Question 1

10 pts

A2Q1.png

A simplified model for the movement of the price of a stock suppose that on each day the stock's price either moves up 1 unit with probability p or moves down 1 unit with probability $1-p$. The changes on different days are assumed to be independent.

(a): What is the probability that after 2 days the stock will be at its original price?

$2 \quad +1\ p \quad -1\ (1-p)$

$-1 \quad +1$

Hint: only 1 correct choice.

- 2p(1-p)
- $(1-p)^2$
- p^2
- p(1-p)

Question 2

10 pts

$3p^2(1-p)$

$+2-1 \qquad = +1$

In Question 1,

(b): What is the probability that after 3 days the stock's price will have increased by 1 unit?

Hint: only 1 correct choice.

- $2p^2(1-p)$
- 2p(1-p)
- 3p(1-p)
- $2p(1-p)^2$
- $3p^2(1-p)$
- $3p(1-p)^2$

Question 3

50 pts

Suppose that each child born to a couple is equally likely to be a boy or a girl, independently of the sex of the other children in the family. For a couple having 5 children, compute the probabilities of the following events:

(a): All children are of the same sex
(b): The 3 eldest are boys and the others are girls
(c): Exactly 3 are boys
(d): The 2 eldest are girls
(e): There is at least 1 girl.

Hint: keep exactly 4 decimal places after the decimal point

(a) 0.0625 $(\frac{1}{2})^5 \times 2$

(b) 0.0312 $(\frac{1}{2})^5$

(c) 0.3125 $\binom{5}{3} \times (\frac{1}{2})^5$

(d) 0.25 $(\frac{1}{2})^2$

(e) 0.9687 $1 - (\frac{1}{2})^5$

Question 4

60 pts

Two balls are chosen randomly from an urn containing 8 white, 4 black, and 2 orange balls. Suppose that we win \$2 for each black ball selected, win nothing for each orange ball selected and lose \$1 for each white ball selected. Let X denote the winnings
Let p(.) denote X's probability mass function and F(.) denote X's cumulative distribution function. Find the following values.

Hint: keep exactly 4 decimal places after the decimal point

p(-2)= 0.3076 $\quad \dfrac{\binom{8}{2}}{\binom{14}{2}}$

p(-1)= 0.1758 $\quad \dfrac{\binom{8}{1}\binom{2}{1}}{\binom{14}{2}}$

p(0)= 0.0109 $\quad \leftarrow \quad \dfrac{\binom{2}{2}}{\binom{14}{2}}$

p(1)= 0.3516 $\quad \dfrac{\binom{4}{1}\binom{8}{1}}{\binom{14}{2}}$ $\quad +2 \quad -1$

p(4)= 0.0659 $\quad \leftarrow \quad \dfrac{\binom{4}{2}}{\binom{14}{2}}$

p(3)= 0 $\quad \leftarrow \quad 0$

F(-2)= 0.3076 $\quad \dfrac{\binom{8}{2}}{\binom{14}{2}}$

F(-1.2)= 0.3076 $\quad \dfrac{\binom{8}{2}}{\binom{14}{2}}$

F(1.8)= 0.8461 $\quad 2 \quad 0$

F(2.1)= 0.9340 $\quad p(2) = \dfrac{\binom{4}{1}\binom{2}{1}}{\binom{14}{2}}$

F(3.9)= 0.9340

F(4)= 1

Question 5

50 pts

Five distinct number are randomly distributed to players A, B,C,D,E. Whenever two players compare their numbers, the one with the larger number is declared the winner. Initially, players A and B compare their numbers; the winner then compares with player C, and followed by D, and then by E. Let X denote the number of times that the player A is a winner. Let p(.) denote X's probability mass function and F(.) denote X's cumulative distribution function. Find the following values.

Hint: keep exactly 4 decimal places after the decimal point

p(0)= $\frac{1}{2}$

p(1)= $\frac{1}{2}$

p(2)= $\frac{1}{2} \times \frac{1}{3}$

p(3)= $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$

p(4)= $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5}$

F(0)=

F(1.1)=

F(2)=

F(3.99)=

F(10)=

A	B	C
2	1	0
3	1	2
3	2	1
4	1	(2,3)
4	2	(1,3)
4	3	(1,2).
5	1	
5	2	
5	3	
5	4	

3 1
3 2
4 1
4

Question 6 — 20 pts

A total of 4 buses carrying 148 students from the same school arrives at a football stadium. The buses carry, respectively, 40, 33, 25, and 50 students. One of the students is randomly selected. Let X denote the number of students that were on the bus carrying the randomly selected student. One of 4 bus drivers is also randomly selected. Let Y denote the number of the students on her bus. Compute the expectations $E(X)$ and $E(Y)$.

Hint: keep exactly 4 decimal places after the decimal point

E(X)=

$\frac{40}{148} \times 40 + \frac{33}{148} \times 33 \ldots$

E(Y)=

$\frac{1}{4} \times \frac{40}{148} + \frac{1}{4} \times \frac{33}{148} + \ldots$

Question 7 — 10 pts

Suppose that 4 fair dice are rolled. Let M be the minimum of 4 numbers rolled. What are the possible values of M. Find $E(M)$. (*Hint: Find* $P(M \geq k)$ *for any* k*, and use the tail sum formula* $E(M) = \sum_{k=1}^{\infty} P(M \geq k)$)

Hint: keep exactly 4 decimal places after the decimal point

E(M)=

$M = 1, 2, 3, 4, 5, 6$

$P(M \geq 6) = \left(\frac{1}{6}\right)^4$

$P(M \geq k) = \left(\frac{1}{6}\right)^4 \left(\binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{5}{4}\right)$

①: 5 $\binom{4}{1} \times \left(\frac{1}{6}\right)^4$

②: 5 $\binom{4}{2} \times \left(\frac{1}{6}\right)^4$

③: 5 $\binom{4}{3}$

④: 5 $\binom{4}{4}$

$P(M \geq 5) =$

$$P(M \geq 6) = 6666 = \left(\frac{1}{6}\right)^4$$

$$P(M \geq 5) = \left(\frac{(6-5+1)}{6}\right)^4$$

$$P(M \geq 4) = \left(\frac{(6-4+1)}{6}\right)^4$$

Question 8

15 pts

Suppose the number of typos on a page of document follows the Poisson distribution with parameter $\lambda = 1$. Answer the following questions.

(a): What is the probability of having no typos on a page?

(b): What is the probability of having at least two typos on a page?

(c): What is the probability of having no typos in a 5-page document?

≥ 2

$1 - < 2$

Hint: keep exactly 4 decimal places after the decimal point

Hint: use the natural number e=2.7183

(a) e^{-1}

(b) $1 - e^{-1} - e^{-1}$

(c) e^{-5}

Question 9

10 pts

The probability of being dealt a full house in a hand of poker is approximately 0.0014. Find an approximation for the probability that in 1000 hands of poker you will be dealt at least 2 full houses.

1.4

Hint: keep exactly 4 decimal places after the decimal point

Hint: use the natural number e=2.7183

$1 - e^{-1.4} - 1.4e^{-1.4}$

Question 10

20 pts

A sample of 3 items is selected at random from a box containing 20 items of which 4 is defective. Find the expected value and variance of defective items in the sample.

Hint: keep exactly 4 decimal places after the decimal point

$X=0: \frac{\binom{16}{3}}{\binom{20}{3}}$

$X=3: \frac{\binom{4}{3}}{\binom{20}{3}}$

Expected value = 0.6

$X=1: \frac{\binom{4}{1}\binom{16}{2}}{\binom{20}{3}}$

Variance = 0.4294

$X=2: \frac{\binom{4}{2}\binom{16}{1}}{\binom{20}{3}}$

Question 11

15 pts

Two unfair coins are to be flipped. The first coin will land on heads with probability 0.6, the second with probability 0.7. Assume that the results of the flips are independent, and let X be the total number of heads that result.

(a): Find $P(X = 1)$.

(b): Determine $E(X)$ and $\text{Var}(X)$.

Hint: keep exactly 4 decimal places after the decimal point

(a) 0.46

$X=1: 0.6\times 0.3 + 0.4\times 0.7$

(b)

E(X)= 1.3

$X=0: 0.4\times 0.3$

Var(X)= 0.45

$X=2: 0.6\times 0.7$

Question 12

15 pts

On a multiple-choice exam with 3 possible answers for each of the 5 questions.

(a): What is the expected value and variance of the correct answers of a student if she/he answers the question just by guessing?

(b): What is the probability that a student will get 3 or more correct answers just by guessing?

Hint: keep exactly 4 decimal places after the decimal point

(a)

$Bin(5, \frac{1}{3})$

Expected value = $\frac{5}{3}$

Variance = $5(\frac{1}{3})(\frac{2}{3})$

$P(X \geq 3) = P(X=3)\ P(X=4)$

(b) 0.2098