Markov Chain Monte Carlo

Homework

RNG and Sampling 1D Distributions

1. **LCG Random Number Generator** (from 2021 quiz)

   Consider a linear congruential generator random number generator (RNG) to sample uniform random numbers ($u_i = n_i/m$). Typically a is an odd integer, and $m = 2^k$, where $k \sim 32$ or 64 for single and double precision numbers, respectively.

   $$n_{i+1} = (an_i) \mod m. \tag{1}$$

   Assume that the initial seed $n_0 = 1$ for all the problems below. You may use a computer, but report all requested work on your answer sheet.

   (i) Suppose $m = 2^k$ with $k = 3$.
      - (a) Generate and report the first 10 random numbers $n_1, n_2, \cdots, n_{10}$ using $a = 2$.
      - (b) Repeat the calculation for any other even $a > 2$.
      - (c) Summarize your observations: what happens to the sequence when a is even?

   (ii) Thus, it might be advisable to set a = odd number.
      - (a) Generate and report the first 10 random numbers $n_1, n_2, \cdots, n_{10}$ using $a = 3$.
      - (b) Repeat the calculation for any other odd $a > 3$.
      - (c) What is the period (length of non-repeating sequence) of the RNG?

   (iii) What is the period for $a = 3$ and $m = 2^4$, and $m = 2^5$?

   (iv) Claim: "the period of a LCG RNG with $m = 2^k$ and odd a is 2^{k-2}". Do your numerical experiments support this claim?

   (v) Based on the claim above, what is the period of the RANDU RNG? Is it more than 1 billion?

2. **Sampling 1D Distribution**: Consider the distribution

   $$f(x) = 6x(1-x), \quad 0 \leq x \leq 1 \tag{2}$$

   Generate $\approx 10^5$ samples from this distribution using all different methods suggested below. Test your solution by plotting the histogram of the samples and $f(x)$.

   (a) the accept-reject method with an appropriate $f_{\max}$.

   (b) the transformation method: you can:
      - a. numerically solve any nonlinear equation that may arise, *or*
      - b. use "inverse linear interpolation"

   Inverse Linear Interpolation:

   - Generate a F versus x table, $\{F_i = F(x_i), x_i\}$, where $F(x)$ is the CDF

- Generate a uniform random number $u \sim U[0, 1]$
- Use the table and linear interpolation to find the x corresponding to $F = u$