

IE2111 ISE Principles & Practice II
Solutions to Assignment #4

- $MARR = 10\%$
- Study period = 5 years

| | Project A | Project B |
|------------------------------------|-----------|-----------|
| Initial investment cost | \$120,000 | \$80,000 |
| Equivalent Uniform Annual Benefits | \$30,000 | \$20,000 |
| Salvage Value | \$18,000 | \$10,000 |
| Useful Life | 5 years | 5 years |

(a) Base Value Analysis:

- $PW(10\%)$ for Project A = $-120,000 + 30,000 [P/A, 10\%, 5] + 18,000 [P/F, 10\%, 5]$
 $= -120,000 + 30,000 (3.7907868) + 18,000 (0.6209213)$
 $= \text{\$ } \underline{\underline{4,900.19}}$
- $PW(10\%)$ for Project B = $-80,000 + 20,000 [P/A, 10\%, 5] + 10,000 [P/F, 10\%, 5]$
 $= -80,000 + 20,000 (3.7907868) + 10,000 (0.6209213)$
 $= \text{\$ } \underline{\underline{2,024.95}}$
- Project A should be selected.

(b) Break-Even Cost of Project B's Initial Cost.

- Based on base values Project A is preferred as it has a higher PW.
- The Initial cost of Project B must to be decreased by at least $\$4,900.19 - \$2,024.95 = \text{\$ } \underline{\underline{2,875.24}}$ to reverse the decision in (a).

(c) Probabilistic Risk Analysis for Project A

- Initial Investment:
Mean = \$120,000 // from base value
Variance = 0 // no uncertainty
- Salvage value: Uniform (\$16,000, \$20,000)
Mean = $(16,000 + 20,000)/2 = \$18,000$ // same as base value
Variance = $(20,000 - 16,000)^2 / 12$
= \$1,333,333.33
- Annual profits: Discrete Distribution

| Cash Flow | Probability |
|-----------|-------------|
| \$25,000 | 0.25 |
| \$30,000 | 0.50 |
| \$35,000 | 0.25 |

$$\begin{aligned}\text{Mean} &= 0.25 (25,000) + 0.5 (30,000) + 0.25 (35,000) = \$30,000 \quad // \text{ same as base value} \\ \text{Variance} &= 0.25 (25,000 - 30,000)^2 + 0.5 (30,000 - 30,000)^2 + 0.25 (35,000 - 30,000)^2 \\ &= \$12,500,000\end{aligned}$$

- $E[PW \text{ of Project } A] = \$ \underline{\underline{4,900.19}}$ // from part (a)
- $Var [PW \text{ of Project } A]$
 $= 0 + 12,500,000 [P/A, 10\%, 5]^2 + 1,333,333.33 [P/F, 10\%, 5]^2$
 $= 0 + 12,500,000 (3.7907868)^2 + 1,333,333.33 (0.6209213)^2$
 $= \$ \underline{\underline{180,139,861.86}}$
- Standard Deviation of PW of Project $A = \sqrt{180,139,861.86} = \$ \underline{\underline{13,421.62}}$

(d) Probabilistic Risk Analysis for Project B

- Salvage value: Triangular (8,000, 12,000, 10,000)
Mean = $(8,000 + 12,000 + 10,000) / 3 = \$10,000$ // same as base value
Var = $(8,000^2 + 12,000^2 + 10,000^2 - 8,000 \times 12,000 - 8,000 \times 10,000 - 12,000 \times 10,000) / 18$
= \$666,666.67
- Equivalent uniform annual profits: Normal (\$20,000, \$5,000)
Mean = \$20,000 // same as base case
Var = $5,000^2 = \$25,000,000$
- $E[PW \text{ of Project } B] = \$ \underline{\underline{2,024.95}}$ // from part (a)
- $Var [PW \text{ of Project } B]$
 $= 0 + 25,000,000 [P/A, 10\%, 5]^2 + 666,666.67 [P/F, 10\%, 5]^2$
 $= 0 + 25,000,000 (3.7907868)^2 + 666,666.67 (0.6209213)^2$
 $= \$ \underline{\underline{359,508,637.14}}$
- Standard Deviation of PW of Project $B = \sqrt{359,508,637.14} = \$ \underline{\underline{18,960.71}}$

(e) **Mean-Variance Dominance Analysis**

- Comparing the Mean and Standard Deviation of PW of Projects A and B :

| | Mean | Standard Deviation |
|-------------|------------|--------------------|
| Project A | \$4,900.19 | \$13,421.62 |
| Project B | \$2,024.95 | \$18,960.71 |

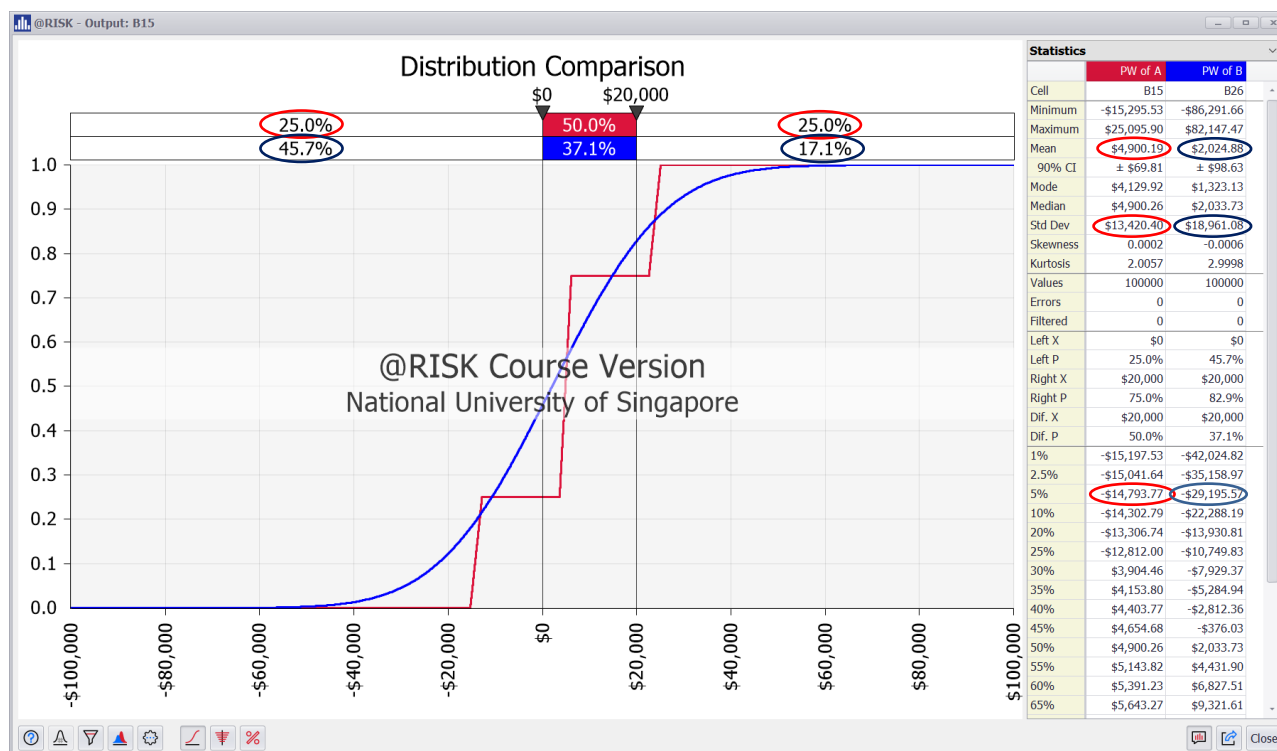
- Choose Project A as it has a higher Expected PW and a Smaller Standard Deviation of PW .

(f) **Risk Analysis using @Risk**

@Risk Model (See Excel Files for details)

| ie2111-24-assign-4-solutions.xlsx - Excel | | | | | | |
|--|---------------|--------|--------------|------------|----------|----------|
| Poh Kim Leng | | | | | | |
| File | Home | Insert | Page Layout | Formulas | Data | Review |
| B15 =RiskOutput()+B9 - PV(\$B\$3, B8, B10, B13, 0) | | | | | | |
| A | B | C | D | E | F | G |
| IE2111 2024 Assignment 4 | | | | | | |
| MARR | 10.000% | | | | | |
| Project A | | | | | | |
| Useful Life (years) | 5 | | Distribution | Parameters | | |
| Initial Investment | -\$120,000.00 | | | | | |
| Annual Benefits | \$30,000.00 | | Discrete | \$25,000 | 0.25 | |
| | | | | \$30,000 | 0.50 | |
| | | | | \$35,000 | 0.25 | |
| Salvage Value | \$17,113.97 | | Uniform | \$16,000 | \$20,000 | |
| PW of A | \$4,350.03 | | | | | |
| Project B | | | | | | |
| Useful Life (years) | 5 | | Distribution | Parameters | | |
| Initial Investment | -\$80,000.00 | | | | | |
| Annual Benefits | \$28,578.34 | | Normal | \$20,000 | \$5,000 | |
| Salvage Value | \$11,667.16 | | Triangular | \$8,000 | \$10,000 | \$12,000 |
| PW of B | \$35,578.79 | | | | | |
| Base Model Breakeven init cost of B @Risk Model | | | | | | |

Risk Profiles Generated by @Risk (100,000 trials)



Stochastic Dominance Analysis

- There is no First Order Stochastic Dominance.
- Need to check for higher orders Stochastic Dominance.

Comparing Simulation and Analytical Results

| Method | EV of Project A | Std Dev of Project A |
|------------------------|-----------------|----------------------|
| Monte Carlo Simulation | \$ 4,900.19 | \$ 13,420.40 |
| Analytical | \$ 4,900.19 | \$ 13,421.62 |
| Method | EV of Project B | Std Dev of Project B |
| Monte Carlo Simulation | \$ 2,024.88 | \$ 18,961.08 |
| Analytical | \$2,024.95 | \$ 18,960.71 |

Downside Risks

- Project A: 25.0%
- Project B: 45.7%

Upside Potentials for $PW=\$20,000$

- Project A: 25.0%
- Project B: 17.1%

Equivalent Present VaR(95%)

- Project A: \$ 14,793.77
- Project B: \$ 29,195.57

Recommendation:

- Although there is no Mean-Var dominance, we would recommend Project A as it has a lower downside risk, a higher upside potential for a realistic target, and a lower VaR.