高一銜接第二單元練習題詳解

類題 P4

1.

$(a-1)(a+1)(a^2-a+1)(a^2+a+1)=[(a-1)(a^2+a+1)][(a+1)(a^2-a+1)]$

$=(a^3-1)(a^3+1)=(2-1)(2+1)=3$

2.

$x^3+\dfrac{1}{x^3}=(x+\dfrac{1}{x})^3-3(x+\dfrac{1}{x})=3^3+3\times3=27-9=18$

3.

(1) $(a+b-c+d)(a+b+c-d)=[(a+b)-(c-d)][(a+b)+(c-d)]$

$=(a+b)^2-(c-d)^2=a^2+2ab+b^2-(c^2-2cd+d^2)$

$=a^2+2ab+b^2-c^2+2cd-d^2$

(2)

利用公式

$x^n-1=(x-1)(x^{n-1}+x^{n-2}+\cdots+x+1)$

$x^n+1=(x+1)(x^{n-1}-x^{n-2}+\cdots-x+1)$（$n$ 為奇數）

$\therefore (x-1)(x^5+x^4+x^3+x^2+x+1)=x^6-1$

4.

(1) $x^4-3x^2y^2+y^4=x^4-2x^2y^2+y^4-x^2y^2=(x^2-y^2)^2-(xy)^2$

$=(x^2-y^2+xy)(x^2-y^2-xy)=(x^2+xy-y^2)(x^2-xy-y^2)$

(2) $[(x^2-2x)+5][(x^2-2x)-7]-13$

$=(x^2-2x)^2-2(x^2-2x)-48$

$=(x^2-2x-8)(x^2-2x+6)$

$=(x+2)(x-4)(x^2-2x+6)$

(3) $(x^2+14x+24)(x^2+11x+24)-4x^2$

$=[(x^2+24)+14x][(x^2+24)+11x]-4x^2$

$=(x^2+24)^2+25x(x^2+24)+150x^2$

$=(x^2+24+10x)(x^2+24+15x)$

$=(x^2+10x+24)(x^2+15x+24)$

$=(x+4)(x+6)(x^2+15x+24)$

5.

$2^{24}-1=(2^{12})^2-1^2=(2^{12}+1)(2^{12}-1)$

$=[(2^4)^3+1](2^6+1)(2^6-1)$

$=(16^3+1)(64+1)(64-1)$

$=(16+1)(16^2-16+1)\times65\times63$

$=17\times241\times65\times63$

$\because$恰有一正整數介於 240 與 250 之間　$\therefore$所以必為 241

類題 P7

1.

(1) $\dfrac{1}{x}=\dfrac{1}{2-\sqrt{3}}=2+\sqrt{3}$

(2) $x+\dfrac{1}{x}=2-\sqrt{3}+\dfrac{1}{2-\sqrt{3}}=2-\sqrt{3}+2+\sqrt{3}=4$

(3) $x-\dfrac{1}{x}=2-\sqrt{3}-\dfrac{1}{2-\sqrt{3}}=2-\sqrt{3}-(2+\sqrt{3})=-2\sqrt{3}$

(4) $x^2+\dfrac{1}{x^2}=(x+\dfrac{1}{x})^2-2=4^2-2=14$

(5) $x^2-\dfrac{1}{x^2}=(x+\dfrac{1}{x})(x-\dfrac{1}{x})=4\cdot(-2\sqrt{3})=-8\sqrt{3}$

(6) $x^3+\dfrac{1}{x^3}=(x+\dfrac{1}{x})^3-3(x+\dfrac{1}{x})=4^3-3\times4=64-12=52$

2.

(1) $\dfrac{\sqrt{5}}{\sqrt{3}}+\dfrac{\sqrt{3}}{\sqrt{5}}=\dfrac{\sqrt{15}}{3}+\dfrac{\sqrt{15}}{5}=\dfrac{5\sqrt{15}+3\sqrt{15}}{15}=\dfrac{8\sqrt{15}}{15}$

(2) $\dfrac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}}-\dfrac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}+\dfrac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$

$=\dfrac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{3}-\dfrac{4\sqrt{3}(\sqrt{6}-\sqrt{2})}{4}+\dfrac{\sqrt{6}(\sqrt{3}-\sqrt{2})}{1}$

$=2\sqrt{3}-\sqrt{6}-3\sqrt{2}+\sqrt{6}+3\sqrt{2}-2\sqrt{3}=0$

(3) $\dfrac{1}{1-\dfrac{\sqrt{2}}{2}}=\dfrac{2}{2-\sqrt{2}}=\dfrac{2(2+\sqrt{2})}{2}=2+\sqrt{2}$

3.

$a^2=(2\sqrt{3}+\sqrt{3})^2=11+4\sqrt{6}=11+2\sqrt{24}$

$b^2=(\sqrt{7}+2)^2=11+4\sqrt{7}=11+2\sqrt{28}$

$c^2=(\sqrt{6}+\sqrt{5})^2=11+2\sqrt{30}$

$\because c^2>b^2>a^2$ 且 a,b,c 均為正數 $\Rightarrow c>b>a$

4.

(1) $\sqrt{11+2\sqrt{18}}=\sqrt{(9+2)+2\sqrt{9\times2}}=\sqrt{(\sqrt{9}+\sqrt{2})^2}=\sqrt{9}+\sqrt{2}=3+\sqrt{2}$

(2) $\sqrt{12-4\sqrt{5}}=\sqrt{12-2\sqrt{20}}=\sqrt{(10+2)-2\sqrt{10\times2}}=\sqrt{(\sqrt{10}-\sqrt{2})^2}=\sqrt{10}-\sqrt{2}$

(3) $\sqrt{5-\sqrt{24}}=\sqrt{5-2\sqrt{6}}=\sqrt{(3+2)-2\sqrt{3\times 2}}=\sqrt{(\sqrt{3}-\sqrt{2})^2}=\sqrt{3}-\sqrt{2}$

(4) $\sqrt{2+\sqrt{3}}=\sqrt{\dfrac{8+4\sqrt{3}}{4}}=\dfrac{\sqrt{8+2\sqrt{12}}}{2}=\dfrac{\sqrt{(6+2)+2\sqrt{6\times 2}}}{2}=\dfrac{\sqrt{(\sqrt{6}+\sqrt{2})^2}}{2}=\dfrac{\sqrt{6}+\sqrt{2}}{2}$

5.

$\sqrt{4+\sqrt{3}}=\sqrt{3}+1\approx 2.\sim$

$\therefore a=2 \Rightarrow b=\sqrt{3}+1-2=\sqrt{3}-1$

$\dfrac{1}{b}-\dfrac{1}{a+b}=\dfrac{1}{\sqrt{3}-1}-\dfrac{1}{2+\sqrt{3}-1}=\dfrac{1}{\sqrt{3}-1}-\dfrac{1}{\sqrt{3}+1}=\dfrac{\sqrt{3}+1}{2}-\dfrac{\sqrt{3}-1}{2}=\dfrac{2}{2}=1$

自我練習題

1.

(1) $(2a-3b)^2$

$=(2a)^3-3\cdot(2a)^2\cdot(3b)+3\cdot(2a)\cdot(3b)^2-(3b)^3$

$=8a^2-36a^2b+54ab^2-27b^3$

(2) $(a-2b+3c)^2$

$=a^2+(-2b)^2+(3c)^2+2[a\cdot(-2b)+(-2b)\cdot(3c)+a\cdot(3c)]$

$=a^2+4b^2+9c^2-4ab-12bc+6ac$

(3) $(a+b-2)(a-b+2)$

$=[a+(b-2)][a-(b-2)]$

$=a^2-(b-2)^2=a^2-(b^2-4b+4)=a^2-b^2+4b-4$

(4) $(a-5)(a+5)(a^2+5a+25)(a^2-5a+25)$

$=[(a-5)(a^2+5a+25)][(a+5)(a^2-5a+25)]$

$=(a^3-5^3)(a^3+5^3)$

$=(a^3)^2-(5^3)^2=a^6-5^6=a^6-15625$

2.

(1) $2024.5^2=(2024+0.5)^2$

$=2024^2+2\times 2024\times 0.5+0.5^2$

$=2024^2+2024+0.25=2024^2+2024.25 \quad \therefore x=2024.25$

(2) $(4-1)(4+1)(4^2+1)(4^4+1)(4^8+1)$

$=(4^2-1)(4^2+1)(4^4+1)(4^8+1)$

$=(4^4-1)(4^4+1)(4^8+1)$

$=(4^8-1)(4^8+1)=4^{16}-1=2^{32}-1 \quad \therefore y=32$

3.

(1) $a^2+b^2=(a+b)^2-2ab$

$\Rightarrow 14=4^2-2ab$

$\Rightarrow 2ab=2$

$\Rightarrow ab=1$

(2) $a^3+b^3=(a+b)^3-3ab(a+b)$

$=4^3-3\times 1\times 4=64-12=52$

(2) a^4+b^4

$=(a^2)^2+(b^2)^2$

$=(a^2+b^2)^2-2a^2b^2$

$=14^2-2\times 1^2$

$=196-2=194$

4.

(1) $x^2+\dfrac{1}{x^2}=(x+\dfrac{1}{x})^2-2$

$\Rightarrow (x+\dfrac{1}{x})^2=9 \Rightarrow x+\dfrac{1}{x}=\pm 3$ (負不合　$\because x>0$)

$\therefore x+\dfrac{1}{x}=3$

(2) $x^2+\dfrac{1}{x^2}=(x-\dfrac{1}{x})^2+2$

$\Rightarrow (x-\dfrac{1}{x})^2=5$

$\Rightarrow x-\dfrac{1}{x}=\pm\sqrt{5}$ (正不合　$\because 0<x<1 \Rightarrow \dfrac{1}{x}>x$)

$\therefore x-\dfrac{1}{x}=-\sqrt{5}$

6.

(1) $\dfrac{1}{x^2-3x+2}+\dfrac{1}{x^2-5x+6}-\dfrac{2}{x^2-4x+3}$

$=\dfrac{1}{(x-2)(x-1)}+\dfrac{1}{(x-2)(x-3)}-\dfrac{2}{(x-3)(x-1)}$

$=\dfrac{x-3+x-1-2(x-2)}{(x-1)(x-2)(x-3)}=\dfrac{0}{(x-1)(x-2)(x-3)}=0$

(2) $\dfrac{2x^2+3x+1}{x^2-1}-\dfrac{2x-1}{x+1}$

$=\dfrac{2x^2+3x+1-(x-1)(2x-1)}{(x+1)(x-1)}$

$=\dfrac{6x}{(x+1)(x-1)}=\dfrac{6x}{x^2-1}$

7.

$$x=\frac{-(\sqrt{5}+\sqrt{3})}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}=\frac{-(8+2\sqrt{15})}{2}=-4-\sqrt{15}$$

$$y=\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}=\frac{8-2\sqrt{15}}{2}=4-\sqrt{15}$$

$$\therefore x-y=-8 \qquad xy=-(4+\sqrt{15})(4-\sqrt{15})=-1$$

$$5x^2+5y^2+6xy=5(x^2+y^2)+6xy=5[(x-y)^2+2xy]+6xy$$
$$=5(x-y)^2+16xy=5\times(-8)^2-16=304$$

8.

(1) $x=\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}\times\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}=\frac{12-2\sqrt{35}}{2}=6-\sqrt{35}$

$$\frac{1}{x}=\frac{1}{6-\sqrt{35}}=6+\sqrt{35}$$

$$\therefore x+\frac{1}{x}=12$$

(2) $x^2+\frac{1}{x^2}=(x+\frac{1}{x})^2-2=12^2-2=142$

(3) $x^3+\frac{1}{x^3}=(x+\frac{1}{x})^3-3(x+\frac{1}{x})=12^3-3\times12=1728-36=1692$

9.

(1)

$$\frac{1}{a}=\frac{1}{\sqrt{5}-2}=\sqrt{5}+2$$

$$\frac{1}{b}=\frac{1}{\sqrt{6}-\sqrt{5}}=\sqrt{6}+\sqrt{5}$$

$$\frac{1}{c}=\frac{1}{\sqrt{7}-\sqrt{6}}=\sqrt{7}+\sqrt{6}$$

$$\therefore \frac{1}{c}>\frac{1}{b}>\frac{1}{a}\Rightarrow c<b<a$$

(2)

$$a^2=(\sqrt{7}-\sqrt{2})^2=9-2\sqrt{14}$$

$$b^2=(2\sqrt{2}-1)^2=9-4\sqrt{2}=9-2\sqrt{8}$$

$$c^2=(\sqrt{6}-\sqrt{3})^2=9-2\sqrt{18}$$

$\because b^2>a^2>c^2$ 且 a,b,c 均為正數 $\Rightarrow b>a>c$

10.

(1) $\sqrt{7+2\sqrt{12}}=\sqrt{(4+3)+2\sqrt{4\times 3}}=\sqrt{(\sqrt{4}+\sqrt{3})^2}=\sqrt{4}+\sqrt{3}=2+\sqrt{3}$

(2) $\sqrt{12-4\sqrt{5}}=\sqrt{12-2\sqrt{20}}=\sqrt{(10+2)-2\sqrt{10\cdot 2}}=\sqrt{(\sqrt{10}-\sqrt{2})^2}=\sqrt{10}-\sqrt{2}$

(3) $\sqrt{5+\sqrt{24}}=\sqrt{5+2\sqrt{6}}=\sqrt{(3+2)+2\sqrt{3\cdot 2}}=\sqrt{(\sqrt{3}+\sqrt{2})^2}=\sqrt{3}+\sqrt{2}$

(4) $\sqrt{8+4\sqrt{3}}=\sqrt{8+2\sqrt{12}}=\sqrt{(6+2)+2\sqrt{6\cdot 2}}=\sqrt{(\sqrt{6}+\sqrt{2})^2}=\sqrt{6}+\sqrt{2}$

(5) $\sqrt{4-\sqrt{7}}=\sqrt{\dfrac{16-4\sqrt{7}}{4}}=\dfrac{\sqrt{16-2\sqrt{28}}}{2}=\dfrac{\sqrt{(14+2)-2\sqrt{14\times 2}}}{2}=\dfrac{\sqrt{(\sqrt{14}-\sqrt{2})^2}}{2}=\dfrac{\sqrt{14}-\sqrt{2}}{2}$

(6) $\sqrt{12-\sqrt{140}}=\sqrt{12-2\sqrt{35}}=\sqrt{(7+5)-2\sqrt{7\times 5}}=\sqrt{(\sqrt{7}-\sqrt{5})^2}=\sqrt{7}-\sqrt{5}$

11.

$\sqrt{11-\sqrt{72}}=\sqrt{11-2\sqrt{18}}=\sqrt{9}-\sqrt{2}=3-\sqrt{2}\approx 1.\sim$

$\therefore a=1\quad b=3-\sqrt{2}-1=2-\sqrt{2}$

所求 $\dfrac{1}{a-b}+b=\dfrac{1}{1-(2-\sqrt{2})}+2-\sqrt{2}$

$=\dfrac{1}{\sqrt{2}-1}+2-\sqrt{2}=\sqrt{2}+1+2-\sqrt{2}=3$

12.

$\sqrt{19+8\sqrt{3}}=\sqrt{19+2\sqrt{48}}=\sqrt{(\sqrt{16}+\sqrt{3})^2}=\sqrt{16}+\sqrt{3}=4+\sqrt{3}$

$\sqrt{21-12\sqrt{3}}=\sqrt{21-2\sqrt{108}}=\sqrt{(\sqrt{12}-\sqrt{9})^2}=\sqrt{12}-\sqrt{9}=2\sqrt{3}-3$

$\therefore$原式： $x+y(4+\sqrt{3})=x(2\sqrt{3}-3)+15\sqrt{3}$

$\Rightarrow \underbrace{(x+4y)}_{a}+\underbrace{y\sqrt{3}}_{b\sqrt{n}}=\underbrace{-3x}_{c}+\underbrace{(2x+15)\sqrt{3}}_{d\sqrt{n}}$

$\therefore \begin{cases} x+4y=-3x \\ y=2x+15 \end{cases} \Rightarrow \begin{cases} x+y=0 \\ y=2x+15 \end{cases} \Rightarrow x=-5$ ， $y=5$

13.

$\because b$ 為小數部分　$\therefore 0\le b<1 \Rightarrow 0\le b^2<1$

又 $a^2+b^2=38 \Rightarrow a^2\approx 37.\sim$　$\therefore a=6.\sim$

設 $a=6+b$ ，代入 $a^2+b^2=38$

$\Rightarrow (6+b)^2+b^2=38 \Rightarrow 2b^2+12b-2=0 \Rightarrow b^2+6b-1=0$

$b=\dfrac{-6\pm\sqrt{40}}{2}=\dfrac{-6\pm 2\sqrt{10}}{2}=-3\pm\sqrt{10}$ (負不合　$\because 0\le b<1$) $\therefore b=-3+\sqrt{10}$

$\therefore a=6+b=6+(-3+\sqrt{10})=3+\sqrt{10}$