Parallel Matrix Multiplication

Final Project

INF236: Parallel Programming

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Introduction

- Complexity: $\mathcal{O}(n^3)$
- Consecutive memory access

Algorithm 1 matrix multiplication

Require: $\mathbf{A}, \mathbf{B}$
Ensure: $\mathbf{C}$ (the resulting matrix)

function MATMUL($\mathbf{A}, \mathbf{B}$)
    for $i = 0, \dots, n-1$ **do**
        for $j = 0, \dots, n-1$ **do**
            $c[i][j] = 0$
        for $k = 0, \dots, n-1$ **do**
            for $j = 0, \dots, n-1$ **do**
                $c[i][j] += a[i][k] \cdot b[k][j]$
    return C

Strassen's Algorithm

- Blockwise matrix multiplication

$$\begin{aligned}\mathbf{A}\cdot\mathbf{B} &= \begin{pmatrix}\mathbf{A}_{00} & \mathbf{A}_{01}\\ \mathbf{A}_{10} & \mathbf{A}_{11}\end{pmatrix}\cdot\begin{pmatrix}\mathbf{B}_{00} & \mathbf{B}_{01}\\ \mathbf{B}_{10} & \mathbf{B}_{11}\end{pmatrix}\\ &= \begin{pmatrix}\mathbf{A}_{00}\mathbf{B}_{00}+\mathbf{A}_{01}\mathbf{B}_{10} & \mathbf{A}_{00}\mathbf{B}_{01}+\mathbf{A}_{01}\mathbf{B}_{11}\\ \mathbf{A}_{10}\mathbf{B}_{00}+\mathbf{A}_{11}\mathbf{B}_{10} & \mathbf{A}_{10}\mathbf{B}_{01}+\mathbf{A}_{11}\mathbf{B}_{11}\end{pmatrix}\\ &= \begin{pmatrix}\mathbf{C}_{00} & \mathbf{C}_{01}\\ \mathbf{C}_{10} & \mathbf{C}_{11}\end{pmatrix} = \mathbf{C}\end{aligned}$$

- Idea: reduce number of multiplications from 8 to 7
- Complexity of Strassen's algorithm: $\mathcal{O}(n^{\log_2 7}) \approx \mathcal{O}(n^{2.8073})$

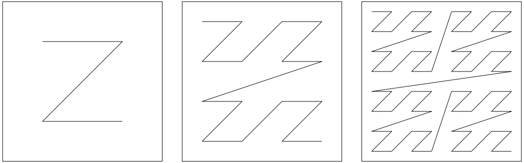
Strassen's Algorithm

Algorithm 2 Strassen's matrix multiplication

Require: $\mathbf{A}, \mathbf{B}$
Ensure: $\mathbf{C}$ (the resulting matrix)

function $\textsc{Strassen}(\mathbf{A}, \mathbf{B}, n)$
  if n == cutoff **then**
    return $\textsc{MatMul}(\mathbf{A}, \mathbf{B})$
  $\mathbf{P}_1 = \textsc{Strassen}(\mathbf{A}_{00} + \mathbf{A}_{11}, \mathbf{B}_{00} + \mathbf{B}_{11}, \frac{n}{2})$
  $\mathbf{P}_2 = \textsc{Strassen}(\mathbf{A}_{10} + \mathbf{A}_{11}, \mathbf{B}_{00}, \frac{n}{2})$
  $\mathbf{P}_3 = \textsc{Strassen}(\mathbf{A}_{00}, \mathbf{B}_{01} - \mathbf{B}_{11}, \frac{n}{2})$
  $\mathbf{P}_4 = \textsc{Strassen}(\mathbf{A}_{11}, \mathbf{B}_{10} - \mathbf{B}_{00}, \frac{n}{2})$
  $\mathbf{P}_5 = \textsc{Strassen}(\mathbf{A}_{00} + \mathbf{A}_{01}, \mathbf{B}_{11}, \frac{n}{2})$
  $\mathbf{P}_6 = \textsc{Strassen}(\mathbf{A}_{10} - \mathbf{A}_{00}, \mathbf{B}_{00} + \mathbf{B}_{01}, \frac{n}{2})$
  $\mathbf{P}_7 = \textsc{Strassen}(\mathbf{A}_{01} - \mathbf{A}_{11}, \mathbf{B}_{10} + \mathbf{B}_{11}, \frac{n}{2})$
  $\mathbf{C}_{00} = \mathbf{P}_1 + \mathbf{P}_4 - \mathbf{P}_5 + \mathbf{P}_7$
  $\mathbf{C}_{01} = \mathbf{P}_3 + \mathbf{P}_5$
  $\mathbf{C}_{10} = \mathbf{P}_2 + \mathbf{P}_4$
  $\mathbf{C}_{11} = \mathbf{P}_1 - \mathbf{P}_2 + \mathbf{P}_3 + \mathbf{P}_6$
  return C

z-ordering



$$\begin{matrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{matrix}$$

1D array in memory:

$a_{00}\ a_{01}\ a_{02}\ a_{03}\ a_{10}\ a_{11}\ a_{12}\ \ldots$

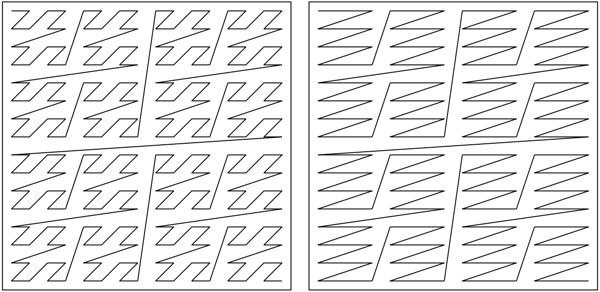
$$\begin{matrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{matrix}$$

1D array in memory:

$a_{00}\ a_{01}\ a_{10}\ a_{11}\ a_{02}\ a_{03}\ a_{12}\ \ldots$

z-ordering

- Matrices at cutoff level are stored row-wise



Experiments

Conclusion