Regularized Regression II

Setup

```r
library(foreach)
library(mlbench)
library(caret)
library(glmnet)
library(gglasso)
```

Data

In this notebook, we use the Boston Housing data set. “This dataset contains information collected by the U.S Census Service concerning housing in the area of Boston Mass. It was obtained from the StatLib archive (http://lib.stat.cmu.edu/datasets/boston), and has been used extensively throughout the literature to benchmark algorithms.”

Source: https://www.cs.toronto.edu/~delve/data/boston/bostonDetail.html

```r
data(BostonHousing2)
head(BostonHousing2)
```

```
##         town tract      lon     lat medv cmedv    crim zn indus chas   nox
## 1     Nahant  2011 -70.9550 42.2550 24.0  24.0 0.00632 18  2.31    0 0.538
## 2 Swampscott  2021 -70.9500 42.2875 21.6  21.6 0.02731  0  7.07    0 0.469
## 3 Swampscott  2022 -70.9360 42.2830 34.7  34.7 0.02729  0  7.07    0 0.469
## 4 Marblehead  2031 -70.9280 42.2930 33.4  33.4 0.03237  0  2.18    0 0.458
## 5 Marblehead  2032 -70.9220 42.2980 36.2  36.2 0.06905  0  2.18    0 0.458
## 6 Marblehead  2033 -70.9165 42.3040 28.7  28.7 0.02985  0  2.18    0 0.458
##      rm  age    dis rad tax ptratio      b lstat
## 1 6.575 65.2 4.0900   1 296    15.3 396.90  4.98
## 2 6.421 78.9 4.9671   2 242    17.8 396.90  9.14
## 3 7.185 61.1 4.9671   2 242    17.8 392.83  4.03
## 4 6.998 45.8 6.0622   3 222    18.7 394.63  2.94
## 5 7.147 54.2 6.0622   3 222    18.7 396.90  5.33
## 6 6.430 58.7 6.0622   3 222    18.7 394.12  5.21
```

```r
names(BostonHousing2)
```

```
##  [1] "town"    "tract"   "lon"     "lat"     "medv"    "cmedv"   "crim"
##  [8] "zn"      "indus"   "chas"    "nox"     "rm"      "age"     "dis"
## [15] "rad"     "tax"     "ptratio" "b"       "lstat"
```

Since we want to compare the performance of some regularized models at the end of the modeling process, we first split the data into a training and a test part. This can be done by random sampling with `sample`.

```r
set.seed(8593)
train <- sample(1:nrow(BostonHousing2), 0.8*nrow(BostonHousing2))
boston_train <- BostonHousing2[train,]
boston_test <- BostonHousing2[-train,]
```

A quick look on our outcome variable for the next sections, which is the Median value of owner-occupied homes in $1000's.

```r
summary(boston_train$medv)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    5.00   17.10   21.40   22.89   26.25   50.00
```

```r
summary(boston_test$medv)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    5.00   16.50   20.10   21.11   23.77   50.00
```

Regularized regression

Now we can prepare our training data for the regularized regression models. The `glmnet` package needs models to be fitted on an X matrix and an y vector, which we need to generate first.

```r
X <- model.matrix(medv ~ . - town - tract - cmedv,
                  boston_train)[,-1]
y <- boston_train$medv
```

Elastic net

In addition to ridge regression and the lasso, the elastic net can be used as a compromise between the former approaches. Here we build a small tuning loop that estimates series of regularized models for three settings of the mixing parameter `alpha`.

```r
a <- c(0.1, 0.5, 0.9)
m1_cv <- foreach(i = a, .combine = rbind) %do% {
  cv <- cv.glmnet(X, y, alpha = i)
  data.frame(cvm = cv$cvm, lambda = cv$lambda, lambda.min = cv$lambda.min, alpha = i)
}
head(m1_cv)
```

```
##        cvm   lambda lambda.min alpha
## 1 87.67830 67.79232 0.07615274   0.1
## 2 85.87417 61.76984 0.07615274   0.1
## 3 83.70787 56.28237 0.07615274   0.1
## 4 81.46490 51.28240 0.07615274   0.1
## 5 79.15610 46.72662 0.07615274   0.1
## 6 76.56625 42.57555 0.07615274   0.1
```

Based on the former CV loop we select the lambda and alpha constellation that is associated with the smallest CV error.

```r
b1_cv <- m1_cv[m1_cv$cvm == min(m1_cv$cvm),]
m1 <- glmnet(X, y, lambda = b1_cv$lambda, alpha = b1_cv$alpha)
coef(m1)
```

```
## 16 x 1 sparse Matrix of class "dgCMatrix"
##                        s0
## (Intercept) -3.732429e+02
## lon         -3.423549e+00
## lat          3.864709e+00
## crim        -1.014115e-01
## zn           3.968064e-02
## indus       -7.425623e-05
## chas1        2.252981e+00
## nox         -1.616388e+01
## rm           4.304516e+00
## age         -8.314217e-03
## dis         -1.509696e+00
## rad          2.471068e-01
## tax         -1.105398e-02
## ptratio     -9.534987e-01
## b            8.451085e-03
## lstat       -4.588219e-01
```

Prediction in test set

Finally, we investigate the performance of our models in the test set. For this task, we construct an X matrix from the test set.

```r
Xt <- model.matrix(medv ~ . - town - tract - cmedv,
                   boston_test)[,-1]
```

This matrix can be used in the `predict` function, along with the respective model that should be used for prediction.

```r
p_net <- predict(m1, newx = Xt)
```

As a last step, let's look at the test set performance of our model.

```r
postResample(p_net, boston_test$medv)
```

```
##      RMSE  Rsquared       MAE
## 4.5473212 0.7223434 3.2898066
```

Group Lasso

In order to run Group Lasso with `gglasso`, the feature groups have to be specified. Here we only consider two groups that differentiate between location (lon, lat) and all other variables.

```r
groups <- c(1,1,2,2,2,2,2,2,2,2,2,2,2,2,2)
```

The groups object can be passed onto `gglasso`, along with the X matrix and the y vector. To keep things simple, we request that only 10 lambda values should be considered.

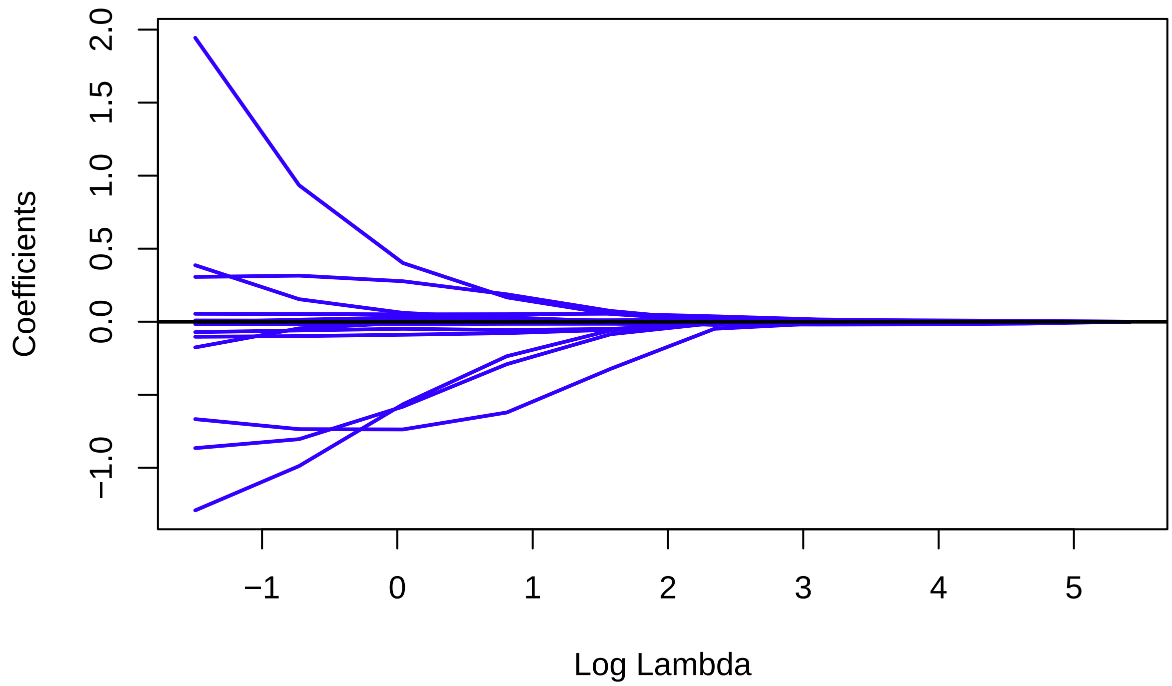
```r
m2 <- gglasso(X, y,
              group = groups,
              loss = 'ls',
              nlambda = 10,
              eps = 1e-04)
```

The lambda values and coefficient paths can be listed (plotted) by simply calling (plotting) the results object.

```r
m2
```

```
##
## Call:  gglasso(x = X, y = y, group = groups, loss = "ls", nlambda = 10,      eps = 1e-04)
##
##    Df   Lambda
## s0  0 224.6000
## s1 13 104.3000
## s2 13  48.3900
## s3 13  22.4600
## s4 13  10.4300
## s5 13   4.8390
## s6 13   2.2460
## s7 13   1.0430
## s8 13   0.4839
## s9 13   0.2246
```

```r
plot(m2)
```



The set of coefficients for specific lambda values are in `m2$beta`.

```
m2$beta[,10]
```

```
##         lon         lat        crim          zn       indus       chas1 
##  0.000000000  0.000000000 -0.103311980  0.054042638 -0.071503099  0.386679116 
##         nox          rm         age         dis         rad         tax 
## -0.175992759  1.943904324 -0.002572343 -1.292021170  0.306791035 -0.016393218 
##     ptratio           b       lstat 
## -0.865834228  0.008347154 -0.667332295 
```

```
m2$beta[,5]
```

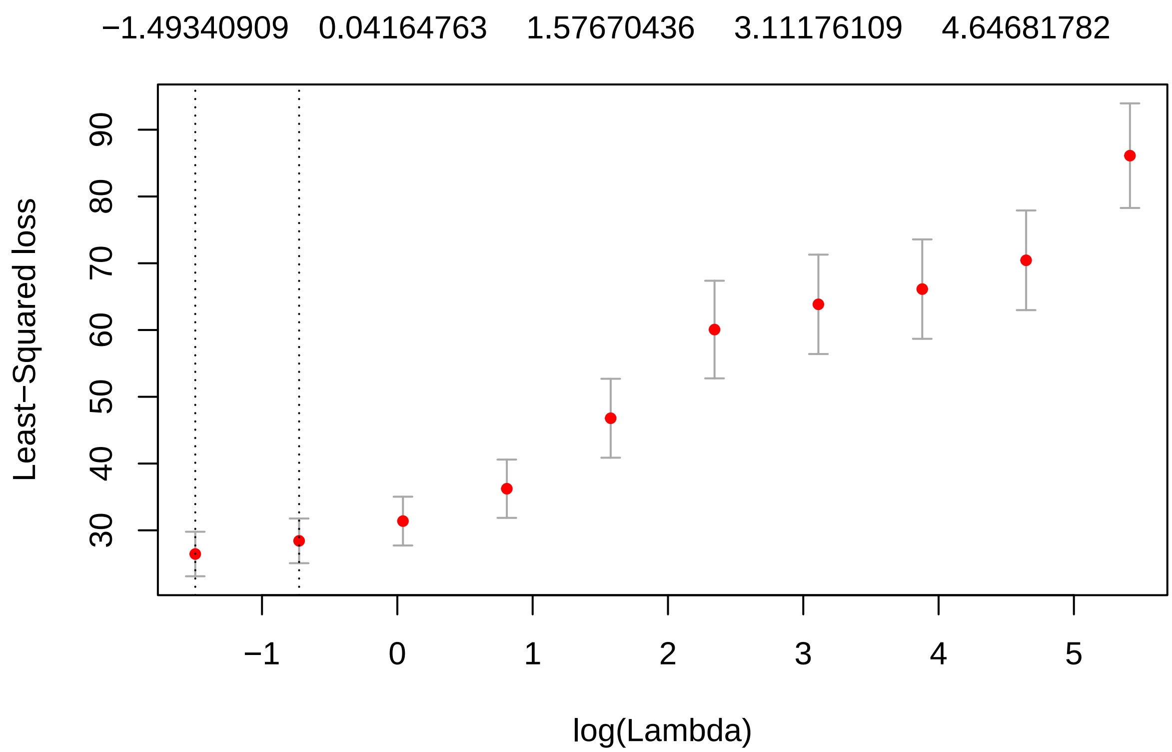
```
##           lon           lat          crim            zn         indus 
##  0.0000000000  0.0000000000 -0.0118729033  0.0367841395 -0.0119060400 
##         chas1           nox            rm           age           dis 
##  0.0006271989 -0.0001038341  0.0064394676 -0.0219291609 -0.0039321927 
##           rad           tax       ptratio             b         lstat 
##  0.0075663394 -0.0177542709 -0.0098060916  0.0128498219 -0.0484268759 
```

```
m2$beta[,1]
```

```
##     lon     lat    crim      zn   indus   chas1     nox      rm     age     dis 
##       0       0       0       0       0       0       0       0       0       0 
##     rad     tax ptratio       b   lstat 
##       0       0       0       0       0 
```

As with **glmnet**, we can run gglasso with Cross-Validation in order to find the best lambda values for prediction. (The following chunk might take some time to run).

```
m2_cv <- cv.gglasso(X, y,
              group = groups,
              loss = 'ls',
              nlambda = 10,
              eps = 1e-04,
              nfolds = 5)
plot(m2_cv)
```



Prediction in test set

Given the CV result, we can use predict directly by referring to `m2_cv$lambda.min` object within `predict` in order to specify which model should be used.

```
p_gglasso <- predict(m2_cv$gglasso.fit, newx = Xt, s = m2_cv$lambda.min)
```

Finally, a quick look at the test set performance of our Group Lasso model.

```
postResample(p_gglasso, boston_test$medv)
```

```
##      RMSE  Rsquared       MAE
## 4.5307547 0.7257953 3.2867874
```

References

- https://web.stanford.edu/~hastie/glmnet/glmnet_alpha.html
- https://cran.r-project.org/web/packages/gglasso/