Chapter 2: Problems:

5. A system is comprised of 5 components, each of which is either working or failed. Consider an experiment that consists of observing the status of each component, and let the outcome of the experiment be given by the vector $(x_1, x_2, x_3, x_4, x_5)$, where x_i is equal to 1 if component i is working and is equal to 0 if component i is failed.

(a) How many outcomes are in the sample space of this experiment?

(b) Suppose that the system will work if components 1 and 2 are both working, or if components 3 and 4 are both working, or if components 1, 3, and 5 are all working. Let W be the event that the system will work. Specify all the outcomes in W.

(c) Let A be the event that components 4 and 5 are both failed. How many outcomes are contained in the event A?

(d) Write out all the outcomes in the event AW.

6. A hospital administrator codes incoming patients suffering gunshot wounds according to whether they have insurance (coding 1 if they do and 0 if they do not) and according to their condition, which is rated as good (g), fair (f), or serious (s). Consider an experiment that consists of the coding of such a patient.

(a) Give the sample space of this experiment.

(b) Let A be the event that the patient is in serious condition. Specify the outcomes in A.

(c) Let B be the event that the patient is uninsured. Specify the outcomes in B.

(d) Give all the outcomes in the event $B^c \cup A$.

10. Sixty percent of the students at a certain school wear neither a ring nor a necklace. Twenty percent wear a ring and 30 percent wear a necklace. If one of the students is chosen randomly, what is the probability that this student is wearing

(a) a ring or a necklace?

(b) a ring and a necklace?

16. Poker dice is played by simultaneously rolling 5 dice. Show that

(a) $P\{\text{no two alike}\} = .0926$;

(b) $P\{\text{one pair}\} = .4630$;

(c) $P\{\text{two pair}\} = .2315$;

(d) $P\{\text{three alike}\} = .1543$;

(e) $P\{\text{full house}\} = .0386$;

(f) $P\{\text{four alike}\} = .0193$;

(g) $P\{\text{five alike}\} = .0008$.

18. Two cards are randomly selected from an ordinary playing deck. What is the probability that they form a blackjack? That is, what is the probability that one of the cards is an ace and the other one is either a ten, a jack, a queen, or a king?

28. An urn contains 5 red, 6 blue, and 8 green balls. If a set of 3 balls is randomly selected, what is the probability that each of the balls will be (a) of the same color? (b) of different colors? Repeat under the assumption that whenever a ball is selected, its color is noted and it is then replaced in the urn before the next selection. This is known as *sampling with replacement*.

35. Seven balls are randomly withdrawn from an urn that contains 12 red, 16 blue, and 18 green balls. Find the probability that

(a) 3 red, 2 blue, and 2 green balls are withdrawn;

(b) at least 2 red balls are withdrawn;

(c) all withdrawn balls are the same color;

(d) either exactly 3 red balls or exactly 3 blue balls are withdrawn.

37. An instructor gives her class a set of 10 problems with the information that the final exam will consist of a random selection of 5 of them. If a student has figured out how to do 7 of the problems, what is the probability that he or she will answer correctly

(a) all 5 problems?

(b) at least 4 of the problems?

39. There are 5 hotels in a certain town. If 3 people check into hotels in a day, what is the probability that they each check into a different hotel? What assumptions are you making?

Chapter 2: Theoretical exercises.

11. If $P(E) = .9$ and $P(F) = .8$, show that $P(EF) \geq .7$. In general, prove Bonferroni's inequality, namely,

$$P(EF) \geq P(E) + P(F) - 1$$

12. Show that the probability that exactly one of the events E or F occurs equals $P(E) + P(F) - 2P(EF)$.

13. Prove that $P(EF^c) = P(E) - P(EF)$.