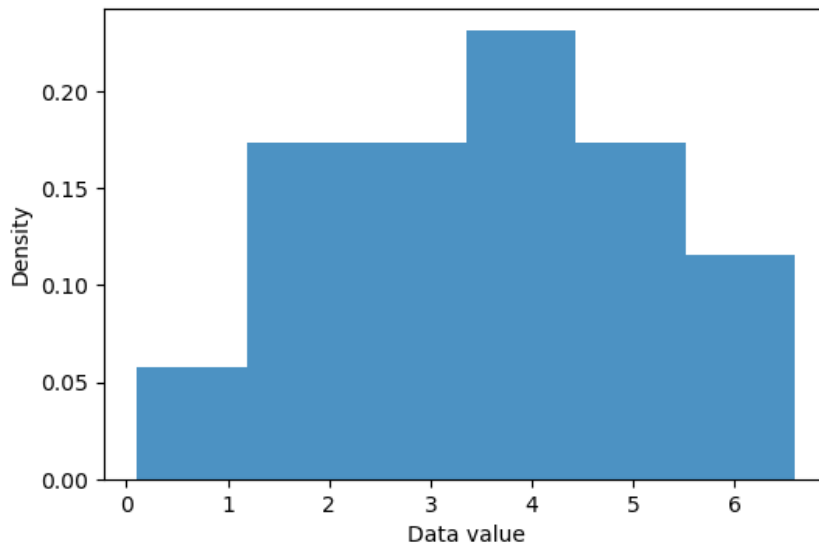


IE5203 Decision Analysis Solutions to Chapter 7 Exercises

P7.1

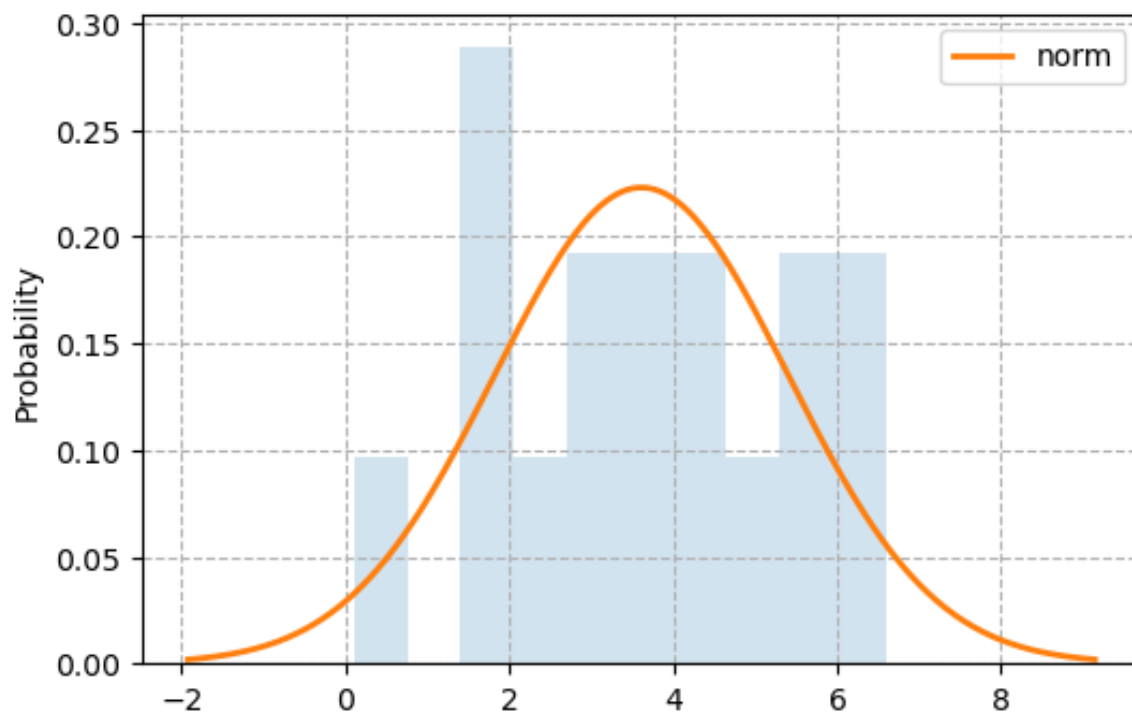


Data Description:

size = 16
minmax = (0.1077, 6.5941)
mean = 3.61086875
var = 3.41809
std dev = 1.7901

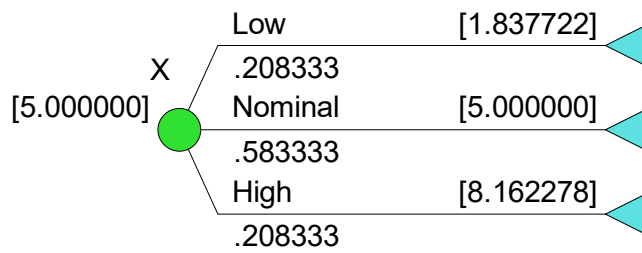
The Maximum Likelihood Estimators (MLE) for the mean and standard deviation of the Normal Distribution are the mean and standard deviation of the observed data.

Hence, we will fit a Normal distribution with mean = 3.611 and standard deviation = 1.790



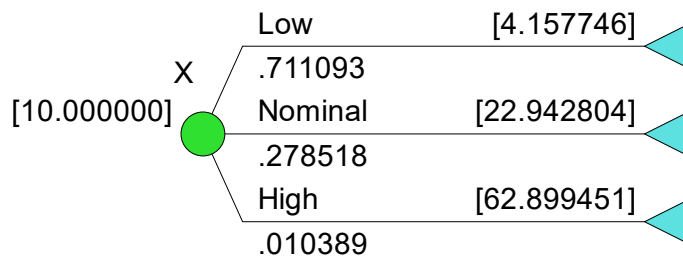
P7.2

(a) Triangular (0, 10, 5)



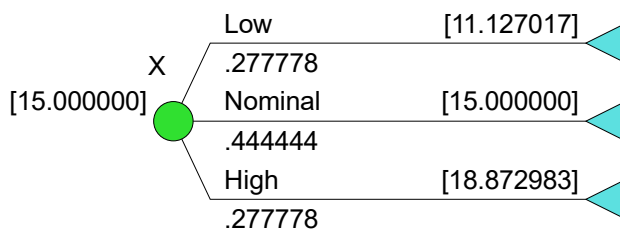
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(b) Exponential (1/10)



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(c) Uniform (10, 20)

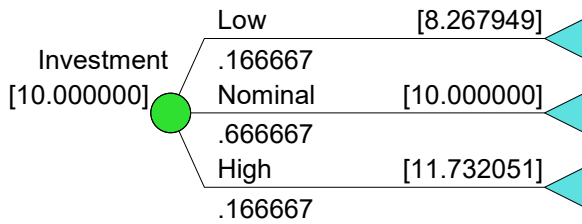


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P7.3

Let the utility function be $u(x) = 1 - e^{-x/5}$ where x is in millions of dollars.

(a) Discrete 3-branch approximation (moments matching) using DPL:



$$E[u(x)] = (1/6) u(8.267849) + (2/3) u(10) + (1/6) u(11.732051) = 0.861931$$

$$CE = u^{-1}(0.861931) = \$ 9.9000 \text{ millions}$$

(b) Stanford/SDG 3-branch quick approximation:

From the CDF, the 10th, 50th, and 90th percentiles are 8.718, 10.0, and 11.282, respectively.

$$E[u(x)] = 0.25 u(8.718) + 0.5 u(10.0) + 0.25 u(11.282) = 0.86243$$

$$CE = u^{-1}(0.86243) = \$ 9.9181 \text{ millions}$$

(c) Pearson-Tukey 3-branch approximation method:

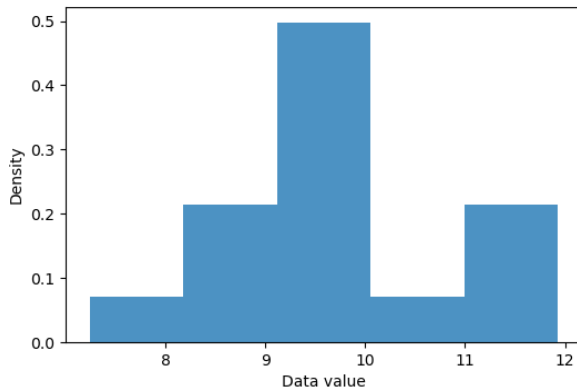
From the CDF, the 5th, 50th, and 95th percentiles are 8.355, 10.0, and 11.645, respectively.

$$E[u(x)] = 0.185 u(8.355) + 0.63 u(10.0) + 0.185 u(11.645) = 0.86193$$

$$CE = u^{-1}(0.86193) = \$ 9.9000 \text{ millions}$$

P7.4

(a) The data:



Data Description:

size = 15

minmax = (7.24, 11.93)

mean = 9.762

var = 1.4419457142857142

The top 5 fitted distributions based on KS:

Distributions: laplace

Parameters = (9.6200, 0.8300)

KS statistic = 0.14891708541566395

KS p-value = 0.8464890809943011

mean = 9.6200

var = 1.3778

std dev = 1.1738

Distribution: beta

Parameters = (1410.7634, 58247502.1299, -33.8063, 1798873.4614)

KS statistic = 0.1960370649094899

KS p-value = 0.5471048303675403

mean = 9.7616

var = 1.3455

std dev = 1.1599

Distribution: lognorm

Parameters = (0.0171, -58.1049, 67.8569)

KS statistic = 0.19629155291299982

KS p-value = 0.5454833079669202

mean = 9.7620

var = 1.3458

std dev = 1.1601

Distribution: gamma

Parameters = (1555.1992, -35.9871, 0.0294)

KS statistic = 0.19633294518248245

KS p-value = 0.5452197271626447

mean = 9.7620

var = 1.3458

std dev = 1.1601

Distribution: norm

Parameters = (9.7620, 1.1601)

KS statistic = 0.19967233371824233

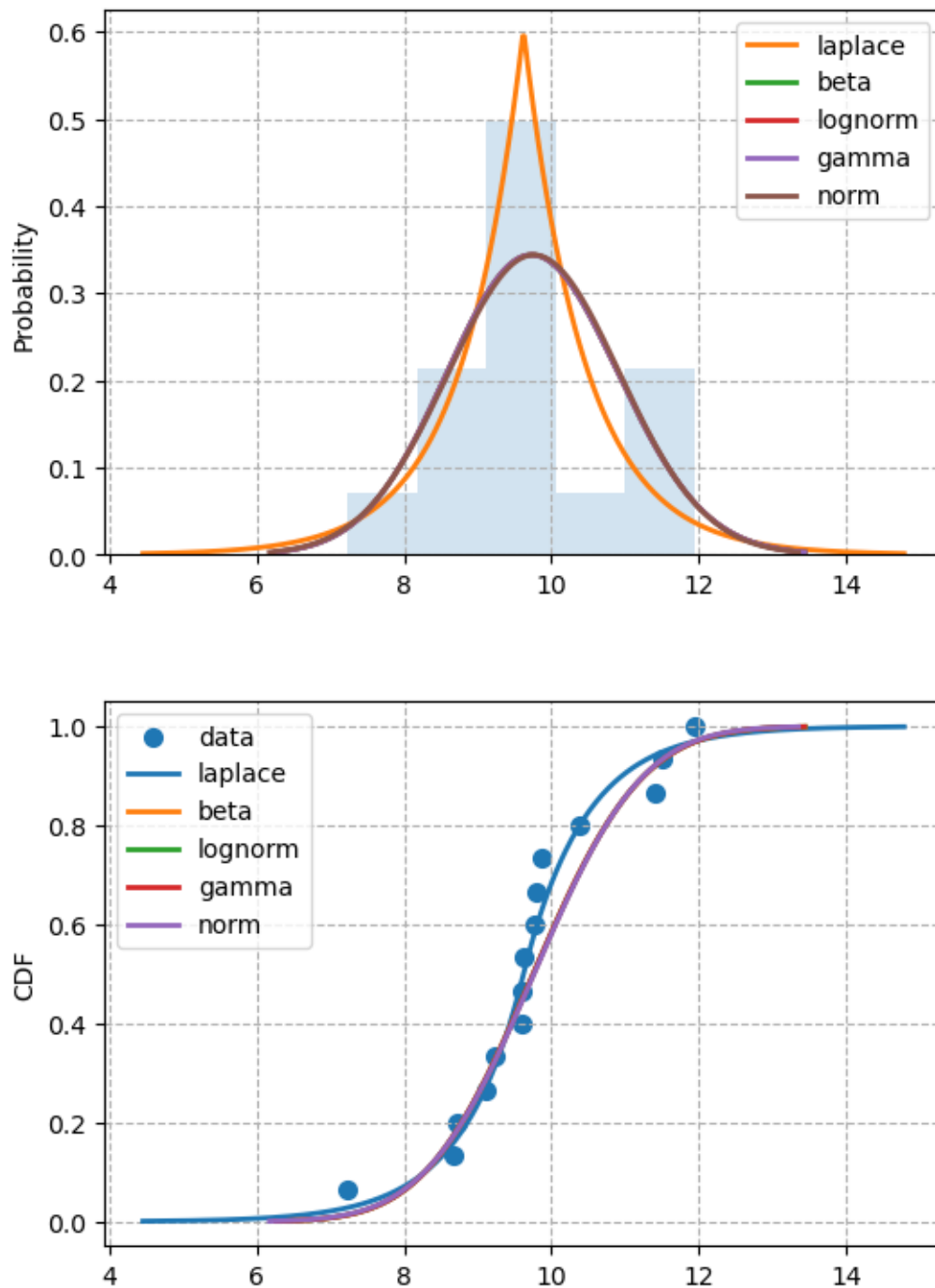
KS p-value = 0.5241052040775288

mean = 9.7620

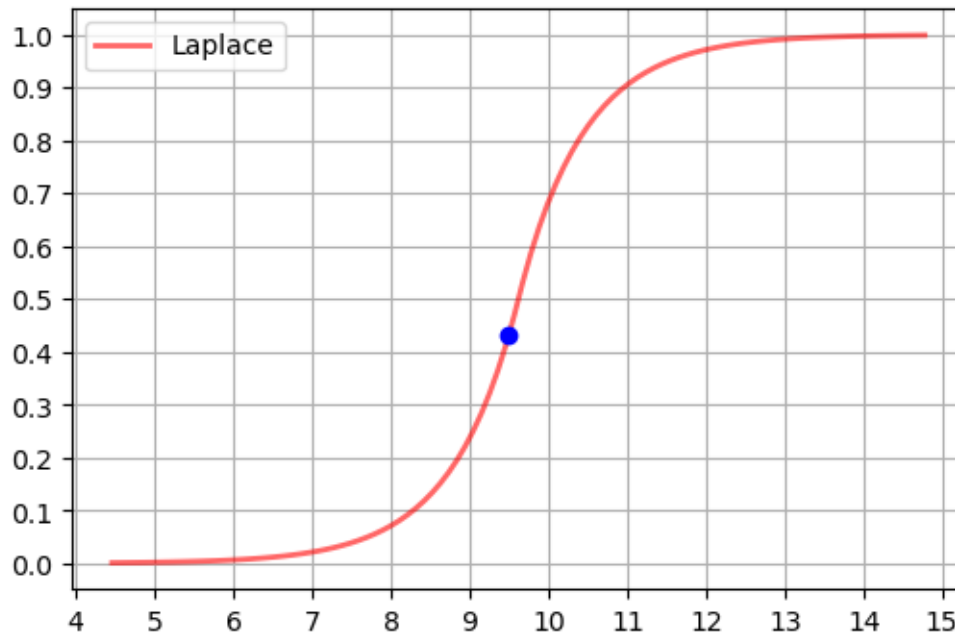
var = 1.3458

std dev = 1.1601

(b) Comparing the PDF and CDF of the fitted distribution with the data.



(c) CDF of the fitted distribution:



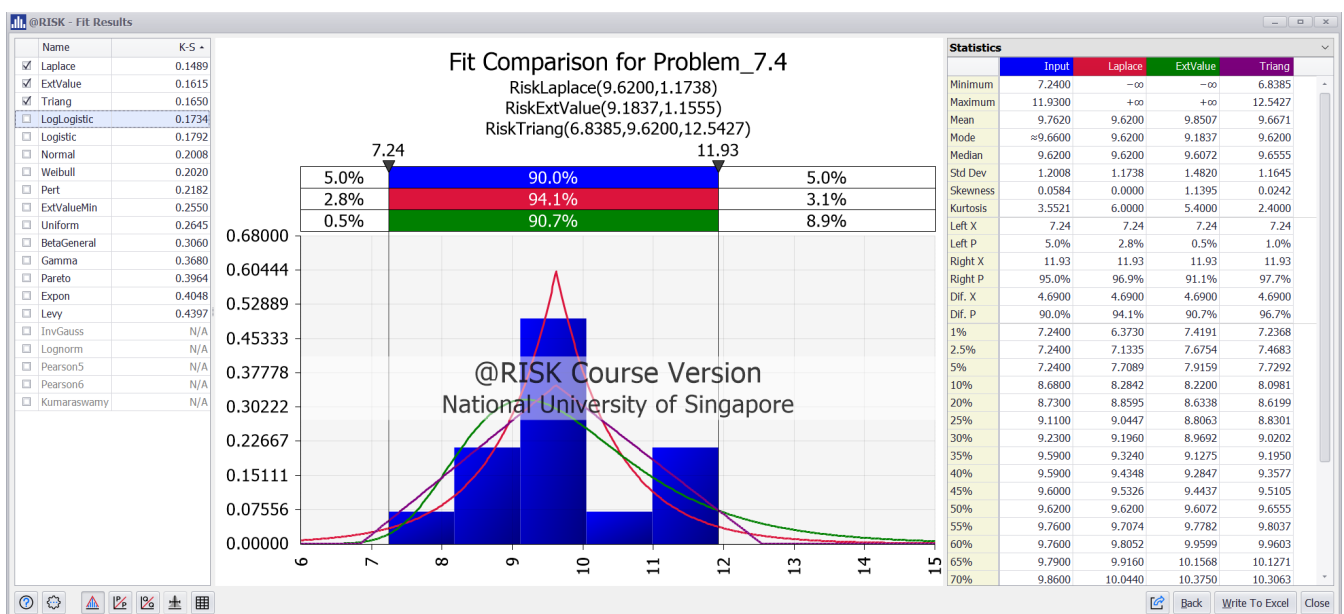
$$F(x) = \frac{1}{2} + \frac{1}{2} \operatorname{sgn}(x - \mu) \left(1 - \exp\left(-\frac{|x - \mu|}{b}\right) \right)$$

where $\mu = 9.6200$, $b = 0.8300$

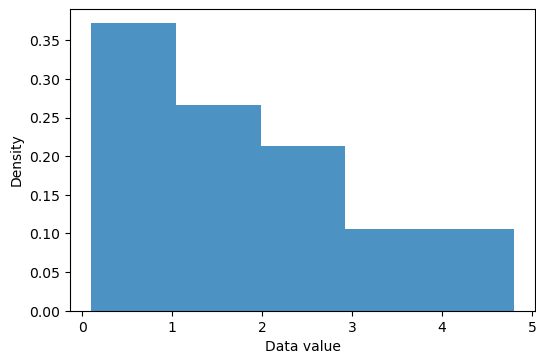
The probability that an animal weights less than 9.5 gram = 0.4327

Python: `scipy.stats.laplace.cdf(9.5, 9.6200, 0.8300)`

Using @Risk, we also get Laplace as the best fit distribution:



P7.5



Data Description:

size = 20

minmax = (0.1, 4.8)

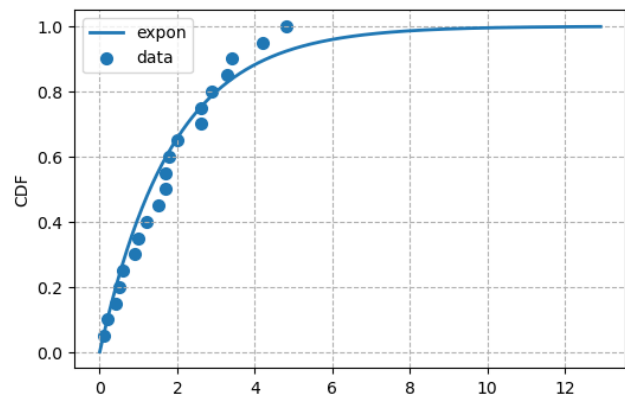
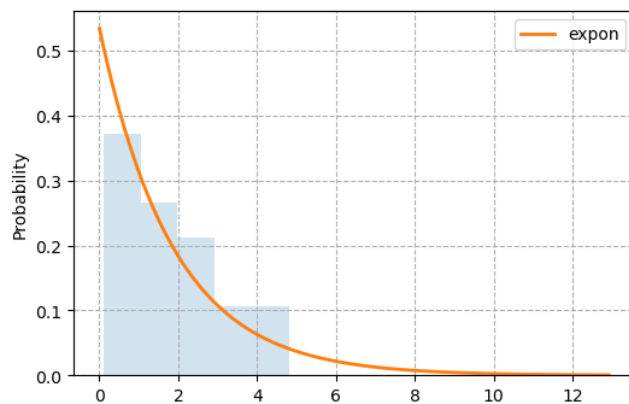
mean = 1.8699999999999999

var = 1.8137894736842104

skewness = 0.5802772894968783

kurtosis = -0.5649727424575377

We will fit an exponential distribution with one parameter (location = 0).



Distribution: expon

Parameters = (0.0000, 1.8700)

KS statistic = 0.15163114014127343

KS p-value = 0.6921603208893784

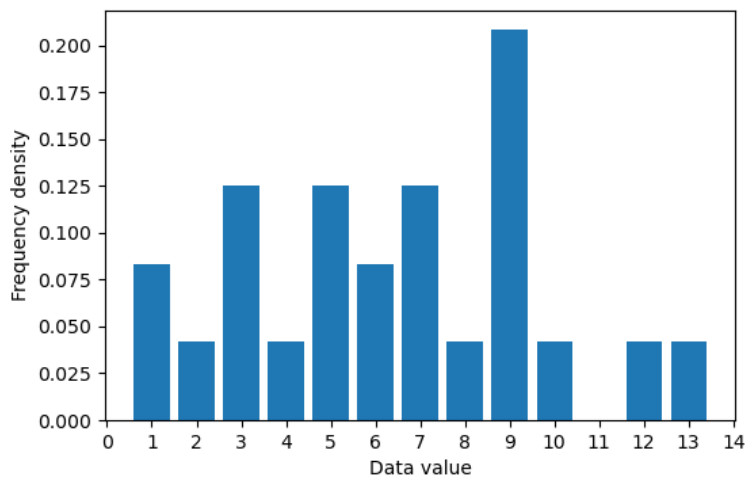
mean = 1.8700

var = 3.4969

std dev = 1.8700

P7.6

(a) Histogram of the data



Data Description:

size = 24
minmax = (1, 13)
mean = 6.375
var = 10.853260869565217
skewness = 0.1004975735577767
kurtosis = -0.7612716019447299

(b) Using DecisionAnalysisPy. DistFit_discrete Class

Top 3 discrete distribution fitted are:

Distribution 1: nbinom
Params = [8.30298378 0.56567604]
KS_stats = 0.17272838916584243
p-value = 0.4233763247639363
mean = 6.375000043564911
var = 11.269701352459267
sd = 3.3570375857978214

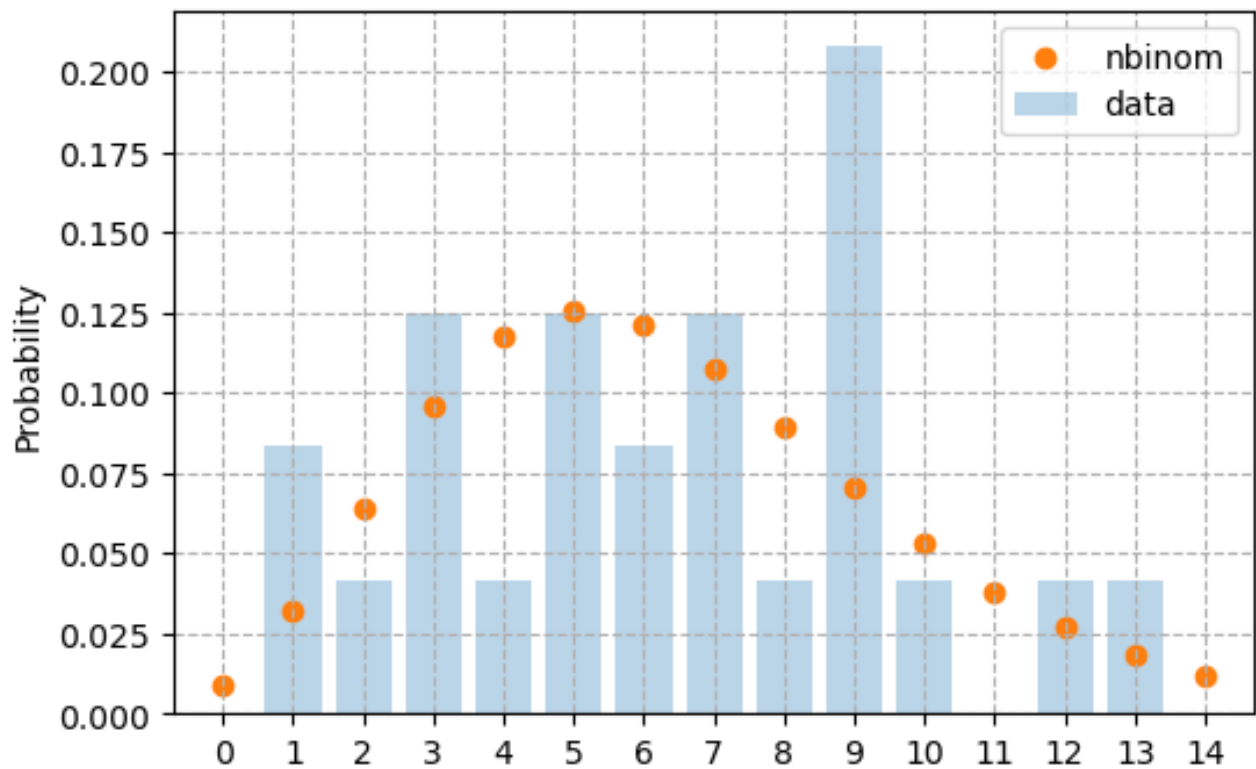
Distribution 2: poisson
Params = [6.37500004]
KS_stats = 0.2211842014782398
p-value = 0.16391064587178117
mean = 6.375000035762788
var = 6.375000035762788
sd = 2.5248762416726067

Distribution 3: binom
Params = [14. 0.5]
KS_stats = 0.24355061848958337
p-value = 0.09726881318566716
mean = 7.0
var = 3.5
sd = 1.8708286933869707

Distribution selected:

- negative binomial with $n = 8.30298378$ and $p = 0.56567604$

(c) Comparing the PMF of the fitted distributions with the data



Using @Risk, we get the same distribution with parameters close.

