

Príklad 1. ZA, $a^n b^n$

Zadanie:

■ Navrhňte zásobníkový automat ZA pre jazyk $L = \{ a^n b^n \mid n \in \mathbb{N} \}$

Riešenie:

$$TM = (K, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

$$K = \{q_0, q_1, q_2\}$$

$$\Sigma = \{a, b\}$$

$$\Gamma = \{Z, Z_0\}$$

$$F = \{q_0, q_3\}$$

$$\delta(q_0, a, Z_0) = (q_1, ZZ_0) \quad \text{push}(Z)$$

$$\delta(q_1, a, Z) = (q_1, ZZ) \quad \text{push}(Z)$$

$$\delta(q_1, b, Z) = (q_2, \varepsilon) \quad \text{pop}$$

$$\delta(q_2, b, Z) = (q_2, \varepsilon) \quad \text{pop}$$

$$\delta(q_2, \varepsilon, Z_0) = (q_3, Z_0) \quad \text{skip}$$

Príklad 2. ZA, wcw^R

Zadanie:

- Navrhните zásobníkový automat ZA pre jazyk $L = \{ wcw^R \mid w \in \{a, b\}^* \}$

Riešenie:

$$TM = (K, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

$$K = \{q_0, q_1, q_2\}$$

$$\Sigma = \{a, b, c\}$$

$$\Gamma = \{Z_0, Z_a, Z_b\}$$

$$F = \{q_2\}$$

$$\delta(q_0, a, Z) = (q_0, Z_a Z) \quad Z \in \Gamma \quad \text{push}(Z_a)$$

$$\delta(q_0, b, Z) = (q_0, Z_b Z) \quad \text{push}(Z_b)$$

$$\delta(q_0, c, Z) = (q_1, Z) \quad \text{skip}$$

$$\delta(q_1, a, Z_a) = (q_1, \varepsilon) \quad \text{pop, ak top} = Z_a$$

$$\delta(q_1, b, Z_b) = (q_1, \varepsilon) \quad \text{pop, ak top} = Z_b$$

$$\delta(q_1, \varepsilon, Z_0) = (q_2, Z_0) \quad \text{akceptuje}$$

Príklad 3. ZA, $c^{3n}b^{2n}$

Zadanie:

■ Navrhňte zásobníkový automat ZA pre jazyk $L = \{ c^{3n}b^{2n} \mid n \in \mathbb{N} \}$

Riešenie:

$$TM = (K, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

$$K = \{q_0, q_1, q_2, q_3, q_4, q_5\}$$

$$\Sigma = \{b, c\}$$

$$\Gamma = \{Z, Z_0\}$$

$$F = \{q_0, q_5\}$$

$$\delta(q_0, c, Z_0) = (q_1, ZZ Z_0) \quad \text{push}(ZZ)$$

$$\delta(q_1, c, Z) = (q_1, ZZZ) \quad \text{push}(ZZ)$$

$$\delta(q_1, b, Z) = (q_2, \varepsilon) \quad \text{1. pop z 1. b}$$

$$\delta(q_2, \varepsilon, Z) = (q_3, \varepsilon) \quad \text{2. pop}$$

$$\delta(q_3, \varepsilon, Z) = (q_4, \varepsilon) \quad \text{3. pop}$$

$$\delta(q_4, b, Z) = (q_2, \varepsilon) \quad \text{1. pop z ďalších b}$$

$$\delta(q_4, \varepsilon, Z_0) = (q_5, Z_0) \quad \text{akcept.}$$

Príklad 4. ZA, $w = uv \mid 2|u| = 3|v|$

Zadanie:

- Navrhните zásob. automat ZA pre jazyk $L = \{w = uv \mid 2|u| = 3|v| \mid w \in \{a, b\}^*, v \in \{c, d\}^*\}$

Riešenie:

$$TM = (K, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

$$K = \{q_0, q_1, q_2, q_3, q_4\}$$

$$\Sigma = \{a, b, c, d\}$$

$$\Gamma = \{Z, Z_0\}$$

$$F = \{q_0, q_5\}$$

slovo u

$$\delta(q_0, a, Z_0) = (q_1, ZZ Z_0) \quad \text{push}(ZZ)$$

$$\delta(q_0, b, Z_0) = (q_1, ZZ Z_0) \quad \text{push}(ZZ)$$

$$\delta(q_1, a, Z) = (q_1, ZZ Z) \quad \text{push}(ZZ)$$

$$\delta(q_1, b, Z) = (q_1, ZZ Z) \quad \text{push}(ZZ)$$

prechod

$$\left. \begin{array}{l} \delta(q_1, c, Z) = (q_2, \varepsilon) \\ \delta(q_1, d, Z) = (q_2, \varepsilon) \end{array} \right\} \quad 1. \text{ pop (1.)}$$

slovo v

$$\left. \begin{array}{l} \delta(q_2, c, Z) = (q_3, \varepsilon) \\ \delta(q_2, d, Z) = (q_3, \varepsilon) \end{array} \right\} \quad 2. \text{ pop}$$

$$\left. \begin{array}{l} \delta(q_3, c, Z) = (q_4, \varepsilon) \\ \delta(q_3, d, Z) = (q_4, \varepsilon) \end{array} \right\} \quad 3. \text{ pop}$$

$$\left. \begin{array}{l} \delta(q_4, c, Z) = (q_2, \varepsilon) \\ \delta(q_4, d, Z) = (q_2, \varepsilon) \end{array} \right\} \quad 1. \text{ pop (každý ďalší 1.)}$$

$$\delta(q_4, \varepsilon, Z_0) = (q_5, Z_0) \quad \text{akcept.}$$

Vysvetlenie:

- keď ide slovo u(Za, b), tak volím push(ZZ) na každý vstupný symbol
- keď príde slovo v(Zc, d), čiže c alebo d začnem vyberať a vyberám po 3 zásobníkové symboly cez 3 stavy

Príklad 5. ZĀ, $w = uv \mid \#(u = \#)v$

Zadanie:

- Navrhňte zásob. automat ZĀ pre jazyk $L = \{w = uv \mid \#(u = \#)v \mid w \in \{(\,, a\}^*, v \in \{a, \,)\}^*\}$

Riešenie:

$$TM = (K, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

$$K = \{q_0, q_1, q_2, q_3\}$$

$$\Sigma = \{(\,, a\}$$

$$\Gamma = \{Z, Z_0\}$$

$$F = \{q_0, q_3\}$$

$\delta(q_0, a, Z_0) = (q_0, Z_0)$	skip
$\delta(q_0, (\,, Z_0) = (q_1, ZZ_0)$	push(Z)
$\delta(q_1, (\,, Z) = (q_1, ZZ)$	push(Z)
$\delta(q_1, a, Z) = (q_1, Z)$	skip
$\delta(q_1, \,), Z) = (q_2, \varepsilon)$	pop
$\delta(q_2, \,), Z) = (q_2, \varepsilon)$	pop
$\delta(q_2, a, Z) = (q_2, Z)$	skip
$\delta(q_2, a, Z_0) = (q_2, Z_0)$	skip
$\delta(q_2, \varepsilon, Z_0) = (q_3, Z_0)$	akcept.

Príklad 6. ZA, $w \in \{(,)\}^*$ je správne uzátvorkovaný vstup

Zadanie:

- Navrhňte zásobníkový automat ZA pre jazyk $L = \{ w \in \{(,)\}^* \mid w \text{ je správne uzátvorkovaný vstup} \}$

Riešenie:

$$TM = (K, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

$$K = \{q_0, q_1\}$$

$$\Sigma = \{(,)\}$$

$$\Gamma = \{Z, Z_0\}$$

$$F = \{q_0\}$$

$$\delta(q_0, (, Z_0) = (q_1, ZZ_0) \quad \text{push}(Z)$$

$$\delta(q_1, (, Z) = (q_1, ZZ) \quad \text{push}(Z)$$

$$\delta(q_1,), Z) = (q_1, \varepsilon) \quad \text{pop}$$

$$\delta(q_1, \varepsilon, Z_0) = (q_0, Z_0) \quad \text{skip, akcept.}$$

Príklad 7. $\text{ZA}, w = a^n w \mid \#_c w = n$

Zadanie:

■ Navrhните zásob. automat ZA pre jazyk $L = \{w = a^n w \mid w \in \{b, c\}^*, \#_c w = n, n \in \mathbb{N}^+\}$

Riešenie:

$$TM = (K, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

$$K = \{q_0, q_1, q_2, q_3\}$$

$$\Sigma = \{a, b, c\}$$

$$\Gamma = \{Z_0, Z_1, Z_2\}$$

$$F = \{q_0, q_3\}$$

$$\delta(q_0, a, Z_0) = (q_1, Z_1 Z_0)$$

$$\delta(q_1, a, Z_1) = (q_1, Z_2 Z_1)$$

$$\delta(q_1, a, Z_2) = (q_1, Z_2 Z_2)$$

$$\delta(q_1, b, Z_2) = (q_2, Z_2)$$

$$\delta(q_1, c, Z_2) = (q_2, \varepsilon)$$

$$\delta(q_1, b, Z_1) = (q_2, Z_1)$$

$$\delta(q_1, c, Z_1) = (q_3, \varepsilon)$$

$$\delta(q_2, b, Z_2) = (q_2, Z_2)$$

$$\delta(q_2, c, Z_2) = (q_2, \varepsilon)$$

$$\delta(q_2, b, Z_1) = (q_2, Z_1)$$

$$\delta(q_2, c, Z_1) = (q_3, \varepsilon)$$

$$\delta(q_3, b, Z_0) = (q_3, Z_0)$$

Príklad 8. ZA, $a^k b^l \mid k \geq l$

Zadanie:

- Navrhните zásobníkový automat ZA pre jazyk $L = \{ a^k b^l \mid k \geq l, k, l \in \mathbb{N} \}$

Riešenie:

$$TM = (K, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

$$K = \{q_0, q_1\}$$

$$\Sigma = \{a, b\}$$

$$\Gamma = \{Z, Z_0\}$$

$$F = \{q_0, q_1\}$$

$$\delta(q_0, a, Z_0) = (q_0, ZZ_0)$$

$$\delta(q_0, a, Z) = (q_0, ZZ)$$

$$\delta(q_0, b, Z) = (q_1, \varepsilon)$$

$$\delta(q_1, b, Z) = (q_1, \varepsilon)$$

Príklad 9. ZA, $\#_a w = \#_b w$

Zadanie:

■ Navrhňte zásobníkový automat ZA pre jazyk $L = \{ w \in \{a, b\}^* \mid \#_a w = \#_b w \}$

Riešenie:

$$TM = (K, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

$$K = \{q_0, q_1\}$$

$$\Sigma = \{a, b\}$$

$$\Gamma = \{A, B, Z_0\}$$

$$F = \{q_0\}$$

$$\delta(q_0, a, Z_0) = (q_1, AZ_0) \quad \text{push}(A)$$

$$\delta(q_0, b, Z_0) = (q_1, BZ_0) \quad \text{push}(B)$$

$$\delta(q_1, a, A) = (q_1, AA) \quad \text{push}(A)$$

$$\delta(q_1, b, B) = (q_1, BB) \quad \text{push}(B)$$

$$\delta(q_1, a, B) = (q_1, \varepsilon) \quad \text{pop}$$

$$\delta(q_1, b, A) = (q_1, \varepsilon) \quad \text{pop}$$

$$\delta(q_1, \varepsilon, Z_0) = (q_0, Z_0) \quad \text{skip, akcept.}$$

Príklad 10. ZA, ww^R

Zadanie:

- Navrhňte zásobníkový automat ZA pre jazyk $L = \{ ww^R \mid w \in \{a, b\}^+ \}$

Riešenie:

$$TM = (K, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

$$K = \{q_0, q_1, q_2\}$$

$$\Sigma = \{a, b\}$$

$$\Gamma = \{Z_0, Z_a, Z_b\}$$

$$F = \{q_2\}$$

číta w

$$\delta(q_0, a, Z_0) = (q_0, Z_a Z_0)$$

$$\delta(q_0, b, Z_0) = (q_0, Z_b Z_0)$$

$$\delta(q_0, a, Z_a) = \{(q_0, Z_a Z_a), (q_1, \varepsilon)\}$$

$$\delta(q_0, b, Z_b) = \{(q_0, Z_b Z_b), (q_1, \varepsilon)\}$$

$$\delta(q_0, a, Z_b) = (q_0, Z_a Z_b)$$

$$\delta(q_0, b, Z_a) = (q_0, Z_b Z_a)$$

číta w^R

$$\delta(q_1, a, Z_a) = (q_1, \varepsilon)$$

$$\delta(q_1, b, Z_b) = (q_1, \varepsilon)$$

koniec

$$\delta(q_1, \varepsilon, Z_0) = (q_2, Z_0)$$

Príklad 11. ZA, prevod $G \rightarrow PDA$

Zadanie:

- Navrhните zásobníkový automat PDA pre jazyk G v Greibachovej normálnom tvare.

$$G = \{N, T, P, S\}$$

$$N = \{S, A\}$$

$$T = \{a, b\}$$

$$P = \{$$

$$S \rightarrow aAA \mid a$$

$$S \rightarrow aSA \mid bS \mid b$$

$$\}$$

Riešenie:

$$PDA = (K, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

$$K = \{q_0, q_1, q_F\}$$

$$\Sigma = \{a, b\}$$

$$\Gamma = \{Z_0, S, A\}$$

$$F = \{q_F\}$$

$$\delta(q_0, \varepsilon, Z_0) = (q_1, SZ_0)$$

$$\delta(q_1, a, S) = \{(q_1, AA), (q_1, \varepsilon)\}$$

$$\delta(q_1, a, A) = (q_1, SA)$$

$$\delta(q_1, b, A) = \{(q_1, S), (q_1, \varepsilon)\}$$

$$\delta(q_1, \varepsilon, Z_0) = (q_F, Z_0)$$