

# Markov Chain Monte Carlo

## Homework

### MC Integration

1. How can you determine an integral using Newton-Cotes or Monte Carlo when the domain is infinite? Suggest change of variables to map,

a.  $[-\infty, \infty] \rightarrow [-1, 1]$

b.  $[0, \infty] \rightarrow [0, 1]$

c.  $[0, \infty] \rightarrow [-1, 1]$

2. **Direct MC in High Dimensions**

The volume of a  $n$ -dimensional hypercube is  $V_C = L^n$  (think line  $\rightarrow$  square  $\rightarrow$  cube).

The volume of a  $n$  dimensional hypersphere of radius  $R$  is

$$V_S = \frac{\pi^{n/2}}{\Gamma(\frac{n}{2} + 1)} R^n,$$

where  $\Gamma(x)$  is the “gamma function”. Consider finding the volume of a hypersphere of radius  $R$  by throwing darts inside a hypercube (in principle, this is far simpler than it sounds!) of edge length  $L = 2R$ .<sup>1</sup> The fraction of darts that land in the sphere is,

$$\frac{V_S}{V_C} = \frac{\pi^{n/2}}{2^n \Gamma(\frac{n}{2} + 1)}.$$

Plot this ratio as a function of  $n$  to understand why MC might not be a silver bullet.

3. **Basic Integration**

Use Monte Carlo integration to find the volume of a right circular cone of height  $H = 2$ , whose base is a unit circle centered at  $(0,0)$ .

That is compute the integral,

$$I = \int_{-1}^1 \int_{-1}^1 g(x_1, x_2) dx_1 dx_2,$$

where,

$$g(x_1, x_2) = \begin{cases} H(1 - \sqrt{x_1^2 + x_2^2}), & \text{if } x_1^2 + x_2^2 \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

Use a suitable number of points. The true volume is  $V = \pi H/3$ .

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<sup>1</sup>You don't have to do an MC simulation for this question.