Satellite orbit and attitude estimation using three-axis magnetometer

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Abstract: The determination of satellite orbital and attitude position and velocity from measurement of a single earth magnetic field (emf) vector without additional measurements, but using a state estimator, is a challenging problem. It is not obvious from first glace whether a solution exists at all – whether the problem is observable with the measurement of only a single emf vector, and an analysis is necessary. This paper performs this analysis for a simple linear system model. Almost circular low earth nearly polar orbits and a dipole emf model are considered. Although these are rather restrictive assumptions they nevertheless provide considerable insight. Both a purely algebraic situation as well as dynamic estimation are studied. It is shown that if the emf induction vector magnitude is used to estimate satellite orbit (position and velocity) and its three projections are used to estimate the attitude, that the situation is sufficiently observable for orbit and attitude determination using just magnetometer measurements. However, for nearly polar orbits, longitude and east velocity are difficult to estimate due to weak observability, and estimation convergence time can be lengthy with poor accuracy.

Keywords: satellite orbit/attitude estimation; magnetometer; Earth magnetic field; satellite navigation; low Earth orbit.

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1 Introduction

The earth's magnetic field has been used in one form or another for terrestrial navigation for many centuries, the most familiar use being the magnetic compass for determining the course of ships for maritime navigation. Similar devices were standard equipment for aircraft navigation in the last century. Since the appearance of earth orbiting space vehicles, the three-axes magnetometer has been used to determine satellite attitude in low earth orbit (LEO), and it has become almost as common for this purpose as the magnetic compass for ships or aircraft.

The use of the three-axis magnetometer for not just vehicle navigation but rather determination of the full set of 12 parameters (linear and angular position and velocity) for a moving vehicle has attracted much attention in the research community, and its potential for both civilian and military applications has been the subject of many publications (Psiaki et al., 1993; Psiaki, 1995; Bar-Itzhack and Deutschmann, 1997; Bar-Itzhack et al., 1998; Natanson et al., 1990; Roh et al., 2007; Cote and de Lafontaine, 2008a, 2008b). Using only a single three-axis magnetometer for determining both satellite attitude and orbit is a very challenging problem and a successful resolution would be a substantial accomplishment. Currently separate instruments are applied for orbit (global positional system – GPS) and attitude (magnetometer – mag and sun sensor – ss or horizon scanner – hs and ss) determination (Sidi, 1997; Vallado, 2007).

Among the potential benefits of a method of combined attitude and orbit determination are the relatively low cost of the required equipment, the potential system autonomy, and the main motivation: the fact that earth's magnetic field has been thoroughly studied, and a very reliable and precise mathematical model of the field (IGRF model) in terms of its expansion in spherical harmonics (Wertz, 1978) is available and in common use. The IGRF model Gaussian coefficients have been empirically determined, based on many years of observation and measurement of the earth's magnetic field and least squares fitting of the measurements. Generally ten terms of this model are sufficient for space navigation. In the IGRF model the measured magnetic field induction vector $\overline{B}$ components in a reference coordinate frame depend on both the location of the magnetometer relative to the earth (latitude, longitude and altitude) and on the orientation (attitude) of the magnetometer with respect to this reference frame. The magnetometer position and/or attitude can be determined using the measurements of the instrument itself together with the IGRF model to provide functional dependence of the vector $\overline{B}$ on magnetometer position and attitude.

Psiaki et al. (1993) suggested a method to determine satellite position with a magnetometer only, and Psiaki (1995) attempted to determine both satellite orbit and the earth magnetic field (emf) model with a magnetometer and star tracker. Bar-Itzhack and Deutschmann (1997) and Bar-Itzhack et al. (1998) investigated the determination of satellite orbit and attitude with magnetometer and gyro measurements using an extended Kalman filter. Natanson et al. (1990) proposed determining satellite attitude using magnetometer measurements only. The appearance of the nonlinear unscented Kalman filter (UKF) stimulated a new series of investigations to determine satellite orbit and attitude with magnetometer measurements only, and Roh et al. (2007) presented encouraging results for determination of satellite orbit using only magnetometer measurement of the $\bar{B}$ vector magnitude and a UKF estimation algorithm. In a general form the problem is presented in Cote and de Lafontaine (2008a, 2008b), where Cote and de Lafontaine described their results on determination of orbit and attitude of the PROBA-2 satellite using magnetometer measurements and a UKF.

Despite the promising results presented in these references the issue of observability (see Bryson and Ho, 1975; Gelb, 1974) has not been fully addressed. Does this method work under any conditions without restriction, and can the entire navigation problem be solved, or just a part of it? What constrains need be accounted for? Specifically: how many of the parameters of satellite motion can be determined using this method? Is it possible to determine the full satellite attitude (all three Euler angles) and position simultaneously or only attitude, or only position? Must the satellite orbit be oriented in some particular fashion with respect to the earth's magnetic field? What is the expected time for convergence, and how accurate is this sort of estimation? Is it possible to calibrate the magnetometer in orbit using only its own measurements?

This article considers these problems for a linear time-invariant system with a dipole model of the emf and a direct method (Kwakernaak and Sivan, 1972), for observability analysis. This direct method is not as well known as the methods related to determination of observability matrix rank (Gelb, 1974). It is based on the direct analysis of the measured noise free signal and grouping the coefficients of similar functions of time, each group representing an observable state vector component in this signal. Both non-dynamic or purely algebraic estimation, which is essentially instantaneous, and dynamic state estimation of the Kalman filter type are considered below.

Practically, for instantaneous attitude determination with a pair of vectors (Wertz, 1978) geometrical factors such as the angle between the two measured vectors can be taken as a figure of merit for achievable accuracy, and for dynamic estimation the observability index (signal/noise ratio) (Kim, 2008) can be used. Note that at least two non-collinear vectors are required for instantaneous three-axis attitude determination. In the case of information redundancy with more than two measured vectors available, the least squares method (batch or recursive least squares) can be used for instantaneous attitude determination for greater accuracy. Also, estimation of deterministic magnetometer errors such as bias, scale factor, and misalignment is better addressed in the case of redundant measurements of more than two vectors of different physical natures being measured (Kim et al., 2004), and is not done here apart from bias during the attitude estimation procedure.

The use of a single, simple and cheap device such as a magnetometer can shift the burden of obtaining a satellite's kinematic parameters from expensive hardware to low cost algorithm implementation, and could be of particular value to low cost nanosat missions to minimise the use of expensive instrumentation based on other sensing methodologies.

2 Observability analysis of satellite orbit and attitude determination using a magnetometer

2.1 Problem formulation

As mentioned above, there have been many studies on attitude and orbit determination using only magnetometer measurements, and of these a number produced encouraging results, which would suggest that the problem is indeed observable. However, to date there has not been an actual examination of the observability problem offering guidance on the conditions for observability and related considerations. This analysis is presented below.

The particular problem considered here is a low earth (h < 1,000 km) nearly circular nearly polar orbit satellite, equipped with a three axis navigation magnetometer. The magnetometer is taken to be accurately aligned and calibrated, with the satellite's own magnetic influences having a negligible effect on the magnetometer measurements. This calibration is not necessary but is assumed here for simplicity.

The magnetometer is assumed to measure three orthogonal components of the emf induction vector $\overline{B}_m$ with respect to its frame with a continuous (analogue) output. The magnetometer provides this output to an on-board real time processor that continuously computes an IGRF model that retains at least the first ten Gaussian harmonic terms of the IGRF $\overline{B}_r$, which is sufficiently precise (Wertz, 1978) for navigation purposes. The satellite position with respect to the earth is approximately known and propagated for limited time periods as Keplerian motion (Bryson, 1993), with an on-board orbit propagator. The task of the computer is to continuously update the current position and velocity of the satellite's centre of mass and its attitude and angular velocity using magnetometer measurements $(\overline{B}_m)$ and the IGRF model $(\overline{B}_r)$. With the knowledge of satellite position and velocity its Keplerian orbital elements can be determined (Montenbruk and Gill, 2000). The satellite position is denoted by the vector $\overline{r}$, velocity by the vector $\overline{V}$, attitude by the direction cosine matrix (DCM) C, and angular velocity by the vector $\overline{\omega}$. The nominal (undisturbed) satellite orbital motion is Keplerian, and the satellite has zero nominal attitude ($C = I$) and zero body rate $(\overline{\omega} = 0)$. Satellite navigation is conducted in the *flight frame*: Z_f axis toward the earth's centre of mass, coinciding with the local gravity vertical, X_f in the orbital plane, tangential to the orbit and positive in the direction of the flight velocity vector, and Y_f normal to the orbital plane, completing a three-axis right-hand orthogonal coordinate system.

In the following, small deviations from the nominal satellite motions caused by space environmental disturbance forces and torques (Sidi, 1997; Bryson, 1993) are considered. These deviations are represented by vectors: $\delta\overline{r}, \delta\overline{V}$ (linear deviations from nominal $\overline{r}$ and $\overline{V}$) and $\overline{\alpha}, \delta\overline{\omega}$ (angular deviations from C and $\overline{\omega}$). As these are small deviations, linear approximations can be used and linear estimates: $\delta\hat{\overline{r}}, \delta\hat{\overline{V}}$ and $\hat{\overline{\alpha}}, \delta\hat{\overline{\omega}}$ of these small deviations in orbital and attitude motion sought.

2.2 Measured and reference magnetic field

The difference between the vectors measured by the magnetometer and the computed magnetic induction vectors (mivs) (based on the IGRF model) is expressed as

$$\Delta\bar{B} = \bar{B}_m - \bar{B}_r \tag{1}$$

where $\bar{B}_m$ is a measured EMF induction vector and $\bar{B}_r$ a reference (computed) EMF induction vector.

The differences in the two vectors are due to errors in the knowledge of the satellite position and attitude, with possibly magnetometer misalignments lumped together with attitude errors. In other words, (1) can be rewritten as follows

$$\Delta\bar{B} = \delta\bar{B} + \bar{v} \tag{2}$$

where $\delta\bar{B}$ is the error caused by satellite position and attitude uncertainty and $\bar{v}$ is the magnetometer instrument error. Alternatively, system errors can be considered to be included in $\delta\bar{B}$. $\bar{v}(t)$ is a purely white Gaussian process as is common in observability analysis (Bryson and Ho, 1975; Gelb, 1974; Kwakernaak and Sivan, 1972) in the absence of more precise information. Assuming that the satellite position and attitude errors are small, the difference between the measured magnetic induction $\bar{B}_m$ and reference vectors $\bar{B}_r$ will also be small, and $\delta\bar{B}$ can be written as the following variation of vector $\bar{B}$

$$\delta\bar{B} = \delta\tilde{B} - \bar{\alpha} \times \bar{B} \tag{3}$$

where $\delta\tilde{B}$ is due to errors in satellite position (local variation of $\bar{B}$) and $\bar{\alpha}$ due to errors in satellite attitude (a small vector-angle error in the magnetometer orientation). The magnetometer instrument frame is assumed to coincide with the satellite body frame, which deviates from the attitude reference frame by this small vector angle.

Substituting (3) into (2) and denoting $\Delta\bar{B}$ by $\bar{Z}$, equation (1) can be rewritten as

$$\bar{Z} = \delta\tilde{B} - \bar{\alpha} \times \bar{B} + \bar{v} \tag{4}$$

where vector $\bar{Z}$ is the magnetometer residual vector, which is referred to as the magnetometer measurement vector below, even though it is actually the residuals. The magnetometer is taken to be free of bias and scale factor errors, or equivalently, the deterministic errors can be lumped together in $\delta\bar{B}$, with $\delta\tilde{B}$ containing the magnetometer bias $\Delta\bar{B}_0$ and $\bar{\alpha}$ its misalignment $\Delta\bar{\alpha}_0$, which will impose limits on the observability of the satellite position and attitude as a consequence of these errors. Ideally the vector $\bar{Z}$ should be zero but is actually non-zero due to measurement noise and errors in the magnetometer orientation, and erroneous computation of the reference miv due to uncertainty in knowledge of satellite position. It is this vector $\bar{Z}$ which would generally be used as the measurement vector in state estimation.

Expression (4), the measurement equation, can be expressed in a number of coordinate systems. One of these, appropriate for this analysis, is the orbital magnetic coordinate frame attached to the satellite centre of mass and defined as Z_m coinciding with the local vertical (geocentric, positive down), X_m perpendicular to Z_m and in the plane of the magnetic meridian (positive toward magnetic north), Y_m perpendicular to X_m

and Z_m, completing a three-axis right-hand orthogonal coordinate frame (perpendicular to magnetic meridian plane, positive toward magnetic east).

The satellite position and orientation are referenced with respect to this frame with the spacecraft in a magnetic polar orbit coinciding with the magnetic meridian, travelling toward the magnetic north pole. In this frame, as indicated below, the magnetic east-west component B_y of the earth's field is negligible. These assumptions also provide for simple analytical results applicable to near polar leo orbits also.

Vector equation (4) can be represented in component form in this frame as

$$\begin{aligned} z_x &= \delta B_x - B_z\alpha_y + v_x, \\ z_y &= \delta B_y + B_z\alpha_x - B_x\alpha_z + v_y, \\ z_z &= \delta B_z + B_x\alpha_y + v_z. \end{aligned} \tag{5}$$

in which $\bar{\alpha}$ is a vector of small angular errors with respect to this frame, with components α_x magnetic roll error, α_y magnetic pitch error, and α_z magnetic yaw error. The nominal attitude is taken to be zero. The terms δx, δy, δz are the satellite position errors with respect to the magnetic field coordinate system which produced δB_x, δB_y, δB_z.

The representation of the earth's magnetic field used here is a simple magnetic dipole $\bar{m}$ with magnetic field vector $\bar{B}_0$ represented as [Wertz, (1978), pp.782–785]:

$$\bar{B}_0 = \frac{M}{r^3}\left[\bar{m} - 3(\bar{n}\cdot\bar{m})\bar{n}\right] \tag{6}$$

where $M = a^3H_0$, $H_0 = 7.943 \cdot 10^{15}$ Wb $\cdot$ m is the total dipole strength, a = 6,371.2 km is the equatorial radius of the earth, r is the distance from the spacecraft to the centre of the earth, $\bar{m}$ is a unit vector along the dipole magnetic north axis and $\bar{n}$ is a unit local vertical vector along $\bar{r}$ (positive up).

The earlier assumption that the satellite orbit coincides with the magnetic meridian (magnetic polar orbit) allows a simple analytical expression useful in the sequel. The angle between the $\bar{m}$ and $\bar{n}$ vectors is denoted as θ (magnetic co-latitude), so that the angle between the magnetic equator and the vector $\bar{n}$ is $\varphi_m = 90° - \theta$ (magnetic latitude).

Projection of the vector $\bar{B}_0$ onto axes $X_mY_mZ_m$ is now

$$\begin{aligned} B_{0x} &= \frac{M}{r^3}\cos\varphi_m, \\ B_{0y} &= 0, \\ B_{0z} &= -2\frac{M}{r^3}\sin\varphi_m. \end{aligned} \tag{7}$$

Assuming that the orbit is close to circular and taking ω_0 as orbital rate (nominally coinciding with magnetic west as flight is towards magnetic north and satisfying $\omega_0 << U_e$), then over a short time period of say a few orbits, $\varphi_m \approx \varphi_{m0} + \omega_0 t$, φ_{m0} is initial magnetic latitude and t is time. Local variations of the measured projections of vector $\bar{B}_0$ can be derived by taking variations of equations (7) as follows:

$$\delta B_{0x} = \frac{-M}{r^3}\left(\frac{\delta r}{r}\cos\varphi_m + \sin\varphi_m \delta\varphi_m\right),$$
$$\delta B_{0y} = 0, \tag{8}$$
$$\delta B_{0z} = 2\frac{M}{r^3}\left(\frac{\delta r}{r}\sin\varphi_m - \cos\varphi_m \delta\varphi_m\right).$$

where δr is the variation in the distance r and $\delta\varphi_m$ is the variation in the magnetic latitude φ_m.

The theoretical value of the magnitude of vector $\bar{B}_0$ at any distance from the earth centre and at any latitude φ_m can also be obtained from (7) via

$$B_0 = \frac{M}{r^3}\left(1 + 3\sin^2\varphi_m\right) \tag{9}$$

The dependence (linear) of δB_0 on small errors in knowledge δr and $\delta\varphi_m$, can be obtained by taking the variation of (9) with respect to distance from the earth centre r and magnetic latitude φ_m

$$\frac{\delta B_0}{B_0} = -\frac{3\delta r}{r} + \frac{3\sin 2\varphi_m \delta\varphi_m}{2\left(1 + 3\sin^2\varphi_m\right)} \tag{10}$$

This simple equation can be used to draw conclusions on satellite position and attitude as will be seen below.

The assumptions above do limit the analysis; however, there are, nevertheless, many satellites in circular, low earth nearly polar orbits, such as the Canadian Radarsat satellite.

2.3 Algebraic approach: simultaneous determination of navigation parameters

The problem now is simultaneous determination of instantaneous satellite position and attitude by algebraic means.

2.3.1 Using the residuals of the miv

2.3.1.1 Position determination

Equations (5) for the three residuals of the magnetometer measurement can be used to determine satellite position variations only. Neglecting the noise $\bar{v}$ in (5) for the moment, and noting (9), results in three linear equations in six variables, which has no unique solution. However, if the attitude errors in (5) were also neglected, measurement of the three magnetic induction projections, would allow the following set of equations to be written and solved for small position errors:

$$\frac{\delta r}{r} = \frac{-r^3\left(2\cos\varphi_m \delta B_{0x} - \sin\varphi_m \delta B_{0z}\right)}{2M},$$
$$\delta\varphi_m = \frac{-r^3\left(2\sin\varphi_m \delta B_{0x} + \cos\varphi_m \delta B_{0z}\right)}{2M}, \tag{11}$$
$$\delta\lambda_m = indetermined \sim (0).$$

Hence, *if the satellite attitude is known*, the altitude and magnetic latitude of the satellite can be instantaneously determined from the three residuals of the magnetometer measurement.

In the general case a more accurate IGRF model in the geocentric tangential frame (X-north, Y-east, Z-local geocentric vertical dawn) would depend on longitude as well and could be represented in the following form: $B_x = B_x(\varphi, \lambda, r)$, $B_y = B_y(\varphi, \lambda, r)$, $B_z = B_z(\varphi, \lambda, r)$, where φ is the latitude, λ is the longitude, and r the vertical distance to the centre of the earth. The variation of the longitude can be found by solving three linear equations of three-axis magnetometer measurements:

$$\begin{aligned} &\frac{\partial B_x}{\partial \varphi}\delta\varphi + \frac{\partial B_x}{\partial \lambda^*}\delta\lambda^* + \frac{\partial B_x}{\partial r}\delta r = \delta B_x, \\ &\frac{\partial B_y}{\partial \varphi}\delta\varphi + \frac{\partial B_y}{\partial \lambda^*}\delta\lambda^* + \frac{\partial B_y}{\partial r}\delta r = \delta B_y, \\ &\frac{\partial B_z}{\partial \varphi}\delta\varphi + \frac{\partial B_z}{\partial \lambda^*}\delta\lambda^* + \frac{\partial B_z}{\partial r}\delta r = \delta B_z. \end{aligned} \tag{12}$$

where $\delta\varphi$ is variation of the latitude, $\delta\lambda$ is variation of the longitude, $\delta\lambda^* = \delta\lambda\cos\varphi$, δr is variation of the distance r.

If the determinant of (12) is non-zero then this system will have a unique solution: $\delta\varphi_0, \delta\lambda_0^*, \delta r_0$.

However, the longitude obtained this way is likely to be less accurate than both latitude and altitude.

2.3.1.2 Attitude determination

Returning now to (5) but this time ignoring the errors in satellite position δB_x, δB_y, δB_z, as well as the noises ν_x, ν_y, ν_z reduces (5) to the following:

$$\begin{aligned} z_x &= -B_z\alpha_y, \\ z_y &= B_z\alpha_x - B_x\alpha_z, \\ z_z &= B_x\alpha_y. \end{aligned} \tag{13}$$

The three satellite attitude angles cannot simultaneously be determined from (13). α_y (magnetic pitch) can be determined anywhere in this polar (magnetic) orbit (from z_x in polar areas, where $B_x \approx 0$, $B_z = B_{z\max}$ and from z_z in equatorial areas, where $B_z \approx 0$, $B_x = B_{x\max}$). However, α_x (magnetic roll) can be determined only in the polar regions ($B_x \approx 0$, $B_z = B_{z\max}$), and α_z (magnetic yaw) only in equatorial areas ($B_z \approx 0$, $B_x = B_{x\max}$). Hence, only two of three Euler angles of the satellite attitude can be determined by this method simultaneously: pitch and yaw in equatorial regions and pitch and roll in polar areas. It can be shown that this conclusion holds for more general cases of satellite orbit and angular motion, and with respect to any coordinate frame.

Equation (4) above, again with the position errors and noise set to zero but for any orbit and orientation of the spacecraft now becomes, since the B_y component is no longer necessarily zero,

$$\begin{aligned} z_x &= -B_z\alpha_y + B_y\alpha_z, \\ z_y &= B_z\alpha_x - B_x\alpha_z, \\ z_z &= -B_y\alpha_x + B_x\alpha_y. \end{aligned} \tag{14}$$

The determinant of (14) is zero:

$$\Delta = B_x B_y B_z - B_x B_y B_z \equiv 0 \tag{15}$$

as is also clear from B^x being a 3 × 3 skew symmetric matrix. Hence, (14) cannot be solved for arbitrary satellite orbital motions, and the three unknowns (α_x, α_y, α_z) cannot be determined this way. Physically this represents the well-known fact that the angle of rotation about the measurement vector cannot be found by measuring only this single vector. Result (15) is also well known and is used below to show that the magnetic navigation observability problem addressed here provides a partial explanation for the positive outcomes mentioned above which used only magnetometer measurements.

2.3.2 Using three projections of the miv and its magnitude

2.3.2.1 Position determination

It was shown above that it is not possible to determine satellite position and attitude simultaneously by measuring only miv (vector $\bar{B}$) projections followed by computation of the residuals (z_x, z_x, z_z). It was also shown that it is possible to determine them separately assuming knowledge of either the satellite position or its attitude. However, there is an option for using the measured vector magnitude B: $Z = B_m - B_r = \delta B + V(t)$, where δB is the difference in miv magnitudes caused by the uncertainty in satellite position $\delta\bar{r}$, and $V(t)$ now denotes measurement noise.

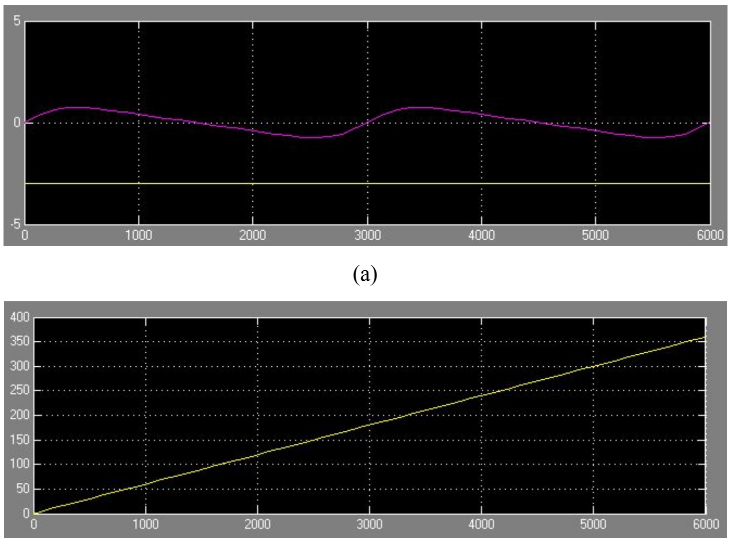
This measurement is invariant with respect to satellite attitude. Hence, satellite position can be determined by measuring only the magnitude of the miv using (12), without knowledge of the satellite attitude.

In the simplest case of representation of miv by the magnetic dipole (6) it is not possible to determine all three parameters of the satellite position this way, but only two – the altitude δr and the magnetic latitude $\delta\varphi_m$. These two quantities also cannot be determined simultaneously: δr can be determined only in equatorial ($\varphi_m = 0$) and polar ($\varphi_m = 90°$) regions and, $\delta\varphi_m$ at mid latitudes ($\varphi_m \approx 45°$) but with a previously determined δr. Simulink plots of the first and the second components of equation (16) [which is (10) above, repeated here for convenience] below, and of change in satellite magnetic latitude with time, are presented in Figure 1 (6,000 sec~1 orbit).

$$\frac{\delta B_0}{B_0} = -3\frac{\delta r}{r} + 3\frac{\sin 2\varphi_m}{2\left(1+3\sin^2\varphi_m\right)}\delta\varphi_m \tag{16}$$

For more accurate representations of the IGRF model (Natanson et al., 1990), and for some orbits that are not necessarily close to polar, the longitude of the satellite will be observable also. However, its estimation as well as the East velocity component will likely require greater time intervals and may be not very accurate.

Figure 1 (a) Ordinate – contributions (weight coefficients) of first (yellow line lower) and second (pink line) terms of formula (16), relative magnitudes of $\frac{\delta B_0}{B_0}$ for $\frac{\delta r}{r}$ (yellow) and $\delta\varphi_m$ (pink) (b) Ordinate – change in the magnetic latitude absolute value $|\varphi_m|$ [deg] for orbital flight (beginning from the magnetic equator) (see online version for colours)



(a)

(b)

Notes: Abscissa – t [sec]. Orbital period $T_0 \approx 6{,}000$ s. Figure 1(b) represents $\delta\varphi_m(t)$ for Figure 1(b).

2.3.2.2 Attitude determination

The results above, for the case of three components of magnetic vector measurements remain correct for this case with the use of the magnetic vector magnitude as well. However, knowledge of satellite position can be obtained a priori based on magnetometer information also. It is therefore assumed that satellite position errors are determined first, based on the magnitude of vector $\overline{B}$: $Z_B = \delta B + V(t)$ (in turn, $\delta r \rightarrow \delta\varphi \rightarrow \delta\lambda\cos\varphi$). With these errors known, three attitude angles (or two simultaneously) can be determined in turn also, using the three projections of the vector of the measurement: $\overline{Z}_{\overline{B}} = \delta\overline{B} + \overline{V}(t)$.

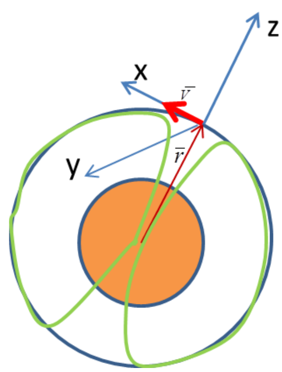
2.4 Dynamic approach: estimation of navigation parameters

This approach entails a dynamical phase delay for estimation of updates following any manoeuvre or change in deterministic (noise free) disturbances. This delay is required for estimation process convergence and can be of significant duration – from a quarter of an orbit to several orbits. Even for estimation of stationary states the estimator introduces a certain amount of dynamic delay that can tend to smooth out information (with low level residual noise), and its use for satellite control in real time is therefore questionable. The usability of these estimates for orbit determination depends on the situation, but for real time attitude control their use is problematic and probably more suitable for ground based post processing of telemetry data or for auxiliary modes such as safe hold. In spite of this potential shortcoming, it is worthwhile to analyse the observability of the problem.

2.4.1 Satellite orbital motion linear model

The linear Hill differential equations (Sidi, 1997; Bryson, 1993) are used to represent the disturbed satellite orbital motion in the orbital (flight) frame (Z-local gravity vertical, positive up, X-orbit tangential, positive in flight direction, Y-orbital plane normal) for small deviations from the nominal Keplerian orbit. The orbital flight direction is also taken to be along the magnetic meridian.

Figure 2 Orbital (flight) frame (see online version for colours)



Note: Orbital plane coincides with the magnetic meridian (green) plane.

Rewriting Hill's equations (Sidi, 1997) in first order form for this reference frame results in

$$\begin{aligned}
&\delta\dot{x} = \delta V_x, \\
&\delta\dot{V}_x = 2\omega_0 \delta V_z + f_x, \\
&\delta\dot{y} = \delta V_y, \\
&\delta\dot{V}_y = -\omega_0^2 \delta y_y + f_y, \\
&\delta\dot{z} = \delta V_z, \\
&\delta\dot{V}_z = 3\omega_0^2 \delta z - 2\omega_0 \delta V_x + f_z.
\end{aligned} \tag{17}$$

Note that the flight frame x, y, z above is different from the orbital X, Y, Z frame of Sidi (1997) and is related to it by $X = -z$, $Y = x$, $Z = -y$.

The equations for the X and Z axes representing satellite longitudinal motion (along track and radial respectively), are coupled. The equation for the Y axis represents the satellite lateral (cross track) motion with respect to the orbital plane. This lateral motion equation is independent of longitudinal motion equations (X and Z). Perturbing specific forces (ratio of disturbance forces to the satellite mass) are represented by f_x, f_y, f_z. The parameter $\omega_0 = \dfrac{V}{r}$ is satellite orbital rate and is assumed constant (nearly circular orbit), the parameters δx, δy, δz are variations in satellite position, and δV_x, δV_y, δV_z are variations in satellite velocity in the flight frame (attached to the nominal unperturbed orbit). The free response, or unforced ($f_x = 0, f_y = 0, f_z = 0$) solution of (17), due to only initial conditions: δx_0, δV_{x0}, δ_{y0}, δV_{y0}, δ_{z0}, δV_{y0} can be written as follows (Sidi, 1997):

$$\begin{aligned}
\delta x(t) &= \delta x_0 - \frac{2\delta V_{z0}}{\omega_0} - 3\left(\delta V_{x0} - 2\omega_0 \delta z_0\right)t \\
&\quad - \frac{2\delta V_{z0}}{\omega_0}\cos\omega_0 t + \frac{4\left(\delta V_{x0} - 6\omega_0 \delta z_0\right)}{\omega_0}\sin\omega_0 t, \\
\delta y(t) &= \delta y_0 \cos\omega_0 t + \frac{\delta V_{y0}}{\omega_0}\sin\omega_0 t, \\
\delta z(t) &= 4\delta z_0 - \frac{2\delta V_{x0}}{\omega_0} + \frac{\delta V_{z0}}{\omega_0}\sin\omega_0 t - \left(3\delta z_0 - \frac{2\delta V_{x0}}{\omega_0}\right)\cos\omega_0 t.
\end{aligned} \tag{18}$$

It can be seen from (18) that the along track coordinate x in the longitudinal channel is unstable and errors $\delta x(t)$ will be unbounded with time.

As discussed above, the magnetic dipole emf model (6) in magnetic spherical coordinates is used here. Assuming a non-rotating earth, the transformations between satellite Cartesian coordinates: δx, δy, δz and magnetic spherical coordinates: $\delta\varphi_m$, $\delta\lambda_m$, δr can be expressed as follows:

$$\begin{aligned}
\delta\varphi_m &= \frac{\delta x}{r}, \\
\delta\lambda_m &= \frac{\delta y}{r\cos\varphi_m}, \\
\delta r &= -\delta z.
\end{aligned} \tag{19}$$

2.4.2 Satellite angular motion linear model

The angular motion of the satellite in the x, y, z flight frame with the assumption of the principal moments of inertia of the satellite not differing greatly from one another, and with small attitude errors (angle and body rate errors), can be approximately represented by the following linear differential equations (Sidi, 1997):

$$\begin{aligned}
\dot{\alpha}_x &= \omega_0\alpha_z + \omega_x, \\
\dot{\omega}_x &= \tau_x, \\
\dot{\alpha}_y &= \omega_y + \omega_0, \\
\dot{\omega}_y &= \tau_y, \\
\dot{\alpha}_z &= -\omega_0\alpha_x + \omega_z, \\
\dot{\omega}_z &= \tau_z,
\end{aligned} \tag{20}$$

where α_x, α_y, α_z are small Euler angles of the satellite attitude error, ω_x, ω_y, ω_z are angular rate errors, τ_x, τ_y, τ_z are small specific disturbance torques, which are the ratios of torque to corresponding moment of inertia i.e., $\left(\tau_x = \frac{T_x}{J_x}, \tau_y = \frac{T_y}{J_y}, \tau_z = \frac{T_z}{J_z}\right)$ and ω_0 is the satellite orbital rate. Note that for small angles, $\alpha_x \approx \phi(roll)$, $\alpha_y \approx \theta(pitch)$, $\alpha_z \approx \psi(yaw)$.

The first and third pairs of (20) are coupled for longitudinal motion (*XZ*) while the second pair, for the lateral motion (*Y*), is independent.

The solution of (20) for free angular motion ($\tau_x = 0$, $\tau_y = 0$, $\tau_z = 0$) can be expressed as follows:

$$\begin{aligned}
\alpha_x(t) &= \alpha_{x0}\cos\omega_0 t + \left(\alpha_{z0} + \frac{\omega_{x0}}{\omega_0}\right)\sin\omega_0 t, \\
\alpha_y(t) &= \alpha_{y0} + \omega_{y0} t, \\
\alpha_z(t) &= \alpha_{z0}\cos\omega_0 t + \left(-\alpha_{x0} + \frac{\omega_{z0}}{\omega_0}\right)\sin\omega_0 t.
\end{aligned} \tag{21}$$

The state differential equations (17) and (20) can be used for synthesis of a linear (Kalman type) filter for estimation of the satellite orbit and attitude with measurements provided by the magnetometer.

Orbit and attitude from magnetometer measurements can only be obtained by using the miv magnitude to determine satellite orbit, then with a known orbit using 3 miv measured projections to determine the satellite attitude.

The solutions of (17) and (20) for free motion: (18), (19), (21), can be used for direct analysis of the observability problem considered next.

2.5 *Analysis of observability*

If a linear Kalman filter (Bryson and Ho, 1975; Kim, 2008), is to be used for satellite orbit (position and velocity) and/or attitude (attitude Euler angles and body rate) estimation, the question of which components of the full state vector $X = (x, y, z, V_x, V_y, V_z, \alpha_x, \alpha_y, \alpha_z, \omega_x, \omega_y, \omega_z)^T$ are observable and can be estimated by the filter must be addressed. This issue is first examined using the direct approach (Kim, 2008), which is based on a measured vector analytical expression and checking whether $Z \equiv 0$ implies $X(0) = X_0 = 0$ (Kwakernaak and Sivan, 1972).

2.5.1 *Orbit determination observability*

5.5.1.1 *Using the three components of the miv*

Here the satellite attitude is assumed to be accurately measured and known [α_x, α_y, α_z terms in (5) are zero], with no noise ($v_x = 0$, $v_y = 0$, $v_z = 0$). The magnetometer measurements for a dipole magnetic model using (11), and (18), (19) can then be written as ($\varphi_{m0} = 0$, equatorial initial position)

$$
\begin{aligned}
\delta z_x &= \frac{-M}{r^4}\Bigg[-2\left(2\delta z_0 + \frac{\delta V_{x0}}{\omega_0}\right)\cos\omega_0 t - 2\left(\frac{3}{2}\delta z_0 - \frac{\delta V_{x0}}{\omega_0}\right)\cos^2\omega_0 t + \\
&\qquad + \left(\delta x_0 - \frac{2\delta V_{z0}}{\omega_0}\right)\sin\omega_0 t - 3\left(\delta V_{x0} - 2\omega_0\delta z_0\right)t\sin\omega_0 t \\
&\qquad - \frac{\delta V_{z0}}{2\omega_0}\sin 2\omega_0 t + \frac{4\left(\delta V_{x0} - 6\omega_0\delta z_0\right)}{\omega_0}\sin^2\omega_0 t\Bigg], \\
\delta z_y &= 0, \\
\delta z_z &= -2\frac{M}{r^4}\Bigg[2\left(2\delta z_0 - \frac{\delta V_{x0}}{\omega_0}\right)\sin\omega_0 t + \frac{\delta V_{z0}}{\omega_0}\sin^2\omega_0 t - \left(\frac{3\delta z_0}{2} - \frac{\delta V_{x0}}{\omega_0}\right)\sin 2\omega_0 t + \\
&\qquad + \left(\delta x_0 - \frac{2\delta V_{z0}}{\omega_0}\right)\cos\omega_0 t - 3\left(\delta V_{x0} - 2\omega_0\delta z_0\right)t\cos\omega_0 t \\
&\qquad - \frac{2\delta V_{z0}}{\omega_0}\cos^2\omega_0 t + \frac{4\left(\delta V_{x0} - 6\omega_0\delta z_0\right)}{2\omega_0}\sin 2\omega_0 t\Bigg].
\end{aligned}
\tag{22}
$$

where δz_x, δz_y, δz_z are differences between measurements of magnetometer and corresponding computed IGRF components in three axes.

It can be seen from the coefficients of (22) that if $z_x \equiv 0$, $z_y \equiv 0$, $z_z \equiv 0$ then following conditions must be satisfied:

$$
\begin{aligned}
&2\delta z_0 + \frac{\delta V_{x0}}{\omega_0} = 0, \\
&\delta V_{z0} = 0, \\
&\frac{3}{2}\delta z_0 - \frac{\delta V_{x0}}{\omega_0} = 0, \\
&\delta x_0 - \frac{2\delta V_{z0}}{\omega_0} = 0, \\
&\delta V_{x0} - 2\omega_0\delta z_0 = 0, \\
&\delta V_{x0} - 6\omega_0\delta z_0 = 0.
\end{aligned}
\tag{23}
$$

Hence, it can be concluded that the variables δx, δz, δV_x, δV_z (longitudinal channel) can be observed and estimated accurately. Variables δy and δV_y (lateral channel) are not observable and cannot be estimated, at least not with this simple representation of the reference magnetic field model and an orbit parallel to the magnetic meridian. Apparently the observability in the lateral channel is much weaker than in the longitudinal. Simulations and flight experiments (Cote and de Lafontaine, 2008a, 2008b), show that for nearly polar circular orbits longitude and flight velocity in the lateral channel are observable. This is due to the IGRF magnetic model's dependence on not only latitude φ and distance r to the centre of the earth, but also on longitude λ (Wertz, 1978). Also, the rotation of the earth changes the inclination of the magnetic pole with respect to orbit inclination. However, this effect does not show up with the simple magnetic dipole model considered here.

The along track (δx) and radial (δz) errors are observable in equatorial areas, while the velocity is observable in polar areas.

2.5.1.2 Using the magnitude of the miv

In this case the measured value is independent of attitude as can be seen from (10). Using the difference between the measured and the modelled (dipole) magnetic vector magnitudes, assuming zero noise and using the satellite position errors (18), the following can obtained from (10)

$$z(t) = \frac{B_0}{r}\left\{-3\left[4\delta z_0 - \frac{2\delta V_{x0}}{\omega_0} + \frac{\delta V_{z0}}{\omega_0}\sin\omega_0 t - \left(3\delta z_0 - \frac{2\delta V_{x0}}{\omega_0}\right)\cos\omega_0 t\right] + \right.$$
$$+\frac{3\sin 2\omega_0 t}{2(1+3\sin^2\omega_0 t)}\left[\delta x_0 - \frac{2\delta V_{z0}}{\omega_0} - 3\left(\delta V_{x0} - 2\omega_0\delta z_0\right)t\right. \tag{24}$$
$$\left.\left.-\frac{2\delta V_{z0}}{\omega_0}\cos\omega_0 t + \frac{4\left(\delta V_{x0} - 6\omega_0\delta z_0\right)}{\omega_0}\right]\sin\omega_0 t\right\}$$

It can be seen from this expression that the following conditions must be satisfied in order to have $z(t) \equiv 0$:

$$4\delta z_0 - \frac{2\delta V_{x0}}{\omega_0} = 0,\ \delta V_{z0} = 0,\ 3\delta z_0 - \frac{2\delta V_{x0}}{\omega_0} = 0 \tag{25}$$
$$\delta x_0 - \frac{2\delta V_{z0}}{\omega_0} = 0,\ \delta V_{x0} - 2\omega_0\delta z_0 = 0,\ \delta V_{z0} = 0,\ -6\delta z_0 + \frac{\delta V_{x0}}{\omega_0} = 0$$

Hence, the position and velocity errors in the longitudinal plane are observable, but not in the lateral plane. This conclusion is a consequence of the EMF dipole model and the magnetic polar orbit.

Note that the radial position error (δz) and the velocity errors (δV_x, δV_z) are observable in the equatorial areas ($\omega_0 t \approx 0$), while the along track error (δx) is observable in the mid-latitudes ($2\omega_0 t \approx 90^0$).

2.5.2 Attitude determination observability

In this case the satellite position is assumed known (perhaps previously determined with the same magnetometer or by some another means such as a GPS) and there are residual constant errors ΔB_x, ΔB_y, ΔB_z (biases) in the magnetometer output. Then taking $\delta B_x = \Delta B_x$, $\delta B_y = \Delta B_y$, $\delta B_z = \Delta B_z$, $v_x = v_y = v_z = 0$ and substituting (21) and (7) into (5), yields

$$z_x(t) = \Delta B_x + \left(2\frac{M}{r^3}\sin\omega_0 t\right)\left(\alpha_{y0} + \omega_{y0} t\right),$$
$$z_y(t) = \Delta B_y - \frac{M}{r^3}\left[\frac{1}{2}\left(\alpha_{x0} + \frac{\omega_{z0}}{\omega_0}\right)\sin 2\omega_0 t\right.$$
$$\left.+2\left(\alpha_{z0} + \frac{\omega_{x0}}{\omega_0}\right)\sin^2\omega_0 t + \alpha_{z0}\cos^2\omega_0 t\right], \tag{26}$$
$$z_z(t) = \Delta B_z + \left(\frac{M}{r^3}\cos\omega_0 t\right)\left(\alpha_{y0} + \omega_{y0} t\right),$$

where φ_{m0} is the initial magnetic latitude (starting from the magnetic equator), $\varphi_{m0} = 0$.

It can be seen from (26) that for $z_x(t) \equiv 0$, $z_y(t) \equiv 0$, $z_z(t) \equiv 0$ the following conditions must be satisfied

$$\begin{aligned}
&\Delta B_x = 0, \alpha_{y0} = 0, \omega_{y0} = 0 \\
&\Delta B_y = 0, \alpha_{z0} = 0, \omega_{x0} = 0, \alpha_{x0} + \frac{\omega_{z0}}{\omega_0} = 0 \\
&\Delta B_z = 0.
\end{aligned} \tag{27}$$

Hence, the observable variables in this satellite attitude estimation problem are the three angles of the attitude error (α_x, α_y, α_z), the three projections of the magnetometer bias (ΔB_x, ΔB_y, ΔB_z) and the three projections of the body rate ω_x, ω_y, ω_z. Quantitative assessment of the time required for these estimates can be made using the observability index presented in Kim (2008).

For approximate coarse analysis (26) can be reduced for short flight times ($\omega_0 t < \delta$)

- near the magnetic equator to:

$$\begin{aligned}
&z_x(t) \approx \Delta B_x, \\
&z_y(t) \approx \Delta B_y - \frac{M}{r^3}\alpha_{z0}, \\
&z_z(t) \approx \Delta B_z + \frac{M}{r^3}\alpha_{0y},
\end{aligned} \tag{28}$$

- near the magnetic poles ($90^0 - \omega t < \delta$) to:

$$\begin{aligned}
&z_x(t) \approx \Delta B_x + 2\frac{M}{r^3}\alpha_{y0} \\
&z_y(t) \approx \Delta B_y, \\
&z_z(t) \approx \Delta B_z,
\end{aligned} \tag{29}$$

From (28) it is clear that in earth equatorial regions only the pitch, α_y, and the yaw, α_z, can be estimated, and then only if the magnetometer bias is compensated by a precise calibration on the ground. From (29) it is seen that only the pitch, α_y can be estimated in the polar areas. From (26) it can also be concluded that the roll, α_x can be estimated in mid-latitudes ($\varphi_m \approx 45^0$). Note that although the subscript 0 indicates initial time, these conclusions hold for any time.

3 Simulation of satellite navigation with magnetometer

To illustrate the main conclusions presented above, a simulation was performed using a linear Kalman filter (Bryson and Ho, 1975) for the simplified linearised model of the satellite orbital and angular motion presented earlier, and for a tenth order reference IGRF model. The simulation was performed for a nearly circular low earth polar orbit at altitude 650 km.

The orbital period is $T_0 = 2\pi\sqrt{\frac{a^3}{\mu_e}}$, $a = r = R_e + h$ is circular orbit radius, $R_e = 678.14$ km is earth radius, $h = 650$ km is satellite attitude, $T_0 = 97.73$ min (5,863.7 s).

In the simulation, satellite position, velocity, attitude, and body rate were estimated separately with two different filters, one for orbital motion and the other for angular motion. The simulation was performed in Matlab Simulink.

3.1 Simulation of position and velocity determination with the magnetometer

The magnetometer provided measurements of $\bar{B}$, and then the computed $\bar{B}$ magnitude was used to estimate satellite position and velocity. The reference miv $\bar{B}$ was modelled by an IGRF model of tenth order. The linear Hill equations (17) of the satellite orbital motion were used with magnetometer measurements Z modelled as small variation of the reference field IGRF vector magnitude with additive white noise. This modelled measurement vector is expressed as

$$\begin{aligned}\delta B = \frac{1}{B}\Bigg[&\left(B_x\frac{\partial B_x}{\partial x} + B_y\frac{\partial B_y}{\partial x} + B_z\frac{\partial B_z}{\partial x}\right)\delta x + \left(B_x\frac{\partial B_x}{\partial y} + B_y\frac{\partial B_y}{\partial y} + B_z\frac{\partial B_z}{\partial y}\right)\delta y \\ &+\left(B_x\frac{\partial B_x}{\partial z} + B_y\frac{\partial B_y}{\partial z} + B_z\frac{\partial B_z}{\partial z}\right)\delta z\Bigg] + V(t)\end{aligned} \tag{30}$$

where $V(t)$ is a white noise process representing the measurement errors in $B = \sqrt{B_x^2 + B_y^2 + B_z^2}$.

The transformation between satellite rectangular and absolute spherical coordinates (19) has variations in the flight frame Cartesian coordinates of:

$$\begin{aligned}&\delta x = r \cdot \delta\varphi, \\ &\delta y = \delta\lambda \cdot r \cdot \cos\varphi, \\ &\delta z = -\delta r\end{aligned} \tag{31}$$

where φ is the latitude, r is the orbit radius, $\delta\varphi$, $\delta\lambda$, δr are small variations in latitude, longitude (absolute) and orbital radius, respectively.

A circular polar orbit of altitude 650 km and flight in the direction from the ascending node toward the north pole was simulated. Simulink plots of the satellite nominal position for ten orbits are presented in Figures 3 to Figure 5 (position and bearing in Figure 3, magnetic field components in Figure 4, and noisy miv magnitude in Figure 5).

Plots of the simulation of satellite position errors δx, δy, δz are given in Figure 6 and velocity errors δV_x, δV_y, δV_z in Figure 7.

Figure 3 Simulation of (from top down) latitude [deg], longitude [deg], bearing [deg] (ordinates) and orbit radius [m] versus time [sec] (abscissa) for ten orbits (~ 58,637 sec) (see online version for colours)

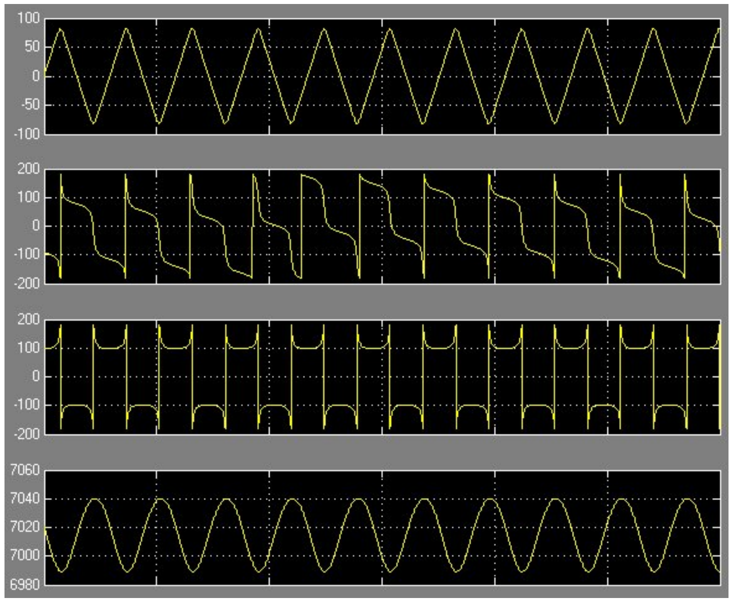
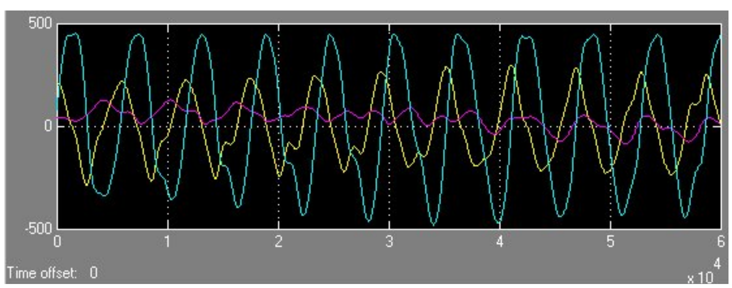
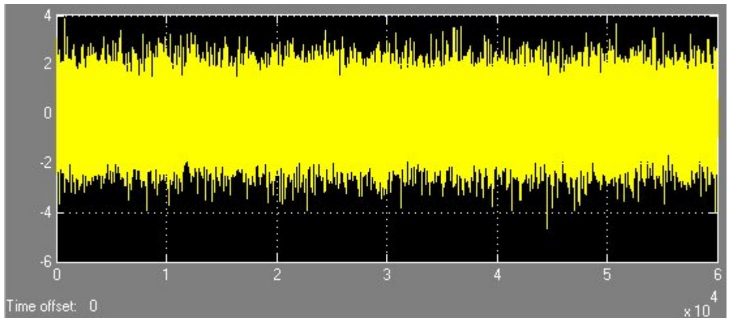


Figure 4 IGRF components (see online version for colours)



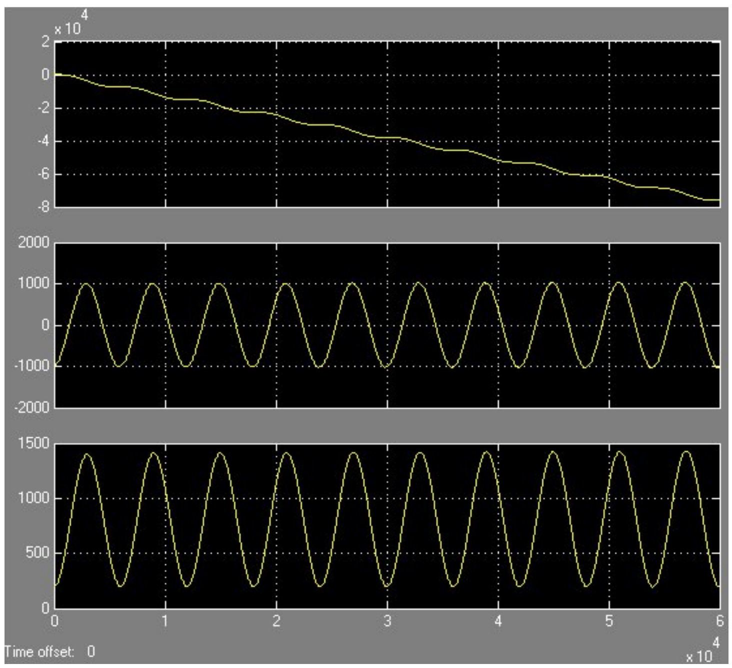
Notes: Yellow B_x, pink B_y, and blue B_z [mG] (ordinate), versus t [sec] (abscissa).

Figure 5 $Z(t)$ measurements [mg] (ordinate), as computed with formula (32) (see online version for colours)



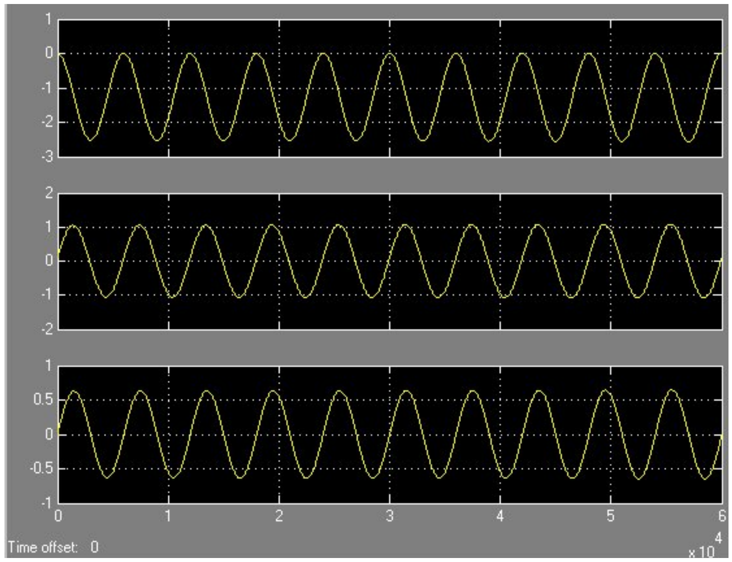
Notes: Noise: σ_v = 1 mG, Δt = 1 sec abscissa t [sec]

Figure 6 Simulated satellite position errors (see online version for colours)



Notes: Top down respectively: ordinate – δx, δy, δz [m] abscissa – t [sec]

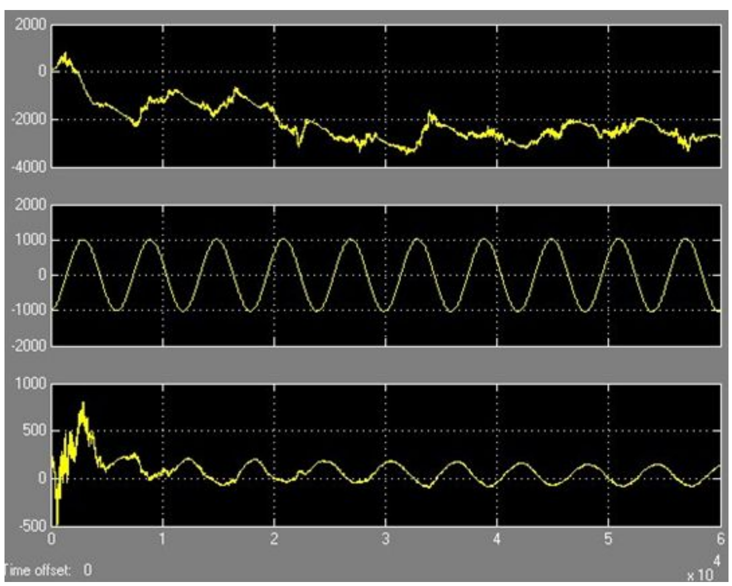
Figure 7 Simulated velocity errors (see online version for colours)



Notes: From top down respectively: ordinate – δV_x, δV_y, δV_z [m/x], abscissa t – [sec].

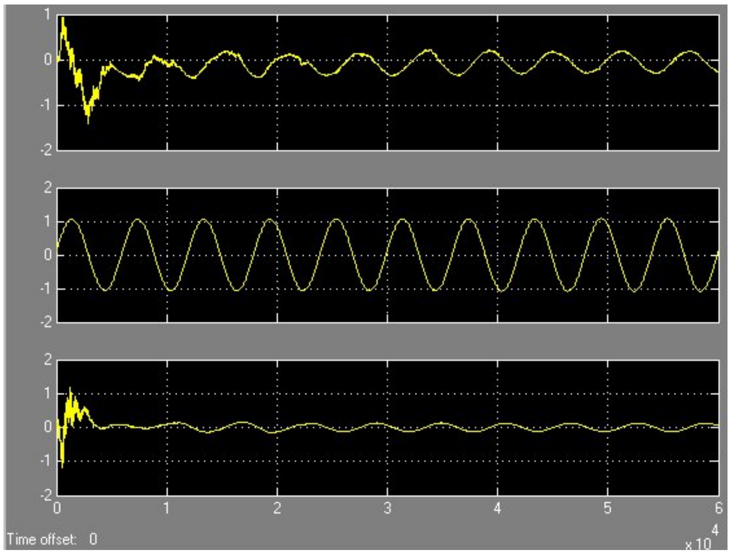
Plots of the estimation results are displayed in Figures 8–9. Figure 8 gives the satellite position estimation errors and Figure 9 velocity errors.

Figure 8 Position estimation errors (see online version for colours)



Notes: From top down: ordinate – $\delta\tilde{x} = \delta\hat{x} - \delta x$, $\delta\tilde{y} = \delta\hat{y} - \delta y$, $\delta z = \delta\hat{z} - \delta z$ [m], abscissa – t[sec].

Figure 9 Velocity estimation errors (see online version for colours)



Notes: From top down: ordinate – $\delta\tilde{V}_x = \delta\hat{V}_x - \delta V_x$, $\delta\tilde{V}_y = \delta\hat{V}_y - \delta V_y$, $\delta\tilde{V}_z = \delta\hat{V}_z - \delta V_z$ [m/s], abscissa t [sec].

As can be seen, the satellite position and velocity in the longitudinal channel (δx, δV_x, δz, δV_z) are estimated, but not in the lateral channel (δy, δV_y). The required observation time for estimate convergence is about one orbit. The estimation accuracy is moderate, being within 2.5–3 km for position and 0.3 m/s for the velocity. This simulation is rather optimistic in comparison with experimental data (Psiaki et al., 1993; Cote and de Lafontaine, 2008a, 2008b).

3.2 Simulation of attitude determination with the magnetometer

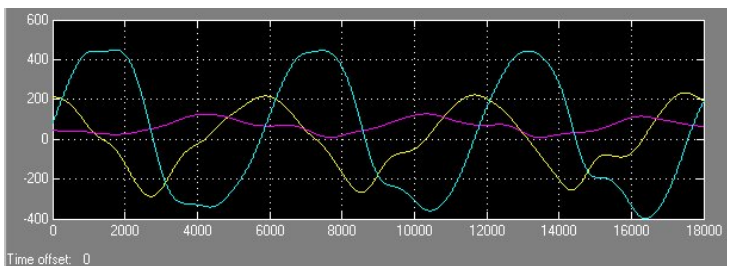
The linear model of satellite angular motion (20) and corresponding Kalman filter were implemented in Matlab Simulink. The magnetometer provided measurements of $\overline{B}$, and the three projections of the miv in the body frame were used to estimate the satellite attitude and attitude rate. The position (required to calculate the IGRF model) was modelled as nominal and known. It should be noted that in the real situation, when a magnetometer is used for satellite position determination and the calculated reference miv contains significant errors, the actual expected attitude determination accuracy is likely to be of lower quality than that presented below.

The same IGRF model as for satellite position estimation and the same polar orbit at 650 km altitude were used for this attitude study. The simulated IGRF model components are presented in Figure 10. The measurements were modelled as in (13) with additive stationary noise (σ_v = 1 mG, Δt = 1 sec). Plots of the measurements during three simulated orbits are presented in Figure 11. The linear satellite attitude model (20) errors were used in the simulation (Figure 12). Estimated attitude errors (small angles

$\hat{\alpha}_x$ – delta-roll, $\hat{\alpha}_y$ – delta pitch, $\hat{\alpha}_z$ – delta yaw) with residual noise are presented in Figure 13. It can be seen that pitch and yaw errors are estimated in the equatorial zone quite rapidly (within a few minutes), *but an accurate estimate for roll requires about 20 minutes*. The estimation accuracy of both angles (yaw and roll) decreases in the polar regions.

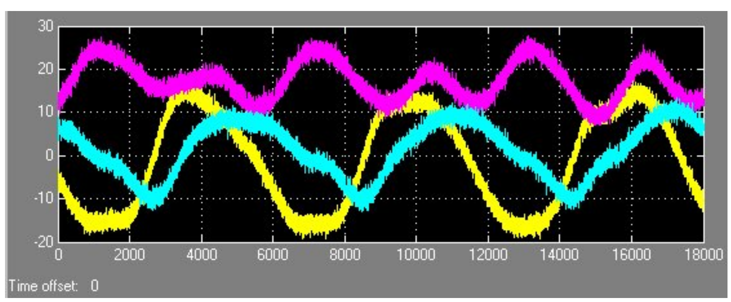
The Kalman filter could not be tuned for estimation of the satellite body rates and the magnetometer biases as it was for attitude, so that only attitude errors were included in the estimated attitude filter state vector and estimated. In spite of the theoretical result above that the body rates and magnetometer biases are observable as well as the attitude, this indicates that the estimator is very sensitive to parameters such as system noise and magnetometer measurement noise. Estimation of satellite body rates and bias using magnetometer measurements only is a subject for future study. Without the bias estimation the body rate estimation becomes much less sensitive.

Figure 10 Simulated IGRF components (see online version for colours)



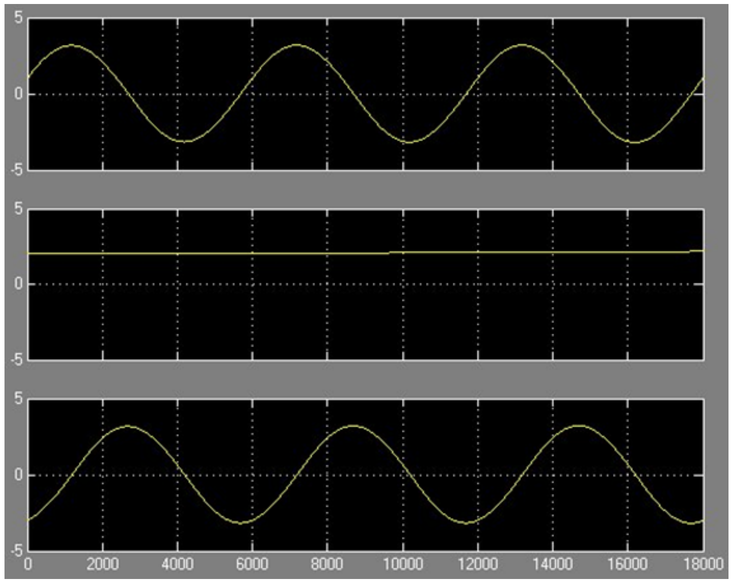
Notes: Ordinate – the yellow B_x, pink B_y, blue B_z [mG], abscissa – t [sec].

Figure 11 Differences between measured and referenced magnetic field plus white noise (see online version for colours)



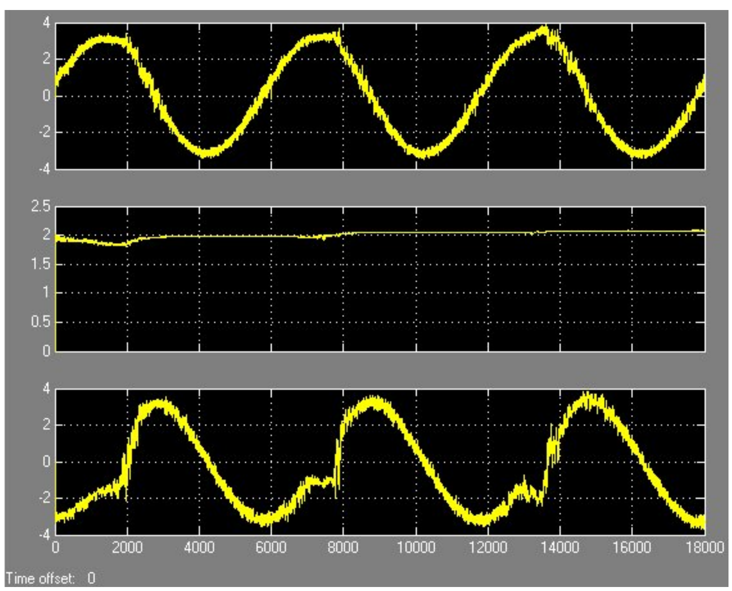
Notes: Ordinate: z_x – yellow, z_y – pink, z_z – blue [mG], abscissa t [sec]

Figure 12 Simulated satellite attitude errors (see online version for colours)



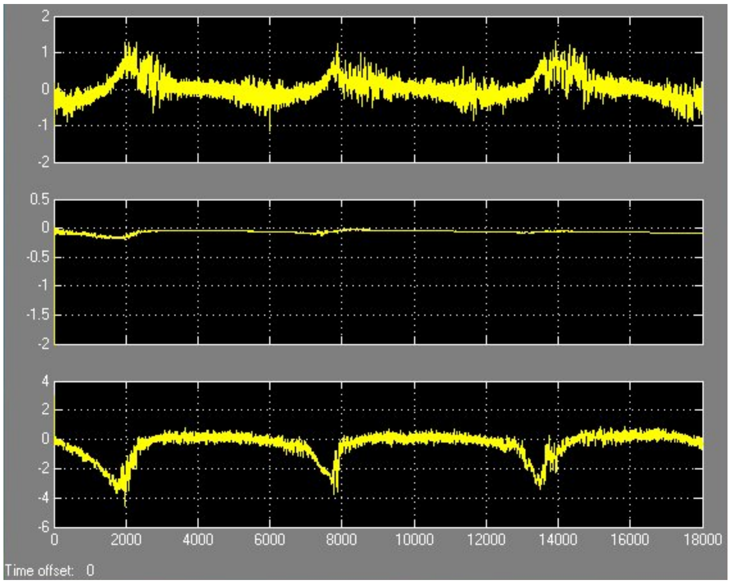
Note: From top down: ordinate – α_x, α_y, α_z [deg], abscissa t [sec]

Figure 13 Attitude error estimates (see online version for colours)



Note: From top down: ordinate – $\hat{\alpha}_x$, $\hat{\alpha}_y$, $\hat{\alpha}_z$ [deg], abscissa t [sec].

Figure 14 Attitude estimation errors (see online version for colours)



Notes: From top down: ordinate $\tilde{\alpha}_x = \hat{\alpha}_x - \alpha_x$, $\tilde{\alpha}_y = \hat{\alpha}_y - \alpha_y$, $\tilde{\alpha}_z = \hat{\alpha}_z - \alpha_z$ [deg], abscissa t [sec].

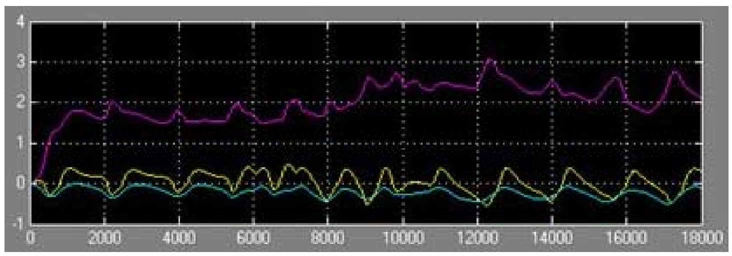
4 Experimental estimation

The Kalman filter that was presented above for simulated attitude estimation was also applied to ground based post-processing of TLM data provided by the attitude control system of the Canadian satellite RADARSAT-1. Two measured and reference vectors were used for this purpose. The pair of sun vector $(\bar{S})$ and earth miv $(\bar{B})$ was used for determination of the 'true' attitude by the conventional TRIAD method, and the single vector $\bar{B}$ for estimation of the attitude with only magnetometer measurements using the Kalman filter.

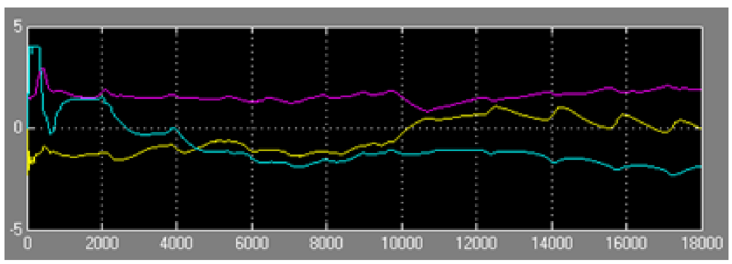
With the pair of the reference and measured vectors the satellite attitude was determined in indication mode (satellite attitude controlled using measurements from the horizon scanner and the sun sensor), and the magnetometer was used in open loop only. To reduce the output noise from the horizon scanner and sun sensor a second order dynamic filter (emulating satellite closed loop control dynamics) was used to obtain the 'true' attitude. After smoothing, the 'true' attitude and attitude estimate were compared.

Plots of the true attitude and its estimate are presented in Figure 15. The initial uncertainty of KF estimates was set to 10 deg (one sigma in all three-axes). After about 18,000 sec (three-orbits) attitude estimates from KF output (using the three projections of the measured miv) converge to the 'true' attitude determined by the TRIAD method (magnetometer and sun sensor). After convergence the residual errors are small. It should be noted that previous to this experiment the magnetometer was precisely calibrated in orbit and its bias and misalignment were compensated. The orbital parameters and ephemeris required for on-board IGRF computation were determined in the CSA mission control centre (ground station St. Hubert, Quebec) and uploaded to the satellite. In other words, an external estimate of the satellite position was used.

Figure 15 RADARSAT-1 attitude in orbit, (a) true attitude (roll-yellow, pitch-pink, and yaw-blue) of RADRSAT-1, using $\overline{S}$ and $\overline{B}$ and the TRIAD method (b) estimates obtained using the magnetometer measurements and the KF over three orbits (see online version for colours)



(a)



(b)

Notes: Orbital period: 100 min; inclination: 98.6 deg; altitude 800 km; sun – synchronous; dawn – dusk circular orbit; ordinate [deg]; abscissa t [sec]

It should also be mentioned that the sampling rate of the TLM data used was not high enough to provide an accurate result; however a qualitative similarity between the true and the estimated attitude can be seen from the figures above.

5 Potential applications

Many Canadian satellites (RADARSAT, SCISAT, QUICKSAT, JC2SAT, and NEOSSAT) were designed and equipped with three axis magnetometers, which are generally viewed as devices for coarse satellite attitude determination. Usually the magnetometer is used in conjunction with some other attitude sensor such as a Sun sensor or an earth sensor (RADARSAT, SCISAT), providing measurements of a second vector to determine attitude by some conventional means (e.g., TRIAD). For this case 3-axis attitude determination can be achieved instantaneously, but with unfiltered noise from the sensors, and body rates can be estimated by a simple differentiating filter, also still retaining significant levels of residuals noise. This noise, however, is usually sufficiently filtered for closed loop attitude control purposes.

Nevertheless, the use of a single, simple and low cost device such as a magnetometer for satellite orbit and attitude estimation together with some sort of dynamic estimator opens up new possibilities for space and ground applications.

Significant steps in this direction were made by the development of the LOCOOS Navigation System presented in Cote and de Lafontaine (2008a, 2008b). Despite the fact that its accuracy is moderate and the estimation time in many cases is significant, there are some situations when the system might be useful. These could be a simple on board attitude control system even of rough quality, on-ground monitoring of the satellite attitude and orbit in safe hold mode with telemetry information provided by an on-board magnetometer, or first reacquisition of a lost satellite. Future novel applications which exploit system observability for space magnetic navigation are also likely to appear.

6 Conclusions

Satellite navigation (orbit and attitude determination) using magnetometer measurements is possible due to the observability of the system. This property is guaranteed if the EMF induction vector magnitude is used to estimate satellite orbit (position and velocity) and its three projections are used to estimate the attitude. However in practical terms at least for near polar orbits, the observability is very weak in the lateral direction. Longitude and East velocity can barely be estimated. Due to magnetometer errors and weak observability, estimates can be quite coarse and significant time can be required to attain convergence. The time required to estimate the three attitude angles probably means that this method would be impractical for real time three axis attitude control.

This method might be considered for coarse navigation of satellites, in safe hold mode and for estimation of the satellite orbit and attitude on the ground. Future studies will be directed towards the use of available experimental data for further analysis of this magnetic navigation method.

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References

Bar-Itzhack, I. and Deutschmann, J. (1997) 'Comprehensive evaluation of attitude and orbit estimation using real earth magnetic field data', *11th AIAA/USU Conference on Small Satellites*, 15–18 September 1997, Logan, UT, USA.

Bar-Itzhack, I., Deutschmann, J. and Harman, R.R. (1998) 'An innovative method for low cost autonomous navigation for low earth orbit satellites', *AIAA/AAS Astrodynamics Specialist Conference*, AIAA-98-4183.

Bryson, A. and Ho, Y-C. (1975) *Applied Optimal Control*, pp.369, 457, Revised Edition, Taylor & Francis, Levitown.

Bryson, A.E. (1993) *Control of Spacecraft and Aircraft*, pp.1–15, Princeton University Press, Princeton, NJ.

Cote, J.D. and de Lafontaine, J. (2008a) 'A low-cost, compact attitude and orbit determination system for small earth satellite', *14th Canadian Astronautics Conference, ASTRO 2008*, 29 April to 1 May 2008, Montreal, QC, Canada.

Cote, J.D. and de Lafontaine, J. (2008b) 'Magnetic only orbit and attitude estimation using the square-root unscented Kalman filter: application to the PROBA-2 spacecraft', *AIAA GN&C Conference*, 18–21 August 2008, AIAA-2008-6293, Honolulu, Hawaii.

Gelb, A.E. (Ed.) (1974) *Applied Optimal Estimation*, p.67, The Analytic Sciences Corporation, MIT Press, Cambridge, USA.

Kim, Y.V. (2008) 'Kalman filter decomposition in the time domain using observability index', *Proceedings of 17th IFAC World Congress*, Seoul, Vol. 17, No. 17, Part 1, pp.12522–12527.

Kim, Y.V., Di Filippo, K.J. and Ng, A. (2004) 'On the autonomous in orbit calibration of satellite attitude sensors', *Proceedings of AIAA International Guidance Navigation and Control Conference*, Providance, RI.

Kwakernaak, H. and Sivan, R. (1972) *Linear Optimal Control Systems*, Wiley-Interscience, a Division of J.Wiley & Sons, Inc., New York.

Montenbruk, O. and Gill, E. (2000) *Satellite Orbits; Models, Methods, Applications*, pp.28–29, Springer-McGraw-Hill, Heidelberg, Germany.

Natanson, G.A., McLaughlin, S.F. and Nicklas, R.C. (1990) 'A method of determining attitude from magnetometer data only', *NASA Conference publication 3102, Proceedings of the flight Mechanic/Estimation Theory Symposium*, May, NASA, Goddard Space Flight Center, Greenbelt, Maryland.

Psiaki, M.L. (1995) 'Autonomous orbit and magnetic field determination using magnetometer and star sensor data', *Journal of Guidance, Control and Dynamics*, May–June, Vol. 18, No. 3, pp.584–592.

Psiaki, M.L., Hwang, L. and Fox, S. (1993) 'Ground tests of magnetometer based autonomous navigation (MAGNAV) for low-earth orbiting spacecraft', *Journal of Guidance, Control and Dynamics*, January–February, Vol. 16, No. 1, pp.206–214.

Roh, K.M., Park, S.Y. and Choi, K.H. (2007) 'Orbit determination using the geomagnetic field measurement via unscented Kalman filter', *Journal of Spacecraft and Rockets*, January–February, Vol. 44, No. 1, pp.246–253.

Sidi, M.J. (1997) *Spacecraft Dynamics and Control*, pp.13–24, 101, 57–63, 101, 110, 328–378, Cambridge University Press, New York, NY.

Vallado, D. (2007) *Fundamentals of Astrodynamics and Applications*, pp.253, 735, 788, 844–897, Springer, NY.

Wertz, J.R. (Ed.) (1978) *Spacecraft Attitude Determination and Control*, pp.118, 779, 693, Kluwer Academic Publishers, Dordrecht.

Nomenclature

ϕ	roll angle
θ	pitch angle
ψ	yaw angle
$\overline{B}$	true earth magnetic field induction vector
$\overline{B}_m$	measured earth magnetic field induction vector

$\overline{B}_r$	referenced earth magnetic field induction vector
h	satellite altitude
h_m	satellite orbital momentum
$\overline{r}$	satellite position vector
$\overline{V}$	satellite linear velocity vector
C	satellite attitude DCM
$\overline{\varpi}$	satellite angular velocity vector (body rate vector)
$\delta\overline{r}$	satellite position deviation (error) vector
$\delta\overline{V}$	satellite linear velocity deviation (error) vector
$\overline{\alpha}$	satellite attitude deviation (error) small angle rotation vector
$\delta\overline{\varpi}$	satellite angular velocity deviation (error) vector
$\delta\hat{\overline{r}}$	satellite position deviation (error) vector estimate
$\delta\hat{\overline{V}}$	satellite linear velocity deviation (error) vector estimate
$\hat{\overline{\alpha}}$	satellite attitude deviation (error) small angle rotation vector estimate
$\delta\hat{\overline{\varpi}}$	satellite angular velocity deviation (error) vector estimate
$\delta\tilde{B}$	local variation of the vector $\overline{B}$
B	magnitude of vector $\overline{B}$
$\overline{b}$	$\overline{B}$ unit vector
$\overline{Z}$	vector of magnetic measurements
$\overline{\varepsilon}$	small rotation angle vector of orientation error of $\overline{B}$
$\overline{v}$	vector of noise in magnetometer measurements
$\overline{z}$	vector of measurements
r	distance from satellite centre of mass to centre of earth
φ_m	magnetic latitude
λ_m	magnetic longitude
δB	variation of B
δr	variation of r
$\delta\varphi_m$	variation of magnetic latitude
$\delta\lambda_m$	variation of magnetic longitude

M	magnetic dipole strength
ω_0	satellite orbital angular velocity
f_x, f_y, f_z	disturbance specific forces acting on satellite
$U_e = 15.041$ deg/h	earth absolute angular rate
U	satellite orbit argument of latitude angle
i	satellite orbit inclination angle
δU	variation in satellite orbit argument of latitude angle
δi	variation in satellite orbit inclination angle
θ_m	miv deviation angle from local horizontal plane
ψ_m	magnetic heading angle
δx	along track position error
δy	across track position error
$\delta z = -\delta r$	radial position error
δV_x	along track velocity error
δV_y	across track velocity error
δV_z	radial velocity error
$\overline{m}$	unit vector of the dipole polar axis
$\overline{n}$	unit vector of radius-vector $\overline{r}$
$\mu = 3.986005 \cdot 10^{14}\ \mathrm{m^3 s^{-4}}$	earth gravity constant
a	satellite orbit semi-major axis
$V_{cir} = \sqrt{\dfrac{\mu}{a}}$	satellite first space velocity
e	satellite orbit eccentricity
θ	orbit true anomaly
ω_π	orbit argument of perigee
Ω	orbit right ascension of ascending node
T	satellite orbital period
E	eccentric anomaly
t	current time
t_π	time at orbit perigee.