HW 4.

Q1.

$$p(0,0) = \frac{8}{13} \times \frac{7}{12}$$

$$p(0,1) = \frac{8}{13} \times \frac{5}{12}$$

$$p(1,0) = \frac{5}{13} \times \frac{8}{12}$$

$$p(1,1) = \frac{5}{13} \times \frac{4}{12}$$

Q2.

$p(1,3) = \quad \times \checkmark \checkmark \times$

$\frac{2}{5} \times \frac{3}{4} \times \frac{2}{3} \times \frac{1}{2}$

$p(2,2) = \quad \checkmark \times \checkmark \times$

$\frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} \times \frac{1}{2}$

$p(3,2) = \quad \checkmark \checkmark \times \checkmark \times$

$\frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} \times \frac{1}{2}$

$p(3,3) = 0$

Q3.

$$\binom{5}{2\,2\,1}(0.45)^2(0.15)(0.4)^2$$

$$= 0.1458$$

Q4.

$$\int_0^\infty \int_{-y}^{y} y^2 e^{-y} - x^2 e^{-y}\,dx\,dy = \frac{1}{c}$$

$$\int_0^\infty 2y^3 e^{-y} - \left[\frac{x^3}{3}e^{-y}\right]_{-y}^{y} dy = \frac{1}{c}$$

$$\int_0^\infty 2y^3 e^{-y} - \left(\frac{2y^3}{3}e^{-y}\right) dy = \frac{1}{c}$$

$$\frac{4}{3}\int_0^\infty \underset{u}{y^3}\,\underset{v'}{e^{-y}}\,dy = \frac{1}{c}$$

$$= \frac{3}{4c}$$

$u' = 3y^2$

$v = -e^{-y}$

$$\left[-y^3 e^{-y}\right]_0^\infty + \int_0^\infty \underset{u}{3y^2}\,\underset{v'}{e^{-y}}\,dy = \frac{3}{4c}$$

$u' = 2y$

$v = -e^{-y}$

$$\left[-y^2 e^{-y}\right]_0^\infty + 2\int_0^\infty \underset{u}{y}\,\underset{v'}{e^{-y}}\,dy = \frac{1}{4c}$$

$u' = 1$

$v = -e^{-y}$

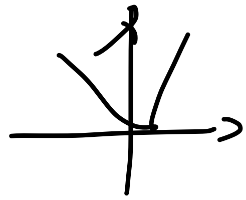
$$\left[-ye^{-y}\right]_0^\infty + \int_0^\infty e^{-y}\,dy = \frac{1}{8c}$$

$$\left[-e^{-y}\right]_0^\infty = \frac{1}{8c}$$

$$f_X(x) = \frac{1}{8}\int_0^\infty y^2 e^{-y} - x^2 e^{-y}\, dy$$

$-y = x$

$y = -x.$



$$\int_0^\infty \underset{u}{y^2}\, \underset{v'}{e^{-y}}\, dy$$

$u' = 2y$

$v = -e^{-y}$

$$\left[-y^2 e^{-y}\right]_0^\infty + 2\int_0^\infty \underset{u}{y}\,\underset{v'}{e^{-y}}\, dy$$

$u' = 1$

$v = -e^{-y}$

$$\left[-ye^{-y}\right]_0^\infty + 2\int_0^\infty e^{-y}\, dy$$

$$= 2\left[-e^{-y}\right]_0^\infty = 2$$

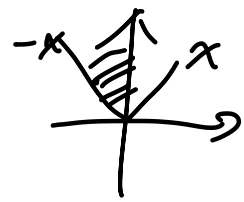
$$\int_0^\infty x^2 e^{-y}\, dy$$

$$= \left[-x^2 e^{-y}\right]_0^\infty$$

$$= x^2$$

$$\frac{1}{8}(2 - x^2)$$

$f_X(x) = \frac{1}{8}\int_{-x}^{\infty} y^2 e^{-y} - x^2 e^{-y}\, dy \; +$



u v'

$\frac{1}{8}\int_{x}^{\infty} y^2 e^{-y} - x^2 e^{-y}\, dy$ $\qquad \int_0^{\infty}\int_{-y}^{y} f(x,y)\,dx\,dy$

$\int\int_{-x}^{\infty} f(x,y)\,dy\,dx$

$u' = 2y$

$v = -e^{-y}$

$= \frac{1}{8}\left(\left[-y^2 e^{-y}\right]_{-x}^{\infty} + \int_{-x}^{\infty} 2y e^{-y}\,dy + \left[x^2 e^{-y}\right]_{-x}^{\infty} \right.$

u v'

$\left. + \frac{1}{8}\left(\left[-y^2 e^{-y}\right]_{x}^{\infty} + \int_{x}^{\infty} 2y e^{-y}\,dy + \left[x^2 e^{-y}\right]_{x}^{\infty} \right) \right)$

$u' = 1$

$v = -e^{-y}$

$= \frac{1}{8}\left(x^2 e^{x} + 2\left(\left[-y e^{-y}\right]_{-x}^{\infty} + \int_{-x}^{\infty} -e^{-y}\,dy \right) + x^2 e^{x} \right)$

$$\int_{-\infty}^{0} \overset{\mathrm{I}}{-\frac{1}{4}x^2 e^x} + \overset{\mathrm{II}}{\frac{1}{4}xe^x}\, dx$$

$$\int_{0}^{\infty} \underset{\mathrm{III}}{\frac{1}{4}x^2 e^{-x}} + \underset{\mathrm{IV}}{\frac{1}{4}xe^{-x}}\, dx$$

$$\mathrm{I} = -\frac{1}{4}\int_{-\infty}^{0} \underset{u}{x^2}\, \underset{v'}{e^x}\, dx \qquad u' = 2x \quad v = e^x$$

$$= -\frac{1}{4}\left(\left[x^2 e^x\right]_{-\infty}^{0} - \int_{-\infty}^{0} 2xe^x\, dx \right)$$

$$= \frac{1}{2}\int_{-\infty}^{0} \underset{u}{x}\, \underset{v'}{e^x}\, dx \qquad u' = 1 \quad v = e^x$$

$$= \frac{1}{2}\left[xe^x\right]_{-\infty}^{0} - \frac{1}{2}\int_{-\infty}^{0} e^x\, dx$$

$$= -\frac{1}{2}(1) = -\frac{1}{2}.$$

II. $\frac{1}{4}\int_{-\infty}^{0} xe^{x}\,dx$

$u \quad v'$

$u'=1$

$v=e^{x}$

$= \frac{1}{4}\left[xe^{x}\right]_{-\infty}^{0} - \frac{1}{4}\int_{-\infty}^{0} e^{x}\,dx$

$= -\frac{1}{4}\left[e^{x}\right]_{-\infty}^{0}$

$= -\frac{1}{4}$

IV: $\frac{1}{4}\int_{0}^{\infty} x^{2}e^{-x}\,dx$

$u \quad v'$

$u'=2x$

$v=-e^{-x}$

$= \frac{1}{4}\left[-x^{2}e^{-x}\right]_{0}^{\infty} + \frac{1}{4}\int_{0}^{\infty} 2xe^{-x}\,dx$

$= \frac{1}{2}\int_{0}^{\infty} xe^{-x}\,dx$

$u \quad v'$

$u'=1$

$v=-e^{-x}$

$=$

$\left[-xe^{-x}\right]_{0}^{\infty} + \frac{1}{2}\int_{0}^{\infty} e^{-x}\,dx$

$=$

$\frac{1}{2}\left[-e^{-x}\right]_{0}^{\infty} = \frac{1}{2}$

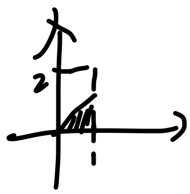
$$f_X(x) = \frac{6}{7}\int_0^2 x^2 + \frac{xy}{2}\, dy$$

$$= \frac{6}{7}\left[x^2 y + \frac{xy^2}{4} \right]_0^2$$

$$= \frac{6}{7}\left(2x^2 + x \right)$$

$$f_{X,Y}(x,y) = \begin{cases} \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) & , 0<x<1,\ 0<y<2 \\ 0 & \text{otherwise} \end{cases}$$

1. $$P(X>Y) = \frac{6}{7}\int_0^1\int_y^1 x^2 + \frac{xy}{2}\, dx\, dy$$



$$= \frac{6}{7}\int_0^1 \left[\frac{x^3}{3} + \frac{x^2 y}{4} \right]_y^1 dy$$

$$= \frac{6}{7}\int_0^1 \frac{1}{3} + \frac{y}{4}\, dy$$

$$= \frac{6}{7}\left[\frac{1}{3}y + \frac{y^2}{8} \right]_0^1$$

$$= \frac{6}{7}\left(\frac{1}{3} + \frac{1}{8} \right) = 0.3928 \left(\frac{11}{28}\right).$$

b). $P(Y>0.5 \mid X<0.5) = \dfrac{P(Y>0.5,\ X<0.5)\ \ (\mathrm{I})}{P(X<0.5)\ \ (\mathrm{II})}$.

I: $\frac{6}{7}\int_{0.5}^{2}\int_{0}^{0.5} x^2 + \frac{xy}{2}\,dx\,dy$

$= \frac{6}{7}\int_{0.5}^{2} \left[\frac{x^3}{3} + \frac{x^2y}{4}\right]_0^{0.5} dy$

$= \frac{6}{7}\int_{0.5}^{2} \frac{1}{24} + \frac{1}{16}y\,dy$

$= \frac{6}{7}\left[\frac{1}{24}y + \frac{1}{32}y^2\right]_{0.5}^{2}$

$= \frac{6}{7}\left(\frac{1}{12} + \frac{1}{8} - \frac{1}{48} - \frac{1}{128}\right)$

$= \frac{69}{448}$

II. $\frac{6}{7}\int_0^{0.5} 2x^2 + x\,dx$

$= \frac{6}{7}\left[\frac{2x^3}{3} + \frac{x^2}{2}\right]_0^{0.5}$

$= \frac{6}{7}\left(\frac{2(0.5)^3}{3} + \frac{0.5^2}{2}\right) = \frac{3}{14}$.

$= 0.71875$

$$\frac{6}{7}\int_0^1 2x^3 + x^2 \, dx$$

$$= \frac{6}{7}\left[\frac{2x^4}{4} + \frac{x^3}{3}\right]_0^1$$

$$= \frac{6}{7}\left(\frac{2}{4} + \frac{1}{3}\right)$$

$Man \sim U(15, 45)$ $\frac{1}{30}$

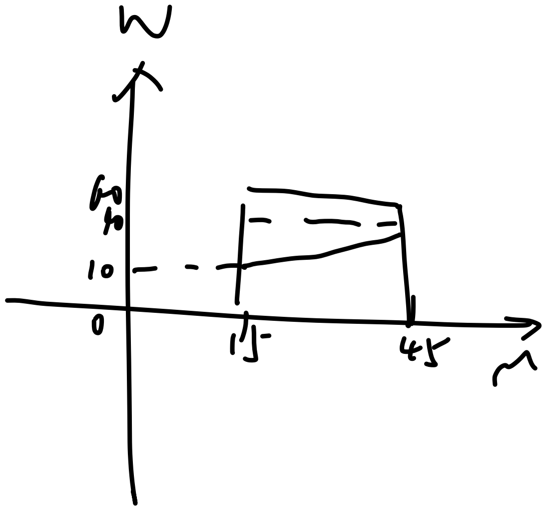
$W \sim U(0, 60)$ $\frac{1}{60}$

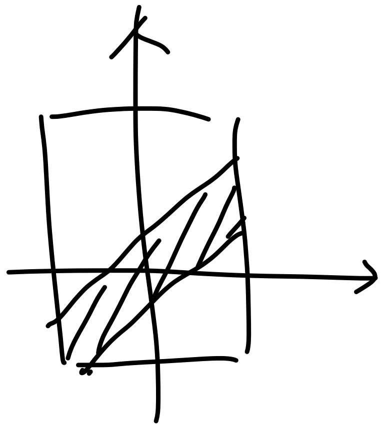
$y > x - 5$

$P(M - W < 5) + P(W - M < 5)$

$W > M - 5$

$= \int\int \frac{1}{1800} \, dM \, dW$

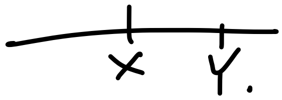




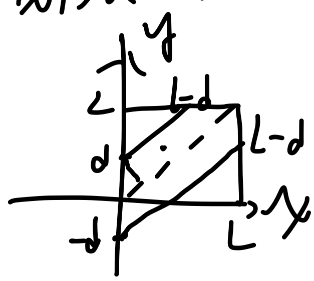
$f_D(d) = |X-Y|$

$X \sim U(0,L)$

$Y \sim U(0,L)$



$|y-x| < d$.



$$\iint_{|x-y|<d} \frac{1}{L^2}\, dx\, dy$$

$$= \frac{1}{L^2} \int \int_{x}^{x+d} dx\, dy$$

$|y-x| < d$

$y - x < d$ or $-(y-x) < d$

$y < x+d$

$x - y < d$

$x - d < y$

$d \frac{d}{L}$

$\left(\sqrt{2d^2} + \sqrt{2L^2}\right)$

$y \leq -x+a$

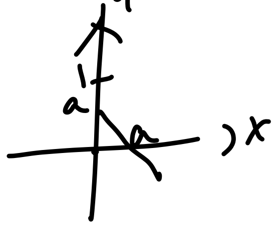
$P(X+Y \leq a) =$

$y = -x+a$

$y-a = -x$

$a-y = x$

$$\int_0^a \int_0^{a-y} \lambda e^{-\lambda x}\, dx\, dy$$

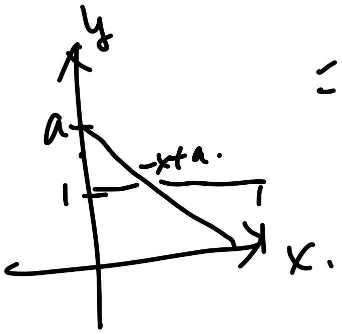


$y \leq -x+a.$

$x>0$

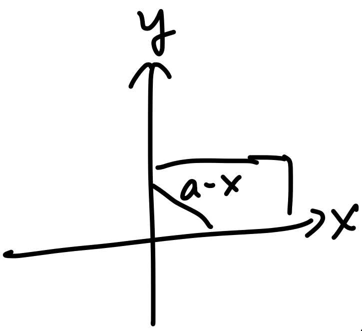
$0<y<1$

$$= \int_0^a \left[-\frac{1}{\lambda}(\lambda) e^{-\lambda x}\right]_0^{a-y} dy$$



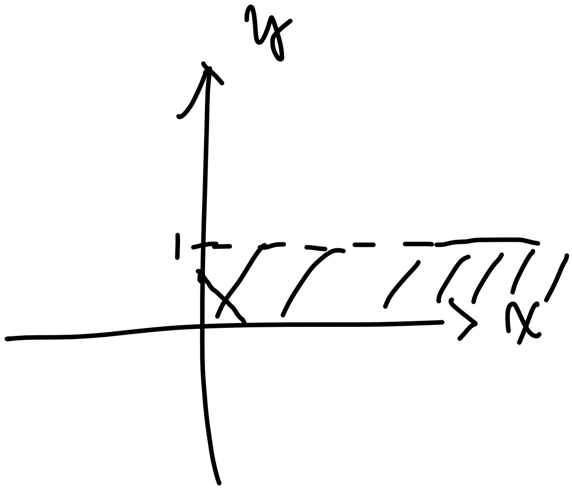
$$= \int_0^a 1 - e^{-\lambda(a-y)}\, dy$$

$$= \int_0^a 1 - e^{-\lambda a} e^{\lambda y}\, dy$$



$$= a - \left[\frac{1}{\lambda} e^{-\lambda a} e^{\lambda y}\right]_0^a$$

$$= a - \frac{1}{\lambda}$$



$y = z - x.$