SRI RAMACHANDRA
INSTITUTE OF HIGHER EDUCATION AND RESEARCH
(Category - I Deemed to be University) Porur, Chennai
SRI RAMACHANDRA ENGINEERING AND TECHNOLOGY

Day-2: 20-10-2020 **MODULE -1 Assignment - 1**

1. **Obtain the asymptotic bound for the recurrence relations given below using Master theorem.**

1. T (n) = 3T (n/2)+ n2

2. T (n) = 4T (n/2)+ n2

3. T (n) = T (n/2) + 2n

4. T (n) = 2nT (n/2) + nn

5. T (n) = 16T (n/4)+ n

6. T (n) = 2T (n/2)+ n log n 7. T (n)

= 2T (n/2)+ n/ log n 8. T (n) = 2T

(n/4)+ n0.51 9. T (n) = 0.5T (n/2)+

1/n 10. T (n) = 16T (n/4)+ n!

11. T (n) = p2T (n/2) + log n 12. T

(n) = 3T (n/2)+ n

13. T (n) = 3T (n/3)+ pn

14. T (n) = 4T (n/2)+ cn

15. T (n) = 3T (n/4)+ n log n 16. T

(n) = 3T (n/3)+ n/2

17. T (n) = 6T (n/3)+ n2 log n

18. T (n) = 4T (n/2)+ n/ log n 19. T (n) =

64T (n/8)− n2 log n 20. T (n) = 7T

(n/3)+ n2

21. T (n) = 4T (n/2)+ log n

2. Solve the following recurrence relation using recursion tree method.

1. $T(n) = T(n/5) + T(4n/5) + n$

2. $T(n) = 3T(n/4) + cn2$

3. $T(n) = cn+2T(n/2)$

Module 2: Combinatorial Optimization

3. Design a greedy algorithmic technique using binary min heap to encode the word ***'mississippi'*** using variable length codeword. Calculate the number of bits may be required for encoding the message 'mississippi'?

1. a=3 b=2 k=2 p=0 logba ≃ 1.5 since logba<k (case 3)

   Answer: θ(n2)

2. a=4 b=2 k=2 p=0 logba ≃ 2 since logba=k (case 1)

   Answer: θ(n2logn)

3. a=1 b=2 k=1 p=1 logba ≃ 0 since logba<k (case 3)

   Answer: θ (2)

4. Masters theorem does not work a not a constant

5. a=16 b=4 k=1 p=0 logba ≃ 2 since logba>k (case 1)

   Answer: θ(n2)

6. a=2 b=2 k=1 p=1 logba ≃ 1 since logba=k (case 2)

   Answer: θ(nlog2n)

7. a=2 b=2 k=1 p=-1 logba ≃ 1since logba=k (case 2)

   Answer: θ(nloglogn)

8. a=2, b=4, k=0.51 p=0 logba ≃ 0.5 since logba<k(case3)

   Answer: θ (n0.51)

9. a=0.5 cannot be solved using masters theorem a<1

10. a=16 b=4 k=c p=1 logba ≃ 2 since logba<k (case 3)

    Answer: θ(n!)

11. a=16 b=4 k=c p=1 logba ≃ 2 since logba<k (case 3)

    Answer: θ(n!)

12. a=3 b=2 k=1 p=0 logba ≃ 1.58 since logba>k (case1)

Answer: θ(nlog23)

13. a=3 b=3 k=0.5 p=0 logba ≃ 1 since logba>k (case 1)

Answer: θ(n)

14. a=4 b=2 k=1 p=0 logba ≃ 2 since logba>k (case 1)

Answer: θ(n2)

15. a=3 b=4 k=1 p=1 logba ≃ 0.79 since logba<k (case 3)

Answer: θ(nlogn)

16. a=3 b=3 k=1 p=0 logba ≃ 1 since logba=k (case 2)

Answer: θ(nlogn)

17. a=6 b=3 k=2 p=1 logba ≃ 1.63 since logba<k(case3)

Answer: θ(n2logn)

18. a=4 b=2 k=1 p=-1 logba ≃ 2 since logba>k (case1)

Answer: θ(n2)

19. a=64 b=8 k=2 p=1 logba ≃ 2 since logba=k (case2)

Answer: θ(n2log2n)

20. a=7 b=3 k=2 p=0 logba ≃ 1.77 since logba<k (case3)

Answer: θ(n2)

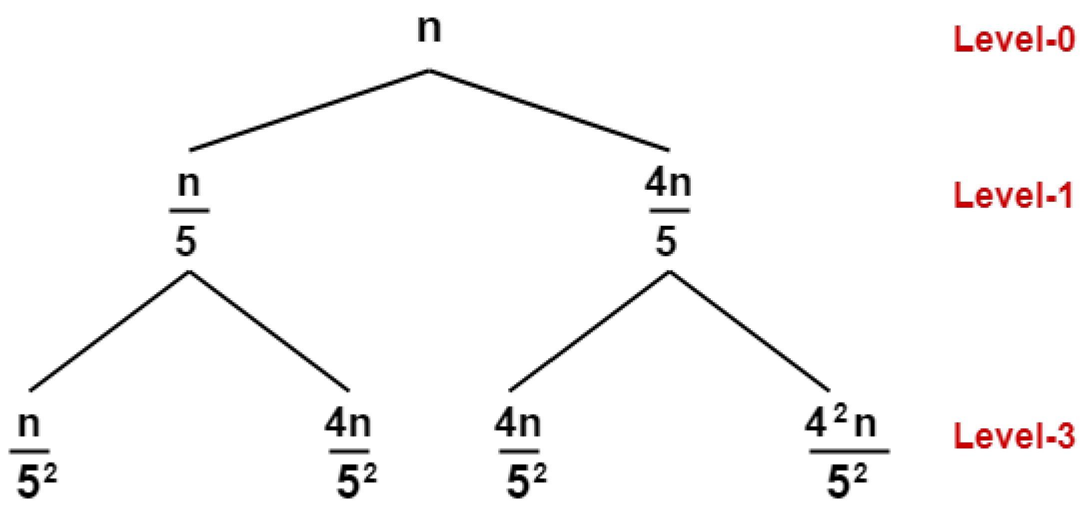
21. a=4 b=2 k=0 p=1 logba ≃ 2 since logba>k (case1)

Answer:

.θ(n2)

2. Solve the following recurrence relation using recursion tree method:

1. $T(n) = T(n/5) + T(4n/5) + n$



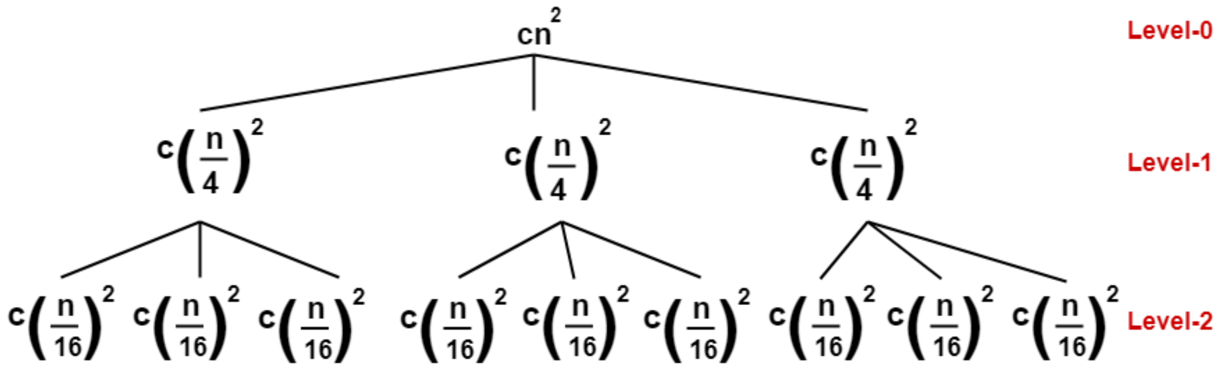
- Cost of level-0 = n
- Cost of level-1 = $n/5 + 4n/5 = n$
- Cost of level-2 = $n/5^2 + 4n/5^2 + 4n/5^2 + 4^2n/5^2 = n$

Cost of last level = $2^{\log_{5/4} n} \times T(1) = \theta(2^{\log_{5/4} n}) = \theta(n^{\log_{5/4} 2})$

$$T(n) = \underbrace{\{\, n + n + n + \ldots\ldots \,\}}_{\text{For } \log_{5/4} n \text{ levels}} + \theta(n^{\log_{5/4} 2})$$

Ans . $\theta(n \log_{5/4} n)$

2. T(n) = 3T(n/4) + cn2



- Cost of level-0 = cn^2
- Cost of level-1 = $c(n/4)^2 + c(n/4)^2 + c(n/4)^2 = (3/16)\ cn^2$
- Cost of level-2 = $c(n/16)^2 \times 9 = (9/16^2)\ cn^2$

Cost of last level = $n^{\log_4 3} \times T(1) = \theta(n^{\log_4 3})$

$$T(n) = \underbrace{\left\{ cn^2 + \frac{3\ cn^2}{16} + \frac{9\ cn^2}{(16)^2} + \cdots\cdots\cdots \right\}}_{\text{For } \log_4 n \text{ levels}} + \theta\left(n^{\log_4 3}\right)$$

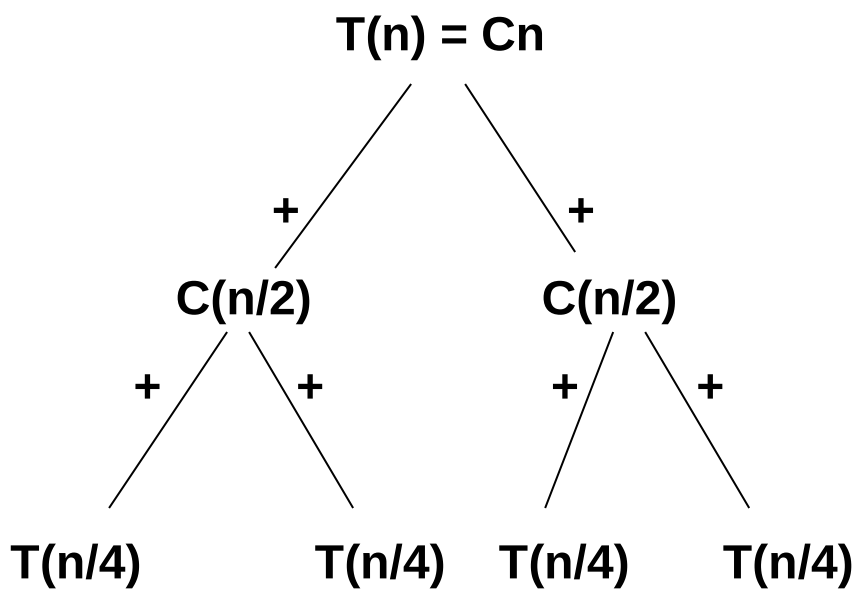
On solving, we get-

$= (16/13)\ cn^2 \{1 - (3/16)^{\log_4 n}\} + \theta(n^{\log_4 3})$

$= (16/13)\ cn^2 - (16/13)\ cn^2\ (3/16)^{\log_4 n} + \theta(n^{\log_4 3})$

Ans. = $O(n^2)$

3.T(n) = cn + 2t(n/2)



nk = θ(nlogn)+c

assume that n/2^k = 1:

n = 2^k

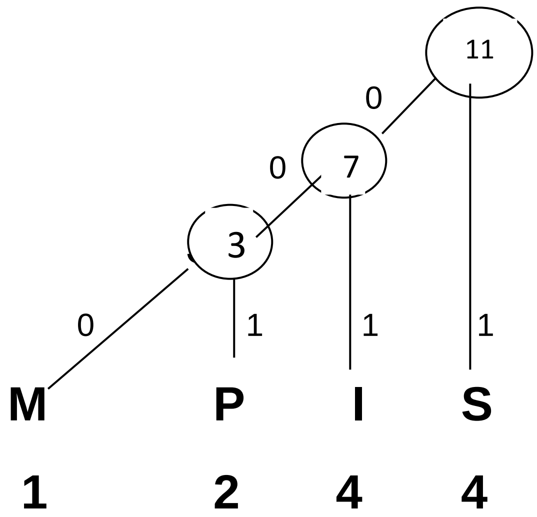
k = logn

3. Design a greedy algorithmic technique using binary min heap to encode the word ***'Mississippi'*** using variable length codeword. Calculate the number of bits may be required for encoding the message 'Mississippi'?

Word: Mississippi

Letter	**Frequency**	**Binary**
M	**1**	**000**
I	**4**	**01**
S	**4**	**1**
P	**2**	**001**

Huffman tree:



Character code = 4 * 8 = 32 bits

Huffman code = (2*3) +2+1 = 9 bits

Total = 41 bits

Total size of message = (1*3) +(4*2) +(4*1) +(2*3) = 21 bits

$\therefore \text{Total compressed size} = 62 \text{ bits}$

- **M.Sathishkumar**
- **E0119052**
- **B. Tech CSE AI & ML**

21-10-2020

X

M.Sathishkumar
Student
Signed by: 69097380-d90a-4c4d-9ad0-9a1a3b909ff2