Chapter 5 Decision Modeling using Influence Diagrams

"The formulation of the problem is often more essential than its solution, which may be merely a matter of mathematical or experimental skill."

Albert Einstein

5.1 Introduction

- In Chapter 4, we use Decision Trees to represent, solve and analyze decision problems.

- A major problem in using a decision tree is that the size of the tree grows exponentially, i.e., becomes very large, when the problem size increases. This makes it very difficult to work on the model when the number of variables is large.

- This chapter introduces Influence Diagram which is more compact way to represent a decision problem using network-based representation.

5.2 Influence Diagrams

5.2.1 Decision Modeling using Influence Diagrams

Definition of Influence Diagram (ID)

- An **Influence Diagram** (also known as a Decision Diagram) is a **Directed Acyclic Graph** representing a decision problem. The nodes and arcs of an influence diagram are described below:

<u>**Chance Node**</u>

- A *Chance Node* (denoted by an oval) in an influence diagram represents an uncertain variable.
- In each chance node, we assess a conditional probability distribution for the uncertain variable it represents, conditioned on its parent nodes.

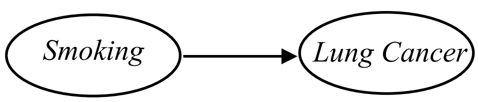
<u>**Relevance Arc**</u>

- An arc between two chance nodes in an influence diagram is known as a *Relevance Arc*.
- It denotes possible probabilistic dependence between the two variables.
- A relevance arc in an influence diagram may be reversed if doing so will not create any directed cycle in the diagram. Note that additional arcs may be added as a result of a relevant arc reversal.

Example

- The risk of having lung cancer is dependent on smoking habits.

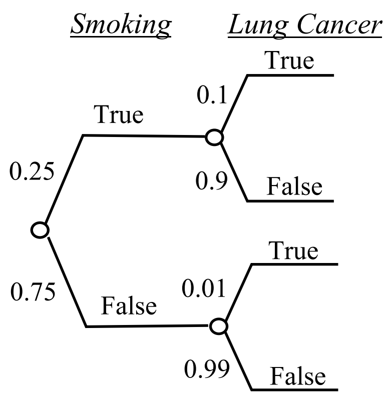
<u>Influence Diagram</u>



Smoking	$p(S \mid \xi)$
True	0.25
False	0.75

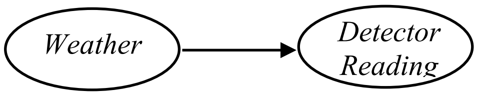
	$p(L \mid S, \xi)$	
	Lung Cancer	
Smoking	True	False
True	0.10	0.90
False	0.01	0.99

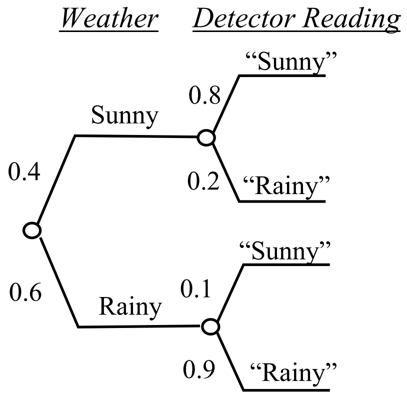
<u>Equivalent Probability Tree</u>



Example

- The detector's reading depends on the actual weather condition.

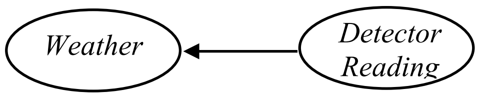


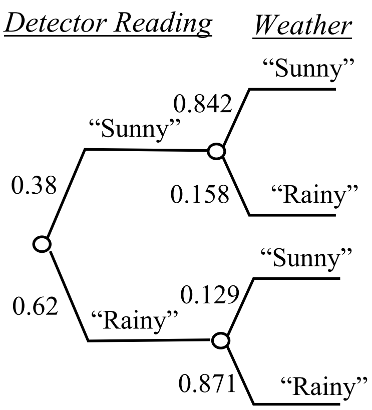


Weather	$p(W \mid \xi)$
Sunny	0.4
Rainy	0.6

	$p(R \mid W, \xi)$	
	Detector Reading	
Weather	"Sunny"	"Rainy"
Sunny	0.8	0.2
Rainy	0.1	0.9

- The relevance arc between weather and detector reading may be reversed and the probabilities in the two chance nodes are updated.





	$p(W \mid R, \xi)$	
	Weather	
Detector Reading	"Sunny"	"Rainy"
"Sunny"	0.842	0.158
"Rainy"	0.129	0.871

Detector Reading	$p(R \mid \xi)$
"Sunny"	0.38
"Rainy"	0.62

Decision Node

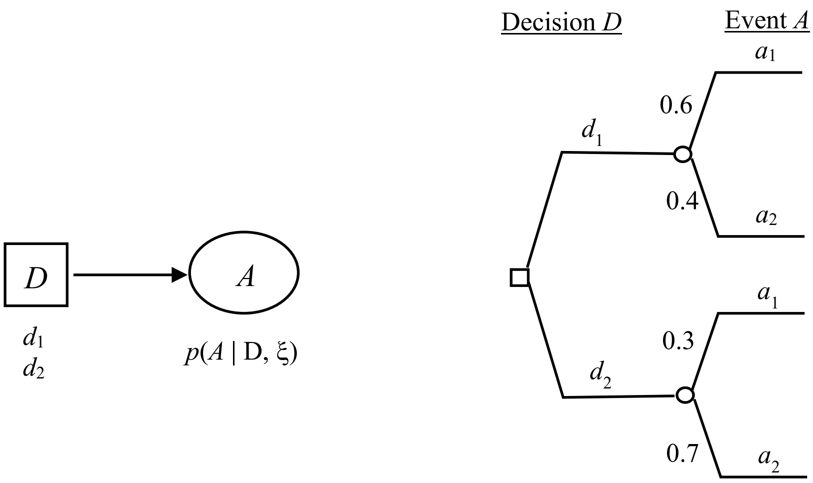
- A *Decision Node* (denoted by a rectangle) in an influence diagram represents a decision variable.
- In each decision node, we indicate a list of alternatives for the decision variable it represents.

Influence Arc

- An arc from a decision node to a chance node in an influence diagram is known as an *Influence Arc*.
- It indicates that the probability of the uncertain variable is dependent on the alternatives of the decision node.
- An influence arc cannot be reversed.

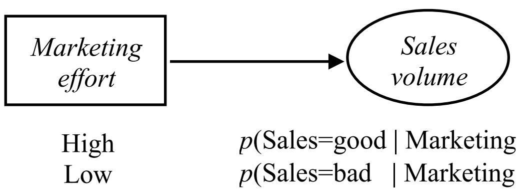
Example

- The probability for the outcomes of *A* depends on the actual decision made at *D*. We therefore assess a conditional probability at *A* conditioned on each alternative at *D*.



Example

- The performance of a product in a market (sales volume) depends on how much marketing was done.



p(Sales=good | Marketing = high, ξ) = 0.9
p(Sales=bad | Marketing = high, ξ) = 0.1

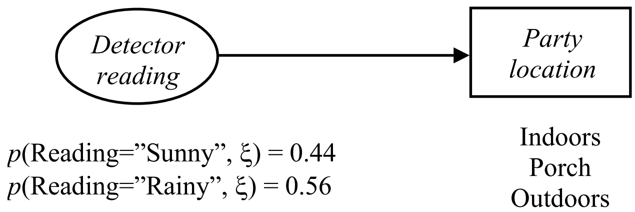
p(Sales=good | Marketing = low, ξ) = 0.3
p(Sales=bad | Marketing = low, ξ) = 0.7

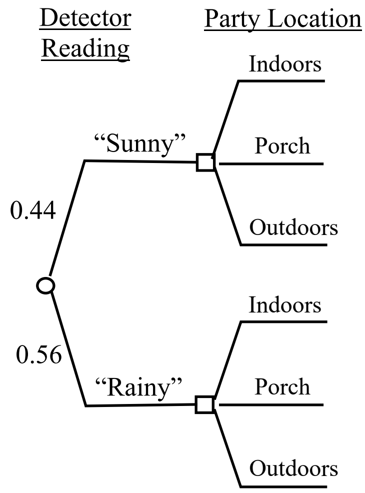
Information Arc

- An arc from a chance node into a decision node in an influence diagram is known as an *Information Arc*.
- It indicates that the outcome of the chance node will be known to the decision maker when the decision is being carried out.
- An information arc cannot be reversed.

Example

- We are currently uncertain about the outcome of "Detector Reading", but we do know its probabilities.
- However, the exact outcome of "Detector Reading" (i.e., "sunny" or "rainy"), will be known to the decision maker when he decides on the party location.

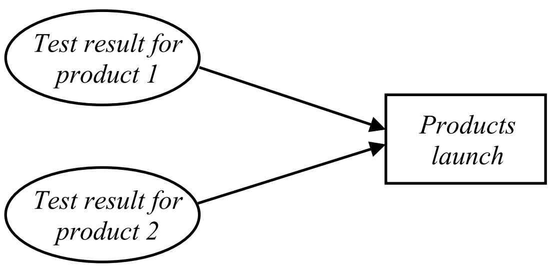


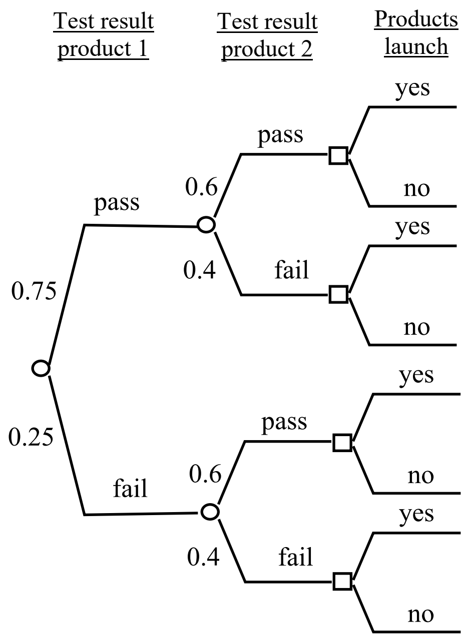


Multiple Information Arcs

- A decision node may have multiple information arcs.

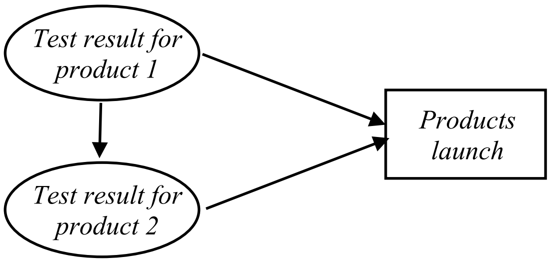
Example

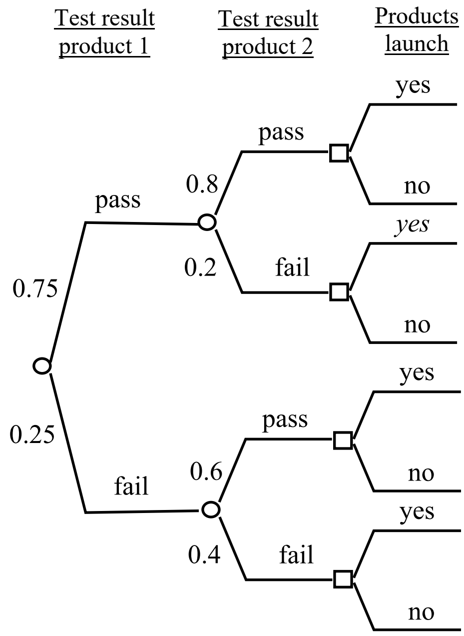




- The results of the two tests are currently uncertain. However, at the time when we carry out the decision to launch products, we will know the outcomes of "Test result for product 1" and "Test result for product 2." Moreover, the two test results are mutually independent.

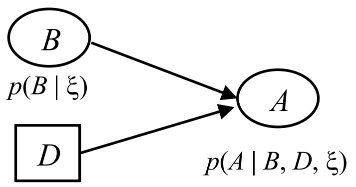
- What about the following influence diagram?

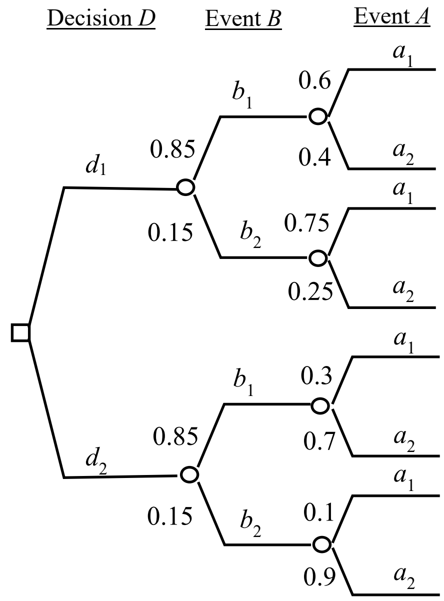




- This is the same as the previous example, except that two test results may not be independent.

Example (Combining influence and relevance arcs)

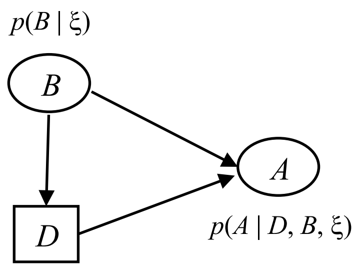


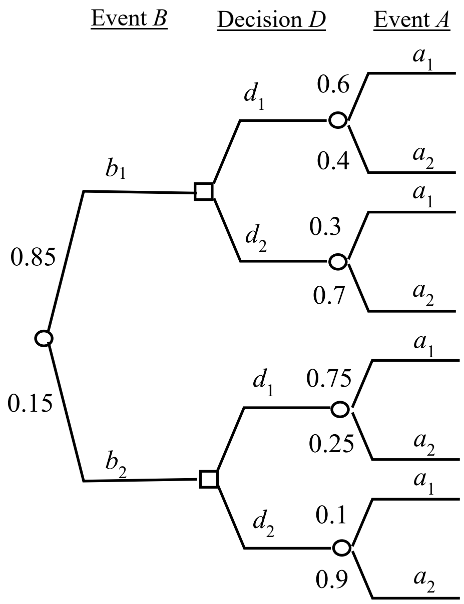


- The outcomes of A and B are uncertain. The probability of A depends on the outcome of B and the decision made at D. At the time decision D is carried out, we do not know the outcomes of B and A.

- Note that the decision tree with the nodes ordering B, D, A is wrong. Why?

Example (Combining influence, relevance and information arcs)



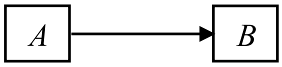


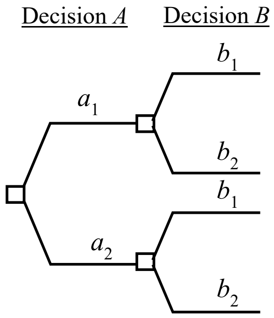
- The outcomes of A and B are uncertain. The probability of A depends on the outcome of B and the decision made at D. At the time decision D is carried out, we know the outcome of B, but not of A.

Chronological Arc

- An arc from a decision node to another decision node in an influence diagram is known as a *Chronological Arc.*
- It indicates the *chronological order* in which the decisions are being carried out.
- A chronological arc cannot be reversed.

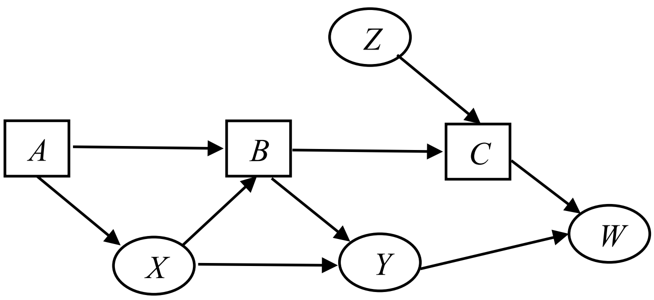
Example





- Decision B is made after Decision A.

Example (Relevance, Influence, Information and Chorological arcs)



- Decision A is carried out before decision B which is carried out before decision C.
- Uncertain variable X is influenced by decision A.
- At the time decision B is carried out, the outcome of X and the choice made at A are known.
- Uncertain variable Y is influenced by decision B and the outcome of X.
- At the time decision C is carried out, the outcome of Z and the choice made at B are known.
- Uncertain variable W is influenced by decision C and the outcome of Y.

- Can you draw (or specify the sequence of nodes for) the equivalent decision tree?

Value Node

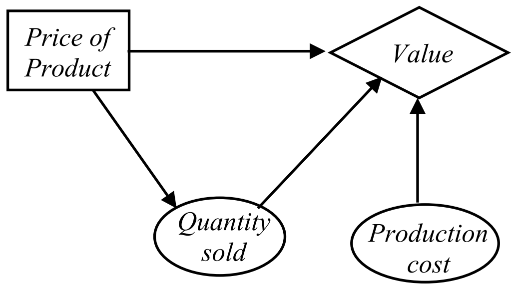
- A *Value Node* (denoted by a diamond) in an influence diagram represents the quantity whose expected value is to be optimized.
- A value node is used to represent the utility or value function of the decision maker.
- A value node must be a sink node, i.e., it has only incoming arcs.
- Only one value node is allowed in a standard influence diagram.

Value Arc

- An arc from any node to the value node is known as a *Value Arc*.
- Value arcs indicate the variables whose outcomes the decision maker cares about. i.e., have direct impact on his utility.
- A value arc cannot be reversed.

Example

- The decision maker is concerned about the profit which is affected by the product price, quantity sold and production cost.



Profit = Price * Quantity Sold – Total Production Cost

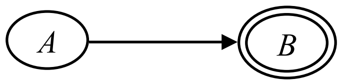
- Can you draw (or specify the sequence of nodes for) the equivalent decision tree?

Deterministic Node

- A *Deterministic Node* (denoted by a double oval) in an influence diagram is a special type chance node.
- It represents a variable whose outcome is known with certainty (i.e., has probability = 1), once the outcomes of other conditioning nodes are known.
- A deterministic node can be used to represent a function.

Example

- Variable B is deterministically dependent on A, i.e., if the outcome of A is known, then the outcome of B is also known exactly.



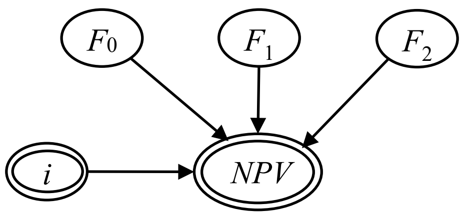
A	$p(A)$
a_1	0.2
a_2	0.7
a_3	0.1

	$p(B \mid A)$	
	b_1	b_2
a_1	1	0
a_2	0	1
a_3	1	0

- Also, B is function A such that $f(a_1) = b_1$, $f(a_2) = b_2$, and $f(a_3) = b_1$.

Example

- F_o, F_1, and F_2 are mutually independent uncertain cash flows at times 0, 1, and 2, respectively. The discounting (interest) rate i is certain.



$$NPV = F_0 + \frac{F_1}{(1+i)} + \frac{F_2}{(1+i)^2}$$

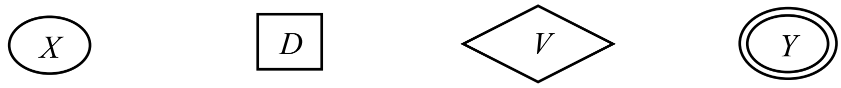
- Note that although NPV is a deterministic node, it is actually a random variable as it is a function of three random variables.

- If the cash flows F_o, F_1, and F_2 are not mutually independent, add relevance arcs between them.

<u>Summary on Influence Diagram</u>

- An **Influence Diagram** is a *Directed Acyclic Graph* (DAG) representing a decision model.

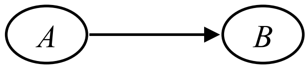
Types of Node:



Chance node | Decision node | Value node | Deterministic node

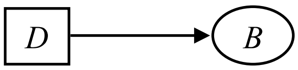
Types of Arc:

Relevance Arc



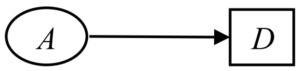
The probability for *B* may be dependent on the outcome of *A*.

Influence Arc



The probability for *B* may be dependent on alternative chosen at *D*.

Information Arc

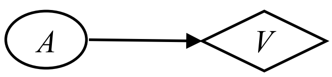


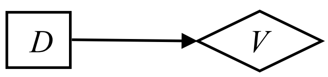
The decision maker knows the outcome of *A* when carrying out decision *D*.

Chronological Arc

Decision *T* is made before decision *D*.

Value Arc





Outcome of *A* has direct impact on Value.

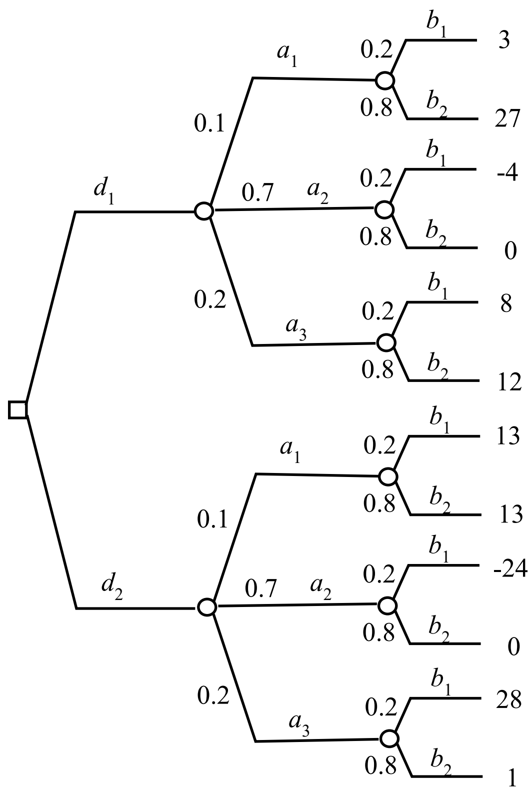
Decision made a *D* has direct impact on Value

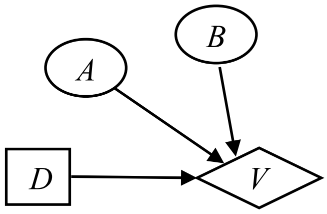
5.2.2 Comparing Influence Diagrams and Decision Trees

	Influence Diagram	Decision Tree
1	**Compact** The size of an influence diagram is equal to the total number of variables.	**Combinatory** The size of a decision tree grows exponentially with the total number of variables. A binary tree with n nodes has 2^n leaf nodes.
2	**Graphical Representation of Independence** Conditional independence relations among the variables are represented by the graphical structure of network. No numerical computations needed to determine conditional independence relations.	**Numerical Representation of Independence** Conditional independence relations among the variables can only be determined through numerical computation using the probabilities.
3	**Non-Directional** The nodes and arcs of an influence diagram may be added or deleted in any order. This makes the modeling process flexible.	**Unidirectional** A decision tree can only be built in the direction from the root to the leaf nodes. The exact sequence of the nodes or events must be known in advance.
4	**Symmetric Model only** The outcomes of all nodes must be conditioned on all outcomes of its parents. This implies that the equivalent tree must be symmetrical.	**Asymmetric Model possible** The outcomes of some nodes may be omitted for certain outcomes of its parent leading to an asymmetrical tree.

Examples

Case 1

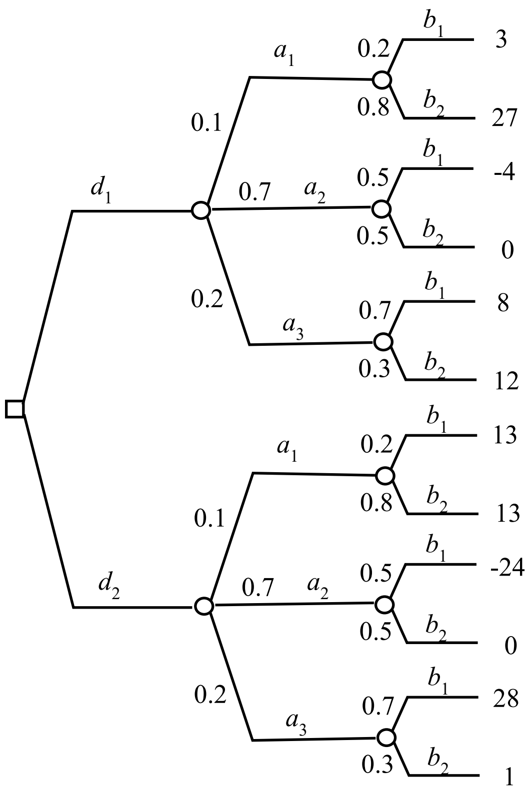


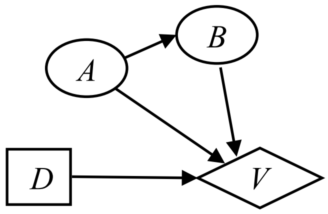


- In the decision tree, we can only infer that $A \perp B$, $A \perp D$, and $B \perp D$ through the numerical probability values.

- In the influence diagram, these independent conditions are explicitly expressed by the graphical structure.

Case 2

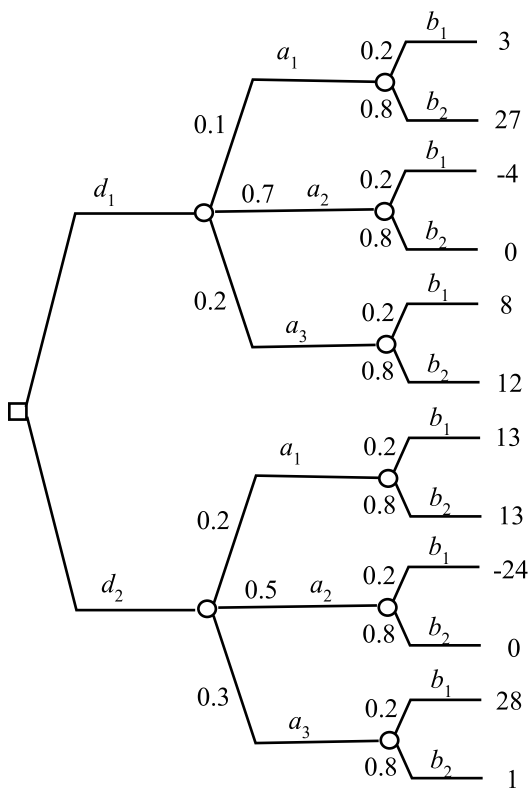


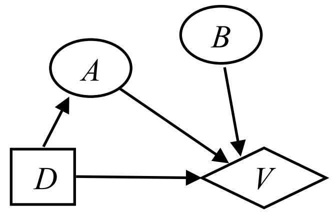


- In the decision tree, we can only infer that $A \perp D$, $B \perp D$, but B is dependent on A through the numerical probability values.

- In the influence diagram, these independent conditions are explicitly expressed by the graphical structure.

Case 3



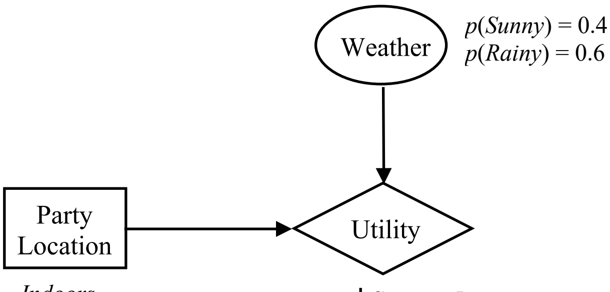


- In the decision tree, we can only infer that $A \perp B$, $B \perp D$, but A is dependent on D through the numerical probability values.

- In the influence diagram, these independent conditions are explicitly expressed by the graphical structure.

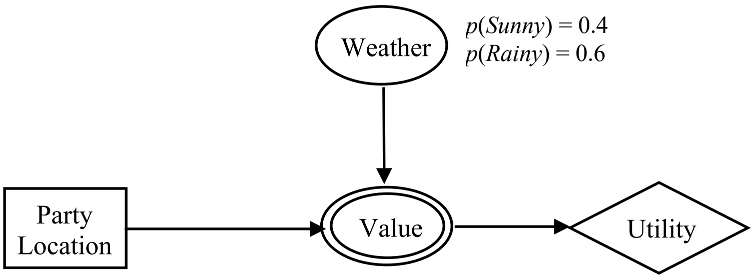
5.3 Modeling the Party Problem using Influence Diagram

Base model for Kim's Party Problem with direct assessment of utilities



	Sunny	*Rainy*
Indoors	0.57	0.67
Porch	0.95	0.32
Outdoors	1	0

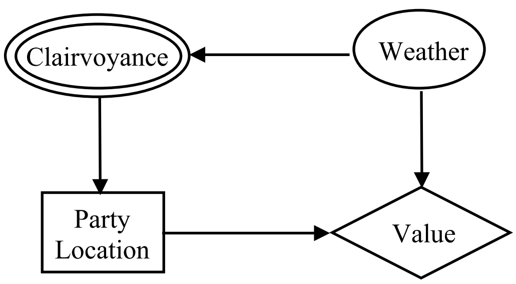
Kim's Party Problem with equivalent dollar values and utility function



	Sunny	*Rainy*
Indoors	40	50
Porch	90	20
Outdoors	100	0

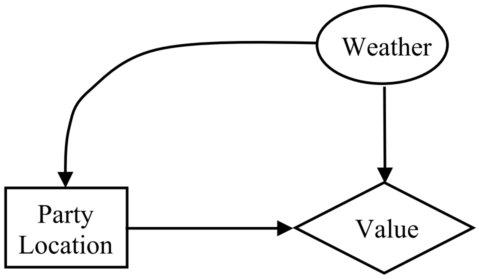
$$u(x) = \frac{4}{3}\left(1 - e^{-x/72.13}\right)$$

Decision Model with Perfect Information on Weather



- Here, "Clairvoyance" is a deterministic node because it is always equal to the Weather outcome with probability 1.

- The same result is obtained if we simply draw an information arc directly from Weather to Party Location:

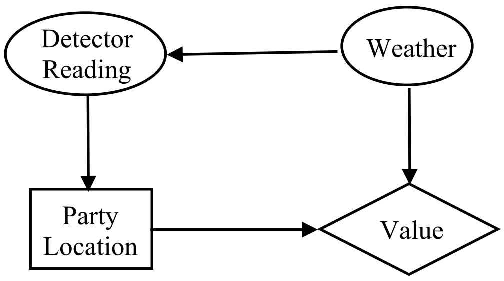


- Having clairvoyance on weather is the same as being able to "observe" the weather prior to deciding on the location.

- Hence in general, to represent the situation where we have perfect information (clairvoyance) on an uncertain event, we simply draw an information arc directly from the said event to the decision node. However, in doing so, we must not create any cycle.

Decision Model with Imperfect Information on Weather

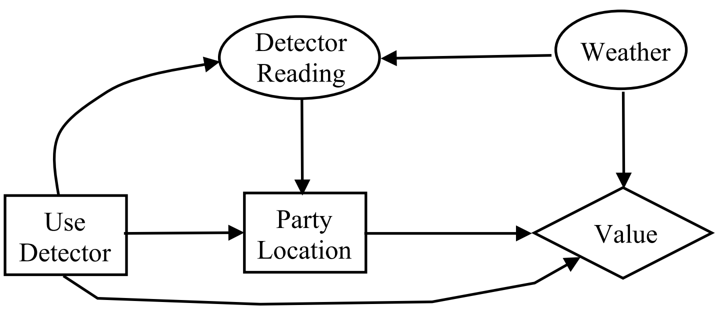
- Suppose we do not have perfect information on weather, but rather, imperfect information via a weather detection device, then the influence diagram representation for the situation is as follows:



- We input the "true-sunny detection rate" and "true-rainy detection rate" in the "Detector Reading" node.
- The detector reading is known before the location decision is carried out. Hence add an *information arc* from the observed node (detector indicator) to the location decision node.

Decision Model with Option to use Weather Detector or not

- If the decision maker has the option to use the rain detector or not we add a decision node at the front:



5.4 Evaluating Influence Diagrams

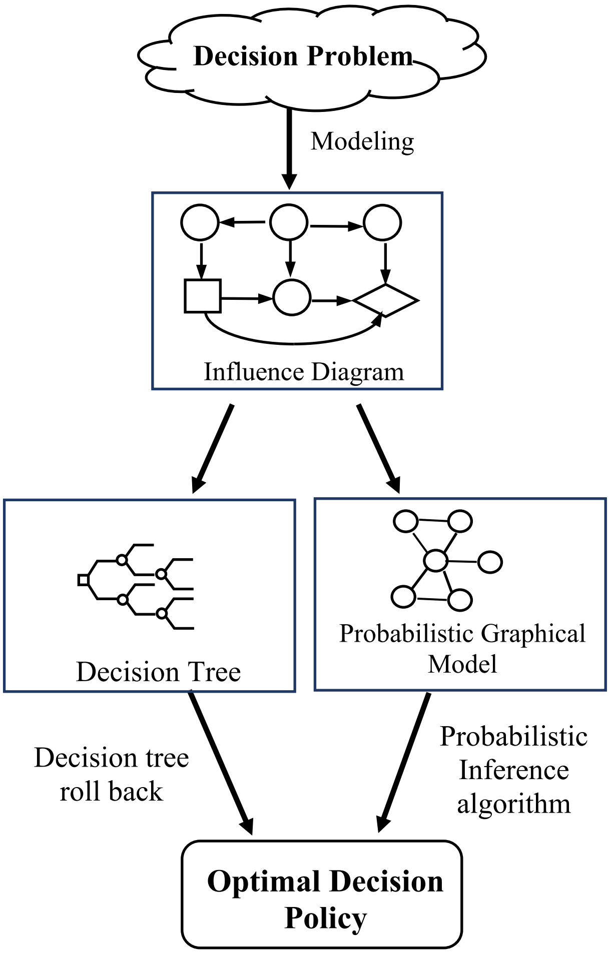
5.4.1 Approaches to Solving an Influence Diagram

- To find the optimal decision policy of a decision problem represented by an influence diagram we need to evaluate it.

- Methods for evaluating an influence diagram:

    1. Convert the influence diagram into an equivalent decision tree and perform tree roll back.

    2. Convert the influence diagram into a probabilistic graphical model (PGM) and apply probabilistic inference algorithms.



- See Shachter (1986) for the first algorithm for evaluation an ID directly without converting it into a decision tree, and books on traditional artificial intelligence for probabilistic graphical models (PGM). Bayesian network (BN) is one type of PGM.

- Note that DPL software uses an influence diagram and a decision tree to model a decision problem. It then automatically generates the optimal decision policy by expanding and solving the decision tree.

5.4.2 The Decision Network Conditions

- An influence diagram may be used to represent a decision problem, but before we can solve it by any methods to find the optimal decision policy, we must ensure that it is properly specified.

- The **Decision Network Conditions** ensure that an influence diagram is solvable, i.e., there is no ambiguity in the information.

- An influence diagram is said to satisfy the **Decision Network Conditions** if the following two conditions are true:

  1. **The Complete Decisions Ordering Condition**

     - There exists a complete chronological order for all the decision nodes.

     - This means that the decisions are made sequentially and there are no independent or parallel decisions involved.

     - This condition is also called the "Single Decision Maker" condition.

  2. **The "No-Forget" Condition**

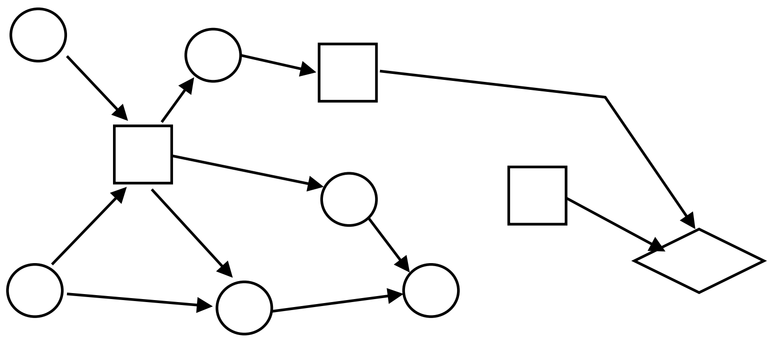
     - All direct predecessors (parents) of a decision node are also direct predecessors (parents) of all decision nodes down the chronological order.

     - This means that information available at one decision point is always available (not forgotten) at subsequent decision points.

Example

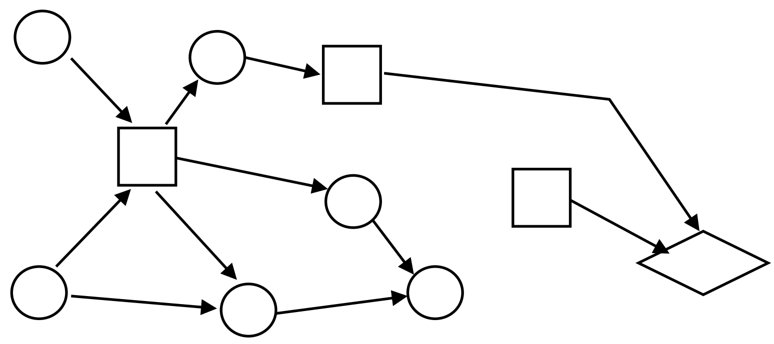
- The following ID is not a decision network. It cannot be solved.

- Can you identify the violations?

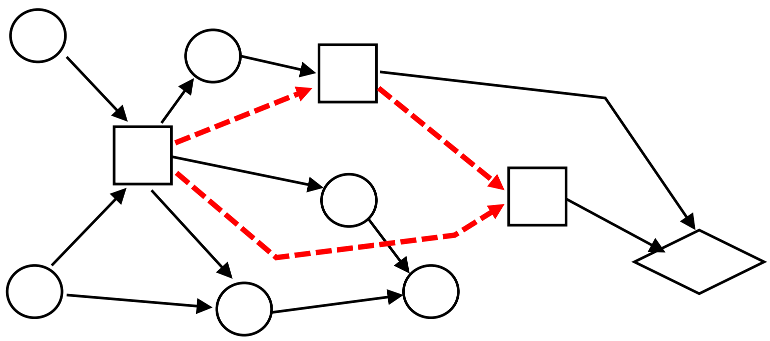


Example

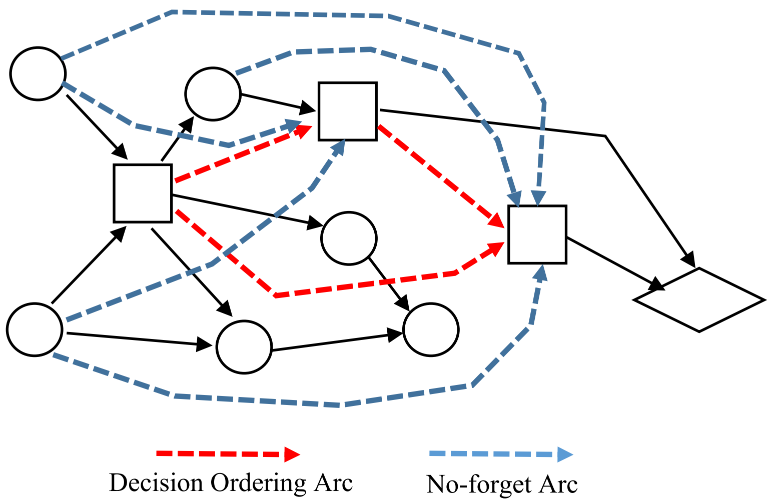
- Convert the previous influence diagram into a Decision Network:



- Adding Decision Ordering (Chorological) Arcs:



- Adding No-Forget Arcs:



- The influence diagram is now a Decision Network and can be solved without ambiguity.

5.4.3 Drawing a Decision Tree from an Influence Diagram

- We would like to convert an influence diagram (ID) into an equivalent decision tree so that we can solve it.

- The equivalent tree must satisfy the following constraints:

  1. The ID must be a decision network.

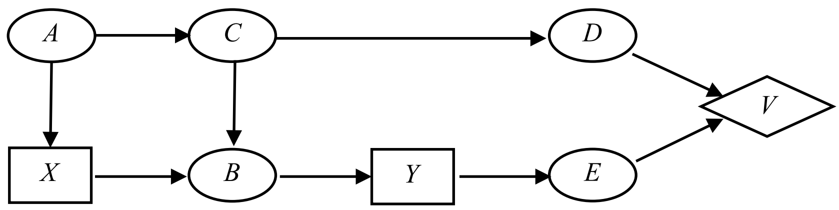
  2. The decision nodes in the DT must follow the same chorological order in the ID.

  3. A node that is a direct predecessor (parent) of a decision node in the ID should appear in the DT just before the decision node that first observed it.

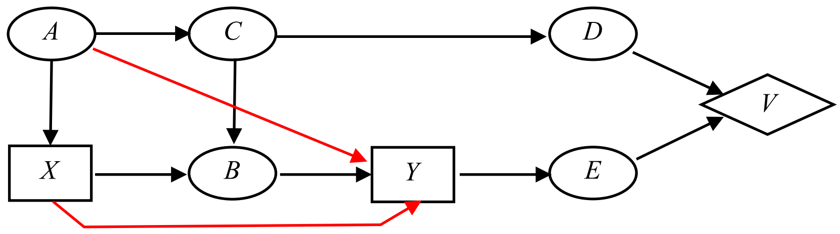
  4. A node that is not direct predecessor (parent) of any decision node in the ID should appear after the last decision node in the DT.

Example 1

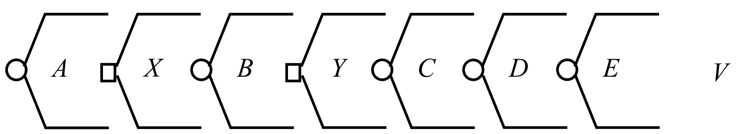
- Given the following influence diagram:



- Convert it into a decision network:



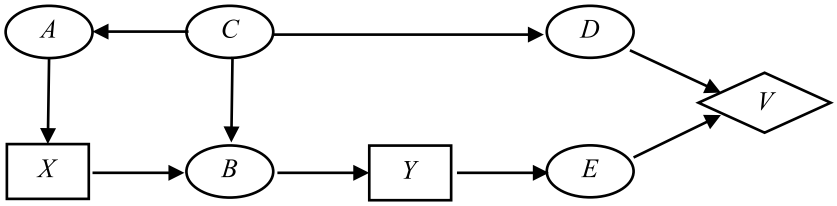
- An equivalent decision tree (in generic format) is as follows:



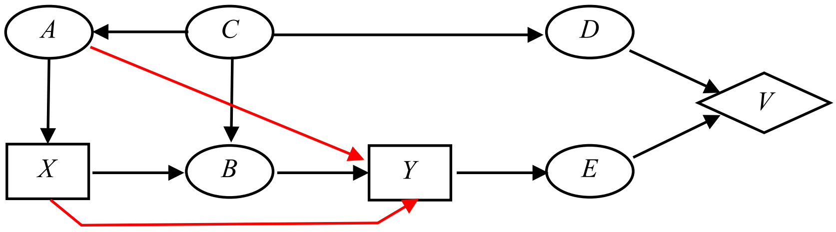
- Any permutation of the node sequence *C*, *D*, *E* is also valid.

Example 2

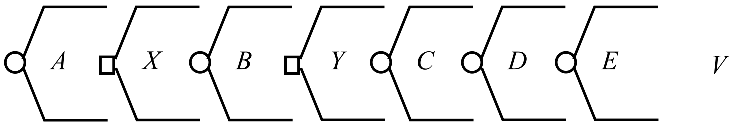
- Given the following influence diagram:



- Convert it into a decision network:

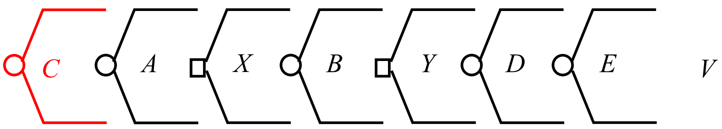


- An equivalent decision tree (in generic format) is as follows:



- Any permutation of the node sequence C, D, E is also valid.

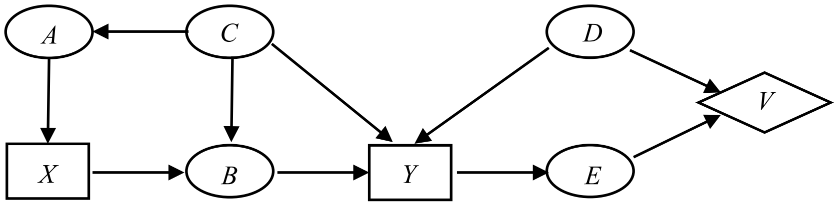
- Note that the following decision tree is **WRONG !**



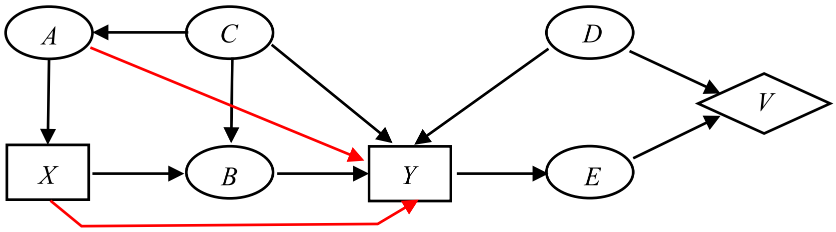
- Even though chance node C is a grandparent of X in the ID, it is not observed by either X or Y as there is no information arc from C to X and Y. Hence C cannot be drawn before the decision nodes X and Y in the DT.

Example 3

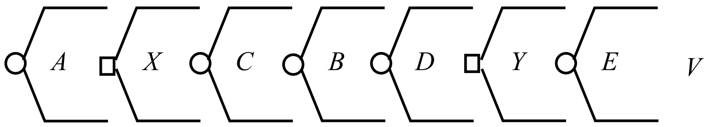
- Given the following influence diagram:



- Convert it into a decision network:



- An equivalent decision tree (in generic format) is as follows:



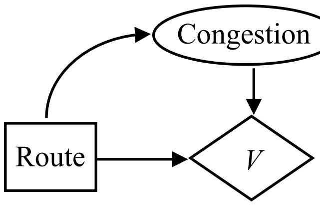
- Any permutation of the node sequence C, B, D is also valid.

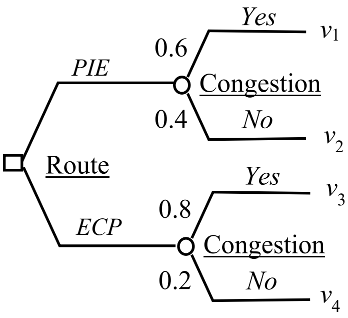
5.5 Influence Diagram in Canonical Form

Influence Diagrams with Influence Arc

- Consider the problem of getting from Changi Airport to the City. You have to decide on which route to take: PIE or ECP.

- Because the probability of encountering a congestion along the way depends on which route you took, you might model the decision problem as follows:

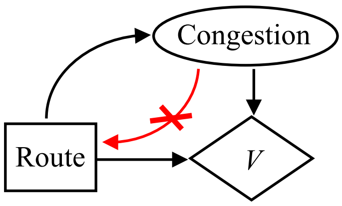


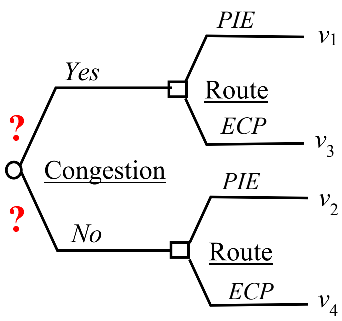


- Using this decision model, the best route can be determined.

- Suppose now you want to determine the expected value of perfect information on congestion, can it be done? Can you create a decision tree or influence diagram with perfect information?

- The value of information on congestion cannot be performed using the decision model created this way.





- If you add an information arc from chance node "Congestion" to decision node "Route" to denote perfect information, a loop will be created because there is already an arc from the "Route" to "Congestion".

- If we avoid the *influence arc* from "Route" to "Congestion", we can perform value of information analysis.

Influence Diagram in Canonical Form

Definition

- An influence diagram is said to be in ***Canonical Form*** if it does not have any chance nodes that are descendants of a decision node.

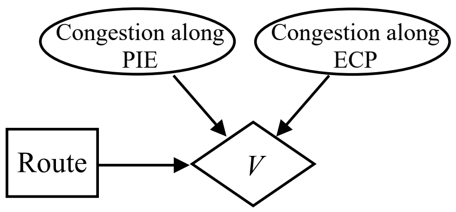
Drawing an Influence Diagram in Canonical Form

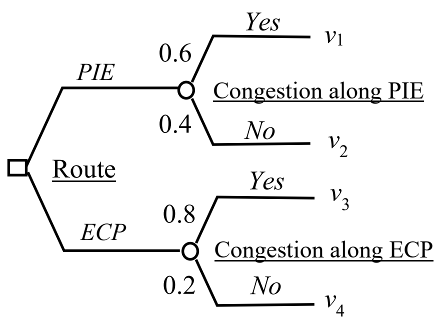
- An influence in non-canonical form can be redrawn into canonical form by replacing the chance node with multiple chance nodes.

Example

- We replace the single chance node "Congestion" by two chance nodes
  1. "Congestion along PIE"
  2. "Congestion along ECP"

- If we assume that congestion along PIE is independent of congestion along ECP, then we can model the problem as follows:



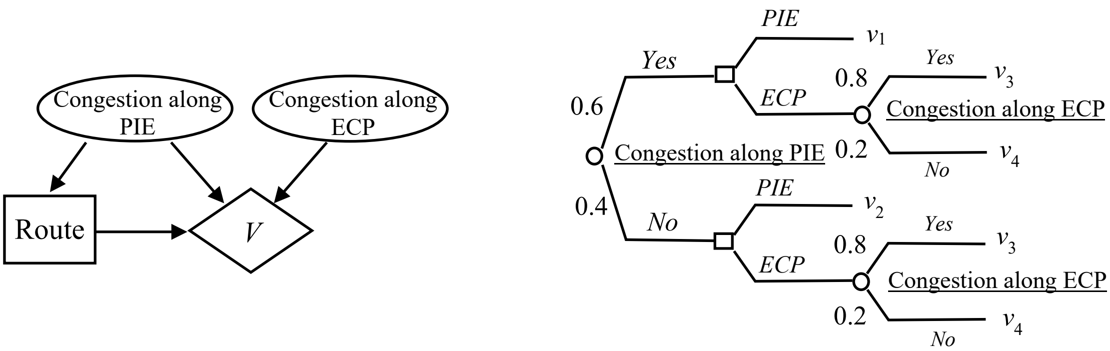


- The diagram is now in ***Canonical Form*** because the decision node does not have any descendant chance nodes.

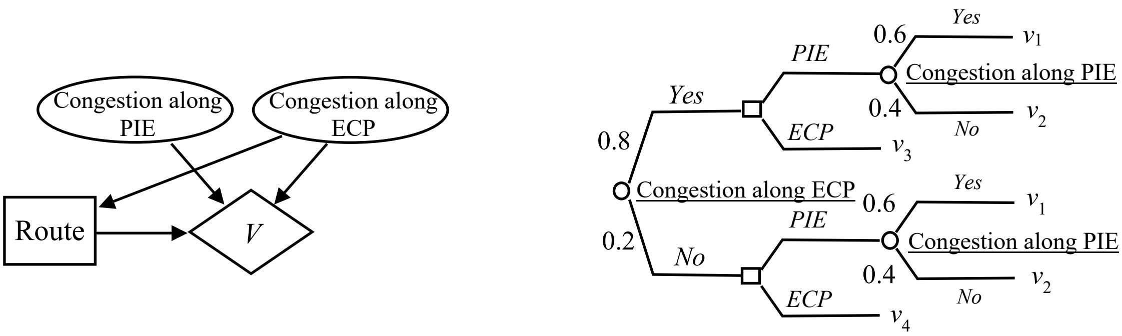
Performing Value of Information Analysis

- We can now perform value of information analysis, but the expected value of information are now computed specific to particular routes:

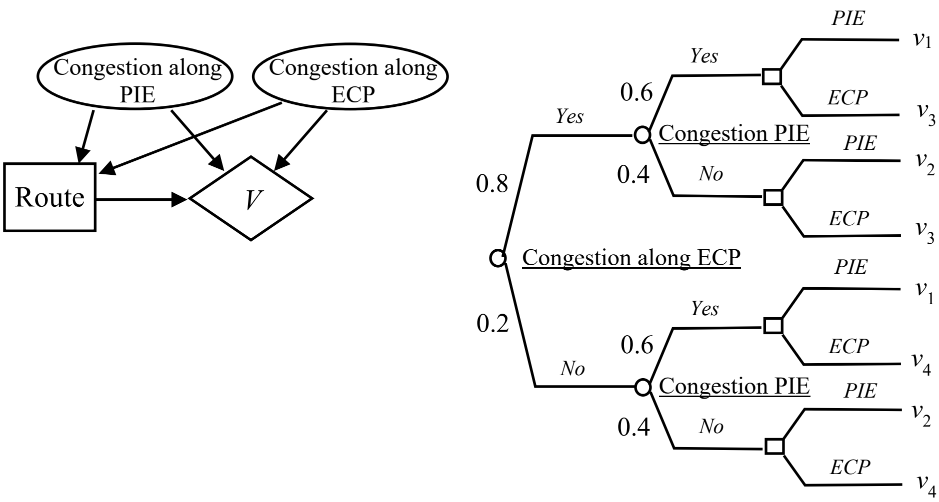
- Decision model with free perfect information on congestion along PIE:



- Decision model with free perfect information on congestion along ECP:



  - Decision model with free joint perfect information on congestion along PIE and congestion along ECP:

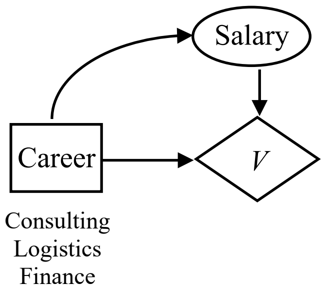


When the decision node has more than two alternatives

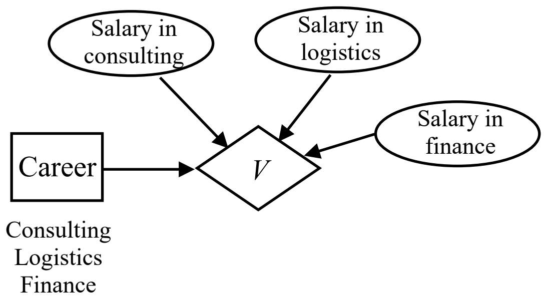
- We can usually draw an influence diagram in canonical form by defining as many chance nodes are there are number of alternatives in the decision node.

Example (Career Planning for ISyE Graduates)

- A popular ISEM graduate's career choice decision problem:



<u>Non-canonical form</u>
Salary depends on your career choice

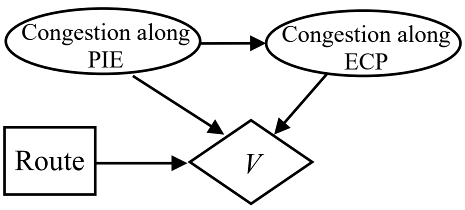


<u>Canonical form</u>
Salaries are different in each job sector and they are mutually independent

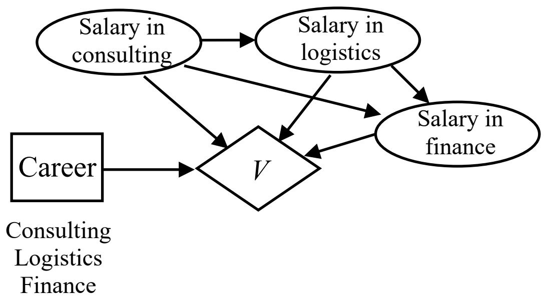
Canonical Form with Non-Mutually Independent Chance Nodes

- In general, the chance nodes do not need to be mutually independent. Relevance arcs can be added between them if needed.

Example



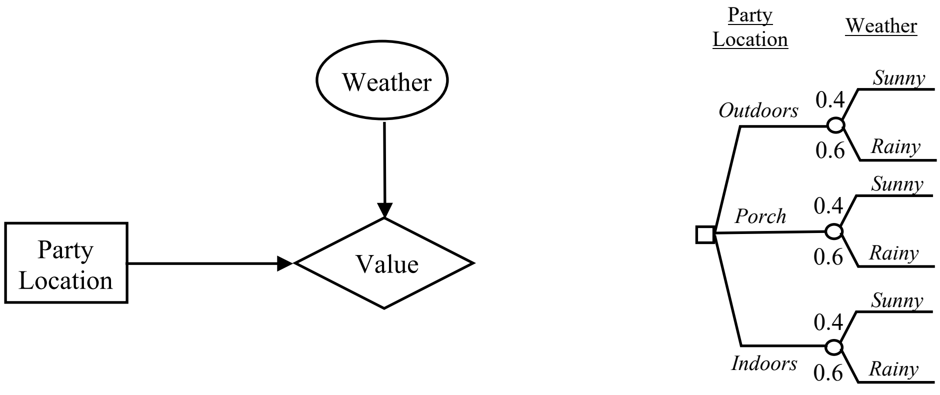
Congestion along ECP and congestion along PIE may not be independent



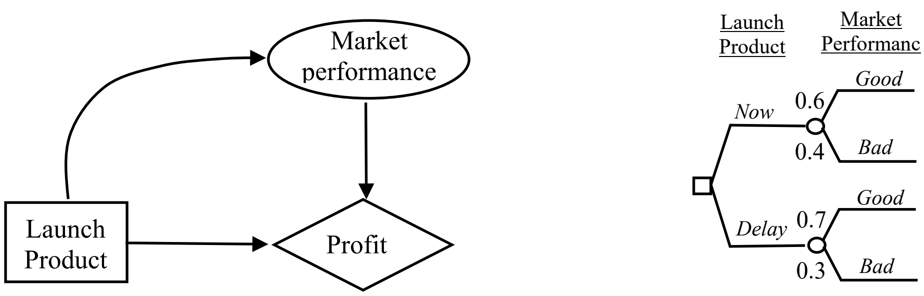
Salaries in the three job sectors may not be mutually independent

5.6 Example Influence Diagrams

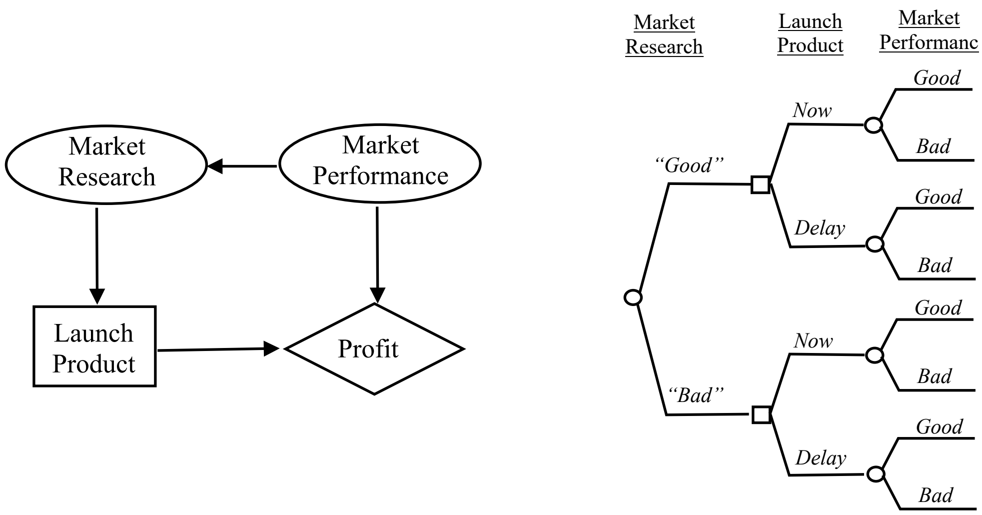
The Party Problem



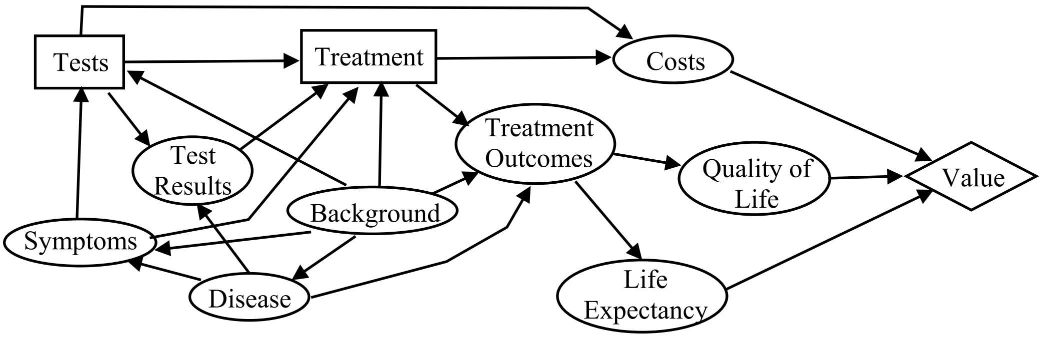
Basic Risky Decision Problem (One-Decision One-Uncertainty)



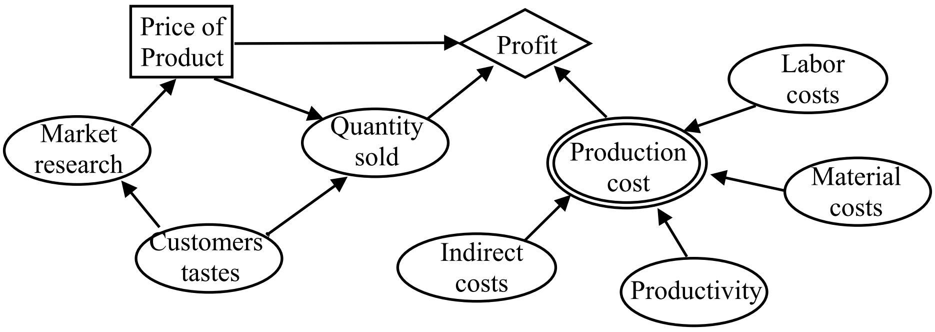
Decision Problem with Free Imperfect Information on the Uncertainty



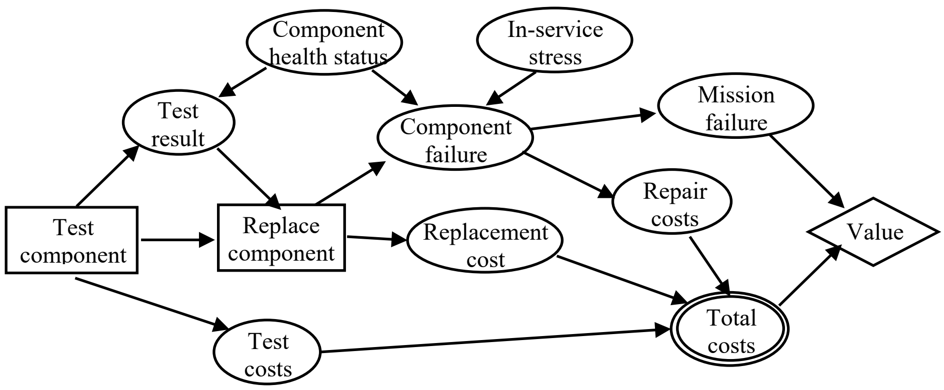
<u>**A Medical Decision Problem**</u>



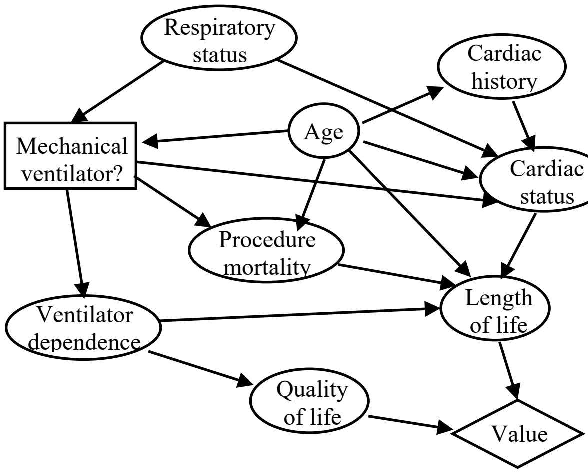
<u>**The Production/Sale Problem**</u>



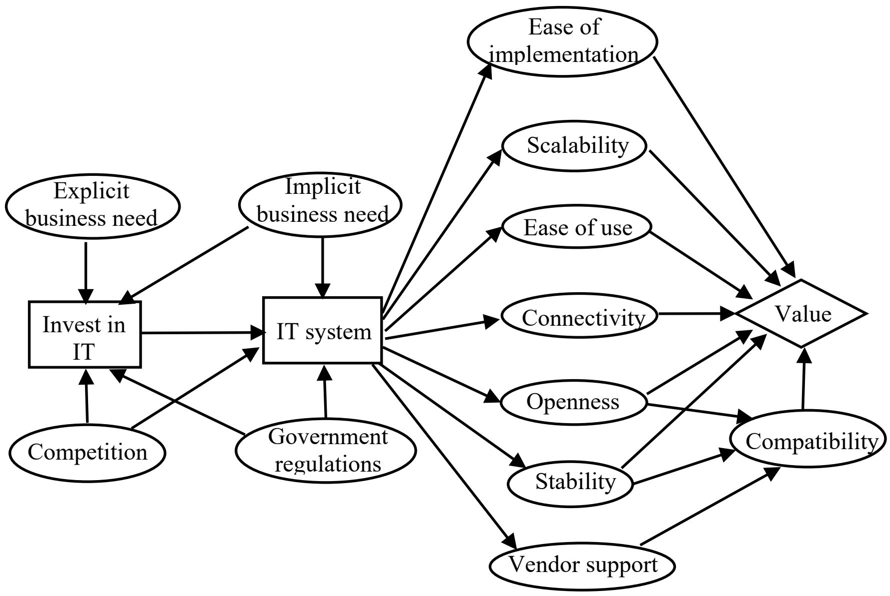
<u>**Critical Component Replacement Maintenance Decision Problem**</u>



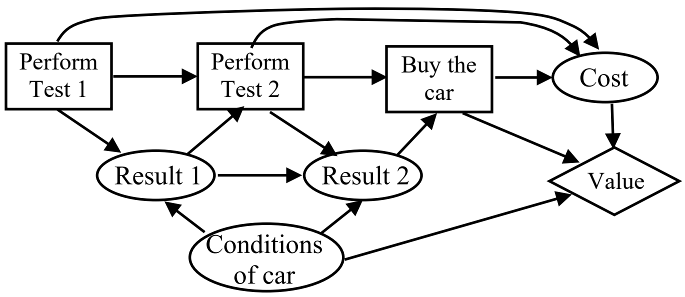
An Intelligent ICU Decision Problem



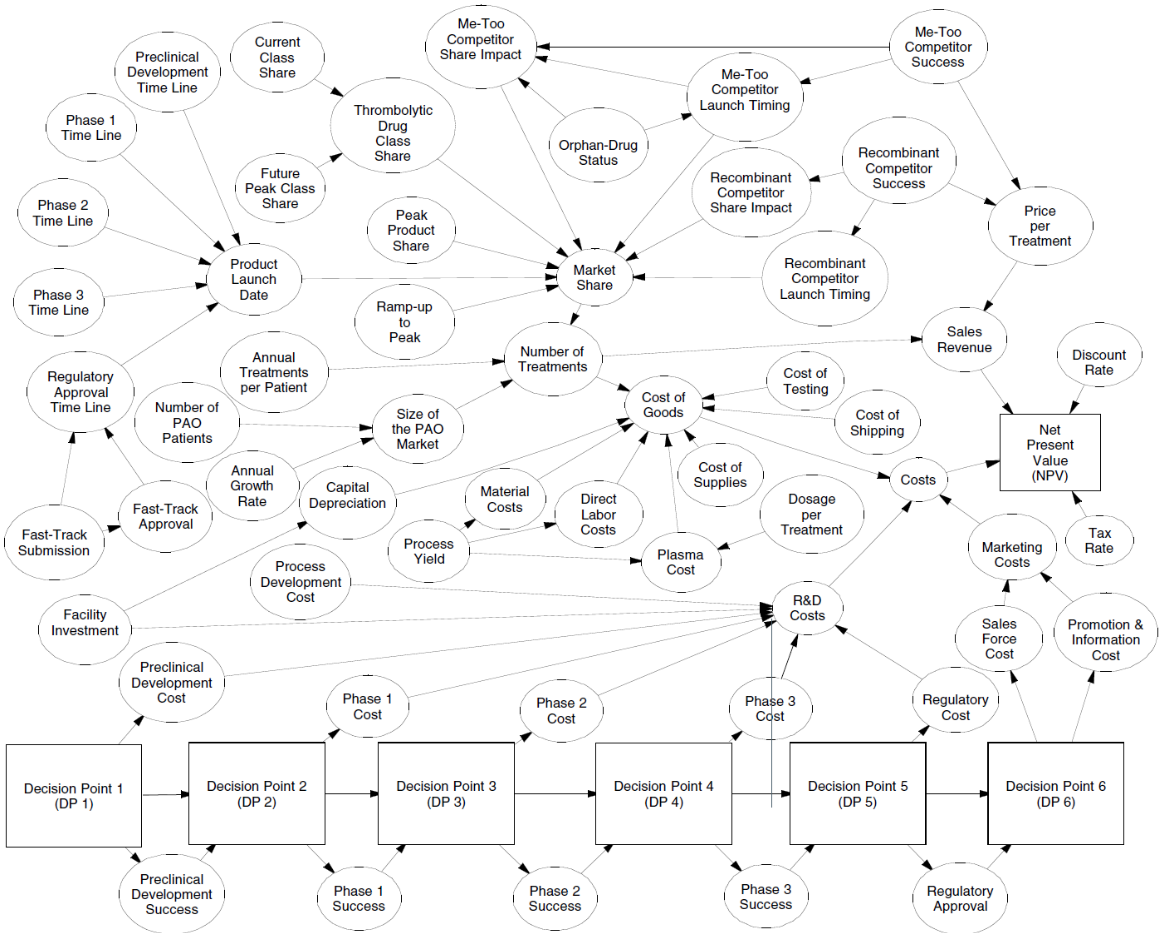
Investment Decision for an IT System



Buying a Used Car

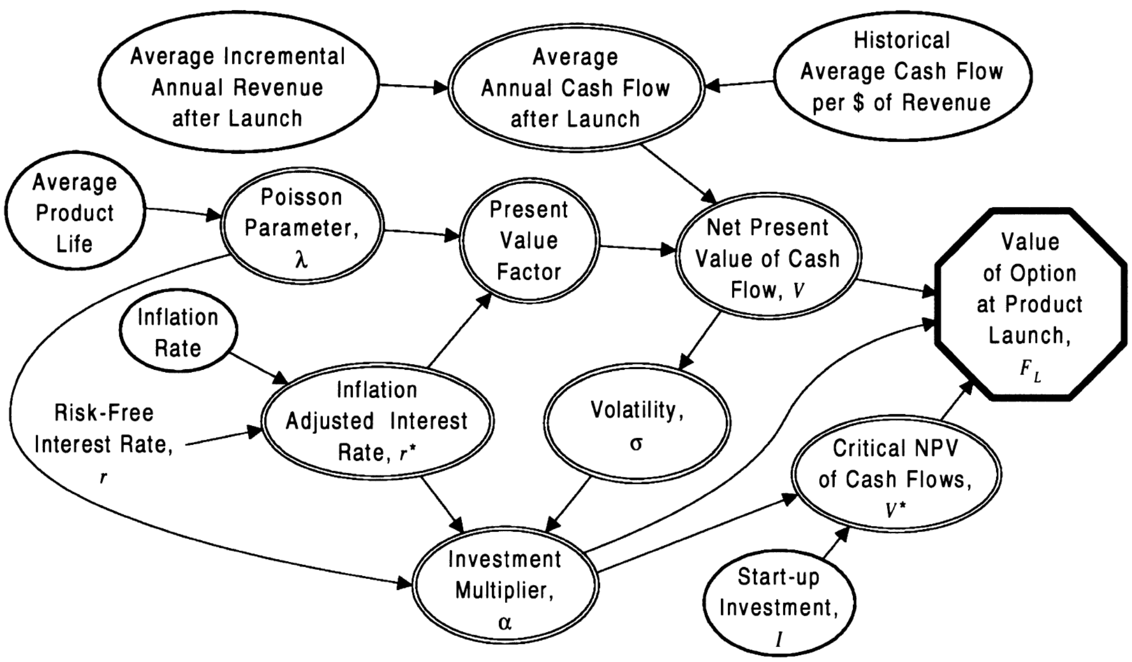


Decisions to Develop New Drugs (Stonebraker 2002)

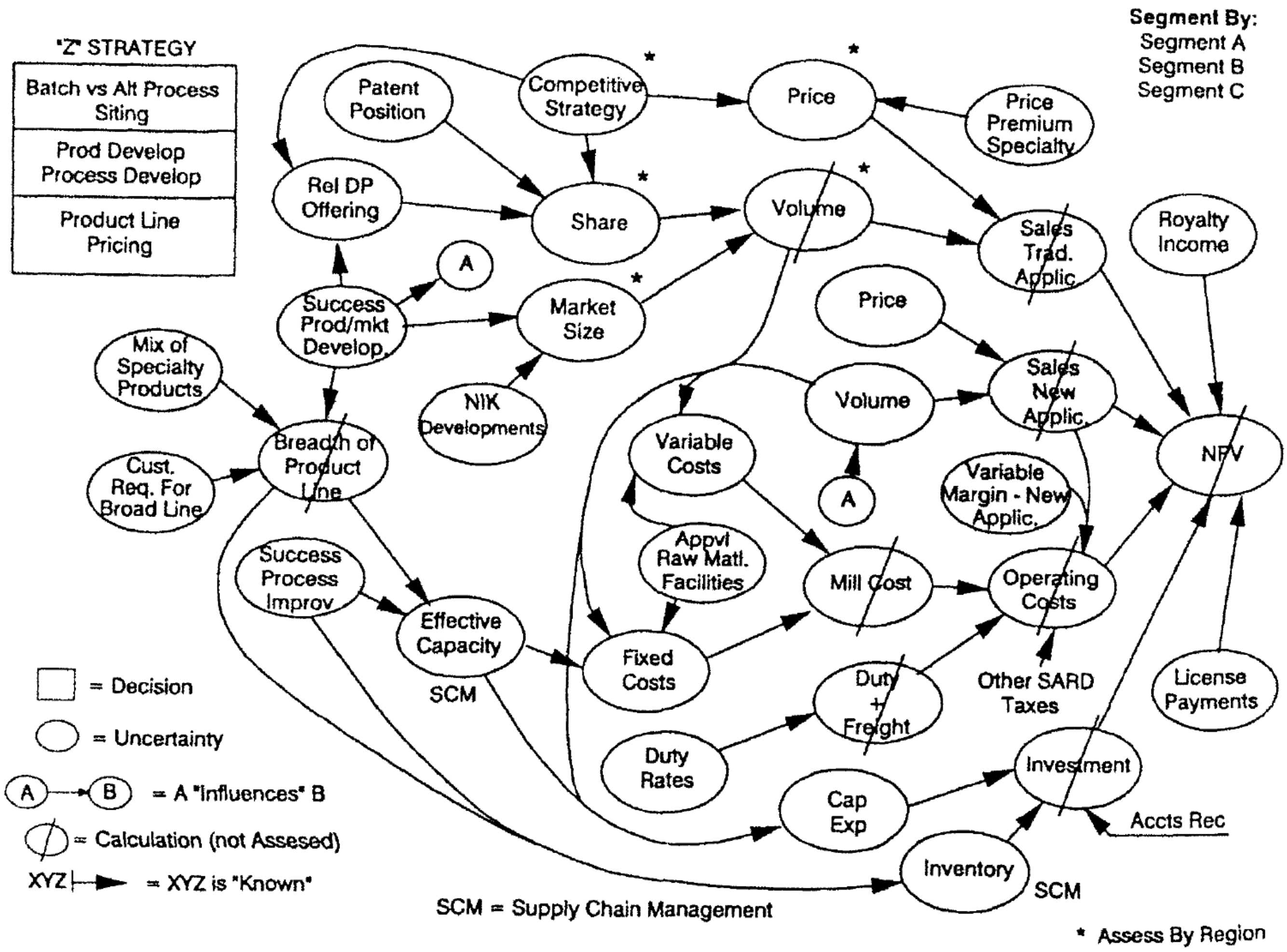


Valuation of R & D Projects using Options Pricing and Decision Analysis Models

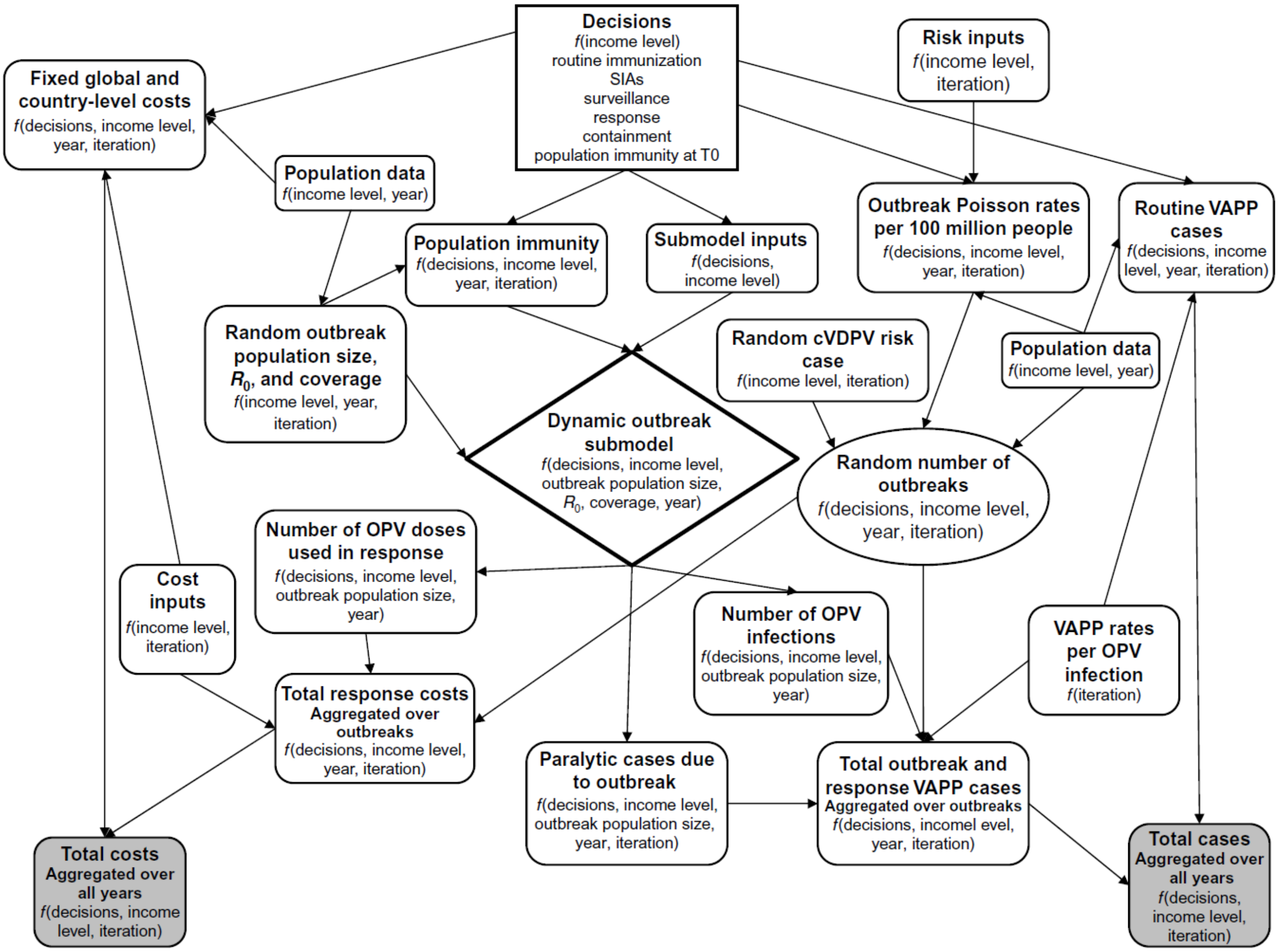
(Perdue et al,1999)



Decision and Risk Analysis in Du Pont (Krumm and Rolle, 1992)



Polio Eradicators Decision Analysis (Thompson *et al*, 2015)



References

1. RA Howard and JE Matheson (1981). Influence Diagrams. In RA. Howard and J.E. Matheson (Editors). *Readings on the Principles and Applications of Decision Analysis*, pp 720 –762. This classical paper been republished in the journal *Decision Analysis* **2**(3):127-143, 2005.

2. RA Howard (1990). From influence to relevance to knowledge. In RM. Oliver and JQ Smith (editors), *Influence diagrams, Belief Nets and Decision Analysis*, pp 3-23, John Wiley.

3. RD Shachter (1986). Evaluating Influence Diagrams, *Operations Research* **34**(6):871-882.

4. J.S. Stonebraker (2002). How Bayer Makes Decisions to Develop New Drugs. *Interfaces* **32**(6):77-90.

5. R.K. Perdue, W.J. McAllister, P.V. King and B.G. Berkey (1999). Valuation of R and D Projects Using Options Pricing and Decision Analysis Models. *Interfaces* **29**(6):57-74.

6. F.V. Krumm, C.F. Roll (1992). Management and Application of Decision and Risk Analysis in Du Pont. *Interfaces* **22**(6):84-93.

7. K.M. Thompson, R.J.D Tebbens, M.A. Pallansch, S.G.F Wassilak and S.L Cochi (2015). Polio Eradicators Use Integrated Analytical Models to Make Better Decisions. *Interfaces* **45**(1):5-25

Exercises

P5.1 (Clement and Reilly 2001, Problem 3.9, p99)
A dapper young decision maker has just purchased a new suit for \$200. On the way out the door, he considers taking an umbrella. With the umbrella on hand, the suit will be protected in the event of rain. Without the umbrella, the suit will be ruined if it rains. On the other hand, if it does not rain, carrying the umbrella is an unnecessary inconvenience.

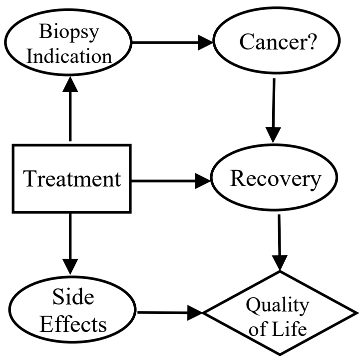
(***a***) Draw a decision tree of this situation.

(***b***) Draw an influence diagram of the situation.

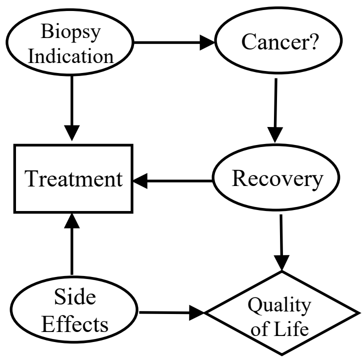
(***c***) Before deciding, the decision maker considers listening to the weather forecast on the radio. Draw an influence diagram that takes into account the weather forecast.

P5.2 You are helping a friend who might have cancer. She has just had a needle biopsy taken and will be meeting with her doctor when the laboratory report becomes available to choose a treatment. Her concerns in making the decision is whether she recovers from any cancer and some serious side effects associated with some of the treatment choices. Which of these influence diagrams correctly and best captures her situation?

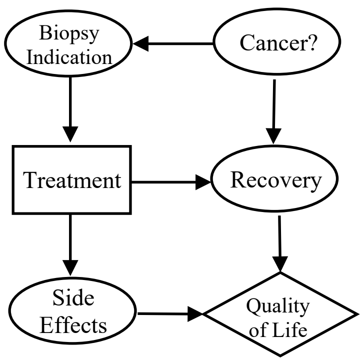
(***a***)



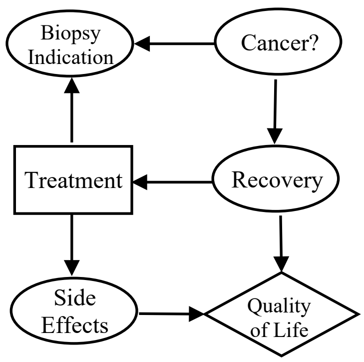
(***b***)



(***c***)



(***d***)



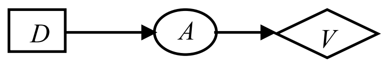
P5.3 (Clement and Reilly 2001, Problem 310, p99)

When patients suffered from hemorrhagic fever, M*A*S*H doctors replaced lost sodium by administering a saline solution intravenously. However, headquarters (HQ) sent a treatment change disallowing the saline solution. With a patient in shock and near death from a disastrously low sodium level, B.J. Hunnicut wanted to administer a low-sodium concentration saline solution as a last ditch attempt to save the patient. Colonel Potter looked at B.J. and Hawkeye and summed up the situation: "OK, let's get this straight. If we go by the new directive from HQ and don't administer saline to replace the sodium, our boy will die for sure. If we try B.J's idea, then he may survive, and we'll know how to treat the next two patients who are getting worse. If we try it and he doesn't make it, we're in trouble with HQ and may get court-martialed. I say we have no choice. Let's try it". (Source: Mr. and Mrs. Who" written by Ronny Graham, directed by Burt Metealfe, 1980)

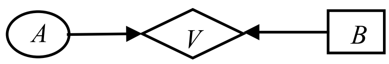
Structure the doctors' decision. What are their objectives? What risks do they face? Draw a decision tree and an influence diagram for their situation.

P5.4 For each of the influence diagram below, draw an equivalent decision tree.

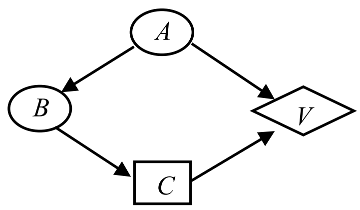
(***a***)



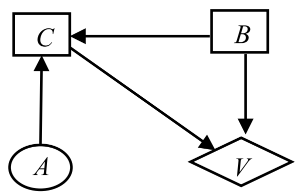
(***b***)



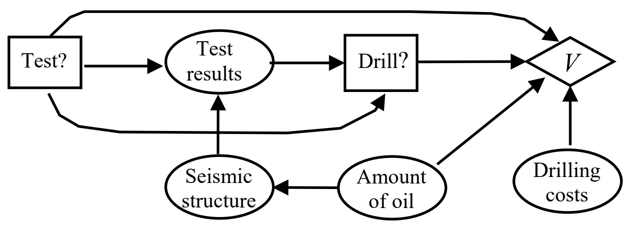
(***c***)



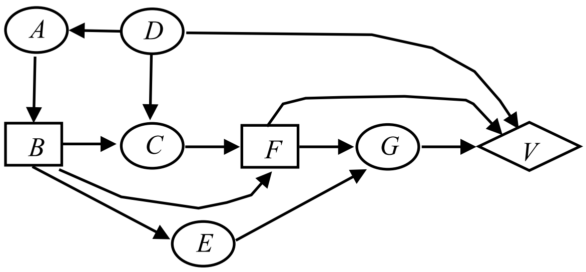
(***d***)



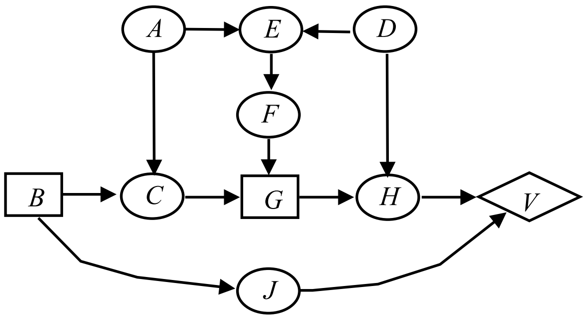
(***e***) The oil cluttering problem



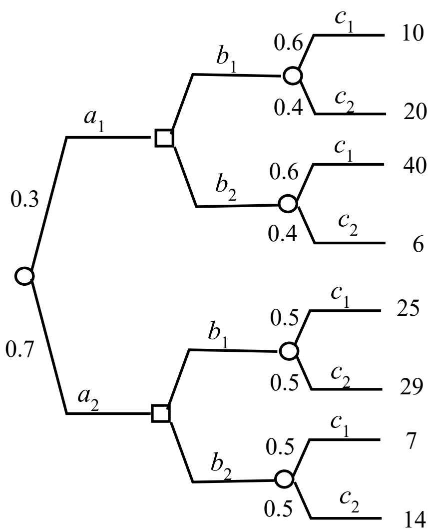
(***f***)



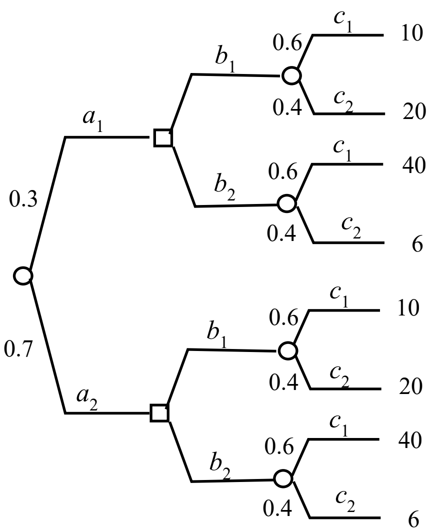
(***g***)



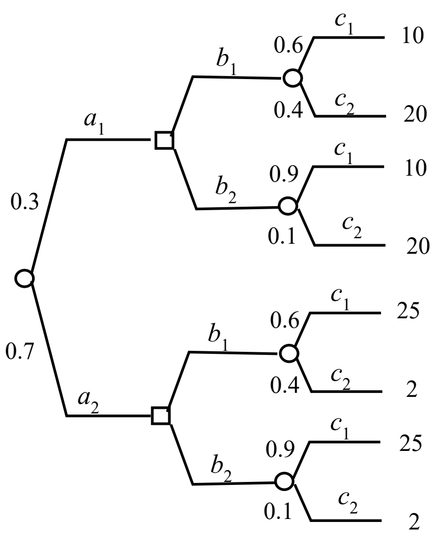
P5.5 For each of the following decision trees, draw an equivalent influence diagram representing the decision situation depicted by the tree. Include only those nodes and arcs that are necessary.



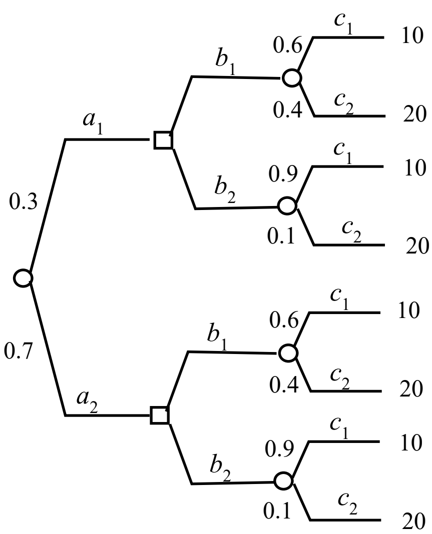
(***a***)



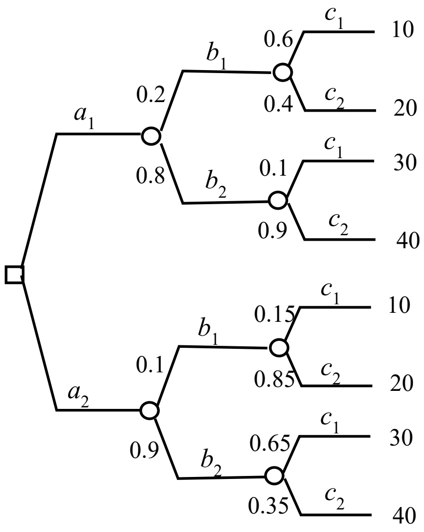
(***b***)



(***c***)

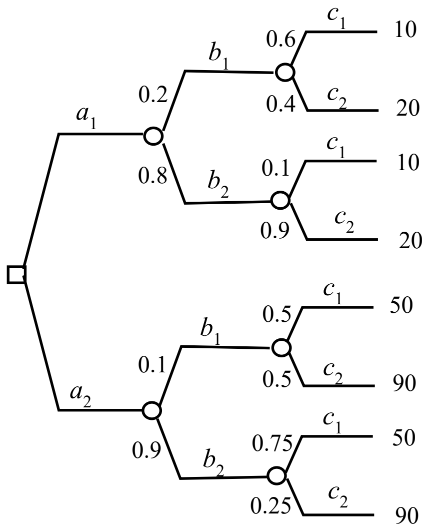


(***d***)



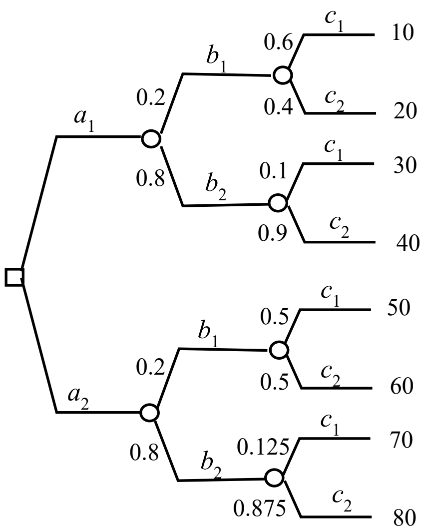
a1
a2
b1
b2
b1
b2
c1
c2
c1
c2
c1
c2
c1
c2
0.2
0.8
0.1
0.9
0.6
0.4
0.1
0.9
0.15
0.85
0.65
0.35
10
20
30
40
10
20
30
40

(*e*)



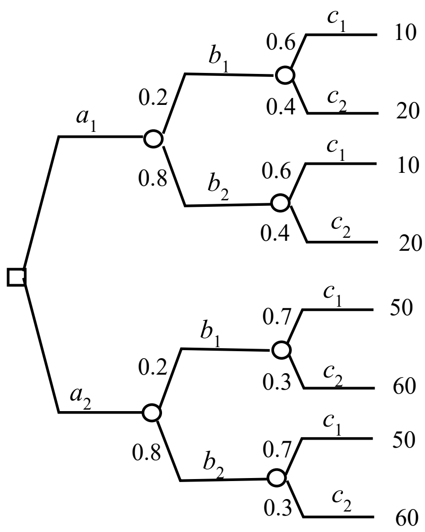
a1
a2
b1
b2
b1
b2
c1
c2
c1
c2
c1
c2
c1
c2
0.2
0.8
0.1
0.9
0.6
0.4
0.1
0.9
0.5
0.5
0.75
0.25
10
20
10
20
50
90
50
90

(*f*)



a1
a2
b1
b2
b1
b2
c1
c2
c1
c2
c1
c2
c1
c2
0.2
0.8
0.2
0.8
0.6
0.4
0.1
0.9
0.5
0.5
0.125
0.875
10
20
30
40
50
60
70
80

(*g*)



a1
a2
b1
b2
b1
b2
c1
c2
c1
c2
c1
c2
c1
c2
0.2
0.8
0.2
0.8
0.6
0.4
0.6
0.4
0.7
0.3
0.7
0.3
10
20
10
20
50
60
50
60

(*h*)

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