

## Exercises for the free-fall collapse problem

1. Show that the free-fall collapse of a homogeneous sphere satisfies the solution:

$$\left(8\pi G \frac{\rho_o}{3}\right)^{1/2} (t - t_o) = \left(1 - \frac{R}{R_o}\right)^{1/2} \left(\frac{R}{R_o}\right)^{1/2} + \arcsin\left(1 - \frac{R}{R_o}\right)^{1/2}$$

2. Run the code to simulate a free-fall collapse. Assume 100 shells,  $R_o = 10^4$  cm,  $\rho_o = 1$  g cm<sup>-3</sup>, and  $\beta = 0.5$ . Plot  $R(t)$ .
3. Estimate the time required to shrink the radius of the sphere to 1% of its initial radius.
4. Represent the density profile throughout the sphere at different times and demonstrate that the sphere remains homogenous during the collapse.
5. Compare the results obtained for different choices of  $\beta = 0, 0.25, 0.5, 0.75$ , and 1.
6. Estimate the final value of  $R$  for which significant differences between the numerical and the analytical solution appear.