Recall rule for square roots (approximation of true root or the definition through peasant math):

$$\sqrt{x} = n + \frac{x - n^2}{2n+1},$$

where $n = \max z | z^2 \leq x$; the $2n+1$ term comes from $(n+1)^2 - n^2$. We can extend to any k-th root similarly with "error" $\frac{x-n^k}{(n+1)^k - n^k}$. Based on this logic, (as long as we know what "exponent" means) we can make a logarithm approximation:

$$\log_a(x) = n + \frac{x - a^n}{a^{n+1} - a^n},$$

where $n = \max z | a^z \leq x$. Perhaps the abstract pattern is clear from this, so that now peasant math accommodates the concept of inverse:

$$f(x) = n + \frac{x - f(n)}{f(n+1) - f(n)},$$

where $n = \max z | f(z) \leq x$ (of course, the denominator cannot be zero, but if a function has an inverse it's one-to-one, so no problems).

Note that these rules "estimate from below"; we can also estimate from above:

$$f(x) = n - \frac{f(n) - x}{f(n+1) - f(n)},$$

where $n = \min z | f(z) \geq x$.

Potential work/questions/comments:

*is any/all this straying too far from the idea of using dots/pictures to understand things intuitively?

*for what functions does this approximation work/fail? Looks like from the root pictures that large $f(n+1) - f(n)$ values are bad. I know that in diff.eq.'s linear (or sublinear) growth conditions are often assumed...

*what if we take the mean/median of the the above/below estimates to see if they give better estimates? or compute the variance?

*connection to the derivative? the fraction part kind of looks like derivative when $f(x) \approx x$, which makes the fraction part resemble $x - n$, so it's a trivial approximation $f(x) \approx n + (x - n) \approx x$. ... does that mean anything?