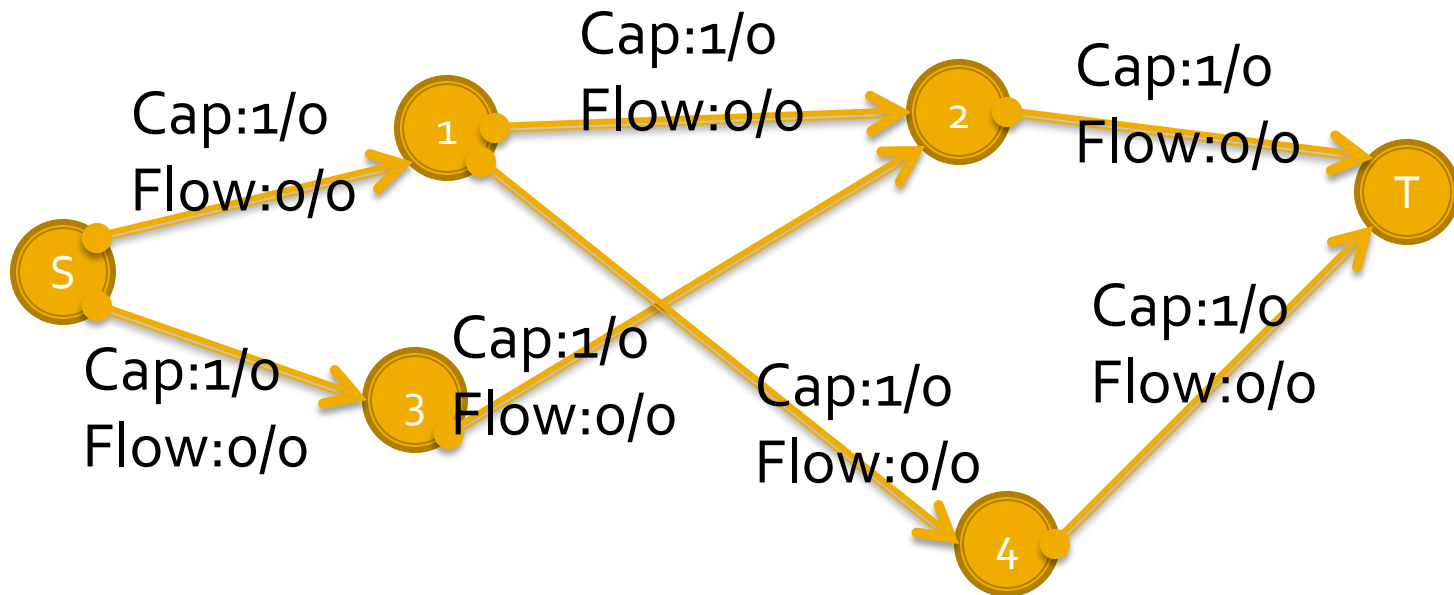


# Maximum flow part 2

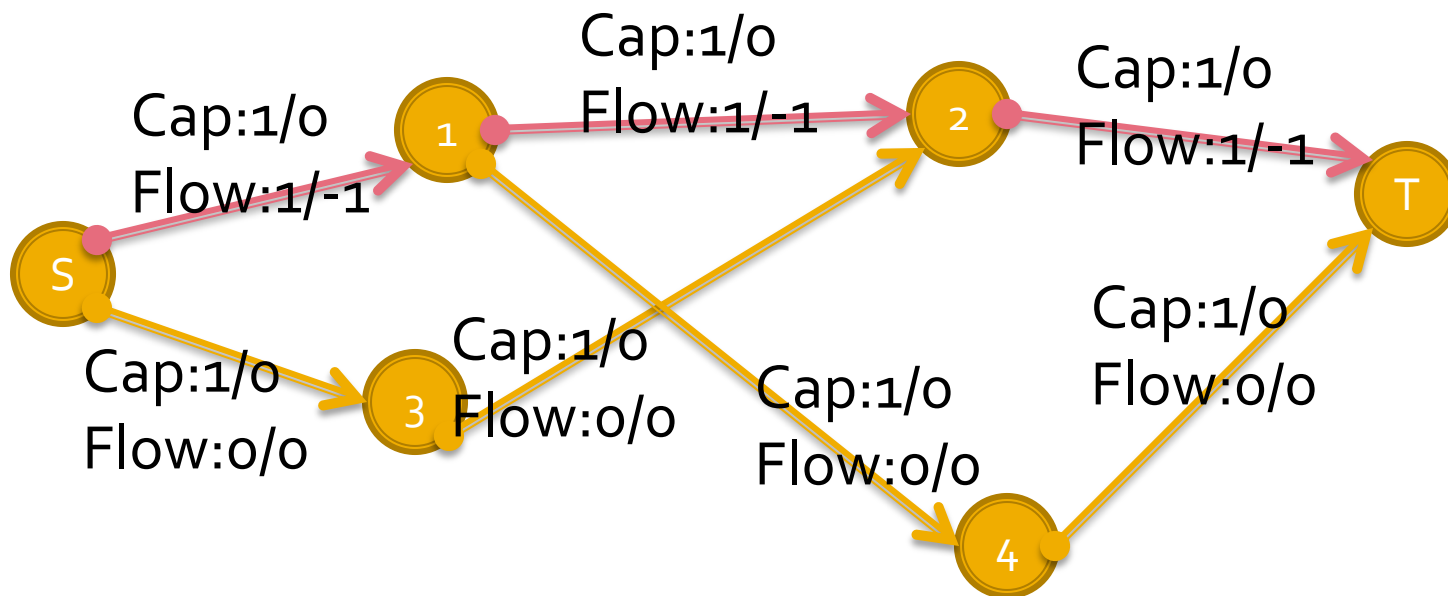
# Maxflow, how to cancel?

- Consider this network



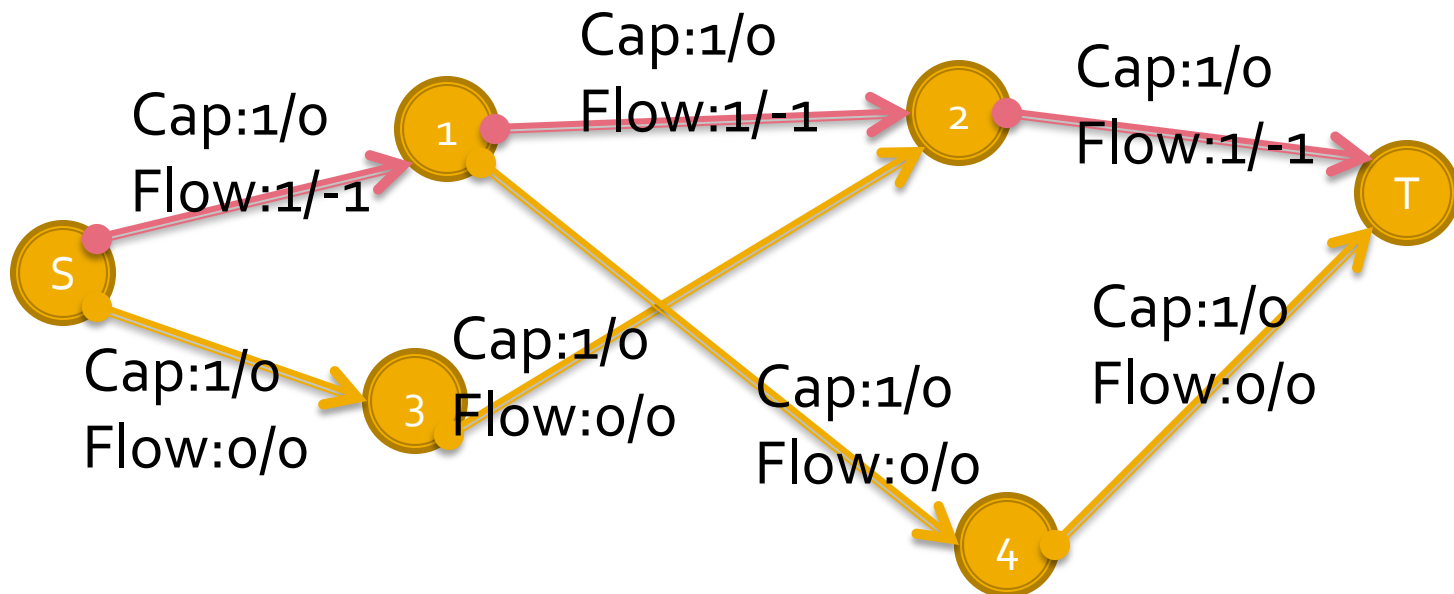
# Maxflow, how to cancel?

- residue flow = cap - flow



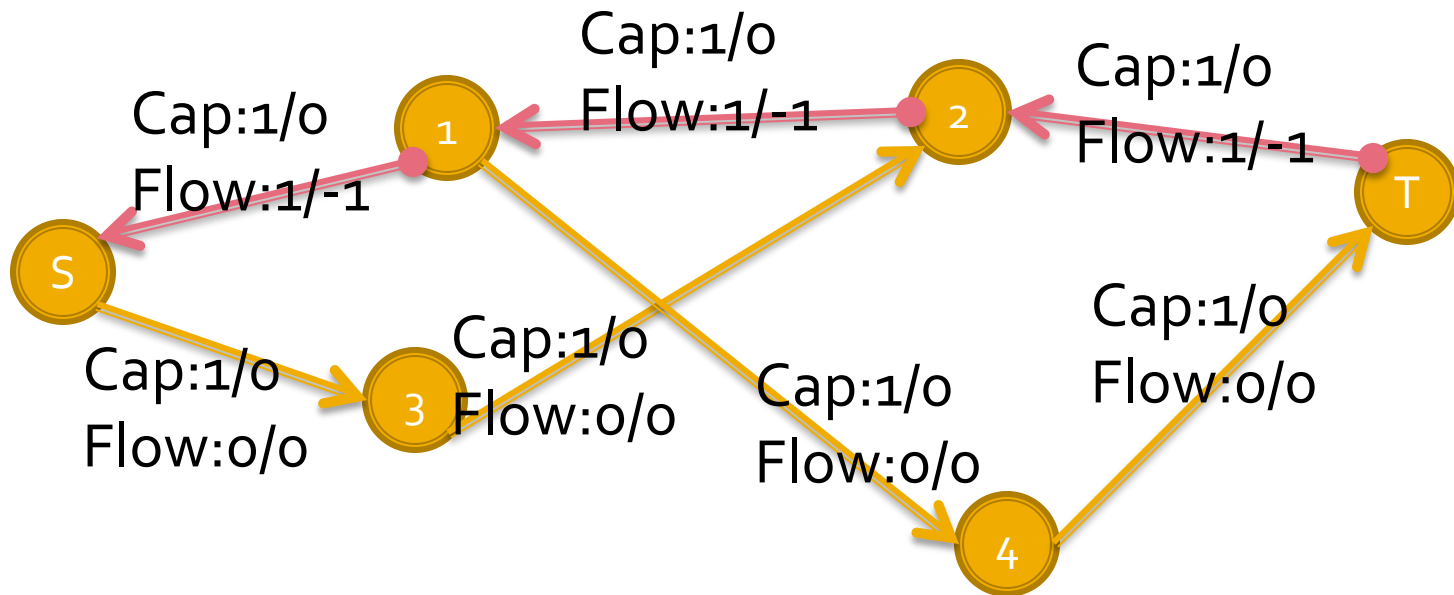
# Maxflow, how to cancel?

- residue flow = cap – flow
- Now...
- residue flow: node 1 to 2 =  $1 - 1 = 0 <$  can't use
- residue flow: node 2 to 1 =  $0 - (-1) = 1 <$  can use



# Maxflow, how to cancel?

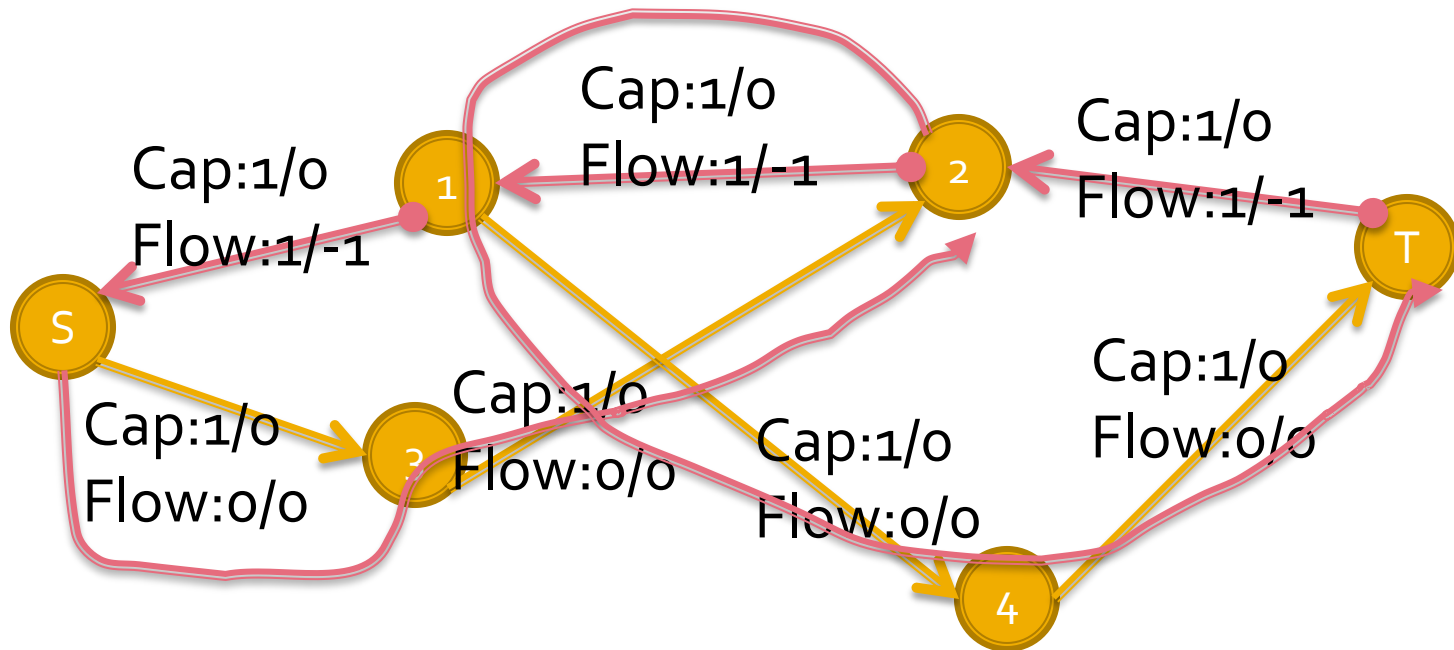
- So, it becomes:



# Maxflow, how to cancel?

- Cancellation

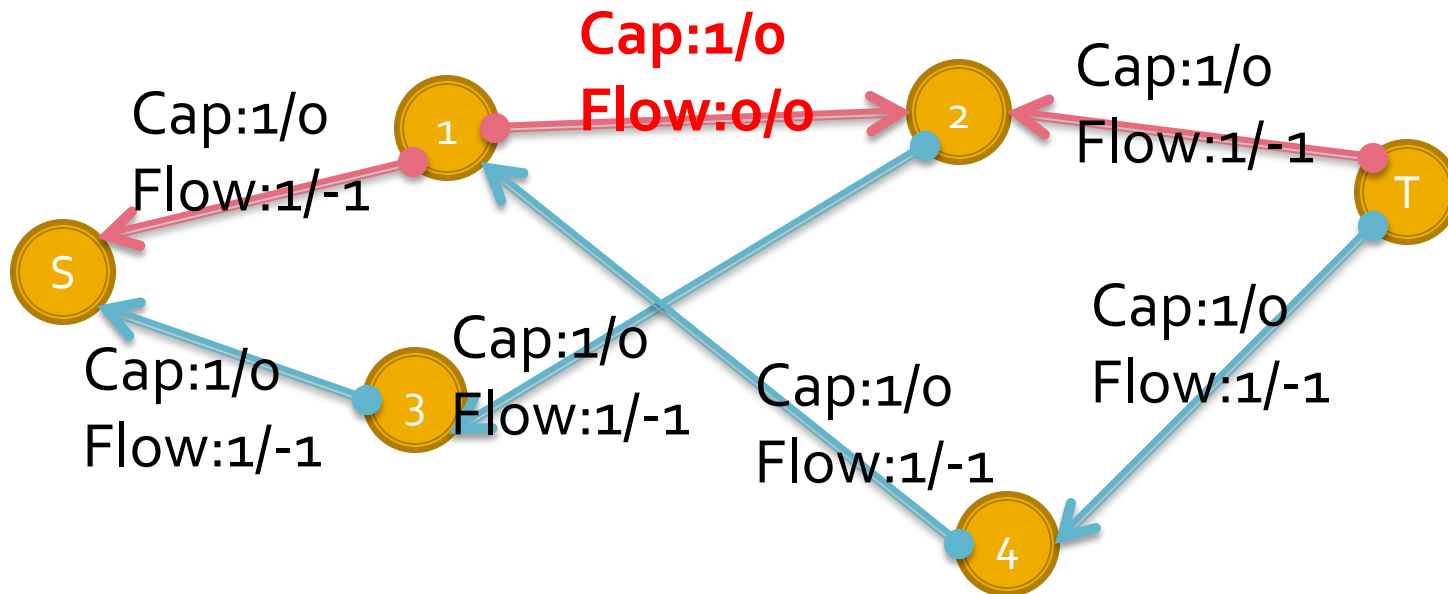
this edge is cancelled.



# Maxflow, how to cancel?

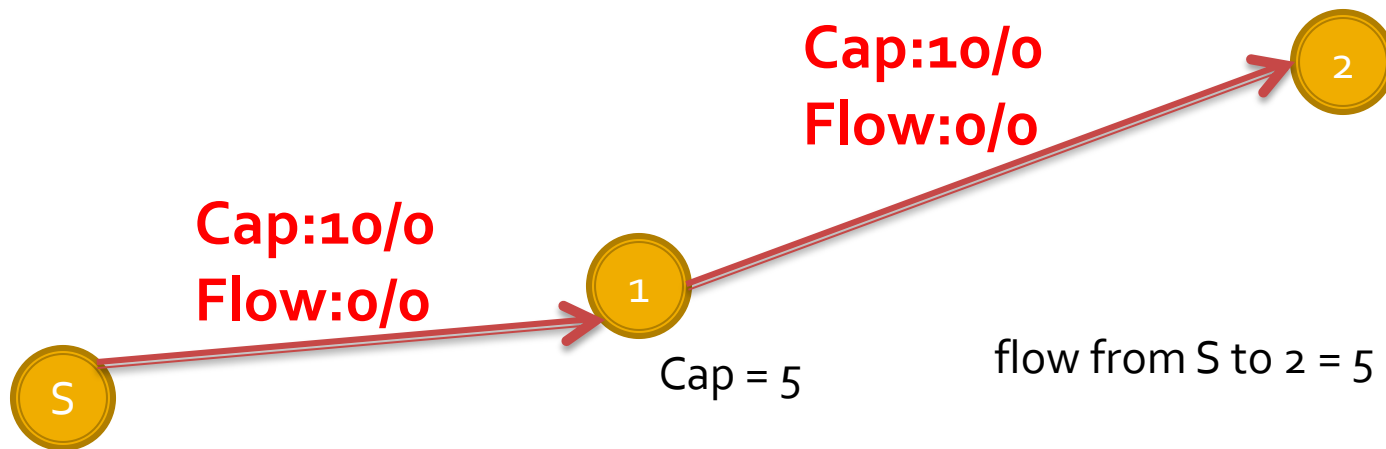
- Cancellation

this edge is cancelled.



# Vertex with capacity...?

- How to handle this...?



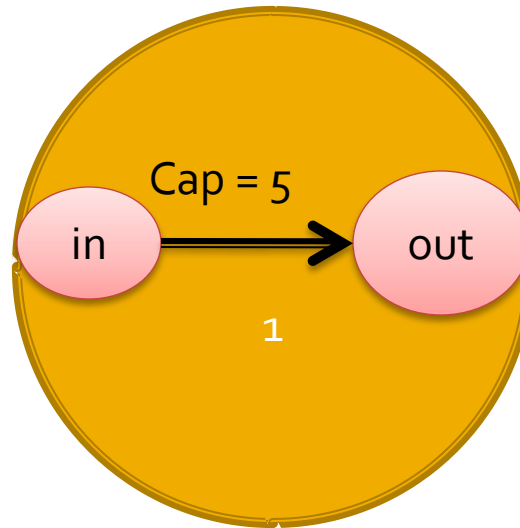


# Vertex with capacity...?

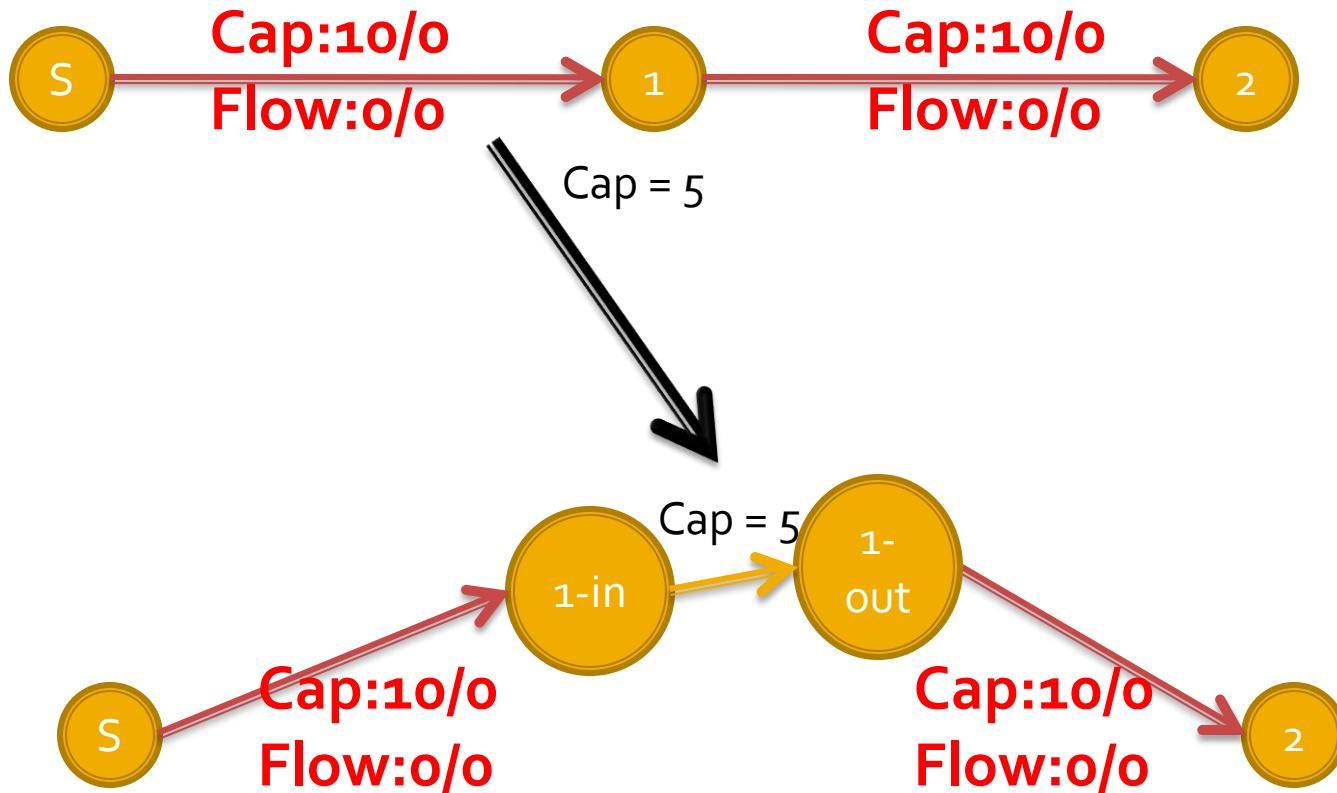
Cap = 5



Transform =>

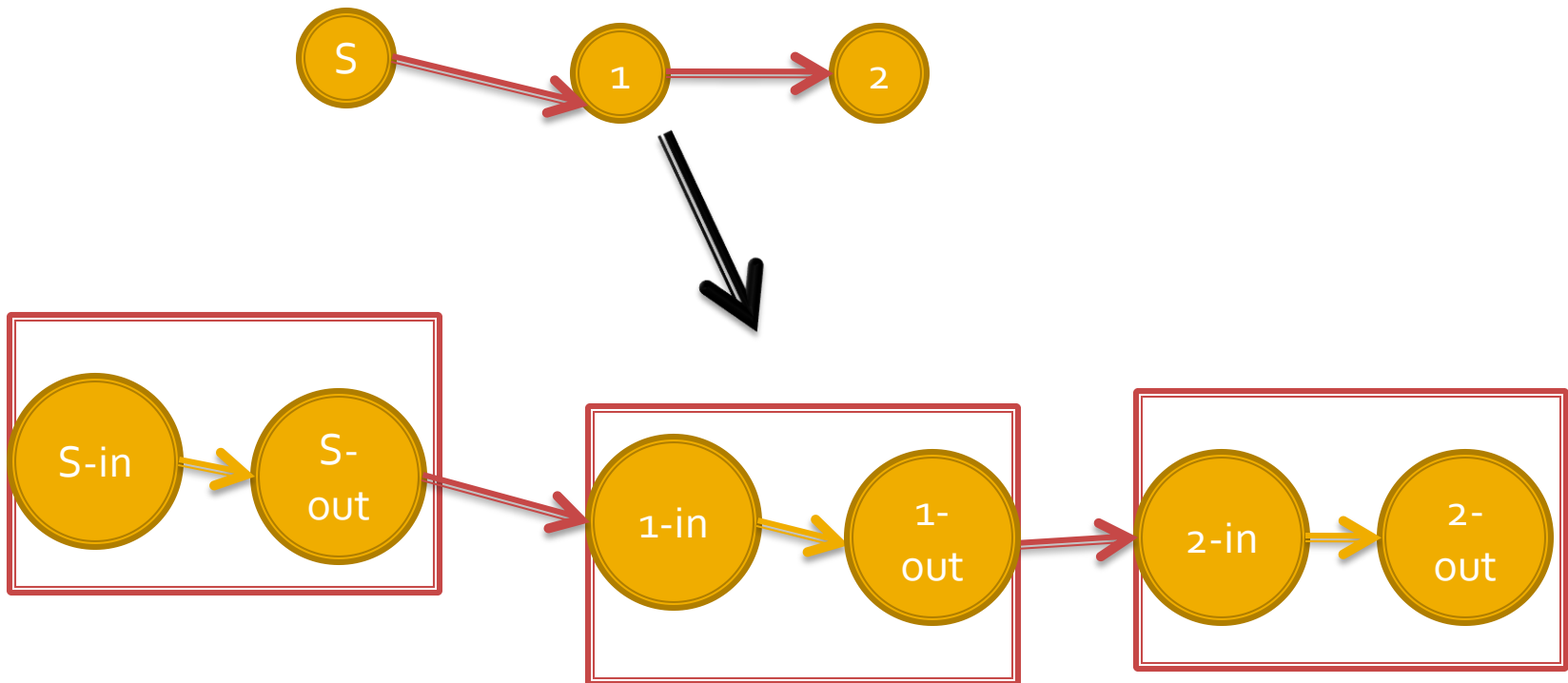


# Vertex with capacity...?



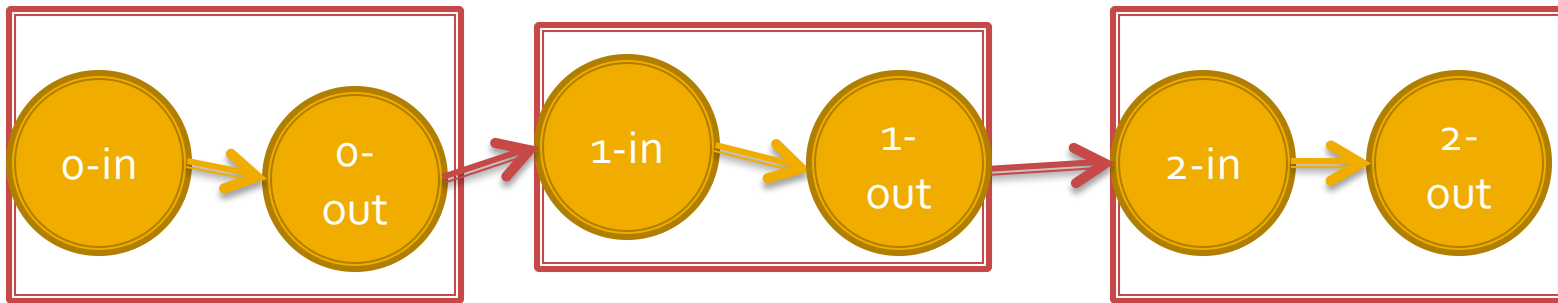
# Vertex with capacity...?

- Using “split vertex”, the size of graph will increase =  $(n * 2)$



# Vertex with capacity...?

- How to code?



- In node: (original node)
- Out node: (original node)+n
- or
- In node: (original node)\*2
- Out node: (original node)\*2+1

# Edge disjoint path

- Suppose you want to send  $k$  large files from  $s$  to  $t$  but never have two files use the same network link (to avoid congestion on the links).

# k Edge-disjoint Paths

- Given directed graph  $G$ , and two nodes  $s$  and  $t$ , find  $k$  paths from  $s$  to  $t$  such that no two paths share an edge.

# k Edge-disjoint Paths

## Reduce the problem to maxflow

1. Given an instance of k-Edge-Disjoint Paths,
2. Create an instance of Maximum Network Flow.
3. The maximum flow will be used to find the k edge disjoint paths.

# k Edge-disjoint Paths

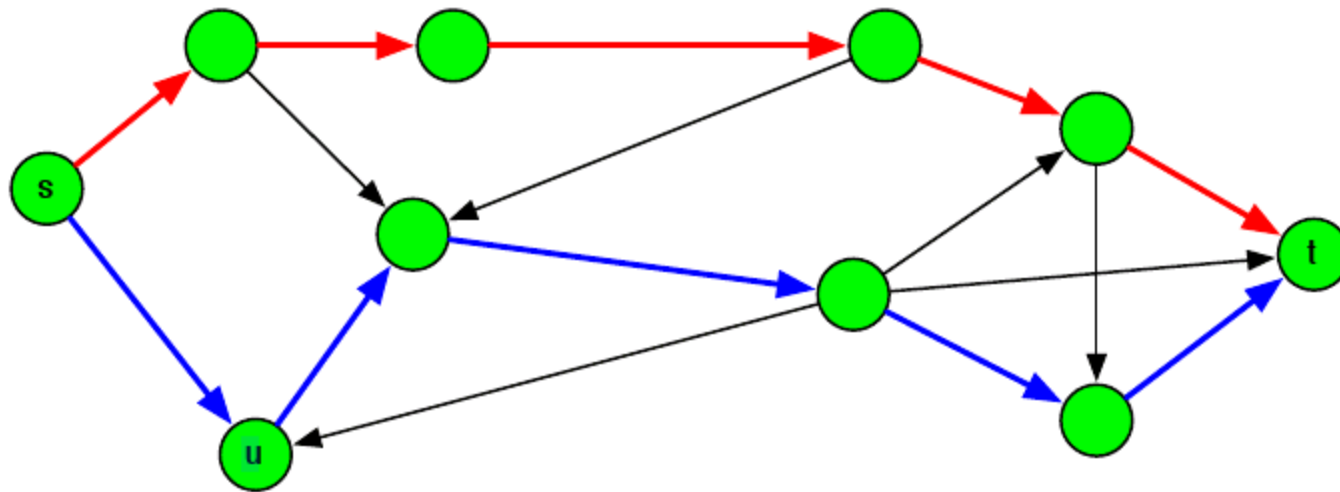
## Reduce the problem to maxflow

- Suppose we had  $k$  edge-disjoint  $s - t$  paths.
- We could send 1 unit of flow along each path without violating the capacity constraints.
- If there are  $k$  edge-disjoint  $s - t$  paths in directed, unit-weight graph  $G$ , then the maximum  $s - t$  flow is  $\geq k$ .



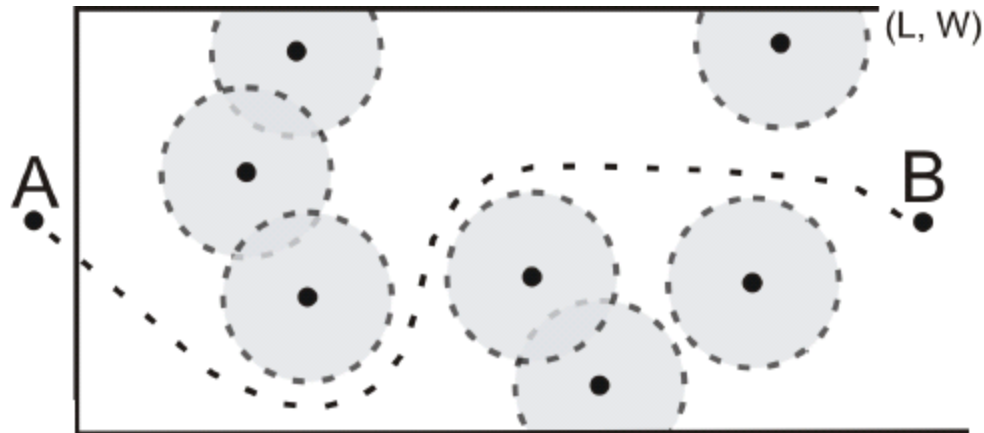
# k Edge-disjoint Paths

## Reduce the problem to maxflow



# What!? This is maxflow!?

- CityU OJ 261



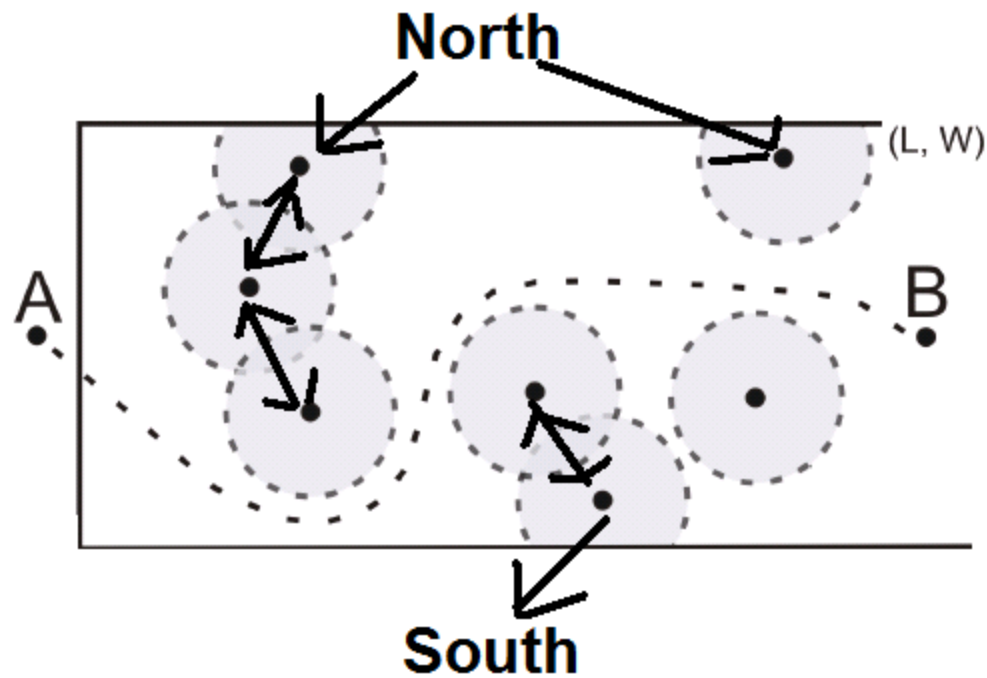
# What!? This is maxflow!?

- If this problem just asks you whether a winger can go to L without being tackled or not...
- Reduce to graph problem!

# Reduce to graph problem

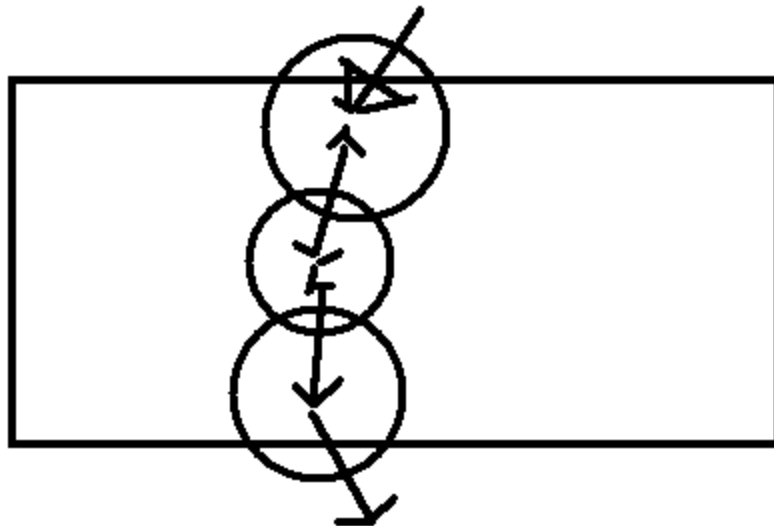
- Let robots, border of North and South be vertex
  1. Connect North  $\rightarrow$  robot if the robot touches/overlap with border of North
  2. Connect robot  $\rightarrow$  South if the robot touches/overlap with border of South
  3. Connect robot<sub>1</sub>  $\leftrightarrow$  robot<sub>2</sub> with undirected edge if two robots touches/overlap each other

# Reduce to graph problem



# Reduce to graph problem

- If there is a path from north to south, it means a winger need to “eat some tackles”



# Transform to maxflow

- The problem now is transformed to “what is the **minimum** number of vertex should be remove in order to **disconnect** North and South”
- “Disconnect” = cut
- “minimum... disconnect” = minimum cut!
- minimum cut = maximum flow!!!

# Transform to maxflow, another version

- The problem now is transformed to “How many paths going from North to South, if vertex can be only used once in the path?”
- similar to Edge disjoint Path
- But now is **Vertex disjoint Path**

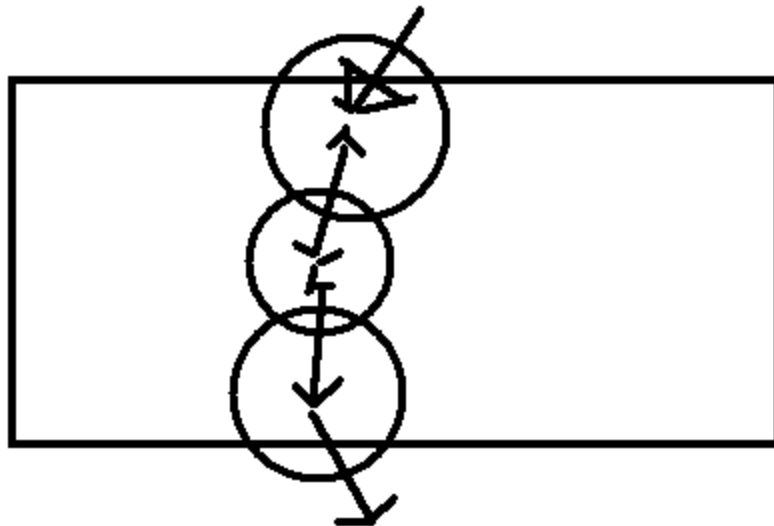


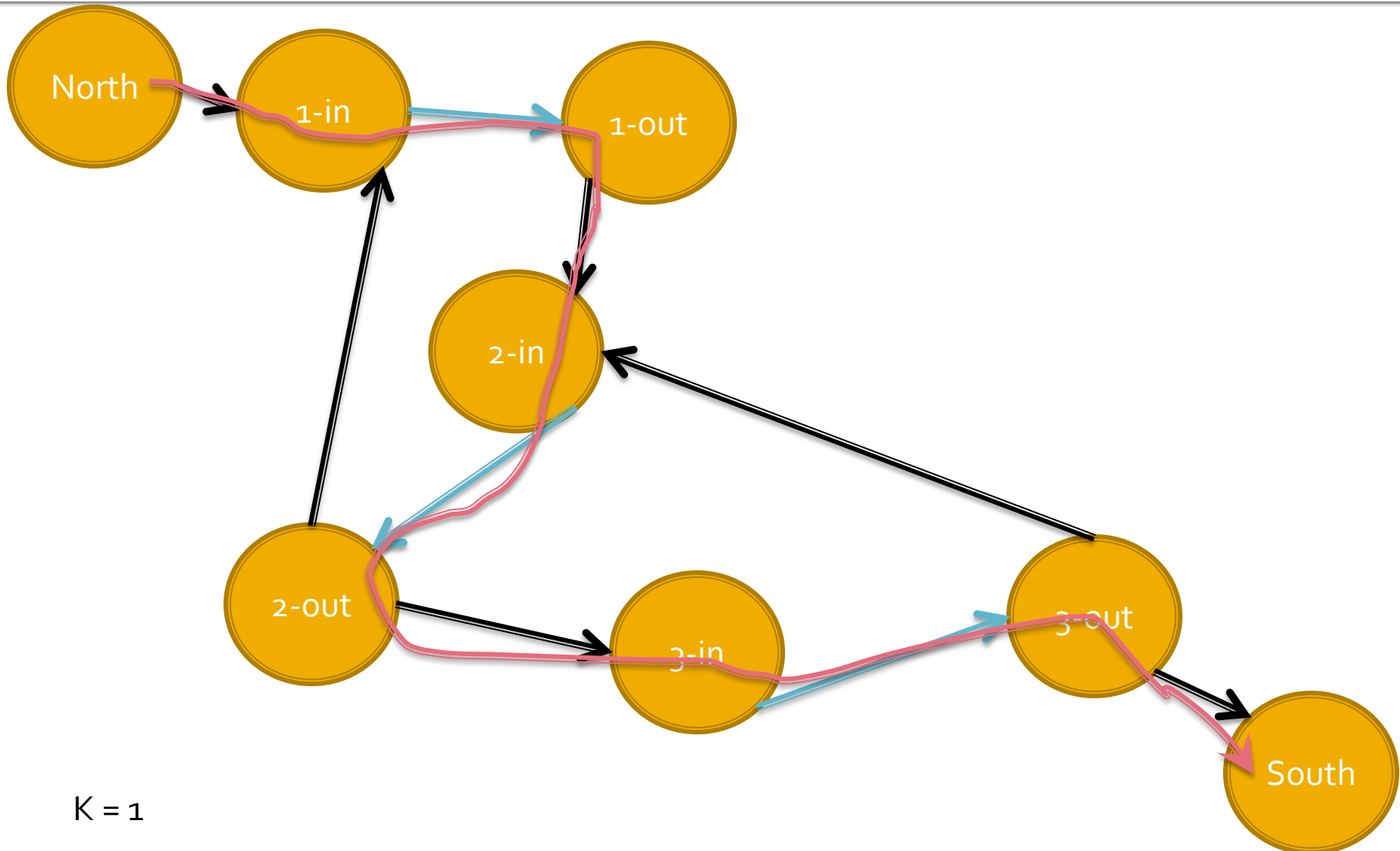
# Transform to maxflow

- Either of these will lead to same algorithm
- Cut vertex, not edge
- $\Rightarrow$  Capacity of vertex is a main concern
- $\Rightarrow$  Capacity of edge can be ignored
- Try to link up with previous topic

# Transform to maxflow

- Set capacity of vertex be 1
  - Be the “bottleneck” of the network graph
- Set capacity of edge be INF



$$K = 1$$


# Transform to maxflow

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- How to know a circle touch/overlap with
  - North border?
  - South border?
  - Another Circle?

---

- Q & A