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Magnetometer-Only Attitude Determination Using Novel Two-Step Kalman Filter Approach

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Determining spacecraft attitude in real time using only magnetometer data presents a challenging filtering problem. A flexible and computationally efficient method for solving the spacecraft attitude using only an inexpensive and reliable magnetometer would be a useful option for satellite missions, particularly those with modest budgets. The primary challenge is that magnetometers only instantaneously resolve two axes of the spacecraft attitude. Typically, magnetometers are used in conjunction with other sensors to resolve all three axes. However, by using a filter over an adequately long orbit arc, the magnetometer data can yield full attitude, and in real time. The method presented solves the problem using a two-step extended Kalman filter. In the first step, the magnetic field data are filtered to obtain the magnetic field derivative vector, which is combined with the magnetic field vector in the second step to fully resolve the attitude. A baseline scenario is developed, and a parametric study is conducted using the parameters of interest.

Nomenclature

$\mathbf{B}$ = magnetic field vector
F = Kalman filter dynamic matrix
H = Kalman filter measurement matrix
I = spacecraft moments of inertia
K_k = Kalman filter gain
$\mathbf{M}$ = external torque
P_k = Kalman filter covariance matrix
q = spacecraft attitude quaternion
q^c = quaternion conjugate
$\boldsymbol{\omega}$ = spacecraft angular velocity

I. Introduction

ATTITUDE determination is a problem that has been examined in-depth over the last several decades. Determining the orientation of an object in three-dimensional space has been an interest in dynamics and control since long before Sputnik launched in 1957. The application to spaceflight, of course, began shortly after Sputnik launched. As with any estimation problem, the challenge is to use the available measurements to estimate the spacecraft attitude. The measurements that have historically been used or experimented with are numerous and can be combined in different ways to achieve the necessary attitude estimation accuracy.

One of the classic early works on spacecraft attitude determination was by Wertz [1]. Wertz's book on spacecraft attitude determination is still a handbook used by many professionals in the field. Wertz covers many aspects of vector-based attitude determination as well as the basic attitude quaternion derivation that is used in this study to reduce the complexity of determining a nine-element direction cosine attitude matrix [1]. Other early attitude determination studies have resulted in the TRIAD method, the QUEST method, and additional solutions to Wabha's problem [2–5].

Work on attitude determination without the benefit of a rate sensor usually includes the analysis of filters using measurements from magnetometers, Sun sensors, star trackers, horizon sensors, and so forth. A new focus on gyro-less spacecraft attitude determination systems has emerged. These studies show that it is possible to estimate both the attitude and the angular rates from a variety of pointing vector measurements [6–8].

One of the first attitude determination studies that used magnetometer-only data was done when the Earth Radiation Budget Experiment mission became the victim of an attitude control anomaly and was lost. The data that were able to be downlinked were used in an attempt to determine the causes of the mission failure through post processing. Among the downlinked telemetry were data from a magnetometer. The magnetometer data were used with a method that was developed by Natanson et al. known as deterministic attitude determination using magnetometer-only data (DADMOD) [9]. The method solved for the attitude and angular rates from the magnetometer data by using finite differencing of the measurements to find the magnetic field derivative. The measurements of the magnetic field and its derivative were then used, along with knowledge of the spacecraft angular acceleration, to estimate the attitude and angular velocities. The equations became quadratic so that there were multiple solutions, and DADMOD selected which of the two solutions was most likely to be the correct attitude [9]. The method worked well for post processing, but as encountered during the course of this study, using noisy, real-time measurements prevents an accurate solution from being found.

Another magnetometer-only attitude determination solution was created by Psiaki. The error magnitudes achieved by Psiaki's Kalman filtering method showed errors of around 2–3 deg after about 100 s with low initial filter offset [10]. By using two nested Kalman filters, the method presented in this study is able to achieve better accuracy than previous Kalman filter-based magnetometer-only methods that were available in the literature.

Often attempts are made to avoid using high power consuming, expensive, and fragile gyroscopes for determining spacecraft attitude. Microelectrical-mechanical systems (MEMS) devices have been created that allow for the creation of solid-state inertia measurement units (IMUs), but most consider them to be too inaccurate and with inadequate resolution to provide the needed performance. One of the most recent attempts at magnetometer-only attitude determination was completed by Ma and Jiang [11]. The authors used an unscented Kalman filter (UKF) with magnetometer measurements to estimate the attitude of the spacecraft and to calibrate the magnetometers [11]. The importance of this method is that it included the ability to account for additional error beyond the specifications of the magnetometer. This calibration could be done on the ground, although the difficulty persists that some residual

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magnetic fields created by the spacecraft could corrupt the measurements, creating more noise. The UKF is much less computationally efficient than the extended Kalman filter (EKF), which presents an important drawback [12]. The method presented in this study, using the two-step EKF, provides a relatively good estimate accuracy, without the need to propagate several state vectors or additional states that cause the covariance propagation to become less computationally efficient. There are additional methods that can be used to calibrate magnetometers online such as shown by Crassidis in [13]. Electromagnetic interference (EMI) and electromagnetic compatibility (EMC) analysis can provide calibration of the magnetometer on the ground before the spacecraft is launched. The use of a Kalman filter with a calibrated sensor can thus provide computational efficiency over the UKF method and the preliminary results show comparable accuracy.

II. Solution Approach

The magnetic field vector can be quantified in terms of two different frames [Earth-centered inertial (ECI) and spacecraft body frame] using a spherical harmonics model for the former and a magnetometer measurement for the latter. The magnetic field vector, when expressed in terms of the body frame, is related to the magnetic field vector expressed in terms of the ECI frame (after first being found using the Earth-centered Earth fixed (ECF) frame as a function of longitude, latitude, and altitude) using the attitude quaternion. The magnetic field derivative with respect to the ECI frame is found by finite differencing the magnetic field model. The first step Kalman filter, or prefilter, is used to estimate the magnetic field derivative from the magnetometer measurements. The reason an approach such as the TRIAD method (used for calculating attitude from two linearly independent pointing vectors described in terms of two different frames) cannot be used in this application is due to the angular velocity of the body frame. The spinning body frame induces uncertainty in the derivative that in turn causes error in the TRIAD method. However, the EKF filter can estimate the attitude in the presence of the uncertainty.

The magnetic field vector and its derivative with respect to the inertial frame are calculated from a spherical harmonics model with coefficients obtained from NOAA's World Magnetic Model [14]. These provide the actual magnetic field values as well as a truth model for comparison. The measurements are simulated by adding noise to the magnetic field vector truth model.

The continuous-discrete EKF is used for both the prefilter and the attitude filter. The Kalman filter provides a method to account for inaccuracies in the dynamic model of a system by combining sensor measurements with knowledge of the system dynamics. A continuous dynamic model is used that describes the system, and measurements are used that can be related to the states of the system, the quantities that are being estimated. Measurements are received and states are updated at discrete intervals. With knowledge of the system model accuracy, as well as knowledge of the sensor measurement accuracy, an estimate of the system states is obtained. The Kalman filter propagates the state dynamics and error covariance forward in time [15].

The EKF calculates the estimate covariance, propagates it, and then uses it to update the states. Consider the system

$$\dot{x} = f(x) + w \tag{1}$$

$$y = h(x) + v \tag{2}$$

where y is the measurement, with w the process noise and v the measurement or sensor noise with a zero mean and a variance of Q and R, respectively. A set of partial derivative matrices is next defined as

$$F = \frac{\partial}{\partial x} f(x) \qquad H = \frac{\partial}{\partial x} h(x) \tag{3}$$

The F and H matrices in Eq. (3) are the Jacobian matrices of the plant and measurement, respectively. The system is numerically integrated including the states and the estimate covariance.

The system dynamics in Eq. (1) are used to propagate the states forward in time. The model can be very accurate or inaccurate, with the process noise, w, used to account for any inaccuracies. The covariance propagation equation for the EKF takes the form

$$\dot{P} = FP + PF^T + Q \tag{4}$$

where P is the estimate covariance, F is the Jacobian of the system dynamics, and Q is the process noise covariance. The results of the integration are known as the *a priori* state estimate and the covariance, and they are designated by a "bar" above the variable. Posteriori estimates are designated with a "caret" above them. The estimate and covariance are then updated using the equations

$$\begin{gathered} K_k = \bar{P}_k H_k^T (H_k \bar{P}_k H_k^T + R_k)^{-1} \\ \hat{x}_k = \bar{x}_k + K_k(y_k - h_k(\bar{x}_k, t_k)) \qquad \hat{P}_k = (I - K_k H_k)\bar{P}_k \end{gathered} \tag{5}$$

Results obtained from a Kalman filter are optimal for linear systems, and the EKF is very accurate and robust in most cases. Another benefit to the EKF is that it is very computationally efficient. If the results presented here were not sufficiently accurate to meet the specific mission requirements, a nonlinear filtering technique such as the UKF or particle filter could be attempted. In the examples considered in this study, the EKF provides acceptable accuracy while minimizing the impact on Command and Data Handling subsystem requirements.

Next, the process by which the magnetic field vector is used as a measurement to the prefilter in order to estimate the magnetic field derivative is described. The Kalman filter uses a third-order Markov process to model the magnetic field. The model is given by

$$\frac{d^3}{dt^3}\mathbf{B} = w \tag{6}$$

where $\mathbf{B}$ is the magnetic field vector and w is white Gaussian process noise. A Markov process is used to model the magnetic field vector because of the unknown spacecraft angular rates. A model could be used with the current estimate of the angular rates, but the Kalman filter has performed adequately without it. The measurement from the magnetometer is assumed to provide the magnetic field vector once per second. The use of a third-order Markov model allows the filter to estimate the first and second derivatives of the magnetic field vector as well as the field vector itself. The third-order Markov model used to estimate the magnetic field derivative is given by

$$\frac{d}{dt}\mathbf{B} = \dot{\mathbf{B}} \qquad \frac{d}{dt}\dot{\mathbf{B}} = \ddot{\mathbf{B}} \qquad \frac{d}{dt}\ddot{\mathbf{B}} = 0 \tag{7}$$

The model in Eq. (7) represents the dynamics of the prefilter. The measurement is represented by

$$y = \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} \tag{8}$$

The state dynamics can be represented as

$$\dot{x} = \frac{d}{dt}\begin{bmatrix} B_x \\ B_y \\ B_z \\ \dot{B}_x \\ \dot{B}_y \\ \dot{B}_z \\ \ddot{B}_x \\ \ddot{B}_y \\ \ddot{B}_z \end{bmatrix} = \begin{bmatrix} \dot{B}_x \\ \dot{B}_y \\ \dot{B}_z \\ \ddot{B}_x \\ \ddot{B}_y \\ \ddot{B}_z \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{9}$$

The F matrix and measurement matrix, H, for the Kalman filter are given by

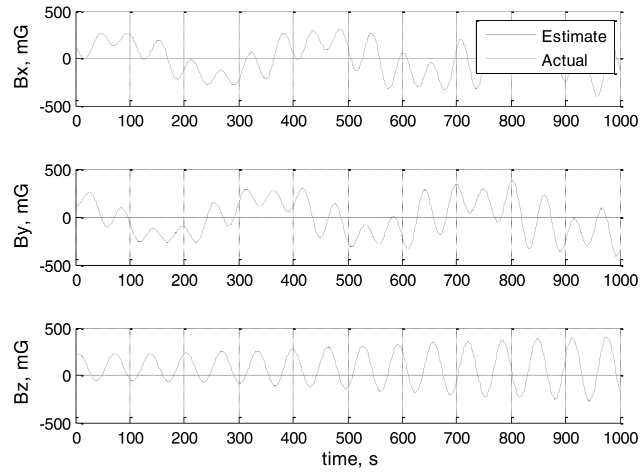

Fig. 1 Estimated and actual magnetic field components.

$$F = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{11}$$

Figures 1 and 2 show the prefilter estimates of the magnetic field vector and derivative for a simulation running 1000 s. The prefilter is evaluated for a baseline case in which a circular orbit is defined with 400 km altitude, 40 deg inclination, and an initial spacecraft angular velocity of [2, 5, 3] deg/s about the body axes. The simulation shows that the magnetic field derivative can be accurately estimated even without knowledge of the satellite rotation in the model. The estimates track very closely to the actual data. The errors in the magnetic field vector and derivative estimates are shown in Figs. 3 and 4.

The prefilter estimates the magnetic field derivative with sufficient accuracy to calculate the spacecraft attitude. The filter can be adjusted using process noise covariance to obtain improved results depending on the specific mission parameters, but the given initial error covariance and process noise covariance matrices typically provide consistent results regardless of the simulation conditions considered.

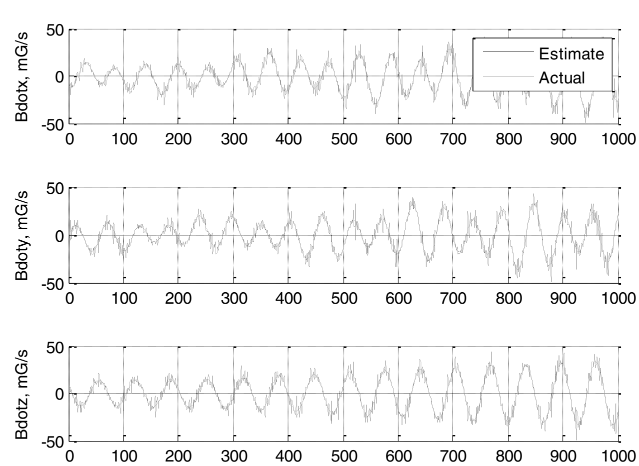

Fig. 2 Estimated and actual magnetic field derivative components.

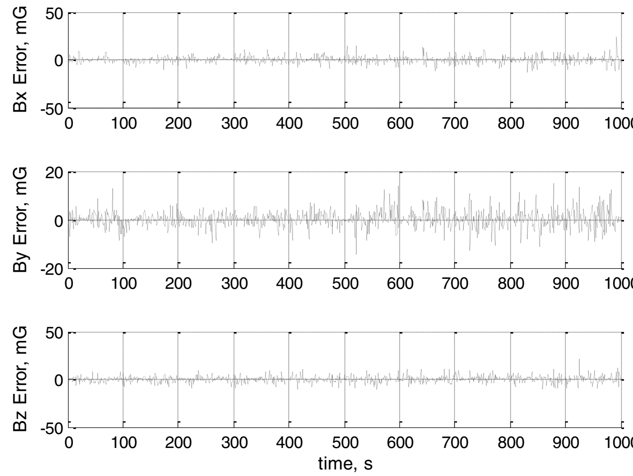

Fig. 3 Magnetic field vector component estimation error.

By using a prefilter to provide the magnetic field vector and its derivative (pseudomeasurements) and knowing the inertially referenced vectors from Earth's magnetic field model, the only unknowns are the attitude and angular rates. By making the attitude and angular rates the state vector for an EKF, the equations can be differentiated to find the measurement matrix.

To summarize the method, first, start by equating the measurement (magnetic field vector) to the desired states of the filter. In this case, it is desirable to estimate the attitude quaternion from the magnetic field measurements. The measurement equation must show the magnetic field calculated as a function of the attitude quaternion, namely,

$$\langle 0, \mathbf{B}_b \rangle = q^c \langle 0, \mathbf{B}_I \rangle q \tag{12}$$

where q is the spacecraft attitude quaternion, and q^c is the attitude quaternion conjugate. The "real" part of the quaternion is the first element and the "vector" part is the second through the fourth element. When simplified, Eq. (12) yields three scalar equations and four unknowns (the elements of the attitude quaternion). The time derivative is taken resulting in

$$\langle 0, \dot{\mathbf{B}}_b \rangle = \dot{q}^c \langle 0, \mathbf{B}_I \rangle q + q^c \langle 0, \dot{\mathbf{B}}_I \rangle q + q^c \langle 0, \mathbf{B}_I \rangle \dot{q} \tag{13}$$

Equations (12) and (13) represent six scalar equations and eight unknowns. By using the fact that the attitude quaternion has unit magnitude, the constraint equations

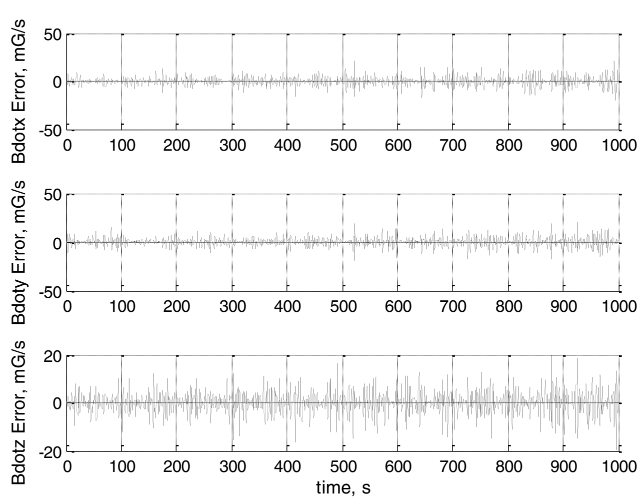

Fig. 4 Magnetic field derivative vector component estimation error.

$$q_0^2 + \mathbf{v}_q \cdot \mathbf{v}_q = 1 \tag{14}$$

$$2q_0\dot{q}_0 + 2q_1\dot{q}_1 + 2q_2\dot{q}_2 + 2q_3\dot{q}_3 = 0 \tag{15}$$

are obtained [16]. The system now has eight equations and eight unknowns; however, it is more beneficial to define the angular velocity as a state rather than the quaternion derivative, so by using the quaternion kinematic equation

$$\dot{q} = \frac{1}{2} q\boldsymbol{\omega}^* \tag{16}$$

where $\boldsymbol{\omega}^*$ is the quaternion representation of the spacecraft angular velocity, $\langle 0, \boldsymbol{\omega}\rangle$, Eq. (13) becomes

$$\langle 0, \dot{\mathbf{B}}_b\rangle = \frac{1}{2} q^c \boldsymbol{\omega}^* \langle 0, \mathbf{B}_I\rangle q + q^c\langle 0, \dot{\mathbf{B}}_I\rangle q + q^c\langle 0, \mathbf{B}_I\rangle \frac{1}{2} q\boldsymbol{\omega}^* \tag{17}$$

Equations (12) and (17) become the measurement equations for the second step filter.

The governing equations for the spacecraft attitude quaternion with respect to the magnetic field vector and derivative are now used to configure the filter. Equations (12–15) are used to relate the states, attitude quaternion and spacecraft angular rates to the pseudomeasurements, the magnetic field vector and its derivative. These attitude equations provide a system with eight equations and eight unknowns, and this system could be solved. However, the quadratic nature of the equations leads to multiple solutions, and the equations are difficult to solve. Another approach uses the system in a filter that uses a sequence of estimates and measurements to find the best estimate without the need to choose between two solutions. The next step, then, is to construct an EKF using the magnetic field vector and its derivative as measurements and estimate the attitude quaternion and rates.

The attitude determination filter is configured to accept the magnetic field and its derivative as measurements with the states for the filter as the attitude quaternion and the spacecraft angular rates. The states are related to the measurement through the H matrix which contains the derivatives of the quaternion equations derived previously. Finite differencing is used to calculate the measurement matrix needed for the EKF filter used in the attitude determination algorithm.

The measurements for the attitude filter are

$$y = \begin{bmatrix} B_{b_x} \\ B_{b_y} \\ B_{b_z} \\ \dot{B}_{b_x} \\ \dot{B}_{b_y} \\ \dot{B}_{b_z} \end{bmatrix} \tag{18}$$

By substituting Eqs. (12), (13), and (16) into Eq. (18), the measurements can be related to the states as

$$\begin{aligned} y &= \begin{bmatrix} q^c\langle 0, B_I\rangle q \\ \dot{q}^c\langle 0, B_I\rangle q + q^c\langle 0, \dot{B}_I\rangle q + q^c\langle 0, B_I\rangle \dot{q} \end{bmatrix} \\ &= \begin{bmatrix} q^c\langle 0, B_I\rangle q \\ \frac{1}{2} q^c\omega^*\langle 0, B_I\rangle q + q^c\langle 0, \dot{B}_I\rangle q + q^c\langle 0, B_I\rangle \frac{1}{2} q\omega^* \end{bmatrix} \end{aligned} \tag{19}$$

Note that in the preceding equation, the multiplications are quaternion multiplications and the brackets around the magnetic field values add a zero as the first element so that the vector becomes a four element vector that can be multiplied with quaternions. It is also assumed that the first element (which is always zero) of each resultant four-element vector is removed after the multiplications (in order to preserve the dimension of y having six elements instead of eight).

The system dynamics are represented using quaternion notation and Euler's equations as

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \frac{1}{2} q\omega^* \\ \frac{M_x - (I_z - I_y)\omega_y\omega_z}{I_x} \\ \frac{M_y - (I_x - I_z)\omega_x\omega_z}{I_y} \\ \frac{M_z - (I_y - I_x)\omega_x\omega_y}{I_z} \end{bmatrix} \tag{20}$$

The filter matrix F becomes

$$F = \begin{bmatrix} 0 & -\frac{1}{2}\omega_x & -\frac{1}{2}\omega_y & -\frac{1}{2}\omega_z & -\frac{1}{2}q_1 & -\frac{1}{2}q_2 & -\frac{1}{2}q_3 \\ \frac{1}{2}\omega_x & 0 & \frac{1}{2}\omega_z & -\frac{1}{2}\omega_y & \frac{1}{2}q_0 & -\frac{1}{2}q_3 & \frac{1}{2}q_2 \\ \frac{1}{2}\omega_y & -\frac{1}{2}\omega_z & 0 & \frac{1}{2}\omega_x & \frac{1}{2}q_3 & \frac{1}{2}q_0 & -\frac{1}{2}q_1 \\ \frac{1}{2}\omega_z & \frac{1}{2}\omega_y & -\frac{1}{2}\omega_x & 0 & -\frac{1}{2}q_2 & \frac{1}{2}q_1 & \frac{1}{2}q_0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-(I_z - I_y)\omega_z}{I_x} & \frac{-(I_z - I_y)\omega_y}{I_x} \\ 0 & 0 & 0 & 0 & \frac{-(I_x - I_z)\omega_z}{I_y} & 0 & \frac{-(I_x - I_z)\omega_x}{I_y} \\ 0 & 0 & 0 & 0 & \frac{-(I_y - I_x)\omega_y}{I_z} & \frac{-(I_y - I_x)\omega_x}{I_z} & 0 \end{bmatrix} \tag{21}$$

The H matrix for the EKF filter is calculated using finite differencing on the equations that represent the measurements in terms of the states as shown in Eq. (19).

III. Results

To conduct a parametric analysis, a baseline set of conditions was defined and the performance of the algorithm assessed. The baseline case defined previously of a circular, 400 km altitude orbit at 40 deg inclination was used. The right ascension of the ascending node was arbitrarily set to 10 deg. Because of the time dependence of the magnetic field, a simulation start date was needed and the epoch 28 March 2011 was arbitrarily assumed. The initial conditions for the attitude quaternion and angular rates were arbitrarily selected as

$$q = \begin{bmatrix} 0.1031 \\ 0.5157 \\ 0.2063 \\ 0.8251 \end{bmatrix} \tag{22}$$

$$\omega = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix} \text{ deg/s} \tag{23}$$

The model was propagated for 1000 s. The attitude rates change slowly over time due to the unequal principle moments of inertia, assumed as (consistent with a microsatellite-class spacecraft)

$$I_{xx} = 0.892 \text{ kg} \cdot \text{m}^2 \qquad I_{yy} = 0.875 \text{ kg} \cdot \text{m}^2$$
$$I_{zz} = 0.618 \text{ kg} \cdot \text{m}^2 \tag{24}$$

The measurements, which consist of the Earth's magnetic field vector components, were generated by adding Gaussian noise to the magnetic field truth model. The noise was assumed to be white noise with a zero norm and a mean of 3 deg. The noise was added in an angular manner because magnetometer specifications are often given in degrees. The measurement was assumed to be random within a 3 deg cone about the true vector. With the truth model simulation process completed, the next step was to initialize the filter and run the attitude determination algorithm. The prefilter was initialized using Eqs. (25) and (26). The prefilter does not need to be adjusted for each individual simulation, and was not for the results presented here. The possibility exists, however, that with certain orbits, adjusting the weights on the Markov model or the measurements could improve the results because the prefilter could be subject to sensitivities in the magnetic field fluctuations. In this study, it is assumed that the prefilter is a standalone add-on to the attitude filter that does not need to be adjusted.

The filter states are composed of the attitude quaternion and the angular rates. The initial estimates used for each of the simulations in the parametric analysis (unless otherwise noted) were assumed as

$$\hat{q} = \begin{bmatrix} 0.28222 \\ 0.56443 \\ 0.18814 \\ 0.75258 \end{bmatrix} \tag{25}$$

$$\hat{\omega} = \begin{bmatrix} 2.2 \\ 5.5 \\ 3.3 \end{bmatrix} \text{ deg/s} \tag{26}$$

These filter initial conditions reflect an 11.48 deg attitude determination error and a 10% error in the initial angular velocity estimate. These estimates can be updated by using data available from the launch vehicle provider on expected tip off rates.

As shown in Fig. 5, the attitude angular estimation error decreases from the initial value of 11.48 deg and reaches steady state in about 900 s. The steady-state error is below 1 deg. The method is able to compute the attitude quaternion very efficiently and quickly converges. Figure 6 shows the filter state covariance with the first four diagonal elements corresponding to the attitude quaternion, and the last three corresponding to the angular velocity. The elements corresponding to the attitude quaternion rise initially because the system would not converge unless the initial covariance diagonal elements were very low. When running the simulation for a longer time span, the covariance elements for the attitude quaternion stay bounded.

After the baseline conditions were set, it was important to conduct tests under varying conditions to determine the robustness of the method. The variables of interest were the angular velocity of the spacecraft, the inclination of the orbit, the altitude of the orbit, and the accuracy of the position estimate of the spacecraft.

The plot of the magnetic field vector components shows that the oscillations in the magnetic field vector are mostly due to the spacecraft rotation. Comparisons between the baseline case and the low and high angular velocity cases show the oscillations increase from very low to very high as the angular velocity increases. The error covariance matrix diagonal elements are initialized very low because of the error spike encountered at the beginning of the simulation. If the covariance was initialized higher, the estimates would diverge immediately. The covariance diagonal elements behave differently than in a typical Kalman filter. Although the error covariance diagonals remain low and bounded (simulations that have been run over longer time spans show the bounded nature), the noise and nonlinearities cause fluctuations in the values. The next section describes how the filter simulation responds to changes in orbit and initial conditions.

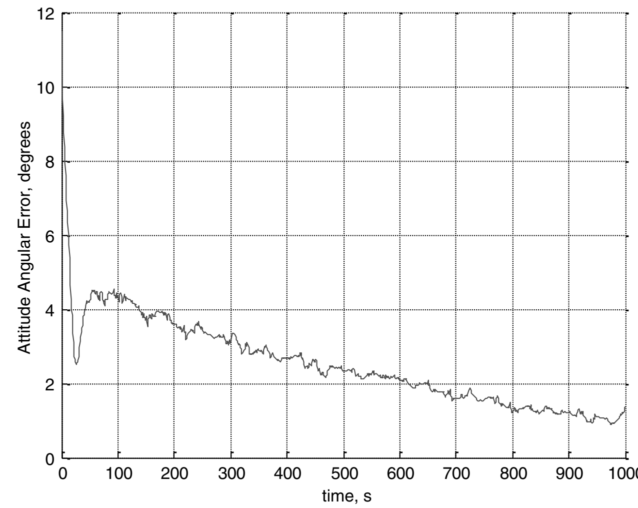

Fig. 5 Angular error in estimated spacecraft attitude.

IV. Parametric Study

With the attitude determination algorithm successively converging for the baseline case, the next step was to evaluate robustness and reliability by varying mission parameters to identify any ambiguities or singularities. The same initial conditions as the baseline case were used, varying only the parameter of interest.

The first parameter varied was the altitude. The spacecraft orbital altitude is of interest because the magnetic field decreases in intensity as altitude increases. It was desired to determine the limit in orbital altitude for which the algorithm performs adequately. Common sense suggested that the algorithm should be more effective at lower altitudes because of the higher magnitudes of the magnetic field vector and because the magnetic field derivative is likely changing more rapidly.

The first simulation was performed with an altitude of 3000 km. The baseline altitude of 400 km is sufficiently low to define the minimum altitude considered. The only change in the simulation

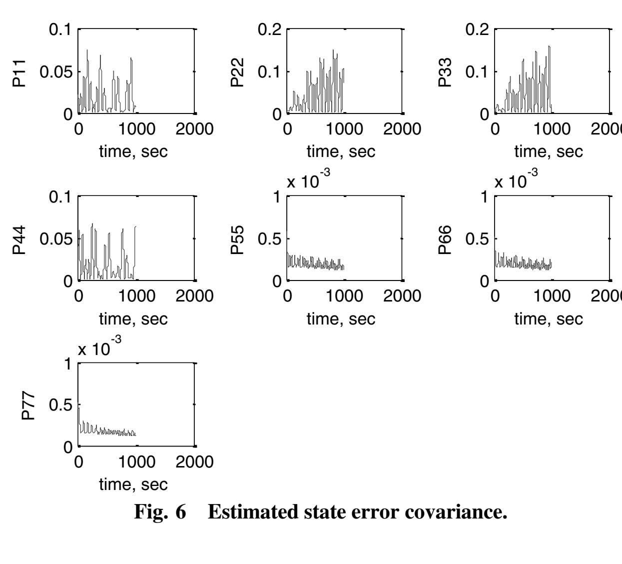

Fig. 6 Estimated state error covariance.

conditions from the baseline case was the altitude increase. The new altitude of 3000 km was chosen because it was the next lowest altitude considered that showed a significant change in the results, shown in Fig. 7.

The 3000 km simulation showed that the estimation error using the prefilter was decreased with the large increase in altitude. There was marginal change in the overall estimated attitude error for the simulations. It was then important to next examine a simulation at a much higher altitude, 10,000 km, to determine if the accuracy trend continued.

The results of the 10,000 km altitude simulation (Fig. 8) show that the accuracy increase of the prefilter continues as the altitude is increased. The estimated attitude angular error reflects the increased pseudomeasurement accuracy for this case. The error decreases faster from the initial offset and is lower than the lower altitude simulations. This confirms that the algorithm accuracy does indeed improve for higher altitudes. The algorithm is expected to eventually diverge when the magnetic field intensity becomes too small to measure.

An explanation can be offered for the (unexpected) improvement with altitude. The magnetometer is assumed to have 1 deg of accuracy along the direction of the magnetic field, and the algorithm adds 3 deg (three sigma) of normally distributed noise to the pointing direction of the "true" magnetic field vector (without changing its magnitude). The filter uses only the vector components for the attitude estimation. By adding 3 deg of noise at a higher altitude when the magnetic field vector magnitude is lower, changes in the magnetic field vector elements are less than when adding 3 deg at a lower altitude where the magnetic field vector magnitude is higher. In effect, the higher altitude case is using higher accuracy measurements of the magnetic field components. In reality, this would probably not be the case. An improved analysis could be made if magnetometer performance for varying altitudes was available, but such information could not be readily found.

Inclination change is the orbit parameter that most significantly affects the variance of the value of the magnetic field vector. The local magnetic field changes significantly as the position of the satellite moves over the poles. In contrast, the magnetic field at the equator is fairly constant as a spacecraft moves along an equatorial orbit. It was deemed important to evaluate the effect of inclination on the attitude determination algorithm. This section shows the results of two more simulations: a polar orbit and an equatorial orbit.

The polar orbit case was expected to exhibit better performance because the magnetic field vector is more dynamic, making the magnetic field derivative more observable. Figure 9 shows the attitude estimation angular error results of this simulation, with the altitude returned to the baseline value of 400 km. Indeed, as expected, convergence is a little faster compared to the baseline case.

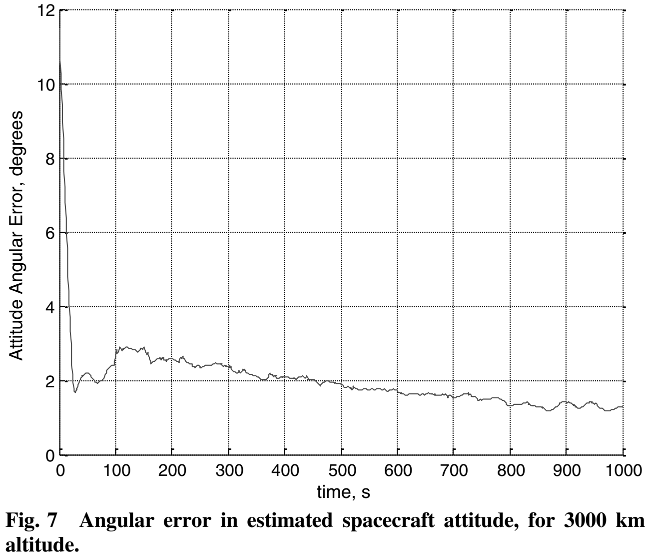

Fig. 7 Angular error in estimated spacecraft attitude, for 3000 km altitude.

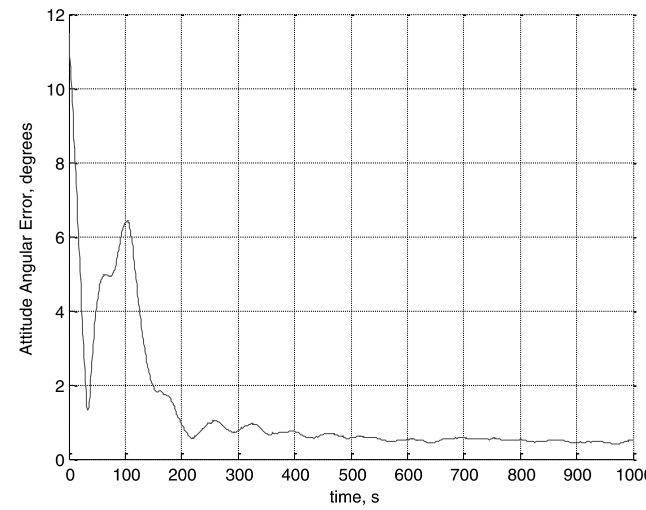

Fig. 8 Angular error in estimated spacecraft attitude, for 10,000 km altitude.

The results for a polar orbit at 3000 km altitude are very similar to the results of the 40 deg inclination baseline case, and are omitted here for brevity. There is little difference in convergence time or steady-state error.

The next simulation shows the baseline simulation with the inclination lowered to an equatorial orbit. This is one of the cases of concern for testing the limits of the algorithm, because the magnetic field does not change as rapidly in an equatorial orbit. The assumed spacecraft attitude rotation was expected to be beneficial to the estimation; however, if the spacecraft rotates about an axis nearly aligned with the magnetic field vector (which is more likely to occur in an equatorial orbit, but still somewhat unlikely) the problem becomes unobservable.

The equatorial orbit inclination does have a significant effect on the estimation of the attitude, as shown in Fig. 10. By examining the magnetic field derivative, it was noted that the rate of change of the magnetic field vector is about half of the previous polar orbit case. Even though the magnetic field is estimated to roughly the same error level, the lower magnitude means that the amount of error will have more significant impact. The angular error is still decreasing at the end of the simulation, so an extended simulation of 6000 s was run to determine if a steady state would be reached as shown in Fig. 11. The error eventually converges to approximately the same order of magnitude as all of the other simulations, but taking approximately ten times longer to reach steady-state. The algorithm appears to be sufficiently robust to successfully converge to the proper attitude

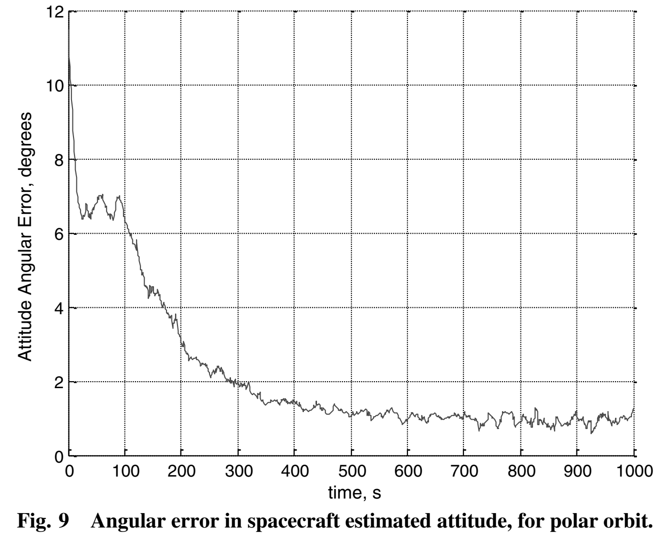

Fig. 9 Angular error in spacecraft estimated attitude, for polar orbit.

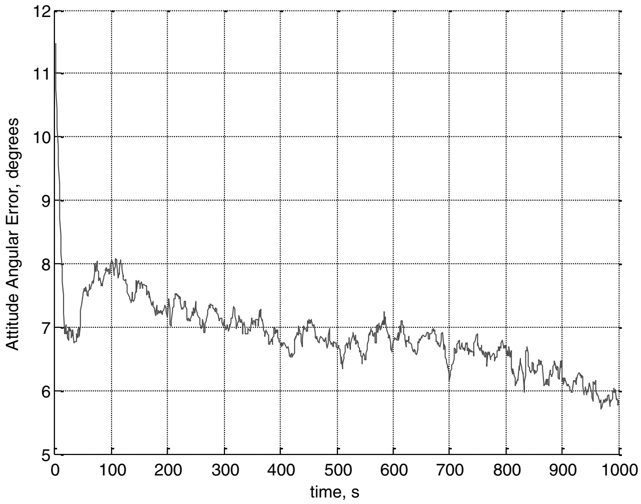

Fig. 10 Angular error in spacecraft attitude estimate, for equatorial orbit.

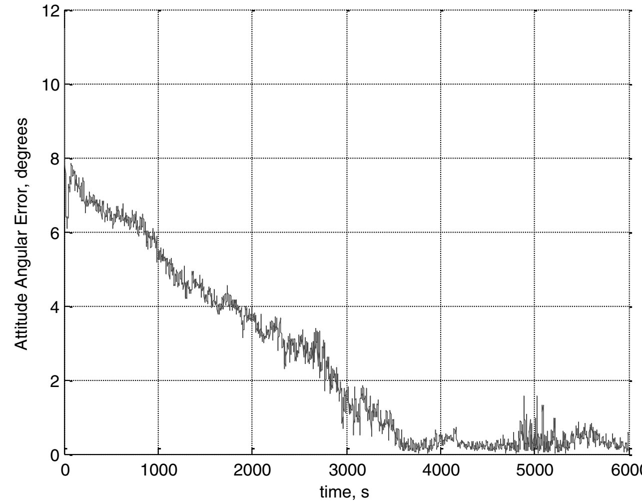

Fig. 11 Angular error in spacecraft attitude estimate, for equatorial orbit and extended simulation duration.

when faced with the observability challenges from equatorial orbits (given a longer period of time). The algorithm does require a longer time history of the magnetic field data in order to converge to its estimate.

The spacecraft angular velocity is a critical parameter in this algorithm. Because there is no measurement of the angular rates, and the measurements of the magnetic field and its derivative vary largely with angular velocity, there is the potential for difficulties with certain angular velocities. There are two expected issues with the angular velocity. It has been determined through previous simulations that a more rapid change in the magnetic field vector leads to better accuracy. It can thus be postulated that as the spacecraft rotates faster the attitude determination accuracy will increase. A low rotation rate lowers the amount that the magnetic field vector changes between each measurement and should thus degrade the observability.

The second issue is with high angular velocity, the higher the angular velocity, the lower the required filter response time. When selecting the measurement covariance of the pseudomeasurements for the filter for the previous cases, the response time of the filter was purposefully slowed to minimize over corrections. This could pose a concern for higher rotation rates, and such a case is included in this study.

To address the first issue, the next simulation was conducted with a decrease in the angular velocity to values that have been determined by trial-and-error to be at the lower limit at which the algorithm can successfully converge. The attitude estimate diverges for zero angular velocity. This failure could be avoided by inducing a spin about an axis that does not affect the mission success criteria. It is also possible that further tuning of the Kalman filters could allow for the solution to be found in this case, and is currently a topic of study. (It was found that by varying the weights, the performance in the zero angular velocity case could be improved; however, the performance in cases with higher angular velocity was then decreased. The attitude filter estimate has a bias in the zero angular velocity case that causes the algorithm to "believe" the spacecraft is rotating slowly.) In this paper, the limiting low spacecraft angular velocity case was assigned the initial value

$$\omega = \begin{bmatrix} 0.02 \\ 0.05 \\ 0.1 \end{bmatrix} \ \mathrm{deg}/\mathrm{s} \tag{27}$$

The results for this low angular velocity case (Fig. 12) show that the attitude error drops to less than 1 deg but requires more time to reach steady state. It is noted that the magnetic field vector does not oscillate as in the other simulations, because most of the variance in the magnetic field vector is due to the spinning of the spacecraft and the angular velocity is greatly reduced in this simulation.

The error in the estimation of the magnetic field derivative was of the same order of magnitude as those in previous simulations, but it required more time to converge. The algorithm successfully converges with a low angular velocity, but diverges with zero angular velocity.

The simulation showed that orbits with low angular velocity are more difficult to solve using magnetometer-only determination. The convergence time of the algorithm was much higher in the low angular velocity case than for the nonzero cases. It is important to note the other differences in this simulation compared to the others presented. The quaternion and angular velocity estimation errors show a bias, or a lack of convergence. The errors in previous simulations quickly converge and oscillate slightly around zero. The covariance diagonal elements also behave differently compared to the other simulations. The diagonal elements vary more slowly, and do not show the bounded behavior seen earlier.

To address the second issue (high rotation rates), the spacecraft rotation rate is increased to 20 deg/s along each axis. As shown in Fig. 13, the attitude estimation performance is enhanced, as the results show an improvement over the baseline case.

The simulation shows that the results improve with increasing angular velocity. This was expected when considering the dynamics

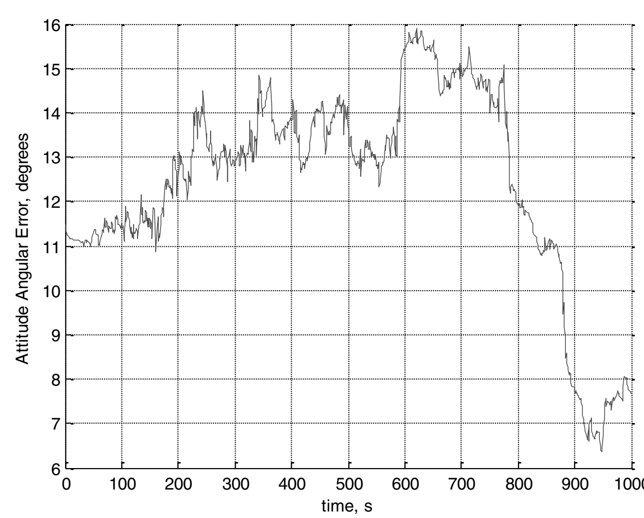

Fig. 12 Angular error in spacecraft attitude estimate, for low angular velocity.

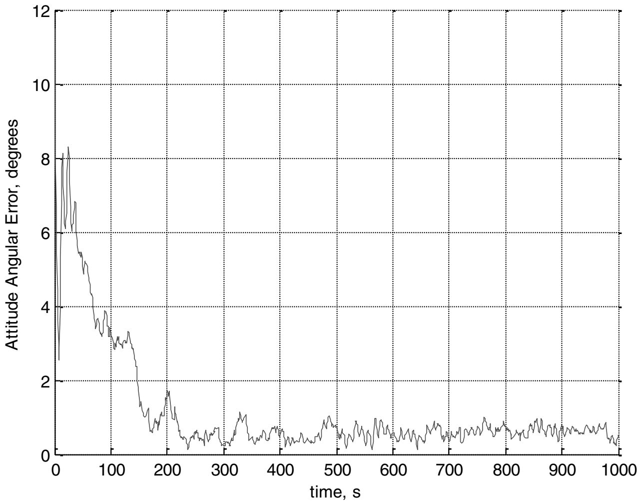

Fig. 13 Angular error in spacecraft attitude estimate, for 20 deg/s angular velocity.

of the problem. As the magnetic field vector components change more rapidly, the vector derivative magnitude increases, and will then be easier to estimate. The angular velocity is thus important to consider when using this algorithm. Spacecraft that do not rotate may encounter difficulty in obtaining an accurate attitude estimate, especially near an equatorial orbit.

The final parameter examined in the parametric analysis was the sensitivity of the algorithm to the spacecraft orbital location, typically provided by Global Positioning Satellite measurements. The magnetometer-only algorithm relies on the position of the satellite being known so that the model can calculate the magnetic field vector and its derivative in terms of the ECI frame at that point in space. The attitude determination filter performance could be degraded by inaccuracies in the spacecraft position estimation. In this study, a simulation was conducted in which half a kilometer of normally distributed, zero mean noise was added to the spacecraft position. As shown in Fig. 14, the simulation converged to a solution without any apparent difficulties.

The 0.5 km position error added to the simulation shows that the relatively small change in the magnetic field is not enough to significantly affect the attitude determination algorithm. The spinning of the spacecraft would cause a much larger change in the magnetic field vector than the position of the spacecraft. Extremely accurate position information is thus not vital for the attitude algorithm to provide a suitable attitude estimate.

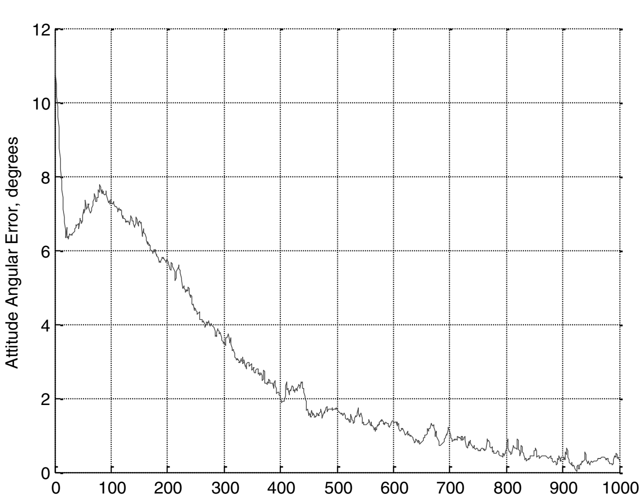

Fig. 14 Angular error in spacecraft attitude estimate (deg), for 0.5 km position error.

V. Conclusions

This paper describes the development of an attitude determination algorithm that can rely solely on magnetometer measurements and achieve accuracies of less than 1 deg. The algorithm was tested through a rigorous parametric study, and conclusions were drawn about the performance of the attitude determination system. The algorithm can be used for low-cost satellites where only a single sensor can be procured, or as a contingency to a more accurate system that may experience a system failure or gap in measurements. The software has been tuned to run efficiently, though there are some improvements suggested.

The method developed is effective when the angular velocity along at least one axis is higher than 0.1 deg/s. Unfortunately, this is higher than the nominal operating conditions for some spacecraft. A solution to this limitation could be to induce a slight rotation about a nonessential axis. The difficulty appears to be induced by an observability issue with the EKF formulation when the angular velocity is zero. However, future modifications or tuning may alleviate the problem. By examining the simulations conducted, it was observed that increasing the initial covariance diagonals slowed or stopped the divergence of the algorithm. Raising the covariance diagonals, however, causes divergence in cases with nonzero angular velocity. The results presented here are intended to demonstrate the robustness of the algorithm to varying conditions, as well as identify its limitations.

The algorithm encounters complications if the spacecraft has zero angular velocity. For nonrotating and slightly rotating spacecraft, the attitude algorithm fails to determine the correct attitude. It may be explained by the fact that the magnetic field vector is changing much more slightly and the derivative is very low for these cases. The algorithm also has occasional convergence difficulties if the spacecraft is in an equatorial orbit. The selection of initial error covariance and the process and measurement noise covariance of the two nested Kalman filters was a very important factor in ensuring convergence of the algorithm to a suitable attitude solution. One issue noticed was when the initial covariance matrix is too large. The correction at the beginning of the simulation causes the estimates to diverge if the initial covariance matrix is too large. Also, if the weights on the pseudomeasurements are too low, the system will perform less accurately. Determining the weights on the pseudomeasurements was a difficult task because there are no sensor data to determine the noise covariance. The first attempt was to use the error covariance of the prefilter as the measurement noise covariance of the attitude filter, but the values were too low for the filter to succeed.

There are a few aspects that can be completed in order to improve the algorithm, either in accuracy or in computational efficiency. The first item that should be addressed is analytically determining the derivative of the magnetic field. The alternative used in this study was to use finite differencing on the magnetic field model to find the derivative. Although this has proven to be effective, the analytical solution would be more computationally efficient.

A second improvement is to add attitude perturbations and control to the system. Difficulties are not anticipated with the algorithm when encountering a changing angular velocity (the asymmetries inherently produce this effect anyway). The algorithm is improved in situations where the magnetic field body frame measurement is more dynamic, so adding an actual attitude model to the system dynamics should improve the results. Such a model would include solar radiation pressure effects, drag, and gravity gradient effects. These can easily be modeled and added to the attitude model that the filter uses to predict the future attitude.

Magnetometer calibration is also a concern for this type of attitude system. The spacecraft will produce residual magnetic fields from electronic components. Also, some spacecraft use magnetic torque coils or rods. These attitude control devices cause control torques by creating a magnetic field that reacts with Earth's magnetic field. The magnetic field vector created by the torque coils will likely interfere with the measurement from the magnetometer. There are a couple of approaches to mitigate these effects. The first method would be to determine the Earth's magnetic field vector from the measurement by using the known magnetic field vector generated by the coils. This would require calibration of the magnetometer inside the completed satellite with the coils activated. Using the known values before the coil is activated, and the measurements after they are activated, will allow for the disturbance to be accounted for and removed. The same process can be used for the residual magnetic field created by the electronic components. The amount of interference may also depend on the operating mode of the spacecraft. Another solution is to add states to the prefilter to calculate and remove the interference. This is only necessary if the filter is unable to filter out the residual magnetic field when it filters the measurement, but only full-scale, fully integrated testing will determine what is necessary.

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