1 Notations and Conventions

Description	Notation	Comment
Body angular rate	$\boldsymbol{\omega} = \boldsymbol{\omega}_{B/I}^{B} = \boldsymbol{\omega}_{I\to B}^{B}$	Angular rotation of the body frame with respect to the inertial frame expressed in the body frame.
Attitude quaternion	$\boldsymbol{q} = \begin{bmatrix} \boldsymbol{q}_{vec} \\ q_{sca} \end{bmatrix} = \boldsymbol{q}_{B/I} = \boldsymbol{q}_{I\to B}$	Scalar-last right-handed unit quaternion representing attitude transformation from inertial to body frame.
Position vector	$\boldsymbol{p}_{V/I}^{I} = \boldsymbol{p}_{I\to V}^{I}$	A position vector from the origin of the I frame to the Vehicle, expressed in the I frame. Similarly to a velocity vector. Note that strictly speaking, a frame has no origin.

2 Common Attitude Functions

$$
\begin{aligned}
[\boldsymbol{a}\times] &\equiv \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} && 3\times 3 \\
\Xi &\equiv \begin{bmatrix} q_{sca} I_3 + [\boldsymbol{q}_{vec}\times] \\ -\boldsymbol{q}_{vec}^{T} \end{bmatrix} && 4\times 3 \\
\Psi &\equiv \begin{bmatrix} q_{sca} I_3 - [\boldsymbol{q}_{vec}\times] \\ -\boldsymbol{q}_{vec}^{T} \end{bmatrix} && 4\times 3 \\
\Omega &\equiv \begin{bmatrix} -[\boldsymbol{\omega}\times] & \boldsymbol{\omega} \\ -\boldsymbol{\omega}^{T} & 0 \end{bmatrix} && 4\times 4 \\
\bar{\Omega} &\equiv \begin{bmatrix} \cos\left(\frac{1}{2}\|\boldsymbol{\omega}\|\Delta t\right) - [\boldsymbol{\psi}\times] & \boldsymbol{\psi} \\ -\boldsymbol{\psi}^{T} & \cos\left(\frac{1}{2}\|\boldsymbol{\omega}\|\Delta t\right) \end{bmatrix} && 4\times 4 \\
\boldsymbol{\psi} &\equiv \frac{\sin\left(\frac{1}{2}\|\boldsymbol{\omega}\|\Delta t\right)\boldsymbol{\omega}}{\|\boldsymbol{\omega}\|} && 3\times 1 \\
\Gamma &\equiv \begin{bmatrix} [\boldsymbol{n}\times] & \boldsymbol{n} \\ -\boldsymbol{n}^{T} & 0 \end{bmatrix} && 4\times 4
\end{aligned}
$$

Note that

- $\Omega^{T}(\boldsymbol{b}) = -\Omega(\boldsymbol{b})$ and $\Gamma^{T}(\boldsymbol{b}) = -\Gamma(\boldsymbol{b})$
- Ω and Γ can be functions to other 3×1 vectors.

3 Attitude Representation

$$
\begin{aligned}
\boldsymbol{q} &= \begin{bmatrix} \boldsymbol{q}_{vec} \\ q_{sca} \end{bmatrix} \\
&= \begin{bmatrix} \hat{\boldsymbol{e}}\sin\left(\frac{\vartheta}{2}\right) \\ \cos\left(\frac{\vartheta}{2}\right) \end{bmatrix}
\end{aligned}
$$