CS145 Howework 3, Part 1: kNN

Important Note: HW3 is due on **11:59 PM PT, Nov 9 (Monday, Week 6)**. Please submit through GradeScope.

Note that, Howework #3 has two jupyter notebooks to complete (Part 1: kNN and Part 2: Neural Network).

Print Out Your Name and UID

Name: Kevin Li, UID: 405200619

Before You Start

You need to first create HW2 conda environment by the given `cs145hw3.yml` file, which provides the name and necessary packages for this tasks. If you have `conda` properly installed, you may create, activate or deactivate by the following commands:

```
conda env create -f cs145hw3.yml
conda activate hw3
conda deactivate
```

OR

```
conda env create --name NAMEOFYOURCHOICE -f cs145hw3.yml
conda activate NAMEOFYOURCHOICE
conda deactivate
```

To view the list of your environments, use the following command:

```
conda env list
```

More useful information about managing environments can be found here (https://docs.conda.io/projects/conda/en/latest/user-guide/tasks/manage-environments.html).

You may also quickly review the usage of basic Python and Numpy package, if needed in coding for matrix operations.

In this notebook, you must not delete any code cells in this notebook. If you change any code outside the blocks (such as hyperparameters) that you are allowed to edit (between `STRART/END YOUR CODE HERE`), you need to highlight these changes. You may add some additional cells to help explain your results and observations.

Download and prepare the dataset

Download the CIFAR-10 dataset (file size: ~163M). Run the following from the HW3 directory:

```
cd hw3/data/datasets
./get_datasets.sh
```

Make sure you put the dataset downloaded under hw3/data/datasets folder. After downloading the dataset, you can start your notebook from the HW3 directory. Note that the dataset is used in both jupyter notebooks (kNN and Neural Networks). You only need to download the dataset once for HW3.

Import the appropriate libraries

In [29]:
```
import numpy as np # for doing most of our calculations
import matplotlib.pyplot as plt# for plotting
from data.data_utils import load_CIFAR10 # function to load the CIFAR-10 datas
et.

# Load matplotlib images inline
%matplotlib inline

# These are important for reloading any code you write in external .py files.
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipyt
hon
%load_ext autoreload
%autoreload 2
```

```
The autoreload extension is already loaded. To reload it, use:
  %reload_ext autoreload
```

Now, to verify that the dataset has been successfully set up, the following code will print out the shape of train/test data and labels. The output shapes for train/test data are (50000, 32, 32, 3) and (10000, 32, 32, 3), while the labels are (50000,) and (10000,) respectively.

In [30]:
```
# Set the path to the CIFAR-10 data
cifar10_dir = './data/datasets/cifar-10-batches-py'
X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

# As a sanity check, we print out the size of the training and test data.
print('Training data shape: ', X_train.shape)
print('Training labels shape: ', y_train.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
```

```
Training data shape:  (50000, 32, 32, 3)
Training labels shape:  (50000,)
Test data shape:  (10000, 32, 32, 3)
Test labels shape:  (10000,)
```

Now we visualize some examples from the dataset by showing a few examples of training images from each class.

In [31]:
```
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'ship', 'truck']
num_classes = len(classes)
samples_per_class = 7
for y, cls in enumerate(classes):
    idxs = np.flatnonzero(y_train == y)
    idxs = np.random.choice(idxs, samples_per_class, replace=False)
    for i, idx in enumerate(idxs):
        plt_idx = i * num_classes + y + 1
        plt.subplot(samples_per_class, num_classes, plt_idx)
        plt.imshow(X_train[idx].astype('uint8'))
        plt.axis('off')
        if i == 0:
            plt.title(cls)
plt.show()
```

In [32]:
```
# Subsample the data for more efficient code execution in this exercise
num_training = 5000
mask = list(range(num_training))
X_train = X_train[mask]
y_train = y_train[mask]

num_test = 500
mask = list(range(num_test))
X_test = X_test[mask]
y_test = y_test[mask]

# Reshape the image data into rows
X_train = np.reshape(X_train, (X_train.shape[0], -1))
X_test = np.reshape(X_test, (X_test.shape[0], -1))
print(X_train.shape, X_test.shape)
```

```
(5000, 3072) (500, 3072)
```

Implement K-nearest neighbors algorithms

In the following cells, you will build a KNN classifier and choose hyperparameters via k-fold cross-validation.

In [33]:
```
# Import the KNN class
from hw3code import KNN
```

In [34]:
```
# Declare an instance of the knn class.
knn = KNN()

# Train the classifier.
#   We have implemented the training of the KNN classifier.
#   Look at the train function in the KNN class to see what this does.
knn.train(X=X_train, y=y_train)
```

Questions

(1) Describe what is going on in the function knn.train().

(2) What are the pros and cons of this training step of KNN?

Answers

1. We are saving the training data points so that we can compare against them later
2. it is quick to train, but the little amount of work done now means inferencing is expensive

KNN prediction

In the following sections, you will implement the functions to calculate the distances of test points to training points, and from this information, predict the class of the KNN.

In [35]:
```
# Implement the function compute_distances() in the KNN class.
# Do not worry about the input 'norm' for now; use the default definition of t
he norm
#   in the ccv ode, which is the 2-norm.
# You should only have to fill out the clearly marked sections.

import time
time_start =time.time()

dists_L2 = knn.compute_distances(X=X_test)

print('Time to run code: {}'.format(time.time()-time_start))
print('Frobenius norm of L2 distances: {}'.format(np.linalg.norm(dists_L2, 'fr
o')))
```

```
Time to run code: 33.31875228881836
Frobenius norm of L2 distances: 7906696.077040902
```

Really slow code?

Note: This probably took a while. This is because we use two for loops. We could increase the speed via vectorization, removing the for loops. Normally it may takes 20-40 seconds.

If you implemented this correctly, evaluating np.linalg.norm(dists_L2, 'fro') should return: ~7906696

KNN vectorization

The above code took far too long to run. If we wanted to optimize hyperparameters, it would be time-expensive. Thus, we will speed up the code by vectorizing it, removing the for loops.

In [45]:
```
# Implement the function compute_L2_distances_vectorized() in the KNN class.
# In this function, you ought to achieve the same L2 distance but WITHOUT any
 for loops.
# Note, this is SPECIFIC for the L2 norm.

time_start =time.time()
dists_L2_vectorized = knn.compute_L2_distances_vectorized(X=X_test)
print('Time to run code: {}'.format(time.time()-time_start))
print('Difference in L2 distances between your KNN implementations (should be
 0): {}'.format(np.linalg.norm(dists_L2 - dists_L2_vectorized, 'fro')))
```

```
Time to run code: 0.1739976406097412
Difference in L2 distances between your KNN implementations (should be 0): 0.
0
```

Speedup

Depending on your computer speed, you should see a 20-100x speed up from vectorization and no difference in L2 distances between two implementations.

On our computer, the vectorized form took 0.20 seconds while the naive implementation took 26.88 seconds.

Implementing the prediction

Now that we have functions to calculate the distances from a test point to given training points, we now implement the function that will predict the test point labels.

In [52]:
```
# Implement the function predict_labels in the KNN class.
# Calculate the training error (num_incorrect / total_samples)
#   from running knn.predict_labels with k=1

error = 1

# ================================================================ #
# START YOUR CODE HERE
# ================================================================ #
#   Calculate the error rate by calling predict_labels on the test
#   data with k = 1.  Store the error rate in the variable error.
# ================================================================ #

y_pred = knn.predict_labels(dists_L2_vectorized)
error = 1 - np.mean(y_pred == y_test)

# ================================================================ #
# END YOUR CODE HERE
# ================================================================ #

print(error)
```

```
0.726
```

If you implemented this correctly, the error should be: 0.726. This means that the k-nearest neighbors classifier is right 27.4% of the time, which is not great.

Questions:

What could you do to improve the accuracy of the k-nearest neighbor classifier you just implemented? Write down your answer in less than 30 words.

Answers:

Optimize my k hyperparameter and find a better value; very low k tends to overfit

Optimizing KNN hyperparameters k

In this section, we'll take the KNN classifier that you have constructed and perform cross validation to choose a best value of k.

If you are not familiar with cross validation, cross-validation is a technique for evaluating ML models by training several ML models on subsets of the available input data and evaluating them on the complementary subset of the data. Use cross-validation to detect overfitting, ie, failing to generalize a pattern. More specifically, in k-fold cross-validation, you evenly split the input data into k subsets of data (also known as folds). You train an ML model on all but one (k-1) of the subsets, and then evaluate the model on the subset that was not used for training. This process is repeated k times, with a different subset reserved for evaluation (and excluded from training) each time.

More details of cross validation can be found here (https://scikit-learn.org/stable/modules/cross_validation.html). However, you are not allowed to use sklean in your implementation.

Create training and validation folds

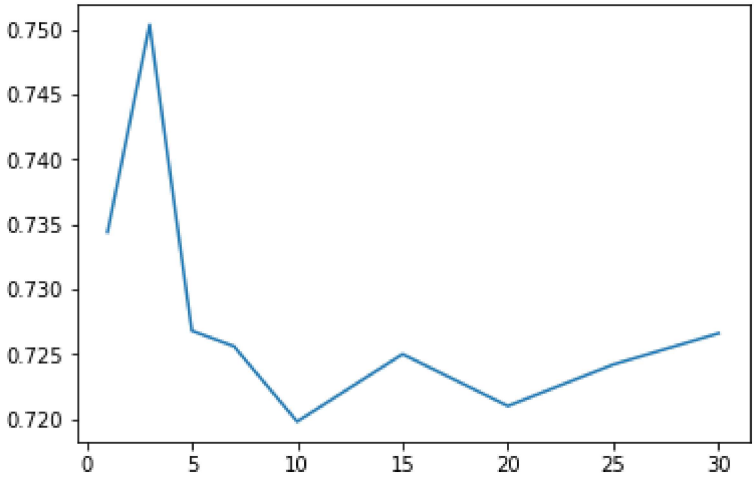
First, we will create the training and validation folds for use in k-fold cross validation.

In [53]:
```
# Create the dataset folds for cross-valdiation.
num_folds = 5

X_train_folds = []
y_train_folds =  []

# ================================================================ #
# START YOUR CODE HERE
# ================================================================ #
#   Split the training data into num_folds (i.e., 5) folds.
#   X_train_folds is a list, where X_train_folds[i] contains the
#      data points in fold i.
#   y_train_folds is also a list, where y_train_folds[i] contains
#      the corresponding labels for the data in X_train_folds[i]
# ================================================================ #

X_train_folds = np.array_split(X_train, num_folds)
y_train_folds = np.array_split(y_train, num_folds)

# ================================================================ #
# END YOUR CODE HERE
# ================================================================ #
```

Optimizing the number of nearest neighbors hyperparameter.

In this section, we select different numbers of nearest neighbors and assess which one has the lowest k-fold cross validation error.

In [63]:
```
time_start =time.time()

ks = [1, 3, 5, 7, 10, 15, 20, 25, 30]

# ================================================================ #
# START YOUR CODE HERE
# ================================================================ #
#   Calculate the cross-validation error for each k in ks, testing
#   the trained model on each of the 5 folds.  Average these errors
#   together and make a plot of k vs. average cross-validation error.
#   Since we assume L2 distance here, please use the vectorized code!
#   Otherwise, you might be waiting a long time.
# ================================================================ #

def cross_fold_validate(model, folds, X_train_folds, y_train_folds, k):
    def validate(i):
        X_train = np.concatenate((*X_train_folds[:i], *X_train_folds[i+1:]))
        y_train = np.concatenate((*y_train_folds[:i], *y_train_folds[i+1:]))

        X_val = X_train_folds[i]
        y_val = y_train_folds[i]

        model.train(X=X_train, y=y_train)
        dists = model.compute_L2_distances_vectorized(X_val)
        y_pred = model.predict_labels(dists, k=k)

        return 1 - np.mean(y_pred == y_val)
    return np.mean([validate(i) for i in range(folds)])



m = KNN()
error = [cross_fold_validate(m, num_folds, X_train_folds, y_train_folds, k) fo
r k in ks]

plt.plot(ks, error)
plt.show()

i = np.argmin(error)
print(f'Best k: {ks[i]}')
print(f'\t5-fold CV error: {error[i]}')
print()

# ================================================================ #
# END YOUR CODE HERE
# ================================================================ #

print('Computation time: %.2f'%(time.time()-time_start))
```




```
Best k: 10
        5-fold CV error: 0.7198

Computation time: 24.89
```

Questions:

(1) Why do we typically choose k as an odd number (for exmple in `ks`)

(2) What value of k is best amongst the tested k's? What is the cross-validation error for this value of k?

Answers

1. We usually choose k odd to avoid ties
2. $k = 10$ was the best result, with a 5-fold CV error of 0.7198

Evaluating the model on the testing dataset.

Now, given the optimal k which you have learned, evaluate the testing error of the k-nearest neighbors model.

In [64]:
```
error = 1

# ================================================================ #
# START YOUR CODE HERE
# ================================================================ #
#   Evaluate the testing error of the k-nearest neighbors classifier
#   for your optimal hyperparameters found by 5-fold cross-validation.
# ================================================================ #

y_pred = knn.predict_labels(dists_L2_vectorized, k=10)
error = 1 - np.mean(y_pred == y_test)

# ================================================================ #
# END YOUR CODE HERE
# ================================================================ #

print('Error rate achieved: {}'.format(error))
```

```
Error rate achieved: 0.718
```

Question:

How much did your error change by cross-validation over naively choosing $k = 1$ and using the L2-norm?

Answers

My error decreased very slightly going from 0.726 to 0.718

End of Homework 3, Part 1 :)

After you've finished both parts the homework, please print out the both of the entire `ipynb` notebooks and `py` files into one PDF file. Make sure you include the output of code cells and answers for questions. Prepare submit it to GradeScope. Do not include any dataset in your submission.

CS145 Howework 3, Part 2: Neural Networks

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Section 1: Backprop in a neural network

Note: Section 1 is "question-answer" style problem. You do not need to code anything and you are required to calculate by hand (with a scientific calculator), which helps you understand the back propagation in neural networks.

In this question, let's consider a simple two-layer neural network and manually do the forward and backward pass. For simplicity, we assume our input data is two dimension. Then the model architecture looks like the following. Notice that in the example we saw in class, the bias term `b` was not explicit listed in the architecture diagram. Here we include the term `b` explicitly for each layer in the diagram. Recall the formula for computing $\mathbf{x}^{(l)}$ in the l-th layer from $\mathbf{x}^{(l-1)}$ in the $(l-1)$-th layer is $\mathbf{x}^{(l)} = \mathbf{f}^{(l)}(\mathbf{W}^{(l)}\mathbf{x}^{(l-1)} + \mathbf{b}^{(l)})$. The activation function $\mathbf{f}^{(l)}$ we choose is the `sigmoid` function for all layers, i.e. $\mathbf{f}^{(l)}(z) = \frac{1}{1+\exp(-z)}$. The final loss function is $\frac{1}{2}$ of the `mean squared error` loss, i.e. $l(\mathbf{y}, \hat{\mathbf{y}}) = \frac{1}{2}||\mathbf{y} - \hat{\mathbf{y}}||^2$.

We initialize our weights as

$$\mathbf{W}^{(1)} = \begin{bmatrix} 0.15 & 0.2 \\ 0.25 & 0.3 \end{bmatrix}, \quad \mathbf{W}^{(2)} = [0.4, 0.45], \quad \mathbf{b}^{(1)} = [0.35, 0.35], \quad \mathbf{b}^{(2)} = 0.6$$

Forward pass

Questions

1. When the input $\mathbf{x}^{(0)} = [0.05, 0.1]$, what will be the value of $\mathbf{x}^{(1)}$ in the hidden layer? (Show your work).
2. Based on the value $\mathbf{x}^{(1)}$ you computed, what will be the value of $\mathbf{x}^{(2)}$ in the output layer? (Show your work).
3. When the target value of this input is $y = 0.01$, based on the value $\mathbf{x}^{(2)}$ you computed, what will be the loss? (Show your work).

Answers:

1.

$$W^1x^0 + b^1 = \begin{bmatrix} 0.15 & 0.2 \\ 0.25 & 0.3 \end{bmatrix} \begin{bmatrix} 0.05 \\ 0.1 \end{bmatrix} + \begin{bmatrix} 0.35 \\ 0.35 \end{bmatrix} = \begin{bmatrix} 0.3775 \\ 0.3925 \end{bmatrix}$$

This is our intermediate value; we then use the activation function to get x^1 (activation function is sigmoid fn i.e. $\frac{1}{1+e^{-x}}$)

$$x^1 = f^1\left(\begin{bmatrix} 0.3775 \\ 0.3925 \end{bmatrix}\right) = \begin{bmatrix} 0.5933 \\ 0.5969 \end{bmatrix}$$

2.

$$W^2x^1 + b^2 = \begin{bmatrix} 0.4 & 0.45 \end{bmatrix} \begin{bmatrix} 0.5933 \\ 0.5969 \end{bmatrix} + 0.6 = 1.106$$
$$x^2 = f^2(1.106) = 0.751$$

3.

$$l(y, \hat{y}) = 1/2(0.01 - 0.751)^2 = 0.275$$

Backward pass

With the loss computed below, we are ready for a backward pass to update the weights in the neural network. Kindly remind that the gradients of a variable should have the same shape with the variable.

Questions

1. Consider the loss l of the same input $\mathbf{x^{(0)}} = [0.05, 0.1]$, what will be the update of $\mathbf{W^{(2)}}$ and $\mathbf{b^{(2)}}$ when we backprop, i.e. $\frac{\partial l}{\partial \mathbf{W^{(2)}}}$, $\frac{\partial l}{\partial \mathbf{b^{(2)}}}$ (Show your work in detailed calculation steps. Answers without justification will not be credited.).
2. Based on the result you computed in part 1, when we keep backproping, what will be the update of $\mathbf{W^{(1)}}$ and $\mathbf{b^{(1)}}$, i.e. $\frac{\partial l}{\partial \mathbf{W^{(1)}}}$, $\frac{\partial l}{\partial \mathbf{b^{(1)}}}$ (Show your work in details calculation steps. Answers without justification will not be credited.).

Answers:

1. We have the derivative of the loss wrt x

$$\frac{\partial l}{\partial x^{(2)}} = \frac{\partial}{\partial x^{(2)}}(x^{(2)} - y)^2 = x^{(2)} - y$$

We take the deriavtive of x wrt z

$$\frac{\partial x^{(2)}}{\partial z^{(2)}} = \frac{\partial}{\partial z^{(2)}}\sigma(z^{(2)}) = \sigma(z^{(2)})(1 - z^{(2)})$$

We additionally take derivative of z wrt W and b to finally find their gradients

$$\frac{\partial z^{(2)}}{\partial W_j^{(2)}} = \frac{\partial}{\partial W_j^{(2)}} W_j^{(2)} x^{(1)} + b^{(2)} = x_j^{(1)}$$

$$\frac{\partial z^{(2)}}{\partial b^{(2)}} = \frac{\partial}{\partial b^{(2)}} W_j^{(2)} x^{(1)} + b^{(2)} = 1$$

To accomplish calculate the update for W^{(2)} and b^{(2)}, we apply these together in the following fashion

$$\frac{\partial l}{\partial W_j^{(2)}} = \frac{\partial l}{\partial x^{(2)}} \frac{\partial x^{(2)}}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial W_j^{(2)}} = \left(x^{(2)} - y\right) \left(\sigma(z^{(2)})(1 - \sigma(z^{(2)}))\right) \left(x_j^{(1)}\right)$$

$$\frac{\partial l}{\partial b^{(2)}} = \frac{\partial l}{\partial x^{(2)}} \frac{\partial x^{(2)}}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial W_j^{(2)}} = \left(x^{(2)} - y\right) \left(\sigma(z^{(2)})(1 - \sigma(z^{(2)}))\right)$$

We substitute our values of $x^{(2)}, y, b^{(2)}$, and $z^{(2)}$ (we get the z from the forward pass of this question) and get the following for both gradients:

$$\frac{\partial l}{\partial W_j^{(2)}} = \begin{bmatrix} 0.082 & 0.083 \end{bmatrix}$$

$$\frac{\partial l}{\partial b^{(2)}} = 0.138$$

1. We now chain backwards another layer. We again take several partial derivatives and chain them together to calculate the $\frac{\partial l}{\partial W_{ij}^{(1)}}$ and $\frac{\partial l}{\partial b_i^{(1)}}$

The first partial derviative we need is $z^{(2)}$ wrt $x_i^{(1)}$

$$\frac{\partial z^{(2)}}{\partial x_i^{(1)}} = \frac{\partial}{\partial x_i^{(1)}} W^{(2)} x^{(1)} + b^{(2)} = W_i^{(2)}$$

We then need the partial of $x_i^{(1)}$ wrt $z_i^{(1)}$

$$\frac{\partial x_i^{(1)}}{\partial z_i^{(1)}} = \sigma(z_i^{(1)})(1 - \sigma(z_i^{(1)}))$$

Then of $z_i^{(1)}$ wrt $W_{ij}^{(1)}$ and $b_i^{(1)}$

$$\frac{\partial z_i^{(1)}}{\partial W_{ij}^{(1)}} = \frac{\partial}{\partial W_{ij}^{(1)}} W_i^{(1)} x^{(0)} + b_i^{(1)} = x_j^{(0)}$$
$$\frac{\partial z_i^{(1)}}{\partial b_i^{(1)}} = \frac{\partial}{\partial b_i^{(1)}} W_i^{(1)} x^{(0)} + b_i^{(1)} = 1$$

We can now chain them together to form the desired gradients

$$\frac{\partial l}{\partial W_{ij}^{(1)}} = \frac{\partial l}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial x_i^{(1)}} \frac{\partial x_i^{(1)}}{\partial z_i^{(1)}} \frac{\partial z_i^{(1)}}{\partial W_{ij}^{(1)}}$$
$$\frac{\partial l}{\partial b_i^{(1)}} = \frac{\partial l}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial x_i^{(1)}} \frac{\partial x_i^{(1)}}{\partial z_i^{(1)}} \frac{\partial z_i^{(1)}}{\partial b_i^{(1)}}$$

We already computed $\frac{\partial l}{\partial z^{(2)}}$ in the steps for the previous computation, and we can simply plug in the rest to calculate yields

$$\frac{\partial l}{\partial W_{ij}^{(1)}} = \frac{\partial l}{\partial z^{(2)}} \left(W_i^{(2)} \right) \left(\sigma(z_i^{(1)})(1 - \sigma(z_i^{(1)})) \right) \left(x_j^{(0)} \right)$$
$$\frac{\partial l}{\partial W_{ij}^{(1)}} = \frac{\partial l}{\partial z^{(2)}} \left(W_i^{(2)} \right) \left(\sigma(z_i^{(1)})(1 - \sigma(z_i^{(1)})) \right)$$

We could, of course, recalculate the first partial but it is unncessary since we previously did it (and this is part of why backprop can be quick - it reuses the previous layer's computation to avoid the majority of the first "half" of calculation so even with a deep neural network, no layer has to do too much calculation)

We now plug in the values we know, and we get the following results:

$$\frac{\partial l}{\partial W^{(1)}} = \begin{bmatrix} 0.000668 & 0.00134 \\ 0.000750 & 0.00150 \end{bmatrix}$$
$$\frac{\partial l}{\partial b^{(1)}} = \begin{bmatrix} 0.0134 & 0.0150 \end{bmatrix}$$

Section 2: Coding a two-layer neural network

Import libraries and define relative error function, which is used to check results later.

In [2]:
```
import random
import numpy as np
from data.data_utils import load_CIFAR10
import matplotlib.pyplot as plt

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0)
%load_ext autoreload
%autoreload 2

def rel_error(x, y):
    """ returns relative error """
    return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))
```

Toy example

Before loading CIFAR-10, there will be a toy example to test your implementation of the forward and backward pass.

In [3]:
```
from hw3code.neural_net import TwoLayerNet
```

In [4]:
```
# Create a small net and some toy data to check your implementations.
# Note that we set the random seed for repeatable experiments.

input_size = 4
hidden_size = 10
num_classes = 3
num_inputs = 5

def init_toy_model():
    np.random.seed(0)
    return TwoLayerNet(input_size, hidden_size, num_classes, std=1e-1)

def init_toy_data():
    np.random.seed(1)
    X = 10 * np.random.randn(num_inputs, input_size)
    y = np.array([0, 1, 2, 2, 1])
    return X, y

net = init_toy_model()
X, y = init_toy_data()
```

Compute forward pass scores

In [5]:
```
## Implement the forward pass of the neural network.

# Note, there is a statement if y is None: return scores, which is why
# the following call will calculate the scores.
scores = net.loss(X)
print('Your scores:')
print(scores)
print()
print('correct scores:')
correct_scores = np.asarray([
    [-1.07260209,  0.05083871, -0.87253915],
    [-2.02778743, -0.10832494, -1.52641362],
    [-0.74225908,  0.15259725, -0.39578548],
    [-0.38172726,  0.10835902, -0.17328274],
    [-0.64417314, -0.18886813, -0.41106892]])
print(correct_scores)
print()

# The difference should be very small. We get < 1e-7
print('Difference between your scores and correct scores:')
print(np.sum(np.abs(scores - correct_scores)))
```

```
Your scores:
[[-1.07260209  0.05083871 -0.87253915]
 [-2.02778743 -0.10832494 -1.52641362]
 [-0.74225908  0.15259725 -0.39578548]
 [-0.38172726  0.10835902 -0.17328274]
 [-0.64417314 -0.18886813 -0.41106892]]

correct scores:
[[-1.07260209  0.05083871 -0.87253915]
 [-2.02778743 -0.10832494 -1.52641362]
 [-0.74225908  0.15259725 -0.39578548]
 [-0.38172726  0.10835902 -0.17328274]
 [-0.64417314 -0.18886813 -0.41106892]]

Difference between your scores and correct scores:
3.381231204052648e-08
```

Forward pass loss

The total loss includes data loss (MSE) and regularization loss, which is,

$$L = L_{data} + L_{reg} = \frac{1}{2N}\sum_{i=1}^{N}(\boldsymbol{y}_{\text{pred}} - \boldsymbol{y}_{\text{target}})^2 + \frac{\lambda}{2}\left(||W_1||^2 + ||W_2||^2\right)$$

More specifically in multi-class situation, if the output of neural nets from one sample is $y_{\text{pred}} = (0.1, 0.1, 0.8)$ and $y_{\text{target}} = (0, 0, 1)$ from the given label, then the MSE error will be
$Error = (0.1-0)^2 + (0.1-0)^2 + (0.8-1)^2 = 0.06$

Implement data loss and regularization loss. In the MSE function, you also need to return the gradients which need to be passed backward. This is similar to batch gradient in linear regression. Test your implementation of loss functions. The Difference should be less than 1e-12.

In [6]:
```
loss, _ = net.loss(X, y, reg=0.05)
correct_loss_MSE = 1.8973332763705641

# should be very small, we get < 1e-12
print('Difference between your loss and correct loss:')
print(np.sum(np.abs(loss - correct_loss_MSE)))
```

```
Difference between your loss and correct loss:
0.0
```

Backward pass (You do not need to implemented this part)

We have already implemented the backwards pass of the neural network for you. Run the block of code to check your gradients with the gradient check utilities provided. The results should be automatically correct (tiny relative error).

If there is a gradient error larger than 1e-8, the training for neural networks later will be negatively affected.

In [7]:
```
from data.gradient_check import eval_numerical_gradient

# Use numeric gradient checking to check your implementation of the backward p
ass.
# If your implementation is correct, the difference between the numeric and
# analytic gradients should be less than 1e-8 for each of W1, W2, b1, and b2.

loss, grads = net.loss(X, y, reg=0.05)

# these should all be less than 1e-8 or so
for param_name in grads:
    f = lambda W: net.loss(X, y, reg=0.05)[0]
    param_grad_num = eval_numerical_gradient(f, net.params[param_name], verbos
e=False)
    print('{} max relative error: {}'.format(param_name, rel_error(param_grad_
num, grads[param_name])))
```

```
W2 max relative error: 8.80091875172355e-11
b2 max relative error: 2.4554844805570154e-11
W1 max relative error: 1.7476665046687833e-09
b1 max relative error: 7.382451041178829e-10
```

Training the network

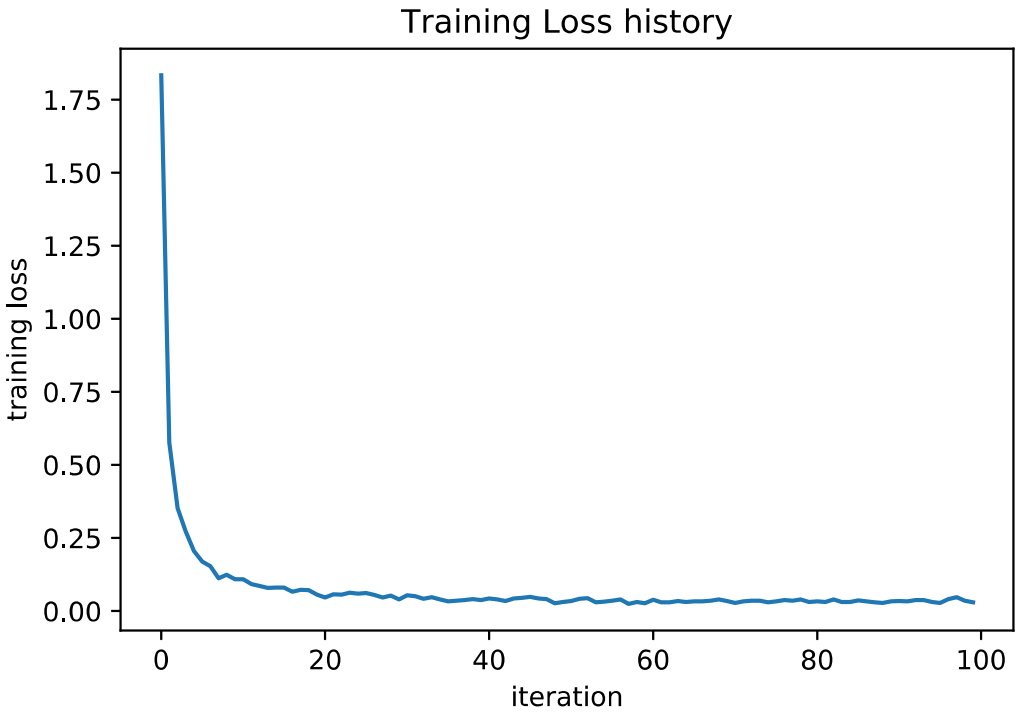
Implement neural_net.train() to train the network via stochastic gradient descent, much like the linear regression.

In [8]:
```
net = init_toy_model()
stats = net.train(X, y, X, y,
            learning_rate=1e-1, reg=5e-6,
            num_iters=100, verbose=False)

print('Final training loss: ', stats['loss_history'][-1])

# plot the loss history
plt.plot(stats['loss_history'])
plt.xlabel('iteration')
plt.ylabel('training loss')
plt.title('Training Loss history')
plt.show()
```

```
Final training loss:  0.02950555626206818
```



Classify CIFAR-10

Do classification on the CIFAR-10 dataset.

In [9]:
```
from data.data_utils import load_CIFAR10

def get_CIFAR10_data(num_training=49000, num_validation=1000, num_test=1000):
    """
    Load the CIFAR-10 dataset from disk and perform preprocessing to prepare
    it for the two-layer neural net classifier. These are the same steps as
    we used for the SVM, but condensed to a single function.
    """
    # Load the raw CIFAR-10 data
    cifar10_dir = './data/datasets/cifar-10-batches-py'
    X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

    # Subsample the data
    mask = list(range(num_training, num_training + num_validation))
    X_val = X_train[mask]
    y_val = y_train[mask]
    mask = list(range(num_training))
    X_train = X_train[mask]
    y_train = y_train[mask]
    mask = list(range(num_test))
    X_test = X_test[mask]
    y_test = y_test[mask]

    # Normalize the data: subtract the mean image
    mean_image = np.mean(X_train, axis=0)
    X_train -= mean_image
    X_val -= mean_image
    X_test -= mean_image

    # Reshape data to rows
    X_train = X_train.reshape(num_training, -1)
    X_val = X_val.reshape(num_validation, -1)
    X_test = X_test.reshape(num_test, -1)

    return X_train, y_train, X_val, y_val, X_test, y_test


# Invoke the above function to get our data.
X_train, y_train, X_val, y_val, X_test, y_test = get_CIFAR10_data()
print('Train data shape: ', X_train.shape)
print('Train labels shape: ', y_train.shape)
print('Validation data shape: ', X_val.shape)
print('Validation labels shape: ', y_val.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
```

```
Train data shape:  (49000, 3072)
Train labels shape:  (49000,)
Validation data shape:  (1000, 3072)
Validation labels shape:  (1000,)
Test data shape:  (1000, 3072)
Test labels shape:  (1000,)
```

Running SGD

If your implementation is correct, you should see a validation accuracy of around 15-18%.

In [10]:
```
input_size = 32 * 32 * 3
hidden_size = 50
num_classes = 10
net = TwoLayerNet(input_size, hidden_size, num_classes)

# Train the network
stats = net.train(X_train, y_train, X_val, y_val,
            num_iters=1000, batch_size=200,
            learning_rate=1e-5, learning_rate_decay=0.95,
            reg=0.1, verbose=True)

# Predict on the validation set
val_acc = (net.predict(X_val) == y_val).mean()
print('Validation accuracy: ', val_acc)

# Save this net as the variable subopt_net for later comparison.
subopt_net = net
test_acc = (subopt_net.predict(X_test) == y_test).mean()
print('Test accuracy (subopt_net): ', test_acc)
```

```
iteration 0 / 1000: loss 0.5000623457905098
iteration 100 / 1000: loss 0.4998246529435278
iteration 200 / 1000: loss 0.4995946718475304
iteration 300 / 1000: loss 0.49933536166627984
iteration 400 / 1000: loss 0.4989962372581251
iteration 500 / 1000: loss 0.49847178744773624
iteration 600 / 1000: loss 0.49758927830530253
iteration 700 / 1000: loss 0.4966248113033766
iteration 800 / 1000: loss 0.4958001901438695
iteration 900 / 1000: loss 0.4939583435911163
Validation accuracy:  0.172
Test accuracy (subopt_net):  0.191
```

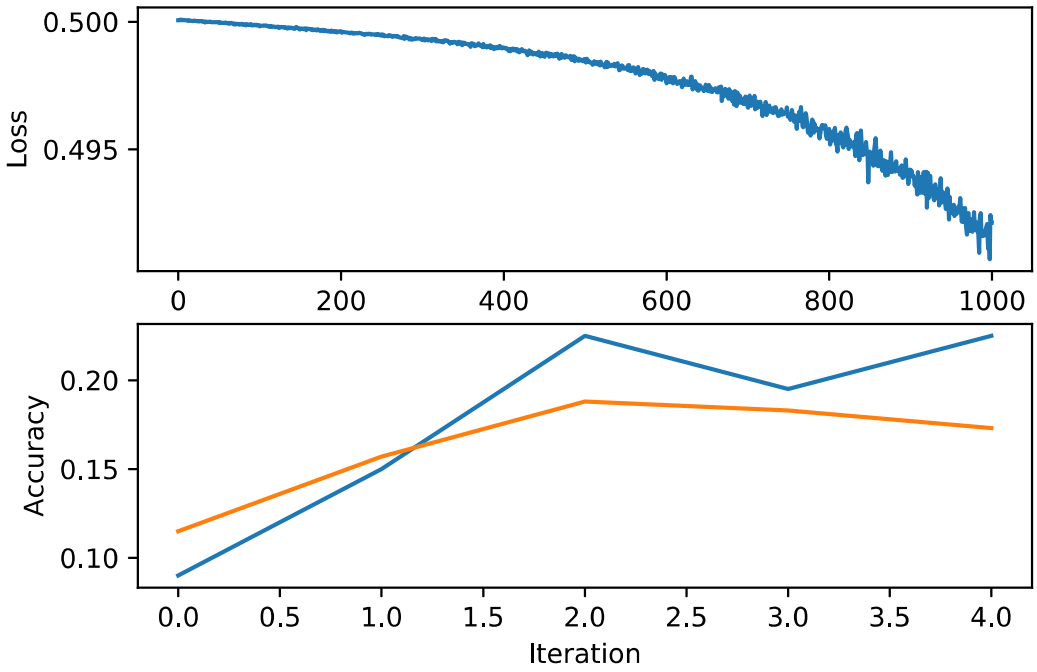
In [11]:
```
stats['train_acc_history']
```

Out[11]:
```
[0.09, 0.15, 0.225, 0.195, 0.225]
```

In [12]:
```
# Plot the loss function and train / validation accuracies
plt.subplot(2, 1, 1)
plt.plot(stats['loss_history'])
plt.xlabel('Iteration')
plt.ylabel('Loss')

plt.subplot(2, 1, 2)
plt.plot(stats['train_acc_history'], label='train')
plt.plot(stats['val_acc_history'], label='val')
plt.xlabel('Iteration')
plt.ylabel('Accuracy')

plt.show()
```



Questions:

The training accuracy isn't great. It seems even worse than simple KNN model, which is not as good as expected.

(1) What are some of the reasons why this is the case? Based on previous observations, please provide at least two possible reasons with justification.

(2) How should you fix the problems you identified in (1)?

Answers:

1. There are several potential problems. The first and most obvious one is that we haven't tuned our hyper parameters - we can and need to tune them to make sure the network converges on the best solution it can find. Additionally, the model is very possibly just not complex enough to approximate the truth (this is why modern networks are often quite deep/complex and involve more complicated layers e.g. convolutions)
2. We can tune the hyperparameters (which we will do) or introduce a more sophisticated network architecture by adding more hidden layers, adding more neurons in the hidden layers, or making different kinds of layers.

Optimize the neural network

Use the following part of the Jupyter notebook to optimize your hyperparameters on the validation set. Store your nets as best_net. To get the full credit of the neural nets, you should get at least **45%** accuracy on validation set.

Reminder: Think about whether you should retrain a new model from scratch every time your try a new set of hyperparameters.

In [14]:
```
best_net = None # store the best model into this

# ================================================================ #
# START YOUR CODE HERE:
# ================================================================ #
#   Optimize over your hyperparameters to arrive at the best neural
#   network.  You should be able to get over 45% validation accuracy.
#   For this part of the notebook, we will give credit based on the
#   accuracy you get.  Your score on this question will be multiplied by:
#      min(floor((X - 23%)) / %22, 1)
#   where if you get 50% or higher validation accuracy, you get full
#   points.
#
#   Note, you need to use the same network structure (keep hidden_size = 50)!
# ================================================================ #

# todo: optimal parameter search (you may use grid search by for-loops )
best_valacc = 0
best_params = {}
best_lr = 0.001
best_lr_decay = 1
best_reg_weight = 0.1

num_iters = 2000
batch_size = 1000

params = {
    'Learning Rate': [
        0.1,
        0.01,
        0.001
    ],
    'Learning Rate Decay': [
        1,
        0.995,
        0.99,
        0.985,
        0.98
    ],
    'Regularizer Weight': [
        0.1,
        0.075,
        0.05,
        0.025
    ]
}

def run_params(learning_rate, learning_rate_decay, reg):
    global best_valacc, best_net, best_params
    print(f'Training and testing {[learning_rate, learning_rate_decay, reg]}')
    net = TwoLayerNet(input_size, hidden_size, num_classes)
    net.train(X_train, y_train, X_val, y_val,
        num_iters=num_iters,
        batch_size=batch_size,
        learning_rate=learning_rate,
        learning_rate_decay=learning_rate_decay,
```

```
            reg=reg,
            verbose=False)

        acc = np.mean(net.predict(X_val) == y_val)
        print(f'Accuracy: {acc}\n')
        if acc > best_valacc:
            best_valacc = acc
            best_net = net
            best_params = {
                'Iterations': num_iters,
                'Batch Size': batch_size,
                'Learning Rate': learning_rate,
                'Learning Rate Decay': learning_rate_decay,
                'Regularizer Weight': reg
            }

def dfs(params_list, params=[]):
    if not params_list:
        run_params(*params)
        return

    param_values = params_list[0]
    for val in param_values:
        params.append(val)
        dfs(params_list[1:], params)
        params.pop()

dfs(list(params.values()))

# ================================================================ #
# END YOUR CODE HERE
# ================================================================ #
# Output your results
print("== Best parameter settings ==")
for k, v in best_params.items():
    print(f'{k} = {v}')
print("Best accuracy on validation set: {}".format(best_valacc))
```

```
Training and testing [0.1, 1, 0.1]
Accuracy: 0.087

Training and testing [0.1, 1, 0.075]
Accuracy: 0.087

Training and testing [0.1, 1, 0.05]
Accuracy: 0.087

Training and testing [0.1, 1, 0.025]
Accuracy: 0.087

Training and testing [0.1, 0.995, 0.1]
Accuracy: 0.087

Training and testing [0.1, 0.995, 0.075]
Accuracy: 0.087

Training and testing [0.1, 0.995, 0.05]
Accuracy: 0.087

Training and testing [0.1, 0.995, 0.025]
Accuracy: 0.087

Training and testing [0.1, 0.99, 0.1]
Accuracy: 0.087

Training and testing [0.1, 0.99, 0.075]
Accuracy: 0.087

Training and testing [0.1, 0.99, 0.05]
Accuracy: 0.087

Training and testing [0.1, 0.99, 0.025]
Accuracy: 0.087

Training and testing [0.1, 0.985, 0.1]
Accuracy: 0.087

Training and testing [0.1, 0.985, 0.075]
Accuracy: 0.087

Training and testing [0.1, 0.985, 0.05]
Accuracy: 0.087

Training and testing [0.1, 0.985, 0.025]
Accuracy: 0.087

Training and testing [0.1, 0.98, 0.1]
Accuracy: 0.087

Training and testing [0.1, 0.98, 0.075]
Accuracy: 0.087

Training and testing [0.1, 0.98, 0.05]
Accuracy: 0.087
```

```
Training and testing [0.1, 0.98, 0.025]
Accuracy: 0.087

Training and testing [0.01, 1, 0.1]
Accuracy: 0.087

Training and testing [0.01, 1, 0.075]
Accuracy: 0.087

Training and testing [0.01, 1, 0.05]
Accuracy: 0.087

Training and testing [0.01, 1, 0.025]
Accuracy: 0.087

Training and testing [0.01, 0.995, 0.1]
Accuracy: 0.087

Training and testing [0.01, 0.995, 0.075]
Accuracy: 0.087

Training and testing [0.01, 0.995, 0.05]
Accuracy: 0.087

Training and testing [0.01, 0.995, 0.025]
Accuracy: 0.087

Training and testing [0.01, 0.99, 0.1]
Accuracy: 0.087

Training and testing [0.01, 0.99, 0.075]
Accuracy: 0.087

Training and testing [0.01, 0.99, 0.05]
Accuracy: 0.087

Training and testing [0.01, 0.99, 0.025]
Accuracy: 0.087

Training and testing [0.01, 0.985, 0.1]
Accuracy: 0.087

Training and testing [0.01, 0.985, 0.075]
Accuracy: 0.087

Training and testing [0.01, 0.985, 0.05]
Accuracy: 0.087

Training and testing [0.01, 0.985, 0.025]
Accuracy: 0.087

Training and testing [0.01, 0.98, 0.1]
Accuracy: 0.087

Training and testing [0.01, 0.98, 0.075]
Accuracy: 0.087
```

```
Training and testing [0.01, 0.98, 0.05]
Accuracy: 0.087

Training and testing [0.01, 0.98, 0.025]
Accuracy: 0.087

Training and testing [0.001, 1, 0.1]
Accuracy: 0.488

Training and testing [0.001, 1, 0.075]
Accuracy: 0.494

Training and testing [0.001, 1, 0.05]
Accuracy: 0.472

Training and testing [0.001, 1, 0.025]
Accuracy: 0.482

Training and testing [0.001, 0.995, 0.1]
Accuracy: 0.464

Training and testing [0.001, 0.995, 0.075]
Accuracy: 0.479

Training and testing [0.001, 0.995, 0.05]
Accuracy: 0.483

Training and testing [0.001, 0.995, 0.025]
Accuracy: 0.502

Training and testing [0.001, 0.99, 0.1]
Accuracy: 0.485

Training and testing [0.001, 0.99, 0.075]
Accuracy: 0.486

Training and testing [0.001, 0.99, 0.05]
Accuracy: 0.49

Training and testing [0.001, 0.99, 0.025]
Accuracy: 0.49

Training and testing [0.001, 0.985, 0.1]
Accuracy: 0.48

Training and testing [0.001, 0.985, 0.075]
Accuracy: 0.493

Training and testing [0.001, 0.985, 0.05]
Accuracy: 0.491

Training and testing [0.001, 0.985, 0.025]
Accuracy: 0.496

Training and testing [0.001, 0.98, 0.1]
Accuracy: 0.497
```

```
Training and testing [0.001, 0.98, 0.075]
Accuracy: 0.489

Training and testing [0.001, 0.98, 0.05]
Accuracy: 0.493

Training and testing [0.001, 0.98, 0.025]
Accuracy: 0.48

== Best parameter settings ==
Iterations = 2000
Batch Size = 1000
Learning Rate = 0.001
Learning Rate Decay = 0.995
Regularizer Weight = 0.025
Best accuracy on validation set: 0.502
```

Quesions

(1) What is your best parameter settings? (Output from the previous cell)

(2) What parameters did you tune? How are they changing the performance of nerural network? You can discuss any observations from the optimization.

Answers

1. 2000 iterations, batch size of 1000, learning rate of 0.001, learning rate decay of 0.995, regularizer weight of 0.025
2. I tuned learning rate, learning rate decay, and regularization weight. Learning rate tuning will help make sure the network converges quickly enough, yet is stable enough to converge on an optimal solution instead of skipping over the optimal solution. Learning rate decay plays into this similary; by decaying the learning rate, the model makes bigger adjustments at first but towards the end makes only very small movements, trying to converge on the solution.
   Regularization aims to prevent overfitting for a noisy dataset, but too high a reg weight will cause the model to underfit (by simply fitting to minimize the regularization penalty).
   I chose not to modify some characteristics like batch size and iteration number because I did not want to have too large a grid to search. Additionally, I didn't change the number of iterations because I would try to prevent overfitting due to a long training period via Early Stopping or by picking the lowest validation error model trained during the iterations. I also didn't change batch size because I believed I chose a batch size large enough to avoid highly stochastic/random behavior yet small enough to allow the training to run fairly quickly.

   If I had access to more resources, could run it on GPU, and had more time I probably would have also used Early Stopping and selected a larger batch size; however, I obviously could not use those tools.

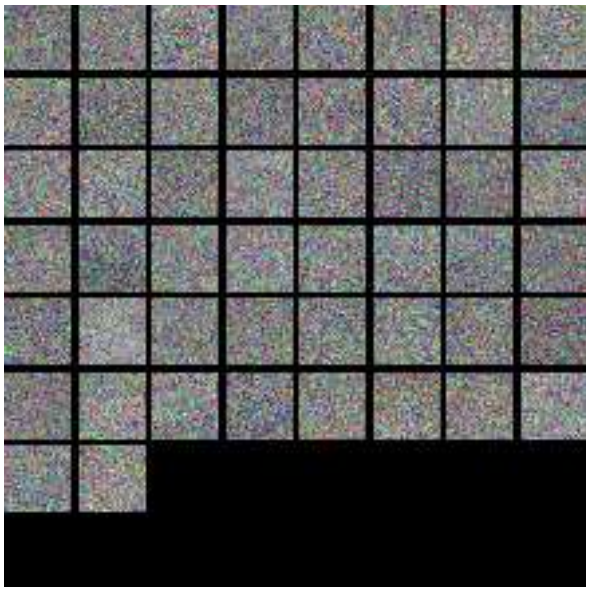
Visualize the weights of your neural networks

In [15]:
```
from data.vis_utils import visualize_grid

# Visualize the weights of the network

def show_net_weights(net):
    W1 = net.params['W1']
    W1 = W1.T.reshape(32, 32, 3, -1).transpose(3, 0, 1, 2)
    plt.imshow(visualize_grid(W1, padding=3).astype('uint8'))
    plt.gca().axis('off')
    plt.show()

show_net_weights(subopt_net)
show_net_weights(best_net)
```



Questions:

What differences do you see in the weights between the suboptimal net and the best net you arrived at? What do the weights in neural networks probably learn after training?

Answer:

The suboptimal net weights look very arbitrary, while the trained net clearly is figuring things out about phsyical locality and starting to make "blobs", indicating that it is looking for patterns and chunks to be similar.

Evaluate on test set

In [16]:
```
test_acc = (best_net.predict(X_test) == y_test).mean()
print('Test accuracy (best_net): ', test_acc)
```

```
Test accuracy (best_net):  0.484
```

Questions:

(1) What is your test accuracy by using the best NN you have got? How much does the performance increase compared with kNN? Why can neural networks perform better than kNN?

(2) Do you have any other ideas or suggestions to further improve the performance of neural networks other than the parameters you have tried in the homework?

Answers:

1. Test accuracy for my best net was 0.484, which is an improvement of over 0.2. One of the best things about neural networks is their far better ability to inference on unseen data than KNN; KNN tends to be very poor at inferencing on data that is dissimilar to the training data (it does a poor job extrapolating to new, different data). Additionally, the network may simply be able to capture more nuance without needing as much training data; with good enough training data, KNN may be able to have a low K value and fit the true distribution very closely but without that, KNN is forced to use a fairly high K to avoid overfitting.
2. As I said before, I would change the architecture of the network. Modern image processing networks (and especially image classification networks) are well researched, and the use of convolutional layers (and their related layers such as pooling layers) and a deeper neural network that can fit a more complex function have already shown themselves to be extremely powerful. Once we have a more complicated architecture, we could also train and tune on a more powerful system and potentially leverage GPUs to allow for better hyperparameter tuning.

Bonus Question: Change MSE Loss to Cross Entropy Loss

This is a bonus question. If you finish this (cross entropy loss) correctly, you will get **up to 10 points** (add up to your HW3 score).

Note: From grading policy of this course, your maximum points from homework are still 25 out of 100, but you can use the bonus question to make up other deduction of other assignments.

Pass output scores in networks from forward pass into softmax function. The softmax function is defined as,

$$p_j = \sigma(z_j) = \frac{e^{z_j}}{\sum_{c=1}^{C} e^{z_c}}$$

After softmax, the scores can be considered as probability of j-th class.

The cross entropy loss is defined as,

$$L = L_{\text{CE}} + L_{reg} = \frac{1}{N}\sum_{i=1}^{N} \log(p_{i,j}) + \frac{\lambda}{2}\left(||W_1||^2 + ||W_2||^2\right)$$

To take derivative of this loss, you will get the gradient as,

$$\frac{\partial L_{\text{CE}}}{\partial o_i} = p_i - y_i$$

More details about multi-class cross entropy loss, please check http://cs231n.github.io/linear-classify/ (http://cs231n.github.io/linear-classify/) and more explanation (https://deepnotes.io/softmax-crossentropy) about the derivative of cross entropy.

Change the loss from MSE to cross entropy, you only need to change you `MSE_loss(x,y)` in `TwoLayerNet.loss()` function to `softmax_loss(x,y)` .

Now you are free to use any code to show your results of the two-layer networks with newly-implemented cross entropy loss. You can use code from previous cells.

In [49]:
```
# Start training your networks and show your results
# ================================================================ #
# START YOUR CODE HERE:
# ================================================================ #
model = TwoLayerNet(input_size, hidden_size, num_classes)

# increased iterations to 3000 since I had more time
model.train(X_train, y_train, X_val, y_val,
    num_iters=3000, batch_size=1000,
    learning_rate=0.001, learning_rate_decay=0.995,
    reg=0.025, verbose=True)
test_acc = (model.predict(X_test) == y_test).mean()
print('Test accuracy: ', test_acc)

# ================================================================ #
# END YOUR CODE HERE
# ================================================================ #
```

```
iteration 0 / 3000: loss 2.3026152191579814
iteration 100 / 3000: loss 1.9243084193238658
iteration 200 / 3000: loss 1.7594292012159314
iteration 300 / 3000: loss 1.6251066827479497
iteration 400 / 3000: loss 1.5632776043852186
iteration 500 / 3000: loss 1.4781723168914798
iteration 600 / 3000: loss 1.4990366648903883
iteration 700 / 3000: loss 1.4744859951901206
iteration 800 / 3000: loss 1.3807530413926155
iteration 900 / 3000: loss 1.3744581974837389
iteration 1000 / 3000: loss 1.4243201032773842
iteration 1100 / 3000: loss 1.37427244978942
iteration 1200 / 3000: loss 1.3334782587134006
iteration 1300 / 3000: loss 1.3136866751448488
iteration 1400 / 3000: loss 1.3126012277186372
iteration 1500 / 3000: loss 1.253111268512218
iteration 1600 / 3000: loss 1.3105484027011365
iteration 1700 / 3000: loss 1.3177762095192393
iteration 1800 / 3000: loss 1.2851925455240112
iteration 1900 / 3000: loss 1.2255751233352978
iteration 2000 / 3000: loss 1.2841311549893708
iteration 2100 / 3000: loss 1.2630641118099908
iteration 2200 / 3000: loss 1.2228576093324488
iteration 2300 / 3000: loss 1.2082941583130393
iteration 2400 / 3000: loss 1.2728758311484343
iteration 2500 / 3000: loss 1.2215092032455557
iteration 2600 / 3000: loss 1.239929665743268
iteration 2700 / 3000: loss 1.237733069122445
iteration 2800 / 3000: loss 1.2010280474689212
iteration 2900 / 3000: loss 1.231346748580564
Test accuracy:  0.513
```

End of Homework 3, Part 2 :)

After you've finished both parts the homework, please print out the both of the entire `ipynb` notebooks and `py` files into one PDF file. Make sure you include the output of code cells and answers for questions. Prepare submit it to GradeScope. Do not include any dataset in your submission.

```python
import numpy as np
import pdb

"""
This code was based off of code from cs231n at Stanford University, and modified for CS145 at UCLA.
"""

class KNN(object):

    def __init__(self):
        pass

    def train(self, X, y):
        """
        Inputs:
        - X is a numpy array of size (num_examples, D)
        - y is a numpy array of size (num_examples, )
        """
        # ========================================================== #
        # START YOUR CODE HERE
        # ========================================================== #
        #   Hint: KNN does not do any further processsing, just store the training
        #   samples with labels into as self.X_train and self.y_train
        # ========================================================== #
        self.X_train = X
        self.y_train = y
        # ========================================================== #
        # END YOUR CODE HERE
        # ========================================================== #

    def compute_distances(self, X, norm=None):
        """
        Compute the distance between each test point in X and each training point
        in self.X_train.

        Inputs:
        - X: A numpy array of shape (num_test, D) containing test data.
        - norm: the function with which the norm is taken.

        Returns:
        - dists: A numpy array of shape (num_test, num_train) where dists[i, j]
          is the Euclidean distance between the ith test point and the jth training
          point.
        """
        if norm is None:
            norm = lambda x: np.sqrt(np.sum(x**2)) #norm = 2

        num_test = X.shape[0]
        num_train = self.X_train.shape[0]
        dists = np.zeros((num_test, num_train))
        for i in np.arange(num_test):

            for j in np.arange(num_train):
            # ========================================================== #
            # START YOUR CODE HERE
            # ========================================================== #
            #   Compute the distance between the ith test point and the jth
            #   training point using norm(), and store the result in dists[i, j].
```

```
            # ================================================================ #

                dists[i, j] = norm(X[i] - self.X_train[j])

            # ================================================================ #
            # END YOUR CODE HERE
            # ================================================================ #

        return dists

    def compute_L2_distances_vectorized(self, X):
        """
        Compute the distance between each test point in X and each training point
        in self.X_train WITHOUT using any for loops.

        Inputs:
        - X: A numpy array of shape (num_test, D) containing test data.

        Returns:
        - dists: A numpy array of shape (num_test, num_train) where dists[i, j]
          is the Euclidean distance between the ith test point and the jth training
          point.
        """
        num_test = X.shape[0]
        num_train = self.X_train.shape[0]
        dists = np.zeros((num_test, num_train))

        # ================================================================ #
        # START YOUR CODE HERE
        # ================================================================ #
        #   Compute the L2 distance between the ith test point and the jth
        #   training point and store the result in dists[i, j].  You may
        #    NOT use a for loop (or list comprehension).  You may only use
        #     numpy operations.
        #
        #     HINT: use broadcasting.  If you have a shape (N,1) array and
        #   a shape (M,) array, adding them together produces a shape (N, M)
        #   array.
        # ================================================================ #

        dists = np.sqrt(np.sum(X**2, axis=-1)[:, None] + np.sum(self.X_train**2,
axis=-1)[None, :] - 2*(X @ self.X_train.T))

        # ================================================================ #
        # END YOUR CODE HERE
        # ================================================================ #

        return dists


    def predict_labels(self, dists, k=1):
        """
        Given a matrix of distances between test points and training points,
        predict a label for each test point.

        Inputs:
        - dists: A numpy array of shape (num_test, num_train) where dists[i, j]
          gives the distance betwen the ith test point and the jth training point.

        Returns:
```

```
        - y: A numpy array of shape (num_test,) containing predicted labels for the
          test data, where y[i] is the predicted label for the test point X[i].
        """
        num_test = dists.shape[0]
        y_pred = np.zeros(num_test)
        for i in range(num_test):
            # A list of length k storing the labels of the k nearest neighbors to
            # the ith test point.

            closest_y = []

            # ========================================================================= #
            # START YOUR CODE HERE
            # ========================================================================= #
            #   Use the distances to calculate and then store the labels of
            #   the k-nearest neighbors to the ith test point.  The function
            #   numpy.argsort may be useful.
            #
            #   After doing this, find the most common label of the k-nearest
            #   neighbors.  Store the predicted label of the ith training example
            #   as y_pred[i].  Break ties by choosing the smaller label.
            # ========================================================================= #
            closest = self.y_train[np.argsort(dists[i])[:k]]
            classes, counts = np.unique(closest, return_counts=True)

            y_pred[i] = classes[np.argmax(counts)]

            # ========================================================================= #
            # END YOUR CODE HERE
            # ========================================================================= #
        return y_pred

```

```python
import numpy as np
import matplotlib.pyplot as plt

class TwoLayerNet(object):
    """
    A two-layer fully-connected neural network. The net has an input dimension of
    N, a hidden layer dimension of H, and performs classification over C classes.
    We train the network with a softmax loss function and L2 regularization on the
    weight matrices. The network uses a ReLU nonlinearity after the first fully
    connected layer.

    In other words, the network has the following architecture:

    input - fully connected layer - ReLU - fully connected layer - MSE Loss

    ReLU function:
    (i) x = x if x >= 0  (ii) x = 0 if x < 0

    The outputs of the second fully-connected layer are the scores for each class.
    """

    def __init__(self, input_size, hidden_size, output_size, std=1e-4):
        """
        Initialize the model. Weights are initialized to small random values and
        biases are initialized to zero. Weights and biases are stored in the
        variable self.params, which is a dictionary with the following keys:

        W1: First layer weights; has shape (H, D)
        b1: First layer biases; has shape (H,)
        W2: Second layer weights; has shape (C, H)
        b2: Second layer biases; has shape (C,)

        Inputs:
        - input_size: The dimension D of the input data.
        - hidden_size: The number of neurons H in the hidden layer.
        - output_size: The number of classes C.
        """
        self.params = {}
        self.params['W1'] = std * np.random.randn(hidden_size, input_size)
        self.params['b1'] = np.zeros(hidden_size)
        self.params['W2'] = std * np.random.randn(output_size, hidden_size)
        self.params['b2'] = np.zeros(output_size)

    def loss(self, X, y=None, reg=0.0):
        """
        Compute the loss and gradients for a two layer fully connected neural
        network.

        Inputs:
        - X: Input data of shape (N, D). Each X[i] is a training sample.
        - y: Vector of training labels. y[i] is the label for X[i], and each y[i] is
          an integer in the range 0 <= y[i] < C. This parameter is optional; if it
          is not passed then we only return scores, and if it is passed then we
          instead return the loss and gradients.
        - reg: Regularization strength.

        Returns:
        If y is None, return a matrix scores of shape (N, C) where scores[i, c] is
        the score for class c on input X[i].

```

```
        If y is not None, instead return a tuple of:
        - loss: Loss (data loss and regularization loss) for this batch of training
          samples.
        - grads: Dictionary mapping parameter names to gradients of those parameters
          with respect to the loss function; has the same keys as self.params.
        """
        # Unpack variables from the params dictionary
        W1, b1 = self.params['W1'], self.params['b1']
        W2, b2 = self.params['W2'], self.params['b2']
        N, D = X.shape

        # Compute the forward pass
        scores = None

        # ============================================================= #
        # START YOUR CODE HERE
        # ============================================================= #
        #   Calculate the output scores of the neural network.  The result
        #   should be (N, C). As stated in the description for this class,
        #   there should not be a ReLU layer after the second fully-connected
        #   layer.
        #   The code is partially given
        #   The output of the second fully connected layer is the output scores.
        #   Do not use a for loop in your implementation.
        #   Please use 'h1' as input of hidden layers, and 'a2' as output of
        #   hidden layers after ReLU activation function.
        #   [Input X] --W1,b1--> [h1] -ReLU-> [a2] --W2,b2--> [scores]
        #   You may simply use np.maximun for implementing ReLU.
        #   Note that there is only one ReLU layer.
        #   Note that plase do not change the variable names (h1, h2, a2)
        # ============================================================= #

        h1 = X @ W1.T + b1
        a2 = h1 * (h1 > 0)
        h2 = a2 @ W2.T + b2
        scores = h2

        # ============================================================= #
        # END YOUR CODE HERE
        # ============================================================= #


        # If the targets are not given then jump out, we're done
        if y is None:
            return scores

        # Compute the loss
        loss = None

        # scores is num_examples by num_classes (N, C)
        def softmax_loss(x, y):
            loss, dx = 0,0
            # ============================================================= #
            # START YOUR CODE HERE (BONUS QUESTION)
            # ============================================================= #
            #   Calculate the cross entropy loss after softmax output layer.
            #   The format are provided in the notebook.
            #   This function should return loss and dx, same as MSE loss function.
            # ============================================================= #

```

```
            n = y.shape[0]
            y_onehot = np.zeros((n, y.max() + 1))
            y_onehot[np.arange(n), y] = 1

            exps = np.exp(x - x.max(axis=-1)[:, None])
            softmax = exps / exps.sum(axis=-1)[:, None]
            log_likelihoods = -np.log(np.sum(softmax * y_onehot, axis=-1))

            loss = np.mean(log_likelihoods)
            dx = (softmax - y_onehot) / n

            # ================================================================ #
            # END YOUR CODE HERE
            # ================================================================ #
            return loss, dx


        def MSE_loss(x, y):
            loss, dx = 0,0
            # ================================================================ #
            # START YOUR CODE HERE
            # ================================================================ #
            #   This function should return loss and dx (gradients ready for back
prop).
            #   The loss is MSE loss between network ouput and one hot vector of
class
            #   labels is required for backpropogation.
            # ================================================================ #
            # Hint: Check the type and shape of x and y.
            #       e.g. print('DEBUG:x.shape, y.shape', x.shape, y.shape)

            n = y.shape[0]
            y_onehot = np.zeros((n, y.max() + 1))
            y_onehot[np.arange(n), y] = 1
            loss = np.sum((x - y_onehot) ** 2) / (2 * n)

            dx = (x - y_onehot) / n

            # ================================================================ #
            # END YOUR CODE HERE
            # ================================================================ #
            return loss, dx

        data_loss, dscore = softmax_loss(scores, y)
        # The above line is for bonus question. If you have implemented
softmax_loss, de-comment this line instead of MSE error.

        # data_loss, dscore = MSE_loss(scores, y) # "comment" this line if you use
softmax_loss
        # ================================================================ #
        # START YOUR CODE HERE
        # ================================================================ #
        #   Calculate the regularization loss. Multiply the regularization
        #   loss by 0.5 (in addition to the factor reg).
        # ================================================================ #
        reg_loss = 0.5 * reg * (np.sum(W1*W1) + np.sum(W2*W2))

        # ================================================================ #
        # END YOUR CODE HERE
        # ================================================================ #
```

```
177         loss = data_loss + reg_loss
178 
179         grads = {}
180 
181         # ============================================================ #
182         # START YOUR CODE HERE
183         # ============================================================ #
184         # Backpropogation: (You do not need to change this!)
185         #   Backward pass is implemented. From the dscore error, we calculate
186         #   the gradient and store as grads['W1'], etc.
187         # ============================================================ #
188         grads['W2'] = a2.T.dot(dscore).T + reg * W2
189         grads['b2'] = np.ones(N).dot(dscore)
190 
191         da_h = np.zeros(h1.shape)
192         da_h[h1>0] = 1
193         dh = (dscore.dot(W2) * da_h)
194 
195         grads['W1'] = np.dot(dh.T,X) + reg * W1
196         grads['b1'] = np.ones(N).dot(dh)
197         # ============================================================ #
198         # END YOUR CODE HERE
199         # ============================================================ #
200 
201         return loss, grads
202 
203     def train(self, X, y, X_val, y_val,
204             learning_rate=1e-3, learning_rate_decay=0.95,
205             reg=1e-5, num_iters=100,
206             batch_size=200, verbose=False):
207         """
208         Train this neural network using stochastic gradient descent.
209 
210         Inputs:
211         - X: A numpy array of shape (N, D) giving training data.
212         - y: A numpy array f shape (N,) giving training labels; y[i] = c means that
213           X[i] has label c, where 0 <= c < C.
214         - X_val: A numpy array of shape (N_val, D) giving validation data.
215         - y_val: A numpy array of shape (N_val,) giving validation labels.
216         - learning_rate: Scalar giving learning rate for optimization.
217         - learning_rate_decay: Scalar giving factor used to decay the learning rate
218           after each epoch.
219         - reg: Scalar giving regularization strength.
220         - num_iters: Number of steps to take when optimizing.
221         - batch_size: Number of training examples to use per step.
222         - verbose: boolean; if true print progress during optimization.
223         """
224         num_train = X.shape[0]
225         iterations_per_epoch = max(num_train / batch_size, 1)
226 
227         # Use SGD to optimize the parameters in self.model
228         loss_history = []
229         train_acc_history = []
230         val_acc_history = []
231 
232         for it in np.arange(num_iters):
233             X_batch = None
234             y_batch = None
235 
236             #   Create a minibatch (X_batch, y_batch) by sampling batch_size
```

```
            #   samples randomly.

            b_index = np.random.choice(num_train, batch_size)
            X_batch = X[b_index]
            y_batch = y[b_index]

            # Compute loss and gradients using the current minibatch
            loss, grads = self.loss(X_batch, y=y_batch, reg=reg)
            loss_history.append(loss)

            # ========================================================== #
            # START YOUR CODE HERE
            # ========================================================== #
            #   Perform a gradient descent step using the minibatch to update
            #   all parameters (i.e., W1, W2, b1, and b2).
            #   The gradient has been calculated as grads['W1'], grads['W2'],
            #   grads['b1'], grads['b2']
            #   For example,
            #   W1(new) = W1(old) - learning_rate * grads['W1']
            #   (this is not the exact code you use!)
            # ========================================================== #

            self.params['W1'] -= learning_rate * grads['W1']
            self.params['b1'] -= learning_rate * grads['b1']
            self.params['W2'] -= learning_rate * grads['W2']
            self.params['b2'] -= learning_rate * grads['b2']

            # ========================================================== #
            # END YOUR CODE HERE
            # ========================================================== #

            if verbose and it % 100 == 0:
                print('iteration {} / {}: loss {}'.format(it, num_iters, loss))

            # Every epoch, check train and val accuracy and decay learning rate.
            if it % iterations_per_epoch == 0:
                # Check accuracy
                train_acc = (self.predict(X_batch) == y_batch).mean()
                val_acc = (self.predict(X_val) == y_val).mean()
                train_acc_history.append(train_acc)
                val_acc_history.append(val_acc)

                # Decay learning rate
                learning_rate *= learning_rate_decay

        return {
          'loss_history': loss_history,
          'train_acc_history': train_acc_history,
          'val_acc_history': val_acc_history,
        }

    def predict(self, X):
        """
        Use the trained weights of this two-layer network to predict labels for
        data points. For each data point we predict scores for each of the C
        classes, and assign each data point to the class with the highest score.

        Inputs:
        - X: A numpy array of shape (N, D) giving N D-dimensional data points to
          classify.
```

```
297
298         Returns:
299         - y_pred: A numpy array of shape (N,) giving predicted labels for each of
300           the elements of X. For all i, y_pred[i] = c means that X[i] is predicted
301           to have class c, where 0 <= c < C.
302         """
303         y_pred = None
304
305         # ========================================================== #
306         # START YOUR CODE HERE
307         # ========================================================== #
308         #   Predict the class given the input data.
309         # ========================================================== #
310
311         scores = self.loss(X)
312         y_pred = np.argmax(scores, axis=-1)
313
314         # ========================================================== #
315         # END YOUR CODE HERE
316         # ========================================================== #
317
318         return y_pred
319
320
321
```