Problem Set 2

Calculus

Problem 1: Derivatives

Find the derivative of each expression with respect to x.

(a) $y = 55x$

$$\frac{dy}{dx} = 55$$

(b) $y = \frac{1}{3}x^3$

$$\frac{dy}{dx} = x^2$$

(c) $y = (2x)^3$

$$y = 8x^3$$

$$\frac{dy}{dx} = 24x^2$$

(d) $y = 3x^7 - 20x^6 + 16x^5 - 4x^4 + 17x^3 - 20x^2 + 8x - 76$

$$\frac{dy}{dx} = 21x^6 - 120x^5 + 80x^4 - 16x^3 + 51x^2 - 40x + 8$$

Problem 2: Examining Graphs of Derivatives

For each expression, graph the original expression and the first two derivatives. Use these to identify:

1. The location (x value) of any roots of the original function or the first two derivatives.
2. The location of any local minima or maxima in the original function or first derivative.
3. Intervals on which the function or its first derivative is increasing or decreasing (use interval notation).
4. The location of any inflection points in the original function
5. Intervals on which the original function is concave up or concave down.

(a) $x^4 - 5x^3 + 4x^2 + x + 2$

- Roots (original function): 1.589, 3.863
- Roots (first derivative): -0.104, 0.782, 3.073
- Roots (second derivative): 0.304, 2.196
- Local Extrema (original function): -0.104 (minimum), 0.782 (maximum), 3.073 (minimum)

- Local Extrema (first derivative): 0.304 (maximum), 2.196 (minimum)
- Increasing (original function): $(-0.104, 0.782) \cup (3.073, \infty)$
- Decreasing (original function): $(-\infty, -0.104) \cup (0.782, 3.073)$
- Increasing (first derivative): $(-\infty, 0.304) \cup (2.196, \infty)$
- Decreasing (first derivative): $(0.304, 2.196)$
- Inflection Points (original function): 0.304, 2.196
- Concave Up (original function): $(-\infty, 0.304) \cup (2.196, \infty)$
- Concave Down (original function): $(0.304, 2.196)$

(b) $\tan x$, considering only the interval $[0, 2\pi]$.

- Roots (original function): $0, \pi, 2\pi$
- Roots (first derivative): None
- Roots (second derivative): $0, \pi, 2\pi$
- Local Extrema (original function): None
- Local Extrema (first derivative): $0, \pi, 2\pi$ (all minima)
- Increasing (original function): $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \frac{3\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi]$
- Decreasing (original function): None
- Increasing (first derivative): $(0, \frac{\pi}{2}) \cup (\pi, \frac{3\pi}{2})$
- Decreasing (first derivative): $(\frac{\pi}{2}, \pi) \cup (\frac{3\pi}{2}, 2\pi)$
- Inflection Points (original function): $0, \pi, 2\pi$
- Concave Up (original function): $(0, \frac{\pi}{2}) \cup (\pi, \frac{3\pi}{2})$
- Concave Down (original function): $(\frac{\pi}{2}, \pi) \cup (\frac{3\pi}{2}, 2\pi)$

(c) e^x

- Roots (original function): None
- Roots (first derivative): None
- Roots (second derivative): None
- Local Extrema (original function): None
- Local Extrema (first derivative): None
- Increasing (original function): $(-\infty, \infty)$
- Decreasing (original function): Nowhere
- Increasing (first derivative):$(-\infty, \infty)$
- Decreasing (first derivative): Nowhere
- Inflection Points (original function): None
- Concave Up (original function): $(-\infty, \infty)$
- Concave Down (original function): Nowhere

(d) $\sin x$, considering only the interval $[0, 2\pi]$.

- Roots (original function): $0, \pi, 2\pi$

- Roots (first derivative): $\frac{\pi}{2}, \frac{3\pi}{2}$
- Roots (second derivative): $0, \pi, 2\pi$
- Local Extrema (original function): $\frac{\pi}{2}$ (maximum), $\frac{3\pi}{2}$ (minimum)
- Local Extrema (first derivative): 0 (maximum), π (minimum), 2π (maximum)
- Increasing (original function): $(0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi)$
- Decreasing (original function): $(\frac{\pi}{2}, \frac{3\pi}{2})$
- Increasing (first derivative):$(\pi, 2\pi)$
- Decreasing (first derivative): $(0, \pi)$
- Inflection Points (original function): $0, \pi, 2\pi$
- Concave Up (original function): $(\pi, 2\pi)$
- Concave Down (original function): $(0, \pi)$

Hint: Desmos can be used to easily graph the original experession (in this case, $\sin x$ and the first two derivatives by graphing:

$$\sin x$$
$$\frac{d}{dx} \sin x$$
$$\frac{d}{dx} \frac{d}{dx} \sin x$$

Bonus: Fun Tasks

1. Without looking it up, use Desmos to determine expressions for the first *four* derivatives of $\cos x$, in terms of $\sin x$ and $\cos x$.

2. A piece of wire 40 inches long is cut into two pieces. One is bent into a circle and the other is bent into a square. How should the pieces be cut to *minimize* the total area of the two shapes?