Question 1   10 pts

Q1: A well-known nursery rhyme starts as follows
" As I was going to St. Ives
I met a man with 7 wives.
Each wife had 7 sacks.
Each sack had 7 cats.
Each cat had 7 kittens..."
How many kittens did the traveler meet?

7^4

Input your answers in the following box (Hint: input an integer)

2401

Question 2   10 pts

Q2: Consider a group of 20 people. If everyone shakes hands once with everyone else, how many handshakes take place?

Input your answers in the following box (Hint: input an integer)

190

A
B
C
D
E

AB BC CD
AC BD CE
AD BE
AE
        DE

4+3+2+1

ABCDEF

19+18+17+ ... +1

$\frac{20(19)}{2}$

Question 3   30 pts

Q3: How many different letter arrangements can be made from the following letters: (a) fluke, (b) propose, (c) arrange.

Input your answers in the following box (**Hint: input an integer**)

(a) $\boxed{5!}$

$= 120$

(b) $\boxed{}$

$\dfrac{7!}{2!\,2!} = 1260$

(c) $\boxed{}$
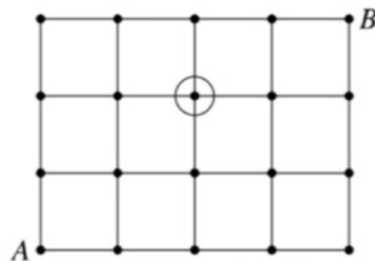
$\dfrac{7!}{2!\,2!} = 1260$

Question 4   30 pts

Q4: Consider the grid of points shown below. Suppose that, starting at the point labelled A, you can go one step up or one step to the right at each move. This procedure is completed until the points labelled B is reached.

(i): How many possible paths from A to B are possible?
(ii): How many possible paths are there from A to B that go through the point circled in the grid?
(iii): How many possible paths are there from A to B that avoids the point circled in the grid?

$\binom{7}{3} = 35$

$\binom{4}{2} \times \binom{3}{2} = 18$

$\binom{7}{3} - \binom{4}{2} \times \binom{3}{2} = 17$

Input your answers in the following box (**Hint: input an integer**)

Question 5   20 pts

Q5: From a group of 8 women and 6 men, a committee consisting of 3 men and 3 women is to be formed. How many different committees are possible if
(i): 2 of men refuse to serve together?
(ii): 1 man and 1 woman refuse to serve together?

Input your answers in the following box (Hint: input an integer)

(i)

(ii)

解 AB - 符

i) $\binom{8}{3}\binom{6}{3} - \binom{8}{3}\binom{4}{1} = 896$

ii) $\binom{8}{3}\binom{6}{3} - \binom{7}{2}\binom{5}{2} = 910$

Question 6   40 pts

Poker dice is played by simultaneously rolling 5 dice. Compute the following probability:

(i) $P(\{\text{no two alike}\})$ (ii) $P(\{\text{one pair}\})$
(iii) $P(\{\text{two pair}\})$ (iv) $P(\{\text{three alike}\})$

Input your answers in the following box (Hint: keep exactly 4 decimal places after the decimal point)

(i) 0.0925

(ii) 0.4629

(iii) 0.4629

(iv) 0.1543

i). $\dfrac{6 \times 5 \times 4 \times 3 \times 2}{6^5} = \dfrac{5}{54}$

ii). $\dfrac{\binom{5}{2} \times 6 \times 5 \times 4 \times 3}{6^5} = \dfrac{25}{54}$

iii). $\dfrac{\binom{5}{2} \times 6 \times \binom{3}{2} \times 6 \times 5 \times 4}{6^5} = \dfrac{25}{54}$

iv). $\dfrac{\binom{5}{3} \times 6 \times 5 \times 4}{6^5}$

Question 7   10 pts

Two fair dice are rolled. What is the probability that the second die lands on a higher value than the first?

Input your answers in the following box (Hint: keep exactly 4 decimal places after the decimal point)

$$\frac{5+(4)+7(4)}{36}$$

0.4166

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1  2        2  3      2  4     4  5     5  6
1  3        2  4      3  5     4  6
1  4        2  5      3  6
1  5        2  6
1  6
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Question 8   10 pts

Two fair dice are rolled. What is the probability that the sum of two numbers is even or larger than 7?

Input your answers in the following box (Hint: keep exactly 4 decimal places after the decimal point)

$$\frac{23+24}{36}$$

0.6388

0-6666

```
11   22   31   42   51   62
13   24   33   44   53   64
15   26   35   46   55   66
          36   45   54   63
                    56   65
```

Question 9   10 pts

Two fair dice are rolled. What is the conditional probability that at least one lands on 6 given that the dice land on different numbers?

Input your answers in the following box (Hint: keep exactly 4 decimal places after the decimal point)

=1.3333

$P(\geqslant 1$

$1 \text{ different number })$

1 1   2 1

1 2   3 1

1 3   4 1

1 4   5 1

1 5   6 1

1 6

$\boxed{36-6}$

$\dfrac{30}{36}.$

$$\dfrac{10}{30} = \dfrac{1}{3}$$

$$= 0.37$$

Question 10   40 pts

Two fair dice are rolled. What is the probability that at least one lands on 6, given that the sum of two dice is $i = 5, 6, 7, 8$?

Input your answers in the following box (Hint: keep exactly 4 decimal places after the decimal point)

i=5, 0

i=6, 0

i=7, $\frac{1}{3} \approx 0.3333$

i=8, $\frac{2}{5} = 0.4$

Question 11   10 pts

Consider 3 urns. Urn A contains 2 white and 4 red balls; urn B contains 8 white and 4 red balls; and urn C contains 1 white and 3 red balls. If 1 ball is selected from each urn, what is the probability that the ball chosen from urn A was white, given that exactly 2 white balls were selected?

Input your answers in the following box (Hint: keep exactly 4 decimal places after the decimal point)

$$P(\quad \geqslant 1 \div 6 \quad | \text{ sum} = 5,6,7,8)$$

$$= \quad \frac{16,61}{} \qquad \frac{2}{6}$$

16
2 5
3 4
4 3
5 2
6 1

$$\frac{26,62}{}$$

2 6
3 5
4 4
5 3
6 2

Consider 3 urns. Urn A contains 2 white and 4 red balls; urn B contains 8 white and 4 red balls: and urn C contains 1 white and 3 red balls. If 1 ball is selected from each urn, what is the probability that the ball chosen from urn A was white, given that exactly 2 white balls were selected?

Input your answers in the following box (Hint: keep exactly 4 decimal places after the decimal point)

0.64?0   0.6363

$P(\ A: white \ |\ 2\ white\ balls\ selected\ )$

$ABC'\quad \frac{2}{6}\times\frac{8}{12}\times\frac{3}{4}\ +$

$ACB'\quad \frac{2}{6}\times\frac{1}{4}\times\frac{4}{12}\ +$

$BCA'\quad \frac{8}{12}\times\frac{1}{4}\times\frac{4}{6}$

$=$

$\dfrac{\frac{11}{36}}{\frac{17}{36}} = \dfrac{11}{17}.$

$= \dfrac{7}{36}+\dfrac{1}{9}$

$= \dfrac{11}{36}.$

Question 12   20 pts

Two cards are randomly chosen without replacement, from an ordinary deck of 52 playing cards. Let B be the event that both cards are aces, let A_s be the event that the ace of spades is chosen, and let A be the event that at least one ace is chosen. Find (i): $P(B|A_s)$, (ii): $P(B|A)$.

Input your answers in the following box (**Hint: keep exactly 4 decimal places after the decimal point**)

(i) $\boxed{\frac{1}{17} \doteq 0.0588}$

$$P(A_s) = \frac{1}{52} + \frac{51}{52} \times \frac{1}{51} = \frac{1}{26}$$

$$P(B \cap A_s) = \frac{1}{52} \times \frac{3}{51} + \frac{3}{52} \times \frac{1}{51} = \frac{1}{442}$$

(ii) $\boxed{\frac{1}{33} \doteq 0.0303}$

$$P(A) = \text{ALL} - 0 \text{ Aces chosen}$$
$$= 1 - \frac{48}{52} \times \frac{47}{51} = \frac{33}{221}$$

$$P(B \cap A) = \frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$$

Question 13   10 pts

An urn initially contains 5 white and 7 black balls. Each time a ball is selected, its color is noted and it is replaced in the urn along with 2 other balls of the same color. Compute the probability that the first 2 balls selected are black and the next 2 are white.

Input your answers in the following box (**Hint: keep exactly 4 decimal places after the decimal point**)

$$\frac{7}{12} \times \frac{9}{14} \times \frac{5}{16} \times \frac{7}{18}$$

$\boxed{0.0455}$