Mean-Variance Portfolio Optimization

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Abstract

The primary goal of this project is to create an automated asset management system. The system comprises two key models: a factor model for in-sample calibration and a portfolio optimization model for out-of-sample investment windows. Our goal is to design an investment strategy that incorporates an ensemble of models to achieve optimal and consistent performance across different market environments. In this paper, we disallow short selling.

The factor model is designed to analyze and explain the variation in expected asset returns. The portfolio optimization model complements the factor model by utilizing the expected returns and correlation matrix estimates obtained from the factor model. The optimization model is responsible for allocating assets within the portfolio based on various criteria such as CVaR, L2 norm etc.

Our initial investment budget for this project is set at $\$100,000$. To proceed with our analysis, we have access to three datasets. Each dataset includes monthly adjusted closing prices for various assets over a 15-year period. Additionally, each dataset also has data on eight different factor returns corresponding to these assets for the same 15-year timeframe. Our investment horizon is 5 years with a rebalancing period of 6 months.

Contents

1 Introduction

The objective of this paper is to develop an automated asset allocation system that leverages factor models and portfolio optimization algorithms explored in the Operational Research course. The aim is to create a system that exhibits the highest level of performance and consistency across various market environments.

The automated asset management system comprises two models: a factor model to estimate expected asset returns and calculate the covariance matrix, and a portfolio optimization model to determine optimal asset allocation based on the estimates.

The factor models this paper focuses on are:

- Ordinary Least Square Regression (OLS) Model
- Least Absolute Shrinkage and Selection Operator (LASSO) Model
- Best Subset Selection (BSS) Model

The portfolio optimization models this paper focuses on are:

- Mean-Variance Optimization (MVO) Model
- Robust Mean-Variance Optimization (Robust MVO) Model
- Conditional Value-at-Risk (CVaR) Optimization Model

The combination of factor models and optimization models will result in a total of 9 different trading models. These models will be implemented and compared to assess their performance and effectiveness.

The report follows a structured framework. In Section 2, the methodology and implementation details are explained for all six models discussed in the paper. Section 3 provides a description of the three performance metrics used to assess model performance. In Section

4, the results and analysis are presented, starting with the in-sample parameter tuning for the LASSO and BSS models. Subsequently, an out-of-sample investment performance comparison is conducted to select the best model combination. Finally, Section 5 concludes the report by summarizing the findings and offering conclusive remarks.

2 Methodology

2.1 Factor Model

Factor models are widely used financial tools that allow investors to identify and manage the investment characteristics that drive the risks and returns of stocks and portfolios. These models aim to explain the variation in stock returns using a set of common factors. In this report, the factor we are interested in are:

Market ('Mkt_RF')	Size ('SMB')	Value ('HML')	Short-term reversal ('ST_Rev')
Profitability ('RMW')	Investment ('CMA')	Momentum ('Mom')	Long-term reversal ('LT_Rev')

Figure 1: List of Factors

2.1.1 Ordinary Least Square Regression Model

The Ordinary Least Square (OLS) model is expressed in the following form and incorporates the following variables:

$$r_{it} = \alpha_i + \sum_{k=1}^{p} \beta_{ik} f_{kt} + \epsilon_{it} \tag{1}$$

where:

- r_{it} represents the rate of return of i^{th} asset at time t. For $i = 1, \ldots, I$ and $t = 1, \ldots, T$.
- α_i represents the intercept of regression for i^{th} asset.
- β_{ik} represents the k^{th} factor loading/coefficient on i^{th} asset. For $k = 1, \ldots, p$.

- f_{kt} represents the k^{th} factor monthly return at time t.
- T represents the total number of time steps.
- I represents the total number of asset types.
- p represents the total number of factor types.

The objective of the OLS model is to find the estimate of all α_i and β_{ik}. To achieve this, we need to define a series of matrices beforehand. First, we construct a matrix named A as follows:

$$A = [1, f_1, f_2, f_3, \ldots, f_p] = [1, f]_{T\times(p+1)} \tag{2}$$

where $[f_i]$ represents a $T \times 1$ column that contains the i^{th} factor return over T time points. 1 is a $(T \times 1)$ matrix that consists of 1 in every element, and it corresponds to each regression intercept.

Furthermore, we need to define a vector of regression coefficients named B in the following form:

$$B = \begin{bmatrix} \alpha_i \\ V_i \end{bmatrix} = \begin{bmatrix} \alpha_i \\ \beta_{i1} \\ \vdots \\ \beta_{ip} \end{bmatrix} \tag{3}$$

Moreover, we also need to obtain the expected factor return on a monthly basis by using geometric mean return:

$$\bar{f} = \left(\prod_{t=1}^{T} 1 + f_{kt} \right)^{\frac{1}{T}} - 1 \tag{4}$$

Our objective is to minimize the sum of $L-2$ norms of the residuals of regression, which can be transformed into the following format:

$$\min_{B_i} ||r_i - XB_i||_2^2 \tag{5}$$

where $|| \cdot ||$ represents the $L - 2$ norm operator.

By basic calculus, the closed form of solution (B_i^*) to the OLS model is:

$$B_i^* = (X^T X)^{-1} X^T r_i \tag{6}$$

Thus, the regression residuals (ϵ_i^*) will be the following form:

$$\epsilon_i^* = r_i - X B_i^* \tag{7}$$

The expected asset return (μ) and asset covariance matrix (Q) take the following form:

$$\mu = \alpha + V^T \bar{f} \tag{8}$$

where $\bar{f}$ represents the vector of excepted monthly factor returns. and

$$Q = V^T F V + D \tag{9}$$

where F represents the factor covariance matrix and D represents the diagonal matrix of residual variance of $\sigma^2_{\epsilon_i}$

2.1.2 Least Absolute Shrinkage and Selection Operator Model

The Least Absolute Shrinkage and Selection Operator (LASSO) Model is a L_1 regularization technique that is commonly used in statistical modeling. This method involves adding a penalty term to the objective function, which helps to regularize the model's parameters. The penalty term penalizes the addition of non-performing and underperforming variables into the OLS model. Compared to other models, LASSO regression provides models with high prediction accuracy by reducing variance and minimizing bias. The strength of the regularization is determined by the value of the penalty term, denoted as λ. The formulation of the LASSO regression model is expressed below:

$$\begin{aligned}
\text{minimize}_{B_i,\lambda} \quad & ||r_i - XB_i||_2^2 + \lambda||B_i||_1 \\
\text{subject to:} \quad & y_i \geq B_i \\
& y_i \geq -B_i \\
\text{where:} \quad & y_i = |B_i| \\
& B_i = \begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix}
\end{aligned} \tag{10}$$

In order to incorporate this equation into the quadprog function, we transformed the minimizing problem into the following form:

$$\begin{aligned}
\text{minimize} \quad & X^T Q X + c^T X \\
\text{subject to:} \quad & AX \leq b \\
& A_{eq} X = b_{eq} \\
& lb \leq X \leq ub
\end{aligned} \tag{11}$$

We started with transforming the objective function into matrix form:

$$\begin{aligned}
||r_i - XB_i||_2^2 + \lambda||B_i||_1 &= r_i^T r_i - 2r_i^T X B_i + B_i^T X^T X B_i + \lambda 1^T y_i \\
\text{where:} \quad X &= [1 \;\; f]
\end{aligned} \tag{12}$$

In order to match the format of the objective function with the formulation derived above, we set the following parameters:

$$Q = X^T X \tag{13}$$

$$C^T = -2r_i^T X + \lambda 1^T \tag{14}$$

Since we need to incorporate the constraint $y_i = |B_i|$ into the minimizing problem, hence we will expand our matrices as follows:

$$B_i^{new} = \begin{bmatrix} B_i \\ y_i \end{bmatrix} \tag{15}$$

$$Q^{new} = \begin{bmatrix} X^T X & 0_{(p+1)\times(p+1)} \\ 0_{(p+1)\times(p+1)} & 0_{(p+1)\times(p+1)} \end{bmatrix} \tag{16}$$

$$X^{new} = \begin{bmatrix} 1_{T\times 1} & f_{T\times p} & 0_{T\times(p+1)} \end{bmatrix} \tag{17}$$

$$1_{new}^T := Z^{new} = \begin{bmatrix} 0_{(p+1)\times 1} \\ 1_{(p+1)\times 1} \end{bmatrix} \tag{18}$$

Then, the objective function can be revised into the following form (since $r_i^T r_i$ is constant, we can remove it from the objective function):

$$\begin{aligned} \text{minimize} \quad & B_i^{new^T} Q^{new} B_i^{new} + C^T B_i^{new} \\ \text{where:} \quad & C^T = -2r_i^T X^{new} + \lambda Z^{new} \end{aligned} \tag{19}$$

For the constraint, we need also to define:

$$\begin{aligned} \text{such that} \quad & B_i - y_i \leq 0 \\ & -B_i - y_i \leq 0 \\ \text{which we can transfer into:} \quad & A_{eq} B_i^{new} = b_{eq} \\ \text{where:} \quad & A_{eq} = \begin{bmatrix} I_{(p+1)\times 1} & -I_{(p+1)\times 1} \\ -I_{(p+1)\times 1} & -I_{(p+1)\times 1} \end{bmatrix} \\ & b_{eq} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}_{2(p+1)\times 1} \end{aligned} \tag{20}$$

To sum up, the finalized version of the minimizing problem is:

$$\begin{aligned} \text{minimize} \quad & B_i^{new^T} Q^{new} B_i^{new} + C^T B_i^{new} \\ \text{where:} \quad & A_{eq} B_i^{new} = b_{eq} \end{aligned} \tag{21}$$

In the later part, we will analyze different λ applied into the equation and how coefficient numbers and values will change.

2.1.3 Best Subset Selection Model

Best Subset Selection (BSS) is a method that involves considering all possible combinations of independent variables to determine the subset that provides the best prediction of the outcome variable. In this project, we will be using the constrained form of the BSS model, which includes all eight factors as inputs. By utilizing this approach, we aim to identify the most effective subset of variables that can accurately predict the outcome variable.

Consider the constrained form of the BBS model:

$$\begin{aligned} \text{minimize} \quad & ||r_i - XB_i||_2^2 \\ \text{where:} \quad & ||B_t||_0 \leq K \end{aligned} \tag{22}$$

We can transform the minimizing problem above into a mixed-integer quadratic program (MIQP) by incorporating an auxiliary binary variable y:

$$\begin{aligned} \text{minimize} \quad & ||r_i - XB_i||_2^2 \\ \text{where:} \quad & Ly_i \leq B_i \leq Uy_i \\ & 1^T y_i = K \\ & X = [1 \quad f] \end{aligned} \tag{23}$$

The auxiliary binary y is a $(p+1) \times 1$ vector with each element indicating whether its corresponding factor is included in the model or not.

L is the lower bound of coefficient and U is the upper bound. We set bounds for our β. We need to ensure that the magnitudes of both L and U are large enough so that the performing explanatory will not be cut off. Therefore, we set $U = 100$ and $L = -100$.

In order to incorporate this equation into the Gurobi function, we transformed the minimizing problem into the following form:

$$\text{minimize} \quad X^T Q X + C^T X \tag{24}$$

We started with transforming the objective function into matrix form:

$$\begin{aligned} \text{minimize} \quad & ||r_i - XB_i||_2^2 \\ \equiv \text{minimize} \quad & r_i^T r_i - 2r_i^T xB_i + B_i^T X^T XB_i \\ \equiv \text{minimize} \quad & -2r_i^T XB_i + B_i^T X^T XB_i \end{aligned} \tag{25}$$

Thus, we can set:

$$Q = X^T X \tag{26}$$

$$C^T = -2r_i^T X \tag{27}$$

Furthermore, we also need to incorporate the auxiliary binary variable into our final minimizing problem so we will expand our matrix as follows:

$$B_i^{new} = \begin{bmatrix} B_i \\ y_i \end{bmatrix} \tag{28}$$

$$Q^{new} = \begin{bmatrix} X^T X & 0_{(p+1)\times(p+1)} \\ 0_{(p+1)\times(p+1)} & 0_{(p+1)\times(p+1)} \end{bmatrix} \tag{29}$$

$$X^{new} = \begin{bmatrix} 1_{T\times 1} & f_{T\times p} & 0_{T\times(p+1)} \end{bmatrix} \tag{30}$$

Then, the objective function can be revised into the following form (since $r_i^T r_i$ is constant, we can remove it from the objective function):

$$\begin{aligned} \text{minimize} \quad & B_i^{new^T} Q^{new} B_i^{new} + C^T B_i^{new} \\ \text{where:} \quad & C^T = -2 r_i^T X^{new} \end{aligned} \tag{31}$$

Consider the first constraint in the original minimizing problem:

$$\begin{aligned} & Ly_i \leq B_i \leq Uy_i \\ \text{which is equivalent to:} & \begin{cases} B_i - Uy_i \leq 0 \\ -B_i - Ly_i \leq 0 \end{cases} \end{aligned} \tag{32}$$

Transform it into matrix form:

$$\begin{aligned} & AB_i^{new} = b \\ \text{where:} \quad A = & \begin{bmatrix} I_{(p+1)\times 1} & -UI_{(p+1)\times 1} \\ -I_{(p+1)\times 1} & LI_{(p+1)\times 1} \end{bmatrix} \\ b = & \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{aligned} \tag{33}$$

Consider the second constraint in the original minimizing problem:

$$1^T y_i = K \tag{34}$$

Transform it into matrix form:

$$
\begin{aligned}
A_{eq} B_i^{new} &= b_{eq} \\
\text{where:} \quad A_{eq} &= \begin{bmatrix} 0_{(p+1)\times 1} \\ 1_{(p+1)\times 1} \end{bmatrix} \\
b_{eq} &= K
\end{aligned} \tag{35}
$$

To sum up, the finalized version of the minimizing problem is:

$$
\begin{aligned}
\text{minimize} \quad & B_i^{new^T} Q^{new} B_i^{new} + C^T B_i^{new} \\
\text{where:} \quad & A_{eq} B_i^{new} = b_{eq}
\end{aligned} \tag{36}
$$

Various K values will be examined during the latter part.

2.2 Portfolio Optimization Model

Once the expected returns for the assets of interest have been modeled, the next step is to construct an investment portfolio using portfolio optimization models. This paper will primarily focus on three models: the nominal mean-variance optimization model, the robust mean-variance optimization model, and the best subset selection model. These models utilize the expected asset returns and the covariance matrix as inputs to optimize the portfolio allocation.

2.2.1 Mean-Variance Optimization Model

Mean-variance portfolio (MVO) optimization is a widely used methodology in finance for constructing optimal portfolios. It is based on the principle of balancing expected returns with portfolio risk. The process involves estimating the expected returns, variances, and covariances of each asset in the portfolio, and then using these estimates to calculate the optimal portfolio weights. The optimal portfolio is the one that maximizes expected returns while minimizing portfolio risk, subject to constraints on the weights of each asset.

$$
\begin{aligned}
\underset{x}{\text{minimize}} \quad & x^T \Sigma x \\
\text{subject to} \quad & x^T \mu \geq r \\
& \sum_{i=1}^{n} x_i = 1 \\
& x_i \geq 0, \forall i = 1, 2, \ldots, n
\end{aligned}
$$

In our project, we use the derived mean and variance matrix from the models above in order to arrive at the optimal weights. With the Q and μ from each factor model as inputs, we use the quadprog in MATLAB to solve the problem.

2.2.2 Robust Mean-Variance Optimization

The regular MVO model has a shortfall in that it relies solely on estimated parameters to make investment decisions, without taking into account the potential errors in those estimates or their impacts on the optimization problem. This oversight can lead to suboptimal investment decisions, as the nominal MVO model may not accurately reflect the actual risks and returns of the investment portfolio. Therefore, it is essential to consider the estimation errors and their consequences when using the MVO model to optimize investment portfolios. This can be achieved through the use of robust optimization techniques or other approaches that explicitly account for estimation errors and their potential impacts.
In the typical MVO problem, it has the constraint:

$$x^T \mu \geq r$$

which is a noisy constraint. Hence, we can introduce a box uncertainty set around the estimated expected returns:

$$\mu^{true} \in U(\mu) = \{\mu^{true} \in \mathbb{R}^n : (\mu_i^{true} - \mu_i)^T \Theta^{-1} (\mu_i^{true} - \mu_i) \leq \epsilon_2^2\}$$

where ϵ_2 is the radius that bounds the standard distance between μ and μ^{true} and $\Theta \in \mathbb{R}^{n*n}$ is the measure of uncertainty that serves to standardize out estimated expected return. Moreover, in this method, we set a confidence level to help define the parameter ϵ_2, a measure of distance. We can define as the following:

$$\epsilon_2^2 = \chi_n^2(\alpha)$$

where $\chi_n^2(\alpha)$ is the inverse cumulative distribution function of the chi-squared distribution of n degrees of freedom.
Θ can be represented as:

$$\Theta = \frac{1}{T} diag(diag(Q)), \text{ where we can get that } (\Theta^{1/2})_{ii} = \frac{\sigma_i}{\sqrt{T}} \text{and } (\Theta^{1/2})_{ij} = 0 \text{ for } i \neq j$$

as a result, Θ is a diagonal matrix with $\Theta_{ii} = \frac{\sigma_i^2}{T}$.The uncertainty can be added as a penalty on our target returns as $\epsilon_2 ||\Theta^{1/2} x||_2$, where $||.||_2$ is the Euclidean norm of a vector. This is equivalent to:

$$\epsilon_2 ||\Theta^{1/2} X||_2 \Leftrightarrow \epsilon_2 \sqrt{X^T \Theta X}$$

We can have an optimal trade-off between risk and penalized expected returns.

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \lambda x^T \Sigma x - \mu^T x + \epsilon_2 ||\Theta^{1/2} x||_2 \\ \text{subject to} \quad & e^T x = 1 \\ & x \geq 0 \end{aligned}$$

2.2.3 Conditional Value-at-Risk Optimization Model

Value-at-Risk (VaR) is a risk measure used to assess the potential loss of an investment portfolio. It focuses on estimating the downside risk of a portfolio, particularly at the tail end of the profit and loss probability distribution. The VaR with a probability of 1 - α is defined as:

$$VaR_\alpha(X) = \min\{\gamma \in \mathbb{R} : \mathbb{P}(X > \gamma \leq (1 - \alpha)\} \tag{37}$$

Since optimizing VaR is not a convex activity, we choose to optimize the $CVaR$ which can be transformed into a convex function. Conditional Value-at-Risk (CVaR), also known as Expected Shortfall (ES), is a risk measure that provides additional information beyond VaR. While VaR estimates the potential loss of an investment portfolio at a specified confidence level, CVaR goes a step further by calculating the expected loss in the tail of the distribution beyond the VaR level. The CVaR with a probability of 1 - α is defined as:

$$CVaR_\alpha(X) := F_\alpha(x, r) = \gamma + \frac{1}{1-\alpha} \int \Big(f(x,\gamma) - \gamma\Big)^+ p(r)\, dr \tag{38}$$

where:

- $F(\cdot, \gamma)$ is a convex function about γ.
- $\gamma \in \mathbb{R}$ is the placeholder for $VaR_\alpha(X)$ during optimization.
- $(f(x, r) - \gamma)^+ = \max(f(x, r) - \gamma, 0)$.
- $f(x, r)$ is the loss of portfolio for a realization of our random asset returns r and given

portfolio weights x.

- x is a vector of portfolio weights.
- r is a vector of random asset returns.
- $p(r)$ is the probability density function of the given vector of random asset return r.

Due to the complexity of obtaining an analytical expression for $p(r)$, a scenario-based representation formula is applied. We assume that the probability of each scenario is the same across the board. Thus, we can use scenario-based representation formula to approximate $F_\alpha(x, r)$:

$$CVaR_\alpha(X) \approx \tilde{F}_\alpha(x, r) = \gamma + \frac{1}{(1-\alpha)S} \sum_{s=1}^{S} \left(f(x, \hat{r}_s) - \gamma\right)^+ \tag{39}$$

where

- $\hat{r}_s \in \mathbb{R}^n$ is the realization of scenario s.
- $f(x, \hat{r}_s) = -\hat{r}^T x$.

Then, we introduce an auxiliary variable z_s for each $s = 1, \cdots, S$, and the optimization problem will become:

$$\begin{aligned} \text{minimize}_{x,z,\gamma} \quad & \gamma + \frac{1}{(1-\alpha)S} \sum_{s=1}^{S} z_s \\ \text{Subject to:} \quad z_s \geq 0 \quad & s = 1, \cdots, S \\ z_s >= f(x, \hat{r}_s) - \gamma \quad & s = 1, \cdots, S \\ x \in \chi & \end{aligned} \tag{40}$$

where:

$x \in \chi$ includes our budget, target return, and any other constraints pertaining to our portfolio weights x.

3 Performance Metrics

To evaluate the performance of the models, we have selected three key performance metrics: Sharpe Ratio, Average Turnover Rate, and Computational Time. These metrics will help assess the effectiveness and efficiency of the models, with their importance ranked in descending order.

3.1 Ex-post Sharpe Ratio

The ex-post Sharpe Ratio is employed to compare the financial performance of portfolios generated by factor models. The Sharpe Ratio serves as a metric for assessing risk-adjusted returns. In this context, a higher Sharpe Ratio is preferable as it signifies that the portfolio achieved greater returns while assuming less risk. During the process of model performance evaluation, we will calculate the Sharpe Ratio with average return and average standard deviation:

$$\text{Sharpe Ratio} = \frac{\bar{R_p} - R_f}{\bar{\sigma_p}} \tag{41}$$

where:

- $\bar{R_p}$ is the average rate of return of the portfolio
- r_f is the risk-free rate of return
- $\bar{\sigma_p}$ is the average standard deviation of excess return to the portfolio

3.2 Average Turnover Rate

The average turnover rate takes into account the transaction fees incurred during the trading of financial assets, which can have a negative impact on portfolio profits. A lower turnover rate is considered more desirable as it helps reduce transaction costs. To calculate the average turnover rate, the sum of changes in asset weights for each rebalancing period is recorded:

$$\text{Turnover Rate for } i^{th} \text{ period} := \bar{X}_i = \sum_i |x_i - x_{i-1}| \tag{42}$$

where:

- $x_i \in \mathbb{R}^n$ is a vector of the asset weight of the n-asset portfolio
- $i \in \{1, \cdots, 20\}$ represents the i^{th} period

This captures the adjustments made to the portfolio over time. The average turnover rate is then determined by calculating the arithmetic average of these values across the 20 rebalancing periods:

$$\text{Average Turnover Rate} = \frac{\sum_{i=1}^{20} \bar{X}_i}{20} \tag{43}$$

3.3 Computation Time

The runtime for each trading algorithm is measured to evaluate the efficiency of the optimization algorithms. Longer runtimes result in increased computational costs. Therefore, the runtime for each optimization conducted by the algorithm is recorded and compared. In this analysis, a runtime exceeding 5 minutes is considered to be high.

4 Results

4.1 λ Selection

In the MVO model, increasing the value of λ leads to an increase in the Sharpe ratio, indicating improved risk-adjusted returns, while simultaneously reducing the turnover rate. Similarly, the CVaR model demonstrates that the Sharpe ratio reaches its peak around a λ value of 0.2 and 0.03, coinciding with the lowest turnover rate observed. Turning to the Robust MVO model, as λ increases, both the turnover rate and the Sharpe ratio increase accordingly. Notably, the turnover rate is the lowest at a lambda value of 0.03, but achieves its minimum value when λ equals 0.08. These findings highlight the importance of λ selection in balancing risk-adjusted returns and turnover, providing valuable insights for model selection and portfolio optimization.

In terms of the Shape Ratio (SR), which measures the risk-adjusted returns, the MVO and Robust MVO models consistently demonstrate higher SR values compared to the CVaR model. This indicates that both MVO and Robust MVO methods offer better risk-adjusted performance, with Robust MVO achieving the highest SR values overall.

On the other hand, considering turnover rate, which represents the frequency of portfolio rebalancing and reflects the trading costs involved, the CVaR model shows lower turnover values compared to MVO and Robust MVO. A lower turnover rate is generally desirable as it indicates a more stable and cost-efficient portfolio management strategy.

When evaluating the runtime, which measures the computational time required to execute the models, the CVaR model demonstrates the lowest runtime values among the three methods. This suggests that the CVaR approach may be computationally more efficient compared to MVO and Robust MVO.

Considering the combined analysis of SR and turnover, it is important to strike a balance between risk-adjusted returns and trading costs. In this regard, various method with a lambda value of 0.03 stands out as a potential choice. It exhibits competitive SR values while also demonstrating a reasonable turnover rate.

Table 1: LASSO with MVO Model Comparison

Lambda	Method	SR	SR Rank	Turnover	Turnover Rank	Runtime	Runtime Rank
0.01	MVO	0.152	10	0.511	13	2.117	11
0.02	MVO	0.135	13	0.466	12	2.238	13
0.03	MVO	0.138	12	0.434	11	2.151	12
0.04	MVO	0.144	11	0.389	10	1.965	8
0.05	MVO	0.154	9	0.345	9	1.900	4
0.06	MVO	0.162	8	0.327	8	1.902	5
0.07	MVO	0.167	7	0.308	7	1.972	9
0.08	MVO	0.172	6	0.296	6	1.938	6
0.09	MVO	0.178	5	0.284	5	1.875	1
0.1	MVO	0.181	4	0.279	4	1.960	7
0.2	MVO	0.190	2	0.229	3	1.895	3
0.25	MVO	0.192	1	0.229	2	2.010	10
0.3	MVO	0.190	3	0.222	1	1.875	2

Table 2: LASSO with CVaR Model Comparison

Lambda	Method	SR	SR Rank	Turnover	Turnover Rank	Runtime	Runtime Rank
0.01	CVaR	0.176	4	0.842	4	2.334	3
0.02	CVaR	0.176	5	0.841	1	2.557	12
0.03	CVaR	0.176	6	0.841	2	2.339	4
0.04	CVaR	0.175	7	0.841	3	2.366	5
0.05	CVaR	0.175	8	0.842	5	2.557	13
0.06	CVaR	0.174	9	0.843	6	2.435	9
0.07	CVaR	0.174	10	0.845	7	2.305	2
0.08	CVaR	0.173	11	0.846	8	2.532	11
0.09	CVaR	0.173	12	0.847	9	2.447	10
0.1	CVaR	0.172	13	0.847	10	2.379	6
0.2	CVaR	0.179	1	0.864	11	2.417	7
0.25	CVaR	0.176	3	0.869	12	2.424	8
0.3	CVaR	0.178	2	0.871	13	2.288	1

Table 3: LASSO with Robust MVO Model Comparison

Lambda	Method	SR	SR Rank	Turnover	Turnover Rank	Runtime	Runtime Rank
0.01	Robust MVO	0.168	13	0.481	2	3.304	12
0.02	Robust MVO	0.171	11	0.479	1	3.255	8
0.03	Robust MVO	0.170	12	0.490	5	3.198	3
0.04	Robust MVO	0.173	10	0.501	9	3.263	9
0.05	Robust MVO	0.178	9	0.499	8	3.201	4
0.06	Robust MVO	0.184	4	0.495	7	3.303	11
0.07	Robust MVO	0.183	6	0.486	3	3.520	13
0.08	Robust MVO	0.180	8	0.487	4	3.233	7
0.09	Robust MVO	0.183	7	0.494	6	3.175	2
0.1	Robust MVO	0.184	5	0.503	10	3.272	10
0.2	Robust MVO	0.189	1	0.504	11	3.225	6
0.25	Robust MVO	0.188	2	0.509	12	3.217	5
0.3	Robust MVO	0.187	3	0.515	13	3.132	1

4.2 K Selection

In the MVO method, K = 2 stands out as a potential choice, as it exhibits a relatively high SR value of 0.17 and a lower turnover rate of 0.46. This combination suggests the potential for better risk-adjusted returns while minimizing transaction costs.

Similarly, in the CVaR method, K = 4 showcases a comparable SR value of 0.17 while demonstrating a lower turnover rate of 0.87. This indicates a potential balance between risk-adjusted returns and transaction costs.

In the Robust MVO method, K = 4 emerges as the preferred choice, boasting the highest SR value of 0.19 alongside a lower turnover rate of 0.37. This highlights the potential for superior risk-adjusted returns and reduced transaction costs.

By selecting K = 2 in both the MVO and Robust MVO methods and K = 5 in the CVaR method, one can aim for higher SR values while maintaining lower turnover rates. In order to strike a judicious equilibrium between variable selection and performance optimization, it appears that opting for K = 4 would be a prudent decision. This choice demonstrates competitive Sharpe Ratio (SR) values while also exhibiting a reasonable turnover rate. By selecting K = 3, one can achieve a desirable balance between the inclusion of an appropriate number of variables in the models and the attainment of satisfactory performance metrics.

Table 4: BSS with MVO Model Comparison

K	Method	SR	SR Rank	Turnover	Turnover Rank	Runtime	Runtime Rank
2	MVO	0.17	5	0.46	1	4.80	7
3	MVO	0.16	6	0.49	2	4.59	6
4	MVO	0.16	7	0.51	3	4.15	5
5	MVO	0.17	3	0.52	7	3.78	4
6	MVO	0.17	4	0.52	6	3.74	3
7	MVO	0.17	2	0.51	5	3.63	2
8	MVO	0.17	1	0.51	4	3.54	1

Table 5: BSS with CVaR Model Comparison

K	Method	SR	SR Rank	Turnover	Turnover Rank	Runtime	Runtime Rank
2	CVaR	0.18	3	0.89	7	4.93	7
3	CVaR	0.18	1	0.86	5	4.62	6
4	CVaR	0.18	2	0.87	6	4.36	5
5	CVaR	0.17	4	0.84	1	3.99	4
6	CVaR	0.17	7	0.84	4	3.88	2
7	CVaR	0.17	5	0.84	3	3.81	1
8	CVaR	0.17	6	0.84	2	3.92	3

Table 6: BSS with Robust MVO Model Comparison

K	Method	SR	SR Rank	Turnover	Turnover Rank	Runtime	Runtime Rank
2	Robust MVO	0.19	1	0.37	7	5.87	7
3	Robust MVO	0.17	7	0.36	6	5.71	6
4	Robust MVO	0.17	2	0.32	5	5.37	5
5	Robust MVO	0.17	6	0.31	4	5.00	3
6	Robust MVO	0.17	5	0.31	3	4.87	1
7	Robust MVO	0.17	4	0.31	2	5.02	4
8	Robust MVO	0.17	3	0.31	1	4.99	2

4.3 Top Performing Models and Models Selections

4.3.1 Across Data sets Comparison

In Dataset 1, the CVaR method with BSS as the factor model stands out as the top performer. It achieves the highest Sharpe Ratio (SR) value of 0.183, indicating the potential for superior risk-adjusted returns. Additionally, BSS CVaR demonstrates the lowest turnover rate of 0.855, suggesting a more efficient portfolio with reduced transaction costs. Another noteworthy option is the OLS factor model in the MVO method, which exhibits a relatively high SR value of 0.172.

Moving to Data set 2, the Robust MVO method with BSS as the factor model shows strong performance. It leads the rankings with the highest SR value of 0.109, indicating the potential for favorable risk-adjusted returns. Moreover, BSS Robust MVO also exhibits the lowest turnover rate of 0.232, suggesting a more efficient portfolio with reduced trading activity. Another factor model worth considering is the MVO method with LASSO, which achieves a competitive SR value of 0.1, placing it in the top ranks.

In Dataset 3, the Robust MVO method with OLS as the factor model stands out with the highest SR value of 0.236, securing the top rank among the factor models. This suggests the potential for superior risk-adjusted returns when utilizing this combination. Additionally, the CVaR method with LASSO as the factor model demonstrates a notable SR value of 0.201, indicating the potential for favorable risk-adjusted returns, although it is associated with a relatively high turnover rate of 0.912.

Table 7: In Sample: Dataset 1 Result

Factor Model	Method	SR	SR Rank	Turnover	Turnover Rank	Runtime	Runtime Rank
BSS	CVaR	0.183	1	0.855	1	4.619	6
LASSO	CVaR	0.176	2	0.841	3	2.339	4
OLS	CVaR	0.173	3	0.844	2	1.072	2
OLS	MVO	0.172	4	0.487	6	0.615	1
LASSO	Robust MVO	0.170	5	0.490	4	3.198	3
BSS	Robust MVO	0.167	6	0.356	8	5.715	6
BSS	MVO	0.164	7	0.489	5	4.592	6
OLS	Robust MVO	0.162	8	0.353	9	1.725	3
LASSO	MVO	0.138	9	0.434	7	2.151	6

Table 8: Out of Sample: Dataset 2 Result

Factor Model	Method	SR	SR Rank	Turnover	Turnover Rank	Runtime	Runtime Rank
BSS	Robust MVO	0.109	1	0.232	9	7.447	1
LASSO	Robust MVO	0.102	2	0.428	7	4.976	4
OLS	Robust MVO	0.102	3	0.292	8	1.855	7
LASSO	MVO	0.100	4	0.563	6	3.448	6
OLS	MVO	0.081	5	0.649	5	0.642	9
BSS	CVaR	0.072	6	0.915	1	6.703	2
BSS	MVO	0.066	7	0.704	4	6.334	3
OLS	CVaR	0.038	8	0.903	2	0.935	8
LASSO	CVaR	0.038	9	0.903	3	4.245	5

Table 9: Out of Sample: Dataset 3 Result

Model	Method	SR	SR Rank	Turnover	Turnover Rank	Runtime	Runtime Rank
OLS	Robust MVO	0.236	1	0.330	9	2.023	7
LASSO	Robust MVO	0.229	2	0.481	7	5.390	4
BSS	Robust MVO	0.227	3	0.363	8	8.325	1
LASSO	CVaR	0.201	4	0.912	2	4.561	5
OLS	CVaR	0.200	5	0.925	1	0.963	8
OLS	MVO	0.193	6	0.550	5	0.612	9
BSS	CVaR	0.191	7	0.878	3	7.297	2
BSS	MVO	0.187	8	0.557	4	6.782	3
LASSO	MVO	0.147	9	0.516	6	4.010	6

4.3.2 Mean Aggregated Performance Metrics Comparison

Upon analyzing the table "Average Performance Metrics: Factor Models", it is observed that the BSS and OLS factor models exhibit similar Sharpe Ratios (SR) of approximately 0.151 and turnover ratios around 0.593. Notably, within their respective subgroups, the combinations of BSS and Robust MVO and OLS and Robust MVO outperform other factor model combinations, showcasing relatively lower transaction costs.

Moving to the table "Average Performance Metrics: Asset Allocation Optimization Models," it becomes evident that the Robust MVO model stands out with a significantly lower turnover rate compared to the other algorithms considered.

To further delve into the selection between BSS and Robust MVO, as well as OLS and Robust MVO, the "Across Dataset Comparison" section provides additional insights. Dataset 1 reflects a stable economy, characterized by relatively lower CVaR optimization results. Dataset 2 suggests a bear market scenario, as evidenced by lower CVaR results compared to other datasets. On the other hand, dataset 3 appears to be in a bullish market, with significantly higher SR returns than the other datasets. In this context, the combination of BSS and Robust MVO performs better in stable and bear markets, while the combination of OLS and Robust MVO performs well in a bullish market. It can be inferred that the BSS model selects fewer factors compared to OLS, allowing for higher weights on equities influenced by those selected factors. However, in a bullish market, the performance dynamic between these two combinations is reversed. Overall, in order to reserve more conservative views, our selected algorithm is BSS with K = 3 and Robust MVO model.

Table 10: Average Performance Metrics: Factor Models

Row Labels	**Average of SR**	**Average of Turnover**	**Average of Runtime**
***BSS**	**0.152**	**0.594**	**6.424**
CVaR	0.149	0.883	6.207
MVO	0.139	0.583	5.902
Robust MVO	0.168	0.317	7.162
***LASSO**	**0.144**	**0.619**	**3.813**
CVaR	0.138	0.885	3.715
MVO	0.128	0.504	3.203
Robust MVO	0.167	0.466	4.521
***OLS**	**0.151**	**0.593**	**1.160**
CVaR	0.137	0.891	0.990
MVO	0.149	0.562	0.623
Robust MVO	0.167	0.325	1.868
***Grand Total**	**0.149**	**0.602**	**3.799**

Table 11: Average Performance Metrics: Asset Allocation Optimization Models

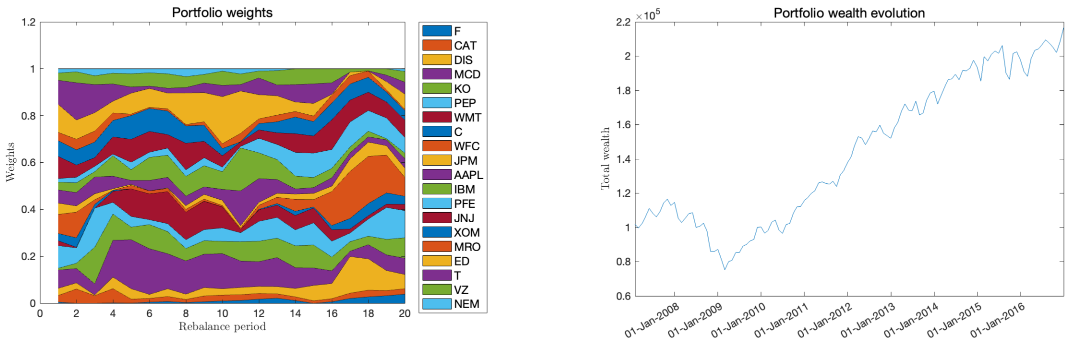
Row Labels	Average of SR	Average of Turnover	Average of Runtime
***CVaR**	**0.141**	**0.886**	**3.637**
BSS	0.149	0.883	6.207
LASSO	0.138	0.885	3.715
OLS	0.137	0.891	0.990
***MVO**	**0.138**	**0.550**	**3.243**
BSS	0.139	0.583	5.902
LASSO	0.128	0.504	3.203
OLS	0.149	0.562	0.623
***Robust MVO**	**0.167**	**0.369**	**4.517**
BSS	0.168	0.317	7.162
LASSO	0.167	0.466	4.521
OLS	0.167	0.325	1.868
***Grand Total**	**0.149**	**0.602**	**3.799**

4.3.3 Portfolio Wealth Evolution and Asset Allocation

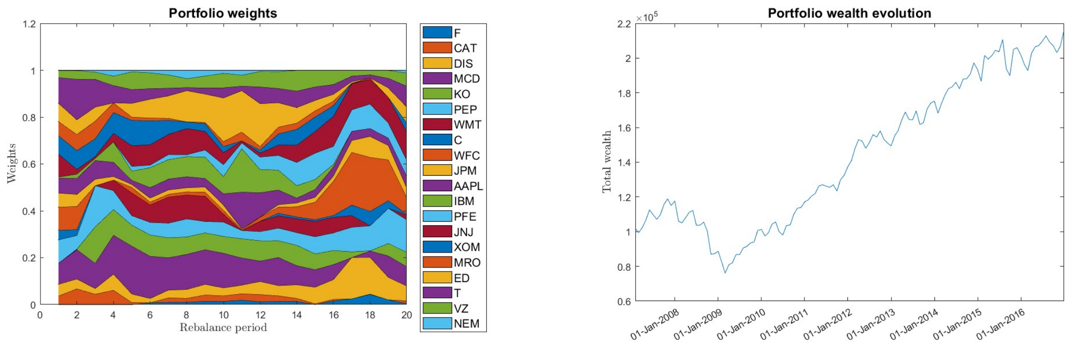
Based on the analysis presented in the "In Sample Comparison between BSS and OLS" table, it can be observed that OLS exhibits a slightly higher turnover rate compared to BSS. This disparity is further manifested in the asset allocation graph, where the stock selection of OLS displays more pronounced changes compared to the relatively smoother and more gradual transitions observed in BSS's graph.

Turning attention to the "BSS + RMVO with K=3 Final Result" table, it becomes apparent that the strategy demonstrates a relatively smooth transition during the re-balancing periods. Notably, during times of crisis such as in 1998 and 2009, the strategy shows trading activities aimed at allocating heavier weights to better-performing stocks. This adaptive approach highlights the strategy's ability to respond to changing market conditions and potentially capitalize on opportunities arising from turbulent market periods.

Table 12: In Sample Comparison between BSS and OLS

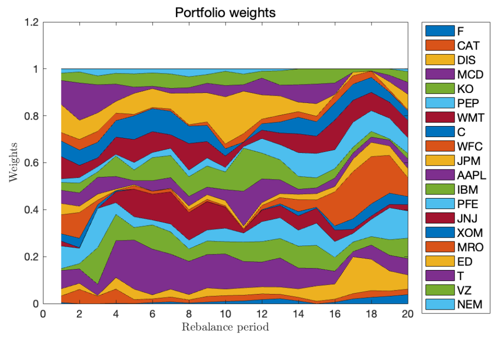


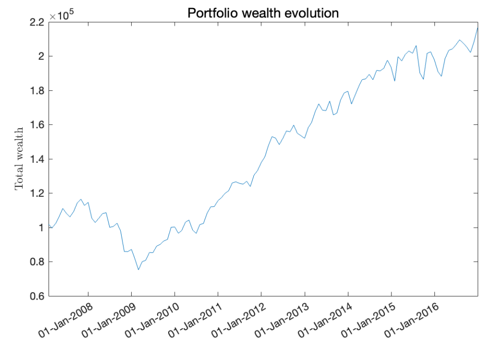
BSS + RMVO



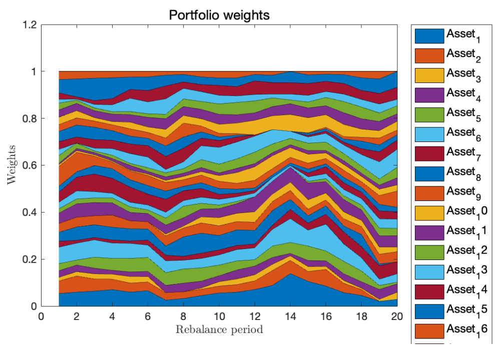
OLS + RMVO

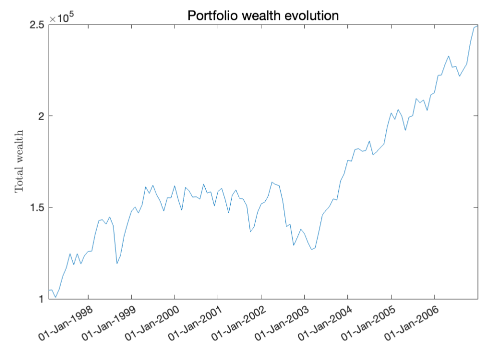
Table 13: BSS + RMVO with K=3 Final Result



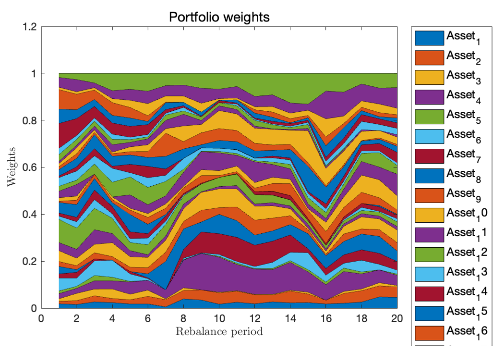


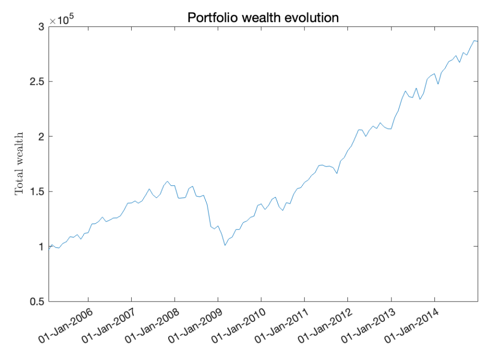
In Sample Dataset 1





Out of Sample Dataset 2





Out of Sample Dataset 3

5 Conclusion

This project aims to design an automated asset management system that utilizes a factor model (OLS, LASSO, BSS) and an optimization model (nominal MVO, robust MVO, CVaR) for determining the optimal portfolio allocation.

First, in pursuit of attaining a equilibrium between a high Sharpe Ratio and a low Asset Turnover rate, parameter fine-tune was undertaken for the LASSO and BSS models. As a result of this endeavor, the optimal values were determined as $\hat{\lambda} = 0.03$ for the LASSO model and $\hat{K} = 3$ for the BSS model.

Subsequently, an across-dataset comparison was conducted. In dataset 1, which signifies a representative stable market condition, the BSS-CVaR and OLS-MVO models demonstrated superior performance. Conversely, dataset 2, characterized as a typical bearish market condition, witnessed the outperformance of the BSS-Robust-MVO and LASSO-MVO models. In dataset 3, mirroring a typical bullish market condition, the OLS-Robust-MVO and LASSO-CVaR models exhibited superior performance.

Following the across-dataset comparison, a mean aggregated performance comparison was conducted, wherein performance metrics were averaged either by factor models or portfolio optimization models. Notably, it was observed that the BBS-Robust-MVO and OLS-Robust-MVO models consistently achieved higher Sharpe Ratios and lower Asset Turnover rates. These results suggest that these models exhibit a desirable combination of risk-adjusted returns and efficient utilization of assets across different market conditions.

In the final stage of model selection, the across-dataset comparison process was revisited to determine the preferred model between BBS-Robust-MVO and OLS-Robust-MVO. It was found that the BBS-Robust-MVO model consistently outperformed the OLS-Robust-MVO model in both bearish and stable market conditions. However, in bullish market conditions, the OLS-Robust-MVO model exhibited superior performance. Taking a conservative approach, the BBS-Robust-MVO model with $K = 3$ was ultimately chosen as the preferred model. This decision ensures a balanced performance across various market conditions while prioritizing risk management.