac{1}{100}\left[\frac{L^2}{2} - 50L\right]_{50}^{75} + \frac{1}{100}\left[100L - \frac{L^2}{2}\right]_{75}^{100}$$

$$= \quad 12.5$$

Q5. $E(X^6) = \int_0^1 x^6 \, dx =$

a6. a). $P(z > \frac{5-10}{6}) = P(z > -0.83)$

$= 0.7967$

b). $P(\frac{4-10}{6} < z < \frac{16-10}{6}) = P(-1 < z < 1)$

$= 0.1587 \times 2$

$= 0.3174$

c). $P(z < \frac{8-10}{6}) = P(z < -0.33)$

$= 0.6293$

d). $P(z < \frac{20-10}{6}) = P(z < 1.67)$

$= \cancel{R} \ 0.9525$

e). $P(z > \frac{16-10}{6}) = P(z > 1)$

$= 1 - 0.8413 =$

7. $p\left(z > \frac{9-5}{\sigma}\right) = 0.2005$

$P\left(z < \frac{4}{\sigma}\right) = 0.7995$

$\frac{4}{\sigma} = 0.84$

$\sigma = 4.76$

8. $P\left(z > \frac{c-12}{2}\right) = 0.1003$

$= 0.8997$

$\frac{c-12}{2} = 1.28$

$=$

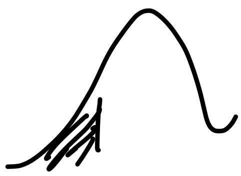
Q9.ii).

$N(50, 5^2)$

$P(48.5 < X < 49.5)$

$= P\left(\frac{48.5-50}{5} < Z < \frac{49.5-50}{5}\right)$

$= P(-0.3 < Z < -0.1)$



$P(Z < -0.1) = 1 - 0.5398$

$= 0.4602$

$P(Z < -0.3) = 1 - 0.6179$

$= 0.3821$

$P(\;) = 0.0781$

(ii) $P(X > 57)$

$= P\left(Z > \dfrac{56.5 - 50}{5}\right)$

$= P(Z > 1.3)$

$= 1 - 0.9032 = 0.0968$

Q10. $f_X(x) = e^{-x}$

$x = e^{y}$

$y = g(x)$.
$y = \log x$

$\frac{d}{dy} g^{-1}(y) = e^{y}$

$f_Y(y) = \begin{cases} f_X(e^{y})\, e^{y} \\ \end{cases}$

$e^{-e^{y}}\, e^{y}$ $\quad e^{y-e^{y}}$