3.7. **(a)** An urn contains n white and m black balls. The balls are withdrawn one at a time until only those of the same color are left. Show that, with probability $n/(n + m)$, they are all white.
Hint: Imagine that the experiment continues until all the balls are removed, and consider the last ball withdrawn.

呢題問：有n白波同m黑波，一直抽波到淨翻相同顏色。
問淨翻嘅全部都係白波嘅概率
答案係n/(n+m).因為如果一直抽到無波為止嘅話，只要最後一個係白波，無論前面係乜，淨翻嘅全部都係白波
所以有n個白波可能被擺在最後被抽到，所以n/(n+m)

3.10. Consider a collection of n individuals. Assume that each person's birthday is equally likely to be any of the 365 days of the year and also that the birthdays are independent. Let $A_{i,j}$, $i \neq j$, denote the event that persons i and j have the same birthday. Show that these events are pairwise independent, but not independent. That is, show that $A_{i,j}$ and $A_{r,s}$ are independent, but the $\binom{n}{2}$ events $A_{i,j}, i \neq j$ are not independent.

呢題證明pairwise independence不等於independence

Aij=1/365,兩人生日一樣

Ars=1/365

Aij and Ars = (1/365)(1/365) 四個人pairwisely一樣

但如果係 $\bigcap_{(i,j)} A_{ij}$, 即係全部人生日一樣 $\left(\frac{1}{365}\right)^{n-1}$

$\prod_{(i,j)} A_{ij}$ 只不過是 $\left(\frac{1}{365}\right)^{\binom{n}{2}}$

(b) A pond contains 3 distinct species of fish, which we will call the Red, Blue, and Green fish. There are r Red, b Blue, and g Green fish. Suppose that the fish are removed from the pond in a random order. (That is, each selection is equally likely to be any of the remaining fish.) What is the probability that the Red fish are the first species to become extinct in the pond?
Hint: Write $P\{R\} = P\{RBG\} + P\{RGB\}$, and compute the probabilities on the right by first conditioning on the last species to be removed.

呢條承上題，問嘅係紅色魚最先絕種的概率

呢個概率等於p(Red|Green 最後死)p(green最後死）+p(Red|blue最後死)p(blue最後死)

而p(green最後死）同上題一樣，就係每一個green魚都可能做最後一個，g/(r+g+b)

P(Red最先死|green最後死) 就係red->blue->green, 就係blue尾二死嘅概率，但留翻隻魚比green做最後=b/(r+g+b-1)

3.12. Show that $0 \leq a_i \leq 1, i = 1, 2, \ldots$, then

$$\sum_{i=1}^{\infty} \left[a_i \prod_{j=1}^{i-1} (1 - a_j) \right] + \prod_{i=1}^{\infty} (1 - a_i) = 1$$

Hint: Suppose that an infinite number of coins are to be flipped. Let a_i be the probability that the ith coin lands on heads, and consider when the first head occurs.

呢題證上述identity
方法係好似hint咁，第一個event係p(第一個head出現係location n) $= a_n \prod_{j=1}^{i-1} (1-a_j)$

解釋：因為前面一定要全tail,第An個係head先得

之後將n延伸至無限，咁樣呢個概率就表示head出現嘅概率
而同樣地，identity第二個term就係表示tail出現嘅概率

3.13. The probability of getting a head on a single toss of a coin is p. Suppose that A starts and continues to flip the coin until a tail shows up, at which point B starts flipping. Then B continues to flip until a tail comes up, at which point A takes over, and so on. Let $P_{n,m}$ denote the probability that A accumulates a total of n heads before B accumulates m. Show that

$$P_{n,m} = pP_{n-1,m} + (1 - p)(1 - P_{m,n})$$

***3.14.** Suppose that you are gambling against an infinitely rich adversary and at each stage you either win or lose 1 unit with respective probabilities p and $1 - p$. Show that the probability that you eventually go broke is

$$\begin{array}{ll} 1 & \text{if } p \leq \frac{1}{2} \\ (q/p)^i & \text{if } p > \frac{1}{2} \end{array}$$

where $q = 1 - p$ and where i is your initial fortune.

let Pn be the probabilitiyes that you eventually go broke in n times

假設一開始贏，prob(破產｜一開始贏)=p*P_{n+1}

prob(破產｜一開始輸)=(1-p)*P_{n-1}

so Pn=p*P_{n+1}+(1-p)*P_{n-1}

而推斷係呢種形式嘅公式下，P_{n}要係a嘅n次先得

所以可以寫成 $$p\alpha^2 - \alpha + (1-p) = 0\,.$$

用quadratic solve之再take正負root即是兩個solution

3.18. Let Q_n denote the probability that no run of 3 consecutive heads appears in n tosses of a fair coin. Show that

$$Q_n = \frac{1}{2}Q_{n-1} + \frac{1}{4}Q_{n-2} + \frac{1}{8}Q_{n-3}$$
$$Q_0 = Q_1 = Q_2 = 1$$

Find Q_8.
Hint: Condition on the first tail.

呢題比左hint係condition on first tail, 純粹擺黎提自己呢題應該咁做

3.21. *The Ballot Problem.* In an election, candidate A receives n votes and candidate B receives m votes, where $n > m$. Assuming that all of the $(n + m)!/n!\,m!$ orderings of the votes are equally likely, let $P_{n,m}$ denote the probability that A is always ahead in the counting of the votes.

(a) Compute $P_{2,1}, P_{3,1}, P_{3,2}, P_{4,1}, P_{4,2}, P_{4,3}$.

(b) Find $P_{n,1}, P_{n,2}$.

(c) On the basis of your results in parts (a) and (b), conjecture the value of $P_{n,m}$.

(d) Derive a recursion for $P_{n,m}$ in terms of $P_{n-1,m}$ and $P_{n,m-1}$ by conditioning on who receives the last vote.

b題黎講呢

P_{n,1}姐係講緊B得一票，A always 超過佢嘅概率

咁只要一開始嘅兩票係A嘅，咁B就唔會超得過A

因為總共有n+1票，概率係n/(n+1) * (n-1)/n = (n-1) / (n+1)

P_{n,2}姐係講緊b有兩票，A只要頭兩票係佢嘅，跟住個票容許b至多有一票，接落黎都係A嘅票就可以

姐係頭兩票係A 後兩票至少一張係A

概率係 n/(n+2) 第一票 * (n-1) / (n+1) 第二票 * (1- 2/n * 1/(n-1)) 順著兩票都係b的

3.23. A bag contains a white and b black balls. Balls are chosen from the bag according to the following method:

1. A ball is chosen at random and is discarded.
2. A second ball is then chosen. If its color is different from that of the preceding ball, it is replaced in the bag and the process is repeated from the beginning. If its color is the same, it is discarded and we start from step 2.

In other words, balls are sampled and discarded until a change in color occurs, at which point the last ball is returned to the urn and the process starts anew. Let $P_{a,b}$ denote the probability that the last ball in the bag is white. Prove that

$$P_{a,b} = \frac{1}{2}$$

Hint: Use induction on $k \equiv a + b$.

題目解釋：step 1:抽波然後丟棄
2：再抽波 如果顏色唔同第一個，回到step 1
如果顏色一樣 丟棄然後再抽波

呢題係演繹法：一開始assume P_{a,b}係1/2， 然後假設依家多左無色嘅波

P_{a,b}= p(最後係白｜頭a個係白）=1/ $\binom{a+b}{a}$
分子係1，因為得頭a白最後白呢個情況中間全部黑色
分母就係

***3.24.** A round-robin tournament of n contestants is a tournament in which each of the $\binom{n}{2}$ pairs of contestants play each other exactly once, with the outcome of any play being that one of the contestants wins and the other loses. For a fixed integer $k, k < n$, a question of interest is whether it is possible that the tournament outcome is such that, for every set of k players, there is a player who beat each member of that set. Show that if

$$\binom{n}{k}\left[1-\left(\frac{1}{2}\right)^k\right]^{n-k} < 1$$

then such an outcome is possible.
Hint: Suppose that the results of the games are independent and that each game is equally likely

to be won by either contestant. Number the $\binom{n}{k}$ sets of k contestants, and let B_i denote the event that no contestant beat all of the k players in the ith set. Then use Boole's inequality to bound $P\left(\bigcup_i B_i\right)$.

首先，假設P(A)為在n個人入面的某個人打唔贏指定嘅subset{k個人}入面某啲人，p(A)=一減佢打得贏呢個subset所有人，所以係1-(1/2)^k
P(B_i)係不在subset入面嘅人打唔贏呢個k subset嘅概率，其實就係p(A)應用係剩下n-k人身上，所以係(1-(1/2)^k)^(n-k)
跟住，有樣嘢叫boole's inequality:

$$P\left(\bigcup_i A_i\right) \leq \sum_i P(A_i)$$

$$P\left(\bigcup_i B_i\right) \leq \left(1-\left(\frac{1}{2}\right)^k\right)^{n-k}$$

所以右邊其實就係作為一個bound,去bound住p(U Bi), 咁呢個事件嘅compliment就係有人打得贏k subset所有人
咁只要右手邊嗰嘢細過1，用一減佢就會得正數
只要正數姐係證明有可能

3.3. How can 20 balls, 10 white and 10 black, be put into two urns so as to maximize the probability of drawing a white ball if an urn is selected at random and a ball is drawn at random from it?

呢題唔知點推嘅

一個袋放9白10黑

一個放1白0黑

PROBLEMS

3.1. Two fair dice are rolled. What is the conditional probability that at least one lands on 6 given that the dice land on different numbers?

3.2. If two fair dice are rolled, what is the conditional probability that the first one lands on 6 given that the sum of the dice is i? Compute for all values of i between 2 and 12.

3.3. Use Equation (2.1) to compute, in a hand of bridge, the conditional probability that East has 3 spades given that North and South have a combined total of 8 spades.

3.4. What is the probability that at least one of a pair of fair dice lands on 6, given that the sum of the dice is i, $i = 2, 3, \ldots, 12$?

3.5. An urn contains 6 white and 9 black balls. If 4 balls are to be randomly selected without replacement, what is the probability that the first 2 selected are white and the last 2 black?

3.6. Consider an urn containing 12 balls, of which 8 are white. A sample of size 4 is to be drawn with replacement (without replacement). What is the conditional probability (in each case) that the first and third balls drawn will be white given that the sample drawn contains exactly 3 white balls?

3.7. The king comes from a family of 2 children. What is the probability that the other child is his sister?

3.8. A couple has 2 children. What is the probability that both are girls if the older of the two is a girl?

3.9. Consider 3 urns. Urn A contains 2 white and 4 red balls, urn B contains 8 white and 4 red balls, and urn C contains 1 white and 3 red balls. If 1 ball is selected from each urn, what is the probability that the ball chosen from urn A was white given that exactly 2 white balls were selected?

3.10. Three cards are randomly selected, without replacement, from an ordinary deck of 52 playing cards. Compute the conditional probability that the first card selected is a spade given that the second and third cards are spades.

3.11. Two cards are randomly chosen without replacement from an ordinary deck of 52 cards. Let B be the event that both cards are aces, let A_s be the event that the ace of spades is chosen, and let A be the event that at least one ace is chosen. Find

(a) $P(B|A_s)$

(b) $P(B|A)$

3.12. A recent college graduate is planning to take the first three actuarial examinations in the coming summer. She will take the first actuarial exam in June. If she passes that exam, then she will take the second exam in July, and if she also passes that one, then she will take the third exam in September. If she fails an exam, then she is not allowed to take any others. The probability that she passes the first exam is .9. If she passes the first exam, then the conditional probability that she passes the second one is .8, and if she passes both the first and the second exams, then the conditional probability that she passes the third exam is .7.

(a) What is the probability that she passes all three exams?

(b) Given that she did not pass all three exams, what is the conditional probability that she failed the second exam?

3.13. Suppose that an ordinary deck of 52 cards (which contains 4 aces) is randomly divided into 4 hands of 13 cards each. We are interested in determining p, the probability that each hand has an ace. Let E_i be the event that the ith hand has exactly one ace. Determine $p = P(E_1E_2E_3E_4)$ by using the multiplication rule.

3.14. An urn initially contains 5 white and 7 black balls. Each time a ball is selected, its color is noted and it is replaced in the urn along with 2 other balls of the same color. Compute the probability that

(a) the first 2 balls selected are black and the next 2 are white;

(b) of the first 4 balls selected, exactly 2 are black.

3.15. An ectopic pregnancy is twice as likely to develop when the pregnant woman is a smoker as it is when she is a nonsmoker. If 32 percent of women of childbearing age are smokers, what percentage of women having ectopic pregnancies are smokers?

3.16. Ninety-eight percent of all babies survive delivery. However, 15 percent of all births involve Cesarean (C) sections, and when a C section is performed, the baby survives 96 percent of the time. If a randomly chosen pregnant woman does not have a C section, what is the probability that her baby survives?

3.17. In a certain community, 36 percent of the families own a dog and 22 percent of the families that own a dog also own a cat. In addition, 30 percent of the families own a cat. What is

(a) the probability that a randomly selected family owns both a dog and a cat?

(b) the conditional probability that a randomly selected family owns a dog given that it owns a cat?

3.18. A total of 46 percent of the voters in a certain city classify themselves as Independents, whereas 30 percent classify themselves as Liberals and 24 percent say that they are Conservatives. In a recent local election, 35 percent of the Independents, 62 percent of the Liberals, and 58 percent of the Conservatives voted. A voter is chosen at random.

Given that this person voted in the local election, what is the probability that he or she is

(a) an Independent?
(b) a Liberal?
(c) a Conservative?
(d) What fraction of voters participated in the local election?

3.19. A total of 48 percent of the women and 37 percent of the men that took a certain "quit smoking" class remained nonsmokers for at least one year after completing the class. These people then attended a success party at the end of a year. If 62 percent of the original class was male,

(a) what percentage of those attending the party were women?
(b) what percentage of the original class attended the party?

3.20. Fifty-two percent of the students at a certain college are females. Five percent of the students in this college are majoring in computer science. Two percent of the students are women majoring in computer science. If a student is selected at random, find the conditional probability that

(a) the student is female given that the student is majoring in computer science;
(b) this student is majoring in computer science given that the student is female.

3.21. A total of 500 married working couples were polled about their annual salaries, with the following information resulting:

	Husband	
Wife	Less than \$25,000	More than \$25,000
Less than \$25,000	212	198
More than \$25,000	36	54

For instance, in 36 of the couples, the wife earned more and the husband earned less than \$25,000. If one of the couples is randomly chosen, what is

(a) the probability that the husband earns less than \$25,000?
(b) the conditional probability that the wife earns more than \$25,000 given that the husband earns more than this amount?
(c) the conditional probability that the wife earns more than \$25,000 given that the husband earns less than this amount?

3.22. A red die, a blue die, and a yellow die (all six sided) are rolled. We are interested in the probability that the number appearing on the blue die is less than that appearing on the yellow die, which is less than that appearing on the red die. That is, with B, Y, and R denoting, respectively, the number appearing on the blue, yellow, and red die, we are interested in $P(B < Y < R)$.

(a) What is the probability that no two of the dice land on the same number?
(b) Given that no two of the dice land on the same number, what is the conditional probability that $B < Y < R$?
(c) What is $P(B < Y < R)$?

3.23. Urn I contains 2 white and 4 red balls, whereas urn II contains 1 white and 1 red ball. A ball is randomly chosen from urn I and put into urn II, and a ball is then randomly selected from urn II. What is

(a) the probability that the ball selected from urn II is white?
(b) the conditional probability that the transferred ball was white given that a white ball is selected from urn II?

3.24. Each of 2 balls is painted either black or gold and then placed in an urn. Suppose that each ball is colored black with probability $\frac{1}{2}$ and that these events are independent.

(a) Suppose that you obtain information that the gold paint has been used (and thus at least one of the balls is painted gold). Compute the conditional probability that both balls are painted gold.
(b) Suppose now that the urn tips over and 1 ball falls out. It is painted gold. What is the probability that both balls are gold in this case? Explain.

3.25. The following method was proposed to estimate the number of people over the age of 50 who reside in a town of known population 100,000: "As you walk along the streets, keep a running count of the percentage of people you encounter who are over 50. Do this for a few days; then multiply the percentage you obtain by 100,000 to obtain the estimate." Comment on this method.

Hint: Let p denote the proportion of people in the town who are over 50. Furthermore, let α_1 denote the proportion of time that a person under the age of 50 spends in the streets, and let α_2 be the corresponding value for those over 50. What quantity does the method suggested estimate? When is the estimate approximately equal to p?

3.26. Suppose that 5 percent of men and .25 percent of women are color blind. A color-blind person is chosen at random. What is the probability of this person being male? Assume that there are an equal number of males and females. What if the population consisted of twice as many males as females?

3.27. All the workers at a certain company drive to work and park in the company's lot. The company is interested in estimating the average number of workers in a car. Which of the following methods will enable the company to estimate this quantity? Explain your answer.

1. Randomly choose n workers, find out how many were in the cars in which they were driven, and take the average of the n values.
2. Randomly choose n cars in the lot, find out how many were driven in those cars, and take the average of the n values.

3.28. Suppose that an ordinary deck of 52 cards is shuffled and the cards are then turned over one at a time until the first ace appears. Given that the first ace is the 20th card to appear, what is the conditional probability that the card following it is the
(a) ace of spades?
(b) two of clubs?

3.29. There are 15 tennis balls in a box, of which 9 have not previously been used. Three of the balls are randomly chosen, played with, and then returned to the box. Later, another 3 balls are randomly chosen from the box. Find the probability that none of these balls has ever been used.

3.30. Consider two boxes, one containing 1 black and 1 white marble, the other 2 black and 1 white marble. A box is selected at random, and a marble is drawn from it at random. What is the probability that the marble is black? What is the probability that the first box was the one selected given that the marble is white?

3.31. Ms. Aquina has just had a biopsy on a possibly cancerous tumor. Not wanting to spoil a weekend family event, she does not want to hear any bad news in the next few days. But if she tells the doctor to call only if the news is good, then if the doctor does not call, Ms. Aquina can conclude that the news is bad. So, being a student of probability, Ms. Aquina instructs the doctor to flip a coin. If it comes up heads, the doctor is to call if the news is good and not call if the news is bad. If the coin comes up tails, the doctor is not to call. In this way, even if the doctor doesn't call, the news is not necessarily bad. Let α be the probability that the tumor is cancerous; let β be the conditional probability that the tumor is cancerous given that the doctor does not call.
(a) Which should be larger, α or β?
(b) Find β in terms of α, and prove your answer in part (a).

3.32. A family has j children with probability p_j, where $p_1 = .1, p_2 = .25, p_3 = .35, p_4 = .3$. A child from this family is randomly chosen. Given that this child is the eldest child in the family, find the conditional probability that the family has
(a) only 1 child;
(b) 4 children.
Redo (a) and (b) when the randomly selected child is the youngest child of the family.

3.33. On rainy days, Joe is late to work with probability .3; on nonrainy days, he is late with probability .1. With probability .7, it will rain tomorrow.
(a) Find the probability that Joe is early tomorrow.
(b) Given that Joe was early, what is the conditional probability that it rained?

3.34. In Example 3f, suppose that the new evidence is subject to different possible interpretations and in fact shows only that it is 90 percent likely that the criminal possesses the characteristic in question. In this case, how likely would it be that the suspect is guilty (assuming, as before, that he has the characteristic)?

3.35. With probability .6, the present was hidden by mom; with probability .4, it was hidden by dad. When mom hides the present, she hides it upstairs 70 percent of the time and downstairs 30 percent of the time. Dad is equally likely to hide it upstairs or downstairs.
(a) What is the probability that the present is upstairs?
(b) Given that it is downstairs, what is the probability it was hidden by dad?

3.36. Stores A, B, and C have 50, 75, and 100 employees, respectively, and 50, 60, and 70 percent of them respectively are women. Resignations are equally likely among all employees, regardless of sex. One woman employee resigns. What is the probability that she works in store C?

3.37. **(a)** A gambler has a fair coin and a two-headed coin in his pocket. He selects one of the coins at random; when he flips it, it shows heads. What is the probability that it is the fair coin?
(b) Suppose that he flips the same coin a second time and, again, it shows heads. Now what is the probability that it is the fair coin?
(c) Suppose that he flips the same coin a third time and it shows tails. Now what is the probability that it is the fair coin?

3.38. Urn A has 5 white and 7 black balls. Urn B has 3 white and 12 black balls. We flip a fair coin. If the outcome is heads, then a ball from urn A is selected, whereas if the outcome is tails, then a ball from urn B is selected. Suppose that a white ball is selected. What is the probability that the coin landed tails?

3.39. In Example 3a, what is the probability that someone has an accident in the second year given that he or she had no accidents in the first year?

3.40. Consider a sample of size 3 drawn in the following manner: We start with an urn containing 5 white and 7 red balls. At each stage, a ball is drawn and its color is noted. The ball is then returned to the urn, along with an additional ball of the same color. Find the probability that the sample will contain exactly

(a) 0 white balls;
(b) 1 white ball;
(c) 3 white balls;
(d) 2 white balls.

3.41. A deck of cards is shuffled and then divided into two halves of 26 cards each. A card is drawn from one of the halves; it turns out to be an ace. The ace is then placed in the second half-deck. The half is then shuffled, and a card is drawn from it. Compute the probability that this drawn card is an ace. *Hint*: Condition on whether or not the interchanged card is selected.

3.42. Three cooks, A, B, and C, bake a special kind of cake, and with respective probabilities .02, .03, and .05, it fails to rise. In the restaurant where they work, A bakes 50 percent of these cakes, B 30 percent, and C 20 percent. What proportion of "failures" is caused by A?

3.43. There are 3 coins in a box. One is a two-headed coin, another is a fair coin, and the third is a biased coin that comes up heads 75 percent of the time. When one of the 3 coins is selected at random and flipped, it shows heads. What is the probability that it was the two-headed coin?

3.44. Three prisoners are informed by their jailer that one of them has been chosen at random to be executed and the other two are to be freed. Prisoner A asks the jailer to tell him privately which of his fellow prisoners will be set free, claiming that there would be no harm in divulging this information because he already knows that at least one of the two will go free. The jailer refuses to answer the question, pointing out that if A knew which of his fellow prisoners were to be set free, then his own probability of being executed would rise from $\frac{1}{3}$ to $\frac{1}{2}$ because he would then be one of two prisoners. What do you think of the jailer's reasoning?

3.45. Suppose we have 10 coins such that if the ith coin is flipped, heads will appear with probability $i/10, i = 1, 2, \ldots, 10$. When one of the coins is randomly selected and flipped, it shows heads. What is the conditional probability that it was the fifth coin?

3.46. In any given year, a male automobile policyholder will make a claim with probability p_m and a female policyholder will make a claim with probability p_f, where $p_f \neq p_m$. The fraction of the policyholders that are male is $\alpha, 0 < \alpha < 1$. A policyholder is randomly chosen. If A_i denotes the event that this policyholder will make a claim in year i, show that

$$P(A_2|A_1) > P(A_1)$$

Give an intuitive explanation of why the preceding inequality is true.

3.47. An urn contains 5 white and 10 black balls. A fair die is rolled and that number of balls is randomly chosen from the urn. What is the probability that all of the balls selected are white? What is the conditional probability that the die landed on 3 if all the balls selected are white?

3.48. Each of 2 cabinets identical in appearance has 2 drawers. Cabinet A contains a silver coin in each drawer, and cabinet B contains a silver coin in one of its drawers and a gold coin in the other. A cabinet is randomly selected, one of its drawers is opened, and a silver coin is found. What is the probability that there is a silver coin in the other drawer?

3.49. Prostate cancer is the most common type of cancer found in males. As an indicator of whether a male has prostate cancer, doctors often perform a test that measures the level of the prostate-specific antigen (PSA) that is produced only by the prostate gland. Although PSA levels are indicative of cancer, the test is notoriously unreliable. Indeed, the probability that a noncancerous man will have an elevated PSA level is approximately .135, increasing to approximately .268 if the man does have cancer. If, on the basis of other factors, a physician is 70 percent certain that a male has prostate cancer, what is the conditional probability that he has the cancer given that

(a) the test indicated an elevated PSA level?
(b) the test did not indicate an elevated PSA level?

Repeat the preceding calculation, this time assuming that the physician initially believes that there is a 30 percent chance that the man has prostate cancer.

3.50. Suppose that an insurance company classifies people into one of three classes: good risks, average risks, and bad risks. The company's records indicate that the probabilities that good-, average-, and bad-risk persons will be involved in an accident over a 1-year span are, respectively, .05, .15, and .30. If 20 percent of the population is a good risk, 50 percent an average risk, and 30 percent a bad risk, what proportion of people have accidents in a fixed year? If policyholder A had no accidents in 1997, what is the probability that he or she is a good or average risk?

3.51. A worker has asked her supervisor for a letter of recommendation for a new job. She estimates that there is an 80 percent chance that she will get the job if she receives a strong recommendation, a 40 percent chance if she receives a moderately good recommendation, and a 10 percent chance if she receives a weak recommendation. She further estimates that the probabilities that the recommendation will be strong, moderate, and weak are .7, .2, and .1, respectively.

(a) How certain is she that she will receive the new job offer?

(b) Given that she does receive the offer, how likely should she feel that she received a strong recommendation? a moderate recommendation? a weak recommendation?

(c) Given that she does not receive the job offer, how likely should she feel that she received a strong recommendation? a moderate recommendation? a weak recommendation?

3.52. A high school student is anxiously waiting to receive mail telling her whether she has been accepted to a certain college. She estimates that the conditional probabilities of receiving notification on each day of next week, given that she is accepted and that she is rejected, are as follows:

Day	$P(\text{mail}\|\text{accepted})$	$P(\text{mail}\|\text{rejected})$
Monday	.15	.05
Tuesday	.20	.10
Wednesday	.25	.10
Thursday	.15	.15
Friday	.10	.20

She estimates that her probability of being accepted is .6.

(a) What is the probability that she receives mail on Monday?

(b) What is the conditional probability that she received mail on Tuesday given that she does not receive mail on Monday?

(c) If there is no mail through Wednesday, what is the conditional probability that she will be accepted?

(d) What is the conditional probability that she will be accepted if mail comes on Thursday?

(e) What is the conditional probability that she will be accepted if no mail arrives that week?

3.53. A parallel system functions whenever at least one of its components works. Consider a parallel system of n components, and suppose that each component works independently with probability $\frac{1}{2}$. Find the conditional probability that component 1 works given that the system is functioning.

3.54. If you had to construct a mathematical model for events E and F, as described in parts (a) through (e), would you assume that they were independent events? Explain your reasoning.

(a) E is the event that a businesswoman has blue eyes, and F is the event that her secretary has blue eyes.

(b) E is the event that a professor owns a car, and F is the event that he is listed in the telephone book.

(c) E is the event that a man is under 6 feet tall, and F is the event that he weighs over 200 pounds.

(d) E is the event that a woman lives in the United States, and F is the event that she lives in the Western Hemisphere.

(e) E is the event that it will rain tomorrow, and F is the event that it will rain the day after tomorrow.

3.55. In a class, there are 4 freshman boys, 6 freshman girls, and 6 sophomore boys. How many sophomore girls must be present if sex and class are to be independent when a student is selected at random?

3.56. Suppose that you continually collect coupons and that there are m different types. Suppose also that each time a new coupon is obtained, it is a type i coupon with probability $p_i, i = 1, \ldots, m$. Suppose that you have just collected your nth coupon. What is the probability that it is a new type?
Hint: Condition on the type of this coupon.

3.57. A simplified model for the movement of the price of a stock supposes that on each day the stock's price either moves up 1 unit with probability p or moves down 1 unit with probability $1 - p$. The changes on different days are assumed to be independent.

(a) What is the probability that after 2 days the stock will be at its original price?

(b) What is the probability that after 3 days the stock's price will have increased by 1 unit?

(c) Given that after 3 days the stock's price has increased by 1 unit, what is the probability that it went up on the first day?

3.58. Suppose that we want to generate the outcome of the flip of a fair coin, but that all we have at our disposal is a biased coin which lands on heads with some unknown probability p that need not be equal to $\frac{1}{2}$. Consider the following procedure for accomplishing our task:

1. Flip the coin.
2. Flip the coin again.
3. If both flips land on heads or both land on tails, return to step 1.
4. Let the result of the last flip be the result of the experiment.

(a) Show that the result is equally likely to be either heads or tails.

(b) Could we use a simpler procedure that continues to flip the coin until the last two flips are different and then lets the result be the outcome of the final flip?

3.59. Independent flips of a coin that lands on heads with probability p are made. What is the probability that the first four outcomes are

(a) H, H, H, H?

(b) T, H, H, H?

(c) What is the probability that the pattern T, H, H, H occurs before the pattern H, H, H, H?

Hint for part (c): How can the pattern H, H, H, H occur first?

3.60. The color of a person's eyes is determined by a single pair of genes. If they are both blue-eyed genes, then the person will have blue eyes; if they are both brown-eyed genes, then the person will have brown eyes; and if one of them is a blue-eyed gene and the other a brown-eyed gene, then the person will have brown eyes. (Because of the latter fact, we say that the brown-eyed gene is *dominant* over the blue-eyed one.) A newborn child independently receives one eye gene from each of its parents, and the gene it receives from a parent is equally likely to be either of the two eye genes of that parent. Suppose that Smith and both of his parents have brown eyes, but Smith's sister has blue eyes.

(a) What is the probability that Smith possesses a blue-eyed gene?

(b) Suppose that Smith's wife has blue eyes. What is the probability that their first child will have blue eyes?

(c) If their first child has brown eyes, what is the probability that their next child will also have brown eyes?

3.61. Genes relating to albinism are denoted by A and a. Only those people who receive the a gene from both parents will be albino. Persons having the gene pair A, a are normal in appearance and, because they can pass on the trait to their offspring, are called carriers. Suppose that a normal couple has two children, exactly one of whom is an albino. Suppose that the nonalbino child mates with a person who is known to be a carrier for albinism.

(a) What is the probability that their first offspring is an albino?

(b) What is the conditional probability that their second offspring is an albino given that their firstborn is not?

3.62. Barbara and Dianne go target shooting. Suppose that each of Barbara's shots hits a wooden duck target with probability p_1, while each shot of Dianne's hits it with probability p_2. Suppose that they shoot simultaneously at the same target. If the wooden duck is knocked over (indicating that it was hit), what is the probability that

(a) both shots hit the duck?

(b) Barbara's shot hit the duck?

What independence assumptions have you made?

3.63. A and B are involved in a duel. The rules of the duel are that they are to pick up their guns and shoot at each other simultaneously. If one or both are hit, then the duel is over. If both shots miss, then they repeat the process. Suppose that the results of the shots are independent and that each shot of A will hit B with probability p_A, and each shot of B will hit A with probability p_B. What is

(a) the probability that A is not hit?

(b) the probability that both duelists are hit?

(c) the probability that the duel ends after the nth round of shots?

(d) the conditional probability that the duel ends after the nth round of shots given that A is not hit?

(e) the conditional probability that the duel ends after the nth round of shots given that both duelists are hit?

3.64. A true–false question is to be posed to a husband-and-wife team on a quiz show. Both the husband and the wife will independently give the correct answer with probability p. Which of the following is a better strategy for the couple?

(a) Choose one of them and let that person answer the question.

(b) Have them both consider the question, and then either give the common answer if they agree or, if they disagree, flip a coin to determine which answer to give.

3.65. In Problem 3.5, if $p = .6$ and the couple uses the strategy in part (b), what is the conditional probability that the couple gives the correct answer given that the husband and wife (a) agree? (b) disagree?

3.66. The probability of the closing of the ith relay in the circuits shown in Figure 3.4 is given by $p_i, i = 1, 2, 3, 4, 5$. If all relays function independently, what is the probability that a current flows between A and B for the respective circuits?

Hint for (b): Condition on whether relay 3 closes.

3.67. An engineering system consisting of n components is said to be a k-out-of-n system ($k \leq n$) if the system functions if and only if at least k of the n components function. Suppose that all components function independently of each other.

(a) If the ith component functions with probability $P_i, i = 1, 2, 3, 4$, compute the probability that a 2-out-of-4 system functions.

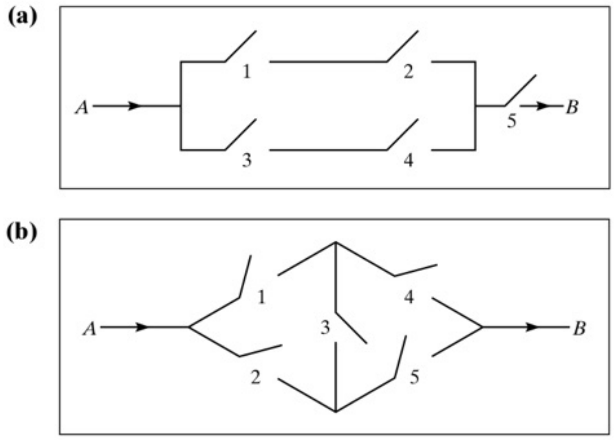


FIGURE 3.4: Circuits for Problem 3.66

(b) Repeat part (a) for a 3-out-of-5 system.

(c) Repeat for a k-out-of-n system when all the P_i equal p (that is, $P_i = p, i = 1, 2, \ldots, n$).

3.68. In Problem 3.65a, find the conditional probability that relays 1 and 2 are both closed given that a current flows from A to B.

3.69. A certain organism possesses a pair of each of 5 different genes (which we will designate by the first 5 letters of the English alphabet). Each gene appears in 2 forms (which we designate by lowercase and capital letters). The capital letter will be assumed to be the dominant gene, in the sense that if an organism possesses the gene pair xX, then it will outwardly have the appearance of the X gene. For instance, if X stands for brown eyes and x for blue eyes, then an individual having either gene pair XX or xX will have brown eyes, whereas one having gene pair xx will have blue eyes. The characteristic appearance of an organism is called its phenotype, whereas its genetic constitution is called its genotype. (Thus, 2 organisms with respective genotypes aA, bB, cc, dD, ee and AA, BB, cc, DD, ee would have different genotypes but the same phenotype.) In a mating between 2 organisms, each one contributes, at random, one of its gene pairs of each type. The 5 contributions of an organism (one of each of the 5 types) are assumed to be independent and are also independent of the contributions of the organism's mate. In a mating between organisms having genotypes aA, bB, cC, dD, eE and aa, bB, cc, Dd, ee what is the probability that the progeny will (i) phenotypically and (ii) genotypically resemble

(a) the first parent?

(b) the second parent?

(c) either parent?

(d) neither parent?

3.70. There is a 50–50 chance that the queen carries the gene for hemophilia. If she is a carrier, then each prince has a 50–50 chance of having hemophilia. If the queen has had three princes without the disease, what is the probability that the queen is a carrier? If there is a fourth prince, what is the probability that he will have hemophilia?

3.71. On the morning of September 30, 1982, the won–lost records of the three leading baseball teams in the Western Division of the National League were as follows:

Team	Won	Lost
Atlanta Braves	87	72
San Francisco Giants	86	73
Los Angeles Dodgers	86	73

Each team had 3 games remaining. All 3 of the Giants' games were with the Dodgers, and the 3 remaining games of the Braves were against the San Diego Padres. Suppose that the outcomes of all remaining games are independent and each game is equally likely to be won by either participant. For each team, what is the probability that it will win the division title? If two teams tie for first place, they have a playoff game, which each team has an equal chance of winning.

3.72. A town council of 7 members contains a steering committee of size 3. New ideas for legislation go first to the steering committee and then on to the council as a whole if at least 2 of the 3 committee members approve the legislation. Once at the full council, the legislation requires a majority vote (of at least 4) to pass. Consider a new piece of legislation, and suppose that each town council member will approve it, independently, with probability p. What is the probability that a given steering committee member's vote is decisive in the sense that if that person's vote were reversed, then the final fate of the legislation would be reversed? What is the corresponding probability for a given council member not on the steering committee?

3.73. Suppose that each child born to a couple is equally likely to be a boy or a girl, independently of the sex distribution of the other children in the family. For a couple having 5 children, compute the probabilities of the following events:
(a) All children are of the same sex.
(b) The 3 eldest are boys and the others girls.
(c) Exactly 3 are boys.
(d) The 2 oldest are girls.
(e) There is at least 1 girl.

3.74. A and B alternate rolling a pair of dice, stopping either when A rolls the sum 9 or when B rolls the sum 6. Assuming that A rolls first, find the probability that the final roll is made by A.

3.75. In a certain village, it is traditional for the eldest son (or the older son in a two-son family) and his wife to be responsible for taking care of his parents as they age. In recent years, however, the women of this village, not wanting that responsibility, have not looked favorably upon marrying an eldest son.
(a) If every family in the village has two children, what proportion of all sons are older sons?
(b) If every family in the village has three children, what proportion of all sons are eldest sons?

Assume that each child is, independently, equally likely to be either a boy or a girl.

3.76. Suppose that E and F are mutually exclusive events of an experiment. Show that if independent trials of this experiment are performed, then E will occur before F with probability $P(E)/[P(E) + P(F)]$.

3.77. Consider an unending sequence of independent trials, where each trial is equally likely to result in any of the outcomes 1, 2, or 3. Given that outcome 3 is the last of the three outcomes to occur, find the conditional probability that
(a) the first trial results in outcome 1;
(b) the first two trials both result in outcome 1.

3.78. A and B play a series of games. Each game is independently won by A with probability p and by B with probability $1 - p$. They stop when the total number of wins of one of the players is two greater than that of the other player. The player with the greater number of total wins is declared the winner of the series.
(a) Find the probability that a total of 4 games are played.
(b) Find the probability that A is the winner of the series.

3.79. In successive rolls of a pair of fair dice, what is the probability of getting 2 sevens before 6 even numbers?

3.80. In a certain contest, the players are of equal skill and the probability is $\frac{1}{2}$ that a specified one of the two contestants will be the victor. In a group of 2^n players, the players are paired off against each other at random. The 2^{n-1} winners are again paired off randomly, and so on, until a single winner remains. Consider two specified contestants, A and B, and define the events $A_i, i \leq n, E$ by

A_i: A plays in exactly i contests:
E: A and B never play each other.

(a) Find $P(A_i), i = 1, \ldots, n$.
(b) Find $P(E)$.
(c) Let $P_n = P(E)$. Show that

$$P_n = \frac{1}{2^n - 1} + \frac{2^n - 2}{2^n - 1}\left(\frac{1}{2}\right)^2 P_{n-1}$$

and use this formula to check the answer you obtained in part (b).
Hint: Find $P(E)$ by conditioning on which of the events $A_i, i = 1, \ldots, n$ occur. In simplifying your answer, use the algebraic identity

$$\sum_{i=1}^{n-1} i x^{i-1} = \frac{1 - n x^{n-1} + (n - 1)x^n}{(1 - x)^2}$$

For another approach to solving this problem, note that there are a total of $2^n - 1$ games played.
(d) Explain why $2^n - 1$ games are played.
Number these games, and let B_i denote the event that A and B play each other in game $i, i = 1, \ldots, 2^n - 1$.
(e) What is $P(B_i)$?
(f) Use part (e) to find $P(E)$.

3.81. An investor owns shares in a stock whose present value is 25. She has decided that she must sell her stock if it goes either down to 10 or up to 40. If each change of price is either up 1 point with probability .55 or down 1 point with probability .45, and the successive changes are independent, what is the probability that the investor retires a winner?

3.82. A and B flip coins. A starts and continues flipping until a tail occurs, at which point B starts flipping and continues until there is a tail. Then A takes over, and so on. Let P_1 be the probability of the coin's landing on heads when A flips and P_2 when B flips. The winner of the game is the first one to get

(a) 2 heads in a row;
(b) a total of 2 heads;
(c) 3 heads in a row;
(d) a total of 3 heads.

In each case, find the probability that A wins.

3.83. Die A has 4 red and 2 white faces, whereas die B has 2 red and 4 white faces. A fair coin is flipped once. If it lands on heads, the game continues with die A; if it lands on tails, then die B is to be used.

(a) Show that the probability of red at any throw is $\frac{1}{2}$.
(b) If the first two throws result in red, what is the probability of red at the third throw?
(c) If red turns up at the first two throws, what is the probability that it is die A that is being used?

3.84. An urn contains 12 balls, of which 4 are white. Three players—A, B, and C—successively draw from the urn, A first, then B, then C, then A, and so on. The winner is the first one to draw a white ball. Find the probability of winning for each player if

(a) each ball is replaced after it is drawn;
(b) the balls that are withdrawn are not replaced.

3.85. Repeat Problem 3.84 when each of the 3 players selects from his own urn. That is, suppose that there are 3 different urns of 12 balls with 4 white balls in each urn.

3.86. Let $S = \{1, 2, \ldots, n\}$ and suppose that A and B are, independently, equally likely to be any of the 2^n subsets (including the null set and S itself) of S.

(a) Show that

$$P\{A \subset B\} = \left(\frac{3}{4}\right)^n$$

Hint: Let $N(B)$ denote the number of elements in B. Use

$$P\{A \subset B\} = \sum_{i=0}^{n} P\{A \subset B | N(B) = i\} P\{N(B) = i\}$$

Show that $P\{AB = \emptyset\} = \left(\frac{3}{4}\right)^n$.

3.87. In Example 5e, what is the conditional probability that the ith coin was selected given that the first n trials all result in heads?

3.88. In Laplace's rule of succession (Example 5e), are the outcomes of the successive flips independent? Explain.

3.89. A person tried by a 3-judge panel is declared guilty if at least 2 judges cast votes of guilty. Suppose that when the defendant is in fact guilty, each judge will independently vote guilty with probability .7, whereas when the defendant is in fact innocent, this probability drops to .2. If 70 percent of defendants are guilty, compute the conditional probability that judge number 3 votes guilty given that

(a) judges 1 and 2 vote guilty;
(b) judges 1 and 2 cast 1 guilty and 1 not guilty vote;
(c) judges 1 and 2 both cast not guilty votes.

Let $E_i, i = 1, 2, 3$ denote the event that judge i casts a guilty vote. Are these events independent. Are they conditionally independent? Explain.

3.90. Suppose that n independent trials, each of which results in any of the outcomes 0, 1, or 2, with respective probabilities p_0, p_1, and p_2, $\sum_{i=0}^{2} p_i = 1$, are performed. Find the probability that outcomes 1 and 2 both occur at least once.

THEORETICAL EXERCISES

3.1. Show that if $P(A) > 0$, then

$$P(AB|A) \geq P(AB|A \cup B)$$

3.2. Let $A \subset B$. Express the following probabilities as simply as possible:

$$P(A|B), \quad P(A|B^c), \quad P(B|A), \quad P(B|A^c)$$

3.3. Consider a school community of m families, with n_i of them having i children, $i = 1, \ldots, k$, $\sum_{i=1}^{k} n_i = m$. Consider the following two methods for choosing a child:

1. Choose one of the m families at random and then randomly choose a child from that family.
2. Choose one of the $\sum_{i=1}^{k} in_i$ children at random.

Show that method 1 is more likely than method 2 to result in the choice of a firstborn child.

Hint: In solving this problem, you will need to show that

$$\sum_{i=1}^{k} in_i \sum_{j=1}^{k} \frac{n_j}{j} \geq \sum_{i=1}^{k} n_i \sum_{j=1}^{k} n_j$$

To do so, multiply the sums and show that, for all pairs i, j, the coefficient of the term $n_i n_j$ is greater in the expression on the left than in the one on the right.

3.4. A ball is in any one of n boxes and is in the ith box with probability P_i. If the ball is in box i, a search of that box will uncover it with probability α_i. Show that the conditional probability that the ball is in box j, given that a search of box i did not uncover it, is

$$\frac{P_j}{1 - \alpha_i P_i} \quad \text{if } j \neq i$$
$$\frac{(1 - \alpha_i)P_i}{1 - \alpha_i P_i} \quad \text{if } j = i$$

3.5. An event F is said to carry negative information about an event E, and we write $F \searrow E$, if

$$P(E|F) \leq P(E)$$

Prove or give counterexamples to the following assertions:

(a) If $F \searrow E$, then $E \searrow F$.
(b) If $F \searrow E$ and $E \searrow G$, then $F \searrow G$.
(c) If $F \searrow E$ and $G \searrow E$, then $FG \searrow E$.

Repeat parts (a), (b), and (c) when $\searrow$ is replaced by $\nearrow$, where we say that F carries positive information about E, written $F \nearrow E$, when $P(E|F) \geq P(E)$.

3.6. Prove that if $E_1, E_2, \ldots, E_n$ are independent events, then

$$P(E_1 \cup E_2 \cup \cdots \cup E_n) = 1 - \prod_{i=1}^{n}[1 - P(E_i)]$$

3.7. **(a)** An urn contains n white and m black balls. The balls are withdrawn one at a time until only those of the same color are left. Show that, with probability $n/(n + m)$, they are all white.
Hint: Imagine that the experiment continues until all the balls are removed, and consider the last ball withdrawn.

(b) A pond contains 3 distinct species of fish, which we will call the Red, Blue, and Green fish. There are r Red, b Blue, and g Green fish. Suppose that the fish are removed from the pond in a random order. (That is, each selection is equally likely to be any of the remaining fish.) What is the probability that the Red fish are the first species to become extinct in the pond?
Hint: Write $P\{R\} = P\{RBG\} + P\{RGB\}$, and compute the probabilities on the right by first conditioning on the last species to be removed.

3.8. Let A, B, and C be events relating to the experiment of rolling a pair of dice.

(a) If

$$P(A|C) > P(B|C) \quad \text{and} \quad P(A|C^c) > P(B|C^c)$$

either prove that $P(A) > P(B)$ or give a counterexample by defining events A, B, and C for which that relationship is not true.

(b) If

$$P(A|C) > P(A|C^c) \quad \text{and} \quad P(B|C) > P(B|C^c)$$

either prove that $P(AB|C) > P(AB|C^c)$ or give a counterexample by defining events A, B, and C for which that relationship is not true.

Hint: Let C be the event that the sum of a pair of dice is 10; let A be the event that the first die lands on 6; let B be the event that the second die lands on 6.

3.9. Consider two independent tosses of a fair coin. Let A be the event that the first toss results in heads, let B be the event that the second toss results in heads, and let C be the event that in both tosses the coin lands on the same side. Show that the events A, B, and C are pairwise independent—that is, A and B are independent, A and C are independent, and B and C are independent—but not independent.

3.10. Consider a collection of n individuals. Assume that each person's birthday is equally likely to be any of the 365 days of the year and also that the birthdays are independent. Let $A_{i,j}$, $i \neq j$, denote the event that persons i and j have the same birthday. Show that these events are pairwise independent, but not independent. That is, show that $A_{i,j}$ and $A_{r,s}$ are independent, but the $\binom{n}{2}$ events $A_{i,j}, i \neq j$ are not independent.

3.11. In each of n independent tosses of a coin, the coin lands on heads with probability p. How large need n be so that the probability of obtaining at least one head is at least $\frac{1}{2}$?

3.12. Show that $0 \leq a_i \leq 1, i = 1, 2, \ldots$, then

$$\sum_{i=1}^{\infty}\left[a_i \prod_{j=1}^{i-1}(1 - a_j)\right] + \prod_{i=1}^{\infty}(1 - a_i) = 1$$

Hint: Suppose that an infinite number of coins are to be flipped. Let a_i be the probability that the ith coin lands on heads, and consider when the first head occurs.

3.13. The probability of getting a head on a single toss of a coin is p. Suppose that A starts and continues to flip the coin until a tail shows up, at which point

B starts flipping. Then B continues to flip until a tail comes up, at which point A takes over, and so on. Let $P_{n,m}$ denote the probability that A accumulates a total of n heads before B accumulates m. Show that

$$P_{n,m} = pP_{n-1,m} + (1 - p)(1 - P_{m,n})$$

***3.14.** Suppose that you are gambling against an infinitely rich adversary and at each stage you either win or lose 1 unit with respective probabilities p and $1 - p$. Show that the probability that you eventually go broke is

$$\begin{array}{ll} 1 & \text{if } p \le \frac{1}{2} \\ (q/p)^i & \text{if } p > \frac{1}{2} \end{array}$$

where $q = 1 - p$ and where i is your initial fortune.

3.15. Independent trials that result in a success with probability p are successively performed until a total of r successes is obtained. Show that the probability that exactly n trials are required is

$$\binom{n-1}{r-1} p^r (1-p)^{n-r}$$

Use this result to solve the problem of the points (Example 4j).
Hint: In order for it to take n trials to obtain r successes, how many successes must occur in the first $n - 1$ trials?

3.16. Independent trials that result in a success with probability p and a failure with probability $1 - p$ are called *Bernoulli trials*. Let P_n denote the probability that n Bernoulli trials result in an even number of successes (0 being considered an even number). Show that

$$P_n = p(1 - P_{n-1}) + (1 - p)P_{n-1} \quad n \ge 1$$

and use this formula to prove (by induction) that

$$P_n = \frac{1 + (1 - 2p)^n}{2}$$

3.17. Suppose that n independent trials are performed, with trial i being a success with probability $1/(2i + 1)$. Let P_n denote the probability that the total number of successes that result is an odd number.

(a) Find P_n for $n = 1, 2, 3, 4, 5$.
(b) Conjecture a general formula for P_n.
(c) Derive a formula for P_n in terms of P_{n-1}.
(d) Verify that your conjecture in part (b) satisfies the recursive formula in part (d). Because the recursive formula has a unique solution, this then proves that your conjecture is correct.

3.18. Let Q_n denote the probability that no run of 3 consecutive heads appears in n tosses of a fair coin. Show that

$$Q_n = \frac{1}{2}Q_{n-1} + \frac{1}{4}Q_{n-2} + \frac{1}{8}Q_{n-3}$$
$$Q_0 = Q_1 = Q_2 = 1$$

Find Q_8.
Hint: Condition on the first tail.

3.19. Consider the gambler's ruin problem, with the exception that A and B agree to play no more than n games. Let $P_{n,i}$ denote the probability that A winds up with all the money when A starts with i and B starts with $N - i$. Derive an equation for $P_{n,i}$ in terms of $P_{n-1,\,i+1}$ and $P_{n-1,\,i-1}$, and compute $P_{7,3}$, $N = 5$.

3.20. Consider two urns, each containing both white and black balls. The probabilities of drawing white balls from the first and second urns are, respectively, p and p'. Balls are sequentially selected with replacement as follows: With probability α, a ball is initially chosen from the first urn, and with probability $1 - \alpha$, it is chosen from the second urn. The subsequent selections are then made according to the rule that whenever a white ball is drawn (and replaced), the next ball is drawn from the same urn, but when a black ball is drawn, the next ball is taken from the other urn. Let α_n denote the probability that the nth ball is chosen from the first urn. Show that

$$\alpha_{n+1} = \alpha_n(p + p' - 1) + 1 - p' \quad n \ge 1$$

and use this formula to prove that

$$\alpha_n = \frac{1 - p'}{2 - p - p'} + \left(\alpha - \frac{1 - p'}{2 - p - p'}\right) \times (p + p' - 1)^{n-1}$$

Let P_n denote the probability that the nth ball selected is white. Find P_n. Also, compute $\lim_{n\to\infty} \alpha_n$ and $\lim_{n\to\infty} P_n$.

3.21. *The Ballot Problem.* In an election, candidate A receives n votes and candidate B receives m votes, where $n > m$. Assuming that all of the $(n + m)!/n!\,m!$ orderings of the votes are equally likely, let $P_{n,m}$ denote the probability that A is always ahead in the counting of the votes.

(a) Compute $P_{2,1}, P_{3,1}, P_{3,2}, P_{4,1}, P_{4,2}, P_{4,3}$.
(b) Find $P_{n,1}, P_{n,2}$.
(c) On the basis of your results in parts (a) and (b), conjecture the value of $P_{n,m}$.
(d) Derive a recursion for $P_{n,m}$ in terms of $P_{n-1,m}$ and $P_{n,m-1}$ by conditioning on who receives the last vote.

(e) Use part (d) to verify your conjecture in part (c) by an induction proof on $n + m$.

3.22. As a simplified model for weather forecasting, suppose that the weather (either wet or dry) tomorrow will be the same as the weather today with probability p. Show that the weather is dry on January 1, then P_n, the probability that it will be dry n days later, satisfies

$$P_n = (2p - 1)P_{n-1} + (1 - p) \qquad n \ge 1$$
$$P_0 = 1$$

Prove that

$$P_n = \frac{1}{2} + \frac{1}{2}(2p - 1)^n \qquad n \ge 0$$

3.23. A bag contains a white and b black balls. Balls are chosen from the bag according to the following method:

1. A ball is chosen at random and is discarded.
2. A second ball is then chosen. If its color is different from that of the preceding ball, it is replaced in the bag and the process is repeated from the beginning. If its color is the same, it is discarded and we start from step 2.

In other words, balls are sampled and discarded until a change in color occurs, at which point the last ball is returned to the urn and the process starts anew. Let $P_{a,b}$ denote the probability that the last ball in the bag is white. Prove that

$$P_{a,b} = \frac{1}{2}$$

Hint: Use induction on $k \equiv a + b$.

***3.24.** A round-robin tournament of n contestants is a tournament in which each of the $\binom{n}{2}$ pairs of contestants play each other exactly once, with the outcome of any play being that one of the contestants wins and the other loses. For a fixed integer $k, k < n$, a question of interest is whether it is possible that the tournament outcome is such that, for every set of k players, there is a player who beat each member of that set. Show that if

$$\binom{n}{k}\left[1 - \left(\frac{1}{2}\right)^k\right]^{n-k} < 1$$

then such an outcome is possible.
Hint: Suppose that the results of the games are independent and that each game is equally likely to be won by either contestant. Number the $\binom{n}{k}$ sets of k contestants, and let B_i denote the event that no contestant beat all of the k players in the ith set. Then use Boole's inequality to bound $P\left(\bigcup_i B_i\right)$.

3.25. Prove directly that

$$P(E|F) = P(E|FG)P(G|F) + P(E|FG^c)P(G^c|F)$$

3.26. Prove the equivalence of Equations (5.11) and (5.12).

3.27. Extend the definition of conditional independence to more than 2 events.

3.28. Prove or give a counterexample. If E_1 and E_2 are independent, then they are conditionally independent given F.

3.29. In Laplace's rule of succession (Example 5e), show that if the first n flips all result in heads, then the conditional probability that the next m flips also result in all heads is $(n + 1)/(n + m + 1)$.

3.30. In Laplace's rule of succession (Example 5e), suppose that the first n flips resulted in r heads and $n - r$ tails. Show that the probability that the $(n + 1)$st flip turns up heads is $(r + 1)/(n + 2)$. To do so, you will have to prove and use the identity

$$\int_0^1 y^n(1 - y)^m dy = \frac{n!m!}{(n + m + 1)!}$$

Hint: To prove the identity, let $C(n,m) = \int_0^1 y^n(1 - y)^m dy$. Integrating by parts yields

$$C(n,m) = \frac{m}{n + 1}C(n + 1, m - 1)$$

Starting with $C(n, 0) = 1/(n + 1)$, prove the identity by induction on m.

3.31. Suppose that a nonmathematical, but philosophically minded, friend of yours claims that Laplace's rule of succession must be incorrect because it can lead to ridiculous conclusions. "For instance," says he, "the rule states that if a boy is 10 years old, having lived 10 years, the boy has probability $\frac{11}{12}$ of living another year. On the other hand, if the boy has an 80-year-old grandfather, then, by Laplace's rule, the grandfather has probability $\frac{81}{82}$ of surviving another year. However, this is ridiculous. Clearly, the boy is more likely to survive an additional year than the grandfather is." How would you answer your friend?

SELF-TEST PROBLEMS AND EXERCISES

3.1. In a game of bridge, West has no aces. What is the probability of his partner's having (a) no aces? (b) 2 or more aces? (c) What would the probabilities be if West had exactly 1 ace?

3.2. The probability that a new car battery functions for over 10,000 miles is .8, the probability that it functions for over 20,000 miles is .4, and the probability that it functions for over 30,000 miles is .1. If a new car battery is still working after 10,000 miles, what is the probability that

(a) its total life will exceed 20,000 miles?
(b) its additional life will exceed 20,000 miles?

3.3. How can 20 balls, 10 white and 10 black, be put into two urns so as to maximize the probability of drawing a white ball if an urn is selected at random and a ball is drawn at random from it?

3.4. Urn A contains 2 white balls and 1 black ball, whereas urn B contains 1 white ball and 5 black balls. A ball is drawn at random from urn A and placed in urn B. A ball is then drawn from urn B. It happens to be white. What is the probability that the ball transferred was white?

3.5. An urn has r red and w white balls that are randomly removed one at a time. Let R_i be the event that the ith ball removed is red. Find

(a) $P(R_i)$
(b) $P(R_5|R_3)$
(c) $P(R_3|R_5)$

3.6. An urn contains b black balls and r red balls. One of the balls is drawn at random, but when it is put back in the urn, c additional balls of the same color are put in with it. Now, suppose that we draw another ball. Show that the probability that the first ball was black, given that the second ball drawn was red, is $b/(b + r + c)$.

3.7. A friend randomly chooses two cards, without replacement, from an ordinary deck of 52 playing cards. In each of the following situations, determine the conditional probability that both cards are aces.

(a) You ask your friend if one of the cards is the ace of spades, and your friend answers in the affirmative.
(b) You ask your friend if the first card selected is an ace, and your friend answers in the affirmative.
(c) You ask your friend if the second card selected is an ace, and your friend answers in the affirmative.
(d) You ask your friend if either of the cards selected is an ace, and your friend answers in the affirmative.

3.8. Show that

$$\frac{P(H|E)}{P(G|E)} = \frac{P(H)}{P(G)}\frac{P(E|H)}{P(E|G)}$$

Suppose that, before new evidence is observed, the hypothesis H is three times as likely to be true as is the hypothesis G. If the new evidence is twice as likely when G is true than it is when H is true, which hypothesis is more likely after the evidence has been observed?

3.9. You ask your neighbor to water a sickly plant while you are on vacation. Without water, it will die with probability .8; with water, it will die with probability .15. You are 90 percent certain that your neighbor will remember to water the plant.

(a) What is the probability that the plant will be alive when you return?
(b) If the plant is dead upon your return, what is the probability that your neighbor forgot to water it?

3.10. Six balls are to be randomly chosen from an urn containing 8 red, 10 green, and 12 blue balls.

(a) What is the probability at least one red ball is chosen?
(b) Given that no red balls are chosen, what is the conditional probability that there are exactly 2 green balls among the 6 chosen?

3.11. A type C battery is in working condition with probability .7, whereas a type D battery is in working condition with probability .4. A battery is randomly chosen from a bin consisting of 8 type C and 6 type D batteries.

(a) What is the probability that the battery works?
(b) Given that the battery does not work, what is the conditional probability that it was a type C battery?

3.12. Maria will take two books with her on a trip. Suppose that the probability that she will like book 1 is .6, the probability that she will like book 2 is .5, and the probability that she will like both books is .4. Find the conditional probability that she will like book 2 given that she did not like book 1.

3.13. Balls are randomly removed from an urn that initially contains 20 red and 10 blue balls.

(a) What is the probability that all of the red balls are removed before all of the blue ones have been removed?
Now suppose that the urn initially contains 20 red, 10 blue, and 8 green balls.
(b) Now what is the probability that all of the red balls are removed before all of the blue ones have been removed?

(c) What is the probability that the colors are depleted in the order blue, red, green?

(d) What is the probability that the group of blue balls is the first of the three groups to be removed?

3.14. A coin having probability .8 of landing on heads is flipped. A observes the result—either heads or tails—and rushes off to tell B. However, with probability .4, A will have forgotten the result by the time he reaches B. If A has forgotten, then, rather than admitting this to B, he is equally likely to tell B that the coin landed on heads or that it landed tails. (If he does remember, then he tells B the correct result.)

(a) What is the probability that B is told that the coin landed on heads?

(b) What is the probability that B is told the correct result?

(c) Given that B is told that the coin landed on heads, what is the probability that it did in fact land on heads?

3.15. In a certain species of rats, black dominates over brown. Suppose that a black rat with two black parents has a brown sibling.

(a) What is the probability that this rat is a pure black rat (as opposed to being a hybrid with one black and one brown gene)?

(b) Suppose that when the black rat is mated with a brown rat, all 5 of their offspring are black. Now what is the probability that the rat is a pure black rat?

3.16. (a) In Problem 3.65b, find the probability that a current flows from A to B, by conditioning on whether relay 1 closes.

(b) Find the conditional probability that relay 3 is closed given that a current flows from A to B.

3.17. For the k-out-of-n system described in Problem 3.67, assume that each component independently works with probability $\frac{1}{2}$. Find the conditional probability that component 1 is working, given that the system works, when

(a) $k = 1, n = 2$;

(b) $k = 2, n = 3$.

3.18. Mr. Jones has devised a gambling system for winning at roulette. When he bets, he bets on red and places a bet only when the 10 previous spins of the roulette have landed on a black number. He reasons that his chance of winning is quite large because the probability of 11 consecutive spins resulting in black is quite small. What do you think of this system?

3.19. Three players simultaneously toss coins. The coin tossed by $A(B)[C]$ turns up heads with probability $P_1(P_2)[P_3]$. If one person gets an outcome different from those of the other two, then he is the odd man out. If there is no odd man out, the players flip again and continue to do so until they get an odd man out. What is the probability that A will be the odd man?

3.20. Suppose that there are n possible outcomes of a trial, with outcome i resulting with probability $p_i, i = 1, \ldots, n, \sum_{i=1}^{n} p_i = 1$. If two independent trials are observed, what is the probability that the result of the second trial is larger than that of the first?

3.21. If A flips $n + 1$ and B flips n fair coins, show that the probability that A gets more heads than B is $\frac{1}{2}$. *Hint*: Condition on which player has more heads after each has flipped n coins. (There are three possibilities.)

3.22. Prove or give counterexamples to the following statements:

(a) If E is independent of F and E is independent of G, then E is independent of $F \cup G$.

(b) If E is independent of F, and E is independent of G, and $FG = \emptyset$, then E is independent of $F \cup G$.

(c) If E is independent of F, and F is independent of G, and E is independent of FG, then G is independent of EF.

3.23. Let A and B be events having positive probability. State whether each of the following statements is (i) necessarily true, (ii) necessarily false, or (iii) possibly true.

(a) If A and B are mutually exclusive, then they are independent.

(b) If A and B are independent, then they are mutually exclusive.

(c) $P(A) = P(B) = .6$, and A and B are mutually exclusive.

(d) $P(A) = P(B) = .6$, and A and B are independent.

3.24. Rank the following from most likely to least likely to occur:

1. A fair coin lands on heads.
2. Three independent trials, each of which is a success with probability .8, all result in successes.
3. Seven independent trials, each of which is a success with probability .9, all result in successes.

3.25. Two local factories, A and B, produce radios. Each radio produced at factory A is defective with probability .05, whereas each one produced at factory B is defective with probability .01. Suppose you purchase two radios that were produced at the same factory, which is equally likely to have been either factory A or factory B. If the first radio that you check is defective, what is the conditional probability that the other one is also defective?

3.26. Show that if $P(A|B) = 1$, then $P(B^c|A^c) = 1$.

3.27. An urn initially contains 1 red and 1 blue ball. At each stage, a ball is randomly withdrawn and replaced by two other balls of the same color. (For instance, if the red ball is initially chosen, then there would be 2 red and 1 blue ball in the urn when the next selection occurs.) Show by mathematical induction that the probability that there are exactly i red balls in the urn after n stages have been completed is $\frac{1}{n+1}, 1 \le i \le n + 1$.

3.28. A total of $2n$ cards, of which 2 are aces, are to be randomly divided among two players, with each player receiving n cards. Each player is then to declare, in sequence, whether he or she has received any aces. What is the conditional probability that the second player has no aces, given that the first player declares in the affirmative, when (a) $n = 2$? (b) $n = 10$? (c) $n = 100$? To what does the probability converge as n goes to infinity? Why?

3.29. There are n distinct types of coupons, and each coupon obtained is, independently of prior types collected, of type i with probability p_i, $\sum_{i=1}^{n} p_i = 1$.

(a) If n coupons are collected, what is the probability that one of each type is obtained?

(b) Now suppose that $p_1 = p_2 = \cdots = p_n = 1/n$. Let E_i be the event that there are no type i coupons among the n collected. Apply the inclusion–exclusion identity for the probability of the union of events to $P(\cup_i E_i)$ to prove the identity

$$n! = \sum_{k=0}^{n} (-1)^k \binom{n}{k} (n - k)^n$$

3.30. Show that, for any events E and F,

$$P(E|E \cup F) \ge P(E|F)$$

Hint: Compute $P(E|E \cup F)$ by conditioning on whether F occurs.

3.1. Two fair dice are rolled. What is the conditional probability that at least one lands on 6 given that the dice land on different numbers?

3.2. If two fair dice are rolled, what is the conditional probability that the first one lands on 6 given that the sum of the dice is i? Compute for all values of i between 2 and 12.

3.3. Use Equation (2.1) to compute, in a hand of bridge, the conditional probability that East has 3 spades given that North and South have a combined total of 8 spades.

3.4. What is the probability that at least one of a pair of fair dice lands on 6, given that the sum of the dice is i, $i = 2, 3, \ldots, 12$?

3.1 $P(\geq 1 : 6 \mid \text{different})$.

$$\frac{\frac{5}{36}}{\frac{30}{36}} = \frac{1}{6}.$$

3.2. $P(2-6) = 0$

$P(7-12) = \frac{1}{36}$.

3.3. ?

3.4. $P(2-6) = 0$

$P(7) = \frac{2}{36}$, $P(8) = \frac{2}{36}$

$P(7-11) = \frac{2}{36}$, $P(12) = \frac{1}{36}$.

3.5. An urn contains 6 white and 9 black balls. If 4 balls are to be randomly selected without replacement, what is the probability that the first 2 selected are white and the last 2 black?

3.6. Consider an urn containing 12 balls, of which 8 are white. A sample of size 4 is to be drawn with replacement (without replacement). What is the conditional probability (in each case) that the first and third balls drawn will be white given that the sample drawn contains exactly 3 white balls?

3.7. The king comes from a family of 2 children. What is the probability that the other child is his sister?

3.8. A couple has 2 children. What is the probability that both are girls if the older of the two is a girl?

3.9. Consider 3 urns. Urn A contains 2 white and 4 red balls, urn B contains 8 white and 4 red balls, and urn C contains 1 white and 3 red balls. If 1 ball is selected from each urn, what is the probability that the ball chosen from urn A was white given that exactly 2 white balls were selected?

3.5.

$$\frac{6}{15} \times \frac{5}{14} \times \frac{9}{13} \times \frac{8}{12}$$

3.6.

$P($ 1, 3rd = white | 3 white $)$

$$= \frac{2 \times \frac{8}{12} \times \frac{4}{12}}{\left(\frac{8}{12}\right)^3 \left(\frac{4}{12}\right)}.$$

without replacement:

$$\frac{2 \times \frac{6}{10} \times \frac{4}{9}}{\binom{8}{3}\binom{4}{1}}$$

sample drawn contains exactly 3 white balls?

3.7. The king comes from a family of 2 children. What is the probability that the other child is his sister?

3.8. A couple has 2 children. What is the probability that both are girls if the older of the two is a girl?

3.9. Consider 3 urns. Urn A contains 2 white and 4 red balls, urn B contains 8 white and 4 red balls, and urn C contains 1 white and 3 red balls. If 1 ball is selected from each urn, what is the probability that the ball chosen from urn A was white given that exactly 2 white balls were selected?

3.7. $\frac{1}{2}$

3.8. $P(\text{both girl} \mid \text{older} = \text{girl})$

$$= \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2}}$$

3.9. $P(\text{urn } A = \text{white} \mid 2 \text{ white})$

$$= \frac{\quad}{\quad}$$

3.10. Three cards are randomly selected, without replacement, from an ordinary deck of 52 playing cards. Compute the conditional probability that the first card selected is a spade given that the second and third cards are spades.

3.11. Two cards are randomly chosen without replacement from an ordinary deck of 52 cards. Let B be the event that both cards are aces, let A_s be the event that the ace of spades is chosen, and let A be the event that at least one ace is chosen. Find

(a) $P(B|A_s)$

(b) $P(B|A)$

3.10.

$$\frac{P(1 = \spadesuit)}{P(2,3 = \spadesuit)}$$

$$\frac{\frac{4}{52} \times \frac{3}{51} \times \frac{2}{50}}{\frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} + \frac{48}{52} \times \frac{4}{51} \times \frac{3}{50}}$$

3.11 a).

$$\frac{P(B \cap A_s)}{P(A_s)} = \frac{\frac{1}{52} \times \frac{3}{51} + \frac{3}{52} \times \frac{1}{51}}{\frac{1}{52} + \frac{51}{52} \times \frac{1}{51}}$$

b).

$$\frac{P(B \cap A)}{P(A)} = \frac{\varphi}{\frac{\binom{4}{2}}{\binom{52}{2}} + \frac{2\binom{4}{1} \times \binom{48}{1}}{\binom{52}{2}}}$$

3.12. A recent college graduate is planning to take the first three actuarial examinations in the coming summer. She will take the first actuarial exam in June. If she passes that exam, then she will take the second exam in July, and if she also passes that one, then she will take the third exam in September. If she fails an exam, then she is not allowed to take any others. The probability that she passes the first exam is .9. If she passes the first exam, then the conditional probability that she passes the second one is .8, and if she passes both the first and the second exams, then the conditional probability that she passes the third exam is .7.

(a) What is the probability that she passes all three exams?

(b) Given that she did not pass all three exams, what is the conditional probability that she failed the second exam?

3.13. Suppose that an ordinary deck of 52 cards (which contains 4 aces) is randomly divided into 4 hands of 13 cards each. We are interested in determining p, the probability that each hand has an ace. Let E_i be the event that the ith hand has exactly one ace. Determine $p = P(E_1E_2E_3E_4)$ by using the multiplication rule.

3.12 a). $.9 \times .8 \times .7 = a$

b). $\dfrac{.9 \times .2}{1-a}$

3.13 $P(E_1)\,P(E_2|E_1)\,P(E_3|E_1E_2)\,P(E_4|E_1E_2E_3)$

$= \dfrac{\binom{4}{1}}{\binom{52}{13}} \times \dfrac{\binom{4}{1}}{\binom{52}{13}}$

3.13. Suppose that an ordinary deck of 52 cards (which contains 4 aces) is randomly divided into 4 hands of 13 cards each. We are interested in determining p, the probability that each hand has an ace. Let E_i be the event that the ith hand has exactly one ace. Determine $p = P(E_1E_2E_3E_4)$ by using the multiplication rule.

$$P(E_1) = \frac{\binom{4}{1}\binom{48}{12}}{\binom{52}{13}} = \frac{9139}{20825} = 0.438847539.$$

$P(E_2|E_1) =$

1st hand 1 ace

2nd hand 1 ace

3.14. An urn initially contains 5 white and 7 black balls. Each time a ball is selected, its color is noted and it is replaced in the urn along with 2 other balls of the same color. Compute the probability that

(a) the first 2 balls selected are black and the next 2 are white;

(b) of the first 4 balls selected, exactly 2 are black.

3.14a).

$$\frac{7}{12} \times \frac{8}{13} \times \frac{5}{14} \times \frac{6}{15} = a$$

b).

$$\binom{4}{2} \times a$$

3.15. An ectopic pregnancy is twice as likely to develop when the pregnant woman is a smoker as it is when she is a nonsmoker. If 32 percent of women of childbearing age are smokers, what percentage of women having ectopic pregnancies are smokers?

3.16. Ninety-eight percent of all babies survive delivery. However, 15 percent of all births involve Cesarean (C) sections, and when a C section is performed, the baby survives 96 percent of the time. If a randomly chosen pregnant woman does not have a C section, what is the probability that her baby survives?

$$P(A) = P(A \cap B) + P(A \cap B').$$

3.15.

$$P(ep \mid smoker) = 2P(ep \mid non\text{-}smoker)$$

$$P(smokers) = 0.32, \quad P(non\text{-}smoker) = 0.68$$

$$P(smokers \mid ep) = ? \quad \frac{a}{P(ep)} = \frac{16}{33}.$$

$$\frac{P(ep \cap Smoker)}{0.32} = 2 \times \frac{P(ep \cap non\text{-}smoker)}{0.68}$$

$$a = \frac{16}{17}(P(ep) - a)$$

$$\frac{33}{17}a = \frac{16}{17}P(ep)$$

$$a = \frac{16}{33}P(ep).$$

0.98

3.16. Ninety-eight percent of all babies survive delivery. However, 15 percent of all births involve Cesarean (C) sections, and when a C section is performed, the baby survives 96 percent of the time. If a randomly chosen pregnant woman does not have a C section, what is the probability that her baby survives?

$P(C) = 0.15$ $\qquad$ $P(\text{Survives}) = 0.98$

$P(\text{survives} \cap C) = 0.96$

$P(\text{survives} \mid C') = ?$

$P(\text{survives} \cap C') = 0.02$

$\cancel{P}\ ? = \dfrac{0.02}{0.85}.$

3.17. In a certain community, 36 percent of the families own a dog and 22 percent of the families that own a dog also own a cat. In addition, 30 percent of the families own a cat. What is

(a) the probability that a randomly selected family owns both a dog and a cat?

(b) the conditional probability that a randomly selected family owns a dog given that it owns a cat?

3.17 a). $P(\text{cat} \mid \text{dog}) = 0.22$

$$\frac{P(\text{cat} \cap \text{dog})}{0.36} = 0.22$$

$P(\text{cat} \cap \text{dog}) = 0.0792$

b). $P(\text{dog} \mid \text{cat}) = \dfrac{0.0792}{0.3}$

$= 0.264$

3.18. A total of 46 percent of the voters in a certain city classify themselves as Independents, whereas 30 percent classify themselves as Liberals and 24 percent say that they are Conservatives. In a recent local election, 35 percent of the Independents, 62 percent of the Liberals, and 58 percent of the Conservatives voted. A voter is chosen at random.

$P(\text{voted} \mid I) = 0.35$

Given that this person voted in the local election, what is the probability that he or she is

(a) an Independent?

(b) a Liberal?

(c) a Conservative?

(d) What fraction of voters participated in the local election? 0.4862

a). $P(\text{voted} \mid I) = \dfrac{P(\text{voted} \cap I)}{P(I)}$

$P(\text{voted} \cap I) = 0.35 \times 0.46$

$P(\text{voted} \cap I) = 0.161$

$P(I \mid \text{voted}) = \dfrac{0.161}{P(\text{voted})}$.

$= \dfrac{0.161}{0.4862} = 0.3311$

$P(\text{voted}) = P(\text{voted} \mid I)\,P(I) + P(\text{voted} \mid L)\,P(L) + P(\text{voted} \mid C)\,P(C)$

$= 0.161 + 0.186 + 0.1392$

$= 0.4862$

b) - $P(L \mid \text{voted}) = \dfrac{0.186}{0.4862} = 0.3826$

c). $\dfrac{0.1392}{0.4862} = 0.2863$

3.19. A total of 48 percent of the women and 37 percent of the men that took a certain "quit smoking" class remained nonsmokers for at least one year after completing the class. These people then attended a success party at the end of a year. If 62 percent of the original class was male, 3

(a) what percentage of those attending the party were women?

(b) what percentage of the original class attended the party?

3.19 a). $P(\text{party} \mid W) = 0.48$ $P(\text{party} \mid \text{men}) = 0.37$

$P(M) = 0.62$

a). $P(W \mid \text{party}) = ?$

$P(\text{party} \cap W) = 0.48 \times 0.38 = 0.1824$

$P(W \mid \text{party}) = \dfrac{0.1824}{P(\text{party})} = 0.4429.$

b). $P(\text{party}) = P(\text{party} \mid W)\, P(W) + P(\text{party} \mid M) P(M)$

$= 0.48 \times 0.38 + 0.62 \times 0.37$

$= 0.1824 + 0.2294 = 0.4118$

3.20. Fifty-two percent of the students at a certain college are females. Five percent of the students in this college are majoring in computer science. Two percent of the students are women majoring in computer science. If a student is selected at random, find the conditional probability that

(a) the student is female given that the student is majoring in computer science;

(b) this student is majoring in computer science given that the student is female.

$0.52 = P(F)$

$0.05 = P(CS)$

$0.02 = P(F \cap CS)$

a). $P(F \mid CS) = \frac{0.02}{0.05} = 0.4$

b). $P(CS \mid F) = \frac{0.02}{0.52} = 0.03846$

3.21. A total of 500 married working couples were polled about their annual salaries, with the following information resulting:

Wife	Husband: Less than $25,000	Husband: More than $25,000
Less than $25,000	212	198
More than $25,000	36	54

For instance, in 36 of the couples, the wife earned more and the husband earned less than $25,000. If one of the couples is randomly chosen, what is

(a) the probability that the husband earns less than $25,000?

(b) the conditional probability that the wife earns more than $25,000 given that the husband earns more than this amount?

(c) the conditional probability that the wife earns more than $25,000 given that the husband earns less than this amount?

a). $\frac{212+36}{500} = 0.496.$

b). $P(\overset{\text{wife}}{>25000} \mid \overset{\text{husband}}{>25000})$

$$= \frac{\frac{54}{500}}{\frac{198+54}{500}} \approx 0.2143$$

c). $P(\text{wife} > 25000 \mid \text{husband} < 25000)$

$$= \frac{\frac{36}{500}}{0.496} = 0.1452$$

3.22. A red die, a blue die, and a yellow die (all six sided) are rolled. We are interested in the probability that the number appearing on the blue die is less than that appearing on the yellow die, which is less than that appearing on the red die. That is,

with B, Y, and R denoting, respectively, the number appearing on the blue, yellow, and red die, we are interested in $P(B < Y < R)$.

(a) What is the probability that no two of the dice land on the same number?

(b) Given that no two of the dice land on the same number, what is the conditional probability that $B < Y < R$?

(c) What is $P(B < Y < R)$?

$\underline{6}\ \underline{\ }\ \underline{\ }$

a). $P(\text{all different}) = \dfrac{6 \times 5 \times 4}{6 \times 6 \times 6} = \dfrac{5}{9}.$

b). $P(B<Y<R \mid \text{no two})$

$$= \frac{\dfrac{\binom{6}{3}}{6^3}}{\dfrac{5}{9}} = \frac{1}{6}.$$

c). $P(B<Y<R) = \dfrac{5}{9} \times \dfrac{1}{6} + P(B<Y<R \mid \geq 2 \text{ same}) = 0.$

$\times \dfrac{4}{9},$

3.23. Urn I contains 2 white and 4 red balls, whereas urn II contains 1 white and 1 red ball. A ball is randomly chosen from urn I and put into urn II, and a ball is then randomly selected from urn II. What is

(a) the probability that the ball selected from urn II is white?

(b) the conditional probability that the transferred ball was white given that a white ball is selected from urn II?

3.23 a). $P(\text{urn II white} \mid \text{urn I white})\, P(\text{urn I white}) + P(\text{urn II white} \mid \text{urn I red})\, P(\text{urn I red})$

$$= \frac{2}{6} \times \frac{2}{3} + \frac{4}{6} \times \frac{1}{3} = \frac{4}{9}.$$

b). $P(\text{urn I white} \mid \text{urn II white})$

$$= \frac{\frac{2}{6} \times \frac{2}{3}}{\frac{4}{9}} = \frac{1}{2}$$

3.24. Each of 2 balls is painted either black or gold and then placed in an urn. Suppose that each ball is colored black with probability $\frac{1}{2}$ and that these events are independent.

(a) Suppose that you obtain information that the gold paint has been used (and thus at least one of the balls is painted gold). Compute the conditional probability that both balls are painted gold.

(b) Suppose now that the urn tips over and 1 ball falls out. It is painted gold. What is the probability that both balls are gold in this case? Explain.

a). $P(\text{both gold} \mid \geq 1 \text{ gold})$

$$= \frac{P(\text{both gold} \cap \geq 1 \text{ gold})}{P(\geq 1 \text{ gold})} = a$$

$P(\geq 1 \text{ gold}) = 1 - (\frac{1}{2})^2 = 0.75$

$P(\text{both gold} \cap \geq 1 \text{ gold}) = \frac{1}{2} \times \frac{1}{2} = 0.25$

$a = \frac{0.25}{0.75}$, $a = \frac{1}{3}$.

b). $P(\text{both gold} \mid \text{one gold}) = \frac{P(\text{both gold})}{P(\text{one gold})} = \frac{0.25}{(1)\frac{1}{2} \times \frac{1}{2}}$

$= \frac{1}{2}$.

3.25. The following method was proposed to estimate the number of people over the age of 50 who reside in a town of known population 100,000: "As you walk along the streets, keep a running count of the percentage of people you encounter who are over 50. Do this for a few days; then multiply the percentage you obtain by 100,000 to obtain the estimate." Comment on this method.

Hint: Let p denote the proportion of people in the town who are over 50. Furthermore, let α_1 denote the proportion of time that a person under the age of 50 spends in the streets, and let α_2 be the corresponding value for those over 50. What quantity does the method suggested estimate? When is the estimate approximately equal to p?

a_2

$= p$

?!!!

3.26. Suppose that 5 percent of men and .25 percent of women are color blind. A color-blind person is chosen at random. What is the probability of this person being male? Assume that there are an equal number of males and females. What if the population consisted of twice as many males as females?

$P(CB/men) = 0.05$

$P(CB/women) = 0.25$

$P(men) = 0.5$

$P(CB) = 0.05 \times 0.5 + 0.25 \times 0.5$

$= 0.15$

$P(M/CB) = \dfrac{0.05 \times 0.5}{0.15} = \dfrac{1}{6}.$

ii). $P(men) = 2\,P(female).$

$a = 2(1-a).$

$a = 2 - 2a$

$3a = 2$

$a = \frac{2}{3}.$

$\dfrac{0.05 \times \frac{2}{3}}{0.05 \times \frac{2}{3} + 0.25 \times \frac{1}{3}}$

$= 0.2857 = \frac{2}{7}.$

3.27. All the workers at a certain company drive to work and park in the company's lot. The company is interested in estimating the average number of workers in a car. Which of the following methods will enable the company to estimate this quantity? Explain your answer.

1. Randomly choose n workers, find out how many were in the cars in which they were driven, and take the average of the n values. ✓
2. Randomly choose n cars in the lot, find out how many were driven in those cars, and take the average of the n values.

$$\frac{p(\text{driven})}{p(\text{workers})} = \frac{a}{n}$$

3.28. Suppose that an ordinary deck of 52 cards is shuffled and the cards are then turned over one at a time until the first ace appears. Given that the first ace is the 20th card to appear, what is the conditional probability that the card following it is the

(a) ace of spades? ♤A.

(b) two of clubs? ♧2.

3.28 a).

$$P(♤A \mid 20\text{th Ace}) = \frac{P(♤A \cap 20\text{th Ace})}{P(20\text{th Ace})}$$

$$P(20\text{th Ace}) = \frac{\binom{48}{19}\binom{4}{4}}{\binom{52}{20}}$$

3.29. There are 15 tennis balls in a box, of which 9 have not previously been used. Three of the balls are randomly chosen, played with, and then returned to the box. Later, another 3 balls are randomly chosen from the box. Find the probability that none of these balls has ever been used.

$$\frac{\binom{9}{3}}{\binom{15}{3}} \times \frac{\binom{6}{3}}{\binom{15}{3}}$$

P(first time no use $\cap$ second time no use).

3.30. Consider two boxes, one containing 1 black and 1 white marble, the other 2 black and 1 white marble. A box is selected at random, and a marble is drawn from it at random. What is the probability that the marble is black? What is the probability that the first box was the one selected given that the marble is white?

$$P(\text{black}) = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{2}{3} = \frac{7}{12}.$$

$$P(\text{first box} \mid \text{white})$$

$$= \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{5}{12}} = \frac{3}{5}$$

3.31. Ms. Aquina has just had a biopsy on a possibly cancerous tumor. Not wanting to spoil a weekend family event, she does not want to hear any bad news in the next few days. But if she tells the doctor to call only if the news is good, then if the doctor does not call, Ms. Aquina can conclude that the news is bad. So, being a student of probability, Ms. Aquina instructs the doctor to flip a coin. If it comes up heads, the doctor is to call if the news is good and not call if the news is bad. If the coin comes up tails, the doctor is not to call. In this way, even if the doctor doesn't call, the news is not necessarily bad. Let α be the probability that the tumor is cancerous; let β be the conditional probability that the tumor is cancerous given that the doctor does not call.

(a) Which should be larger, α or β?

(b) Find β in terms of α, and prove your answer in part (a).

$\alpha = P(\text{cancer} \mid \text{call})P(\text{call}) + \quad \beta \quad P(\text{call}')$

$P(\text{call}) = \frac{1}{2}P(\text{cancer})$

$$\alpha = \frac{1}{2}\alpha + \beta\left(1 - \frac{1}{2}\alpha\right)$$

$$\alpha = \frac{1}{2}\alpha + \beta - \frac{1}{2}\alpha\beta$$

$$\frac{1}{2}\alpha = \beta - \frac{1}{2}\alpha\beta$$

$$\frac{1}{2}\alpha = \beta\left(1 - \frac{1}{2}\alpha\right)$$

$$\frac{\frac{1}{2}\alpha}{1 - \frac{1}{2}\alpha} = \beta.$$

???

3.32. A family has j children with probability p_j, where $p_1 = .1, p_2 = .25, p_3 = .35, p_4 = .3$. A child from this family is randomly chosen. Given that this child is the eldest child in the family, find the conditional probability that the family has

(a) only 1 child;
(b) 4 children.
Redo (a) and (b) when the randomly selected child is the youngest child of the family.

???

3.33. On rainy days, Joe is late to work with probability .3; on nonrainy days, he is late with probability .1. With probability .7, it will rain tomorrow.

(a) Find the probability that Joe is early tomorrow.

(b) Given that Joe was early, what is the conditional probability that it rained?

p(late | rainy) = 0.3

P(late | non rainy) = 0.1

P(rainy) = 0.7

a). p(late') = p(late' | rainy) P(rainy) + p(late' | non-rainy) P(non-rainy)

= 0.7 × 0.7 + 0.9 × 0.3

= 0.76

b). P(rained | late')

= $\dfrac{0.7 \times 0.7}{0.76}$ = 0.6447.

3.34. In Example 3f, suppose that the new evidence is subject to different possible interpretations and in fact shows only that it is 90 percent likely that the criminal possesses the characteristic in question. In this case, how likely would it be that the suspect is guilty (assuming, as before, that he has the characteristic)?

3.35. With probability .6, the present was hidden by mom; with probability .4, it was hidden by dad. When mom hides the present, she hides it upstairs 70 percent of the time and downstairs 30 percent of the time. Dad is equally likely to hide it upstairs or downstairs.

(a) What is the probability that the present is upstairs?

(b) Given that it is downstairs, what is the probability it was hidden by dad?

a). $P(\text{upstairs}) = P(\text{upstairs} \mid \text{mom})P(\text{mom}) + P(\text{upstairs} \mid \text{dad})P(\text{dad})$

$= 0.6 \times 0.7 + 0.4 \times 0.5 = 0.62$

b). $P(\text{dad} \mid \text{downstairs}) = \dfrac{P(\text{downstairs} \mid \text{dad})P(\text{dad})}{0.38}$

$= \dfrac{0.5 \times 0.4}{0.38}$

$= 0.5263$

3.36. Stores A, B, and C have 50, 75, and 100 employees, respectively, and 50, 60, and 70 percent of them respectively are women. Resignations are equally likely among all employees, regardless of sex. One woman employee resigns. What is the probability that she works in store C?

$$P(C \mid \text{women resign})$$

$$= \frac{P(C \cap \text{women resign})}{P(\text{women resign})} = \frac{\frac{100 \times 0.7}{225}}{\frac{50 \times 0.5 + 75 \times 0.6 + 100 \times 0.7}{225}}$$

$$= \frac{1}{2}.$$

3.37. **(a)** A gambler has a fair coin and a two-headed coin in his pocket. He selects one of the coins at random; when he flips it, it shows heads. What is the probability that it is the fair coin?

(b) Suppose that he flips the same coin a second time and, again, it shows heads. Now what is the probability that it is the fair coin?

(c) Suppose that he flips the same coin a third time and it shows tails. Now what is the probability that it is the fair coin?

3.37 a). $P(\text{fair coin} \mid \text{head})$

$$= \frac{P(\text{fair coin} \cap \text{head})}{P(\text{head})}$$

$P(\text{head}) = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2}.$

$= 0.75.$

$$\frac{0.25}{0.75} = \frac{1}{3}.$$

3.37 b). $P(\text{fair coin} \mid E)$

$$= \frac{P(\text{fair coin} \cap E)}{P(E)}.$$

E = event that head → head

$$P(E) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2}.$$

$$= 0.625.$$

$$P(\text{fair coin} \mid E) = \frac{\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}}{0.625} = \frac{1}{5}.$$

c). $P(\text{fair coin} \mid F) = 1$.

F = head → head → tail

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

3.38. Urn A has 5 white and 7 black balls. Urn B has 3 white and 12 black balls. We flip a fair coin. If the outcome is heads, then a ball from urn A is selected, whereas if the outcome is tails, then a ball from urn B is selected. Suppose that a white ball is selected. What is the probability that the coin landed tails?

$P(B \mid \text{white})$.

$P(\text{white}) = \frac{1}{2} \times \frac{5}{12} + \frac{1}{2} \times \frac{3}{15}$

$$\frac{\frac{1}{2} \times \frac{3}{15}}{\frac{1}{2} \times \frac{5}{12} + \frac{1}{2} \times \frac{3}{15}}.$$

3.40. Consider a sample of size 3 drawn in the following manner: We start with an urn containing 5 white and 7 red balls. At each stage, a ball is drawn and its color is noted. The ball is then returned to the urn, along with an additional ball of the same color. Find the probability that the sample will contain exactly

(a) 0 white balls;
(b) 1 white ball;
(c) 3 white balls;
(d) 2 white balls.

a). RRR.

$$\frac{7}{12} \times \frac{8}{13} \times \frac{9}{14}$$

b).

$$\frac{5}{12} \times \frac{7}{13} \times \frac{8}{14} \times \binom{3}{1}$$

c). WWW

$$\frac{5}{12} \times \frac{6}{13} \times \frac{7}{14}$$

d). WWR.

$$\frac{5}{12} \times \frac{6}{13} \times \frac{7}{14} \times \binom{3}{2}$$

3.41. A deck of cards is shuffled and then divided into two halves of 26 cards each. A card is drawn from one of the halves; it turns out to be an ace. The ace is then placed in the second half-deck. The half is then shuffled, and a card is drawn from it. Compute the probability that this drawn card is an ace. *Hint*: Condition on whether or not the interchanged card is selected.

P(Ace$^{\text{second drawn}}$ | selected interchanged) P(selected interchanged) +

P(Ace | selected interchanged$'$) P(selected$'$)

$= \frac{1}{26} \times$

3.42. Three cooks, A, B, and C, bake a special kind of cake, and with respective probabilities .02, .03, and .05, it fails to rise. In the restaurant where they work, A bakes 50 percent of these cakes, B 30 percent, and C 20 percent. What proportion of "failures" is caused by A?

$$P(A \mid \text{fail})$$

$$= \frac{P(A \cap \text{fail})}{P(\text{fail})}$$

$$= \frac{0.02 \times 0.5}{0.02 \times 0.5 + 0.03 \times 0.3 + 0.05 \times 0.2}$$

$$= 0.3448$$

3.43. There are 3 coins in a box. One is a two-headed coin, another is a fair coin, and the third is a biased coin that comes up heads 75 percent of the time. When one of the 3 coins is selected at random and flipped, it shows heads. What is the probability that it was the two-headed coin?

$$\frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times 0.75}$$

3.44. Three prisoners are informed by their jailer that one of them has been chosen at random to be executed and the other two are to be freed. Prisoner A asks the jailer to tell him privately which of his fellow prisoners will be set free, claiming that there would be no harm in divulging this information because he already knows that at least one of the two will go free. The jailer refuses to answer the question, pointing out that if A knew which of his fellow prisoners were to be set free, then his own probability of being executed would rise from $\frac{1}{3}$ to $\frac{1}{2}$ because he would then be one of two prisoners. What do you think of the jailer's reasoning?

≥ 1 of 2

$P(\text{executed} \mid 1 \text{ of other } 2 = \text{free})$

$$= \frac{\frac{1}{3}}{1 - \frac{1}{3}}$$

3.45. Suppose we have 10 coins such that if the ith coin is flipped, heads will appear with probability $i/10, i = 1, 2, \ldots, 10$. When one of the coins is randomly selected and flipped, it shows heads. What is the conditional probability that it was the fifth coin?

$p(5\text{th} \mid \text{heads})$

$p(\text{head}) = \frac{1}{10} \sum_{i=1}^{10} \frac{i}{10} = 0.55$

$\frac{p(5\text{th} \cap \text{heads})}{0.55} = \frac{\frac{1}{10} \times \frac{5}{10}}{0.55} = \frac{1}{11}$

$= 0.0909$

3.46. In any given year, a male automobile policyholder will make a claim with probability p_m and a female policyholder will make a claim with probability p_f, where $p_f \neq p_m$. The fraction of the policyholders that are male is $\alpha, 0 < \alpha < 1$. A policyholder is randomly chosen. If A_i denotes the event that this policyholder will make a claim in year i, show that

$$P(A_2|A_1) > P(A_1)$$

Give an intuitive explanation of why the preceding inequality is true.

$$P(A_1) = p_m\alpha + (1-\alpha)p_f$$

$$= p_m\alpha + p_f - \alpha p_f$$

$$P(A_2 \cap A_1) =$$

3.47. An urn contains 5 white and 10 black balls. A fair die is rolled and that number of balls is randomly chosen from the urn. What is the probability that all of the balls selected are white? What is the conditional probability that the die landed on 3 if all the balls selected are white?

①: $\frac{5}{15}$

②: $\frac{\binom{5}{2}}{\binom{15}{2}}$

a). $\frac{5}{2} \over 11$ $\quad \frac{\binom{5}{i}}{\binom{15}{i}} = \frac{5}{11}$

b). $\frac{\frac{\binom{5}{3}}{\binom{11}{3}}}{\frac{5}{11}} = \frac{22}{455} = 0.04835$

3.48. Each of 2 cabinets identical in appearance has 2 drawers. Cabinet A contains a silver coin in each drawer, and cabinet B contains a silver coin in one of its drawers and a gold coin in the other. A cabinet is randomly selected, one of its drawers is opened, and a silver coin is found. What is the probability that there is a silver coin in the other drawer?

$P(A \mid \text{silver})$

$P(\text{silver}) = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = 0.375$

$P(A / \text{silver}) = \dfrac{\frac{1}{2} \times \frac{1}{2}}{0.375} = \frac{2}{3}$

3.49. Prostate cancer is the most common type of cancer found in males. As an indicator of whether a male has prostate cancer, doctors often perform a test that measures the level of the prostate-specific antigen (PSA) that is produced only by the prostate gland. Although PSA levels are indicative of cancer, the test is notoriously unreliable. Indeed, the probability that a noncancerous man will have an elevated PSA level is approximately .135, increasing to approximately .268 if the man does have cancer. If, on the basis of other factors, a physician is 70 percent certain that a male has prostate cancer, what is the conditional probability that he has the cancer given that

(a) the test indicated an elevated PSA level?

(b) the test did not indicate an elevated PSA level?

Repeat the preceding calculation, this time assuming that the physician initially believes that there is a 30 percent chance that the man has prostate cancer.

$P(\text{elevated} \mid \text{cancer}') = 0.135$

$P(\text{elevated} \mid \text{cancer}) = 0.268$

$P(\text{cancer}) = 0.7$

a). $P(\text{cancer} \mid \text{elevated}) = \dfrac{0.268 \times 0.7}{0.2281} = 0.8224$

$P(\text{elevated}) = 0.7 \times 0.268 + 0.3 \times 0.135 = 0.2281$

b). $P(\text{cancer} \mid \text{elevated}') = \dfrac{0.135 \times 0.3}{0.7719} = 0.05246$

3.50. Suppose that an insurance company classifies people into one of three classes: good risks, average risks, and bad risks. The company's records indicate that the probabilities that good-, average-, and bad-risk persons will be involved in an accident over a 1-year span are, respectively, .05, .15, and .30. If 20 percent of the population is a good risk, 50 percent an average risk, and 30 percent a bad risk, what proportion of people have accidents in a fixed year? If policyholder A had no accidents in 1997, what is the probability that he or she is a good or average risk?

a). $P(\text{accident}) = 0.05 \times 0.2 + 0.15 \times 0.5 + 0.3 \times 0.3$

$= 0.175$

b). $P(\text{good} \cup \text{average} \mid \text{accident}') = \dfrac{0.2 \times 0.95 + 0.5 \times 0.85}{0.825}$

$= 0.7455 \ (\frac{41}{55})$.

3.51. A worker has asked her supervisor for a letter of recommendation for a new job. She estimates that there is an 80 percent chance that she will get the job if she receives a strong recommendation, a 40 percent chance if she receives a moderately good recommendation, and a 10 percent chance if she receives a weak recommendation. She further estimates that the probabilities that the recommendation will be strong, moderate, and weak are .7, .2, and .1, respectively.

(a) How certain is she that she will receive the new job offer?

(b) Given that she does receive the offer, how likely should she feel that she received a strong recommendation? a moderate recommendation? a weak recommendation?

(c) Given that she does not receive the job offer, how likely should she feel that she received a strong recommendation? a moderate recommendation? a weak recommendation?

a). $0.8 \times 0.7 + 0.4 \times 0.2 + 0.1 \times 0.1$

$= 0.65$

b). $\frac{0.8 \times 0.7}{0.65} = 0.8615$

$\frac{0.4 \times 0.2}{0.65} = 0.1231$

$\frac{0.1 \times 0.1}{0.65} = 0.01538$

$\frac{0.7 \times 0.2}{0.35} = 0.4$

$\frac{0.2 \times 0.6}{0.35} = 0.3429 \left(\frac{12}{35}\right)$

$\frac{0.1 \times 0.9}{0.35} = 0.2571 \left(\frac{9}{35}\right)$

3.52. A high school student is anxiously waiting to receive mail telling her whether she has been accepted to a certain college. She estimates that the conditional probabilities of receiving notification on each day of next week, given that she is accepted and that she is rejected, are as follows:

Day	$P(\text{mail}\mid\text{accepted})$	$P(\text{mail}\mid\text{rejected})$	P(mail)
Monday	.15	.05	0.11
Tuesday	.20	.10	0.16
Wednesday	.25	.10	0.19
Thursday	.15	.15	0.15
Friday	.10	.20	0.14
			0.75

She estimates that her probability of being accepted is .6.

(a) What is the probability that she receives mail on Monday?

(b) What is the conditional probability that she received mail on Tuesday given that she does not receive mail on Monday?

(c) If there is no mail through Wednesday, what is the conditional probability that she will be accepted?

(d) What is the conditional probability that she will be accepted if mail comes on Thursday? ??

(e) What is the conditional probability that she will be accepted if no mail arrives that week?

3.52 a). $0.15 \times 0.6 + 0.05 \times 0.4 = 0.11$

b). $\dfrac{0.6 \times 0.2 + 0.4 \times 0.1}{0.89} = 0.1798$

c). P(accepted | no mail on wed) = 0.6

d). = 0.6

e). = 0.6

3.53. A parallel system functions whenever at least one of its components works. Consider a parallel system of n components, and suppose that each component works independently with probability $\frac{1}{2}$. Find the conditional probability that component 1 works given that the system is functioning.

$$P(\text{system function}) = 1 - (\tfrac{1}{2})^n$$

$$\frac{\frac{1}{2}}{1-(\frac{1}{2})^n} = \frac{\frac{1}{2}}{1-\frac{1}{2^n}} = \frac{\frac{1}{2}}{\frac{2^n-1}{2^n}}$$

$$\frac{1}{2} \cdot \frac{2^n}{(2^n-1)} = \frac{2^{n-1}}{2^n-1}$$

$$=$$

3.54. If you had to construct a mathematical model for events E and F, as described in parts (a) through (e), would you assume that they were independent events? Explain your reasoning.

(a) E is the event that a businesswoman has blue eyes, and F is the event that her secretary has blue eyes.

(b) E is the event that a professor owns a car, and F is the event that he is listed in the telephone book.

(c) E is the event that a man is under 6 feet tall, and F is the event that he weighs over 200 pounds.

(d) E is the event that a woman lives in the United States, and F is the event that she lives in the Western Hemisphere.

(e) E is the event that it will rain tomorrow, and F is the event that it will rain the day after tomorrow.

3.54 a).

3.55. In a class, there are 4 freshman boys, 6 freshman girls, and 6 sophomore boys. How many sophomore girls must be present if sex and class are to be independent when a student is selected at random?

(4).

P(freshmen ∩ men) = P(fresh) P(men)

$$4 = \left(\frac{10}{6+a}\right)\left(\frac{10}{6+a}\right)$$

3.56. Suppose that you continually collect coupons and that there are m different types. Suppose also that each time a new coupon is obtained, it is a type i coupon with probability $p_i, i = 1, \ldots, m$. Suppose that you have just collected your nth coupon. What is the probability that it is a new type?
Hint: Condition on the type of this coupon.

??

3.57. A simplified model for the movement of the price of a stock supposes that on each day the stock's price either moves up 1 unit with probability p or moves down 1 unit with probability $1 - p$. The changes on different days are assumed to be independent.

(a) What is the probability that after 2 days the stock will be at its original price?

(b) What is the probability that after 3 days the stock's price will have increased by 1 unit?

(c) Given that after 3 days the stock's price has increased by 1 unit, what is the probability that it went up on the first day?

a). $2p(1-p)$.

b). $3p^2(1-p)$.

c). $\dfrac{2p(1-p)}{3p^2(1-p)}$

3.58. Suppose that we want to generate the outcome of the flip of a fair coin, but that all we have at our disposal is a biased coin which lands on heads with some unknown probability p that need not be equal to $\frac{1}{2}$. Consider the following procedure for accomplishing our task:

1. Flip the coin.
2. Flip the coin again.
3. If both flips land on heads or both land on tails, return to step 1.
4. Let the result of the last flip be the result of the experiment.

(a) Show that the result is equally likely to be either heads or tails.

(b) Could we use a simpler procedure that continues to flip the coin until the last two flips are different and then lets the result be the outcome of the final flip?

a). $P(\text{Head}) = (1-p)p$

$p(\text{tails}) = p(1-p)$

b). HT $p(1-p)$ TH

HHT

HHHT

$$p(T) = \sum_{i=1}^{\infty} p^i (1-p) = (1-p)\frac{1}{1-p}$$

$$p(H) = \sum_{i=1}^{\infty} (1-p)^i p$$

3.59. Independent flips of a coin that lands on heads with probability p are made. What is the probability that the first four outcomes are

(a) H, H, H, H?

(b) T, H, H, H?

(c) What is the probability that the pattern T, H, H, H occurs before the pattern H, H, H, H?

Hint for part (c): How can the pattern H, H, H, H occur first?

a). p^4

b). $(1-p)p^3$

c). THHH HHHH

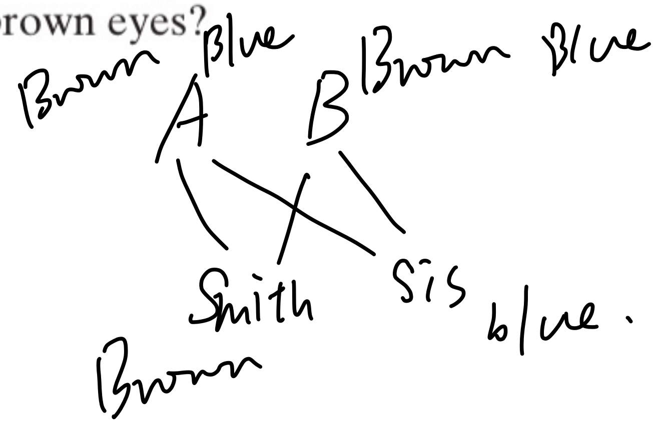
3.60. The color of a person's eyes is determined by a single pair of genes. If they are both blue-eyed genes, then the person will have blue eyes; if they are both brown-eyed genes, then the person will have brown eyes; and if one of them is a blue-eyed gene and the other a brown-eyed gene, then the person will have brown eyes. (Because of the latter fact, we say that the brown-eyed gene is *dominant* over the blue-eyed one.) A newborn child independently receives one eye gene from each of its parents, and the gene it receives from a parent is equally likely to be either of the two eye genes of that parent. Suppose that Smith and both of his parents have brown eyes, but Smith's sister has blue eyes.

(a) What is the probability that Smith possesses a blue-eyed gene?

(b) Suppose that Smith's wife has blue eyes. What is the probability that their first child will have blue eyes?

(c) If their first child has brown eyes, what is the probability that their next child will also have brown eyes?

a).



$P(\text{Smith blue}) = P(\text{Smith blue} \mid AB$

$\frac{1}{2} \times \frac{1}{2}$

b). $P(\text{kids} = \text{blue} \mid \text{wife} = \text{blue}) =$

3.61. Genes relating to albinism are denoted by A and a. Only those people who receive the a gene from both parents will be albino. Persons having the gene pair A, a are normal in appearance and, because they can pass on the trait to their offspring, are called carriers. Suppose that a normal couple has two children, exactly one of whom is an albino. Suppose that the nonalbino child mates with a person who is known to be a carrier for albinism.

(a) What is the probability that their first offspring is an albino?

(b) What is the conditional probability that their second offspring is an albino given that their firstborn is not?

$\{A,a\},\{\cancel{a},A\}$ $\{A,a\}$

3.61(a). a

ϕ

P(白化 | 非白化 + 白化 carrier.)

父 A,a 母 A,a.

不ϕ PK

$\{a,a\}$ ϕ (receive a from both).

$P(\phi) = P(\phi \mid PK \text{ is } AA)P(AA) + P(\phi \mid PK \text{ is } Aa)P(Aa)$

+PK爸 is Aa

PK爸 is Aa

$P(a) = 0 + \left(\frac{1}{2}\times\frac{1}{2}\right)\left(2\times\frac{1}{2}\times\frac{1}{2}\right)$

$= \frac{1}{2}.$

b). $P($第二中$|$第一不中$)$

$$\frac{P(\text{第二中} \cap \text{第1不中})}{\frac{1}{2}}$$

$\frac{1}{2}\times\frac{1}{2}$

$P($不中 → 中 $|$ PK is AA, PK wife is Aa$)\,P($PK is AA$)$

$+\,P($不中 → 中 $|$ PK is Aa, PK wife is Aa$)$

$P($PK is Aa$)$.

$$= \frac{2\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}}$$

3.62. Barbara and Dianne go target shooting. Suppose that each of Barbara's shots hits a wooden duck target with probability p_1, while each shot of

Dianne's hits it with probability p_2. Suppose that they shoot simultaneously at the same target. If the wooden duck is knocked over (indicating that it was hit), what is the probability that

(a) both shots hit the duck?

(b) Barbara's shot hit the duck?

What independence assumptions have you made?

a). $p_1 p_2$

b). $p_1 (1 - p_2)$

They are independent in shooting

3.63. A and B are involved in a duel. The rules of the duel are that they are to pick up their guns and shoot at each other simultaneously. If one or both are hit, then the duel is over. If both shots miss, then they repeat the process. Suppose that the results of the shots are independent and that each shot of A will hit B with probability p_A, and each shot of B will hit A with probability p_B. What is

(a) the probability that A is not hit?

(b) the probability that both duelists are hit?

(c) the probability that the duel ends after the nth round of shots?

(d) the conditional probability that the duel ends after the nth round of shots given that A is not hit?

(e) the conditional probability that the duel ends after the nth round of shots given that both duelists are hit?

a). A is not hit

A	B	A	B	A	B
✓	✗	✗	✗	✓	✗

$$\sum_{i=0}^{\infty} \left[(1-p_A)(1-p_B)\right]^i (p_A)(1-p_B).$$

$$p_A(1-p_B) \quad \frac{1}{1-(1-p_A)(1-p_B)}$$

$$\frac{p_A - p_A p_B}{1-(1-p_B-p_A+p_A p_B)}$$

$$= \frac{p_A - p_A p_B}{p_B + p_A - p_A p_B}$$

b). $\sum_{i=0}^{\infty} [(1-P_A)(1-P_B)]^i \ P_A P_B$

AB AB
× × ✓ ✓

$$= \frac{P_A P_B}{1-(1-P_A)(1-P_B)}$$

$$= \frac{P_A P_B}{1-(1-P_A-P_B+P_A P_B)}$$

$$= \frac{P_A P_B}{P_A+P_B-P_A P_B}.$$

c). after n round. (n-1) round both AB
× ×

$$\sum_{i=0}^{n-1} [(1-P_A)(1-P_B)]^i$$

$$= \frac{[(1-P_A)(1-P_B)]^n-1}{(1-P_A)(1-P_B)-1}$$

c). n round 结束：A 中，B 中，双方中.

$$P_A(1-P_B)+P_B(1-P_A)+P_AP_B$$

$$= P_A-P_AP_B+P_B-P_AP_B+P_AP_B$$

$$= P_A+P_B-P_AP_B.$$

$$\frac{[(1-P_A)(1-P_B)]^n-1}{P_AP_B-P_A-P_B}\,(P_A+P_B-P_AP_B)$$

$$= 1-[(1-P_A)(1-P_B)]^n$$

d). P(ends n round | A not hit)

$$= \frac{\dfrac{[(1-P_A)(1-P_B)]^n-1}{P_AP_B-P_A-P_B}\,P_A(1-P_B)}{\dfrac{P_A-P_AP_B}{P_A+P_B-P_AP_B}}$$

$$= \left(\frac{[(1-P_A)(1-P_B)]^n-1}{P_AP_B-P_A-P_B}\right)\cdot\frac{P_A(1-P_B)}{P_A-P_AP_B}\cdot(P_A+P_B-P_AP_B)$$

$$= 1-[(1-P_A)(1-P_B)]^n$$

e).
$$\frac{[(1-P_A)(1-P_B)]^n-1}{P_AP_B-P_A-P_B}\,P_AP_B \times \frac{P_A+P_B-P_AP_B}{P_AP_B}$$

$$= 1-[(1-P_A)(1-P_B)]^n.$$

3.64. A true–false question is to be posed to a husband-and-wife team on a quiz show. Both the husband and the wife will independently give the correct answer with probability p. Which of the following is a better strategy for the couple?

(a) Choose one of them and let that person answer the question.

(b) Have them both consider the question, and then either give the common answer if they agree or, if they disagree, flip a coin to determine which answer to give.

a). $\frac{1}{2} \times p + \frac{1}{2} \times p = p$

b). TT/FF → P(correct | TT/FF) P(TT/FF) +

TF/FT → P(Correct | TF/FT) P(TF/FT)

P(correct / TT) P(TT) +

P(correct | FF) P(FF) +

P(corr | TF) P(TF) + P(corr | FT) P(FT)

$= \frac{1}{4} \times p + \frac{1}{4} \times p + \frac{1}{4} \times \frac{1}{2} \times p + \frac{1}{4} \times \frac{1}{2} \times p$

= ??

3.65 ??

3.66. The probability of the closing of the ith relay in the circuits shown in Figure 3.4 is given by $p_i, i = 1, 2, 3, 4, 5$. If all relays function independently, what is the probability that a current flows between A and B for the respective circuits?
Hint for (b): Condition on whether relay 3 closes.

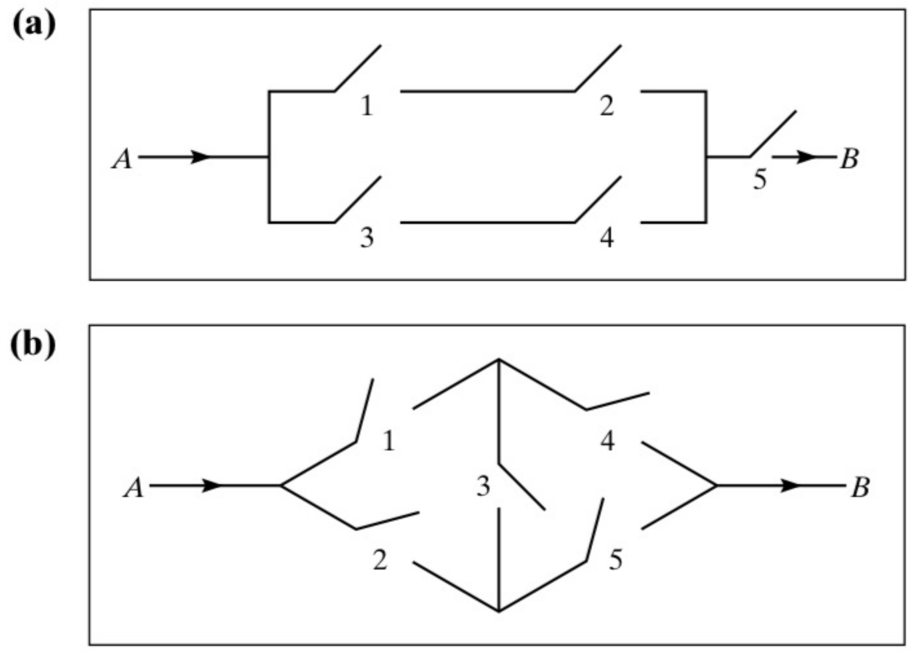


FIGURE 3.4: Circuits for Problem 3.66

a. work: 1 ✓ 2 ✓ 3 ✓/× 4 ✓/× + 5 ✓

3 ✓ 4 ✓ 1 ✓/× 2 ✓/× 5 ✓

$$p_1p_2p_5\left(\cancel{p_3p_4} + \cancel{p_4} - \cancel{p_3p_4} + \cancel{p_3} - \cancel{p_3p_4} + 1 - \cancel{p_3} - \cancel{p_4} + \cancel{p_3p_4}\right) +$$

$$p_3p_4p_5\left(p_1p_2 + (1-p_1)p_2 + p_1(1-p_2) + (1-p_1)(1-p_2)\right)$$

$$= p_1p_2p_5 + p_3p_4p_5.$$

$P(\text{work} \mid 3 \text{ open})(1-p_3) + P(\text{work} \mid 3 \text{ open})\, p_3$

$P(\text{work} \mid 3 \text{ open}) =$ 1 ✓ 4 ✓ 2 ✓/x 5 ✓/x 3 x

2 ✓ 5 ✓ 1 ✓/x 4 ✓/x 3 x

$$\frac{p_1 p_4 (1-p_3) + p_2 p_5 (1-p_3)}{1-p_3} = (p_1 p_4 + p_2 p_5)$$

$P(\text{work} \mid 3 \text{ close}) =$ 1 ✓ 5 ✓ 2 4 3 ✓

2 ✓ 4 ✓ 3 ✓

1 ✓ 4 ✓ 3 ✓

2 ✓ 5 ✓ 3 ✓

$$p_3(p_1 p_5 + p_2 p_4 + p_1 p_4 + p_2 p_5) +$$

$$(p_1 p_4 + p_2 p_5)(1-p_3)$$

$$= p_1 p_3 p_5 + p_2 p_3 p_4 + p_1 p_4 + p_2 p_5$$

3.67. An engineering system consisting of n components is said to be a k-out-of-n system ($k \le n$) if the system functions if and only if at least k of the n components function. Suppose that all components function independently of each other.

(a) If the ith component functions with probability $P_i, i = 1, 2, 3, 4$, compute the probability that a 2-out-of-4 system functions.

(b) Repeat part (a) for a 3-out-of-5 system.

(c) Repeat for a k-out-of-n system when all the P_i equal p (that is, $P_i = p, i = 1, 2, \ldots, n$).

a). $P_1P_2 + P_1P_4 + P_1P_3 + P_2P_4 + P_2P_3 + P_3P_4$

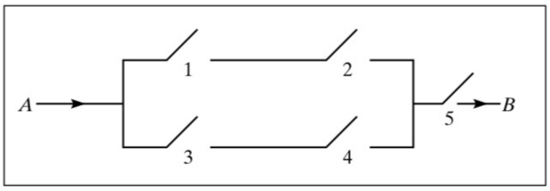
b).

123
124
125
134
135
145
234
235
245
345

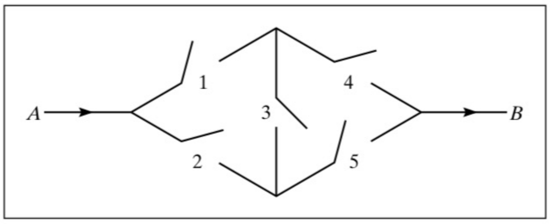
c). $\binom{n}{k} p^k$

3.68. In Problem 3.65a, find the conditional probability that relays 1 and 2 are both closed given that a current flows from A to B.

(a)



(b)



$p(1 \& 2^{\text{closed}} \mid A \to B)$

1✓ 2✓ 5✓

3✓ 4✓ 5✓

$$\frac{p_1 p_2 p_5}{p_1 p_2 p_5 + p_3 p_4 p_5}$$

3.69. A certain organism possesses a pair of each of 5 different genes (which we will designate by the first 5 letters of the English alphabet). Each gene appears in 2 forms (which we designate by lowercase and capital letters). The capital letter will be assumed to be the dominant gene, in the sense that if an organism possesses the gene pair xX, then it will outwardly have the appearance of the X gene. For instance, if X stands for brown eyes and x for blue eyes, then an individual having either gene pair XX or xX will have brown eyes, whereas one having gene pair xx will have blue eyes. The characteristic appearance of an organism is called its phenotype, whereas its genetic constitution is called its genotype. (Thus, 2 organisms with respective genotypes aA, bB, cc, dD, ee and AA, BB, cc, DD, ee would have different genotypes but the same phenotype.) In a mating between 2 organisms, each one contributes, at random, one of its gene pairs of each type. The 5 contributions of an organism (one of each of the 5 types) are assumed to be independent and are also independent of the contributions of the organism's mate. In a mating between organisms having genotypes aA, bB, cC, dD, eE and aa, bB, cc, Dd, ee what is the probability that the progeny will (i) phenotypically and (ii) genotypically resemble

(a) the first parent?

(b) the second parent?

????

3.70. There is a 50–50 chance that the queen carries the gene for hemophilia. If she is a carrier, then each prince has a 50–50 chance of having hemophilia. If the queen has had three princes without the disease, what is the probability that the queen is a carrier? If there is a fourth prince, what is the probability that he will have hemophilia?

P(Queen) = 0.5

P(Prince have | Queen has) = 0.5

P(Queen | prince 1 have' ∩ prince 2 have' ∩ prince 3 have')

P(prince have) = 0.5 × 0.5 + P(prince have | Queen haven't)

= 0.25

$$\frac{1}{2} \times \left(\frac{1}{2}\right)^3$$

0.753

3.71. On the morning of September 30, 1982, the won–lost records of the three leading baseball teams in the Western Division of the National League were as follows:

Team	Won	Lost
Atlanta Braves	87	72
San Francisco Giants	86	73
Los Angeles Dodgers	86	73

Each team had 3 games remaining. All 3 of the Giants' games were with the Dodgers, and the 3 remaining games of the Braves were against the San Diego Padres. Suppose that the outcomes of all remaining games are independent and each game is equally likely to be won by either participant. For each team, what is the probability that it will win the division title? If two teams tie for first place, they have a playoff game, which each team has an equal chance of winning.

For AB: 有可能: 90
89
88
87.

2x 对应可能

SFG		vs	CAD	
89	3√		3x	86
88	2√1X		2x 1√	87
87	1√2x		1x 2√	88
86	3x		3√	89

AB 90: $(\frac{1}{2})^3$

STG / CAD 89/86 AB win

AB 89: $(\frac{1}{2})^3 \times 3 \times (\frac{1}{2})^3 \times 2 \times \frac{1}{2}$

STG/CAD 88/87 +

$(\frac{1}{2})^3 \times 3 \times (\frac{1}{2})^3 \times 3 \times 2$

STG / CAD 88/87 +

AB 88: $(\frac{1}{2})^3 \times 3 \times (\frac{1}{2})^3 \times 3 \times 2 \times \frac{1}{2}$

3.72. A town council of 7 members contains a steering committee of size 3. New ideas for legislation go first to the steering committee and then on to the council as a whole if at least 2 of the 3 committee members approve the legislation. Once at the full council, the legislation requires a majority vote (of at least 4) to pass. Consider a new piece of legislation, and suppose that each town council member will approve it, independently, with probability p. What is the probability that a given steering committee member's vote is decisive in the sense that if that person's vote were reversed, then the final fate of the legislation would be reversed? What is the corresponding probability for a given council member not on the steering committee?

P(decisive of steering)

P(decisive ∩ 大圈子办了 | 小圈子过到) P(小圈子过到)

✓✓✓ⓧ

✓✓✗

✓✗✗ =

3.73. Suppose that each child born to a couple is equally likely to be a boy or a girl, independently of the sex distribution of the other children in the family. For a couple having 5 children, compute the probabilities of the following events:

(a) All children are of the same sex.
(b) The 3 eldest are boys and the others girls.
(c) Exactly 3 are boys.
(d) The 2 oldest are girls.
(e) There is at least 1 girl.

BBBGG

a). $2\times\left(\frac{1}{2}\right)^5$ b). $\left(\frac{1}{2}\right)^5$

c). $\binom{5}{3}\left(\frac{1}{2}\right)^5$ d). $\left(\frac{1}{2}\right)^2$

GG - - -

1- d). $1-\left(\frac{1}{2}\right)^5$

74. A and B alternate rolling a pair of dice, stopping either when A rolls the sum 9 or when B rolls the sum 6. Assuming that A rolls first, find the probability that the final roll is made by A.

$P(9) =$ 54, 45, 36, 63 $\quad \frac{4}{36}$ $\qquad P(6) =$ 15, 51, 24, 42, 33 $\quad \frac{5}{36}.$

A $\qquad\qquad\qquad\qquad$ B

$$\frac{4}{36} + \frac{32}{36} \times \frac{31}{36} \times \frac{4}{36} + \left(\frac{32}{36} \times \frac{31}{36}\right)^3 \frac{4}{36}$$

$$\frac{4}{36} \sum_{i=0}^{\infty} \left(\frac{32}{36} \times \frac{31}{36}\right)^i$$

$$= \frac{4}{36} \left(\frac{1}{1 - \frac{32}{36} \times \frac{31}{36}} \right)$$

3.75. In a certain village, it is traditional for the eldest son (or the older son in a two-son family) and his wife to be responsible for taking care of his parents as they age. In recent years, however, the women of this village, not wanting that responsibility, have not looked favorably upon marrying an eldest son.

(a) If every family in the village has two children, what proportion of all sons are older sons?

(b) If every family in the village has three children, what proportion of all sons are eldest sons?

Assume that each child is, independently, equally likely to be either a boy or a girl.

BG (b) BGG

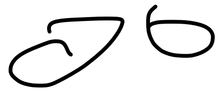
$(\frac{1}{2})^2$ $(\frac{1}{2})^3$? ?

3.76. Suppose that E and F are mutually exclusive events of an experiment. Show that if independent trials of this experiment are performed, then E will occur before F with probability $P(E)/[P(E) + P(F)]$.

$p(E \cap F) = 0$

E before F.

E F.



3.77. Consider an unending sequence of independent trials, where each trial is equally likely to result in any of the outcomes 1, 2, or 3. Given that outcome 3 is the last of the three outcomes to occur, find the conditional probability that

(a) the first trial results in outcome 1;

(b) the first two trials both result in outcome 1.

(a).

1 - 3

P(first 1 有 1 | 3 last)

$$\frac{\frac{1}{3} \times \frac{1}{3}}{\frac{1}{3}} = \frac{1}{3}$$

b). $P(113 \mid 3 \text{ last}) = \left(\frac{1}{3}\right)^2$

3.78. *A* and *B* play a series of games. Each game is independently won by *A* with probability *p* and by *B* with probability $1 - p$. They stop when the total number of wins of one of the players is two greater than that of the other player. The player with the

$a > b+2$

greater number of total wins is declared the winner of the series.

(a) Find the probability that a total of 4 games are played.

(b) Find the probability that A is the winner of the series.

ABAA
BAAA

4. $2 \times p^3 (1-p) +$
$2 \times (1-p)^3 p$

a).

b).

2 AA p^2

3 $\times$

4 $2 \times p^3 (1-p)$

5 $\times$

6 $2 \times p^4 (1-p)^2$

~~AB~~~~AB~~~~A~~A
~~A~~BBAAA

$p^2 + 2 \sum_{i=1}^{\infty} p^{2+i} (1-p)^i$??

3.79. In successive rolls of a pair of fair dice, what is the probability of getting 2 sevens before 6 even numbers?

6 even.

7 7 ⌒⌒

2-12.

2	4	6	8	10	12
11	13	15	26	55	
	31	51	62	64	
	22	24	35	66	
		42	53		
		33	44		
1	3	5	5	3	1

$\left(\frac{6}{36}\right)^2$ $\left(\frac{18}{36}\right)^6$

3.80. In a certain contest, the players are of equal skill and the probability is $\frac{1}{2}$ that a specified one of the two contestants will be the victor. In a group of 2^n players, the players are paired off against each other at random. The 2^{n-1} winners are again paired off randomly, and so on, until a single winner remains. Consider two specified contestants, A and B, and define the events $A_i, i \le n, E$ by

A_i: A plays in exactly i contests:

E: A and B never play each other.

(a) Find $P(A_i), i = 1, \ldots, n$.

(b) Find $P(E)$.

(c) Let $P_n = P(E)$. Show that

$$P_n = \frac{1}{2^n - 1} + \frac{2^n - 2}{2^n - 1}\left(\frac{1}{2}\right)^2 P_{n-1}$$

and use this formula to check the answer you obtained in part (b).

Hint: Find $P(E)$ by conditioning on which of the events $A_i, i = 1, \ldots, n$ occur. In simplifying your answer, use the algebraic identity

$$\sum_{i=1}^{n-1} i x^{i-1} = \frac{1 - nx^{n-1} + (n-1)x^n}{(1-x)^2}$$

For another approach to solving this problem, note that there are a total of $2^n - 1$ games played.

(d) Explain why $2^n - 1$ games are played.

Number these games, and let B_i denote the event that A and B play each other in game $i, i = 1, \ldots, 2^n - 1$.

(e) What is $P(B_i)$?

(f) Use part (e) to find $P(E)$.

a). $P(A_1) = \frac{1}{2}$ $P(A_2) = \frac{1}{2} \times \frac{1}{2}$

$P(A_i) = \left(\frac{1}{2}\right)^i$

(b) $P(E)$:

3.81. An investor owns shares in a stock whose present value is 25. She has decided that she must sell her stock if it goes either down to 10 or up to 40. If each change of price is either up 1 point with probability .55 or down 1 point with probability .45, and the successive changes are independent, what is the probability that the investor retires a winner?

$$(0.55)^{15} + \left(\binom{17}{1} - 1\right)(0.55)^{16}\, 0.45 + \left(\binom{19}{2} - 1\right)(0.55)^{17}\, 0.45^{2}$$

$$(0.55)^{15} + \sum_{i=1}^{\infty} \left(\binom{15+2i}{i} - 1\right)(0.55)^{15+i}\, 0.45^{i}$$

3.1. Show that if $P(A) > 0$, then

$$P(AB|A) \geq P(AB|A \cup B)$$

3.2. Let $A \subset B$. Express the following probabilities as simply as possible:

$$P(A|B), \quad P(A|B^c), \quad P(B|A), \quad P(B|A^c)$$

3.3. Consider a school community of m families, with n_i of them having i children, $i = 1, \ldots, k$, $\sum_{i=1}^{k} n_i = m$. Consider the following two methods for choosing a child:

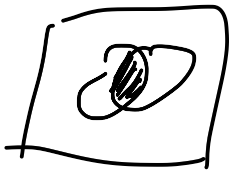
1. Choose one of the m families at random and then randomly choose a child from that family.
2. Choose one of the $\sum_{i=1}^{k} in_i$ children at random.

Show that method 1 is more likely than method 2 to result in the choice of a firstborn child.
Hint: In solving this problem, you will need to show that

$$\sum_{i=1}^{k} in_i \sum_{j=1}^{k} \frac{n_j}{j} \geq \sum_{i=1}^{k} n_i \sum_{j=1}^{k} n_j$$

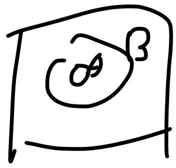
$$\frac{P(AB \cap A)}{P(A)} \qquad \frac{P(AB \cap (A \cup B))}{P(A \cup B)}$$

$$\frac{P(AB)}{P(A)} \qquad \frac{P(AB)}{P(A \cup B)}$$



since $P(A \cup B) \geq P(A)$

$$\frac{P(AB)}{P(A)} \geq \frac{P(AB)}{P(A \cup B)}.$$



3.2 $\frac{P(AB)}{P(B)} = \frac{P(A)}{P(B)}$, $\frac{P(A \cap B^c)}{P(B^c)} = 0$, $P(B|A) = \frac{P(AB)}{P(A)} = 1$

$$P(B|A^c) = \frac{P(B \cap A^c)}{P(A^c)} = \frac{P(B) - P(A)}{P(A^c)}$$

3.3. Consider a school community of m families, with n_i of them having i children, $i = 1, \ldots, k$, $\sum_{i=1}^{k} n_i = m$. Consider the following two methods for choosing a child:

n_1 有 1

n_2 有 2

$\vdots$

n_k 有 k

1. Choose one of the m families at random and then randomly choose a child from that family.
2. Choose one of the $\sum_{i=1}^{k} in_i$ children at random.

Show that method 1 is more likely than method 2 to result in the choice of a firstborn child.
Hint: In solving this problem, you will need to show that

$$\sum_{i=1}^{k} in_i \sum_{j=1}^{k} \frac{n_j}{j} \geq \sum_{i=1}^{k} n_i \sum_{j=1}^{k} n_j$$

To do so, multiply the sums and show that, for all pairs i, j, the coefficient of the term $n_i n_j$ is greater in the expression on the left than in the one on the right.

1. $\frac{n_1}{m} \cdot \frac{1}{1} + \frac{n_2}{m} \cdot \frac{1}{2} + \cdots + \frac{n_k}{m} \frac{1}{k}$

$= \sum_{i=1}^{k} \frac{n_i}{m} \frac{1}{i}$

$\frac{1}{m} \sum_{i=1}^{k} \frac{n_i}{i}$

$= \sum_{i=1}^{k} n_i \sum_{i=1}^{k} \frac{n_i}{i} =$

3.4. A ball is in any one of n boxes and is in the ith box with probability P_i. If the ball is in box i, a search of that box will uncover it with probability α_i. Show that the conditional probability that the ball is in box j, given that a search of box i did not uncover it, is

$$\frac{P_j}{1 - \alpha_i P_i} \quad \text{if } j \neq i$$

$$\frac{(1 - \alpha_i)P_i}{1 - \alpha_i P_i} \quad \text{if } j = i$$

if $j = i$:

$$P(\text{ball in box } i \mid \text{did not uncover in } i)$$

$$\frac{P(\text{ball in box } i \text{ and did not uncover in } i)}{P(\text{did not uncover in } i)}$$

$$= \frac{P_i(1-\alpha_i)}{1-\alpha_i P_i}$$

$1-\alpha_i P_i$ ← 要搜到一定要在+搜到.

$$P(\text{ball in box } j \mid \text{did not uncover in } i)$$

$$= \frac{P_j}{1-\alpha_i P_i}$$

3.5. An event F is said to carry negative information about an event E, and we write $F \searrow E$, if

$$P(E|F) \le P(E)$$

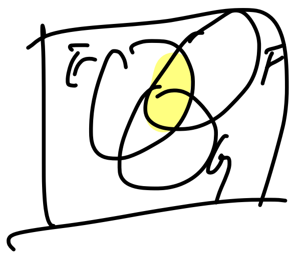
Prove or give counterexamples to the following assertions:

(a) If $F \searrow E$, then $E \searrow F$.

(b) If $F \searrow E$ and $E \searrow G$, then $F \searrow G$.

(c) If $F \searrow E$ and $G \searrow E$, then $FG \searrow E$.

Repeat parts (a), (b), and (c) when $\searrow$ is replaced by $\nearrow$, where we say that F carries positive information about E, written $F \nearrow E$, when $P(E|F) \ge P(E)$.



$$\frac{P(E\cap F)}{P(F)} \le P(E)$$

$$P(E\cap F) \le P(E)\,P(F)$$

a). if $P(E|F) \le P(E)$

True

$$P(E\cap F) \le P(E)P(F)$$

$$P(F|E) = \frac{P(F\cap E)}{P(E)} \le \frac{P(E)P(F)}{P(E)} = P(F)$$

False

b). if $P(E|F) \le P(E)$

$$P(E\cap F) \le P(E)P(F)$$

$$P(G|E) \le P(G)$$

$$P(G\cap E) \le P(G)P(E)$$

$$P(G|F) = \frac{P(G\cap F)}{P(F)}$$

if $P(G\cap F) > P(G)P(F)$

c)

$$P(E|FG) = \frac{P(E \cap FG)}{P(FG)}$$

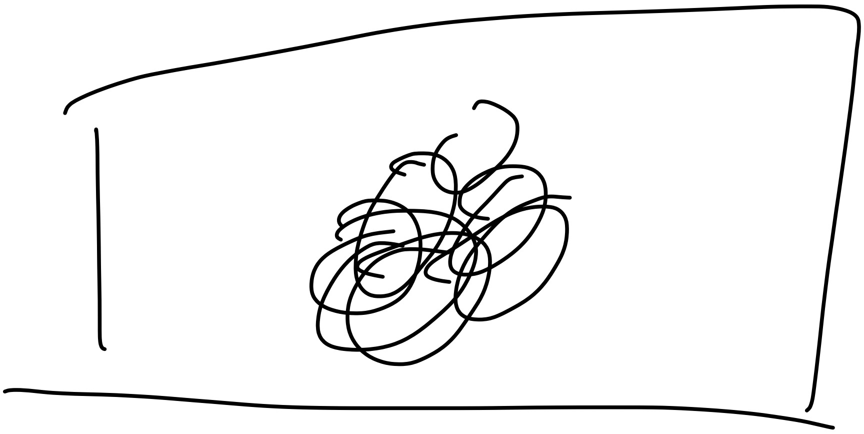
$$= \frac{P((E \cap F) \cap (G \cap E))}{P(FG)}$$

$=$

3.6. Prove that if $E_1, E_2, \ldots, E_n$ are independent events, then

$$P(E_1 \cup E_2 \cup \cdots \cup E_n) = 1 - \prod_{i=1}^{n}[1 - P(E_i)]$$

$P(E_1E_2) = P(E_1)P(E_2)$



$$= 1 - P(E_1^c \cap E_2^c \cap E_3^c \ldots\ldots)$$

$$= 1 - \prod_{i=1}^{n} P(E_i^c)$$

$$= 1 - \prod_{i=1}^{n} (1 - P(E_i))$$

3.7. (a) An urn contains n white and m black balls. The balls are withdrawn one at a time until only those of the same color are left. Show that, with probability $n/(n+m)$, they are all white.

Hint: Imagine that the experiment continues until all the balls are removed, and consider the last ball withdrawn.

(b) A pond contains 3 distinct species of fish, which we will call the Red, Blue, and Green fish. There are r Red, b Blue, and g Green fish. Suppose that the fish are removed from the pond in a random order. (That is, each selection is equally likely to be any of the remaining fish.) What is the probability that the Red fish are the first species to become extinct in the pond?

Hint: Write $P\{R\} = P\{RBG\} + P\{RGB\}$, and compute the probabilities on the right by first conditioning on the last species to be removed.

$P(R) = P(R$ 灭绝先$, BG) + P(R$先$, GB)$

$= P($

3.8. Let A, B, and C be events relating to the experiment of rolling a pair of dice.

(a) If

$$P(A|C) > P(B|C) \quad \text{and} \quad P(A|C^c) > P(B|C^c)$$

either prove that $P(A) > P(B)$ or give a counterexample by defining events A, B, and C for which that relationship is not true.

(b) If

$$P(A|C) > P(A|C^c) \quad \text{and} \quad P(B|C) > P(B|C^c)$$

either prove that $P(AB|C) > P(AB|C^c)$ or give a counterexample by defining events A, B, and C for which that relationship is not true.

Hint: Let C be the event that the sum of a pair of dice is 10; let A be the event that the first die lands on 6; let B be the event that the second die lands on 6.

3.8a). $\dfrac{P(A\cap C)}{P(C)} > \dfrac{P(B\cap C)}{P(C)}$

$P(A\cap C) > P(B\cap C)$,

$P(A\cap C^c) > P(B\cap C^c)$

$P(A) = P(A\cap C) + P(A\cap C^c)$

$P(B) = P(B\cap C) + P(B\cap C^c)$

$P(A) > P(B)$.

(b) If

$$P(A|C) > P(A|C^c) \quad \text{and} \quad P(B|C) > P(B|C^c)$$

either prove that $P(AB|C) > P(AB|C^c)$ or give a counterexample by defining events A, B, and C for which that relationship is not true.

Hint: Let C be the event that the sum of a pair of dice is 10; let A be the event that the first die lands on 6; let B be the event that the second die lands on 6.

$$\frac{P(ABC)}{P(C)} > \frac{P(ABC^c)}{P(C^c)}$$

$$\frac{P(A\cap C)}{P(C)} > \frac{P(A\cap C^c)}{P(C^c)}, \quad \frac{P(B\cap C)}{P(C)} > \frac{P(B\cap C^c)}{P(C^c)}$$

$$\frac{a}{c} > \frac{A-a}{1-c}, \quad \frac{b}{c} > \frac{B-b}{1-c}$$

$P(C) = \begin{matrix} 5\ 5 \\ 4\ 6 \\ 6\ 4 \end{matrix} = \frac{3}{36}$

$P(A\cap C) = \frac{1}{36}$ $\qquad \frac{1}{3} > \frac{5}{33}.$

$P(C^c) = \frac{33}{36}$ $\qquad P(ABC$

$P(A\cap C^c) = \frac{5}{36}$

if $P(ABC) = 0$,

3.9. Consider two independent tosses of a fair coin. Let A be the event that the first toss results in heads, let B be the event that the second toss results in heads, and let C be the event that in both tosses the coin lands on the same side. Show that the events A, B, and C are pairwise independent—that is, A and B are independent, A and C are independent, and B and C are independent—but not independent.

$P(A) = \frac{1}{2}$ $\quad P(B) = \frac{1}{2}$ $\quad P(A \cap B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P(A \cap B) = P(A)\, P(B)$.

$P(C) = \frac{1}{2} \times \frac{1}{2} \times 2 = \frac{1}{2}$ $\quad P(A \cap C) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P(B \cap C) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P(A \cap B \cap C) = \frac{1}{2} \times \frac{1}{2} \neq P(A)\, P(B)\, P(C)$.

3.10. Consider a collection of n individuals. Assume that each person's birthday is equally likely to be any of the 365 days of the year and also that the birthdays are independent. Let $A_{i,j}$, $i \neq j$, denote the event that persons i and j have the same birthday. Show that these events are pairwise independent, but not independent. That is, show that $A_{i,j}$ and $A_{r,s}$ are independent, but the $\binom{n}{2}$ events $A_{i,j}, i \neq j$ are not independent.

$P(A_{r,j}) =$

$P(A_{i,j}) = \frac{1}{365}$ $\quad P(A_{r,s}) = \frac{1}{365}$

3.11. In each of n independent tosses of a coin, the coin lands on heads with probability p. How large need n be so that the probability of obtaining at least one head is at least $\frac{1}{2}$?

≥ 1 H,

0H.

$$1 - (1-p)^n \geq \frac{1}{2}$$

$$\frac{1}{2} \geq (1-p)^n$$

$$\log \frac{1}{2} \geq n \log (1-p)$$

$$n \leq \frac{\log (1-p)}{\log \frac{1}{2}}$$

3.12. Show that $0 \leq a_i \leq 1, i = 1, 2, \ldots,$ then

$$\sum_{i=1}^{\infty}\left[a_i \prod_{j=1}^{i-1}(1 - a_j)\right] + \prod_{i=1}^{\infty}(1 - a_i) = 1$$

Hint: Suppose that an infinite number of coins are to be flipped. Let a_i be the probability that the ith coin lands on heads, and consider when the first head occurs.

$p(a_i) =$

$p(a_i) = p(a_i \mid ?-1\, H)\, p(\bar{i-1}) + p(a_i \mid$

***3.14.** Suppose that you are gambling against an infinitely rich adversary and at each stage you either win or lose 1 unit with respective probabilities p and $1 - p$. Show that the probability that you eventually go broke is

$$\begin{cases} 1 & \text{if } p \le \frac{1}{2} \\ (q/p)^i & \text{if } p > \frac{1}{2} \end{cases}$$

where $q = 1 - p$ and where i is your initial fortune.

i 次之后

eventually go broke :

第 n 次破产:

输在 i 次

输 i+1, 赢 1

i 次: 全输 $(1-p)^i$

i+2 次: $\binom{i+2}{1} p (1-p)^{i+1}$

i+4 次: $\binom{i+4}{2} p^2 (1-p)^{i+2}$

$\vdots$

$$\sum_{j=i}^{\infty} \binom{i+}{}$$

3.22. As a simplified model for weather forecasting, suppose that the weather (either wet or dry) tomorrow will be the same as the weather today with probability p. Show that the weather is dry on January 1, then P_n, the probability that it will be dry n days later, satisfies

$$P_n = (2p - 1)P_{n-1} + (1 - p) \qquad n \geq 1$$
$$P_0 = 1$$

Prove that

$$P_n = \frac{1}{2} + \frac{1}{2}(2p - 1)^n \qquad n \geq 0$$

下一日 dry.　　　　下一日 rain

dry $p \cdot P_{n-1} + (1-p)(1-P_{n-1})$

$= pP_{n-1} + 1 - P_{n-1} - p + pP_{n-1}$

$P_n = (2p-1)P_{n-1} + (1-p)$

$P_n = (2p-1)((2p-1)P_{n-2} + 1-p) + 1-p$

$+(2p-1)(1-p) + 1-p$

$= (2p-1)^n + (1-p)\sum_{i=1}^{n-1}(2p-1)^i$

$= (2p-1)^n + (1-p) \times \frac{1-(2p-1)^n}{2(1-p)}$

$= (2p-1)^n + \frac{1}{2}(1-(2p-1)^n)$

$= \frac{1}{2} + \frac{1}{2}(2p-1)^n.$

3.23. A bag contains a white and b black balls. Balls are chosen from the bag according to the following method:

1. A ball is chosen at random and is discarded.
2. A second ball is then chosen. If its color is different from that of the preceding ball, it is replaced in the bag and the process is repeated from the beginning. If its color is the same, it is discarded and we start from step 2.

In other words, balls are sampled and discarded until a change in color occurs, at which point the last ball is returned to the urn and the process starts anew. Let $P_{a,b}$ denote the probability that the last ball in the bag is white. Prove that

$$P_{a,b} = \frac{1}{2}$$

p = discard

Hint: Use induction on $k \equiv a + b$.

$$\frac{a}{a+b} \cdot p$$

$$\frac{P(E\cap F\cap G)}{P(F\cap G)}\frac{P(G\cap F)}{P(F)}+\frac{P(E\cap F\cap G^c)}{P(F\cap G^c)}\frac{P(G^c\cap F)}{P(F)}$$

$$=\frac{P(E\cap F\cap G)}{P(F)}+\frac{P(E\cap F\cap G^c)}{P(F)}$$

$$=\frac{P(E\cap F\cap G)+P(E\cap F\cap G^c)}{P(F)}$$

$$=\frac{P(E\cap F)}{P(F)}$$

$$=P(E|F).$$

3.1. X

3.2. a). $P(>20000 \mid >10000) = \dfrac{0.4+0.1}{0.8}$

$= 0.625$

b). $\dfrac{0.1}{0.8} = 0.125$

3.3. $\dfrac{a}{n} \times \dfrac{1}{2} + \dfrac{1}{2} \times \dfrac{10-a}{20-n} = p$

$\dfrac{1}{2}\left(\dfrac{a}{n} + \dfrac{10-a}{20-n}\right)$

$\dfrac{a(20-n) + 10n - an}{(20-n)n}$

$\dfrac{20a + 10n - 2an}{20n - n^2} = 2p$

$20a + 10n - 2an = 2p(20n - n^2)$

3.4.

P(transferred white | drawn is white)

$$= \frac{P(\text{trans white} \cap \text{drawn white})}{\frac{2}{3}\times\frac{2}{7} + \frac{1}{3}\times\frac{1}{7}}$$

$$= \frac{\frac{2}{3}\times\frac{2}{7}}{\frac{5}{21}} = 0.8 = \frac{4}{5}.$$

3.5 a). $P(R_5) =$

b). $\dfrac{P(R_5 \cap R_3)}{P(R_3)} =$

$P(R_3) =$

3.6. $P(\text{first black} \mid \text{second red})$

$$= \frac{P(\text{first black} \cap \text{second red})}{P(\text{second red})}.$$

$P(\text{second red}) = P(\text{second red} \mid \text{first black})\, P(\text{first black}) +$

$P(\text{second red} \mid \text{first red})\, P(\text{first red})$

$$= \frac{r}{r+b+c} \times \frac{b}{r+b} + \frac{r+c}{r+b+c} \times \frac{r}{r+b}$$

$$\frac{\dfrac{rb}{(r+b)(r+b+c)}}{\dfrac{rb}{(r+b)(r+b+c)} + \dfrac{r(r+c)}{(r+b)(r+b+c)}} \qquad \frac{rb}{rb + r(r+c)} = \frac{b}{b+r+c}.$$

3.7 a). $\dfrac{P(\text{both card } \cap \text{ one of cards} = \{A\})}{P(\text{one of cards} = \{A\})}$

$= \dfrac{\dfrac{2\times 3}{\binom{52}{2}}}{\dfrac{2\times 51}{\binom{52}{2}}}$ $\quad \underline{A}\ \underline{51}. = \dfrac{3}{51}.$

$\dfrac{3}{51}\times\dfrac{4}{52} = \dfrac{3}{51}$

b). $\dfrac{P(\text{both card } A \cap \text{ first} = A)}{P(\text{first} = A)}$ $\quad \underline{A}\ \_$

$\dfrac{4}{52}.$

c). $\dfrac{\frac{4}{52}\times\frac{3}{51}}{\frac{4}{52}\times\frac{3}{51}+\frac{48}{52}\times\frac{4}{51}} = \frac{1}{17} = 0.0588$ $\quad \_\ \underline{A}$

d). $\dfrac{\frac{4}{52}\times\frac{3}{51}+\frac{3}{51}\times\frac{4}{52}}{\frac{4}{52}\times\frac{3}{51}+\frac{48}{52}\times\frac{4}{51}+\frac{4}{52}} = ?$

3.8.

$$\frac{3}{2}$$

3.9 a). $0.9 \times 0.15 + 0.1 \times 0.8 = 0.215$

b).

$$\frac{0.1 \times 0.2}{1 - 0.215} = 0.02548$$

3.10 a). $$1 - \frac{\binom{22}{6}}{\binom{30}{6}} = 0.874341$$

b).

$$\frac{\binom{10}{2}\binom{12}{4}}{\binom{22}{6}} = 0.2985$$

3.11. a). $\frac{8}{14} \times 0.7 + \frac{6}{14} \times 0.4 = \frac{4}{7}$

b). $$\frac{\frac{8}{14} \times 0.3}{\frac{3}{7}} = 0.4.$$

3.12. ✓

3.13 a). $\frac{10}{30}$ b). $\frac{8}{37}$

c). $\frac{10}{36}$ d). P(blue 先 | 绿 lost) P(绿 lost) + P(blue 先 | 红 lost) P(红 lost)

$$= \frac{20}{37} \times \frac{8}{38} + \frac{8}{37} \frac{20}{38}$$

3.14 a).

$$0.8 \times 0.4 \times 0.5 + 0.8 \times 0.6 + 0.2 \times 0.4 \times 0.5$$

$$= 0.68$$

b). p(correct)

p(correct)

$$= (0.8 \times 0.4 \times 0.5 + 0.8 \times 0.6) + (0.2 \times 0.4 \times 0.5 + 0.2 \times 0.6)$$

$$= 0.8$$

c).

$$\frac{0.8 \times 0.4 \times 0.5 + 0.8 \times 0.6}{0.68} = 0.9412.$$

3.15.

Black Black.

B Brown

4). ???

5).

3.18.

$$\frac{P(\text{连续 11次} \cap \text{前10次黑})}{P(\text{前10次黑})}$$

useless.

3.18

HTT
THH

A 赢 H

$P_n = p_1(1-p_2)(1-p_3) + (1-p_1)p_2p_3 +$ 没人: $(p_1p_2p_3)(P_n) +$

AHT

$P_{\alpha,n} = p_1(1-p_2)(1-p_3) + (1-p_1)p_2p_3 +$

$(p_1p_2p_3)P_{\alpha,n-1} + ((1-p_1)(1-p_2)(1-p_3))P_{\alpha,n-1} +$

HHH HHH TTT TTT
TTT HHH TTT HHH

$p_1^2p_2^2p_3^2 P_{\alpha,n-2} + 2p_1p_2p_3(1-p_1)(1-p_2)(1-p_3) +$

$(1-p_1)(\cancel{1-p_2})(\cancel{1-p_3}) +$ $(1-p_2-p_3+p_2p_3)$

$1-p_2-p_3+p_2p_3-p_1+p_1p_2+p_1p_3-p_1p_2p_3$

3.20 $\frac{1}{2}$.

3.21. $P(A>B) = P(A>B \mid A>B \text{ in } n)\, P(A>B \text{ in } n) +$

$P(A>B \mid B>A \text{ in } n)\, P(B>A \text{ in } n) +$

$P(A>B \mid A>B \text{ in } n)\, P(A=B \text{ in } n)$.

$P(A>B \text{ in } n) : \frac{1}{2}$?

$P(A>B \mid A>B \text{ in } n) = 1$.

$P(A>B \mid B>A \text{ in } n) = 0$.

$P(A>B \mid A>B \text{ in } n) = \frac{1}{2}$

$P(A=B \text{ in } n)$.

HTTTT $\left(\binom{5}{1}\left(\frac{1}{2}\right)^5\right)^2$

TTTHT.

$$\sum_{i=1}^{n}\left[\binom{n}{i}\right]^2\left(\frac{1}{2}\right)^{2n}$$

3.22 例,

$P(EF) = P(E)P(F)$

$P(EG) = P(E)P(G)$

$P(E \cap (F \cup G)) = P(E \cap F) + P(E \cap G) - P(EFG)$

$= P(E)P(F) + P(E)P(G) - P(EFG)$

$= P(E)(P(F) + P(G)) - P(EFG)$

X.

w_1 ~~F~~ w_2 E

黑 白

$\frac{1}{2}$ $\frac{1}{2}$.

↑

若黑 flip 两次 G.

黑只说 Head.

$P(F \cap G) = \frac{1}{2} \times \frac{1}{2}$

$P(F) = \frac{1}{2}$

$P(G) = \frac{1}{2} \times \frac{1}{2} + 1$

b). $P(EF) = P(E)P(F)$

$P(EG) = P(E)P(G)$

$P(FG) = \phi$

$P(EFG) = \phi$

$$P(E \cap (F \cup G)) = P(EF) + P(EG) - P(EFG)$$

$$= P(E)P(F) + P(E)P(G)$$

$$= P(E)(P(F) + P(G))$$

$$= P(E)(P(F) + P(G) - P(FG))$$

$$= P(E)(P(F \cup G))$$

True.

3.22 c).

$$P(EF) = P(E)P(F)$$

$$P(FG) = P(F)P(G)$$

$$P(EFG) = P(E)P(FG)$$
$$= P(E)P(F)P(G) = P(G)P(EF)$$

$$P(GEF) = P(G)P(EF)$$

True.

3.23. a). $P(A \cap B) = \phi$

$P(A)P(B) \neq P(A \cap B)$ false.

b). $P(AB) = P(A)P(B)$ false.

c). $P(A) + P(B) - P(A \cap B) < 1$

$1.2 - P(A \cap B) < 1$

$0.2 < P(A \cap B)$

false

d). possibly true

3.24. 1. $\frac{1}{2}$

2. $(0.8)^3$

3. $(0.9)^7$

3.25. P(both defective | first = defective)

P(first = defective) = $\frac{1}{2}\times 0.05+\frac{1}{2}\times 0.01$

$= 0.03$

P(both defective ∩ first = defective) = $\frac{1}{2}\times 0.05^2+\frac{1}{2}\times 0.01^2$

$= 0.04333$

2.26. $\frac{P(A\cap B)}{P(B)} = 1$

$P(A^c \cap B^c) = P(A^c)$

$P(A\cap B) = P(B)$

$1 - P(A\cup B) = 1 - P(A)$

$P(A\cup B) =$

$P(A\cup B) = P(A)$

$P(A)+P(B)-P(A\cap B)$

$P(A)+P(B)-P(A\cap B) = P(A)$

$= P(A)$

$=$

$1 - P(A\cup B) = 1 - P(A)$

$P(A^c\cap B^c) = P(A^c)$

$\frac{P(A^c\cap B^c)}{P(A^c)} = 1$

2.27. exact 1 balls after 1 step =

$\frac{1}{2}$. (抽蓝).

Assume correct.

进行多一個 step:

exactly 1 ball after 2 step =

$\frac{1}{2} \times \frac{2}{3}$

exactly 2 ball after 2 step:

红蓝 + 蓝红.

$\frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{3} = \frac{1}{3}$.

3.28.

P(second no ace | first play has ≥ 1.

$$\frac{\binom{2n-1}{n-1}\binom{2}{1}}{\binom{2n}{n}} + \frac{\binom{2n-2}{n-2}}{\binom{2n}{n}}$$

$$\frac{\cancel{(2n-1)!}}{\cancel{(n-1)!}\,\cancel{n!}}\,\cancel{(2)} \quad \frac{\cancel{n!}\,\cancel{n!}\,\cancel{n}}{\cancel{(2n)!}\;\cancel{2n}}$$

$1 +$

3.24 (3).

$P_1 P_2 P_3 P_4 \cdots P_n$.

(4).

$$\frac{P(E \cap (E \cup F))}{P(E \cup F)} = \frac{P(E)}{P(E \cup F)}$$

$$P(E \mid E \cup F)\, P(E \cup F) + P(E \mid (E \cup F)^c)\, P((E \cup F)^c) =$$

$$P(E)$$