American Economic Review

A Test for Relative Efficiency and Application to Indian Agriculture

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Economic efficiency is an elusive concept in which the economist, the engineer, and the policy maker all have great stakes.[1] The policy implications of economic efficiency permeate both the micro- and the macroeconomic level. Suppose, for example, that we can measure the efficiency of small and large farms. We can then determine by how much a given set of farms could be expected to increase its output through appropriate reorganization without absorbing additional resources in the aggregate.[2] We can also draw policy recommendations in connection with land ceilings, land redistribution, and land groupings under cooperative farming and other forms of agrarian organization.[3]

The difficulties with the existing approaches to efficiency are both conceptual and empirical. We will illustrate by pointing out the ambiguities of the conventional variants of efficiency, which we will classify as economic efficiency, price or allocative efficiency and technical efficiency.[4]

The simplest—and most naive—measure of economic efficiency is a partial productivity index, usually that of labor, although occasionally farm land (see Morton Paglin, 1965). This approach ignores the presence of other factors which affect average (and marginal) productivity.[5] A more sophisticated approach constructs indexes of efficiency that consist of a weighted average of inputs (the weights being either relative prices or relative factor shares) which is compared to output. Such an index is basically an output-cost ratio (see Paglin 1965, Robert Bennett). This approach runs into the usual index number problems that have been so aptly summarized by Evsey Domar.

Price or allocative efficiency traditionally rests on an index of marginal product and opportunity cost.[6] A number of problems arise in connection with this approach to price efficiency. First, it is an absolute concept which is of doubtful usefulness when one compares different groups of firms, even after allowing for differences in production functions and input prices. If among all inputs, the ratios of marginal products to opportunity costs are equal to one, a firm is price-efficient. Direct comparison among firms that satisfy

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[1] For a discussion of the conceptual and theoretical complications that arise in connection with efficiency, see Margaret Lady Hall and Christopher Winsten.

[2] This is the approach of Harvey Leibenstein and William Comanor.

[3] Most frequently studies of efficiency lead to this kind of microeconomic policy implication. Specific examples are supplied below.

[4] In our own formulation below each of these terms will be given specific and rigorous conceptual and empirical content.

[5] These other factors can be ignored with impunity only under certain special circumstances.

[6] For examples, see Theodore Schultz, W. David Hopper, Yotopoulos (1968 a and b).

this equality to different degrees is almost impossible, as the literature on the second best has demonstrated. Second, it is a rather rigid concept that does not allow for possible differences in the initial endowment of fixed factors.

The conventional measurement of technical efficiency concentrates on the neutral displacement of the production function either between groups of firms or over time (see Irving Hoch, Yair Mundlak). Contrary to price efficiency which is purely a behavioral concept, technical efficiency is purely an engineering concept. It entirely abstracts from the effect of prices. An alternative approach to technical efficiency has been suggested by Michael Farrell. Under the assumption of constant returns to scale Farrell derives the "pessimistic" unit-isoquant, i.e., the isoquant which envelops the observations in the inputs-unit output space in such a way that no observation lies between the pessimistic isoquant and the origin. The index of efficiency is then constructed by measuring the deviation of a specific observation from the pessimistic isoquant. Besides also ignoring the effects of relative prices, the Farrell approach has the additional disadvantage that arises when one attempts to describe a stochastic universe by a deterministic process. The pessimistic isoquant is extremely sensitive to "outliers."[7]

The deficiencies of the existing approaches to measuring efficiency should dictate the minimum requirements that a new concept of relative *economic efficiency* should meet if it is to be at all useful. (i) It should account for firms that produce different quantities of output from a given set of measured inputs of production. This is the component of differences in *technical efficiency*. (ii) It should take into account that different firms succeed to varying degrees in maximizing profits, i.e., in equating the value of the marginal product of each variable factor of production to its price. This is the component of *price efficiency*. (iii) The test should take into account that firms operate at different sets of market prices. The decision rule on profit maximization yields actual profits (as well as quantity of output supplied and quantities of variable inputs demanded) as a function, *inter alia*, of input prices. It is clear that two firms of equal technical efficiency which have successfully maximized profits would still have different value of profits as long as they face different prices.

The interrelationships of the concepts of technical efficiency, price efficiency and economic efficiency can be explained in an intuitive way. Consider two firms with production functions identical up to a neutral displacement parameter,

$$V^1 = A^1F(X^1); \qquad V^2 = A^2F(X^2),$$

where V is the output, A the technical efficiency parameter, F the production function, X the vector of inputs employed, and the superscript denotes firm.

A firm is considered more technical-efficient than another, if, given the same quantities of measurable inputs, it consistently produces a larger output. Firm 1 is more technical-efficient than firm 2 if $A^1 > A^2$.

A firm is price-efficient if it maximizes profits, i.e., it equates the value of the marginal product of each variable input to its price. A firm which fails to maximize profits is, by definition, price inefficient. Consider now two complications in connection with the definition of price efficiency. First, assume that the prices of inputs are different for each firm. Firms now equate the value of the marginal product of each factor to its firm-specific opportunity cost. Second, firms may not maxi-

[7] The approach has been modified to partly account for this shortcoming by Dennis Aigner and S. F. Chu and C. Peter Timmer.

mize profits. For such firms the usual marginal conditions do not hold. It is assumed that these firms equate the value of the marginal product of each factor to a constant (which may be firm- and factor-specific) proportion of the respective firm-specific factor prices, i.e., and for firm 1,

$$p\frac{\partial V^1}{\partial X_i^1} = k_i^1 c_i^1, \qquad k_i^1 \geqq 0$$

In this case k_i^1 indexes the decision rule that describes the firm's "profit-maximizing" behavior with respect to factor i. It encompasses perfect profit maximization as a special case when $k_i^1 = 1$ for all i. Now consider two price-inefficient firms of equal technical efficiency and facing identical output and input prices. The firm with the higher profits within a certain range of prices is considered the *relatively* more *price-efficient* firm (within the same range of prices).

Economic efficiency combines both technical and price efficiency. For this purpose consider two firms of *varying degrees of technical and price efficiency* but facing identical prices. The firm with the higher profits within a certain range of prices is considered the *relatively* more *economic-efficient* firm.

The concept of the profit function, as first introduced by D. L. McFadden, becomes operationally the ideal tool for our approach.[8] In Section I, we develop the theory of the profit function in its general form, without introducing firm-specific (and input-specific) price efficiency decision rules or firm-specific technical efficiency parameters. These are introduced in Section II which formulates the test of relative economic efficiency for the general case. In Section III we make the test of Section II operational by casting it in the framework of a Cobb-Douglas function.

Section IV is based on data from the *Farm Management Studies* (*The Studies*) of the Indian Ministry of Food and Agriculture. A number of researchers have used the same body of data to draw efficiency implications between small and large farms in India: A. K. Sen (1964, 1966); Paglin (1965); Bennett; G. S. Sahota; A. M. Khusro; to mention only a few. The findings are generally contradictory and inconclusive.[9] This comes as small surprise, given the divergent and often ambiguous concepts of efficiency that the authors have used.

A warning is in order at this point. We share the reservations of the previous authors about the limitations and the reliability of the data of *The Studies*. The reader, therefore, is urged to interpret cautiously our finding that small farms are economically more efficient than large farms. We intend our empirical application as an illustration of a method of measuring relative efficiency. It is a method that is based on the precepts of economic theory, it is more general than the existing alternatives, it is operational and it is parsimonious from the point of view of data requirements. Needless to say, the usefulness of our test is not restricted to agriculture nor is it specific for comparing small and large farms. Actually much more important insights into the form of economic organization might be forthcoming if one compares different groupings, such as owners versus share tenants, leaseholders versus tenants, adopters of new varieties versus nonadopters. Similar ramifications can be suggested for other fields of economics.

[8] Marc Nerlove first proposed a measurement of relative economic efficiency based on a profit function. However his approach is different from ours.

[9] For a summary of this discussion see Jagdish Bhagwati and Sukhamoy Chakravarty, and Yotopoulos, Lau, and Kutlu Somel.

I. The Profit Function

Consider a firm with a production function with the usual neoclassical properties

$$(1) \quad V = F(X_1, \ldots, X_m; Z_1, \ldots, Z_n)$$

where V is output, X_i represents variable inputs, and Z_i represents fixed inputs of production. Profit (defined as current revenues less current total variable costs) can be written

$$(2) \quad P' = pF(X_1, \ldots, X_m; Z_1, \ldots, Z_n) - \sum_{i=1}^{m} c_i' X_i$$

where P' is profit, p is the unit price of output, and c_i' is the unit price of the ith variable input. The fixed costs are ignored since, as it is well known, they do not affect the optimal combination of the variable inputs.

Assume that a firm maximizes profits given the levels of its technical efficiency and fixed inputs. The marginal productivity conditions for such a firm are

$$(3) \quad p\frac{\partial F(X;Z)}{\partial X_i} = c_i', \qquad i = 1, \ldots, m$$

By using the price of the output as numeraire we may define $c_i \equiv c_i'/p$ as the normalized price of the ith input. We can then write (3) as

$$(4) \quad \frac{\partial F}{\partial X_i} = c_i, \qquad i = 1, \ldots, m$$

By similar deflation by the price of output we can rewrite (2) as (5) where we define P as the "Unit-Output-Price" profit (or *UOP* profit)

$$(5) \quad P = \frac{P'}{p} = F(X_1, \ldots, X_m; Z_1, \ldots, Z_n) - \sum_{i=1}^{m} c_i X_i$$

Equation (4) may be solved for the optimal quantities of variable inputs, denoted X_i^*'s, as functions of the normalized prices of the variable inputs and of the quantities of the fixed inputs,[10]

$$(6) \quad X_i^* = f_i(c, Z), \qquad i = 1, \ldots, m$$

where c and Z are the vectors of normalized input prices and quantities of fixed inputs, respectively.

By substitution of (6) into (2) we get the *profit function*,[11]

$$(7) \quad \Pi = p\left[F(X_1^*, \ldots, X_m^*; Z_1, \ldots, Z_n) - \sum_{i=1}^{m} c_i X_i^*\right] = G(p, c_1', \ldots, c_m'; Z_1, \ldots, Z_n)$$

The profit function gives the *maximized* value of the profit for each set of values $\{p;\ c_1', \ldots, c_m';\ Z_1, \ldots, Z_n\}$. Observe that the term within square brackets on the right-hand side of (7) is a function only of c and Z. Hence we can write

$$(8) \quad \Pi = pG^*(c_1, \ldots, c_m; Z_1, \ldots, Z_n)$$

The *UOP* profit function is therefore given by

$$(9) \quad \Pi^* = \frac{\Pi}{p} = G^*(c_1, \ldots, c_m; Z_1, \ldots, Z_n)$$

Observe also that maximization of profit in (2) is equivalent to maximization of *UOP* profit in (5) in that they yield identical values for the optimal X_i^*'s. Hence Π^* in (9) indeed gives the maximized value of *UOP* profit in (5). We employ the *UOP* profit function Π^* because it is easier to work with than Π. It is evident that given Π^* one can always find Π, and vice versa.

[10] The unsubscripted variables X, Z, c', c, X^i, Z^i, c^i, and k^i are used to denote vectors. Superscripts, as above, denote firms.

[11] One should be careful to distinguish between profit as defined in (2) and the profit function in (7).

On the basis of a priori theoretical considerations we know that the *UOP* profit function is decreasing and convex in the normalized prices of variable inputs and increasing in quantities of fixed inputs. It follows also that the *UOP* profit function is increasing in the price of the output.

A set of dual transformation relations connects the production function and the profit function.[12] The most important one, from the point of view of our application here, is what is sometimes referred to as the Shephard-Uzawa-McFadden Lemma, as shown in equations (10) and (11).

$$X_i^* = -\frac{\partial \Pi^*(c, Z)}{\partial c_i}, \quad i = 1, \ldots, m, \tag{10}$$

$$V^* = \Pi^*(c, Z) - \sum_{i=1}^{m} \frac{\partial \Pi^*(c, Z)}{\partial c_i} c_i \tag{11}$$

where V^* is the supply function.

At this point we should emphasize the advantages of working with the *UOP* profit function instead of the traditional production function. First, the Shephard-Uzawa-McFadden Lemma allows us to derive the firm's supply function, V^*, and the firm's factor demand functions, X_i^*'s, directly from the *UOP* profit function of (9) instead of solving equation (4) which involves the production function.[13] Second, it is clear that the supply function and the factor demand functions may be obtained by simply starting with an *arbitrary UOP* profit function which is decreasing and convex in the normalized prices of the variable inputs and increasing in the fixed inputs. In addition, by duality, as McFadden has shown, there exists a one-to-one correspondence between the set of concave production functions and the set of convex profit functions.[14] Every concave production function has a dual which is a convex profit function, and vice versa.[15] Hence, without loss of generality, one can consider for profit-maximizing, price-taking firms, only profit functions in the analysis of their behavior *without* an explicit specification of the corresponding production function. This provides a great deal of flexibility in empirical analysis. Third, by starting from a profit function, we are assured by duality that the resulting system of supply and factor demand functions is obtainable from the maximization of a concave production function subject to given fixed inputs and under competitive markets. Fourth, the profit function, the supply function, and the derived demand functions so obtained are functions only of the normalized input prices and the quantities of fixed inputs, variables that are normally considered to be determined independently of the firm's behavior. Econometrically, this implies that these variables are exogenous variables, and by estimating these functions we avoid the problem of simultaneous equations bias to the extent that it is present.

II. Relative Economic Efficiency

The discussion of the profit function in Section I is general. It does not consider differences in technical efficiency and differences in price efficiency that might exist between firms. The purpose of this section is to introduce such differences and to combine them into the concept of rela-

[12] These relations are given and proven in McFadden and Lau.

[13] One practical advantage of using *UOP* profit functions as opposed to deriving the factor demand equations directly from equation (4) is that in many cases equation (4) cannot be solved in closed form.

[14] There are additional regularity conditions on the production and profit functions which are spelled out in detail in McFadden. Since we are interested in the empirical application of profit functions, we will not be concerned with the finer details. It suffices to say that almost all continuous production functions in current use which are concave will give rise to a well-behaved profit function.

[15] We rule out constant returns to scale in the variable factors, which, as is well known, would lead to indeterminate output and input levels. See Lau.

tive economic efficiency. Our approach is straightforward. Given comparable endowments, identical technology, and normalized input prices, the *UOP* profits of two firms should be identical if they have both maximized profits. To the extent that the one firm is more price-efficient, or more technically efficient, than the other, the *UOP* profits will differ even for the same normalized input prices and endowments of fixed inputs.

Let us represent the situation as follows. For each of two firms the production function is given by

$$V^1 = A^1F(X^1, Z^1); \tag{12}$$
$$V^2 = A^2F(X^2, Z^2)$$

where superscripts identify firms. The marginal conditions are given by

$$\frac{\partial A^1F(X^1, Z^1)}{\partial X_j^1} = \overset{1}{k_j}\overset{1}{c_j} \tag{13}$$
$$\frac{\partial A^2F(X^2, Z^2)}{\partial X_j^2} = \overset{2}{k_j}\overset{2}{c_j}$$
$$\overset{1}{k_j} \geqq 0,\ \overset{2}{k_j} \geqq 0,\ j = 1, \ldots, m$$

At this point it is useful to reiterate the basic differences in approach that equations (12) and (13) introduce, as compared to Section I. The formulation of Section I was general while now it becomes firm-specific. We can talk about relative efficiency only by comparing two or more firms. We allow for neutral differences in the production functions in terms of the firm-specific technical efficiency parameters, A^1 and A^2. They represent differences in environmental factors, in managerial ability and in other nonmeasurable fixed factors of production. If the two firms are equally technical-efficient, $A^1=A^2$. Furthermore, we now allow for a firm to be unsuccessful in its attempts to equate values of the marginal products of its inputs to their respective normalized prices. This is introduced through the firm-specific and variable input-specific k's.[16] If, and only if, two firms are equally price-efficient with respect to all variable inputs, then $k_i^1=k_i^2$, $i=1, \ldots, m$. We have defined economic efficiency to encompass both technical and price efficiency. In terms of our notation, therefore, the null hypothesis of equal relative economic efficiency for firm 1 and firm 2 implies that $A^1=A^2$ and $k^1=k^2$. The purpose of this section, therefore, is to develop a method to enable us to make this comparison.

In our formulation, the k's reflect a general systematic rule of behavior—a decision rule that gives the profit-maximizing marginal productivity conditions as a special case. That the decision rule for the firm consists of equating the marginal product to a constant times the normalized price of each input may be rationalized as follows: i) Consistent over- or under-valuation of the opportunity costs of the resources by the firm; ii) Satisficing behavior; iii) Divergence of expected and actual normalized prices; iv) Divergence of the subjective probability distribution of the normalized prices from the objective distribution of normalized prices; v) The elements of k^i may be interpreted as the first-order coefficients of a Taylor's series expansion of arbitrary decision rules of the type

$$\frac{\partial F}{\partial X_j^i} = f_j^i(c_j^i), \qquad i = 1, 2; j = 1, \ldots, m$$

where $f_j^i(0)=0$ and $f_j^{i\prime}(c_j^i) \geqq 0$. A wide class of decision rules may be encompassed under v). Observe that the right-hand sides of equation (13) may be interpreted as the "effective" prices facing the two firms. The behavior of the two firms can

[16] Of course, if a firm is perfectly successful in equalizing the normalized price of an input i to its opportunity cost, k_i assumes the value of one for that specific input.

then be viewed as profit-maximization subject to these effective prices and can be represented by the *behavioral* *UOP* profit function.

Let $G^*(c, Z)$ be the *UOP* profit function corresponding to $F(X, Z)$. By a well-known theorem proved in McFadden, the *UOP* profit function corresponding to a production function

$$V = AF(X, Z) \quad \text{is}$$

$$\Pi^* = AG^*(c/A, Z) \tag{14}$$

Recall that the $k_j^i c_j^i$'s may be interpreted as the effective prices. Thus we may write for the behavioral *UOP* profit functions of the two firms, respectively,

$$\begin{aligned} \Pi_b^1 &= A^1 G^*(k_1^1 c_1^1/A^1, \ldots, k_m^1 c_m^1/A^1; \\ &\qquad Z_1^1, \ldots, Z_n^1) \\ \Pi_b^2 &= A^2 G^*(k_1^2 c_1^2/A^2, \ldots, k_m^2 c_m^2/A^2; \\ &\qquad Z_1^2, \ldots, Z_n^2) \end{aligned} \tag{15}$$

As in the previous section, the demand functions are given by the Shephard-Uzawa-McFadden Lemma. We now, however, differentiate the behavioral *UOP* profit functions with respect to the effective prices $k_j^1 c_j^1$'s and $k_j^2 c_j^2$'s. We write[17]

$$\begin{aligned} X_j^i &= -A^i \frac{\partial G^*(k^i c^i/A^i; Z^i)}{\partial k_j^i c_j^i} \\ &= \frac{-A^i}{k_j^i} \frac{\partial G^*(k^i c^i/A^i; Z^i)}{\partial c_j^i}, \\ &\qquad i = 1, 2; j = 1, \ldots, m \end{aligned} \tag{16}$$

By correspondence from (11) the supply functions are now given by

$$\begin{aligned} V^i &= A^i G^*(k^i c^i/A^i; Z^i) \\ &\quad - A^i \sum_{j=1}^{m} k_j^i c_j^i \frac{\partial G^*(k^i c^i/A^i; Z^i)}{\partial k_j^i c_j^i} \\ &= A^i G^*(k^i c^i/A^i; Z^i) \\ &\quad - A^i \sum_{j=1}^{m} c_j^i \frac{\partial G^*(k^i c^i/A^i; Z^i)}{\partial c_j^i}, \\ &\qquad i = 1, 2 \end{aligned} \tag{17}$$

It should be emphasized at this point that X_j^i and V^i as given in (16) and (17) are the actual quantities of inputs demanded and output supplied by firm i given the firm-specific A^i and k^i. When appropriate functional forms are specified for G, statistical tests can be devised to test the null hypothesis of equal economic efficiency, i.e., $A^1 = A^2$ and $k^1 = k^2$, although not all of the parameters may be independently identified and estimated.

An alternative approach to looking at the demand and supply functions is to examine the *actual* *UOP* profit function. From (16) and (17) we can obtain the actual *UOP* profit functions by using equation (5),

$$\begin{aligned} \Pi_a^i &= V^i - \sum_{j=1}^{m} c_j^i X_j^i \\ &= A^i G^*(k^i c^i/A^i; Z^i) \\ &\quad + A^i \sum_{j=1}^{m} \frac{(1 - k_j^i) c_j^i}{k_j^i} \\ &\quad \cdot \frac{\partial G^*(k^i c^i/A^i; Z^i)}{\partial c_j^i}, \qquad i = 1, 2 \end{aligned} \tag{18}$$

Observe that i) $\partial \pi_a^i / \partial A^i > 0$, i.e., actual profit always increases with the level of technical efficiency for given normalized input prices and k^i; ii) When $k_j^i = 1$ for $j = 1, \ldots, m$, i.e., the firm is a true profit maximizer, the actual and behavioral *UOP* profit functions coincide; iii) When $A^1 = A^2$ and $k^1 = k^2$, the actual *UOP* functions of the two firms coincide with each other. Therefore one can also test the null hypothesis of equal relative economic efficiency by comparing the actual *UOP* profit functions of the two firms when appropriate functional forms are specified

[17] To simplify notation we omitted the asterisks from the demand and supply functions.

for G. This is the approach that will be employed in our empirical analysis.[18]

An additional test becomes relevant if we reject the joint hypothesis that $(A^1, k^1) = (A^2, k^2)$. In this case an overall indication of the relative efficiency between the two firms within a specified range of normalized prices for variable inputs may be obtained by comparing the actual values of the UOP profit functions within this range. If $\Pi_a^1 \geqq \Pi_a^2$ for all normalized prices within a specified range, then clearly, the first firm is relatively more efficient within the price range. If some knowledge on the probability distribution of the future prices is available, a choice may be made as to the relative efficiency of the two firms.

One can also test the hypothesis that the fixed inputs command equal rent on the two firms by computing the first derivatives of the actual UOP profit functions with respect to the fixed inputs and testing for their equality. This may have important implications for the optimal form of economic organization in terms of the distribution of fixed inputs.

III. The Formulation of the Cobb-Douglas Case

In this section we proceed to specify the appropriate functional form of the profit function and formulate empirically the test of relative economic efficiency. For this purpose one can start from a Cobb-Douglas, or for that matter, from any other form of a function. We cast our analysis in terms of the Cobb-Douglas function because it appears superior through tests of alternative functional forms.[19]

A Cobb-Douglas production function with decreasing returns in the m variable inputs and with n fixed inputs is given by[20]

$$V = A\left(\prod_{i=1}^{m} X_i^{\alpha_i}\right)\left(\prod_{j=1}^{n} Z_j^{\beta_j}\right)$$

where $$\mu = \sum_{i=1}^{m} \alpha_i < 1$$

The UOP profit function is given by

$$\Pi^* = A^{(1-\mu)^{-1}}(1-\mu) \cdot\left(\prod_{i=1}^{m} (c_i/\alpha_i)^{-\alpha_i(1-\mu)^{-1}}\right) \cdot\left(\prod_{j=1}^{n} Z_j^{\beta_j(1-\mu)^{-1}}\right) \tag{19}$$

By direct computation, the actual UOP profit functions and the demand functions for this Cobb-Douglas production function are given in equations (20) and (21). It is clear that the actual UOP profit functions of the two firms differ by a constant factor, which is a function of the k_j^i's and A^i's. In addition, all the demand functions differ by constant factors. A test of equal economic efficiency will be based on the

[18] Note that by the profit identity, one of the system of profit, supply and demand functions is redundant and should be ignored in the actual estimation of the system. Otherwise the system variance-covariance matrix will be singular.

[19] These tests are presented in Yotopoulos, Lau, and Somel.

[20] The value of $\mu<1$ is required since constant or increasing returns in the variable inputs are inconsistent with profit maximization.

$$\Pi_a^i = \left[(A^i)^{(1-\mu)^{-1}}\left(1-\sum_{j=1}^{m}\alpha_j/k_j^i\right)\right]\left[\prod_{j=1}^{m}(k_j^i)^{-\alpha_j(1-\mu)^{-1}}\right]\left[\prod_{j=1}^{m}\alpha_j^{-\alpha_j(1-\mu)^{-1}}\right] \cdot\left[\prod_{j=1}^{m}(c_j^i)^{-\alpha_j(1-\mu)^{-1}}\right]\left[\prod_{j=1}^{n}(Z_j^i)^{\beta_j(1-\mu)^{-1}}\right], \quad i = 1, 2 \tag{20}$$

$$X_l^i = (A^i)^{(1-\mu)^{-1}}(\alpha_l/k_l^i c_l^i)\left[\prod_{j=1}^m (k_j^i)^{-\alpha_j(1-\mu)^{-1}}\right]\left[\prod_{j=1}^m \alpha_j^{-\alpha_j(1-\mu)^{-1}}\right] \tag{21}$$
$$\cdot\left[\prod_{j=1}^m (c_j^i)^{-\alpha_j(1-\mu)^{-1}}\right]\left[\prod_{j=1}^n (Z_j^i)^{\beta_j(1-\mu)^{-1}}\right], \qquad i = 1, 2; \quad l = 1, \ldots, m$$

null hypothesis that all the constant factors of difference are ones.

Observe that the terms in the first three brackets of equation (20) involve constants. We thus define equation (22).

$$A_*^i \equiv (A^i)^{(1-\mu)^{-1}}\left(1 - \sum_{j=1}^m \alpha_j/k_j^i\right) \tag{22}$$
$$\left[\prod_{j=1}^m (k_j^i)^{-\alpha_j(1-\mu)^{-1}}\right]\left[\prod_{j=1}^m \alpha_j^{-\alpha_j(1-\mu)^{-1}}\right], \qquad i = 1, 2$$

Then the actual *UOP* profit functions are given by

$$\Pi_a^i = (A_*^i)\left[\prod_{j=1}^m (c_j^i)^{-\alpha_j(1-\mu)^{-1}}\right] \cdot \left[\prod_{j=1}^n (Z_j^i)^{-\beta_j(1-\mu)^{-1}}\right], \qquad i = 1, 2 \tag{23}$$

By writing A_*^2 and A_*^1 for firm 2 and firm 1, respectively, and taking the ratio of the constant terms we have

$$\frac{A_*^2}{A_*^1} = \left[\frac{A^2}{A^1}\right]^{(1-\mu)^{-1}} \frac{\left(1 - \sum_{j=1}^m \alpha_j/k_j^2\right)}{\left(1 - \sum_{j=1}^m \alpha_j/k_j^1\right)} \cdot \left[\prod_{j=1}^m \left[\frac{k_j^2}{k_j^1}\right]^{-\alpha_j(1-\mu)^{-1}}\right] \tag{24}$$

Thus one may write, from equation (20)

$$\Pi_a^1 = A_*^1\left[\prod_{j=1}^m (c_j^1)^{-\alpha_j(1-\mu)^{-1}}\right] \cdot\left[\prod_{j=1}^n (Z_j^1)^{\beta_j(1-\mu)^{-1}}\right] \tag{25}$$

$$\Pi_a^2 = A_*^1\,(A_*^2/A_*^1)\left[\prod_{j=1}^m (c_j^2)^{-\alpha_j(1-\mu)^{-1}}\right] \cdot\left[\prod_{j=1}^n (Z_j^2)^{\beta_j(1-\mu)^{-1}}\right] \tag{26}$$

Further defining

$$\alpha_j^* \equiv -\alpha_j(1-\mu)^{-1}; \tag{27}$$

and

$$\beta_j^* \equiv \beta_j(1-\mu)^{-1} \tag{28}$$

and taking natural logarithms of equations (25) and (26), we have

$$ln\ \Pi_a^1 = ln\ A_*^1 + \sum_{j=1}^m \alpha_j^*\ ln\ c_j^1 + \sum_{j=1}^n \beta_j^*\ ln\ Z_j^1, \tag{29}$$

$$ln\ \Pi_a^2 = ln\ A_*^1 + ln\frac{A_*^2}{A_*^1} + \sum_{j=1}^m \alpha_j^*\ ln\ c_j^2 + \sum_{j=1}^n \beta_j^*\ ln\ Z_j^2 \tag{30}$$

We note that if $A^1 = A^2$ *and* $k^1 = k^2$, then $A_*^1 = A_*^2$ and the two functions Π_a^1 and Π_a^2($ln\ \Pi_a^1$ and $ln\ \Pi_a^2$) should be identical. This implies that $ln\ A_*^2/A_*^1 = 0$. We can therefore test the equal relative efficiency hypothesis by utilizing a firm dummy variable in the logarithmic *UOP* profit function and examining if its value is equal to zero. It should be noted that for the Cobb-Douglas production function case, differences in technical efficiency and relative differences in price efficiency cannot be separately identified from the actual *UOP* profit functions.

IV. Empirical Implementation and Statistical Results

In this section we use data from *The Studies* of the Indian Ministry of Food and Agriculture to estimate the *UOP* profit functions for the small and large farms and to apply the test of equal economic efficiency for the two groups. *The Studies*, which have proven to be a bountiful data source for many researchers,[21] are based on cost-accounting records of 2,962 holdings in the six states of India and cover the three-year period, 1955–57.[22] All the data are, however, reported only in terms of averages of farms of a given size for each state. From the available raw data we proceed to specify as follows the variables of our analysis.

Output is given in terms of revenue V per farm in rupees; land T represents cultivable land per farm in acres,[23] and capital K is defined in terms of interest charges paid or imputed on the quantity of fixed capital per farm.[24] Labor is given in terms of labor days employed per farm as well as in terms of a labor cost per farm concept (i.e., cost of hired labor plus imputed cost of family labor). By dividing the latter labor concept through by the former we define the money wage rate per day, w'. Only three inputs are distinguished: labor, capital, and land. We treat labor as the variable input of production and land and capital as fixed inputs.[25] It appears reasonable, from both institutional reasons and from the periodic nature of the agricultural technology, that the latter may be considered as fixed inputs in the short run. Finally, from the revenue we subtract the total cost of variable inputs per farm, i.e., the wage bill, in order to define the profit variable, Π. It should be recalled that in the *UOP* profit function formulation of the preceding sections both Π^* and w are expressed in real terms. Unavailability of the prices for deflation poses a problem that will be discussed below.

For the Cobb-Douglas case, the profit function is given by (29) as

$$(31) \qquad ln\ \Pi_a^1 = ln\ A_*^1 + \alpha_1^* \ ln\ w + \beta_1^* \ lnK + \beta_2^* \ ln\ T$$

[21] Besides the literature based on *The Studies* that is surveyed in Bhagwati and Chakravarty (especially pp. 40 ff.), one should also notice Paglin, Sahota and Yotopoulos, Lau and Somel.

[22] For this analysis we utilize data from the following states and years: West Bengal, Madras, Uttar Pradesh, Punjab, 1955–56; Madhya Pradesh, 1956–57. The latter is chosen because the 1955–56 report of *The Studies* for Madhya Pradesh does not contain comparable information as the others.

[23] It is assumed that the land input is homogeneous at least within states across farm sizes. This hypothesis was tested in Yotopoulos, Lau, and Somel.

[24] This definition of the capital concept is especially disturbing. Inasmuch as the interest rate used in the imputation—3 percent—is uniform throughout the states and the years, the true quantity of fixed capital will be proportional to our measure. This implicitly assumes that the flow of capital services as a ratio of the stock of capital is constant across farms. Such assumptions, as demonstrated by Yotopoulos (1967, 1968a), may lead to unreliable estimates of the coefficient of capital in a production function formulation. It appears that this may be the case with our estimated capital coefficient.

[25] Total other costs (i.e., costs other than labor costs, interest on fixed capital and land rent) should also be treated as a variable input of production. This is impossible in our profit function formulation due to the fact that we lack the "price" of other costs which is necessary for the *UOP* profit function. To the extent that the price of other costs varies only across states, its effect is captured by the state dummies. An alternative rationalization is that the other costs are employed in fixed proportions to output.

$$\text{(32)} \quad ln\ \Pi_a^2 = ln\ A_*^1 + ln\ (A_*^2/A_*^1) + \alpha_1^*\ ln\ w + \beta_1^*\ ln\ K + \beta_2^*\ ln\ T$$

where Π_a^i is actual *UOP* profit (total revenue less total variable cost, divided by the price of output), w is normalized wage rate, K is interest on fixed capital, and T is cultivable land. A maintained hypothesis is that the production function is identical on large and small farms up to a neutral efficiency parameter. This implies that the coefficients corresponding to $ln\ w$, $ln\ K$, and $ln\ T$ are identical for large and small farms. A problem arises at this point. Our formulation of the *UOP* profit function is in terms of normalized input prices. However, in our empirical application these normalized input prices are not available since the data on money prices of output are rather poor. Fortunately, we note that equations (31) and (32) may be rewritten

$$\begin{aligned} ln\ \Pi_a^1 &= ln\ \Pi'^1 - ln\ p \\ &= ln\ A_*^1 + \alpha_1^*\ ln\ w' - \alpha_1^*\ ln\ p \\ &\quad + \beta_1^*\ ln\ K + \beta_2^*\ ln\ T \end{aligned}$$

or

$$\begin{aligned} ln\ \Pi^1 &= ln\ A_*^1 + (1 - \alpha_1^*)\ ln\ p \\ &\quad + \alpha_1^*\ ln\ w' + \beta_1^*\ ln\ K \\ &\quad + \beta_2^*\ ln\ T \\ ln\ \Pi^2 &= ln\ A_*^1 + (1 - \alpha_1^*)\ ln\ p \\ &\quad + ln\left(\frac{A_*^2}{A_*^1}\right) + \alpha_1^*\ ln\ w' \\ &\quad + \beta_1^*\ ln\ K + \beta_2^*\ ln\ T \end{aligned}$$

where Π'^i is money profit in rupees, w', is the money wage rate in rupees per day, and p is the price of the output in rupees.

If the prices of outputs differ only across states, then one can insert state dummy variables to capture the effect of differences due to $(ln\ A_* + (1-\alpha_1^*) ln\ p)$. This also allows for interstate differences in the efficiency parameter in A_*. Hence our final estimating equation consists of

$$\text{(33)} \quad \begin{aligned} ln\ \Pi &= \alpha_0 + S + \sum_{i=1}^{4} \delta_i^* D_i + \alpha_1^*\ ln\ w' \\ &\quad + \beta_1^*\ ln\ K + \beta_2^*\ ln\ T \end{aligned}$$

where Π is farm profit in rupees (excluding interest on capital and land rent), w' is money wage rate, K is interest on fixed capital, T is cultivable land, the D_i's are state dummy variables and S is a dummy variable with value 1 for large farms and 0 for small farms. Large farms are defined as those with cultivable land greater than ten acres per farm.

A remark about the stochastic specification of the model is appropriate at this point. Not much is known about how disturbance terms in general should be introduced into economic relationships although Hoch, Mundlak and Hoch, and subsequently Zellner, Kmenta and Drèze have proposed one possible assumption that is workable in the Cobb-Douglas case. Here we assume that the error in the profits is due to climatic variations, divergence of the expected output price from the realized output price, imperfect knowledge of the technical efficiency parameter of the farm, and differences in technical efficiency among farms within the same size class. The demand functions are exact, or, in any case, if they are subject to error, the errors are uncorrelated with the errors of the logarithmic profit function, which are assumed to have zero expectation. Hence one can estimate the natural logarithms of the profit function alone with the least squares estimator, which in this case turns out to be minimum variance, linear and unbiased. This specification is similar to the one used by Marc Nerlove in his pioneering study of empirical cost functions.

The results of the estimation are presented in Table 1. The F-value indicates that the hypothesis that all coefficients other than α_0 are zeroes should be rejected. The coefficient of the wage rate is negative while the coefficient of land is positive, in accord with a priori economic theory: the *UOP* profit function is decreasing in w' and increasing in T. The negative coefficient of capital can only be attributed to the misspecification of this variable that is due to the implicit assumption of proportionality between capital service flow and capital stock (Yotopoulos, 1967, 1968a). In addition, the second derivative of the *UOP* profit function with respect to the wage rate is

$$\frac{\partial^2\Pi^*}{\partial w^2} = \frac{\alpha_1^{*2}\Pi^*}{w^2} - \frac{\alpha_1^*\Pi^*}{w^2}$$

$$= \alpha_1^*(\alpha_1^* - 1)\frac{\Pi^*}{w^2}$$

$$= -2.141\,(-3.141)\frac{\Pi^*}{w^2} \geqq 0$$

as $\Pi > 0$ and hence $\Pi^* > 0$ for our whole sample. This also confirms the convexity assumption of the *UOP* profit function.

The estimates of Table 1 imply, by (27) and (28), estimates of the input elasticities of the production function. These are presented in Table 2. The elasticities appear reasonable by comparison with other available estimates of Cobb-Douglas agricultural production functions for India and other parts of the world.

We note that the capital coefficient as estimated directly also has a negative sign. Finally, the sum of elasticities obtained from the indirect estimates is somewhat larger than that obtained from the direct

TABLE 1—COBB-DOUGLAS PROFIT FUNCTION AND RELATED STATISTICS

Parameter	All Farms (n=34)
α_0	4.582
	(0.548)
S	−0.567
	(0.253)
δ_1^*	1.614*
	(0.549)
δ_2^*	−1.359*
	(1.274)
δ_3^*	−0.588*
	(0.485)
δ_4^*	0.296
	(0.715)
α_1^*	−2.141**
	(1.200)
β_1^*	−0.588
	(0.274)
β_2^*	1.797
	(0.233)
$\hat{\sigma}^2$	0.185
$\bar{R}^2$	0.896
F-Statistic	36.4

Source: Farm Management Studies

Notes: The estimating equation is

$$ln\,\Pi = \alpha_0 + S + \sum_{i=1}^{4}\delta_i^* D_i + \alpha_1^*\,ln\,w' + \beta_1^*\,ln\,K + \beta_2^* ln\,T$$

where

Π = profit, i.e., total revenue less total variable costs

w' = the money wage rate

S = dummy variable for farm size with value of one for large farms (greater than ten acres) and zero for small farms (less than ten acres)

D_i = regional dummy variable with D_1, D_2, D_3, D_4 taking the value of one for West Bengal, Madras, Madhya Pradesh, and Uttar Pradesh and zero elsewhere, respectively

K = interest on fixed capital

T = cultivable land in acres

$\hat{\sigma}^2$ = the estimate of the variance of the error in the equation

* Starred coefficients are not significantly different from zero at a probability level ≧ 95 percent

** Double-starred coefficients are not significantly different from zero at a probability level ≧ 95 percent; but they are significantly different from zero at a probability level ≧ 90 percent.

All other coefficients are significantly different from zero at a probability level ≧ 95 percent.

Two-tail test applies to the coefficients of the dummy variables; one-tail test to all other variables.

The standard errors of the estimated parameters are given in parentheses.

TABLE 2—COMPARISON OF DIRECT AND INDIRECT ESTIMATES OF INPUT ELASTICITIES OF THE PRODUCTION FUNCTION

Parameters	Direct Estimates	Indirect Estimates
α_1	.606	.682
β_1	−.103	−.187
β_2	.365	.572
$(\alpha_1+\beta_1+\beta_2)$	0.868	1.067

Source: Farm Management Studies.
Notes: The direct estimates are obtained by ordinary least squares regression of the natural logarithm of output on the natural logarithms of the three inputs, the farm size dummy, the four state dummies and the constant term. The indirect estimates of the parameters are derived from equations (27) and (28). Other notations as in Table 1.

estimates. It is slightly larger than one.[26] One may also add that the estimates obtained by fitting the profit function are statistically consistent, as opposed to those obtained directly from the production function by ordinary least squares, which are in general inconsistent because of the existence of simultaneous equations bias.

As we have indicated in the previous section the hypothesis of relative efficiency can be cast in terms of the constant term by which the two profit functions, one for small and one for large farms, differ. The null hypothesis is that the constant factor is equal to one. Furthermore, if one takes natural logarithms before estimating the profit function, the constant term becomes the coefficient of a dummy variable that differentiates the two groups of farms and the test becomes that the coefficient of the dummy variable is not significantly different from zero. Our results, therefore, reject the hypothesis of equal efficiency between the two groups. Furthermore, the sign of the dummy variable indicates that small farms are more profitable, i.e., more efficient, at all observed prices of the variable input, given the distribution of the fixed factors of production.

[26] An exact linear restriction is available for the testing of the hypothesis of constant returns to scale within the profit function framework. See Lau, also Lau and Yotopoulos.

Given the actual profit functions for the two groups of farms, one can estimate the rate of return on the fixed inputs by computing the partial derivatives of the *UOP* profit function with respect to both capital and land. Suppressing interstate differences, one has

$$\frac{\partial \Pi}{\partial K} = \beta_1^* \frac{\Pi}{K} = -0.588\frac{\Pi}{K} \tag{34}$$

$$\frac{\partial \Pi}{\partial T} = \beta_2^* \frac{\Pi}{T} = 1.797\frac{\Pi}{T} \tag{35}$$

These rates of returns are computed at the geometric mean of the large and small farms separately, and reported in Table 3. It is seen that both the rates of return on fixed capital and on land are larger on the small farms at the existing set of normalized prices faced by these farms.

V. Summary and Conclusion

In our formulation of the test of equal relative economic efficiency we use McFadden's profit function, which expresses a firm's maximized profit as a function of the prices of output and variable inputs of production and of the quantities of the

TABLE 3—COMPARISON OF THE RATES OF RETURN ON FIXED CAPITAL AND LAND BETWEEN LARGE AND SMALL FARMS

	Large Farms	Small Farms
Geometric Means		
$\bar{\Pi}$ (rupees)	2,184.62	493.90
$\bar{K}$ (rupees)	51.35	22.89
$\bar{T}$ (acres)	23.81	3.99
Rates of Return		
$\frac{\partial \Pi}{\partial K}$ (rupees per rupee)	−25.02	−12.69
$\frac{\partial \Pi}{\partial T}$ (rupees per acre)	164.88	222.44

Source: Farm Management Studies.

fixed factors. In the Cobb-Douglas formulation the comparison of relative efficiency of two groups of firms is simply made by examining the coefficient of the group dummy variable.

A crucial feature of the profit function analysis is that it assumes firms behave according to certain decision rules, which include the profit maximization rules, given the price regime for output and variable inputs and given the quantities of their fixed factors of production. For the purposes of this paper, the existence of these systematic decision rules is a maintained hypothesis. It can, however, be tested directly within the framework developed by Wise and Yotopoulos.

The conclusion of the test of relative economic efficiency is in favor of the small farms (i.e., farms of less than ten acres). It appears that, given the fixed factors of production (land and fixed capital) and within the ranges of the observed prices of output and variable inputs (labor), the small farms of the sample of *The Studies* have higher actual profits. In the context of our analysis, this finding means that the small farms attain higher levels of price efficiency (i.e., of optimal price behavior) and/or they operate at higher levels of technical efficiency. This finding, should it be confirmed by similar tests with other sets of data, may imply that in agriculture the supervisory role of the owner-manager of the farm may be crucial for attaining high levels of economic efficiency. Within the context of this hypothesis, the test would draw the limits of the supervisory capacity of the manager at the ten acre farms.

REFERENCES

D. J. Aigner and S. F. Chu, "On Estimating the Industry Production Function," *Amer. Econ. Rev.*, Sept. 1968, *58*, 826–39.

R. L. Bennett, "Surplus Agricultural Labor and Development—Facts and Theories: Comment," *Amer. Econ. Rev.*, Mar. 1967, *57*, 194–202.

J. N. Bhagwati and S. Chakravarty, "Contributions to Indian Economic Analysis: A Survey," *Amer. Econ. Rev. Supp.*, Sept. 1969, *59*, 1–73.

W. S. Comanor and H. Leibenstein, "Allocative Efficiency, X-Efficiency and the Measurement of Welfare Losses," *Economica*, Aug. 1969, *36*, 304–09.

E. Domar, "On Total Productivity and All That," *J. Polit. Econ.*, Dec. 1962, *70*, 597–608.

M. J. Farrell, "The Measurement of Productive Efficiency," *J. Roy. Statist. Soc.*, 1957, Part 3, *120*, 253–82.

M. L. Hall and C. B. Winsten, "The Ambiguous Notion of Efficiency," *Econ. J.*, Mar. 1959, *69*, 71–86.

I. Hoch, "Estimation of Production Function Parameters and Testing for Efficiency," *Econometrica*, 1955, *23*, 325–26.

———, "Simultaneous Equation Bias in the Context of the Cobb-Douglas Production Function," *Econometrica*, Oct. 1958, *26*, 556–78.

W. D. Hopper, "Allocation Efficiency in a Traditional Indian Agriculture," *J. Farm Econ.*, Aug. 1965, *47*, 611–24.

A. M. Khusro, "Returns to Scale in Indian Agriculture," *Indian J. Agr. Econ.*, Oct. 1964, *19*, 51–80.

L. J. Lau, "Applications of Profit Functions," in D. L. McFadden, ed., *The Econometric Approach to Production Theory*, Amsterdam 1971, forthcoming.

——— **and P. A. Yotopoulos,** "Profit, Supply and Factor Demand Functions with Application to Indian Agriculture," Memorandum No. 98, Research Center in Economic Growth, Stanford Univ., 1970.

H. Leibenstein, "Allocative Efficiency vs. 'X-Efficiency'," *Amer. Econ. Rev.*, June 1966, *56*, 392–415.

D. L. McFadden, "Cost, Revenue, and Profit Functions," in D. L. McFadden, ed., *The Econometric Approach to Production Theory*, Amsterdam 1971, forthcoming.

Y. Mundlak, "Empirical Production Function Free of Management Bias," *J. Farm Econ.*, Feb. 1961, *43*, 44–56.

——— and I. Hoch, "Consequences of Alternative Specifications in Estimation of Cobb-Douglas Production Functions," *Econometrica*, Oct. 1965, *33*, 814–28.

M. Nerlove, "Returns to Scale in Electricity Supply," in C. F. Christ et al., eds., *Measurement in Economics: Studies in Mathematical Economics and Econometrics in Memory of Yehuda Grunfeld*, Stanford 1960, 167–98.

———, *Estimation and Identification of Cobb-Douglas Production Functions*, Chicago, 1965.

M. Paglin, " 'Surplus' Agricultural Labor and Development—Facts and Theories," *Amer. Econ. Rev.*, Sept. 1965, *55*, 815–34.

———, "Surplus Agricultural Labor and Development—Facts and Theories: Reply," *Amer. Econ. Rev.*, Mar. 1967, *57*, 202–09.

G. S. Sahota, "Efficiency of Resource Allocation in Indian Agriculture," *Amer. J. Agr. Econ.*, Aug. 1968, *50*, 584–605.

T. W. Schultz, *Transforming Traditional Agriculture*, New Haven 1964.

A. K. Sen, "Size of Holdings and Productivity," *Econ. Weekly*, Feb. 1964, *16*, 323–26.

———, "Peasants and Dualism With or Without Surplus Labor," *J. Polit. Econ.*, Oct. 1966, *74*, 425–50.

R. W. Shephard, *Cost and Production Functions*, Princeton 1953.

C. P. Timmer, "On Measuring Technical Efficiency," unpublished doctoral dissertation, Harvard Univ. 1969.

H. Uzawa, "Duality Principles in the Theory of Cost and Production," *Int. Econ. Rev.*, May 1964, *5*, 216–20.

J. Wise and P. A. Yotopoulos, "The Empirical Content of Economic Rationality: A Test for a Less Developed Economy," *J. Polit. Econ.*, Nov. 1969, *77*, 976–1004.

P. A. Yotopoulos, (1968a) *Allocative Efficiency in Economic Development: A Cross Section Analysis of Epirus Farming*, Center of Planning and Economic Research, Athens 1968.

———, (1968b) "On the Efficiency of Resource Utilization in Subsistence Agriculture," *Food Res. Inst. Stud. Agr. Econ., Trade, Devel.*, 1968, *8*, 125–35.

———, "From Stock to Flow Capital Inputs for Agricultural Production Functions: A Microanalytic Approach," *J. Farm Econ.*, May 1967, *49*, 476–91.

——— and L. J. Lau, "A Simultaneous Equation Approach to Relative Economic Efficiency," Memorandum No. 104, Research Center in Economic Growth, Stanford University 1970.

———, ———, and K. Somel, "Labor Intensity and Relative Efficiency in Indian Agriculture," *Food Res. Inst. Stud. Agr. Econ., Trade, Develop.*, 1970, *9*, 43–55.

A. Zellner, J. Kmenta and J. Drèze, "Specification and Estimation of Cobb-Douglas Production Function Models," *Econometrica*, Oct. 1966, *34*, 784–95.

The Government of India, Ministry of Food and Agriculture, *Studies in the Economics of Farm Management*, Delhi 1957–62. Reports for the year 1955–1956: Madras, Punjab, Uttar Pradesh, West Bengal; Report for the year 1956–1957: Madhya Pradesh.

Appendix

Table 1—Data for Indian Agriculture[a]

State	T	K	L	w'	Π	V
West Bengal	12.15	127.33	402.41	1.54	923.28	1,811.56
	16.96	116.00	628.37	1.61	772.36	2,403.23
	.64	7.44	39.05	1.60	187.78	129.41
	1.81	14.84	97.96	1.49	373.03	352.59
	3.11	25.19	173.10	1.59	555.87	547.05
	4.47	33.30	213.58	1.53	1,948.21	809.07
	6.18	41.59	321.42	1.45	813.20	1,158.13
	8.15	37.89	323.80	1.54	955.08	1,401.80
Madras	11.81	86.21	336.58	.54	1,653.61	907.01
	17.35	93.69	395.58	.56	2,215.54	1,174.59
	22.97	103.36	560.41	.62	2,248.45	1,683.70
	43.78	205.76	897.49	.55	5,838.73	3,607.47
	1.61	39.60	179.35	.62	426.00	354.04
	3.66	37.69	229.85	.52	716.90	751.03
	6.02	67.42	276.92	.56	2,045.88	947.55
	8.83	98.89	342.60	.56	763.14	1,190.28
Madhya Pradesh	12.44	9.57	294.70	1.08	1,709.28	1,479.12
	17.19	11.86	403.45	1.00	6,718.47	1,693.21
	24.25	14.55	470.21	1.11	40.53	2,616.57
	34.77	31.64	756.25	1.04	144.37	3,689.10
	45.17	41.10	1,084.08	1.11	157.86	4,458.28
	93.36	82.15	1,831.72	1.15	334.62	10,017.53
	2.95	3.42	101.13	.94	513.87	422.73
	7.38	8.63	190.40	1.06	729.34	849.44
Uttar Pradesh	12.00	78.00	602.40	1.06	7.57	2,448.00
	16.90	95.99	765.57	1.06	320.98	3,380.00
	27.58	148.93	1,073.14	1.01	384.68	5,653.90
	3.33	31.00	209.16	1.01	411.48	922.41
	7.68	64.97	432.84	.98	227.80	1,843.20
Punjab	14.50	19.57	450.22	1.51	448.19	2,463.55
	28.45	20.48	701.86	1.38	124.41	4,056.97
	81.19	30.85	1,484.96	1.92	391.14	12,957.92
	3.98	8.95	158.96	1.33	129.43	702.47
	7.45	7.37	270.88	1.40	377.94	1,270.22

[a] For identification of variables, see Section IV of text.