GSE Math Camp Day 1: Software and Pre-Calculus Review

Klint Kanopka

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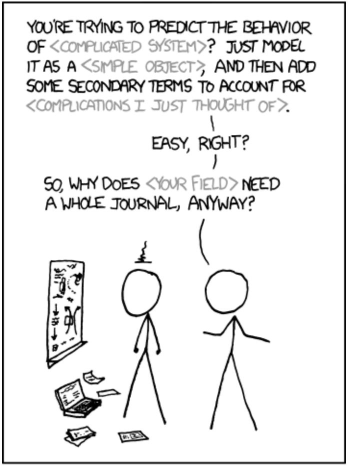
Tuesday, September 3, 2019

About Me



LIBERAL-ARTS MAJORS MAY BE ANNOYING SOMETIMES, BUT THERE'S *NOTHING* MORE OBNOXIOUS THAN A PHYSICIST FIRST ENCOUNTERING A NEW SUBJECT.

Personal Information

- From Philadelphia
- Taught physics in the Philadelphia School District for 8 years
- 3rd year PhD Student (DAPS)
- Also doing MSCS (AI track)
- Interested in measurement, assessment, and the intersection of AI and psychometrics
- I'm willing to answer almost any question you have (personal or otherwise)

Math Camp Materials

All materials for the first week are hosted at:
`http://github.com/klintkanopka/gsemathcamp`

Outline

1. Software Packages of Interest
   - Document Processing
   - Citation Managers
   - Quantitative Software
   - VPN
2. Review of Algebra and Notation
   - Order of Operations
   - Working with Units
   - Exponents
   - The Coordinate Plane
   - Lines
   - Polynomials
3. Limits
   - Properties

Document Processing

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- Microsoft Word (the standard)

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- Markdown (newer, least used, but gaining steam)

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- Stanford library and UToronto library both have good websites describing the different options

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- If you travel overseas often (and plan on working during that time), also consider a physical 2-factor authentication token (available at ID card office for free)

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Often you'll see generalized sums:

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Exercises

Try these and then check with a partner:

1. Evaluate: $\sum_{x=1}^{5} x$
2. Solve for x: $50 = 5(x + 2) + 5$
3. Evaluate: $\prod_{x=1}^{3} (x - 1)$

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- 1 Dog $+$ 1 Dog $=$ 2 Dogs!

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- Proportions have no units!

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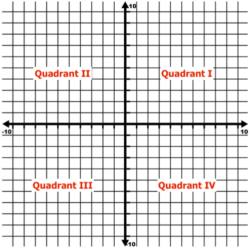
Exercises

1. Evaluate: $(5 \times 4)^2$
2. Evaluate: $((1+3)^3)^2 \times 2 + 5$

Coordinate Plane

There are four **quadrants** on a coordinate plane. The horizontal axis is known as the x-axis and the vertical axis is known as the y-axis.



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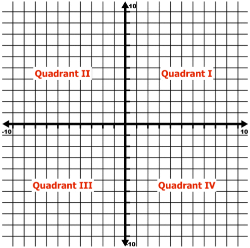

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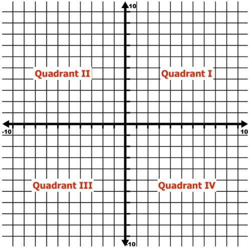

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- **Distance Formula:** The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ in the x-y plane is given by:

$$|P_1 P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Exercises

1. Can you plot the points (1,2) and (3,-2) on the coordinate plane and find the distance between them?
2. Demonstrate that the Pythagorean Theorem and the distance formula are equivalent. Remember, the PT says: in a right triangle, where a and b are the lengths of sides next to the right angle, and c is the length of the hypotenuse, the lengths have the relationship

$$a^2 + b^2 = c^2$$

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  Two lines are parallel if and only if they have the same slope.
  Two lines are perpendicular if and only if their slopes, m_1 and m_2, are negative reciprocals, i.e. $m_1 m_2 = -1$, $m_1 = \frac{-1}{m_2}$

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- To learn more about this, take EDUC400B, EDUC430A, EDUC430B, or equivalent courses in other departments.

Exercises

1. Plot the line through the points $(1, 2)$ and $(4, 3)$.
    1. What is the slope?
    2. What is the intercept?
2. A researcher runs a regression of SAT Math score on age and finds the relationship:

$$SAT_{Math,i} = 30 \times AGE_i$$

    .

    1. What does this relationship mean, in words?
    2. What problems do you see with the results of this regression?

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To find the roots (or zeros) of the quadratic polynomial equation $y = ax^2 + bx + c$, use the **quadratic equation**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercises

1. Find the roots of: $y = x^2 - 6x + 8$

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- **Precise definition:** Let f be a function defined on some open interval that contains the number a (the function need not be defined at a itself). Then we say the limit of $f(x)$ as x approaches a is L and write

$$\lim_{x \to a} f(x) = L$$

  if for every number $\varepsilon > 0$ there is a corresponding number $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - a| < \delta$.

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- **Right-hand limit of** $f(x)$ Writing $\lim_{x\to a^+} f(x) = L$ means that the right-hand limit of $f(x)$ as x approaches a is equal to L if we can make values of $f(x)$ as close to L as we like by taking x to be sufficiently close to a and x greater than a.

Left and Right-handed Limits

- **Left-hand limit of** $f(x)$ Writing $\lim_{x\to a^-} f(x) = L$ means that the left-hand limit of $f(x)$ as x approaches a is equal to L if we can make values of $f(x)$ as close to L as we like by taking x to be sufficiently close to a and x less than a.
- **Right-hand limit of** $f(x)$ Writing $\lim_{x\to a^+} f(x) = L$ means that the right-hand limit of $f(x)$ as x approaches a is equal to L if we can make values of $f(x)$ as close to L as we like by taking x to be sufficiently close to a and x greater than a.
- **Recognize:**

$$\lim_{x\to a} f(x) = L \quad \textbf{if and only if} \lim_{x\to a^-} f(x) = L \textbf{ and } \lim_{x\to a^+} f(x) = L$$

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10. $\lim_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$, where n is a positive integer. Note: If n is even, we assume that $a > 0$.

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11. $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim\limits_{x\to a} f(x)}$, where n is a positive integer. If n is even, we assume that $\lim_{x\to a} f(x) > 0$.

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- **Infinite Limits:** $\lim_{x \to a} f(x) = \infty$ means that values of $f(x)$ can be made arbitrarily large by taking x sufficiently close to a but not equal to a.
- **Limits at Infinity:** Let f be a function defined on some interval (a, ∞). Then $\lim_{x \to \infty} f(x) = L$ means that values of $f(x)$ can be made as close to L as we like by taking x sufficiently large.

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- **Continuous on an interval:** A function is continuous on an interval if it is continuous at every number in the interval.

Exercises

1. Let $y = x$. Does the limit as $x \to 4$ exist? If it exists, find the limit.
2. Let
$$y = \begin{cases} 1, & \text{if } x < 1 \\ x^2, & \text{if } x \geq 1 \end{cases}$$
Does the limit as $x \to 1$ exist? If it exists, find the limit.
3. Let
$$y = \begin{cases} x, & \text{if } x \neq 2 \\ 4, & \text{if } x = 2 \end{cases}$$
Does the limit as $x \to 2$ exist? If it exists, find the limit.

Done for today!

Thank you!
Homework will be posted on GitHub immediately after class.
See you tomorrow!