Assignment 1 - Integration

August 25, 2022

[11]:
```
import numpy as np
import matplotlib.pyplot as plt
```

0.1 Question 1

1. How can you determine an integral using Newton-Cotes or Monte Carlo when the domain is infinite? Suggest change of variables to map,

   a. $[-\infty, \infty] \to [-1, 1]$

   b. $[0, \infty] \to [0, 1]$

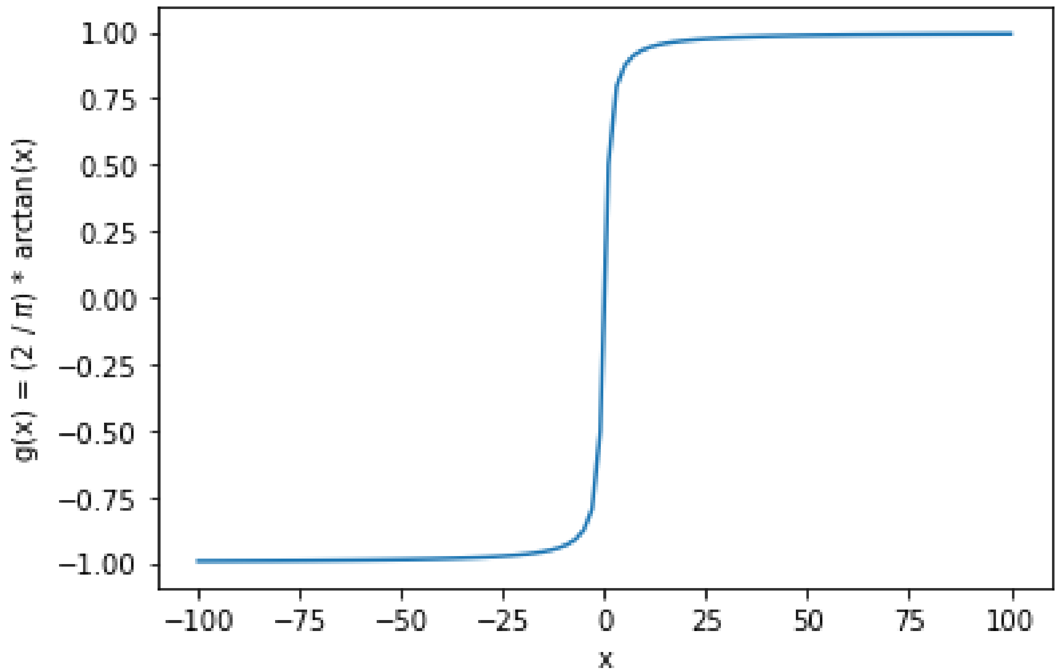
   c. $[0, \infty] \to [-1, 1]$

0.1.1 a. Let u = $\frac{2}{\pi}\arctan(x)$ => x = $\tan\left(\frac{\pi u}{2}\right)$

Need $\frac{\pi}{2}$ to scale properly between [-1,1]

[120]:
```
x1 = np.linspace(-1e2, 1e2, 100)
gx1 = (2 / np.pi) * np.arctan(x1)
plt.plot(x1,gx1)
plt.ylabel("g(x) = (2 / $\pi$) * arctan(x)")
plt.xlabel("x")
plt.show()

gx1_a = (2 / np.pi) * np.arctan(-np.inf)
gx1_b = (2 / np.pi) * np.arctan(np.inf)
print("u(-inf) =", gx1_a)
print("u(inf) = ", gx1_b)
```

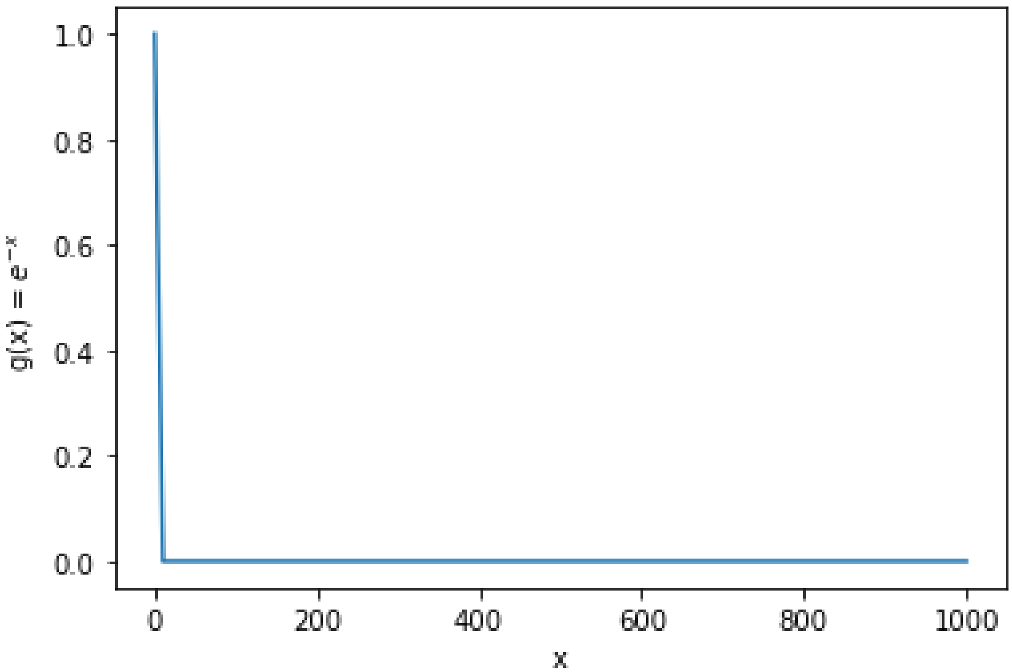



```
u(-inf) = -1.0
u(inf) =  1.0
```

0.1.2 b. Let u = e^{-x} => x = $-\log(u)$

[119]:
```
x2 = np.linspace(0, 1e3, 100)
gx2 = np.exp(-x2)
plt.plot(x2, gx2)
plt.xlabel("x")
plt.ylabel("g(x) = $e^{-x}$")
plt.show()

gx2_a = np.exp(-0)
gx2_b = np.exp(-np.inf)
print("u(0) =", gx2_a)
print("u(inf) =", gx2_b)
```



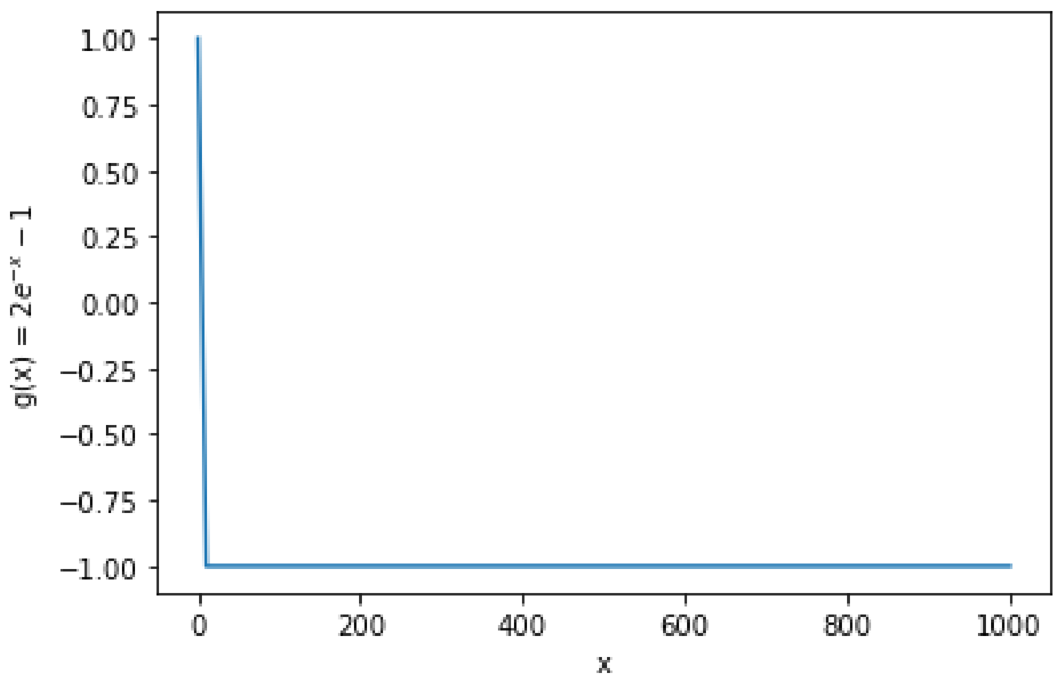

```
u(0) = 1.0
u(inf) = 0.0
```

0.1.3 c. Use a linear transformation on the change of variables from (b.):

```
$y = mu + b$
$y(0) = -1 = m(0) + b \rightarrow b = -1$
$y(1) = 1 = m(1) - 1 \rightarrow m = 2$
$y = 2u - 1 \rightarrow y = 2e^{-x} - 1$
```

[123]:
```
x3 = np.linspace(0, 1e3, 100)
gx3 = 2*np.exp(-x3) - 1
plt.plot(x3, gx3)
plt.xlabel("x")
plt.ylabel("g(x) = $2e^{-x} - 1$")
plt.show()

gx3_a = 2*np.exp(-0) - 1
gx3_b = 2*np.exp(-np.inf) - 1
print("y(0) =", gx3_a)
print("y(inf) =", gx3_b)
```




```
y(0) = 1.0
y(inf) = -1.0
```

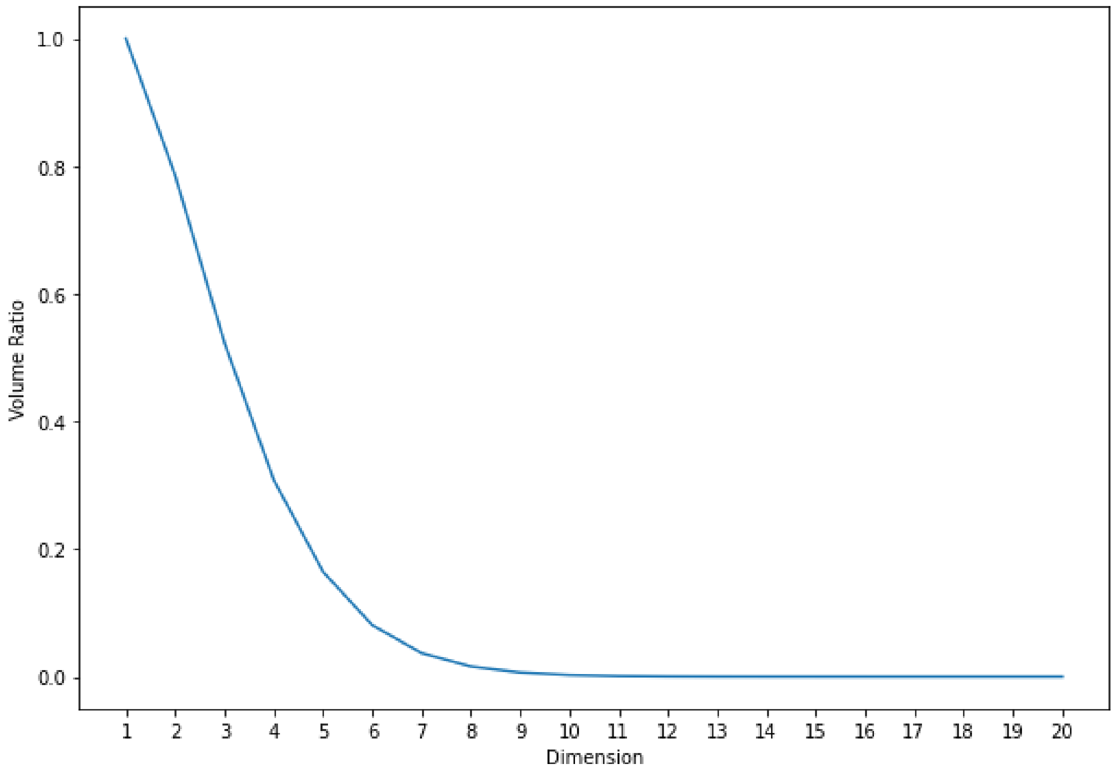
1 Question 2: Direct MC in High Dimensions

##

$\frac{V_S}{V_C} = \frac{\pi^{n/2}}{2^n\Gamma(\frac{n}{2}+1)}$

[140]:
```
from scipy.special import gamma
dims = np.arange(1,21)
ratio = (np.pi**(dims/2)) / ((2**dims)*gamma((dims/2)+1))

plt.figure(figsize=(10,7))
plt.plot(dims, ratio)
plt.xticks(dims)
plt.xlabel('Dimension')
plt.ylabel('Volume Ratio')
plt.show()
```



2 Question 3: Basic Integration

[376]:
```
def averageConeVolume(H, a1, b1, a2, b2, npts):
    x1 = np.random.uniform(a1, b1, size=npts)
    x2 = np.random.uniform(a2, b2, size=npts)
    cond = np.where(x1**2 + x2**2 <= 1, H * (1-np.sqrt(x1**2 + x2**2)), 0)
    return np.mean(cond)
```

[377]:
```
H = 2
trials = 1000
npts = 1000
print("Single-Trial Monte Carlo Volume Estimate:", 4*averageConeVolume(H, -1,␣
 ↪1, -1, 1, npts))
print(str(trials)+"-Trial Monte Carlo Mean Volume Estimate", np.
 ↪mean([4*averageConeVolume(H, -1, 1, -1, 1, 1000) for i in range(100)]))
print('True Volume:', np.pi*H/3)
```

```
Single-Trial Monte Carlo Volume Estimate: 2.0873628482363547
1000-Trial Monte Carlo Mean Volume Estimate 2.092436209833027
True Volume: 2.0943951023931953
```