Chapter 10 Multiple Criteria Decision Making II
Multiple Attribute Decision Making

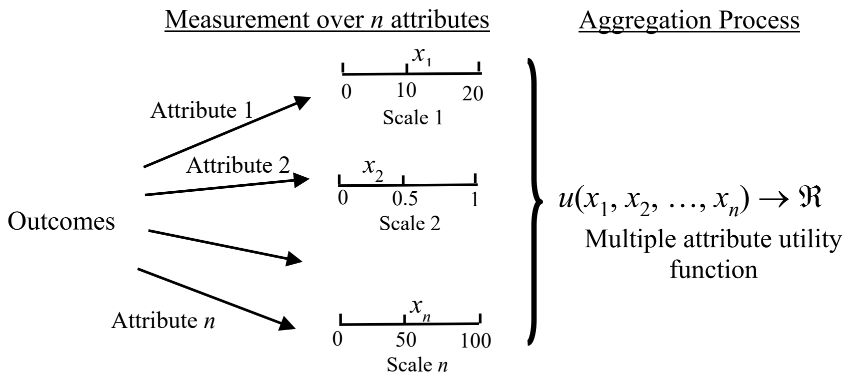
10.1 Multiple Attribute Utility Theory

10.1.1 Evaluating Outcomes Using Multiple Attributes

- In some decision problems, it may not be possible to measure all outcomes using economic dollar values or discounted cash flows.

- In these cases, it may be necessary to measure the outcomes using different attributes on different scales which cannot be directly added together.

- The Multiple Attribute Decision Making Approach is illustrated below:



1. The possible outcomes of a decision problem are first evaluated or measured using n different attribute scales. An n-dimensional vector of attribute values $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$ is obtained.

2. The n attribute values are then aggregated by a **Multiple Attribute Utility function** $u(x_1, x_2, \ldots, x_n)$ to arrive at a utility value.

3. The principle of maximum expected utility is then applied to solve the decision problem.

10.1.2 Decomposable Multiple Attribute Utility Function

- The multiple attribute utility function $u(x_1, x_2, \ldots, x_n)$ is normally very difficult to access directly.

- A decomposition approach is often used to simplify the assessment process.

- Here, each attribute value x_i is first measured by n individual single-attribute utility functions $u_i(x_i)$, and then the results are combined by an aggregation function f as follows:

$$\left.\begin{array}{l} x_1 \longrightarrow u_1(x_1)\\ x_2 \longrightarrow u_2(x_2)\\ \qquad \cdots \\ x_n \longrightarrow u_n(x_n)\end{array}\right\} f(u_1(x_1), u_2(x_2), \ldots, u_n(x_n)) \to \Re$$

Definition: Decomposable Utility Function

- A multiple attribute utility function $u(x_1, x_2, \ldots, x_n)$ is **Decomposable** if it can be written in the form

$$u(x_1, x_2, \ldots, x_n) = f(u_1(x_1), u_2(x_2), \ldots, u_n(x_n))$$

where

$u_i(x_i)$ is a **single-attribute utility function** for attribute i, $i = 1$ to n.

$f()$ a function that aggregates the individual utilities into a single real value.

- There are 3 common types of decomposable multiple attribute utility functions:

 1. Additive utility function
 2. Multiplicative utility function
 3. Multi-linear utility function

1. Additive Utility Function

- The additive utility function has the form:

$$u(x_1, x_2, \cdots, x_n) = \sum_{i=1}^{n} k_i u_i(x_i) \qquad \text{where} \qquad \sum_{i=1}^{n} k_i = 1$$

Examples

- $u(x_1, x_2) = k_1\, u_1(x_1) + (1 - k_1)\, u_2(x_2)$
- $u(x_1, x_2, x_3) = k_1\, u_1(x_1) + k_2\, u_2(x_2) + (1 - k_1 - k_2)\, u_3(x_3)$.

2. Multiplicative Utility Function

- The multiplicative utility function has the form:

$$u(x_1, x_2, \cdots, x_n) = \frac{1}{K}\left[\prod_{i=1}^{n}(1 + K\, k_i\, u_i(x_i)) - 1\right]$$

where $\quad -1 < K < 0 \quad$ or $\quad K > 0$

and $\quad 1 + K = \prod_{i=1}^{n}(Kk_i + 1)$.

- When $K \to 0$, the additive form is obtained.

Examples

$$u(x_1, x_2) = \frac{1}{K}\left[(1 + Kk_1u_1(x_1))(1 + Kk_2u_2(x_2)) - 1\right]$$
$$= k_1u_1(x_1) + k_2u_2(x_2) + Kk_1k_2u_1(x_1)u_2(x_2)$$

where $k_1 + k_2 + K\,k_1k_2 = 1$

$$u(x_1, x_2, x_3) = \frac{1}{K}\left[(1 + Kk_1u_1(x_1))(1 + Kk_2u_2(x_2))(1 + Kk_3u_3(x_3)) - 1\right]$$
$$= k_1u_1(x_1) + k_2u_2(x_2) + k_3u_3(x_3) + Kk_1k_2u_1(x_1)u_2(x_2) + Kk_1k_3u_1(x_1)u_3(x_3)$$
$$+ Kk_2k_3u_2(x_2)u_3(x_3) + K^2k_1k_2k_3u_1(x_1)u_2(x_2)u_3(x_3)$$

where $k_1 + k_2 + k_3 + K\,k_1\,k_2 + K\,k_1\,k_3 + K\,k_2\,k_3 + K^2\,k_1\,k_2\,k_3 = 1$

3. Multi-Linear Utility Function

- The multi-linear utility function has the form:

$$u(x_1, \ldots, x_n) = \sum_{i=1}^{n} k_iu_i(x_i) + \sum_{i=1}^{n}\sum_{j>i} k_{ij}u_i(x_i)u_j(x_j) + \sum_{i=1}^{n}\sum_{j>i}\sum_{l>j} k_{ijl}u_i(x_i)u_j(x_j)u_l(x_l)$$
$$+ \cdots + k_{123\cdots n}u_1(x_1)u_2(x_2)\cdots u_n(x_n)$$

- This is a more general form of the multiplicative utility function with more freedom in the values of the constants.

- For the two attributes case, the multiplicative form is identical to the multi-linear form.

Examples

$u(x_1, x_2) = k_1\,u_1(x_1) + k_2\,u_2(x_2) + k_{12}\,u_1(x_1)\,u_2(x_2)$

where $k_1 + k_2 + k_{12} = 1$

$u(x_1, x_2, x_3) = k_1\,u_1(x_1) + k_2\,u_2(x_2) + k_3\,u_3(x_3) + k_{12}\,u_1(x_1)\,u_2(x_2)$
$+ k_{13}\,u_1(x_1)\,u_3(x_3) + k_{23}\,u_2(x_2)\,u_3(x_3) + k_{123}\,u_1(x_1))\,u_2(x_2)\,u_3(x_3)$

where $k_1 + k_2 + k_3 + k_{12} + k_{13} + k_{23} + k_{123} = 1$

When are the above decomposable utility functions valid?

- The followings are the key conditions:

  1. Preference Independence $\Rightarrow$ Utility function is decomposable.

  2. Utility Independence $\Rightarrow$ Utility function is multiplicative.

  3. Additive Independence $\Rightarrow$ Utility function is additive.

- See Keeney & Raiffa (1976) for definitions of the above conditions and details.

Boundary conditions for multiple attribute utility functions

- Given an n-attribute utility function

  $$u(x_1, x_2, \ldots, x_n) = u(u_1(x_1), u_2(x_2), \ldots, u_n(x_n))$$

- We assume for all attributes i =1 to n:

  $u_i(x_i = \text{worst}) = 0$ // attribute i is at its worst
  $u_i(x_i = \text{best}\ ) = 1$ // attribute i are at their best

  and

  $u(0, 0, 0, \ldots, 0) = 0$ // all attributes are at their worst
  $u(1, 1, 1, \ldots, 1) = 1$ // all attributes are at their best

Assessment of Multiplicative Utility Function

- The constants in the n-attribute multiplicative utility function may be determined using the reference lottery (preference probability) approach.

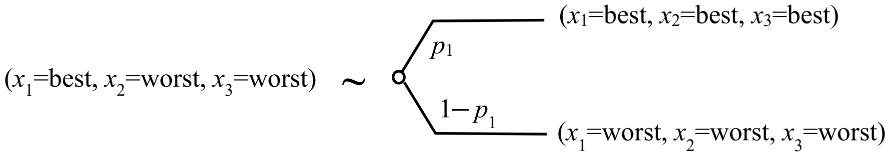
- For example, consider the 3-attribute multiplicative utility function:

$$\begin{aligned} u(x_1, x_2, x_3) &= \frac{1}{K}\left[(1 + Kk_1u_1(x_1))(1 + Kk_2u_2(x_2))(1 + Kk_3u_3(x_3)) - 1\right] \\ &= k_1u_1(x_1) + k_2u_2(x_2) + k_3u_3(x_3) + Kk_1k_2u_1(x_1)u_2(x_2) + Kk_1k_3u_1(x_1)u_3(x_3) \\ &\quad + Kk_2k_3u_2(x_2)u_3(x_3) + K^2k_1k_2k_3u_1(x_1)u_2(x_2)u_3(x_3) \end{aligned}$$

  where $k_1 + k_2 + k_3 + K\,k_1\,k_2 + K\,k_1\,k_3 + K\,k_2\,k_3 + K^2\,k_1\,k_2\,k_3 = 1$

- The constants k_1, k_2 and k_3 can be assessed using the following steps:

- To determine k_1, the decision maker is asked to indicate for what value of p_1 is he/she indifferent between the followings:



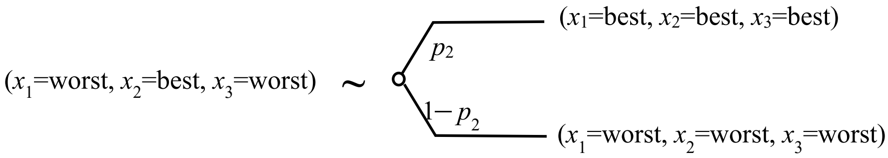
u(best, worst, worst) = p_1 u(best, best, best) + $(1 - p_1)$ u(worst, worst, worst)

$k_1 u_1$(best) + $k_2 u_2$(worst) + $k_3 u_3$(worst) + $K k_1 k_2 u_1$(best) u_2(worst) + $K k_1 k_3 u_1$(best) u_3(worst) + $K k_2 k_3 u_2$(worst) u_3(worst) = p_1 u(best, best, best) + $(1 - p_1)$ u(worst, worst, worst)

- Applying the boundary conditions: u_1(best) = 1, u_2(worst) =0, u_3(worst) = 0, u(best, best, best) = 1 and u(worst, worst, worst) = 0

  we obtain $k_1 = p_1$

- Similarly, to determine k_2, the decision maker is asked to indicate for what value of p_2 is he/she indifferent between the followings:



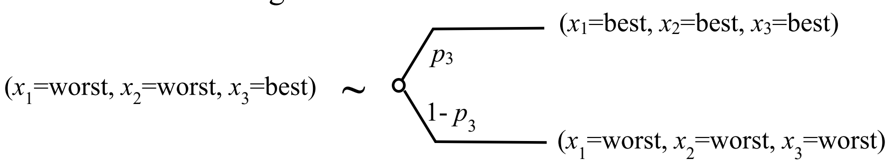
u(worst, best, worst) = p_2 u(best, best, best) + $(1 - p_2)$ u(worst, worst, worst)

$k_1 u_1$(worst) + $k_2 u_2$(best) + $k_3 u_3$(worst) + $K k_1 k_2 u_1$(worst) u_2(best) + $K k_1 k_3 u_1$(worst) u_3(worst) + $K k_2 k_3 u_2$(best) u_3(worst) = p_2 u(best, best, best) + $(1 - p_2)$ u(worst, worst, worst)

- Applying the boundary conditions: u_1(worst) = 0, u_2(best) =1, u_3(worst) = 0, u(best, best, best) = 1 and u(worst, worst, worst) = 0

  we obtain $k_2 = p_2$

- Similarly, to determine k_3, the decision maker is asked to indicate for what value of p_3 is he/she indifferent between the followings:



u(worst, worst, best) = p_3 u(best, best, best) + $(1 - p_3)$ u(worst, worst, worst)

$k_1 u_1$(worst) + $k_2 u_2$(worst) + $k_3 u_3$(best) + $K k_1 k_2 u_1$(worst) u_2(worst) + $K k_1 k_3 u_1$(worst) u_3(best) + $K k_2 k_3 u_2$(worst) u_3(best) = p_3 u(best, best, best) + $(1 - p_3)$ u(worst, worst, worst)

- Applying the boundary conditions: u_1(worst) = 0, u_2(worst) =0, u_3(best) = 1, u(best, best, best) = 1 and u(worst, worst, worst) = 0

  we obtain $k_3 = p_3$

10.1.3 Applications of Multiple Attribute Decision Theory

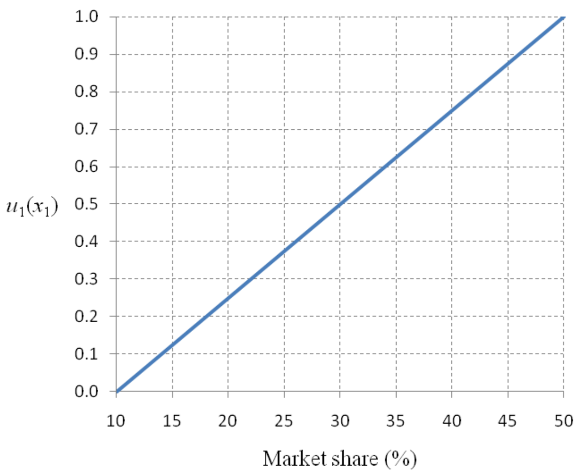
Example 1

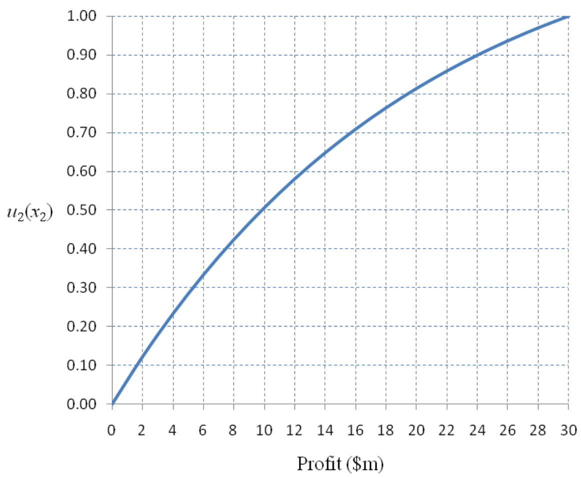
- A company is certain that its market share for a product for the coming year will be between 10% and 50% of the market.
- It is also sure that its profits during the coming year will between \$0 million and \$30 million. The company's multiple attribute utility function is:

$$u(x_1, x_2) = 0.6\, u_1(x_1) + 0.5\, u_2(x_2) - 0.1\, u_1(x_1)\, u_2(x_2)$$

where x_1 = company's market share (%) $(10 \le x_1 \le 50)$
x_2 = company's profit (\$ million) $(0 \le x_2 \le 30)$

$u_1(x_1) = \frac{1}{40}(x_1 - 10)$ and $u_2(x_2) = 1.2872(1 - e^{-x_2/20})$ are single-attribute utility functions for attributes x_1 and x_2, respectively.





- The company would like to decide if it should mount a small or large advertising campaign during the coming year.

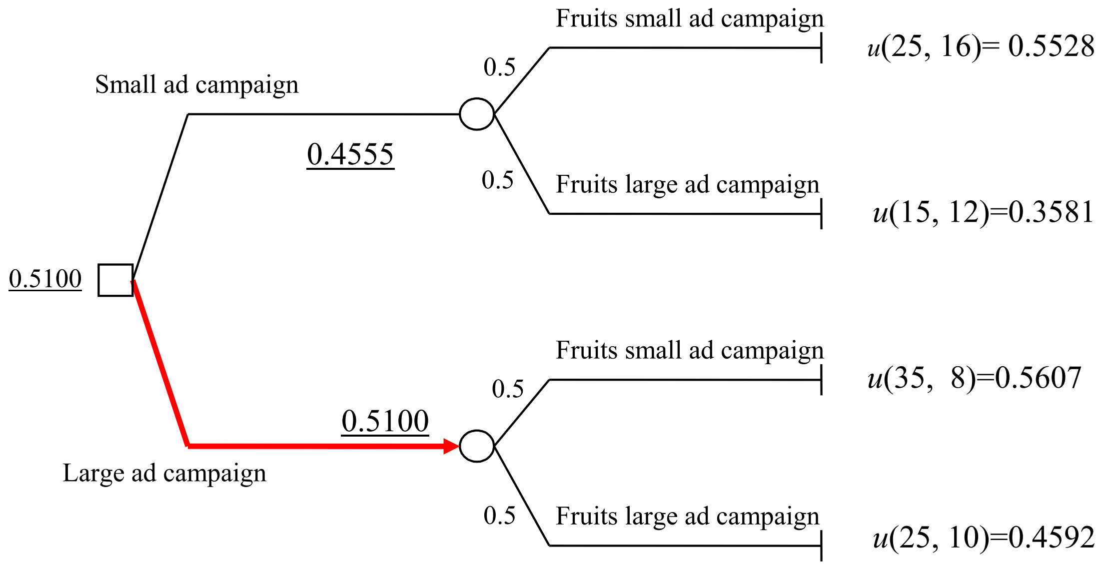
- The company believes that its main rival Fruits Inc. will mount either a small or large TV advertising campaign with a 50-50 chance.

- The resulting market share and profits (\$million) for the company depending on Fruit's action are:

Company's decision	Fruit's Decision	
	Small ad campaign	Large ad campaign
Small ad campaign	25%, \$16	15%, \$12
Large ad campaign	35%, \$8	25%, \$10

- What is the company's best decision?

- The company's decision problem may be represented by the following decision tree:



- The terminal values for the decision trees were computed using the multi-attribute utility function as follows:

$$\begin{aligned} u(25\%, \$16) &= 0.6\, u_1(25\%) + 0.5\, u_2(\$16) - 0.1\, u_1(25\%)\, u_2(\$16) \\ &= 0.6\,(0.375) + 0.5\,(0.7088) - 0.1\,(0.375)\,(0.7088) \\ &= 0.5528 \end{aligned}$$

$$\begin{aligned} u(15\%, \$12) &= 0.6\, u_1(15\%) + 0.5\, u_2(\$12) - 0.1\, u_1(15\%)\, u_2(\$12) \\ &= 0.6\,(0.125) + 0.5\,(0.5808) - 0.1\,(0.125)\,(0.5808) \\ &= 0.3581 \end{aligned}$$

$$\begin{aligned} u(35\%, \$8) &= 0.6\, u_1(35\%) + 0.5\, u_2(\$8) - 0.1\, u_1(35\%)\, u_2(\$8) \\ &= 0.6\,(0.625) + 0.5\,(0.4244) - 0.1\,(0.625)\,(0.4244) \\ &= 0.5607 \end{aligned}$$

$$\begin{aligned} u(25\%, \$10) &= 0.6\, u_1(25\%) + 0.5\, u_2(\$10) - 0.1\, u_1(25\%)\, u_2(\$10) \\ &= 0.6\,(0.375) + 0.5\,(0.5065) - 0.1\,(0.375)\,(0.5065) \\ &= 0.4592 \end{aligned}$$

- The expected utilities for the two alternatives are
  - Small advertisement campaign: 0.4555
  - Large advertisements campaign: **0.5100**

- The company should mount a large advertisement campaign.

Example 2

- Lisa is planning an overseas vacation and has been considering various international resorts. After spending many hours of surfing the Internet, she finally ended up with two options which are roughly equivalent as far as cost is concerned.

- The main differences between the two options are along two attributes:
  1. x_1: Amenities (e.g., available activities, facilities, etc.)
  2. x_2: Accessibility to local attractions (e.g., day tours, nearby shopping, etc.)

- Lisa will assess both attributes on a scale of 0 (worst) to 10 (best), and use the following 2-attribute utility function to make her decision:

  $$u(x_1, x_2) = k_1\, u_1(x_1) + k_2\, u_2(x_2) + (1 - k_1 - k_2)\, u_1(x_1)\, u_2(x_2)$$

  where k_1 and k_2, are constants; $u_1(x_1) = \frac{x_1}{10}$ and $u_2(x_2) = \frac{x_2^2}{100}$ are single-attribute utility functions for attributes x_1 and x_2 respectively.

- Lisa indicates that she is indifferent between the following two deals:

  1. A sure outcome of **best** amenities and **worst** accessibility.

  2. With probability 0.7, receives the **best** amenities and **best** accessibility
     With probability 0.3, receives the **worst** amenities and **worst** accessibility

  Hence, $k_1 = 0.7$

- Lisa also indicates that she is indifferent between the following two deals:

  1. A sure outcome of **worst** amenities and **best** accessibility.

  2. With probability 0.4, receives the **best** amenities and **best** accessibility
     With probability 0.6, receives the **worst** amenities and **worst** accessibility

  Hence, $k_2 = 0.4$

- Hence Lisa's utility function is

  $$u(x_1, x_2) = 0.7\, u_1(x_1) + 0.4\, u_2(x_2) - 0.1\, u_1(x_1)\, u_2(x_2)$$

  $$u_1(x_1) = \frac{x_1}{10} \quad 0 \le x_1 \le 10 \qquad\qquad u_2(x_2) = \frac{x_2^2}{100} \quad 0 \le x_2 \le 10$$

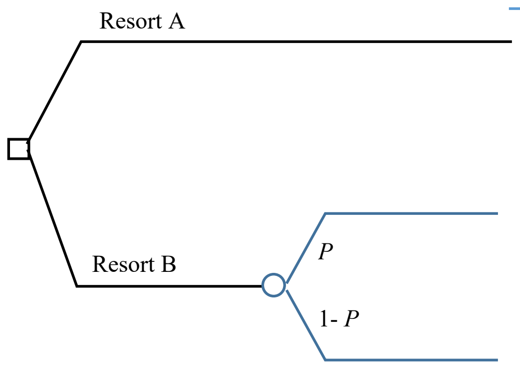
- Lisa's two alternatives are as follows:

  1. Resort A is where one of Lisa's close friends spent a great vacation. Lisa thinks that this resort should be assigned $x_1 = 7$ and $x_2 = 6$ with no uncertainty.

  2. Resort B is an exotic but somewhat risky choice. Based on the skimpy website information, Lisa thinks that with probability P it is an isolated place with great amenities so $x_1 = 9$ and $x_2 = 2$, and with probability $1 - P$ it has poor amenities but good access to nearby attractions so $x_1 = 1$ and $x_2 = 8$.

- What is the range of P for which Lisa will prefer Resort B to Resort A?

- Decision tree for Lisa:

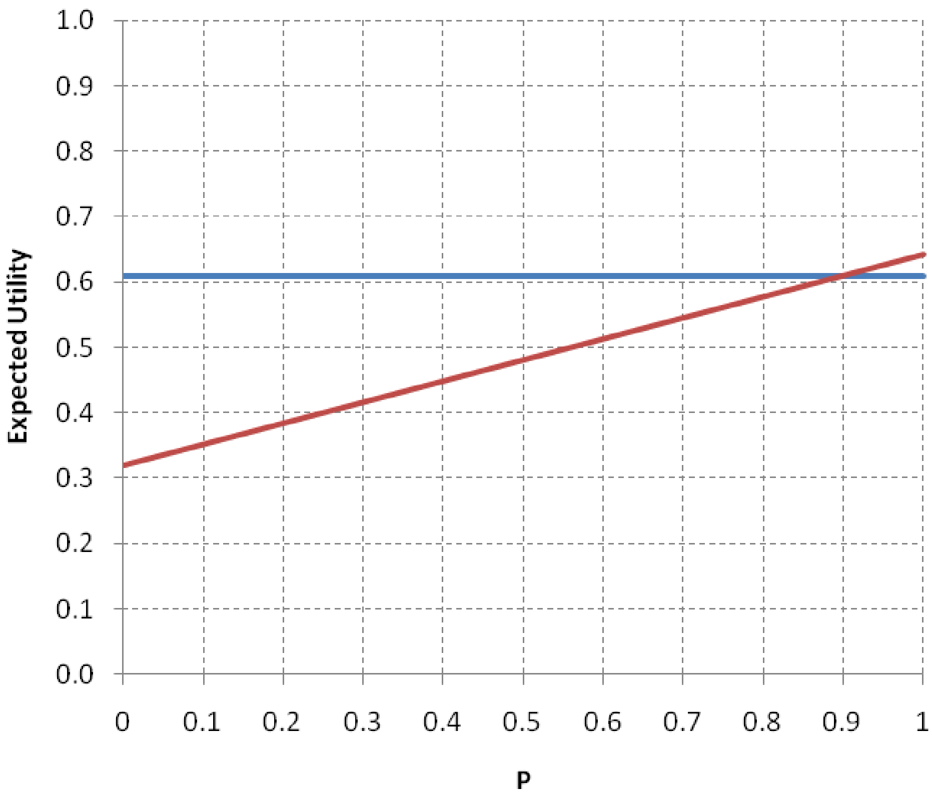


(x_1, x_2)	$u_1(x_1)$	$u_2(x_2)$	$u(x_1, x_2)$
(7, 6)	0.7	0.36	0.6088
(9, 2)	0.9	0.04	0.6424
(1, 8)	0.1	0.64	0.3196

- From the decision tree:
  - EU(Resort A) = 0.6088
  - EU(Resort B) = P (0.6424) + (1 – P)(0.3196) = 0.3196 + 0.3228 P
- Hence Resort B is preferred to Resort A if 0.3196 + 0.3228 P > 0.6088

  $\Rightarrow \quad 0.895911 < P \leq 1$

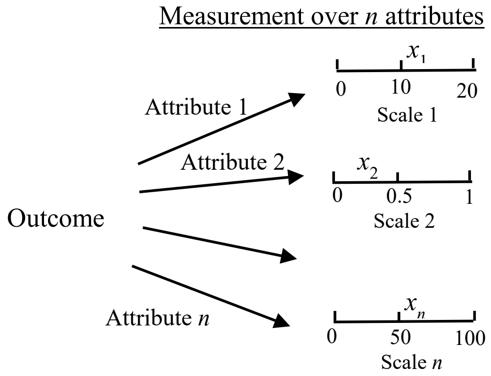
- The range of P is illustrated in the rainbow diagram:



10.2 Multiple Attribute Trade-off Analysis

10.2.1 Multiple Attribute Dominance Analysis

- Suppose we can evaluate the alternative attribute values, but do not have the multiple attribute utility function to complete the analysis.



- The multiple attribute trade-off analysis is performed using only the alternatives' attribute values.

Definition: Multiple Attribute Dominance (*n*-attribute)

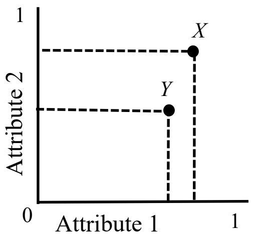
- Let alternative X has outcome values $(x_1, x_2, \ldots, x_n)$ and alternative Y has values $(y_1, y_2, \ldots, y_n)$. X **Dominates** Y if and only if

$$x_i \succeq y_i \qquad \forall\, i \in \{1, 2, \ldots, n\} \qquad \text{and}$$
$$x_j \succ y_j \qquad \exists\, j \in \{1, 2, \ldots, n\}$$

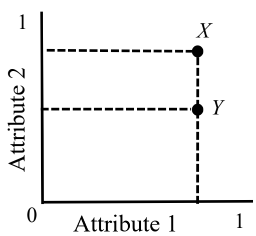
- That is, the outcomes of alternative X dominate those of alternative Y if and only if X is at least as preferred as Y in all attributes, and strictly more preferred in at least one attribute.

Examples (2-Attribute Case)

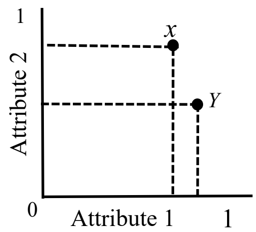
- WLOG, we assume that the decision-maker prefers more to less for all attributes.



X strongly dominates Y



X weakly dominates Y



No dominance

Definition: Multiple Attribute Efficient Solution

- Let $S = \{A_1, A_2, \ldots A_m\}$ be a set of feasible alternatives.
- An alternative $A_j \in S$ is **Efficient, Non-dominated** or **Pareto-optimal** with respect to S and the n attributes if and only if there does not exist another alternative $A_k \in S$ that dominates A_j.

Non-Efficient or Dominated Alternatives can be Eliminated from further consideration

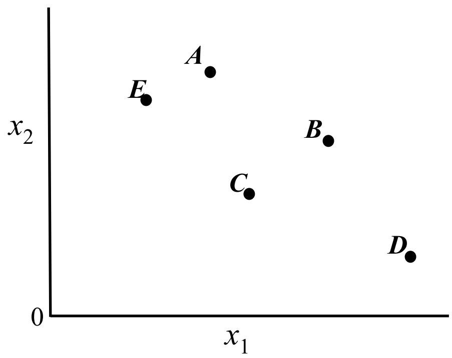
- Given a set of alternatives, a decision maker should not choose a dominated one from the set as he/she can always be better off by choosing another alternative from the set that performs better in at least one other attribute without degrading any other attributes.

10.2.2 Efficient Frontier Analysis

- Given a set of alternatives, the set of efficient alternatives lies on the **Efficient Frontier** of the multiple attribute value space.

Example

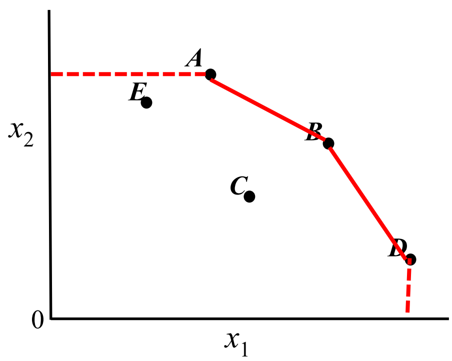
- Consider a 2-attribute problem with five alternatives (A to E) and we prefer more to less for all attributes. Let the 2-attribute values be denoted by x_1 and x_2 and are plotted as follows:



- Observations:
  - *C* is dominated by *B*: *C* is worse off than *B* in both x_1 and x_2.
  - *E* is dominated by *A*: *E* is worse off than *A* in both x_1 and x_2.
- Removing the dominated alternatives *C* and *E*, the efficient set = {*A*, *B*, *D*}.
- Alternatives *E* and *C* can be eliminated from further consideration.
- The decision maker needs to consider only alternatives *A*, *B,* or *D*.

- If we connect up all the efficient solutions in the 2-attribute space, we obtain the **Efficient Frontier:**

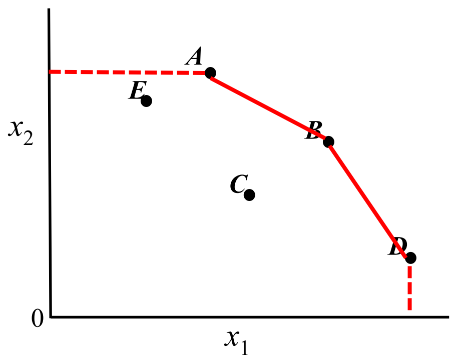
The Efficient Frontier



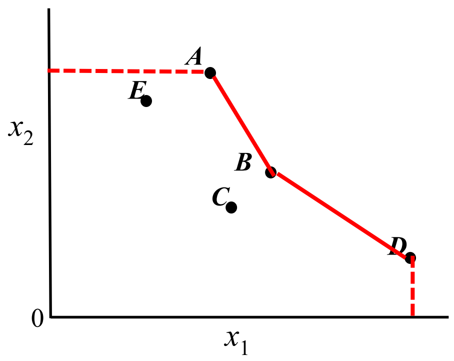
- All alternatives that lie in the interior of the efficient frontier are dominated by at least one other alternative on the efficient frontier.
- Multiple attribute dominance analysis allows us to eliminate inefficient alternatives from further. Only those on the efficient frontier are further considered using other information.

Non-Convex Efficient Frontier

- Note that an Efficient Frontier is not necessarily "convex" in shape.
- In the two diagrams below, all three alternatives A, B, and D are non-dominated.



A "Convex" Efficient Frontier

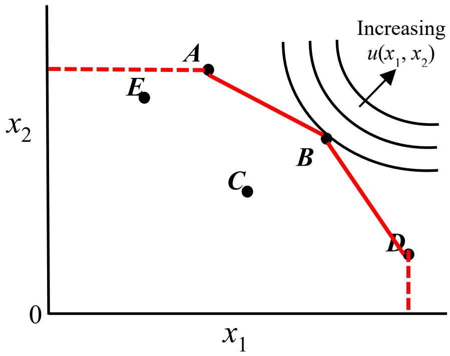


A "Non-Convex" Efficient Frontier

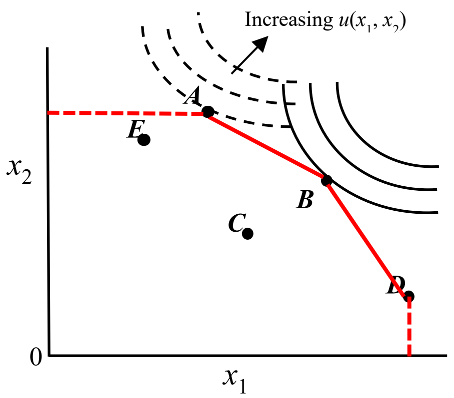
The Optimal Decision

- In principle, the optimal choice depends on the decision maker's preference trade-off between the two attributes. This is described by the two-attribute utility function $u(x_1, x_2)$.

- Suppose the decision maker's utility function $u(x_1, x_2)$ is known, and the contours for equal utility values (also called indifference curves) are shown by the curves below, then the optimal choice is *B*.



- Note that another decision maker with a different utility function will choose differently.
- For example, a decision maker with utility indifference curves corresponding to the dashed lines will choose alternative *A* instead of *B*.



- When the exact multiple attribute utility function is not known, a satisfactory solution to the decision maker may be found using other information or process.

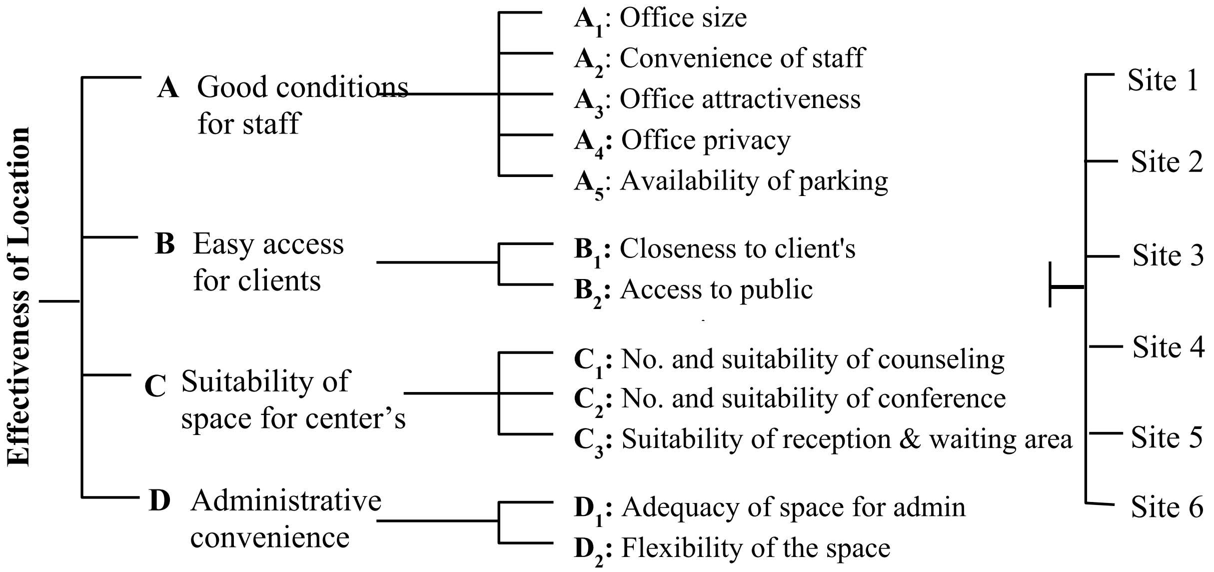
10.3 Cost-Effectiveness Analysis

- In cost-effectiveness analysis, "Effectiveness" and "Cost" are treated as two attributes, and a two-attribute trade-off analysis is performed.

Example: **Drug Counseling Center Relocation Problem** (Adapted from Edward & Newman)

- A drug counseling center needs to relocate and it has identified six possible sites.
- The effectiveness of the six locations is first evaluated without consideration of costs using AHP.

AHP Hierarchy for Effectiveness of Location (without consideration of cost)



- Pairwise comparison of main criteria with respect to Goal

	Good conditions for staff	Easy access for clients	Suitability of space for center's function	Admin convenience	Priority Weight
Good conditions for staff	1	2	2	3	0.42578
Easy access for clients		1	1	2	0.23124
Suitability of space for center's function			1	1	0.19455
Administrative convenience				1	0.14843

$\lambda = 4.0458$, CI = 0.01527, CR = 0.01697 < 0.1

- Pairwise comparison of sub-criteria for "Good Conditions for Staff"

	Office size	Convenience of staff commuting	Office attractiveness	Office privacy	Availability of parking	Priority Weight
Office size	1	2	3	3	3	0.39454
Convenience of staff commuting		1	2	2	2	0.23431
Office attractiveness			1	1	1	0.12372
Office privacy				1	1	0.12372
Availability of parking					1	0.12372

$\lambda = 5.00996$, CI = 0.00249, CR = 0.00222 < 0.1

- Pairwise comparison of sub-criteria for “Easy Access for Clients”

	Closeness to client's homes	Access to public transportation	Priority Weight
Closeness to client's homes	1	1	0.5
Access to public transportation		1	0.5

λ = 2, CI = 0, CR = 0

- Pairwise comparison of sub-criteria for “Suitability of Space for Center's Functions”

	Number and suitability of counseling rooms	Number and suitability of conference rooms	Suitability of reception and waiting area	Priority Weight
Number and suitability of counseling rooms	1	2	2	0.49339
Number and suitability of conference rooms		1	2	0.31081
Suitability of reception and waiting area			1	0.19580

λ = 3.0536, CI = 0.0268, CR = 0.0462 < 0.1

- Pairwise comparison of sub-criteria for “Administrative Convenience”

	Adequacy of space for admin work	Flexibility of the space layout	Priority Weight
Adequacy of space for admin work	1	2	0.66667
Flexibility of the space layout		1	0.33333

λ = 2, CI = 0, CR = 0

- Pairwise comparison of alternatives with respect to “Office Size”

	Site 1	Site 2	Site 3	Site 4	Site 5	Site 6	Priority Weight
Site 1	1	2	9	2	9	2	0.36533
Site 2		1	5	1	5	1	0.18933
Site 3			1	1/5	1	1/4	0.04001
Site 4				1	5	1	0.18933
Site 5					1	1/4	0.04001
Site 6						1	0.17601

λ = 6.00754, CI = 0.001508, CR = 0.001216 < 0.1

- Pairwise comparison of alternatives with respect to “Convenience of Staff Commuting”

	Site 1	Site 2	Site 3	Site 4	Site 5	Site 6	Priority Weight
Site 1	1	2	1/2	5	9	2	0.24672
Site 2		1	1/3	3	6	1	0.13972
Site 3			1	9	9	3	0.39786
Site 4				1	2	1/3	0.04793
Site 5					1	1/6	0.02806
Site 6						1	0.13972

λ = 6.0662, CI = 0.0132, CR = 0.01068 < 0.1

- Pairwise comparison of alternatives with respect to “Office Attractiveness”

	Site 1	Site 2	Site 3	Site 4	Site 5	Site 6	Priority Weight
Site 1	1	1/3	1/2	3	1/3	1/3	0.08379
Site 2		1	1	8	1	1	0.23118
Site 3			1	7	1	1	0.16518
Site 4				1	1/9	1/8	0.04989
Site 5					1	1	0.23878
Site 6						1	0.23118

$\lambda = 6.561$, CI = 0.1122, CR = 0.0905 < 0.1

- Pairwise comparison of alternatives with respect to “Office Privacy”

	Site 1	Site 2	Site 3	Site 4	Site 5	Site 6	Priority Weight
Site 1	1	3	2	9	3	2	0.36027
Site 2		1	1	3	1	1/2	0.12362
Site 3			1	4	1	1	0.15518
Site 4				1	1/3	1/5	0.04007
Site 5					1	1	0.13865
Site 6						1	0.18220

$\lambda = 6.1200$, CI = 0.0240, CR = 0.0193 < 0.1

- Pairwise comparison of alternatives with respect to “Availability of Parking”

	Site 1	Site 2	Site 3	Site 4	Site 5	Site 6	Priority Weight
Site 1	1	1/6	1/3	1	1/9	1/5	0.03947
Site 2		1	2	6	1/2	1	0.21943
Site 3			1	3	1/3	1/2	0.11498
Site 4				1	1/9	1/5	0.03947
Site 5					1	2	0.38035
Site 6						1	0.20630

$\lambda = 6.0135$, CI = 0.0027, CR = 0.00218 < 0.1

- Pairwise comparison of alternatives with respect to “Closeness to Client's Homes”

	Site 1	Site 2	Site 3	Site 4	Site 5	Site 6	Priority Weight
Site 1	1	1	3	1/2	1/3	1	0.11971
Site 2		1	3	1/2	1/3	1	0.11971
Site 3			1	1/5	1/9	1/3	0.04114
Site 4				1	1/2	2	0.22211
Site 5					1	3	0.37762
Site 6						1	0.11971

$\lambda = 6.01$, CI = 0.0020, CR = 0.0016 < 0.1

- Pairwise comparison of alternatives with respect to “Access to Public Transportation”

	Site 1	Site 2	Site 3	Site 4	Site 5	Site 6	Priority Weight
Site 1	1	1	1	1	7	1	0.19277
Site 2		1	1	1	7	1	0.19277
Site 3			1	2	9	1	0.22917
Site 4				1	5	1	0.16464
Site 5					1	1/7	0.02789
Site 6						1	0.19277

$\lambda = 6.056$, CI = 0.0111, CR = 0.0090 < 0.1

- Pairwise comparison of alternatives with respect to “Number and Suitability of Counseling Rooms”

	Site 1	Site 2	Site 3	Site 4	Site 5	Site 6	Priority Weight
Site 1	1	1/8	2	1/5	1/9	1/5	0.03685
Site 2		1	9	2	1	2	0.29502
Site 3			1	1/9	1/9	1/9	0.02416
Site 4				1	1/2	1	0.17145
Site 5					1	2	0.30108
Site 6						1	0.17145

$\lambda = 6.081$, CI = 0.0162, CR = 0.013 < 0.1

- Pairwise comparison of alternatives with respect to “Number and Suitability of Conference Rooms”

	Site 1	Site 2	Site 3	Site 4	Site 5	Site 6	Priority Weight
Site 1	1	1	6	6	1/2	2	0.219426
Site 2		1	5	5	1/2	2	0.206299
Site 3			1	1	1/9	1/3	0.039471
Site 4				1	1/9	1/3	0.039471
Site 5					1	3	0.380349
Site 6						1	0.114984

$\lambda = 6.0135$, CI = 0.0027, CR = 0.00218 < 0.1

- Pairwise comparison of alternatives with respect to “Suitability of Reception and Waiting Area”

	Site 1	Site 2	Site 3	Site 4	Site 5	Site 6	Priority Weight
Site 1	1	1	1	5	2	2	0.24509
Site 2		1	1	4	1/2	1	0.15564
Site 3			1	5	1/2	2	0.17885
Site 4				1	1/9	1/3	0.03635
Site 5					1	3	0.27558
Site 6						1	0.10849

$\lambda = 6.240$, CI = 0.0481, CR = 0.039 < 0.1

- Pairwise comparison of alternatives with respect to “Adequacy of Space for Admin Work”

	Site 1	Site 2	Site 3	Site 4	Site 5	Site 6	Priority Weight
Site 1	1	1/7	1/5	1/9	1/5	1/6	0.03006
Site 2		1	1	1	1	1	0.19496
Site 3			1	1/2	1	1	0.16144
Site 4				1	2	2	0.28571
Site 5					1	1	0.16144
Site 6						1	0.16640

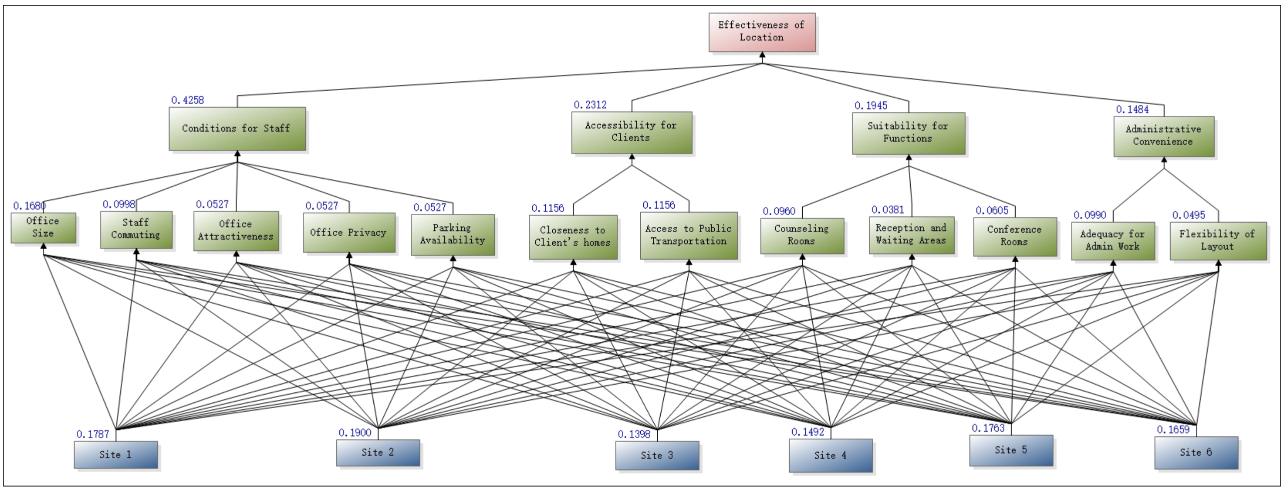
$\lambda = 6.054$, CI = 0.0108, CR = 0.0087 < 0.1

- Pairwise comparison of alternatives with respect to “Flexibility of the Space Layout”

	Site 1	Site 2	Site 3	Site 4	Site 5	Site 6	Priority Weight
Site 1	1	1/4	1/5	1/9	1	1/4	0.04234
Site 2		1	1	2	4	1	0.24475
Site 3			1	1/2	5	1	0.19148
Site 4				1	9	1	0.27886
Site 5					1	1/4	0.04234
Site 6						1	0.20022

$\lambda = 6.263$, CI = 0.052, CR = 0.042 < 0.1

Summary of Priority Weights



Global Weight of Alternatives

Alternative	Global Weight
Site 1	0.17880
Site 2	0.19024
Site 3	0.13756
Site 4	0.15042
Site 5	0.17679
Site 6	0.16619

- The sites in order of decreasing effectiveness weights are as follows:

Alternative	Effectiveness Weight
Site 2	0.19024
Site 1	0.17880
Site 5	0.17679
Site 6	0.16619
Site 4	0.15042
Site 3	0.13756

- Site 2 is the best, but it is the most cost-effective? Similarly, Site 1 is just below Site 2 in effectiveness, but how does it cost compared to Site 2?

- We need to take costs into consideration.

Taking Costs into Consideration

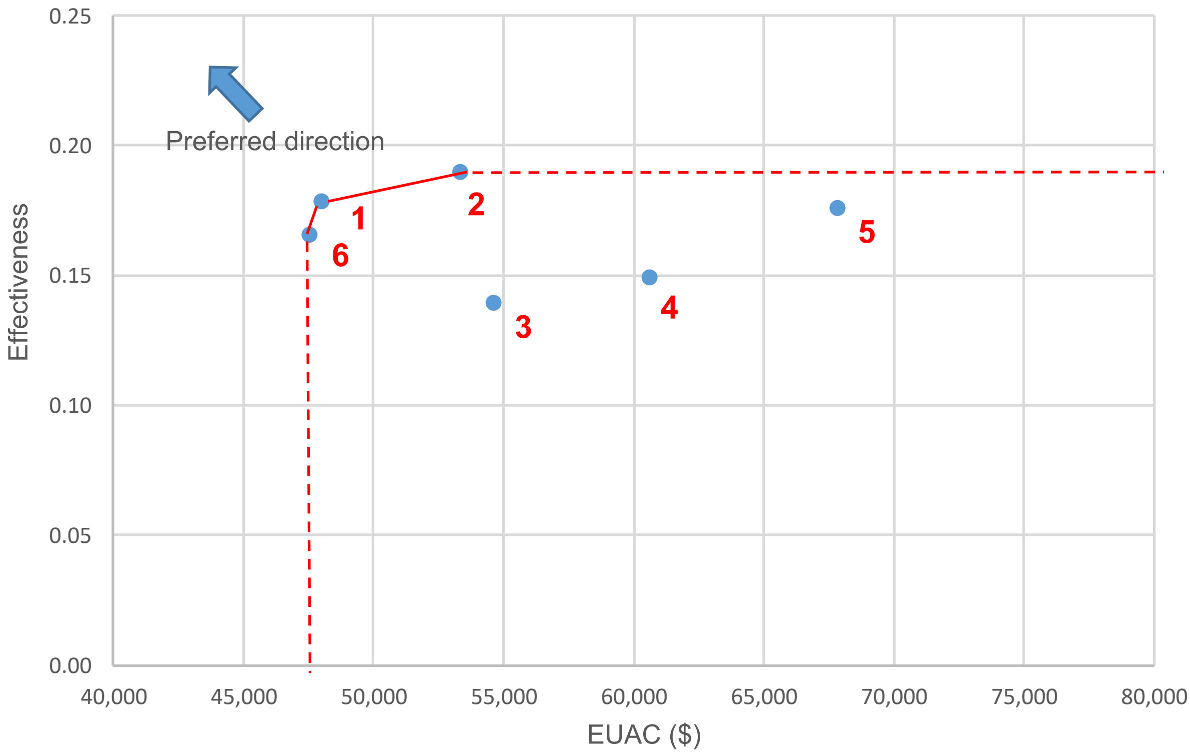
- The equivalent uniform annual rental cost (EUAC) for each of the six sites are given below:

Alternative	EUAC ($)	Effectiveness
Site 6	47,500	0.16619
Site 1	48,000	0.17880
Site 2	53,300	0.19024
Site 3	54,600	0.13756
Site 4	60,600	0.15042
Site 5	67,800	0.17679

- Which alternative is the most cost-effective?

Cost-Effectiveness and Efficient Frontier Analysis

- We plot the Location Effectiveness vs. EUAC in dollars:
- The preferred direction is one with increasing effectiveness and decreasing cost in the direction of the blue arrow.



Observations:

- **Dominated alternatives**: Sites 3, 4 & 5.
- **Efficient or Non-dominated alternatives**: Sites 1, 2 & 6.
- We need more preference information to choose among sites 1, 2 & 6.

- If Cost is of greatest concern then Site 6 which requires the minimum cost should be chosen.
- If the Effectiveness of the site for clients and staff is of greatest concern, and there is sufficient budget, then Site 2 which has the highest effectiveness should be chosen.
- Site 1 provides a good trade-off between Cost and Effectiveness.

Dealing with Uncertain Costs

- In the above example, we have assumed that there is no uncertainty in the costs.

- When costs are uncertain, create uncertain cash flow models for each alternative:

  1. If the cash flows depend on other uncertain factors (e.g., economic condition, foreign exchange rate, etc.), model them using an influence diagram/decision tree.

  2. Generate the risk profiles of the EUAC for each alternative.

  3. Risk and Stochastic dominance analysis on the EUAC of each alternative may be performed.

- The Cost-Effectiveness Trade-off Analysis may be performed as follows:

  - If the decision maker is risk neutral determine the Expected EUAC for each alternative, and use them in the Cost-Effective trade-off and efficient frontier analysis.

  - If the decision maker is not risk-neutral, use a utility function to determine the Certainty Equivalent of the EUAC for each alternative, and use them in the Cost-Effective and efficient frontier trade-off analysis.

  - If a utility function is not available, determine the Value-at-Risk of EUAC at some confidence level for each alternative and use them in the Cost-Effective trade-off and efficient frontier analysis.

10.4 Integrated Multiple Criteria Decision Analysis

- The DA Process or Cycle using Influence diagrams and decision trees (with DPL software) is most suitable for analyzing decision problems with outcomes that may be quantified as cash flows.

- The AHP method is suitable for analyzing problems involving multiple criteria and when outcomes are not easily quantifiable.

- A complex decision problem involving both qualitative and quantitative factors and outcomes may be solved by performing integrated decision analysis.

Method 1: Separation of Qualitative and Quantitative Factors

1. Use DA Process/Cycle with DPL to analyze only the quantitative factors and determine the risk profile of NPV of cash flows.

2. Use AHP to model only the qualitative factors and determines the global weights for the effectiveness or performance of the alternatives.

3. Plot efficient frontier (Effectiveness vs. EV/CE/VaR from the risk profile) and perform incremental trade-off analysis similar to the counseling center relocation problem.

Method 2 (Global AHP model with DPL sub-model)

1. Use AHP to model ALL factors (Qualitative and Quantitative) in one single hierarchy.

2. Use DA Process or Cycle with DPL to analyze selected financial factors.

3. Use the results from DA Process or Cycle to perform pairwise comparisons of financial factors in the AHP model.

4. AHP global weights determine the final decision.

Method 3 (Global DPL model with AHP sub-model)

1. Use DA Process or Cycle with DPL to analyze all the factors.

2. Use AHP to determine weights for multiple attribute utility functions.

3. Final decision is made with DPL.

References

1. R.L. Keeney and H. Raiffa, *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*, John Wiley, New York, 1976.

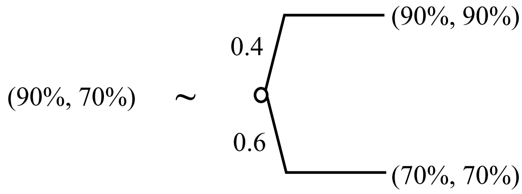
2. Craig W. Kirkwood, *Strategic Decision Making: Multi-objective Decision Analysis with Spreadsheets*, International Thomson, 1997.

Exercises

P10.1 Mathematics & Science have been identified as the most important knowledge for workers in a knowledge-based economy. To give school graduates a good foundation in the two subjects, the Education Ministry is revising its method of teaching the two subjects in their primary schools. It has to make a choice between two teaching techniques and will use a two-attribute additive utility function of the form $u(x_1,x_2) = k_1\ u_1(x_1) + (1 - k_1)\ u_2(x_2)$, where $u_1(x_1)$ and $u_2(x_2)$ are single-attribute utility function for two attributes defined as follows:

- Attribute x_1: Average score of students on a Mathematics test where $70\% \le x_1 \le 90\%$.
- Attribute x_2: Average score of students on a Science test where $70\% \le x_1 \le 90\%$.

(***a***) If the Ministry is indifferent between the following deals, determine its two-attribute utility function.

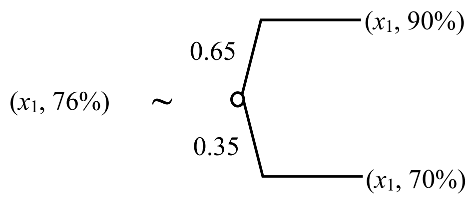


(***b***) The two alternative teaching techniques have uncertain outcomes:

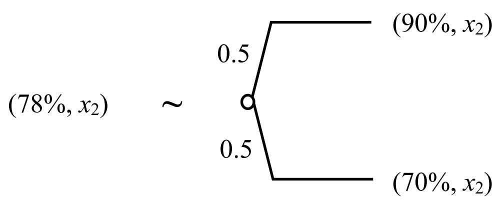
1. Teaching technique 1 is equally likely to produce either average test scores of 78% and 76% for mathematics and science respectively, or average test scores of 70% and 90% for mathematics and science respectively.

2. Teaching technique 2 has a 60% chance of achieving average test scores of 90% for both mathematics and science, and a 40% chance of achieving average test scores of 70% for both subjects.

For any value x_1 of Attribute 1, the Ministry is indifferent between



For any value x_2 of Attribute 2, the Ministry is indifferent between



Which is the best teaching technique for the Education Ministry?

P10.2 A field hospital set up for a relief operation in a remote area must determine at the beginning of each week how many pints of blood should be ordered from its home base. Any blood left over at the end of the week will be outdated and cannot be used. The field hospital considers the following two attributes to be important:

1. *Weekly blood shortage* (x_1): This is the number of pints of blood by which ordered blood falls short of the week's demand. This quantity is known to be always between 0 and 10 pints.

2. *Weekly blood outdated* (x_2): This is the number of pints of blood that are outdated. This quantity is known to be always between 0 and 10 pints.

The hospital's utility function is $u(x_1, x_2) = 0.4\,u_1(x_1) + 0.5\,u_2(x_2) + 0.1\,u_1(x_1)\,u_2(x_2)$

where $u_1(x_1) = 0.582\left[\exp(1 - \frac{x_1}{10}) - 1\right]$ and $u_2(x_2) = 1 - \frac{x_2^2}{100}$.

Suppose that each week there is a 0.5 chance that the demand for blood will be 25 pints and a 0.5 chance it will be 35 pints. Would the blood bank be better off ordering 28 pints, 30 pints, or 32 pints?

P10.3 (Based on Clemen and Reilly 2001, Exercise 15.16, p 631).

You are an up-and-coming developer in downtown Seattle and are interested in constructing a building on a site that you own. You have collected four bids from prospective contractors. The bids include both a cost (millions of dollars) and a time to completion (months):

Contractor	Cost	Time
A	100	20
B	80	25
C	79	28
D	82	26

The problem now is to decide which contractor to choose. You prefer the project to be completed sooner than later, and also at a lower cost.

Contractor B has indicated that for another \$20 million he could do the job in 18 months, and you have said that you would be indifferent between that and the original proposal.

In talking with Contractor C, you have indicated that you would be just as happy to pay her an extra \$4 million if she could get the job done in 26 months.

Who gets the job? Explain your reasoning. (It may be convenient to plot the four alternatives on a graph.)

P10.4 A project manager is faced with the problem of evaluating a number of alternative system designs. He has determined that the four main criteria that determine his choice are human productivity, economics, design, and operations. The pairwise comparison matrix for these four criteria is as follows:

	Human Productivity	Economics	Design	Operations
Human productivity	1	3	3	7
Economics		1	2	5
Design			1	7
Operations				1

Three alternatives are being considered. The pairwise comparison for the three alternatives with respect to each of the evaluation criteria are as follows:

For human productivity:

	System A	System B	System C
System A	1	3	5
System B		1	2
System C			1

For Economics:

	System A	System B	System C
System A	1	1/3	1/2
System B		1	3
System C			1

For Design:

	System A	System B	System C
System A	1	1/2	1/7
System B		1	1/5
System C			1

For Operation:

	System A	System B	System C
System A	1	3	1/5
System B		1	1/9
System C			1

(*a*) Use the AHP method to determine the best system based on the four criteria. Approximate computational methods may be used.

(*b*) If the equivalent uniform annual costs (EUAC) of the three alternatives over the life cycle are as given below, determine the set of efficient cost-effective alternatives. Illustrate your answers with a graphical plot.

Alternative	EUAC ($)
System A	100,000
System B	80,000
System C	110,000

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