Homework: MPI for 2D Heat Equation

In this homework, we are going to solve the 2D Heat Equation by MPI, with part of the coding introduced in class.

Suppose in a thin rectangle of dimension $[0,3] \times [0,4]$, there is temperature $H(x,y,t)$, which satisfies the following Heat Equation:

$$\frac{\partial H}{\partial t} - \left(\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2}\right) = 0$$

The time $t \in (0,T)$. Suppose at time $t = 0$, $H(x,y,0) = 0$ for all x,y. And the boundary condition: Except on the bottom, the temperature keeps at 0, i.e., $H(x, y = 0, t) = 0$, on all other three edges, the temperature is fixed at 1. i.e., $H(x = 0, y, t) = H(x = 3, y, t) = H(x, y = 4, t) = 1.0$. (Two corners, $H(0,0,t) = H(3,0,t) = 0$)

Please write an MPI code to solve this equation. You can select your grid size, for example, 100×100. Also, select a small time step, say 0.0001 to ensure the stability.

Output the temperature $H(x, y)$ of all grid points at a time $T = 20$ to a text file. (Different process can output the solution into different files.)

Optional: One can use Matlab or Python to read the text files and draw the graph.

Try different number of processes, and report the scaling of your MPI code.