4.14. Five distinct numbers are randomly distributed to players numbered 1 through 5. Whenever two players compare their numbers, the one with the higher one is declared the winner. Initially, players 1 and 2 compare their numbers; the winner then compares her number with that of player 3, and so on. Let X denote the number of times player 1 is a winner. Find $P\{X = i\}, i = 0, 1, 2, 3, 4$.

首先，p(X=0)係player 1一開始就輸比2，概率係1/2
p(X=1)係player1贏2，概率係1/2，之後輸比3，概率係1/3，因為

4.15. The National Basketball Association (NBA) draft lottery involves the 11 teams that had the worst won–lost records during the year. A total of 66 balls are placed in an urn. Each of these balls is inscribed with the name of a team: Eleven have the name of the team with the worst record, 10 have the name of the team with the second-worst record, 9 have the name of the team with the third-worst record, and so on (with 1 ball having the name of the team with the 11th-worst record). A ball is then chosen at random, and the team whose name is on the ball is given the first pick in the draft of players about to enter the league. Another ball is then chosen, and if it "belongs" to a team different from the one that received the first draft pick, then the team to which it belongs receives the second draft pick. (If the ball belongs to the team receiving the first pick, then it is discarded and another one is chosen; this continues until the ball of another team is chosen.) Finally, another ball is chosen, and the team named on the ball (provided that it is different from the previous two teams) receives the third draft pick. The remaining draft picks 4 through 11 are then awarded to the 8 teams that did not "win the lottery," in inverse order of their won–lost records. For instance, if the team with the worst record did not receive any of the 3 lottery picks, then that team would receive the fourth draft pick. Let X denote the draft pick of the team with the worst record. Find the probability mass function of X.

這題目講嘅係一個箱入面有66個波，11個係比一號嘅，
10個係比2號嘅如此類推，第一抽就出第一個幸運兒，但
第二抽如果抽到同第一抽一樣，要棄波直至抽到不同為
止
X係一號仔，問x的pmf

15. $P\{X=1\} = 11/66$

$$P\{X=2\} = \sum_{j=2}^{11}\left(\frac{12-j}{66}\right)\left(\frac{11}{54+j}\right)$$

$$P\{X=3\} = \sum_{\substack{k\neq 1\\ k\neq j}}\sum_{j=2}\left(\frac{12-j}{66}\right)\left(\frac{12-k}{54+j}\right)\left(\frac{11}{42+j+k}\right)$$

$$P\{X=4\} = 1-\sum_{i=1}^{3}P\{X=1\}$$

x=1簡單
x=2係因為 1/66*11/54抽中11號仔再到一號仔 +
2/66*（1/55*11/64+11/65）抽中10號仔之後抽到10
號仔嘅概率+，然後約簡發現其實係2/66*（11/

$$\frac{1}{66}\cdot\frac{11}{65}+\frac{2}{66}\left(\frac{1}{65}\times\frac{11}{64}+\frac{11}{65}\right)$$

$$\left(\frac{11+11\cdot 64}{64\cdot 65}\right)$$

$$\frac{1}{66}\cdot\frac{11}{65}+\frac{2}{66}\,\frac{11}{64}$$

4.15. The National Basketball Association (NBA) draft lottery involves the 11 teams that had the worst won–lost records during the year. A total of 66 balls are placed in an urn. Each of these balls is inscribed with the name of a team: Eleven have the name of the team with the worst record, 10 have the name of the team with the second-worst record, 9 have the name of the team with the third-worst record, and so on (with 1 ball having the name of the team with the 11th-worst record). A ball is then chosen at random, and the team whose name is on the ball is given the first pick in the draft of players about to enter the league. Another ball is then chosen, and if it "belongs" to a team different from the one that received the first draft pick, then the team to which it belongs receives the second draft pick. (If the ball belongs to the team receiving the first pick, then it is discarded and another one is chosen; this continues until the ball of another team is chosen.) Finally, another ball is chosen, and the team named on the ball (provided that it is different from the previous two teams) receives the third draft pick. The remaining draft picks 4 through 11 are then awarded to the 8 teams that did not "win the lottery," in inverse order of their won–lost records. For instance, if the team with the worst record did not receive any of the 3 lottery picks, then that team would receive the fourth draft pick. Let X denote the draft pick of the team with the worst record. Find the probability mass function of X.

$P(x=1)=\frac{11}{66}$

①

$P(x\geq 2)=$ P(最终抽中你 ∩ 没抽中(1)) +

~~P(最…… ∩ 不中(1))~~

$P(①)=\frac{2}{66}\times\left(\frac{1}{65}\times\frac{11}{64}+\frac{11}{65}\right)$

$\frac{1}{66}\times\frac{11}{65}+\frac{2}{66}\times\frac{11}{64}+$

$\frac{3}{66}\times\left(\frac{2}{65}\times\frac{1}{64}\times\frac{11}{63}+\frac{2}{65}\times\frac{11}{64}+\frac{11}{63}\right)$

4.24. A and B play the following game: A writes down either number 1 or number 2, and B must guess which one. If the number that A has written down is i and B has guessed correctly, B receives i units from A. If B makes a wrong guess, B pays $\frac{3}{4}$ unit to A. If B randomizes his decision by guessing 1 with probability p and 2 with probability $1 - p$, determine his expected gain if (a) A has written down number 1 and (b) A has written down number 2.

What value of p maximizes the minimum possible value of B's expected gain, and what is this maximin value? (Note that B's expected gain depends not only on p, but also on what A does.)

Consider now player A. Suppose that she also randomizes her decision, writing down number 1 with probability q. What is A's expected loss if (c) B chooses number 1 and (d) B chooses number 2?

What value of q minimizes A's maximum expected loss? Show that the minimum of A's maximum expected loss is equal to the maximum of B's minimum expected gain. This result, known as the minimax theorem, was first established in generality by the mathematician John von Neumann and is the fundamental result in the mathematical discipline known as the theory of games. The common value is called the value of the game to player B.

4.27. An insurance company writes a policy to the effect that an amount of money A must be paid if some event E occurs within a year. If the company estimates that E will occur within a year with probability p, what should it charge the customer in order that its expected profit will be 10 percent of A?

我列的式：

$$p(A-c)+(1-p)C = 0.1A$$

實際 ans：

$$C - Ap = 0.1A.$$

4.66. A total of $2n$ people, consisting of n married couples, are randomly seated (all possible orderings being equally likely) at a round table. Let C_i denote the event that the members of couple i are seated next to each other, $i = 1, \ldots, n$.

(a) Find $P(C_i)$.

(b) For $j \neq i$, find $P(C_j|C_i)$.

(c) Approximate the probability, for n large, that there are no married couples who are seated next to each other.

a). 坐圓圈：分母係 $(2n-1)!$，分子是 fix 2個人，其他 $(2n-2)!$ 亂排. $\therefore \frac{2}{2n-1}$.

b). $$\frac{P(C_j C_i)}{P(C_i)} = \frac{2\times 2\times \frac{(2n-3)!}{(2n-1)!}}{\frac{2}{2n-1}} = \frac{2\times 2}{(2n-1)(2n-2)} \cdot \frac{2n-1}{2} = \frac{2}{2n-2}$$

c). mean $= n\left(\frac{2}{2n-1}\right)$

$$e^{-\frac{2n}{2n-1}}$$

4.67. Repeat the preceding problem when the seating is random but subject to the constraint that the men and women alternate.

4.69. A fair coin is flipped 10 times. Find the probability that there is a string of 4 consecutive heads by

(a) using the formula derived in the text;

(b) using the recursive equations derived in the text.

(c) Compare your answer with that given by the Poisson approximation.

69. With P_j equal to the probability that 4 consecutive heads occur within j flips of a fair coin, $P_1 = P_2 = P + 3 = 0$, and

$P_4 = 1/16$
$P_5 = (1/2)P_4 + 1/16 = 3/32$
$P_6 = (1/2)P_5 + (1/4)P_4 + 1/16 = 1/8$
$P_7 = (1/2)P_6 + (1/4)P_5 + (1/8)P_4 + 1/16 = 5/32$
$P_8 = (1/2)P_7 + (1/4)P_6 + (1/8)P_5 + (1/16)P_4 + 1/16 = 6/32$
$P_9 = (1/2)P_8 + (1/4)P_7 + (1/8)P_6 + (1/16)P_5 + 1/16 = 111/512$
$P_{10} = (1/2)P_9 + (1/4)P_8 + (1/8)P_7 + (1/16)P_6 + 1/16 = 251/1024 = .2451$

The Poisson approximation gives

$$P_{10} \approx 1 - \exp\{-6/32 - 1/16\} = 1 - e^{-.25} = .2212$$

4.72. Two athletic teams play a series of games; the first team to win 4 games is declared the overall winner. Suppose that one of the teams is stronger than the other and wins each game with probability .6, independently of the outcomes of the other games. Find the probability, for $i = 4, 5, 6, 7$, that the stronger team wins the series in exactly i games. Compare the probability that the stronger team wins with the probability that it would win a 2-out-of-3 series.

72. $$P\{\text{wins in } i \text{ games}\} = \binom{i-1}{3}(.6)^4(.4)^{i-4}$$

因為第i場一定是win,

i-1場選3場即可.

START

4.4. $P(x=1)=9!/10! = 1/10$
$P(x=2)=8!/10!$
$P(X=5)=5!/10!$
$P(x=6)=0$

4.5
$-n,-n+2,......n-2,n$

4.6
$P(X=-3)=1/8$
$P(X=-1)=3*1/8$
$P(X=1)=3*1/8$
$P(x=3)=1/8$

4.11a).floor(10^3/3)/10^3,larger = 1/3
b)?????

4.12

A

伸,出
1,1
1,2
2,1
2,2

B

伸,出
1,1
1,2
2,1
2,2

b) 0

1111 → 0
1112 → 0
1121 → 2
1122 → 0

1211 → 3
1212 → 0
1221 → 0
1222 → 0

2111 → 1
2112 → 0
2121 → 0
2122 → 3

2211 → 0
2212 → 3
2221 → 0
2222 → 0

X: 0 2 3

$\frac{12}{16}$ $\frac{1}{16}$ $\frac{3}{16}$

b).

$X = 0 \quad 500 \quad 1000 \quad 1500 \quad 2000$

$2\times(0.3\times\frac{1}{2}+0.6\times\frac{1}{2}) \quad 0.3\times\frac{1}{2}+0.6\times\frac{1}{2}$

$P(X=1000) =$ 1 appent $\to 1000$ $\quad 0.3\times\frac{1}{2}\times0.4+$

$0.7\times\frac{1}{2}\times0.6+$

2 appent $\to 500$ $\quad 0.3\times0.6\times(\frac{1}{2})^2$

$P(X=500) =$ 1 appent $\to 500$ $\quad 0.3\times\frac{1}{2}\times0.4+$

$0.7\times\frac{1}{2}\times0.6$

$P(X=0)$ $\quad 1-2\times50\%$ $\quad 0.7\times0.4.$

4.14.

$P(A>B>C)$ $\binom{5}{3}\times 2$

$P(A>C>B)$

$P(X=1)=\frac{11}{66}$

$P(X=2)=\frac{55}{66}\times\frac{11}{65}+$

1st team choose fan zi gei:0
2nd=2/66*1/65
3rd=3/66*2/65
4th=4/66*3/65
sum 1 to 10= $\sum_{i=2}^{10}\frac{i}{66}\cdot\frac{i-1}{65}=\sum_{i=2}^{10}\frac{i^2-i}{66\cdot 65}=$

4.15

4.16. In Problem 15, let team number 1 be the team with the worst record, let team number 2 be the team with the second-worst record, and so on. Let Y_i denote the team that gets draft pick number i. (Thus, $Y_1 = 3$ if the first ball chosen belongs to team number 3.) Find the probability mass function of (a) Y_1, (b) Y_2, and (c) Y_3.

4.17. Suppose that the distribution function of X is given by

$$F(b) = \begin{cases} 0 & b < 0 \\ \dfrac{b}{4} & 0 \le b < 1 \\ \dfrac{1}{2} + \dfrac{b-1}{4} & 1 \le b < 2 \\ \dfrac{11}{12} & 2 \le b < 3 \\ 1 & 3 \le b \end{cases}$$

(a) Find $P\{X = i\}, i = 1, 2, 3$.
(b) Find $P\{\frac{1}{2} < X < \frac{3}{2}\}$.

4.18. Four independent flips of a fair coin are made. Let X denote the number of heads obtained. Plot the probability mass function of the random variable $X - 2$.

4.19. If the distribution function of X is given by

$$F(b) = \begin{cases} 0 & b < 0 \\ \frac{1}{2} & 0 \le b < 1 \\ \frac{3}{5} & 1 \le b < 2 \\ \frac{4}{5} & 2 \le b < 3 \\ \frac{9}{10} & 3 \le b < 3.5 \\ 1 & b \ge 3.5 \end{cases}$$

4.

calculate the probability mass function of X.

4.20. A gambling book recommends the following "winning strategy" for the game of roulette: Bet \$1 on red. If red appears (which has probability $\frac{18}{38}$), then take the \$1 profit and quit. If red does not appear and you lose this bet (which has probability $\frac{20}{38}$ of occurring), make additional \$1 bets on red on each of the next two spins of the roulette wheel and then quit. Let X denote your winnings when you quit.

(a) Find $P\{X > 0\}$.

(b) Are you convinced that the strategy is indeed a "winning" strategy? Explain your answer!

(c) Find $E[X]$.

$$P(X=1) = \frac{18}{38} + \frac{20}{38} \times \left(\frac{18}{38}\right)^2 = 0.5917$$

b). NO.

c). $E(x) = \quad 1 \times 0.5917 - \frac{20}{38}(1)$

$\Big(\qquad\qquad\qquad \left(\frac{20}{38}\right)^2 \times 2 \Big) = \quad 0.3$

WL $\qquad$ LW $\qquad$ LL

???

4.21. $P(X=40) = \frac{40}{148} \times 40$

$=$

$P(X=33)$

$P(X=25)$

$P(X=50)$

$E(X)$ 39.28

$\frac{1}{4} \times 40 + \frac{1}{4} \times 33 + \frac{1}{4} \times 25 + \frac{1}{4} \times 50$

$E(Y)$. 37.

4.22.

(a).

X denote no. of game played

when $i=2$,

$p(X=2) = p^2 + (1-p)^2$

ABA

BAA

BAB

ABB

$p(X=3) = 2p(1-p)^2 + 2p^2(1-p)$

$p(X=4) = 2p^2(1-p)^2$

ABAB

$2p^2 + 2(1-p)^2 + 6p(1-p)^2 + 6p^2(1-p) + 8p^2(1-p)^2$

$$2p^2+2(1-2p+p^2)+6p(1-2p+p^2)+6p^2-6p^3+8p^2(1-2p+p^2)$$

$$= 2p^2+2-4p+2p^2+6p-12p^2+6p^3+6p^2-6p^3+8p^2-16p^3+8p^4$$

$$= 8p^4+16p^3+6p^2+2p+2$$

$$32p^3+48p^2+12p+2=0$$

$$32p^3+48p^2+12p=-2$$

4.23.

$p(X=-1)=\frac{1}{2}$

$p(X=-4)=\frac{1}{2}$.

$E(X)=-\frac{1}{2}+(-4)\times\frac{1}{2}$

$=-\frac{5}{2}$.

Expected money > 2, remain.

b). now = 500 ounce.

$p(X=1000)=\frac{1}{2}$

$p(X=250)=\frac{1}{2}$

$E(X)=\frac{1}{2}\times1000+\frac{1}{2}\times250$

$=500+125=625$

Expected ounce > 500,

~~buy~~ wait

4.24. $P(\text{guess correct} \mid A=1)$

$$= \frac{p}{\frac{1}{2}} = 2p$$

$P(\text{guess wrong} \mid A=1)$

$$= \frac{1-p}{\frac{1}{2}} = 2-2p.$$

$$E(X_1) = 2p \times 1 - (2-2p)\left(\frac{3}{4}\right)$$

$$= 2p - \frac{3}{2} + \frac{3}{2}p$$

$$= \frac{7}{4}p - \frac{3}{2}$$

$$P(\text{guess correct} \mid A=2) = \frac{1-p}{\frac{1}{2}} = 2-2p$$

$$P(\quad " \quad A=1) = 2p$$

$$E(X_2) = (2-2p) \times 2 - 2p\left(\frac{3}{4}\right)$$

$$= 4 - 4p - \frac{3}{2}p$$

$$= 4 - \frac{11}{2}p$$

4.24.

if 0). $p - (1-p)\frac{3}{4} = p - \frac{3}{4} + \frac{3}{4}p = \frac{7}{4}p - \frac{3}{4}$

if 2 b). $2(1-p) - p\frac{3}{4} = 2 - 2p - \frac{3}{4}p = 2 - \frac{11}{4}p$

$$\frac{7}{4}p - \frac{3}{4} = 2 - \frac{11}{4}p$$

$$\frac{18}{4}p = \frac{11}{4}$$

$$18p = 11$$

$$p = \frac{11}{18}$$

? ?

4.25. a).

$$0.6\times0.3 + 0.7\times0.4 = 0.46$$

$$P(X=0) = 0.4\times0.3 = 0.12$$

$$P(X=2) = 0.6\times0.7 = 0.42.$$

b). $E(X) = 0.46 + 2\times0.42 = 1.3$

4.26 a).

$$P(X=1) = \frac{1}{10}$$

$$P(X=2) = \frac{1}{10} \qquad E(X) = \frac{55}{10}$$

$P(X=1) = 0$ $\qquad$ $10 \xrightarrow{1} 5 \xrightarrow{2} 2 \xrightarrow{3} 1$

$P(X=$

3 $\qquad$???

4.27.

4.27.

$$p(A-c) + (1-p)C = 0.1A$$

$$Ap - cp + C - Cp = 0.1A$$

$$Ap + c - 2cp = 0.1A$$

$$c(1-2p) = 0.1 - Ap$$

$$C = \frac{0.1 - Ap}{1-2p}$$

X A X
A A A
X X A
X X A
X A A

4.28.

$$P(X=0) = \frac{\binom{16}{3}}{\binom{20}{3}}$$

$$P(X=1) = \frac{\binom{16}{2}\binom{4}{1}}{\binom{20}{3}}$$

$$P(X=2) = \frac{\binom{16}{1}\binom{4}{2}}{\binom{20}{3}}$$

$$P(X=3) = \frac{\binom{4}{3}}{\binom{20}{3}}$$

$$E(X) = 0.6.$$

4.29.

4.29. There are two possible causes for a breakdown of a machine. To check the first possibility would cost C_1 dollars, and, if that were the cause of the breakdown, the trouble could be repaired at a cost of R_1 dollars. Similarly, there are costs C_2 and R_2 associated with the second possibility. Let p and $1 - p$ denote, respectively, the probabilities that the breakdown is caused by the first and second possibilities. Under what conditions on $p, C_i, R_i, i = 1, 2$, should we check the first possible cause of breakdown and then the second, as opposed to reversing the checking order, so as to minimize the expected cost involved in returning the machine to working order?

$P(Y = C_1 + C_2 + R_2) = 1-p$

$P(X = C_1 + R_1) = p$

$(C_1 + C_2 + R_2)(1-p) + p$

$(C_1 +$

Check 两个

$E(X)$. compare.

4.30. A person tosses a fair coin until a tail appears for the first time. If the tail appears on the nth flip, the person wins 2^n dollars. Let X denote the player's winnings. Show that $E[X] = +\infty$. This problem is known as the St. Petersburg paradox.

(a) Would you be willing to pay \$1 million to play this game once?

(b) Would you be willing to pay \$1 million for each game if you could play for as long as you liked and only had to settle up when you stopped playing?

4.31. Each night different meteorologists give us the probability that it will rain the next day. To judge how well these people predict, we will score each of them as follows: If a meteorologist says that it will rain with probability p, then he or she will receive a score of

$$\begin{array}{ll} 1 - (1 - p)^2 & \text{if it does rain} \\ 1 - p^2 & \text{if it does not rain} \end{array}$$

We will then keep track of scores over a certain time span and conclude that the meteorologist with the highest average score is the best predictor of weather. Suppose now that a given meteorologist is aware of our scoring mechanism and wants to maximize his or her expected score. If this person truly believes that it will rain tomorrow with probability p^*, what value of p should he or she assert so as to maximize the expected score?

4.32. To determine whether they have a certain disease, 100 people are to have their blood tested. However, rather than testing each individual separately, it has been decided first to place the people into groups of 10. The blood samples of the 10 people in each group will be pooled and analyzed together. If the test is negative, one test will suffice for the 10 people, whereas if the test is positive, each of the 10 people will also be individually tested and, in all, 11 tests will be made on this group. Assume that the probability that a person has the disease is .1 for all people, independently of each other, and compute the expected number of tests necessary for each group. (Note that we are assuming that the pooled test will be positive if at least one person in the pool has the disease.)

$X=0$: $\binom{10}{0}\left(\frac{2}{3}\right)^{10} \rightarrow -10 \times a$

$X=1$: $\binom{10}{1}\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^{9} \quad -10 \times (a-1)+15$

$15\left(\frac{10}{3}\right)-10a$

9 4

Expected sell

Buy

$\frac{150}{3}-10a=0$

$5=a$

4.35(a). $P(\text{same})=\dfrac{\binom{5}{2}\times 2}{\binom{10}{2}}=\dfrac{4}{9}$

$E(X)=1.1\times\frac{4}{9}-1\times\frac{5}{9}=-\frac{1}{15}.$

$Var=\left(1.1^2\times\frac{4}{9}-1\times\frac{5}{9}\right)-\left(-\frac{1}{15}\right)^2$

$=$

$-\frac{1}{45}$

4.38 $E(4+4x+x^2)$

$= 4+4(1)+E(x^2)$

$= 4+4+6$

$= 14.$

$Var(x) = 5$

$E(x^2) - E(x)^2 = 5$

$E(x^2) = 6$

(b) $Var(4+3\cdot x) = Var(3x) = 9\,Var(x)$

$= 9 \times 5 = 45$

4.39.

$$\binom{4}{2}\left(\frac{3}{6}\right)\left(\frac{2}{5}\right)\times\left(\frac{3}{4}\right)$$

4.40.

$$p(X\geq 4) = p(X=4)+p(X=5)$$

$$= \binom{5}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)+\left(\frac{1}{3}\right)^5$$

$$= \frac{11}{243} = 0.04527$$

4.41. $P(\text{would have} \geq 7 \mid \text{no ESP})$

$$= p(X=7)+p(X=8)+p(X=9)+p(X=10)$$

$$= \binom{10}{7}\left(\frac{1}{2}\right)^{10}+\binom{10}{8}\left(\frac{1}{2}\right)^{10}+\binom{10}{9}\left(\frac{1}{2}\right)^{10}+\left(\frac{1}{2}\right)^{10}$$

$$= \left(\frac{1}{2}\right)^{10}\left(\binom{10}{7}+\binom{10}{8}+\binom{10}{9}+1\right)$$

$$= 0.171875$$

4.47. $P(\text{success for } 5) = P(X=3) + P(X=4) + P(X=5)$

$$= \binom{5}{3} p^3 (1-p)^2 +$$

$$\binom{5}{4} p^4 (1-p) +$$

$$p^5$$

$P(\text{success for } 3) = P(X=2) + P(X=3)$

$$= \binom{3}{2} p^2 (1-p) +$$

$$p^3$$

$$p^2 (1-p) \left(\binom{5}{3} p (1-p) + \binom{5}{4} p^2 \right) + p^3 (p^2)$$

$$10p - 10p^2 + 5p^2$$

$$10p - 5p^2 > 3$$

$$1 > p^2$$

$P(\text{fail}) = 0, 1, 2..$

$$\binom{5}{0}(1-p)^5 + \binom{5}{1}p(1-p)^4 + \binom{5}{2}p^2(1-p)^3$$

$3 - 6p > 0$

$3 > 6p$

$\frac{1}{2} > p$

$p < \frac{1}{2}.$

$P(\text{fail for } 3) = 0, 1$

$$(1-p)^3 + \binom{3}{1}p(1-p)^2$$

$$(1-p)^5 + 5p(1-p)^4 + 10p^2(1-p)^3 > (1-p)^3 + 3p(1-p)^2$$

$$(1-p)^3 + 5p(1-p)^2 + 10p^2(1-p) > (1-p) + 3p$$

$$1 - 3p + 3p^2 - p^3 + 5p(1-2p+p^2) + 10p^2 - 10p^3 > 1 + 2p$$

$$\cancel{1-3p} + 3p^2 - p^3 \cancel{+5p} \cancel{-10p^2} + 5p^3 + \cancel{10p^2} - 10p^3 > \cancel{1+2p}$$

$$3p^2 - 6p^3 > 0$$

4.43. $P(X=3) + P(X=4) + P(X=5)$

$\binom{5}{3} 0.2^3\, 0.8^2 + \binom{5}{4} 0.2^4\, 0.8 + 0.2^5$

۴.۴۲.

$$\geq k$$

$$\alpha \sum_{j=k}^{n} \binom{n}{j} p_1^{j} (1-p_1)^{n-j} + (1-\alpha) \sum_{j=k}^{n} \binom{n}{j} p_2^{j} (1-p_2)^{n-j}$$

4.45

$$p(\text{day on}) = 2\, p(\text{day off})$$

$$p(\text{day on}) = 2(1 - p(\text{day on}))$$

$$3\, p(\text{day on}) = 2$$

$$p(\text{day on}) = \frac{2}{3}$$

(3):

$$\frac{2}{3}\left(\binom{3}{1} 0.8^2 \times 0.2 + 0.8^3\right) +$$

$$\frac{1}{3}\left(\binom{3}{1} \times 0.4^2 \times 0.6 + 0.4^3\right)$$

$$= 0.71467 = \frac{268}{375}.$$

(5):

$$\frac{2}{3}\left(\binom{5}{3} 0.8^3 \times 0.2^2 + \binom{5}{4} 0.8^4 \times 0.2 + 0.8^5\right) +$$

$$\frac{1}{3}\left(\binom{5}{3} 0.4^3 \times 0.6^2 + \binom{5}{4} \times 0.4^4 \times 0.6 + 0.4^5\right)$$

$$= 0.4779 = \frac{896}{1875}$$

4.46. $P(\text{guilty}) = 0.05$

$P(\text{votes guilty} \mid \text{innocent}) = 0.1$

$P(\text{votes innocent} \mid \text{guilty}) = 0.2$

4.48. P(one package 无害):

$P(X \leq 1) = P(X=0) + P(X=1)$

$= 0.9^{10} + \binom{10}{1} \times 0.9^{9} \times 0.1$

$= 0.7360989$

P(1 of 3 package 有害)

$\binom{3}{1}(1-0.7360989)(0.7360989)^2$

$\approx 0.4290.$

4.49 a).

$$\frac{1}{2} \times \binom{10}{7} \times 0.4^7 \times 0.6^3 +$$

$$\frac{1}{2} \times \binom{10}{7} \times 0.7^7 \times 0.3^3$$

b). $P(\text{exactly } \frac{7}{10} \mid \text{first} = \text{heads})$

$$= \frac{\frac{1}{2} \times \binom{9}{6} \times 0.4^7 \times 0.6^3 + \frac{1}{2} \times \binom{9}{6} \times 0.7^7 \times 0.3^3}{\frac{1}{2} \times 0.4 + \frac{1}{2} \times 0.7}$$

$= 0.1968.$

4.50. $P(\text{htt} \mid 6h)$.

$$= \frac{p(1-p)^2 \cdot \binom{7}{5} p^5 (1-p)^2}{\binom{10}{6} p^6 (1-p)^4}$$

$$= \frac{\binom{7}{5}}{\binom{10}{6}} = 0.1$$

b). $P(\text{tht} \mid 6h) = 0.1$

4.51 . a)

$np = 0.2$

$p^n =$

Poisson ....

4.5 Lat

≥ 2

$e^{-3.5}$

$1-P(X<2)$

$=1-P(X=0)-P(X=1)$

$=1-e^{-3.5}-e^{-3.5}(3.5)$

$=0.8641$

$\leq 1. = 4.5e^{-3.5}$

4.55.

$$\left(\frac{1}{365}\right)^2$$

$$1-\left(\frac{365}{366}\right)^{10000}$$

$$1-\left(\frac{364}{365}\right)^n \geq \frac{1}{2}$$

4.57. a).

4.59 a). $50 \times \frac{1}{100} = 0.5$

4.60. $$\underbrace{0.25 \times \frac{e^{-3}\, 3^2}{2!}}_{\text{[illegible]}} + 0.25 \times \frac{e^{-5} \times 5^2}{2!}$$

4.65 a). $500 \times \frac{1}{10^3} = 0.5$

$\geq 1 = 1 - <1 = 1 - e^{-0.5}$

b). $P(>1 \mid \geq 1)$

$= \frac{1 - \leq 1}{1 - <1} = \frac{1 - e^{-0.5} - 0.5e^{-0.5}}{1 - e^{-0.5}}$

c). $P(>1 \mid$ 知自己$)$

$= 1 - \leq 1 \qquad 1 - \left(\frac{10^3 - 1}{10^3}\right)^{499}$ ~~会~~张.

d). remaining ~~钱~~:

$1 - \left(\frac{999}{1000}\right)^{500 - i}$

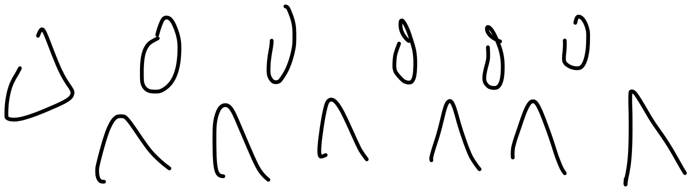
4.66

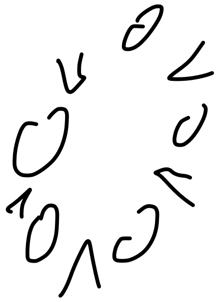
$$p(C_1) = \frac{2!\,(2n-2)!}{(2n-1)!}$$

b).

4.67

$$\frac{n!\,n!}{(2n-1)!}$$





4.68. In response to an attack of 10 missiles, 500 antiballistic missiles are launched. The missile targets of the antiballistic missiles are independent, and each antiballstic missile is equally likely to go towards any of the target missiles. If each antiballistic missile independently hits its target with probability .1, use the Poisson paradigm to approximate the probability that all missiles are hit.

$E(X) = \quad 0.1 \times 500 \times 0.1 = 5$

$$\frac{e^{-5} \, 5^{10}}{10!}$$

4.69. A fair coin is flipped 10 times. Find the probability that there is a string of 4 consecutive heads by

(a) using the formula derived in the text;

(b) using the recursive equations derived in the text.

(c) Compare your answer with that given by the Poisson approximation.

HHHHT _ _ _ _ _

4.69a). T HHHH.

THHHH T _ _ _ _

$$\frac{7\times(\frac{1}{2})}{(\frac{1}{2})^{10}}$$

THHHH

b). $P_n = \frac{1}{2}P_{n-1} + \frac{1}{4}P_{n-2} + \frac{1}{8}P_{n-3}$

4.70. At time 0, a coin that comes up heads with probability p is flipped and falls to the ground. Suppose it lands on heads. At times chosen according to a Poisson process with rate λ, the coin is picked up and flipped. (Between these times the coin remains on the ground.) What is the probability that the coin is on its head side at time t? *Hint* What would be the conditional probability if there were no additional flips by time t, and what would be it be if there were additional flips by time t?

p

$pe^{-\lambda t} + (1-e^{-\lambda t})$

4.71. Consider a roulette wheel consisting of 38 numbers 1 through 36, 0, and double 0. If Smith always bets that the outcome will be one of the numbers 1 through 12, what is the probability that

(a) Smith will lose his first 5 bets;

(b) his first win will occur on his fourth bet?

4.72. Two athletic teams play a series of games; the first team to win 4 games is declared the overall winner. Suppose that one of the teams is stronger than the other and wins each game with probability .6, independently of the outcomes of the other games. Find the probability, for $i = 4, 5, 6, 7$, that the stronger team wins the series in exactly i games. Compare the probability that the stronger team wins with the probability that it would win a 2-out-of-3 series.

第 i 场赢.

WWWLW

4.71: win: $\frac{12}{38}$

lose: $\frac{26}{38}$

$\left(\frac{26}{38}\right)^5$

b). $\left(\frac{26}{38}\right)^3 \frac{12}{38}$

前4有4W

( )

前五有4W

$\binom{5}{4}$

4.72. $P(Y=i) = 0.6^4$

$P(Y=5) = \left(\binom{5}{1}-1\right) 0.6^4\ 0.1$

$P(Y=6) = \left(\binom{6}{2}-\binom{5}{4}\right) .6^4 0.1^2$

$P(Y=7) = \left(\binom{7}{2}-\binom{6}{4}\right) 0.$

WWWWLL

WWWLW

WWW.

4.73. Suppose in Problem 72 that the two teams are evenly matched and each has probability $\frac{1}{2}$ of winning each game. Find the expected number of games played.

A X B X A X B

X A X B X A X B.

用2乘以product

4.74. An interviewer is given a list of people she can interview. If the interviewer needs to interview 5 people, and if each person (independently) agrees to be interviewed with probability $\frac{2}{3}$, what is the probability that her list of people will enable her to obtain her necessary number of interviews if the list consists of (a) 5 people and (b) 8 people? For part (b), what is the probability that the interviewer will speak to exactly (c) 6 people and (d) 7 people on the list?

4.75. A fair coin is continually flipped until heads appears for the 10th time. Let X denote the number of tails that occur. Compute the probability mass function of X.

4.76. Solve the Banach match problem (Example 8e) when the left-hand matchbox originally contained

4.74. (a). $\left(\frac{2}{3}\right)^5$ (b). (d)

$\left(\frac{2}{3}\right)^8 + \binom{8}{1}\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^7 +$

$\binom{8}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^6$ (c) $+$

$\binom{8}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^5$

4.75.

$P(X=0) = \left(\frac{1}{2}\right)^{10}$

$P(X=1) = \binom{11}{1}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)$

$P(X=2) = \binom{12}{2}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^2$

$P(X=i) = \binom{10+i}{i}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^i$

other box contains exactly k matches.

4.78. An urn contains 4 white and 4 black balls. We randomly choose 4 balls. If 2 of them are white and 2 are black, we stop. If not, we replace the balls in the urn and again randomly select 4 balls. This continues until exactly 2 of the 4 chosen are white. What is the probability that we shall make exactly n selections?

4.79. Suppose that a batch of 100 items contains 6 that are defective and 94 that are not defective. If X is the number of defective items in a randomly drawn sample of 10 items from the batch, find (a) $P\{X = 0\}$ and (b) $P\{X > 2\}$.

4.78. $\left(1-\dfrac{\binom{4}{2}\binom{4}{2}}{\binom{8}{4}}\right)^{n-1} \quad \dfrac{\binom{4}{2}\binom{4}{2}}{\binom{8}{4}}$

4.79. $\dfrac{\binom{94}{10}}{\binom{100}{10}}$

b). $1-P(X\le 2)$

$= 1-\dfrac{\binom{94}{10}}{\binom{100}{10}}-\dfrac{\binom{6}{1}\binom{94}{9}}{\binom{100}{10}}-\dfrac{\binom{6}{2}\binom{94}{8}}{\binom{100}{10}}$

4.80. A game popular in Nevada gambling casinos is Keno, which is played as follows: Twenty numbers are selected at random by the casino from the set of numbers 1 through 80. A player can select from 1 to 15 numbers; a win occurs if some fraction of the player's chosen subset matches any of the 20 numbers drawn by the house. The payoff is a function of the number of elements in the player's selection and the number of matches. For instance, if the player selects only 1 number, then he or she wins if this number is among the set of 20, and the payoff is \$2.2 won for every dollar bet. (As the player's probability of winning in this case is $\frac{1}{4}$, it is clear that the "fair" payoff should be \$3 won for every \$1 bet.) When the player selects 2 numbers, a payoff (of odds) of \$12 won for every \$1 bet is made when both numbers are among the 20,

(a) What would be the fair payoff in this case?
Let $P_{n,k}$ denote the probability that exactly k of the n numbers chosen by the player are among the 20 selected by the house.

(b) Compute $P_{n,k}$

(c) The most typical wager at Keno consists of selecting 10 numbers. For such a bet the casino pays off as shown in the following table. Compute the expected payoff:

Keno Payoffs in 10 Number Bets

Number of matches	Dollars won for each \$1 bet
0–4	-1
5	1
6	17
7	179
8	1,299
9	2,599
10	24,999

4.82. A purchaser of transistors buys them in lots of 20. It is his policy to randomly inspect 4 components from a lot and to accept the lot only if all 4 are nondefective. If each component in a lot is, independently, defective with probability .1, what proportion of lots is rejected?

P(accepted) = P(4个是 non defective)

P(rejected) = $1 - 0.9^4$