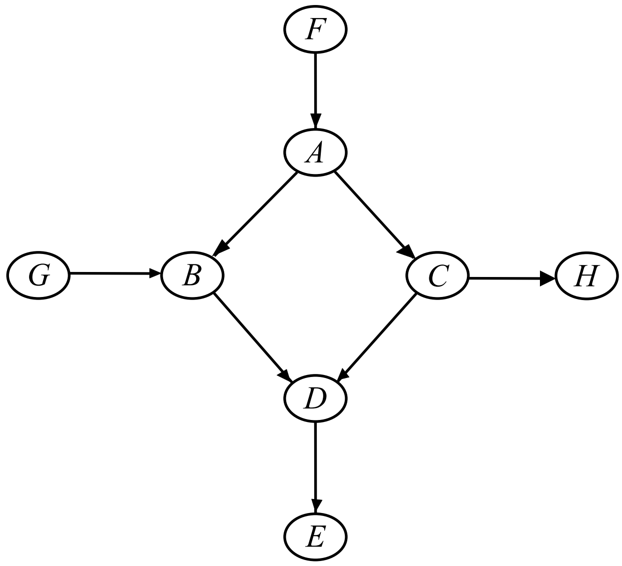
IE5203 Decision Analysis
Solutions to Chapter 5 Exercises

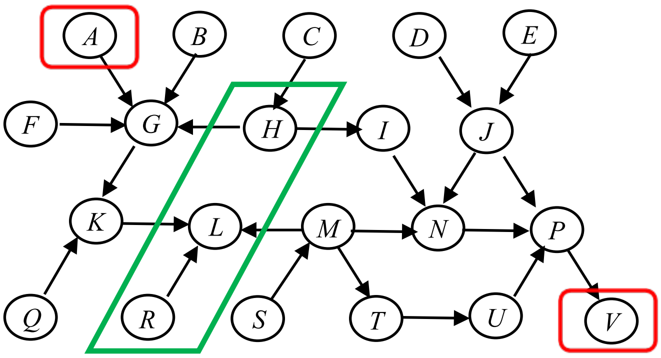
P5.1



(*a*)	$B \perp C \mid F$	False	*B-D-C* is blocked but *B-A-C* is not blocked.
(*b*)	$B \perp C \mid A$	True	*B-A-C* is blocked and *B-D-C* is blocked.
(*c*)	$B \perp C \mid \{A, E\}$	False	*B-A-C* is blocked but *B-D-C* is not blocked.
(*d*)	$B \perp C \mid \{A, D\}$	False	*B-A-C* is blocked but *B-D-C* is not blocked.
(*e*)	$A \perp D \mid \{B, G\}$	False	*A-B-D* is blocked but *A-C-D* is not blocked.
(*f*)	$A \perp D \mid \{B, C\}$	True	*A-B-D* is blocked and *A-C-D* is blocked.
(*g*)	$E \perp F \mid \{B, C\}$	True	*F-A-B-D-E* is blocked and *F-A-C-D-E* is blocked.

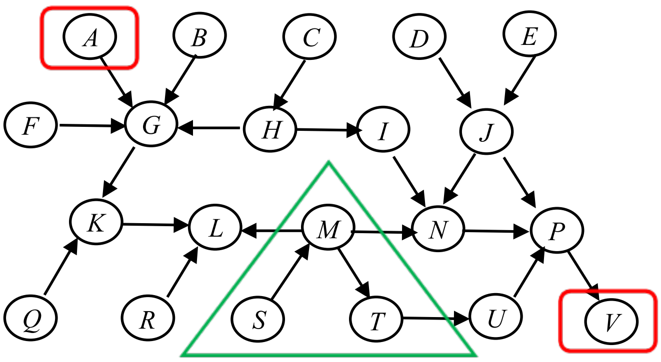
P5.2

(*a*) $\{A\} \perp \{V\} \mid \{H, L, R\}$ **<u>False</u>**



- There exists a path *A-G-K-L-M-N-P-V* that is not blocked as no node along this path satisfies either Condition (1) or (2):
  - G is H2T but $G \notin \{H, L, R\}$.
  - K is H2T but $K \notin \{H, L, R\}$.
  - L is H2H but $L \in \{H, L, R\}$.
  - M is T2T but $M \notin \{H, L, R\}$.
  - N is H2T but $N \notin \{H, L, R\}$.
  - P is H2T but $P \notin \{H, L, R\}$.
- Since there exists a path from $\{A\}$ to $\{V\}$ that is not blocked by $\{H, L, R\}$ it follows that the statement is False.

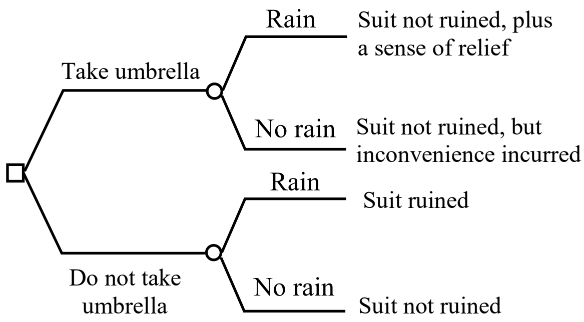
(*b*) $\{A\} \perp \{V\} \mid \{M, S, T\}$ **<u>True</u>**



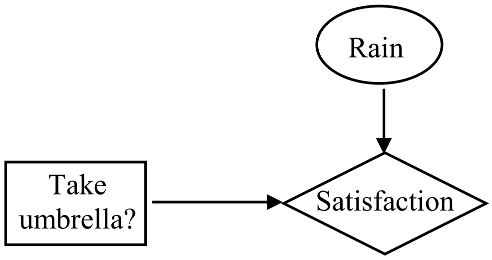
- All paths from $\{A\}$ to $\{V\}$ are blocked by $\{M, S, T\}$:
  - *A-G-H-I-N-P-V* is blocked // G is H2H and G, K & $L \notin \{M,S,T\}$
  - *A-G-H-I-N-J-P-V* is blocked // G is H2H and G, K & $L \notin \{M,S,T\}$
  - *A-G-H-I-N-M-T-U-P-V* is blocked // G is H2H and G, K & $L \notin \{M,S,T\}$
  - *A-G-K-L-M-N-P-V* is blocked // M is T2T and $M \in \{M,S,T\}$
  - *A-G-K-L-M-N-J-P-V* is blocked // M is T2T and $M \in \{M,S,T\}$
  - *A-G-K-L-M-T-U-P-V* is blocked // M is T2T and $M \in \{M,S,T\}$
- Since all the paths from $\{A\}$ to $\{V\}$ are blocked by $\{M, S, T\}$ it follows that the statement is True.

P5.3

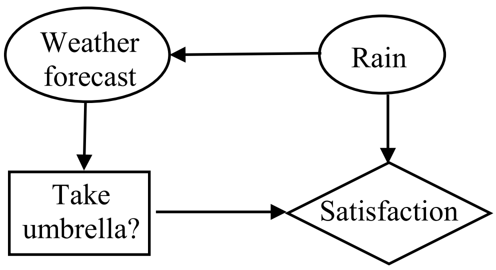
(*a*)


Take umbrella
Do not take umbrella
Rain
No rain
Rain
No rain
Suit not ruined, plus a sense of relief
Suit not ruined, but inconvenience incurred
Suit ruined
Suit not ruined

(*b*)

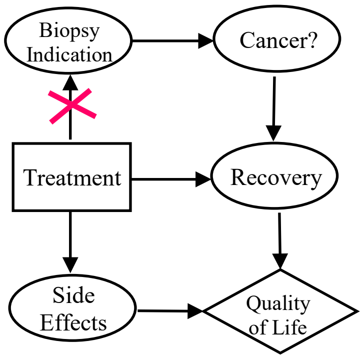

Rain
Take umbrella?
Satisfaction

(*c*)

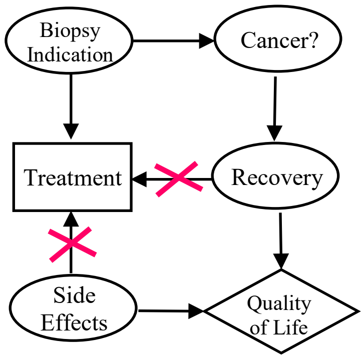

Weather forecast
Rain
Take umbrella?
Satisfaction

P5.4 The correct influence diagram is (*c*). The other three diagrams are not valid due to the offending arcs indicated.

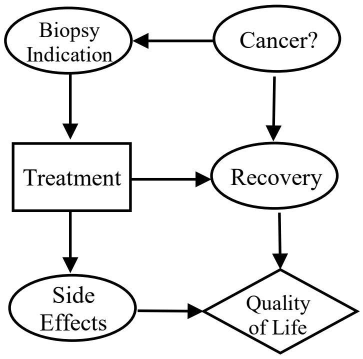
(*a*)



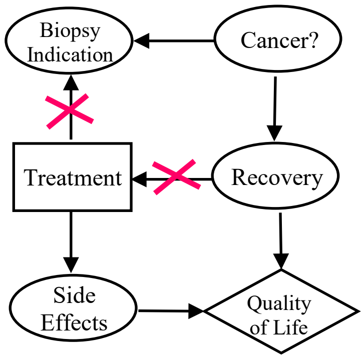
(*b*)



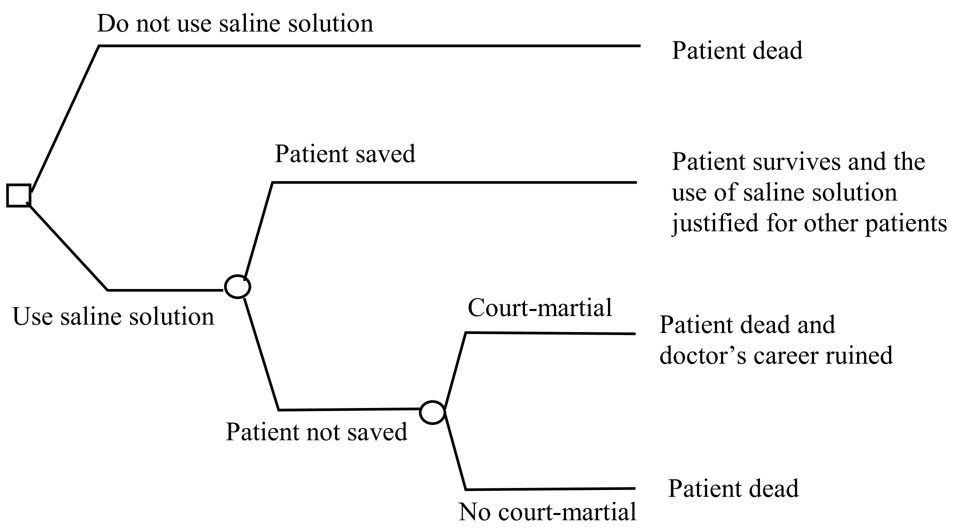
(*c*)



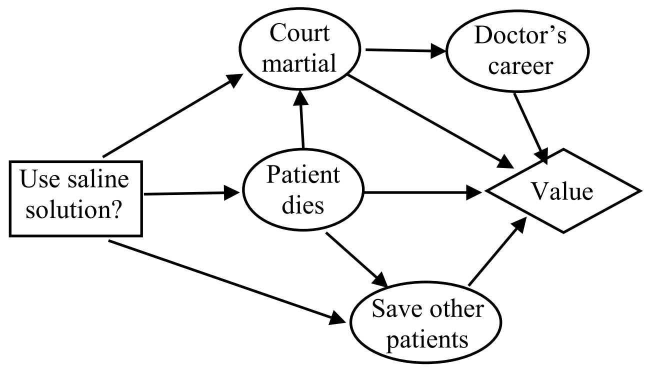
(*d*)



P5.5



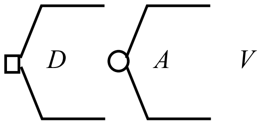
Do not use saline solution
Patient dead
Patient saved
Patient survives and the use of saline solution justified for other patients
Use saline solution
Court-martial
Patient dead and doctor's career ruined
Patient not saved
Patient dead
No court-martial



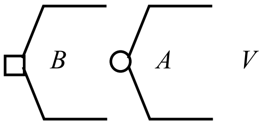
Court martial
Doctor's career
Use saline solution?
Patient dies
Value
Save other patients

P5.6

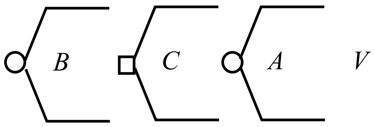
(*a*)


D
A
V

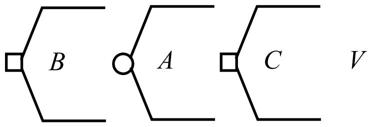
(*b*)


B
A
V

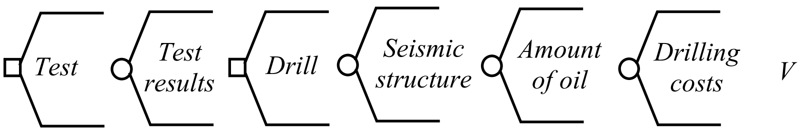
(*c*)


B
C
A
V

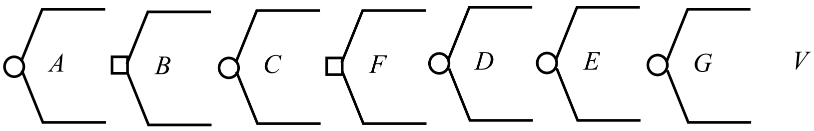
(*d*)


B
A
C
V

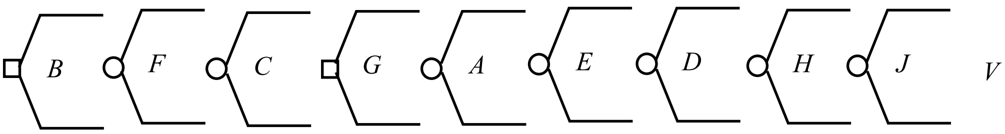
(*e*)


Test
Test results
Drill
Seismic structure
Amount of oil
Drilling costs
V

(*f*)


A
B
C
F
D
E
G
V

(*g*)


B
F
C
G
A
E
D
H
J
V

P5.7

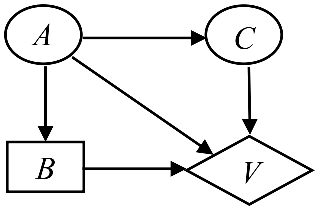
(*a*) Observations:

- Info on A is available before decision B.
- C is dependent on A since $p(C_1|A_1) = 0.6 \neq p(C_1|A_2) = 0.5$
- C is independent of decision B given information on A:

  $p(C_1|B_1,A_1) = p(C_1|B_2,A_1) = 0.6$

  $p(C_1|B_1,A_2) = p(C_1|B_2,A_2) = 0.5$
- Value is dependent on all A, B, C (all the numbers are different).

The influence diagram:



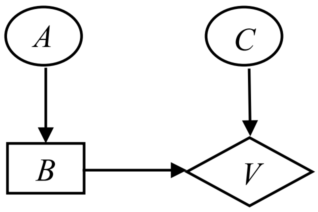
(*b*) Observations:

- Info on A is available before decision B.
- C is independent of A and B

  $p(C_1|A_1) = p(C_1|A_2) = 0.6$

  $p(C_1|B_1) = p(C_1|B_2) = 0.6$
- Value is independent of A.

The influence diagram:



(*c*) Observations:

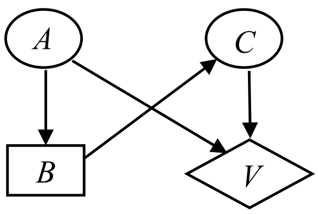
- Info on A is available before decision B.
- C is dependent on decision B

  $p(C_1|B_1) = 0.6 \neq p(C_1|B_2) = 0.9$
- C is independent of A given B

  $p(C_1|A_1,B_1) = p(C_1|A_2,B_1) = 0.6$

  $p(C_1|A_1,B_2) = p(C_1|A_2,B_2) = 0.9$
- Value is independent of B.

The influence diagram:



(*d*) Observations:

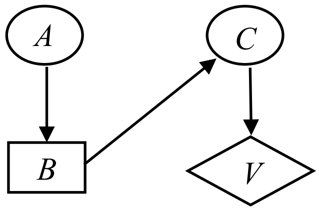
- Info on A is available before decision B.
- C is dependent on decision B

  $p(C_1|B_1) = 0.6 \neq p(C_1|B_2) = 0.9$
- C is independent of A given B

  $p(C_1|A_1,B_1) = p(C_1|A_2,B_1) = 0.6$

  $p(C_1|A_1,B_2) = p(C_1|A_2,B_2) = 0.9$
- Value is independent of A and B.

The influence diagram:



(*e*) Observations:

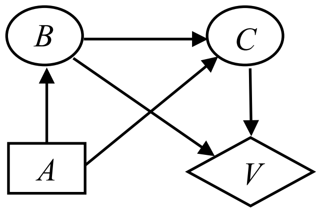
- B is dependent on decision A

  $p(B_1|\mathrm{A}_1) = 0.2 \neq p(B_1|A_2) = 0.1$
- C is dependent on B

  $p(C_1|B_1,A_1) = 0.6 \neq p(C_1|B_2,A_1) = 0.1$
- C is dependent on A

  $p(C_1|A_1) = (.2)(.6)+(.8)(.1) = 0.2 \neq \quad p(C_1|A_2) = (.1)(.15)+(.9)(.65) = 0.6$
- Value is independent of A.

The influence diagram:



(*f*) Observations:

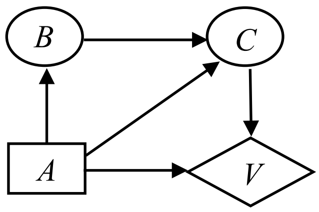
- B is dependent on decision A

  $p(B_1|A_1) = 0.2 \neq p(B_1|A_2) = 0.1$
- C is dependent on B

  $p(C_1|B_1,A_1) = 0.6 \neq p(C_1|B_2,A_1) = 0.1$
- C is dependent on A

  $p(C_1|A_1) = (.2)(.6)+(.8)(.1) = 0.2 \neq \quad p(C_1|A_2) = (.1)(.5)+(.9)(.75) = 0.725$
- Value is independent of B.

The influence diagram:



(*g*) Observations:

- B is independent of decision A

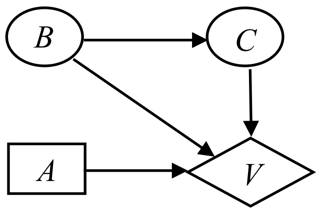
  $p(B_1|A_1) = 0.2 = p(B_1|A_2)$
- C is independent of decision A

  $p(C_1|A_1) = (.2)(.6)+(.8)(.1) = 0.2$

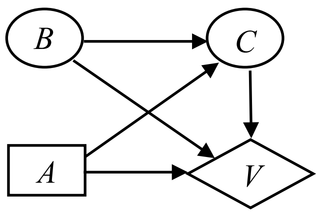
  $p(C_1|A_2) = (.2)(.5)+(.8)(.125) = 0.2$
- C is dependent on B

  $p(C_1|B_1,A_1) = 0.6 \neq p(C_1|B_2,A_1) = 0.1$
- Value is dependent on all A, B, C.

- Based on the above computations, the most compact influence diagram should be:



- However, the above diagram requires $p(C \mid B)$ which cannot be obtained from the decision tree because A is a decision node. $p(C \mid B)$ can only be obtained by a direct assessment between B and C either by the expert or by using the data. If such information is not available, then the following diagram may be drawn using probabilities directly available from the tree. But in so doing, we lose the fact that C is also independent of A.



(*h*) Observations:

- B is independent of decision A

  $p(B_1|A_1) = 0.2 = p(B_1|A_2)$
- C is dependent on decision A

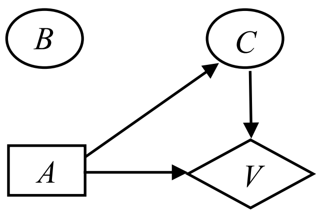
  $p(C_1|A_1) = (.2)(.6)+(.8)(.6) = 0.6 \neq$

  $p(C_1|A_2) = (.2)(.7)+(.8)(.7) = 0.7$
- C is independent of B given A

  $p(C_1|B_1,A_1) = 0.6 = p(C_1|B_2,A_1)$

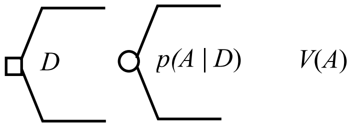
  $p(C_1|B_1,A_2) = 0.7 = p(C_1|B_2,A_2)$
- Value is independent of B.

The influence diagram:

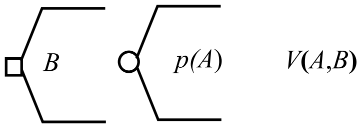


P5.8 (Optional Problem)

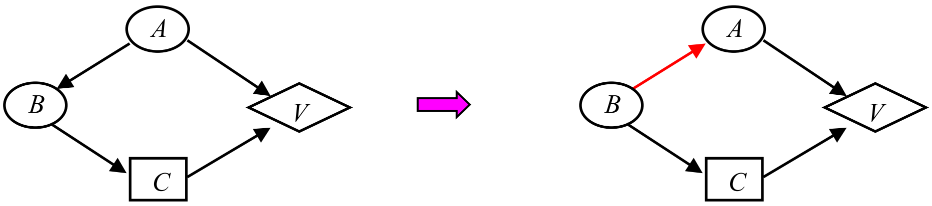
(***a***) Tree Sequence = $\{D, A, V\}$



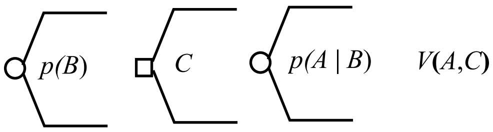
(***b***) Tree Sequence = $\{B, A, V\}$



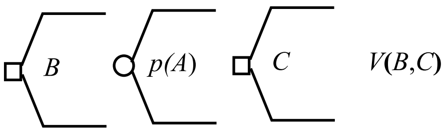
(***c***) Convert to decision tree network:



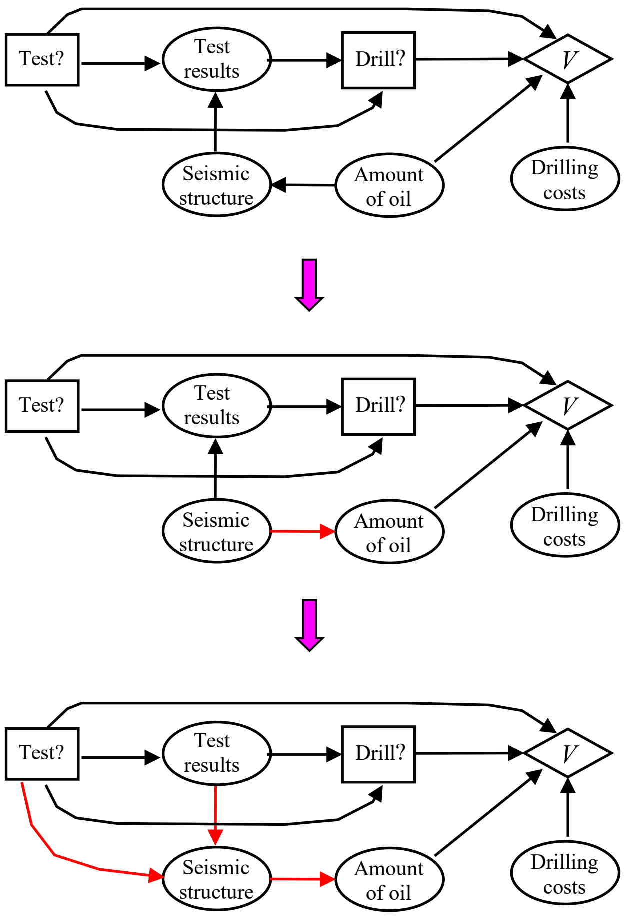
Tree Sequence = $\{B, C, A, V\}$



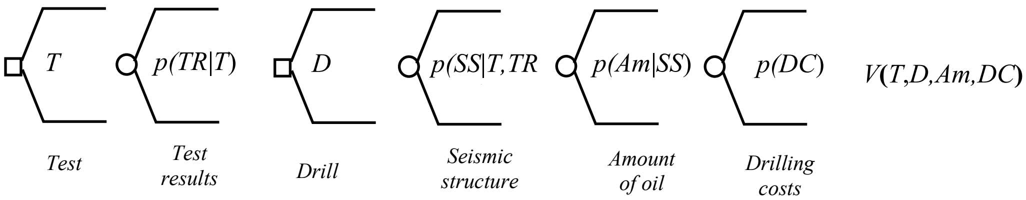
(***d***) Tree Sequence = $\{B, A, C, V\}$



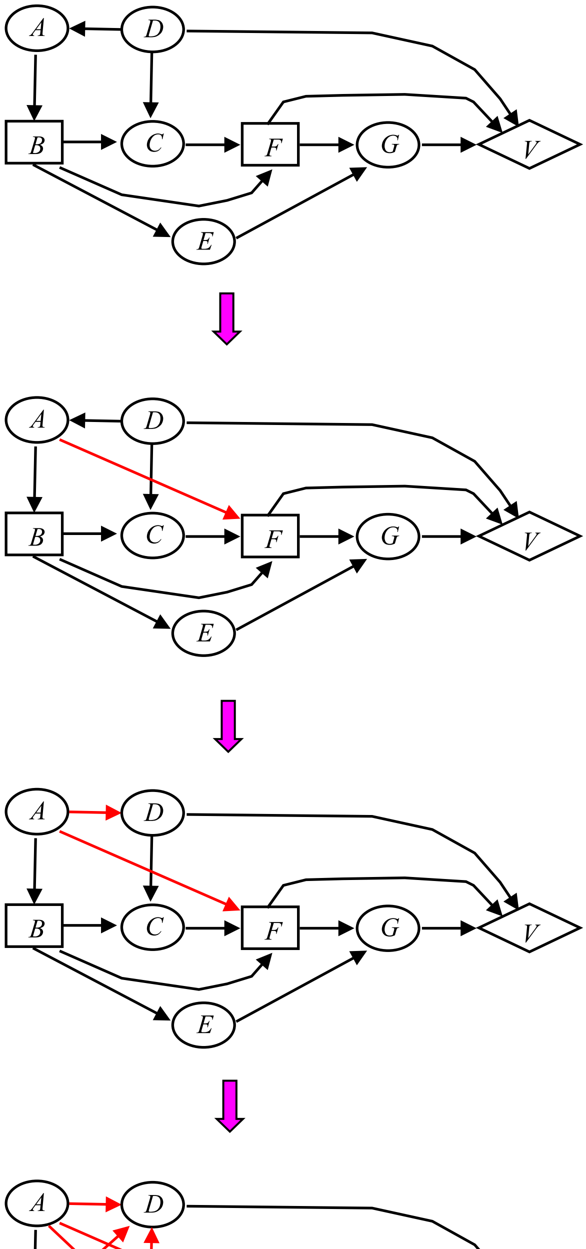
(*e*) Convert to decision tree network:



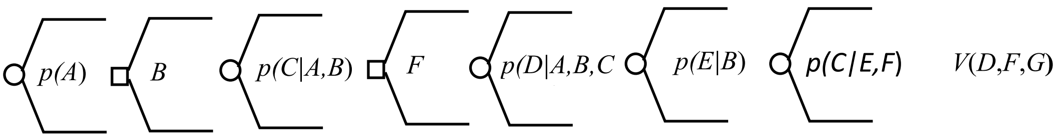
Sequence = {Test, Test results, Drill, Seismic structure, Amount of oil, Drilling costs, V }



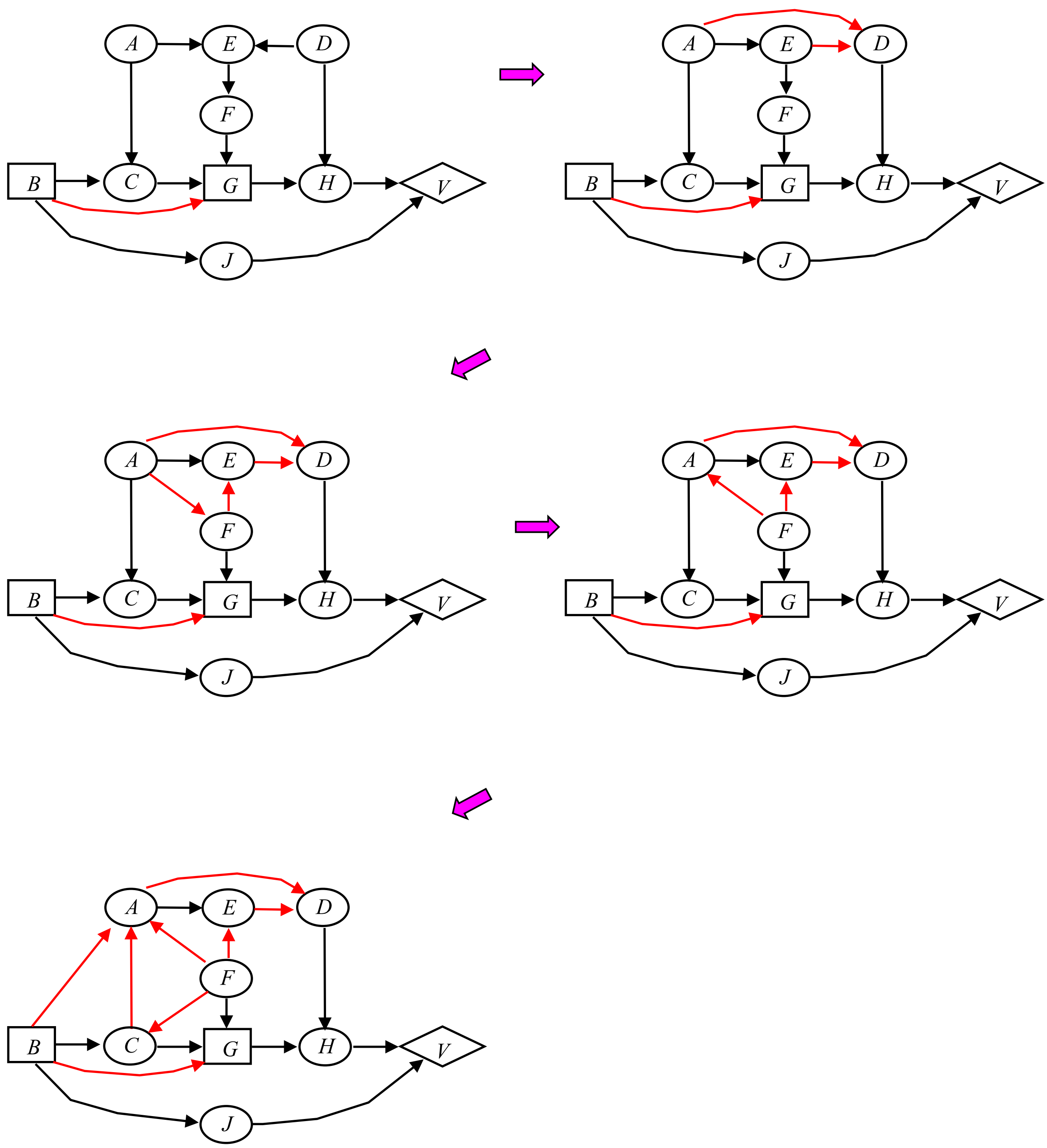
(***f***) Convert to decision tree network:



Tree Sequence = $\{A, B, C, F, D, E, G, V\}$



(g) Convert to decision tree network:



Sequence = { B, F, C, G, A, E, D, H, J ,V }