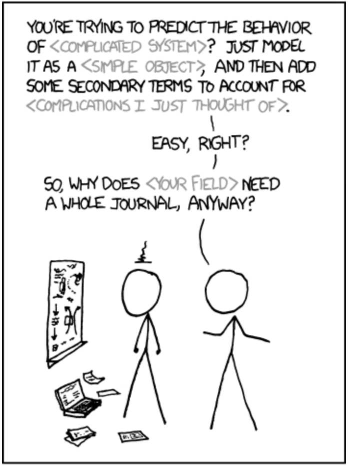
GSE Math Camp Day 1: Software and Pre-Calculus Review

Klint Kanopka

kkanopka@stanford.edu

Tuesday, September 3, 2019

About Me



LIBERAL-ARTS MAJORS MAY BE ANNOYING SOMETIMES, BUT THERE'S *NOTHING* MORE OBNOXIOUS THAN A PHYSICIST FIRST ENCOUNTERING A NEW SUBJECT.

Personal Information

- From Philadelphia
- Taught physics in the Philadelphia School District for 8 years
- 3rd year PhD Student (DAPS)
- Also doing MSCS (AI track)
- Interested in measurement, assessment, and the intersection of AI and psychometrics
- I'm willing to answer almost any question you have (personal or otherwise)

Math Camp Materials

All materials for the first week are hosted at:
`http://github.com/klintkanopka/gsemathcamp`

Outline

1. Software Packages of Interest
   - Document Processing
   - Citation Managers
   - Quantitative Software
   - VPN
2. Review of Algebra and Notation
   - Order of Operations
   - Working with Units
   - Exponents
   - The Coordinate Plane
   - Lines
   - Polynomials
3. Limits
   - Properties

Document Processing

- Microsoft Word (the standard)
- LaTeX(old faithful for scientific papers - and these slides)
- Markdown (newer, least used, but gaining steam)

Citation Managers

These organize your academic papers and help you generate reference lists for papers you're working on.

- Mendeley
- Zotero
- Endnote
- Stanford library and UToronto library both have good websites describing the different options

Quantitative Software

- Microsoft Excel
- SPSS (Statistical Package for Social Science)
- SAS (Statistical Analysis Software)
- Stata
- R
- Python

VPN

Set up Cisco VPN!

- Allows access to journals, data, library materials, networks from off-campus
- Absolutely necessary if you live off campus or travel
- For instructions: uit.stanford.edu/service/vpn
- If you travel overseas often (and plan on working during that time), also consider a physical 2-factor authentication token (available at ID card office for free)

Basic Algebraic Symbols

- Addition $(+)$
- Subtraction $(-)$
- Multiplication $(\times, *)$
- Division $(\div, /)$
- Sum $(\sum)$
- Product $(\prod)$

Sums

$$\sum_{n=1}^{3} n = 1 + 2 + 3 = 6$$

$$N = \{1, 2, 3\}$$

$$\sum_{n \in N} n = 1 + 2 + 3 = 6$$

$$\sum_{i=1}^{3} n_i = 1 + 2 + 3 = 6$$

Often you'll see generalized sums:

$$\sum_{i=1}^{N}$$

Products

$$\prod_{n=1}^{3} n = 1 \times 2 \times 3 = 6$$

$$N = \{1, 2, 3\}$$

$$\prod_{n \in N} n = 1 \times 2 \times 3 = 6$$

$$\prod_{i=1}^{3} n_i = 1 \times 2 \times 3 = 6$$

Often you'll see generalized products:

$$\prod_{i=1}^{N}$$

Exercises

Try these and then check with a partner:

1. Evaluate: $\sum_{x=1}^{5} x$
2. Solve for x: $50 = 5(x+2) + 5$
3. Evaluate: $\prod_{x=1}^{3} (x-1)$

Working with Units

You're only allowed to add, subtract, or compare quantities of the same unit

- 1 English Bulldog $+$ 1 Dachshund $=$??
- 1 Dog $+$ 1 Dog $=$ 2 Dogs!

Working with Units

Units multiply and divide!

- Driving 60 miles per hour for 2 hours takes you 120 miles

$$60\frac{miles}{hour} \times 2hours = 120miles$$

- A rectangular neighborhood that is 2km by 3km has an area of 6 square km

$$2km \times 3km = 6km^2$$

- Proportions have no units!

Exponents

Rules of exponents:

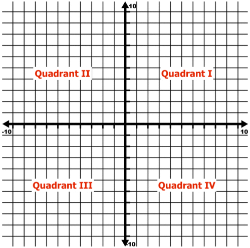
1. $a^n = \underbrace{a \times a \times \cdots \times a}_{n}$
2. $a^0 = 1$
3. $a^{-1} = \frac{1}{a}$
4. $a^{x+y} = a^x a^y$
5. $a^{x-y} = \frac{a^x}{a^y}$
6. $(a^x)^y = a^{xy}$
7. $(ab)^x = a^x b^x$
8. $(\frac{a}{b})^x = \frac{a^x}{b^x}$
9. $a^{\frac{1}{2}} = \sqrt{a}$
10. $\sqrt[x]{a} = a^{\frac{1}{x}}$

Exercises

1. Evaluate: $(5 \times 4)^2$
2. Evaluate: $((1+3)^3)^2 \times 2 + 5$

Coordinate Plane

There are four **quadrants** on a coordinate plane. The horizontal axis is known as the x-axis and the vertical axis is known as the y-axis.



- Points on the coordinate plane are represented as couples of numbers, e.g. $P_1 = (x_1, y_1)$
- **Distance Formula:** The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ in the x-y plane is given by:

$$|P_1 P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Exercises

1. Can you plot the points (1,2) and (3,-2) on the coordinate plane and find the distance between them?
2. Demonstrate that the Pythagorean Theorem and the distance formula are equivalent. Remember, the PT says: in a right triangle, where a and b are the lengths of sides next to the right angle, and c is the length of the hypotenuse, the lengths have the relationship

$$a^2 + b^2 = c^2$$

Lines

- **Slope**: The slope of a line passing through the points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_2} = \frac{rise}{run}$$

- **Equation of a Line:**
  1. **Point-Slope Form:** Given a point, $P_1 = (x_1, y_1)$, and a slope, m, you can plot the line:

  $$y - y_1 = m(x - x_1)$$

  2. **Slope-Intercept Form:** Given a slope, m, and the y-intercept, (0,b), you can plot the line:

  $$y = mx + b$$

  Two lines are parallel if and only if they have the same slope.
  Two lines are perpendicular if and only if their slopes, m_1 and m_2, are negative reciprocals, i.e. $m_1 m_2 = -1$, $m_1 = \frac{-1}{m_2}$

Linear Regression:

- Linear relationships form the basis of much quantitative education research. For a dependent variable, Y, with a single independent variable, X, we can write:

$$Y = \beta_0 + \beta_1 X$$

- We interpret this as "A one unit change in X is associated with a β_1 unit change in Y."
- This works for multiple independent variables, too!
- To learn more about this, take EDUC400B, EDUC430A, EDUC430B, or equivalent courses in other departments.

Exercises

1. Plot the line through the points $(1, 2)$ and $(4, 3)$.
    1. What is the slope?
    2. What is the intercept?
2. A researcher runs a regression of SAT Math score on age and finds the relationship:

    $$SAT_{Math,i} = 30 \times AGE_i$$

    .

    1. What does this relationship mean, in words?
    2. What problems do you see with the results of this regression?

Polynomials

Definition: A polynomial is a mathematical expression of one or more terms, where each term is a constant multiplied by one or more variables each raised to a nonnegative integer power. For example: $ax^2 + bx + c$.

Terminology:

- **Degree:** The degree of a polynomial is the highest degree of its terms. The degree of a term is the sum of the exponents on the variables. For example, the degree of $ax^2 + bx + c$ is 2.
- **Root:** The roots of a polynomial function $f(x)$ are the values of the variable x where $f(x) = 0$.

To find the roots (or zeros) of the quadratic polynomial equation $y = ax^2 + bx + c$, use the **quadratic equation**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercises

1. Find the roots of: $y = x^2 - 6x + 8$

Limits

- **Intuitive definition:** The **limit** of $f(x)$, as x approaches a, equals L if, as we take values closer and closer to a (just a bit bigger and a bit smaller, but not equal to a), we can make the values of $f(x)$ arbitrarily close to L.
- **Precise definition:** Let f be a function defined on some open interval that contains the number a (the function need not be defined at a itself). Then we say the limit of $f(x)$ as x approaches a is L and write

$$\lim_{x \to a} f(x) = L$$

if for every number $\varepsilon > 0$ there is a corresponding number $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - a| < \delta$.

Left and Right-handed Limits

- **Left-hand limit of** $f(x)$ Writing $\lim_{x\to a^-} f(x) = L$ means that the left-hand limit of $f(x)$ as x approaches a is equal to L if we can make values of $f(x)$ as close to L as we like by taking x to be sufficiently close to a and x less than a.
- **Right-hand limit of** $f(x)$ Writing $\lim_{x\to a^+} f(x) = L$ means that the right-hand limit of $f(x)$ as x approaches a is equal to L if we can make values of $f(x)$ as close to L as we like by taking x to be sufficiently close to a and x greater than a.
- **Recognize:**

$$\lim_{x\to a} f(x) = L \quad \textbf{if and only if} \lim_{x\to a^-} f(x) = L \textbf{ and } \lim_{x\to a^+} f(x) = L$$

Limit Laws

Suppose that c is a constant and that the limits
$\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist. Then, the following rules hold:

1. $\lim_{x \to a}[f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
2. $\lim_{x \to a}[f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$
3. $\lim_{x \to a}[cf(x)] = c \lim_{x \to a} f(x)$
4. $\lim_{x \to a}[f(x)g(x)] = \lim_{x \to a} f(x) \times \lim_{x \to a} g(x)$
5. $\displaystyle\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)}$ if $\lim_{x \to a} g(x) \neq 0$
6. $\lim_{x \to a}[f(x)]^n = [\lim_{x \to a} f(x)]^n$, where n is a positive integer.
7. $\lim_{x \to a} c = c$
8. $\lim_{x \to a} x = a$
9. $\lim_{x \to a} x^n = a^n$, where n is a positive integer.
10. $\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$, where n is a positive integer. Note: If n is even, we assume that $a > 0$.
11. $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim\limits_{x \to a} f(x)}$, where n is a positive integer. If n is even, we assume that $\lim_{x \to a} f(x) > 0$.

Limit Properties

- **Direct Substitution Property:** If f is a polynomial or a rational function and a is in the domain of f, then

$$\lim_{x \to a} f(x) = f(a)$$

- **Infinite Limits:** $\lim_{x \to a} f(x) = \infty$ means that values of $f(x)$ can be made arbitrarily large by taking x sufficiently close to a but not equal to a.
- **Limits at Infinity:** Let f be a function defined on some interval (a, ∞). Then $\lim_{x \to \infty} f(x) = L$ means that values of $f(x)$ can be made as close to L as we like by taking x sufficiently large.

Continuity

A function f is continuous at a number a if

$$\lim_{x \to a} f(x) = f(a)$$

Special types of continuity:

- **Continuous from the right:** A function f is continuous *from the right* at a number a if

$$\lim_{x \to a^+} f(x) = f(a)$$

- **Continuous from the left:** A function f is continuous *from the left* at a number a if

$$\lim_{x \to a^-} f(x) = f(a)$$

- **Continuous on an interval:** A function is continuous on an interval if it is continuous at every number in the interval.

Exercises

1. Let $y = x$. Does the limit as $x \to 4$ exist? If it exists, find the limit.
2. Let
$$y = \begin{cases} 1, & \text{if } x < 1 \\ x^2, & \text{if } x \geq 1 \end{cases}$$
Does the limit as $x \to 1$ exist? If it exists, find the limit.
3. Let
$$y = \begin{cases} x, & \text{if } x \neq 2 \\ 4, & \text{if } x = 2 \end{cases}$$
Does the limit as $x \to 2$ exist? If it exists, find the limit.

Done for today!

Thank you!

Homework will be posted on GitHub immediately after class.

See you tomorrow!