Problem Set 1: Pre-Calculus

Due: September 4, 2019 SOLUTIONS

Problem 1

Solve for x:

1. $4x + 5 = 17$
   $x = 12$

2. $13 - 3x = 27x - 2$
   $x = \frac{1}{2}$

3. $0 = x^2 - x - 6$
   Use the Quadratic Formula with $a = 1, b = -1, c = -6$:

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{25}}{2}$$

$$x = \{-2, 3\}$$

4. $x^2 = 36$

$$x = \pm 6$$

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Problem 2

1. Find the equation of the line that passes through (9,3) and (4,5).

$$m = \frac{5-3}{4-9} = -\frac{2}{5}$$

   Use point-slope form:

$$y - 5 = -\frac{2}{5}(x - 4)$$

$$y = -\frac{2}{5}x + \frac{33}{5}$$

2. Do the lines $y = x$ and $y = x^2 + 3$ intersect? If so, where?
   No. Plotting both curves in Desmos (or similar) will show this quickly. Alternately, you could set the curves equal to each other:

$$x = x^2 + 3$$

Rearrange:

$$0 = x^2 - x + 3$$

And then using the quadratic formula will provide no real roots, therefore there are no real values of x where the two curves share the same y value.

3. Find the distance between $(1, 2)$ and $(-4, -3)$. Use the distance formula:

$$d = \sqrt{(-4-1)^2 + (-3-2)^2}$$

$$d = \sqrt{50}$$

4. For the function $f(x) = 3x - x^2$, evaluate $f(-2)$.

$$f(-2) = 3(-2) - (-2)^2 = -6 - 4 = -10$$

5. Plot the polynomial $y = -3 + 2x^2$. For what values of x is this function increasing? Decreasing?

   This function is decreasing on the interval $(-\infty, 0)$ and increasing on the interval $(0, \infty)$

Problem 3

Evaluate:

1.

$$\sum_{x=1}^{5} x^2 - x$$

$$0 + 2 + 6 + 12 + 20 = 40$$

2.

$$\sum_{x=0}^{10} (-1)^x (3x + 2)$$

$$2 - 5 + 8 - 11 + 14 - 17 + 20 - 23 + 26 - 29 + 32 = 17$$

3.

$$\sum_{x=1}^{5} 5$$

$$5 + 5 + 5 + 5 + 5 = 25$$

4.

$$\prod_{x=0}^{5} 3x^3 - x$$

$$0 \times \cdots = 0$$

5.

$$\prod_{x=1}^{3} (-1)^x (x - 2x^2)$$

$$1 \times -6 \times 15 = -90$$

6.

$$\prod_{x=1}^{5} 5$$

$$5 \times 5 \times 5 \times 5 \times 5 = 5^5 = 3125$$

Problem 4

Explain in your own words what is meant by the equation

$$\lim_{x \to 2} f(x) = 5$$

Is it possible for this statement to be true and yet have $f(2) = 3$? Explain.

As x gets closer and closer to 2, the value of $f(x)$ gets closer and closer to 5. This can be true even if $f(2) = 3$. As we saw in class, if there is a "hole" in the function at $x = 2$, the limit can still exist even if the value of the function at that point is different.

Problem 5

The **greatest** integer function is defined by $\lfloor x \rfloor$ = the largest integer that is less than or equal to x. (For instance, $\lfloor 4 \rfloor = 4$, $\lfloor 4.8 \rfloor = 4$, $\lfloor \pi \rfloor = 3$, $\lfloor \sqrt{2} \rfloor = 1$, $\lfloor -\frac{1}{2} \rfloor = -1$.) Show that $\lim_{x \to 3} \lfloor x \rfloor$ does not exist. Hint: You may want to draw a graph.

Sidenote: The greatest integer function is also called the *floor* function - it just rounds down to the nearest integer. Contrast this with the *ceiling* function, which just rounds up to the nearest integer.

Graphing the function shows that:

$$\lim_{x \to 3^-} \lfloor x \rfloor = 2$$

While

$$\lim_{x \to 3^+} \lfloor x \rfloor = 3$$

Because

$$\lim_{x \to 3^-} \lfloor x \rfloor \neq \lim_{x \to 3^+} \lfloor x \rfloor$$

We conclude that $\lim_{x \to 3} \lfloor x \rfloor$ does not exist.

Bonus

Solve for n:

$$\sum_{x=1}^{n}(5x^2 - 7x + 1) = 7871$$