

Astrophysics & Cosmology:

Project work 1: Numerical integration of an elliptic orbit

Purpose: Study the elliptical orbit of a planet/asteroid/comet around the Sun by considering the $r(\theta)$ analytical solution for the trajectory:

$$r(\theta) = \frac{a(1 - e^2)}{1 + e \cos \theta} \quad (1)$$

where a is the major semiaxis, e the eccentricity and θ is the polar angle. The temporal evolution $r(t)$ is, however, obtained from the numerical integration of the ordinary differential equation (ODE):

$$\ddot{r} - \frac{J^2}{m^2 r^3} + G \frac{M_\odot}{r^2} = 0 \quad (2)$$

where J is the moduli of the angular momentum of the orbiting mass m . Use the *Matlab* (or any other) facility to numerically integrate the Equation (2) making sure that the integration of the second order ODE is precise enough (choose a high ≥ 2 order integrator).

Take the initial conditions from well known astronomical bodies. Make use of the information at several WEB pages (for example, the WEB of the NASA and ESA agencies contains a lot of practical information).

Once the trajectory $r(\theta)$ and the temporal evolution $r(t)$ have been obtained:

- a) Make a draw the of complete orbit.
- b) Analyze the analytical relationship between the geometrical orbital parameters a, b, e position of the aphelion and perihelion and physical magnitudes such as the total energy E and angular momentum J .
- c) Analyze the conservation of energy and angular momentum in one or several complete orbits of the model. Can you give the % deviation from perfect conservation ?
- d) Are the Kepler laws fulfilled ?
- e) Any other idea or comment you have concerning your results

The Report

The report of your work should contain:

- 1) A brief introduction to the topic.
- 2) The mathematical foundations of the problem you want to solve and the chosen initial conditions.
- 3) The results of your computations
- 4) A few lines stating the main conclusions of the work done.
- 5) If necessary, include the Bibliography and consulted WEB pages.

Delivering the Report

The report has to be delivered by all the groups not later than **October the 5th**. You can send a pdf of the report to domingo.garcia@upc.edu with copy to jordi.jose@upc.edu.

The following groups have to give a power-point public mini-lecture of the project on Wednesday 10th of October. Of around 10 minutes long.

Public lecture of Project Work 1 by:

Mireia Lamaison and Andrea Fontanet

Pau Riera and Guillem Megias