Decision Analysis
Solutions to Homework #5

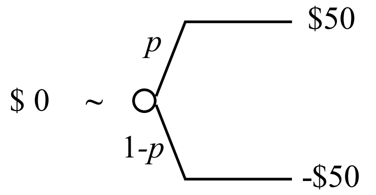
Question 1

Given that John has the utility function $u(x) = 1 - 3^{-x/50}$ over the range of x = -\$50 to \$5,000.
The utility function may be rewritten as $u(x) = 1 - 3^{-x/50} = 1 - e^{-x\ln 3/50}$

(*a*) The utility function is increasing and concave. Hence, John is risk-averse in attitude.

(*b*) John's risk tolerance = \$ 50 / *ln* 3.
Hence John's degree of risk aversion = 1 / risk tolerance = *ln* 3 / 50 = 0.0219722 $\$^{-1}$

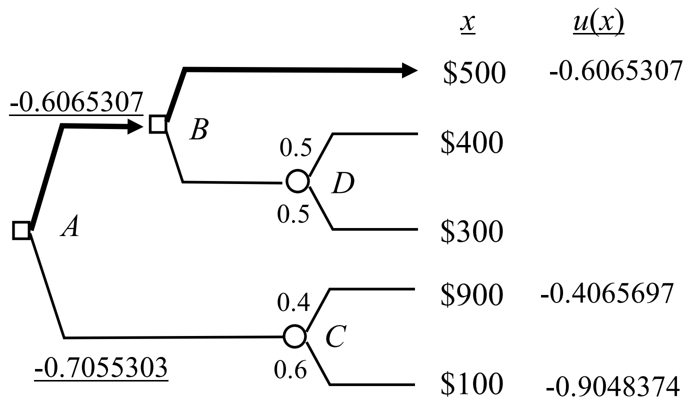
(*c*) We want to find p such that the certainty equivalent of the deal is zero.



$$
\begin{aligned}
u(0) &= p\,u(50) + (1-p)\,u(-50) \\
1 - 3^0 &= p\,(1 - 3^{-1}) + (1-p)\,(1 - 3^1) \\
0 &= p\,(2/3) + (1-p)(-2) \\
p &= 3/4
\end{aligned}
$$

Question 2

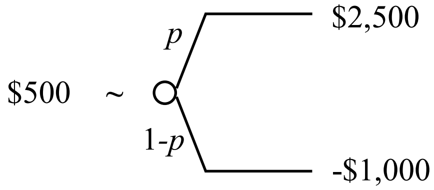
Since George has a constant risk tolerance of \$1,000, it follows that he has constant absolute risk aversion or the delta property. Hence, his utility function is exponential in form.
We let $u(x) = -\exp(-x/1{,}000)$.



Maximum Expected Utility at A = -0.6065307

Certainty Equivalent at $A = u^{-1}(-0.60653)$ = \$500

The required preference probability is p, such that



Hence $\quad u(500) = p\ u(2{,}500) + (1 - p)\ u(-1{,}000)$

$$\Rightarrow \qquad -e^{\frac{-500}{1,000}} = (p)(-e^{\frac{-2,500}{1,000}}) + (1-p)(-e^{\frac{1,000}{1,000}})$$

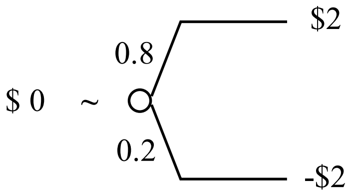
$$\Rightarrow \qquad p = 0.80106$$

Note that we would have got the same answer if we had used the utility function $u(x) = 1 - exp(-x/1000)$, or if we had assumed $u(x) = a - b\ exp(-x/1000)$ and fitted the constants a and b to the boundary conditions $u(\text{-}\$1{,}000) = 0$ and $u(\$2{,}500) = 1$. Make sure you understand why this is so.

Question 3

(*a*) Susan satisfies delta property $\Rightarrow$ utility function is of the form $u(x) = a - b\ e^{-x/\rho}$ where ρ is the risk tolerance.

Given



$u(0) = 0.8\ u(2) + 0.2\ u(-2)$

$a - b = 0.8\ (a - b\ e^{-2/\rho}) + 0.2\ (a - b\ e^{2/\rho})$

$1 = 0.8\ e^{-2/\rho} + 0.2\ e^{2/\rho}$

Let $\quad x = e^{2/\rho}$

$x^2 - 5x + 4 = 0 \ \Rightarrow\ (x - 1)(x - 4) = 0 \ \Rightarrow\ x = 1$ or $x = 4$.

Hence $= e^{2/\rho} = 4 \ \Rightarrow\ \rho = \dfrac{1}{\ln 2} = \$1.44$

Susan risk tolerance = \$1.44

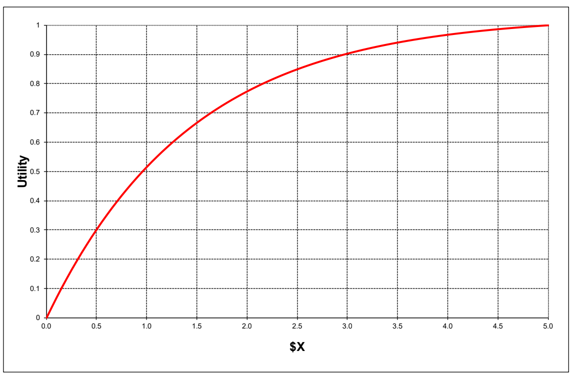
(*b*) Susan's risk attitude is risk averse since her risk tolerance is positive.

(*c*) Susan's utility function such that $u(L = 0) = 0$ and $u(H = 5) = 1$ is

$$u(x) = \frac{1 - e^{-(x-L)/\rho}}{1 - e^{-(H-L)/\rho}}$$

$$= 1.0322581\ (1 - e^{-x\ln 2})$$

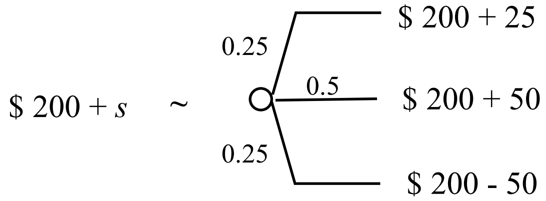
$$= 1.0322581\ (1 - 2^{-x})$$



Question 4

Current wealth = \$200. Wealth Utility function $u(w) = \dfrac{w^2}{2000}$, $w \geq 0$.

(*a*) Let s = personal indifferent selling price.



$u(200 + s) = 0.25\, u(200+25) + 0.5\, u(200 + 50) + 0.25\, u(200 - 50)$

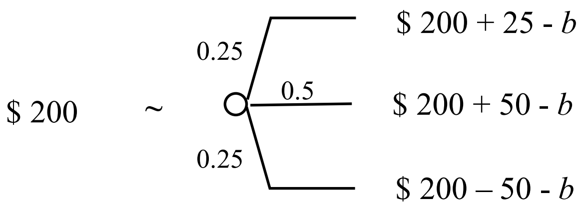
$$\frac{(200+s)^2}{2000} = 0.25\left(\frac{225^2}{2000}\right) + 0.5\left(\frac{250^2}{2000}\right) + 0.25\left(\frac{150^2}{2000}\right)$$

$$(200+s)^2 = 0.25\,(225)^2 + 0.5\,(250)^2 + 0.25\,(150)^2$$

$$s = \$\,22.5562$$

Hence Susan's PISP = <u>**\$22.56**</u>

(*b*) Let b = personal indifferent buying price.



$u(200) = 0.25\, u(200+25 - b) + 0.5\, u(200 + 50 - b) + 0.25\, u(200 - 50 - b)$

$$\frac{200^2}{2000} = 0.25\left(\frac{(225-b)^2}{2000}\right) + 0.5\left(\frac{(250-b)^2}{2000}\right) + 0.25\left(\frac{(150-b)^2}{2000}\right)$$

$$4(200)^2 = (225-b)^2 + 2(250-b)^2 + (150-b)^2$$

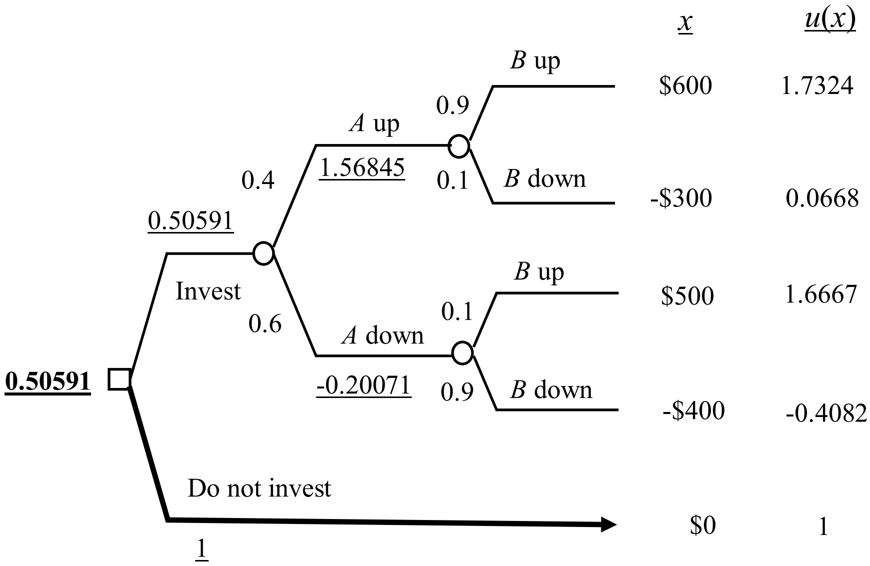
$$4b^2 - 1750b + 38125 = 0$$

Solving: b = \$22.99425 (okay) or \$414.5057 (rejected)

Hence Susan's PIBP = <u>**\$22.99**</u>

Question 5

Kay utility function is $u(x) = 2 - 9^{\frac{-x}{1000}} \Rightarrow$ Delta property.



(*a*) Expected utility for "Invest" = 0.50591

Expected utility for "Do not invest" = 1

(*b*) Expected dollar value for "Invest"
$= (0.4)(0.9 \times 600 + 0.1 \times -300) + (0.6)(0.1 \times 500 + 0.9 \times -400) = \$18.$

Expected dollar value for "Do not invest" = $0.

(*c*) Since *EU*(Do not invest) > *EU*(Invest), Kay's best decision is "Do not invest"
Certainty Equivalent = $0.

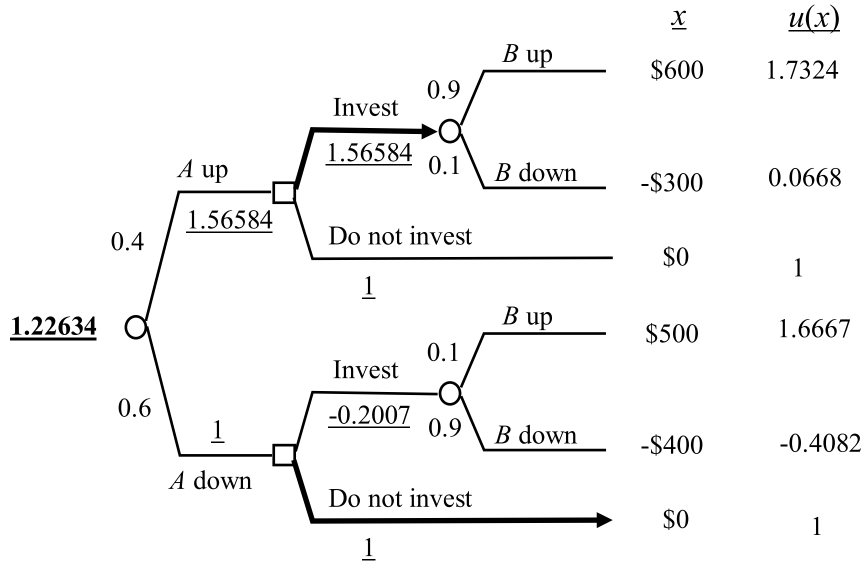
(*d*) We use the answers found in part (*a*) and not part (*b*) because Kay is not risk neutral (even though she satisfies the delta property).

We have to compare expected utilities and not expected dollar values. It would be okay to use expected dollar values if Kay was risk neutral.

If we had used expected dollar values instead, we would have ignored Kay's risk attitude and may get a wrong answer since expected dollar values are not the same as the certainty equivalents.

(*e*) Since Kay satisfies the delta property, we can easily find her value of clairvoyance on *A* using the difference of CEs method.

Decision model with free clairvoyance on Stock *A*:

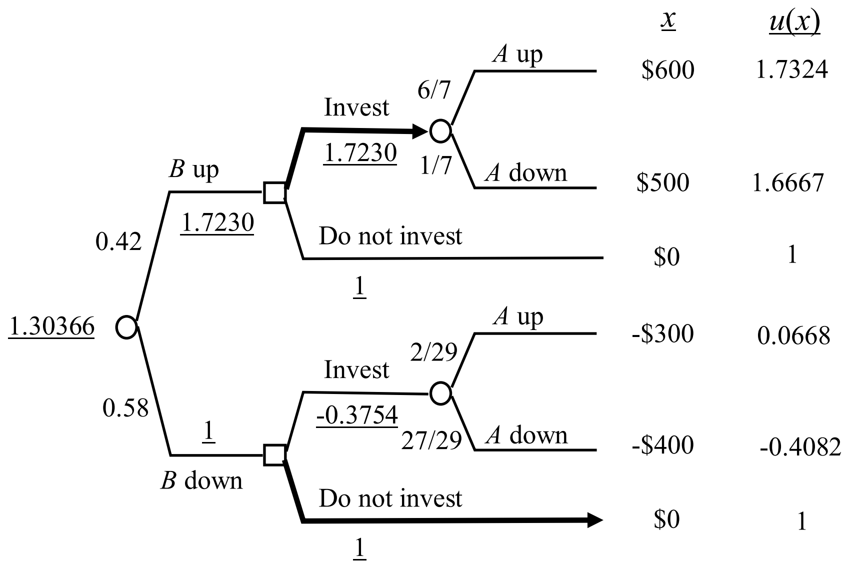


Expected Utility with free clairvoyance = 1.22634.

To find the certainty equivalent: $2-9^{-\frac{x}{1000}} = 1.22634 \Rightarrow x = \$116.8$

Certainty equivalent with free clairvoyance on *A* = \$116.80
Hence, the value of clairvoyance on *A* = \$116.80 –\$ 0 = \$116.80 > \$10.
Therefore, Kay should pay \$10 for clairvoyance on *A*.

(f) Decision model with free clairvoyance on Stock *B*:



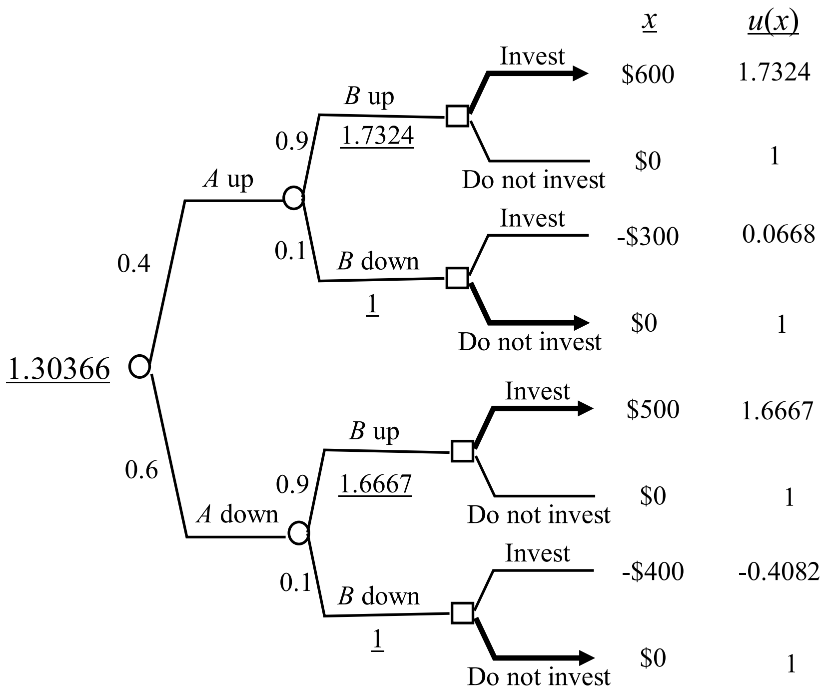
Expected utility with free clairvoyance on *B* = 1.30366

$2-9^{-\frac{x}{1000}} = 1.30366 \Rightarrow x = \$164.71$

Certainty equivalent with free clairvoyance on *B* = \$164.71

Hence, the value of clairvoyance on *B* = \$164.71 –\$ 0 = \$164.71 > \$10.
Therefore, Kay should pay \$10 for clairvoyance on *B*.

(*g*) Decision model with free joint clairvoyance on Stock *A* & Stock *B*:



Expected utility with free clairvoyance on both *A* and *B* = 1.30366

Certainty Equivalent with free clairvoyance on both *A* and *B* = u^{-1} (1.30366) = \$164.71

Kay's value of clairvoyance on *A* and *B* together = \$164.71 –\$ 0 = \$164.71.