Chapter 1: Problems:

4. John, Jim, Jay, and Jack have formed a band consisting of 4 instruments. If each of the boys can play all 4 instruments, how many different arrangements are possible? What if John and Jim can play all 4 instruments, but Jay and Jack can each play only piano and drums?

6. A well-known nursery rhyme starts as follows:

"As I was going to St. Ives
I met a man with 7 wives.
Each wife had 7 sacks.
Each sack had 7 cats.
Each cat had 7 kittens..."
How many kittens did the traveler meet?

8. How many different letter arrangements can be made from the letters
- **(a)** Fluke?
- **(b)** Propose?
- **(c)** Mississippi?
- **(d)** Arrange?

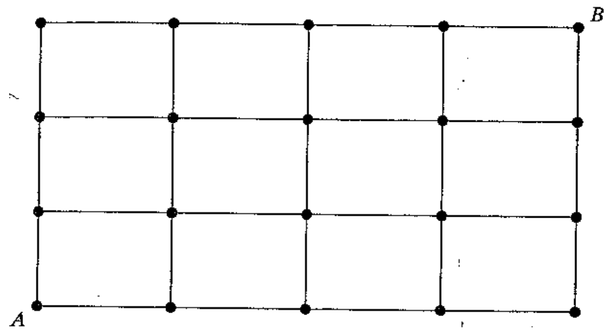
11. In how many ways can 3 novels, 2 mathematics books, and 1 chemistry book be arranged on a bookshelf if
- **(a)** the books can be arranged in any order?
- **(b)** the mathematics books must be together and the novels must be together?
- **(c)** the novels must be together, but the other books can be arranged in any order?

13. Consider a group of 20 people. If everyone shakes hands with everyone else, how many handshakes take place?

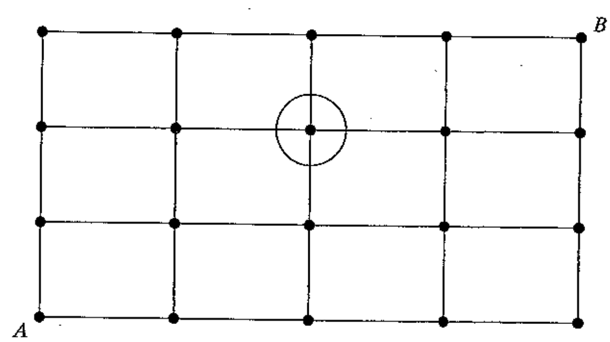
18. A committee of 7, consisting of 2 Republicans, 2 Democrats, and 3 Independents, is to be chosen from a group of 5 Republicans, 6 Democrats, and 4 Independents. How many committees are possible?

19. From a group of 8 women and 6 men, a committee consisting of 3 men and 3 women is to be formed. How many different committees are possible if
- **(a)** 2 of the men refuse to serve together?
- **(b)** 2 of the women refuse to serve together?
- **(c)** 1 man and 1 woman refuse to serve together?

21. Consider the grid of points shown here. Suppose that, starting at the point labeled A, you can go one step up or one step to the right at each move. This procedure is continued until the point labeled B is reached. How many different paths from A to B are possible?
Hint: Note that to reach B from A, you must take 4 steps to the right and 3 steps upward.



22. In Problem 21, how many different paths are there from A to B that go through the point circled in the following lattice?



Chapter 1: Theoretical exercises:

8. Prove that

$$\binom{n+m}{r} = \binom{n}{0}\binom{m}{r} + \binom{n}{1}\binom{m}{r-1} + \cdots + \binom{n}{r}\binom{m}{0}$$

Hint: Consider a group of n men and m women. How many groups of size r are possible?

9. Use Theoretical Exercise 8 to prove that

$$\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$$