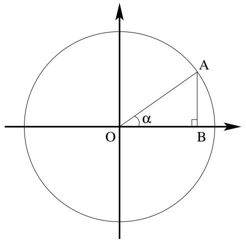
Trigonometry Basics



$$\sin\alpha = \frac{AB}{OA},\quad \cos\alpha = \frac{OB}{OA},$$
$$\tan\alpha = \frac{AB}{OB},\quad \cot\alpha = \frac{OB}{AB},$$
$$\sec\alpha = \frac{OA}{OB},\quad \csc\alpha = \frac{OA}{AB}$$

Trigonometry Basic Formulas

$$\sin^2\alpha + \cos^2\alpha = 1 \qquad \tan\alpha = \frac{1}{\cot\alpha}$$
$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} \qquad \cot\alpha = \frac{\cos\alpha}{\sin\alpha}$$
$$\sec\alpha = \frac{1}{\cos\alpha} \qquad \csc\alpha = \frac{1}{\sin\alpha}$$
$$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} = \sec^2\alpha$$
$$1 + \cot^2\alpha = \frac{1}{\sin^2\alpha} = \csc^2\alpha$$

Popular Angles

	0	$\frac{\pi}{6}(30^o)$	$\frac{\pi}{4}(45^o)$	$\frac{\pi}{3}(60^o)$	$\frac{\pi}{2}(90^o)$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\pm\infty$
cot	$+\infty$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0
sec	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	$\pm\infty$
csc	$+\infty$	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1

Double-Angle

$$\sin 2\alpha = 2\sin\alpha\cos\alpha \qquad \cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$
$$\cos 2\alpha = 1 - 2\sin^2\alpha \qquad \cos 2\alpha = 2\cos^2\alpha - 1$$
$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} \qquad \cot 2\alpha = \frac{\cot^2\alpha - 1}{2\cot\alpha}$$
$$\sin 2\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha} \qquad \cos 2\alpha = \frac{1-\tan^2\alpha}{1+\tan^2\alpha}$$
$$\tan\alpha + \cot\alpha = \frac{2}{\sin 2\alpha} \qquad \cot\alpha - \tan\alpha = \frac{2}{\tan 2\alpha}$$
$$1 - \tan^2\alpha = \frac{\cos 2\alpha}{\cos^2\alpha} \qquad 1 - \cot^2\alpha = -\frac{\cos 2\alpha}{\sin^2\alpha}$$
$$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha \qquad \cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$$

Half-Angle

$$\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}} \qquad \cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}}$$
$$\sin\alpha = \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \qquad \cos\alpha = \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$\tan\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} \qquad \tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}$$
$$\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha}$$
$$\cot\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}} = \frac{1+\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1-\cos\alpha}$$
$$\tan\alpha\tan\frac{\alpha}{2} = \sec\alpha - 1 = \frac{1-\cos\alpha}{\cos\alpha}$$
$$1 + \sin\alpha = 2\cos^2(\frac{\pi}{4} - \frac{\alpha}{2}) \qquad 1 + \cos\alpha = 2\cos^2\frac{\alpha}{2}$$
$$1 - \sin\alpha = 2\sin^2(\frac{\pi}{4} - \frac{\alpha}{2}) \qquad 1 - \cos\alpha = 2\sin^2\frac{\alpha}{2}$$
$$1 + \tan\alpha = \sqrt{2}\,\frac{\sin(\frac{\pi}{4}+\alpha)}{\cos\alpha}$$
$$1 - \tan\alpha = \sqrt{2}\,\frac{\sin(\frac{\pi}{4}-\alpha)}{\cos\alpha}$$

Sums of Angles

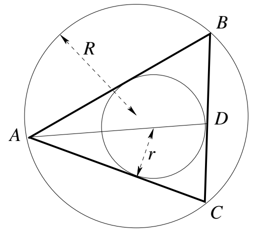
$$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\tan(\alpha\pm\beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$
$$\cot(\alpha\pm\beta) = \frac{\cot\alpha\cot\beta \mp 1}{\cot\alpha \pm \cot\beta}$$
$$\sin(\alpha+\beta)\sin(\alpha-\beta) = \sin^2\alpha - \sin^2\beta$$
$$\cos(\alpha+\beta)\cos(\alpha-\beta) = \cos^2\alpha - \sin^2\beta$$

Phase Shift

	$-\alpha$	$\frac{\pi}{2}-\alpha$	$\frac{\pi}{2}+\alpha$	$\pi-\alpha$	$\pi+\alpha$	$\frac{3\pi}{2}-\alpha$	$\frac{3\pi}{2}+\alpha$	$2\pi-\alpha$
sin	$-\sin\alpha$	$\cos\alpha$	$\cos\alpha$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$
cos	$\cos\alpha$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$	$\sin\alpha$	$\cos\alpha$
tan	$-\tan\alpha$	$\cot\alpha$	$-\cot\alpha$	$-\tan\alpha$	$\tan\alpha$	$\cot\alpha$	$-\cot\alpha$	$-\tan\alpha$
cot	$-\cot\alpha$	$\tan\alpha$	$-\tan\alpha$	$-\cot\alpha$	$\cot\alpha$	$\tan\alpha$	$-\tan\alpha$	$-\cot\alpha$

$$\cos(\alpha+\pi n) = (-1)^n\cos\alpha$$
$$\sin(\alpha+\pi n) = (-1)^n\sin\alpha$$

Sine/Cosine Theorem



$$\frac{BC}{\sin\widehat{BAC}} = \frac{AC}{\sin\widehat{ABC}} = \frac{AB}{\sin\widehat{ACB}} = 2R$$
$$\sin\frac{\widehat{BAC}}{2} = \sqrt{\frac{(P-AC)(P-AB)}{AB\ AC}}, \quad P = \frac{AB+AC+BC}{2}$$
$$\cos\frac{\widehat{BAC}}{2} = \sqrt{\frac{P(P-BC)}{AB\ AC}}$$
$$BC^2 = AC^2 + AB^2 - 2\ AC\ AB\cos\widehat{BAC}$$
$$S_{ABC} = \frac{1}{2}BC\ AC\sin\widehat{ACB} = 2R^2\sin\widehat{BAC}\ \sin\widehat{ABC}\ \sin\widehat{ACB} =$$
$$= \sqrt{P(P-BC)(P-AC)(P-AB)} = \frac{AC\ AB\ BC}{4R} = Pr$$
$$m_{BC} = \sqrt{\frac{BC^2}{4} + AB^2 - BC\ AB\cos\widehat{ABC}} = \frac{1}{2}\sqrt{2AB^2 + 2AC^2 - BC^2}$$
$$\beta_{BC} = \frac{\sin\widehat{ABC}}{\sin\frac{\widehat{BAC}}{2}}\,\frac{AB\ BC}{AC+AB} = \sqrt{AC\ AB - DB\ DC}$$

Inverse Trigonometric Functions

$$\sin(\arcsin\alpha) = \alpha \qquad \cos(\arccos\alpha) = \alpha$$
$$\tan(\arctan\alpha) = \alpha \qquad \arcsin(\sin\alpha) = \pi k + (-1)^k\alpha$$
$$\arcsin a = \arccos\sqrt{1-a^2} = \arctan\sqrt{\frac{a^2}{1-a^2}}$$
$$\arccos a = \arcsin\sqrt{1-a^2} = \cot^{-1}\sqrt{\frac{a^2}{1-a^2}}$$
$$\arctan a = \cot^{-1}\frac{1}{a} = \arcsin\sqrt{\frac{a}{1+a^2}}$$
$$\arcsin a + \arccos a = \frac{\pi}{2} = \arctan a + \cot^{-1} a$$