FLORIDA STATE UNIVERSITY
MAD 5403-0001

Fall Semester	Foundations of Computational Mathematics I	2022

HW 2

Due Date Oct 21st, 2022
Points 100 pts

1. Following is often referred as the equivalence of the norms. For any square matrix $A \in \mathbb{R}^{n\times n}$, prove the following relations:

   (a)
   $$\frac{1}{n}K_2(A) \leq K_1(A) \leq nK_2(A)$$

   (b)
   $$\frac{1}{n^2}K_1(A) \leq K_\infty(A) \leq n^2 K_1(A)$$

2. Let A be a sparse matrix of order n. Prove that the computational cost of the LU factorization of A is given by
   $$\frac{1}{2}\sum_{k=1}^{n} l_k(A)(l_k(A)+3) \text{ flops },$$
   where $l_k(A)$ is the number of active rows at the $k-$th step of the factorization (i.e, the number of rows of A with $i > k$ and $a_{ik} \neq 0$, and having accounted for all the nonzero entires.

3. (Extra Credit 5pts) Prove. If A is strictly diagonally dominant matrix by rows, then the Jacobi and Gauss-Seidel are convergent.

4. (Extra Credit 5pts) Will
   $$A = \begin{bmatrix} 4 & 0 & 0 & -1 \\ -1 & 4 & -1 & 0 \\ 0 & -1 & 4 & 0 \\ -1 & 0 & 0 & 4 \end{bmatrix}$$
   converge with Jacobi and Gauss-Seidel?