Chapter 8 Solving Complex Decision Problems: The Decision Analysis Process / Cycle

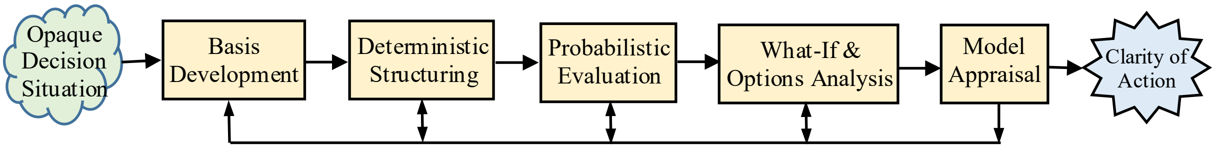
"Good plans shape good decisions. That's why good planning helps to make elusive dreams come true."

Lester R. Bittel
The Nine Master Keys of Management

8.1 Introduction

- The Decision Analysis Process/Cycle is a procedure for analyzing complex decision problems.

- The aim is to ensure that all essential and necessary steps for a complete analysis are consciously and systematically carried out.



- The decision analysis process/cycle comprises a number of iterative steps:

  1. Basis Development
     a. Bound decision
     b. Identify alternatives
     c. Establish outcomes
     d. Identify system variables
     e. Create a decision diagram / generic decision tree

  2. Deterministic Structuring
     a. Create a value model for each alternative using base values (cash flow models)
     b. Identify uncertainty and estimate their variations
     c. Perform one-way range sensitivity analysis (tornado diagrams)
     d. Identify sensitive and insensitive variables
     e. Perform tornado dominance analysis
     f. Reduce the model

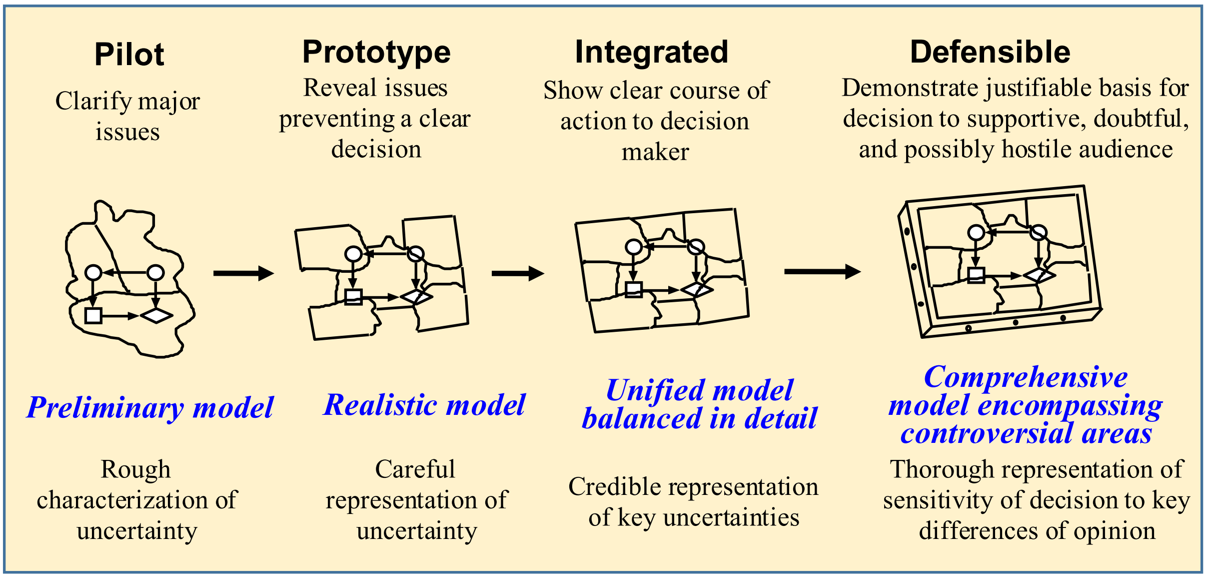
  3. Probabilistic Evaluation
     a. Encode uncertainty on sensitive variables
     b. Solve decision model and generate optimal decision policy (risk-neutral)
     c. Perform value of information analysis (risk-neutral)
     d. Generate risk profiles
     e. Perform stochastic dominance analysis
     f. Encode risk preference
     g. Solve decision model and generate optimal decision policy (risk averse)
     h. Perform risk preference sensitivity
     i. Perform value of information analysis

  4. What-If and Options Analysis
     a. Perform sensitivity analysis on key factors (rainbow diagrams)
     b. Perform value of embedding options or managerial flexibility

  5. Model Appraisal
     a. Assess the Quality of decision model
     b. Assess the Clarity of decisions

- Repeat any of the above steps for another pass of the DA Process/Cycle until a clear and defensible course of action is obtained.

Evolution of Decision Models over DA Cycles



- As the DA process/cycle proceeds, the decision problem is being captured in an increasingly complex decision model. In the initial phase, a pilot model may be constructed which helps to clarify major issues.

- The preliminary model contains a rough characterization of uncertainty. Through iterations, the model may evolve into a prototype model which starts to reveal issues that had earlier on prevented a clear decision. In this realistic model, there is now a careful representation of uncertainty.

- Next, the model becomes an integrated one and it starts to show a clear course of action to the decision-maker. This unified model is balanced in detail and includes a credible representation of key uncertainties.

- Finally, a defensible model is reached and it demonstrates a justifiable basis for the decision to a supportive, doubtful, and possibly hostile audience. This final comprehensive model encompassing controversial areas includes a thorough representation of the sensitivity of decisions to key differences of opinion.

- We will use case studies to illustrate the DA Process/Cycle.

8.2 Case Study 1: Exxoff Research & Development Problem

8.2.1 Problem Description

Exxoff Corp. is considering whether or not to conduct research into developing a new additive for motor oil. The purpose of the additive is to treat motor oil so that it flows smoothly in cold weather. The production cost *c*, of the new additive (per gallon) is uncertain. The effectiveness of the new product measured by a potency index *p*, is also uncertain. Exxoff considers these uncertainties to be independent.

The research needed to develop the new additive will take five years. It will cost the equivalent of $7 million paid out at the end of year 5. At the end of the five years of research, Exxoff will know the *production cost* and the *potency index* of the new additive. Depending on whether the new product is profitable, Exxoff will decide whether to market the new product. There are two additional uncertainties faced by Exxoff in projecting its future cash inflows: the *market size* for oil additives and the *lifetime* of the product.

The *market size* for oil additives is a fraction, *f*, of the 100 million barrels of motor oil that are known to be used each year. The fraction is uncertain. The *lifetime* of the product *L* is the number of years the product will survive in the marketplace before being knocked out by another new additive. All the uncertainties are considered to be independent.

The price of the current additive in the market is $1.00 per gallon, and it takes 1 gallon of the old additive to treat 1 barrel of motor oil. The potency index tells how many units of the new additive are equivalent to one unit of the old additive. A potency index of 1/2 means that 1/2 gallon of the new additive treats as well as one gallon of the old additive; a potency index of 1/4 means that 1/4 gallon of the new additive treats as well as one gallon of the old additive; a potency index of 2 means that 2 gallons of the new additive treat as well as one gallon of the old additive, etc. Thus the smaller the potency index, the better is the additive.

With the old additive, at its price of $1.00 per gallon, the consumer's cost per treated barrel is $1.00. If the price of the new additive is $1.00 per gallon, the consumer's new cost per treatment is a fraction of this price; that fraction is exactly the potency index of the new product. For example, if the new price is $2.00 per gallon and the potency index is 1/2, the consumer is paying the same amount per treated barrel.

Exxoff will price the new additive such that it will take as profit per barrel of motor oil half the potential savings to the consumer. The potential savings to the consumer is the difference between the old cost per gallon and the product of the production cost per gallon and the potency index.

The annual profit is determined by the profit per barrel of motor oil and the market size. This annual profit is taken as constant over the lifetime of the product.

Exoff has the ***option of not marketing*** the product (i.e. terminate the project) after the R&D phase.

The *MARR* or *WACC* for Exxoff is 10%. Assume Exxoff is risk-neutral for the purpose of a preliminary analysis.

Questions to be Addressed:

1. Should Exxoff invest in R&D for the new oil additive?
2. If it does, how should it decides whether to market or not to market the product
3. What happens if Exxoff must always market the product if it does invest in the R&D

8.2.2 First Pass through the DA Cycle

Decision Variables (Alternatives)

- Conduct research? // Primary decision
- Market new product? // Option to terminate after R&D

Uncertain System Variables

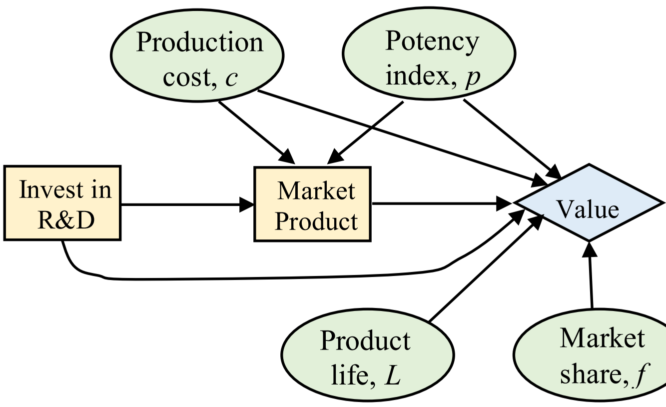
- Production cost c ($ per gallon of new additive)
- Potency Index p (gallons of new additive/gallon of old additive)
- Market share for new additive f (%)
- Product lifetime: L (years)

Values and Preferences

- NPV of total profits at *MARR* or WACC= 10%
- Risk Neutral (for preliminary analysis)

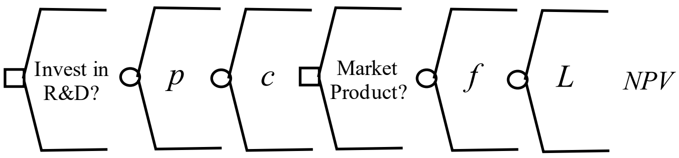
Preliminary Influence Diagram

- The preliminary influence diagram shows the possible relationships among the variables.
- We do not need to worry about the probabilities at this stage.



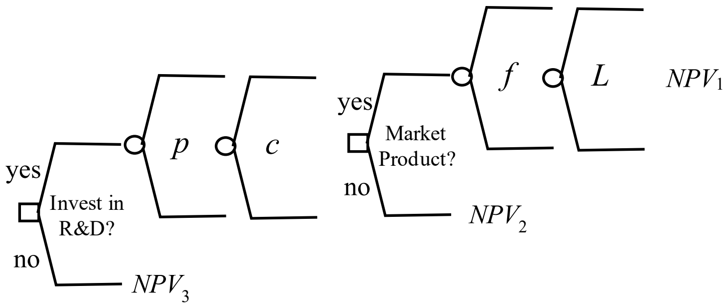
Generic Decision Tree (Symmetric)

- The generic decision tree shows the chorological order of the decisions and events more clearly than the influence diagram.



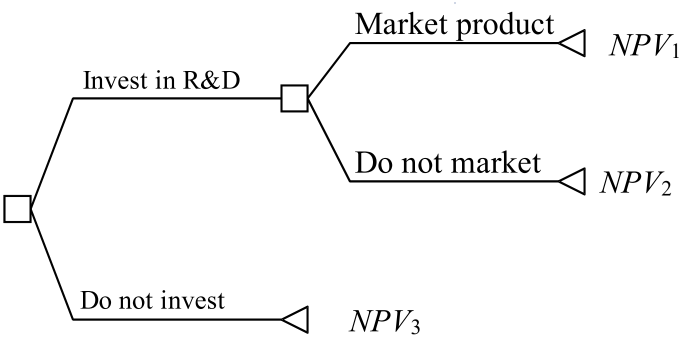
Asymmetric Generic Decision Tree

- The asymmetric generic decision tree shows clearly the asymmetric structure of the decision model.



Deterministic Decision Tree

- A deterministic decision tree assumes that there is no uncertainty in all the state variables. i.e., the value of p, c, f, and L are known with certainty:



- The deterministic decision tree is useful for generating **decision strategies** or **scenarios.**

Decision Strategies or Scenarios

- We observe that there are three decision strategies or scenarios:

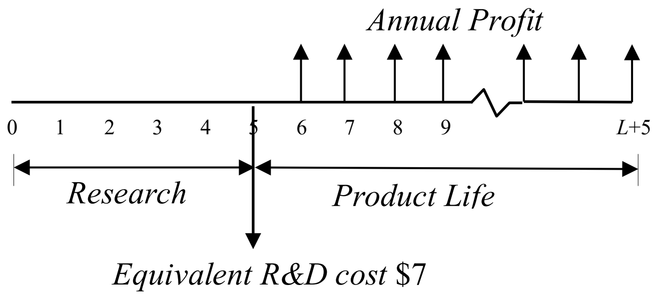
  Strategy 1: Invest in R&D and Market the product.
  Strategy 2: Invest in R&D and the Product is not marketed
  Strategy 3: Do not invest in R&D

- We shall develop the value model for each of the three decision strategies or scenarios and denote them by NPV_1, NPV_2 and NPV_3 respectively.

Value or Business Models and Cash Flow Diagrams

Strategy 1: Invest in R&D and Market Product

Cash Flow Diagram:



- Exxoff's Annual Profit if the product is marketed:
  - Customer potential saving per gallon = $(1 - pc)$ ($)
  - Exxoff's profit per barrel = (1/2) $(1 - p\,c)$ ($)
  - Exxoff's annual profit

    A = Profit/barrel × Market share × Total market volume

    = $(1/2)(1-pc)\ f$ ($ millions)

    = $50 f(1 - pc)$

Value Model:

$$NPV_1 = \left[-7 + \sum_{k=1}^{L} \frac{A}{(1+0.1)^k}\right] \frac{1}{(1+0.1)^5}$$

$$NPV_1 = \left[-7 + A\left(\frac{(1+0.1)^L - 1}{0.1(1+0.1)^L}\right)\right] \frac{1}{(1+0.1)^5}$$

$$NPV_1 = \left[-7 + 50f(1-pc)\left(\frac{(1+0.1)^L - 1}{0.1(1+0.1)^L}\right)\right] \frac{1}{(1+0.1)^5}$$

- Engineering Economy notations:

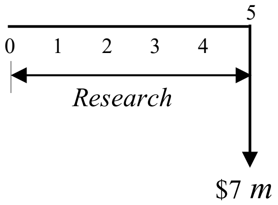
  NPV_1 = (–7 + $50 f(1\text{-}pc)$ [P/A, 10%, L]) [P/F, 10%, 5]

- DPL value expressions:

  MARR = 0.1
  A = 50*f*(1 - p*c)
  NPV1 = (-7 + @pv(A, MARR, L)) / (1 + MARR)^5

Strategy 2: **<u>Invest in R&D and the Product is Not Marketed</u>**

Cash Flow Diagram:



Value Model:

$$NPV_2 = \frac{-7}{(1+0.1)^5}$$

NPV_2 = - 4.346449 ($m)

- Engineering Economy notations:

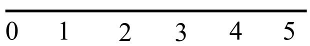
NPV_2 = – 7 [P/F, 10%, 5]

- DPL value expressions:

MARR = 0.1
NPV2 = -7 / (1 + MARR)^5

Strategy 3: **<u>Do Not Invest in R&D</u>**

Cash Flow Diagram:



Value Model:

$NPV_3 = 0$

Deterministic Base-Value Model Solution

- The base or nominal values for each variable is estimated by the company's experts as follows:

Variable	Base or Nominal Value
c	\$0.88 per gal
p	0.95
f	0.48
L	20 years

- The base-value *NPV* of each scenario are:
  - Invest in R&D and Market Product

$$NPV_1 = \left[-7 + 50\times0.48(1 - 0.95\times0.88)\left(\frac{(1+0.1)^{20}-1}{0.1(1+0.1)^{20}}\right)\right]\frac{1}{(1+0.1)^5}$$

$$= 16.460244 \ \ (\$m)$$

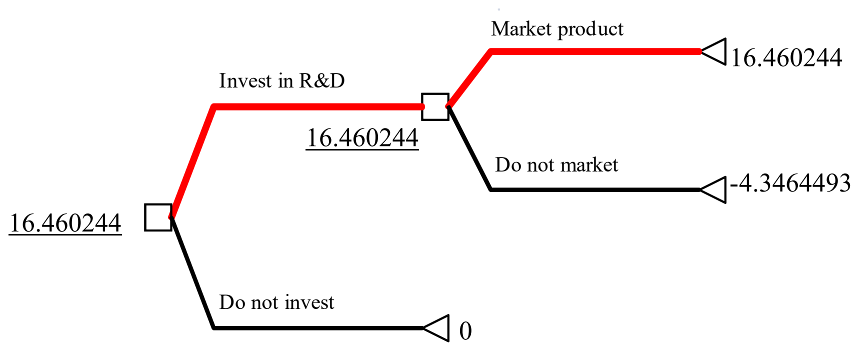
  - Invest in R&D and Do Not Market Product

$$NPV_2 = -4.3464493 \ \ (\$m)$$

  - Do Not Invest in R&D:

$$NPV_3 = 0$$

- These values are shown on the deterministic decision tree:



- The deterministic base-value optimal solution is to Invest in R&D and Market the Product with an NPV of 16.460244 \$m.

Deterministic Sensitivity Analysis

- This is also known as One-Way Range Sensitivity Analysis. The objective here is to determine how the variability of each uncertain variable may affect the value of the decisions.
- For each variable, we estimate its “low”, “base”, and “high” values. These low and high values may be based on the percentiles, e.g., (10%, 90%), (5%, 95%), (1%, 99%), etc.
- We vary one variable at a time from its “low” to “high” value while keeping all the other variables at their base or nominal values, and compute the value of NPV at each extreme.
- The following Deterministic or One-Way Range Sensitivity Tables are obtained

One-Way Range Sensitivity Table for Invest in R&D and Market Product

Var	Low value (10-percentile)	Base or nominal value	High value (90-percentile)	NPV_1			Sensitive ?
				One extreme	Other extreme	$\lvert\Delta\rvert$	
c	0.65	0.88	1.11	44.181356	– 11.260869	55.442224	*yes*
p	0.62	0.95	1.28	53.303314	– 20.382827	73.686141	*yes*
f	0.42	0.48	0.54	13.859407	19.061080	5.201673	*no*
L	14	20	26	13.657339	18.042410	4.385070	*no*

One-Way Range Sensitivity Table for Invest in R&D and Don’t Market Product

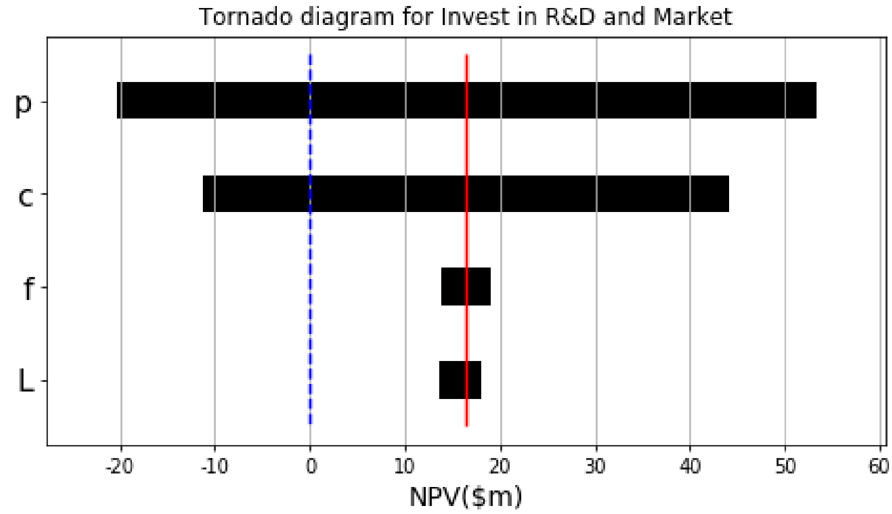
Var	Low value (10-percentile)	Base or nominal value	High value (90-percentile)	NPV_2			Sensitive ?
				One extreme	Other extreme	$\lvert\Delta\rvert$	
c	0.65	0.88	1.11	– 4.346449	– 4.346449	0	*no*
p	0.62	0.95	1.28	– 4.346449	– 4.346449	0	*no*
f	0.42	0.48	0.54	– 4.346449	– 4.346449	0	*no*
L	14	20	26	– 4.346449	– 4.346449	0	*no*

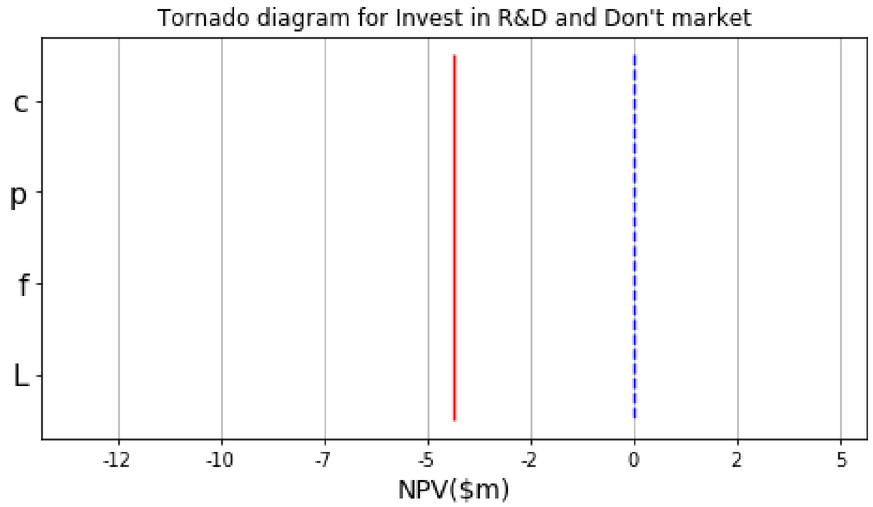
One-Way Range Sensitivity Table for Do Not Invest in R&D

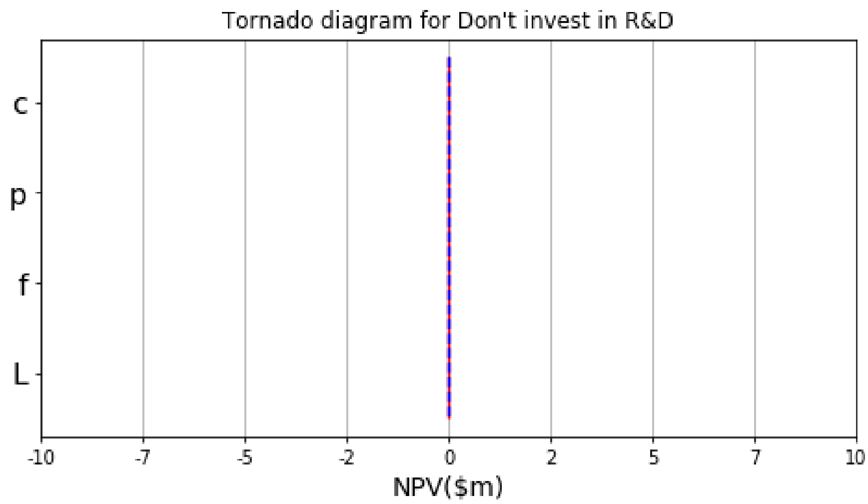
Var	Low value (10-percentile)	Base or nominal value	High value (90-percentile)	NPV_3			Sensitive ?
				One extreme	Other extreme	$\lvert\Delta\rvert$	
c	0.65	0.88	1.11	0	0	0	*no*
p	0.62	0.95	1.28	0	0	0	*no*
f	0.42	0.48	0.54	0	0	0	*no*
L	14	20	26	0	0	0	*no*

Tornado Diagrams Generation

- A **Tornado Diagram** (also known as a one-way range-sensitivity diagram) shows how the *NPV* of an alternative vary (or swing) as each of the uncertain variables is varied over its entire range of possible values.



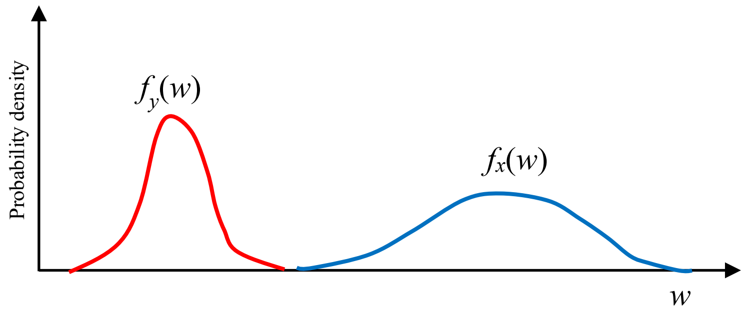




- The tornado diagrams allow us to identify approximate deterministic and tornado dominance.

Deterministic Dominance

- Recall from Chapter 4 that the outcomes of alternative X **Deterministically Dominate** the outcomes of alternative Y if and only if the minimum possible value of X is greater than the maximum possible value of Y. Under this situation, alternative X is preferred to alternative Y regardless of the probability distributions of X and Y.



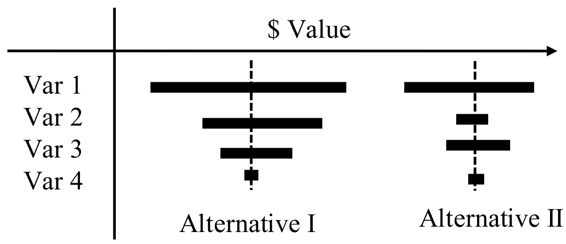
- Deterministic dominance can be identified only by considering all possible variations of the state variables. However, this is not practical and is often approximated by **Tornado Dominance**.

Tornado Dominance

- **Definition**: An alternative is said to exhibit **Tornado Dominance** over another alternative if the tornado diagram of the first alternative is always to the right of the second alternative.

Example:

- In the tornado diagrams below, Alternative II exhibits tornado dominance over Alternative I.



Relationship between Deterministic and Stochastic Dominance

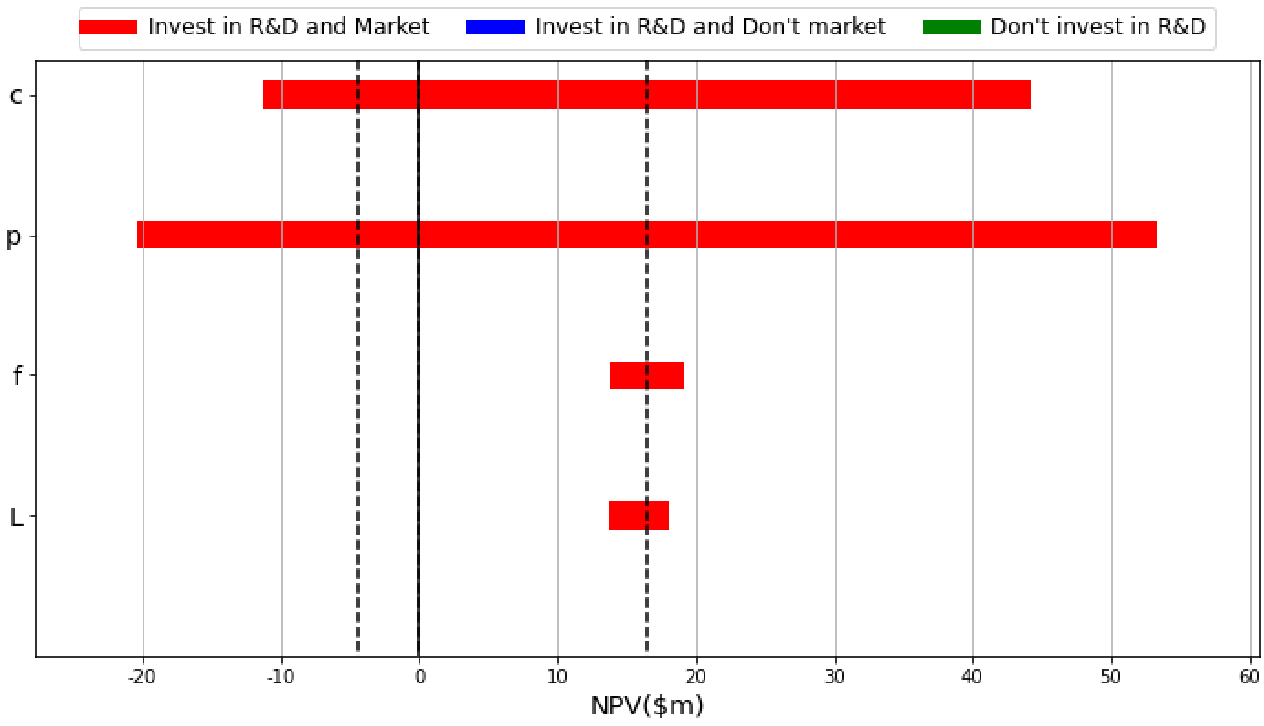
- Deterministic Dominance

  $\Rightarrow$ 1st Order Stochastic Dominance

  $\Rightarrow$ 2nd Order Stochastic Dominance.

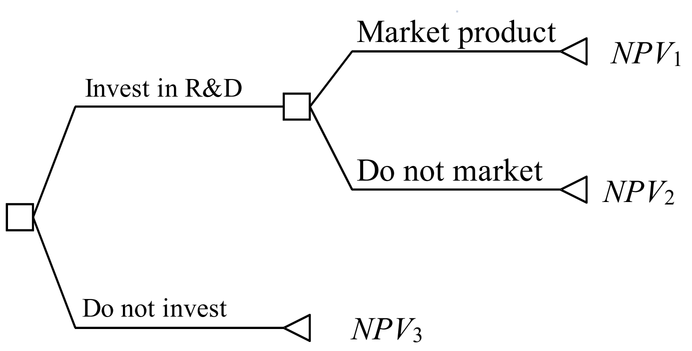
- The converses of the above statements are not necessarily true.

Tornado Dominance Analysis for Decision Strategies



- "Do Not Invest in R&D" tornado dominates "Invest in R&D and Don't Market Product".

Tornado Dominance Analysis for Initial Decision Alternatives



- The two initial decision alternatives are "Invest in R&D" and "Do Not Invest in R&D".

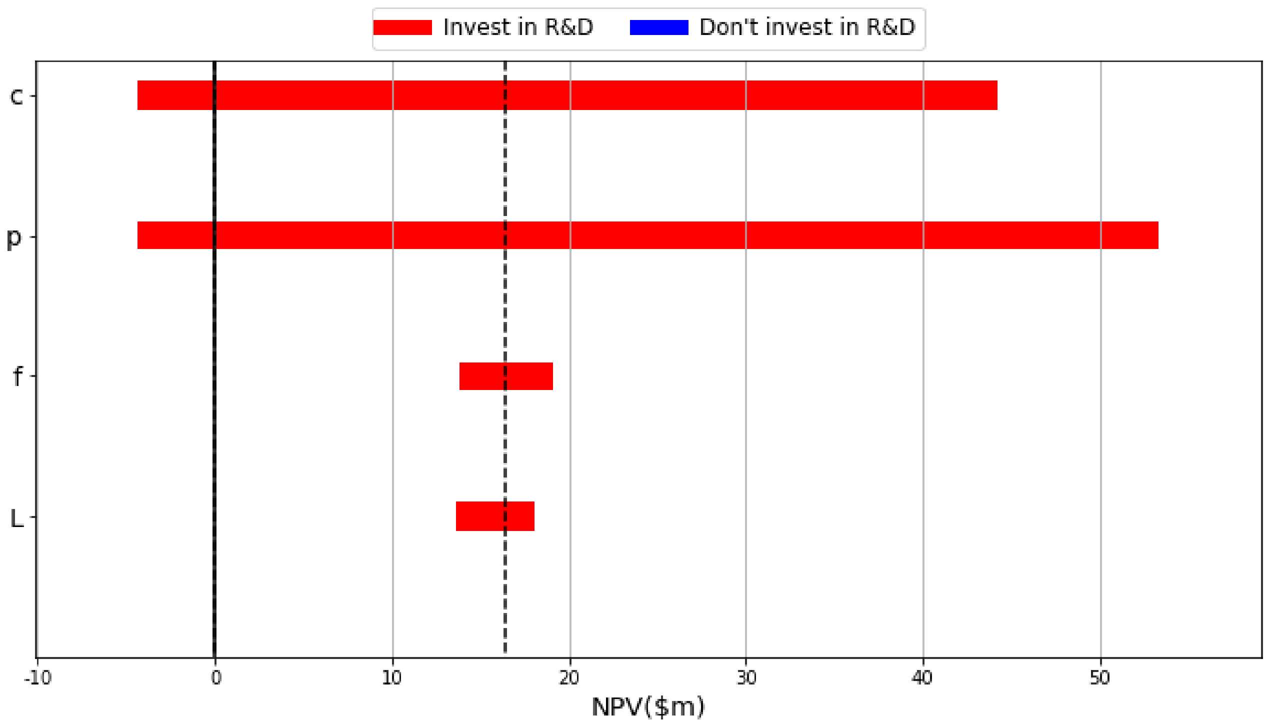
    1. NPV(Invest in R&D) = max(NPV_1, NPV_2)
    2. NPV(Do Not Invest in R&D) = NPV_3

One-Way Range Sensitivity Table for Alternative Invest in R&D

Var	Low value (10-percentile)	Base or nominal value	High value (90-percentile)	NPV(Invest in R&D) = max(NPV1,NPV2)			Sensitive ?
				One extreme	Other extreme	$\|\Delta\|$	
c	0.65	0.88	1.11	44.181356	– 4.346449	48.527805	*yes*
p	0.62	0.95	1.28	53.303314	– 4.346449	57.649763	*yes*
f	0.42	0.48	0.54	13.859407	19.061080	5.201673	*no*
L	14	20	26	13.657339	18.042410	4.385070	*no*

One-Way Range Sensitivity Table for Alternative Do Not Invest in R&D

Var	Low value (10-percentile)	Base or nominal value	High value (90-percentile)	NPV(Do not invest) = NPV_3			Sensitive ?
				One extreme	Other extreme	$\|\Delta\|$	
c	0.65	0.88	1.11	0	0	0	*no*
p	0.62	0.95	1.28	0	0	0	*no*
f	0.42	0.48	0.54	0	0	0	*no*
L	14	20	26	0	0	0	*no*



- The is no tornado dominance between the two initial alternatives “Invest in R&D” and “Do Not Invest in R&D”.

Conclusion:

- We cannot eliminate any initial alternatives at this stage.

Identification of Sensitive Variables

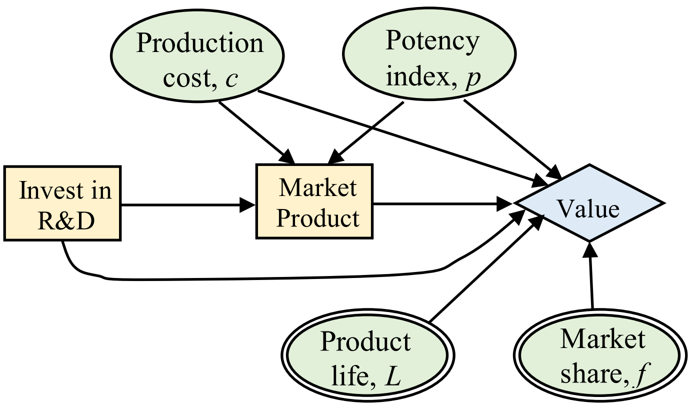
- From the tornado diagram or range-sensitivity table we can identify the sensitive and non-sensitive variables.

  - Variables c and p have significant changes (swings) in NPV over their possible range of values. They are classified as **Sensitive Variables.**
    All sensitive variables will have their probability distribution assessed in the next phase of the DA process/cycle.

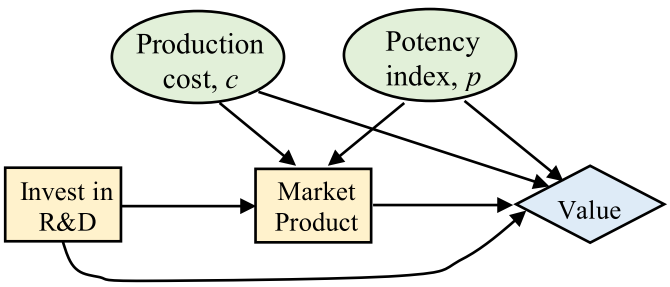
  - Variables f and L have significant changes (swings) in NVP over their possible range of values. They are classified as **Non-Sensitive Variables**.
    All non-sensitive variables will have their value fixed at the nominal or base values for the rest of the analysis, i.e., $f = 0.48$ and $L = 20$ years.

Simplified Influence Diagram

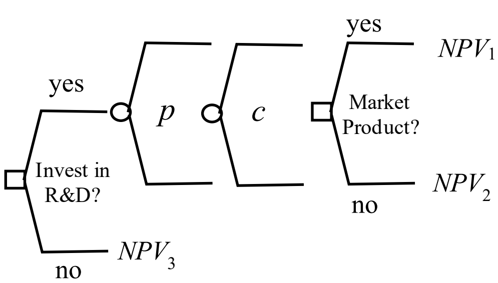
- L and f become "fixed" or deterministic nodes:



- Further simplification can be made to the influence diagram by making L and f implicit in the model:



- Generic Asymmetric Decision Tree:



Assessment of the Probability Distributions for the Sensitive Variables

- The next step is to access the probability distributions for all the sensitivity variables that were identified in the previous step.

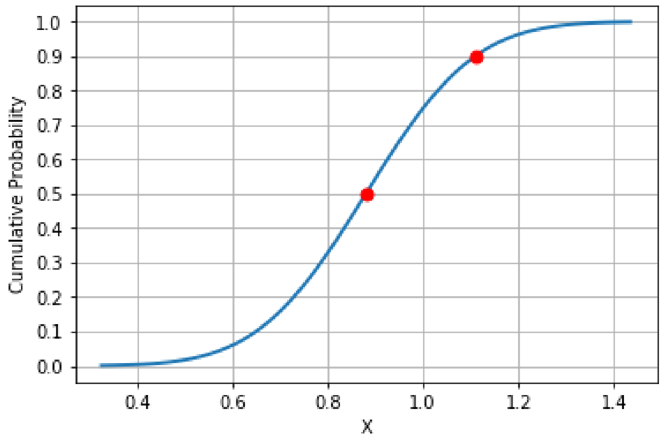
- The probability distribution for each sensitive variable can be assessed using methods discussed in Chapter 7.

- In the Exoff case, we shall assume that the two variables Cost of Production and Potency Index may be approximated by *normal distributions*.

- Based on their 10th, 50th, and 90th percentiles used in the one-way range sensitivity we first fit a normal distribution to each of them and then discretize them using a 3-branch approximation.

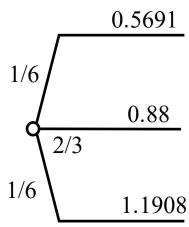
Cost of Production

- Distribution to fit: Normal
- Known parameters:
  - 50-percentile = 0.88
  - 90-percentile = 1.11



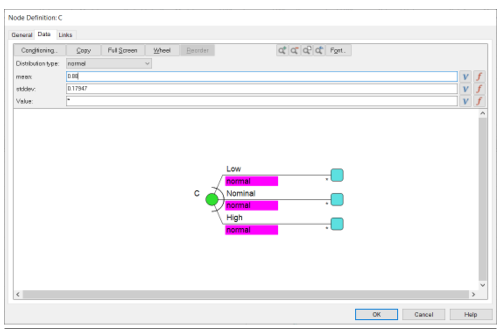
- Fitted distribution: $N(0.88, 0.17947)$

- 3-branch discrete approximation:



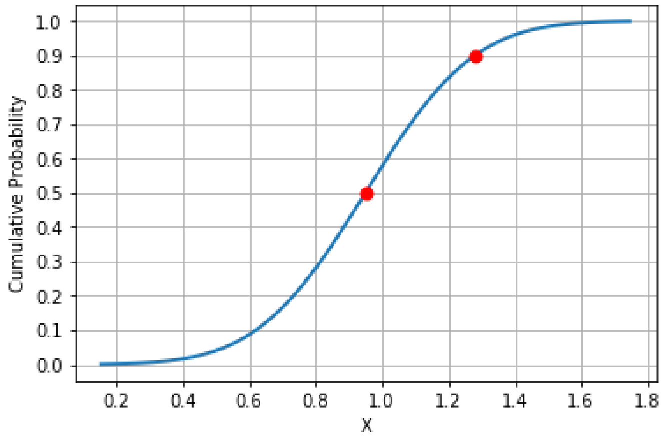
  - x_1=0.5691, p_1= 0.166667
  - x_2=0.8800, p_2= 0.666667
  - x_3=1.1909, p_3= 0.166667

- Note that in DPL, you can just create a discrete chance node with 3 branches and specify a normal distribution with mean = 0.88 and standard deviation = 0.17947. The discretization is automatically done by DPL.



Potency Index

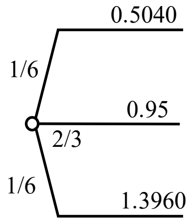
- Distribution to fit: Normal
- Known parameters:
  - 50-percentile = 0.95
  - 90-percentile = 1.28



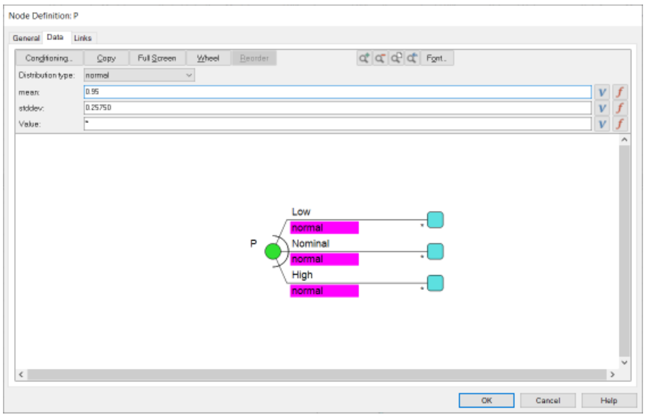
- Fitted distribution: $N(0.95, 0.25750)$

- 3-branch discrete approximation:

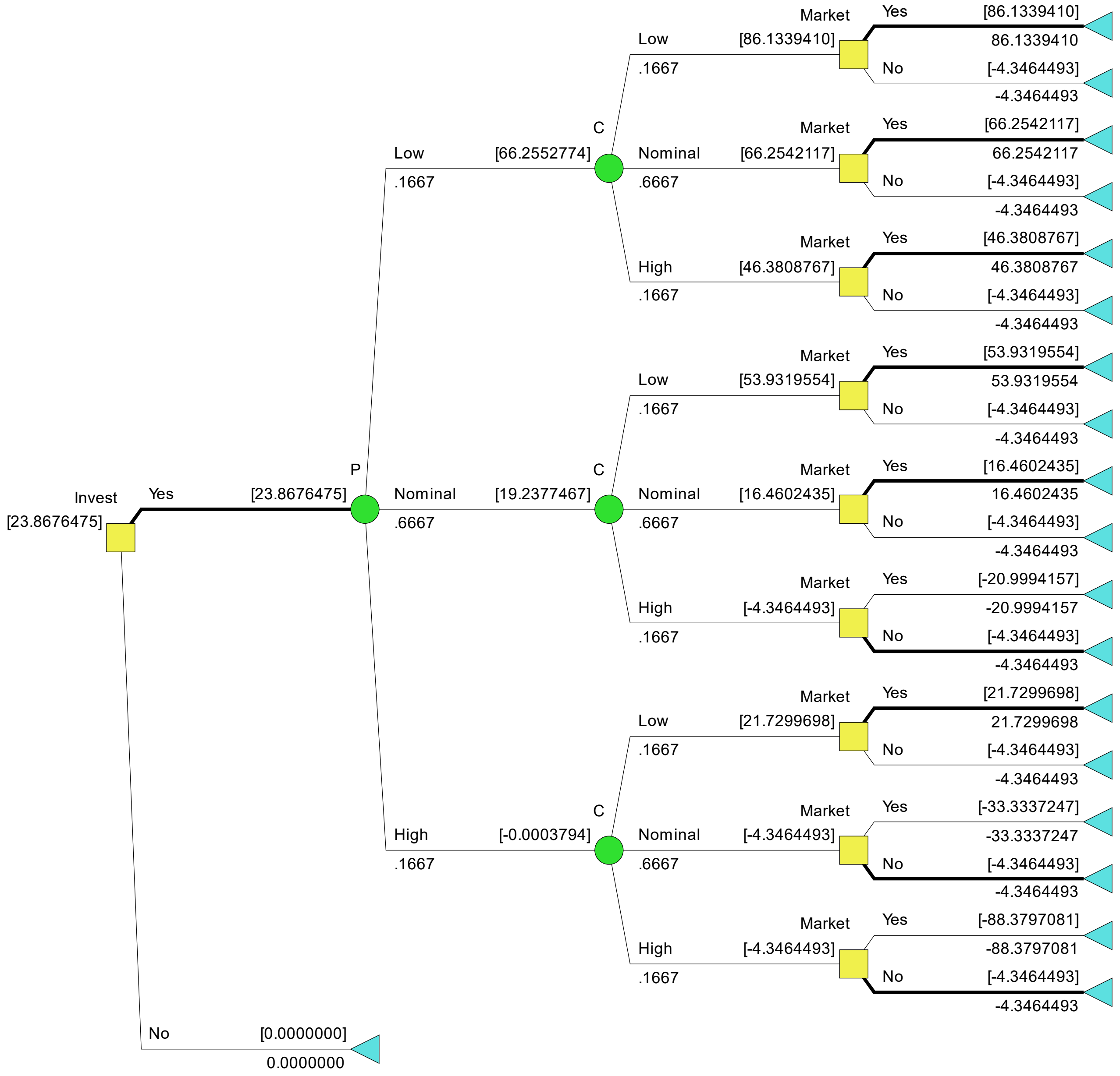
  - x_1=0.5040, p_1= 0.166667
  - x_2=0.9500, p_2= 0.666667
  - x_3=1.3960, p_3= 0.166667



- Note that in DPL, you can just create a discrete chance node with 3 branches and specify a normal distribution with mean = 0.95 and standard deviation = 0.25750. The discretization is automatically done by DPL



Finding the Optimal Decision Policy



- Optimal Decision is to Invest in R&D now.
- At the end of the R&D

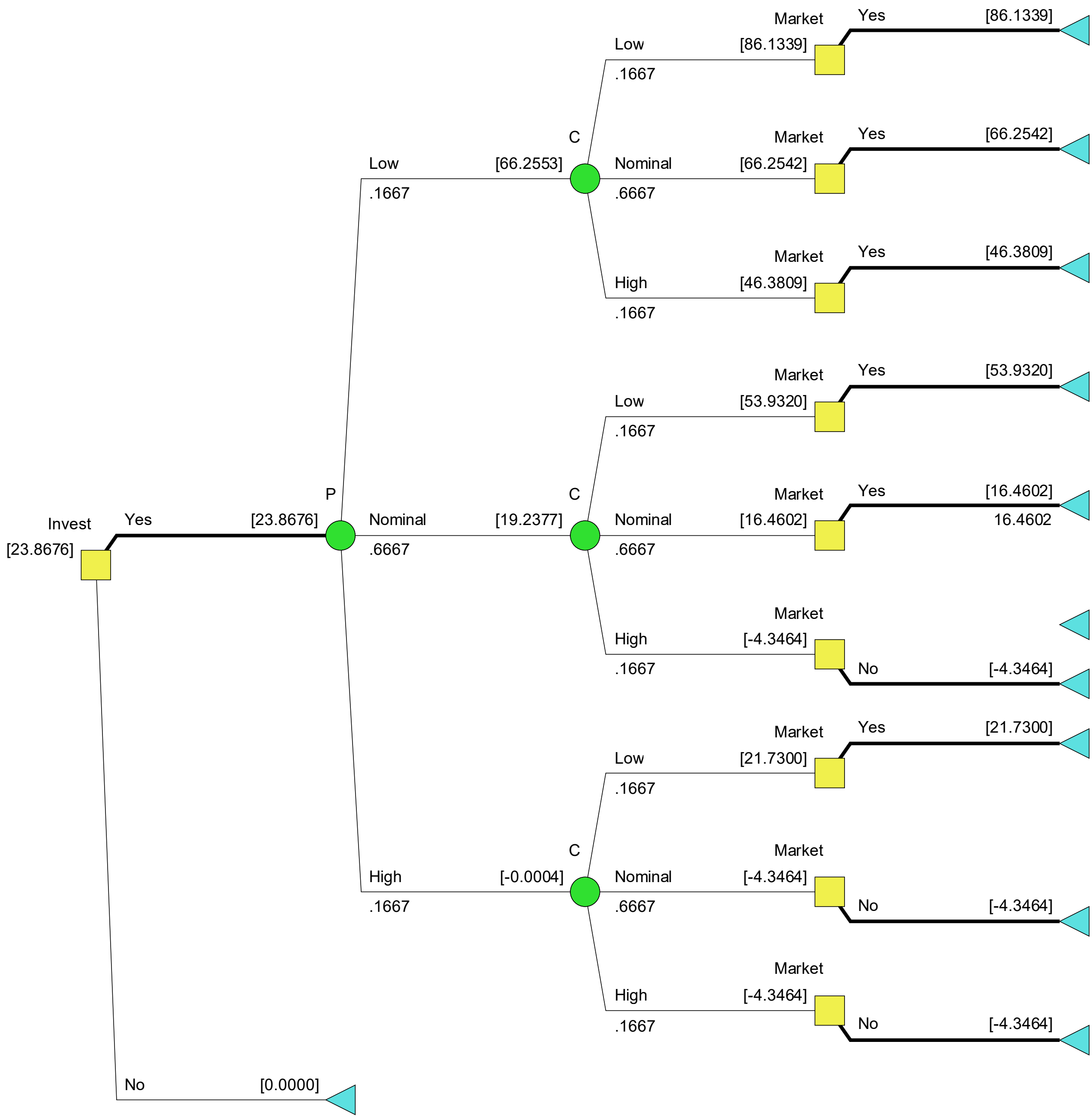
  If Potency Index*Cost of Production < 1
  Then Market the product.
  Else Don't market the product.

- Expected NPV = **$ 23.8676475** million

Risk Profiles and Stochastic Dominance Analysis

Generating Risk Profiles under Optimal Decision Policy

- To generate the risk profiles under optimal decision policy for the two initial alternatives, we first prune the non-optimal sub-trees:



Alternative "Invest in R&D"

- Extract the joint probability of each endpoint NPVs:

Endpoint	NPV $million	Joint Probability
1	86.1339410	(1/6)*(1/6) = 0.0278
2	66.2542117	(1/6)*(2/3) = 0.1111
3	46.3808767	(1/6)*(1/6) = 0.0278
4	53.9319554	(2/3)*(1/6) = 0.1111
5	16.4602435	(2/3)*(2/3) = 0.4444
6	– 4.3464493	(2/3)*(1/6) = 0.1111
7	21.7299698	(1/6)*(1/6) = 0.0278
8	– 4.3464393	(1/6)*(2/3) = 0.1111
9	– 4.3464493	(1/6)*(1/6) = 0.0278

- Sort the end-points by value:

Endpoint	NPV $million	Joint Probability
1	– 4.3464393	0.1111
2	– 4.3464393	0.1111
3	– 4.3464393	0.0278
4	16.4602435	0.4444
5	21.7299698	0.0278
6	46.3808767	0.0278
7	53.9319554	0.1111
8	66.2542117	0.1111
9	86.1339410	0.0278

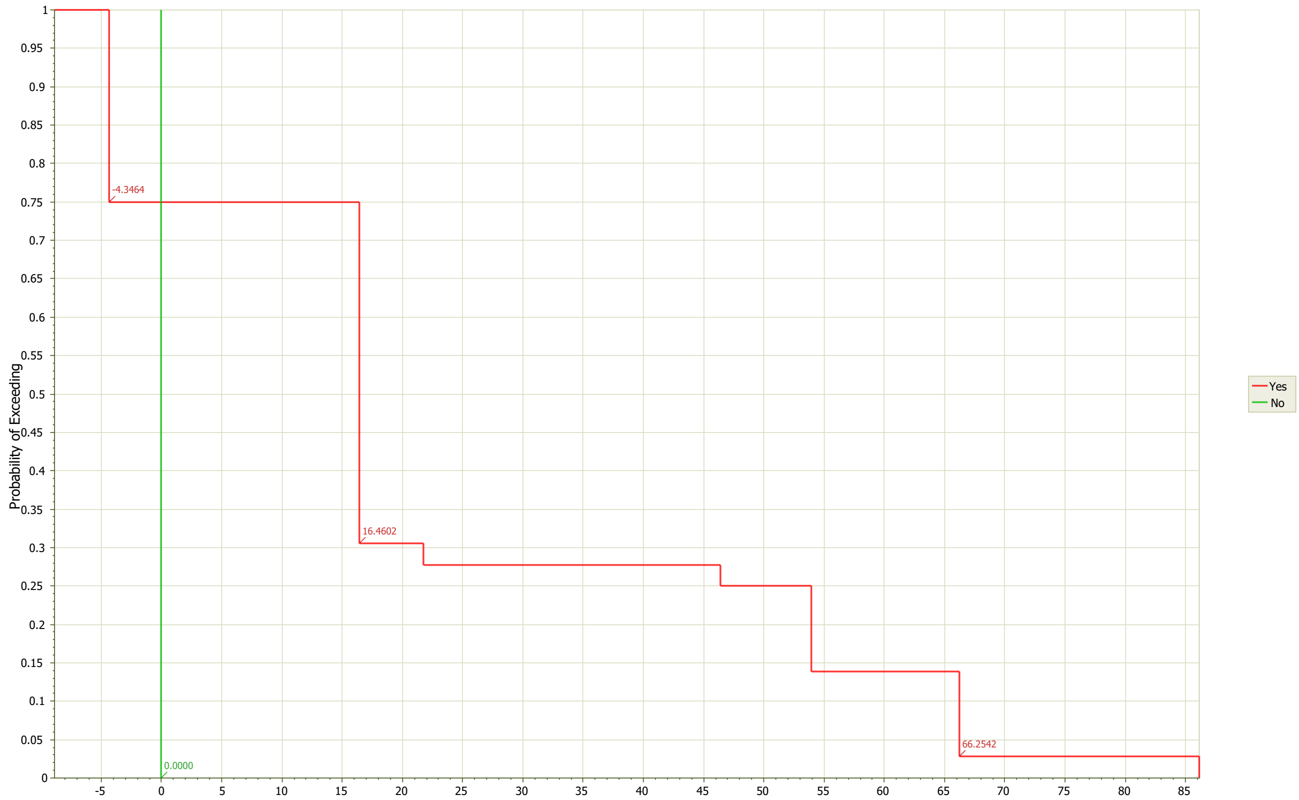
- Sort end-points by value and merge end-points with the same value. Compute their cumulative and excess probability.

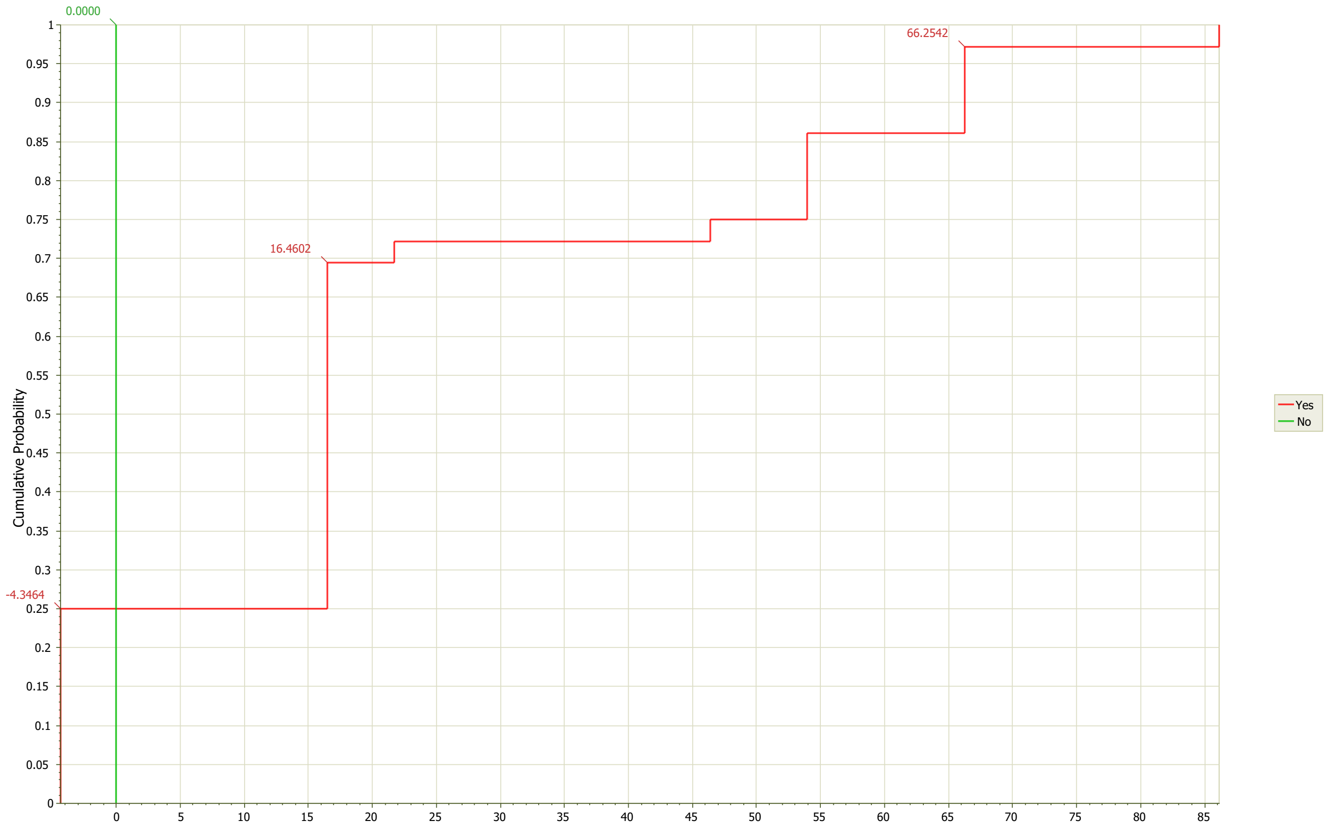
NPV $million	Probability	Cumulative Probability	Excess Probability
-4.3464393	0.25000	0.2500	0.7500
16.4602435	0.4444	0.6944	0.3056
21.7299698	0.0278	0.7222	0.2778
46.3808767	0.0278	0.7500	0.2500
53.9319554	0.1111	0.8611	0.1389
66.2542117	0.1111	0.9722	0.0278
86.1339410	0.0278	1.0000	0.0000

Alternative: "Do Not Invest"

Value $m	Probability	Cumulative Probability	Excess Probability
0	1.0000	1	0

Risk Profiles for the two initial alternatives under optimal decision policy.



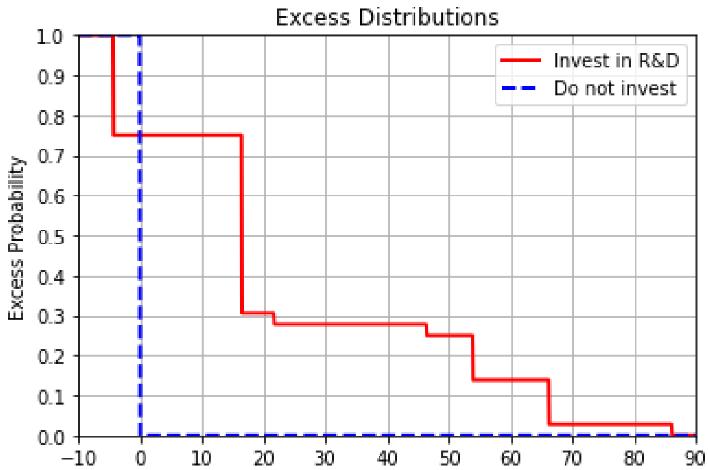


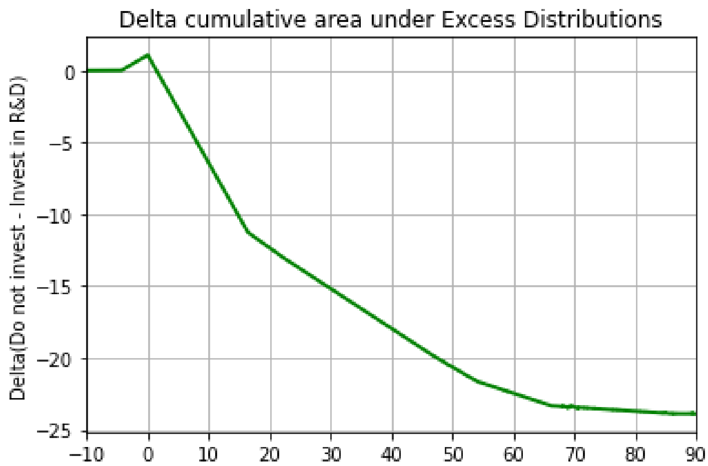
First-Order Stochastic Dominance Analysis

- There is no first-order stochastic dominance as the two risk profiles cross each other.

Second-Order Stochastic Dominance Analysis

- We observe that alternative "Do not Invest" may 2SD "Invest in R&D", but not via versa.
- It is also obvious that "Do not Invest" does not 2SD "Invest in R&D" by visual inspection of the risk profiles. There is only one cross-over point.
- The difference in cumulative areas under the excess distributions for "Do not invest" and "Invest in R&D" for all outcomes is given below:



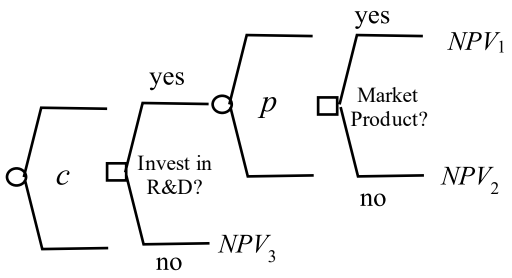


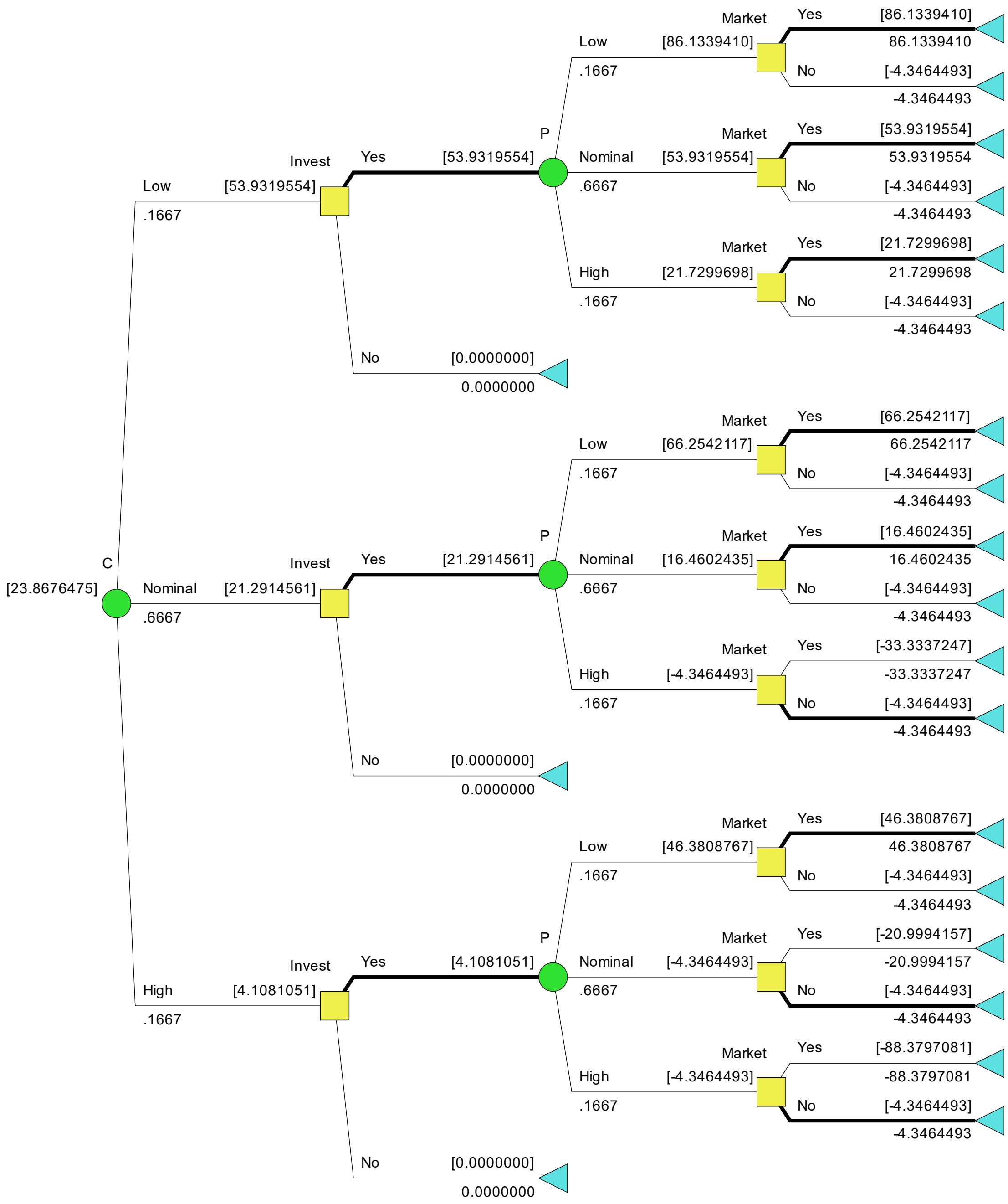
- At this stage, the optimal decision is to invest in R&D if Exxoff is risk neutral.
- But if Exxoff is not risk-neutral, we do not have enough information at this stage to eliminate any alternative without knowing the company's utility function.

Value of Information Analysis

EVPI Analysis for Production Cost

- Decision tree with free clairvoyance on Production Cost:

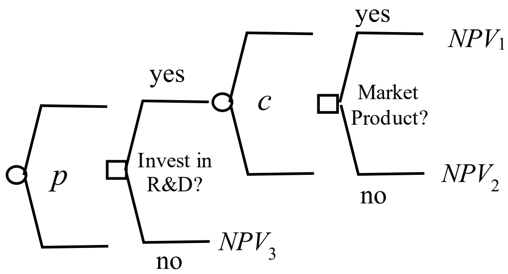


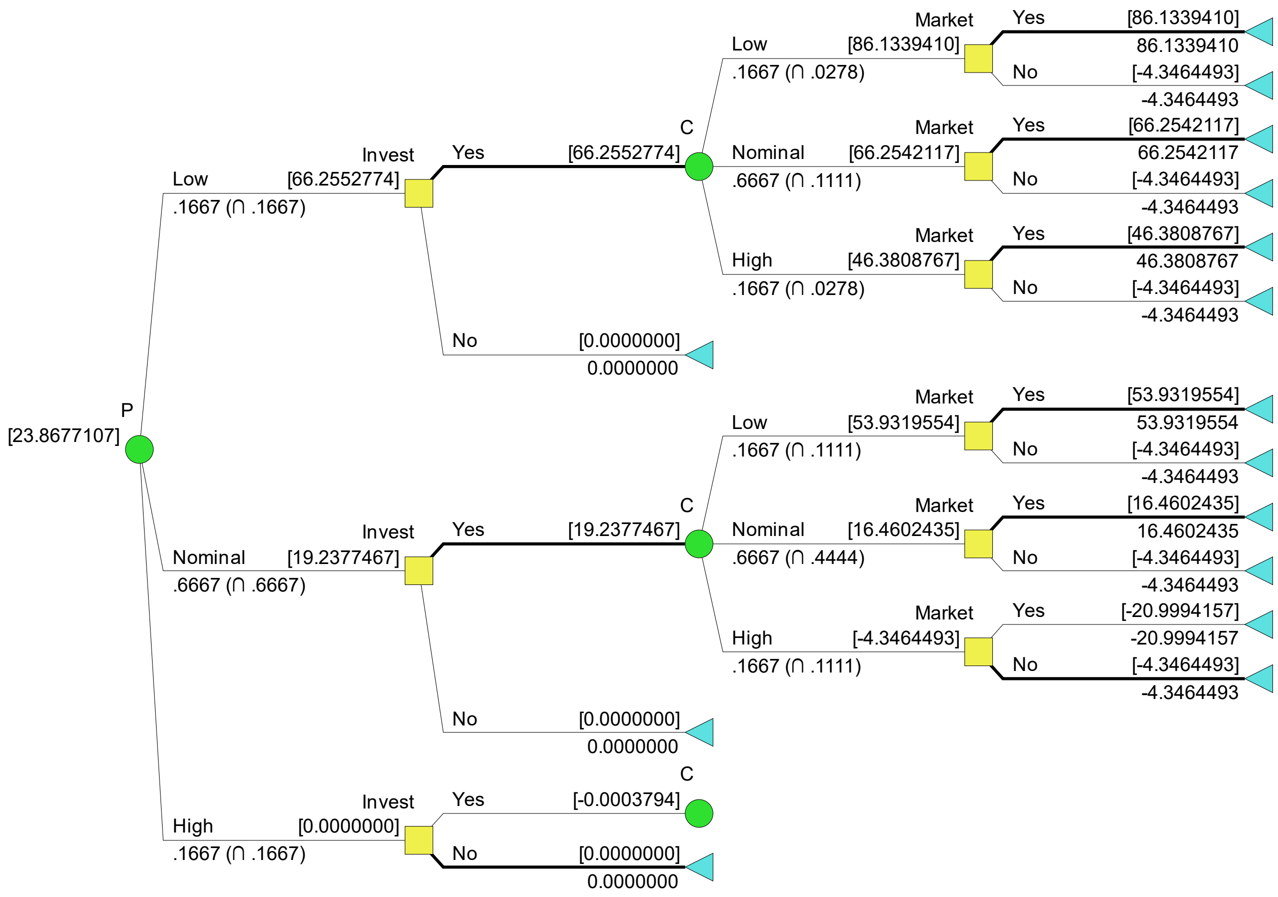


- Expected NPV with free information on Production Cost = $23.8676475 million
- Expected NPV with no information = $23.8676475 million
- EVPI for Production Cost = **$0**.

EVPI Analysis for Potency Index

- Decision tree with free clairvoyance on Potency Index:

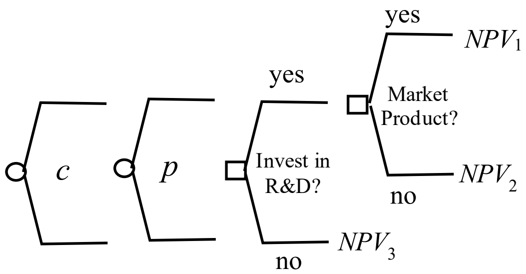


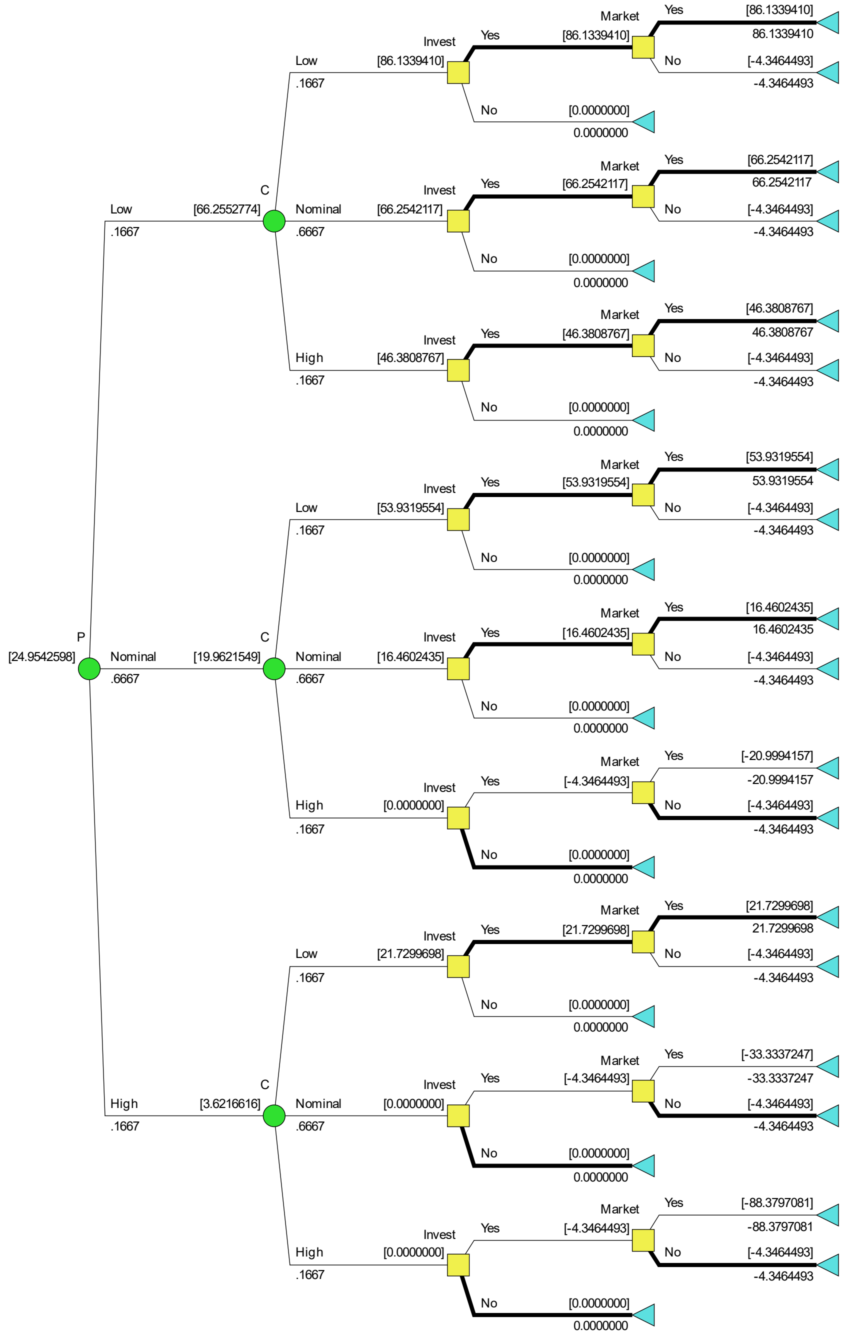


- Expected NPV with free information on Potency Index = \$23.8677107 million
- Expected NPV with no information = \$23.8676475 million
- EVPI for Potency Index = \$23.8677107 million – \$23.8676475 million
  = **$ 0.0000632 million**

Joint-EVPI Analysis on Production Cost and Potency Index

- Decision tree with free joint clairvoyance on Production cost Potency Index:





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- Expected NPV with free joint perfect information on Production Cost and Potency Index
  = \$24.9542598 million
- Expected NPV with no information = \$23.8676475 million
- Joint-EVPI on Production Cost and Potency Index
  = \$24.9542598 million – \$23.8676475 million = **\$1.0866123 million**

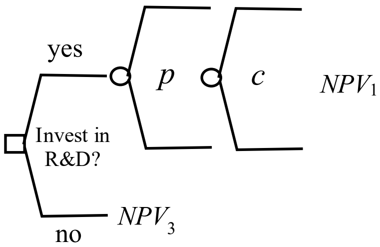
Value of Option to Terminate Project after R&D

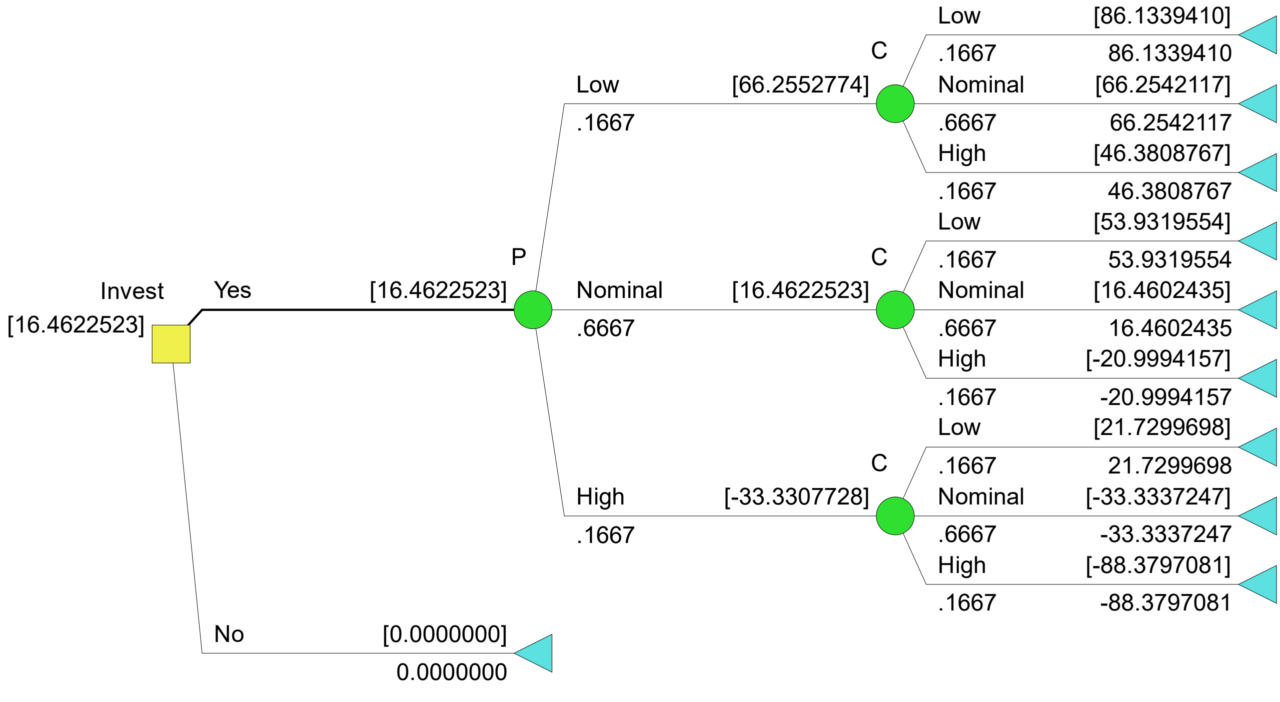
- Exxoff has the option to decide whether market or not to market the product after the R&D phase.

- This is known as an option. It is a right but not an obligation to carry out a business decision.

- There are two questions we want to address for Exxoff:
  1. What would Exxoff's R&D investment decision be if it did not have the option to terminate the project after the R&D result is known, but must always market the product?
  2. What is the value to Exxoff in having this option?

- Decision model with no option to terminate the project after R&D:





- Expected NPV without the option to terminate the project (i.e. always market) = \$16.4622523 million

- The decision is also to invest (for the risk-neutral case).

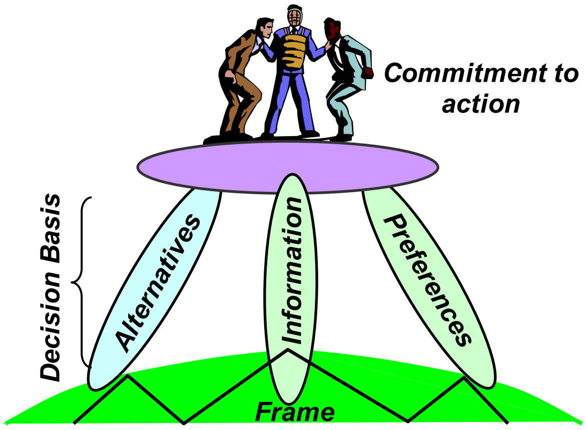
- Value of Option to Terminate after R&D
  = Expected *NPV* with Option – Expected *NPV* without Option
  = \$ 23.8676475 million – \$16.4622523
  = **\$ 7.4053952 million**

- If Exxoff did not have this option, it should be willing to spend additionally up to this amount to have this *managerial flexibility*.

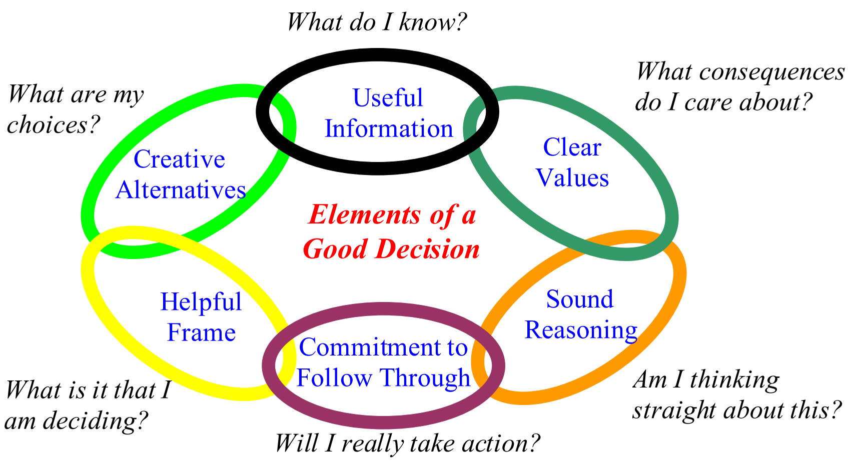
- Note that the above computational procedure requires the delta property.

Decision Appraisal

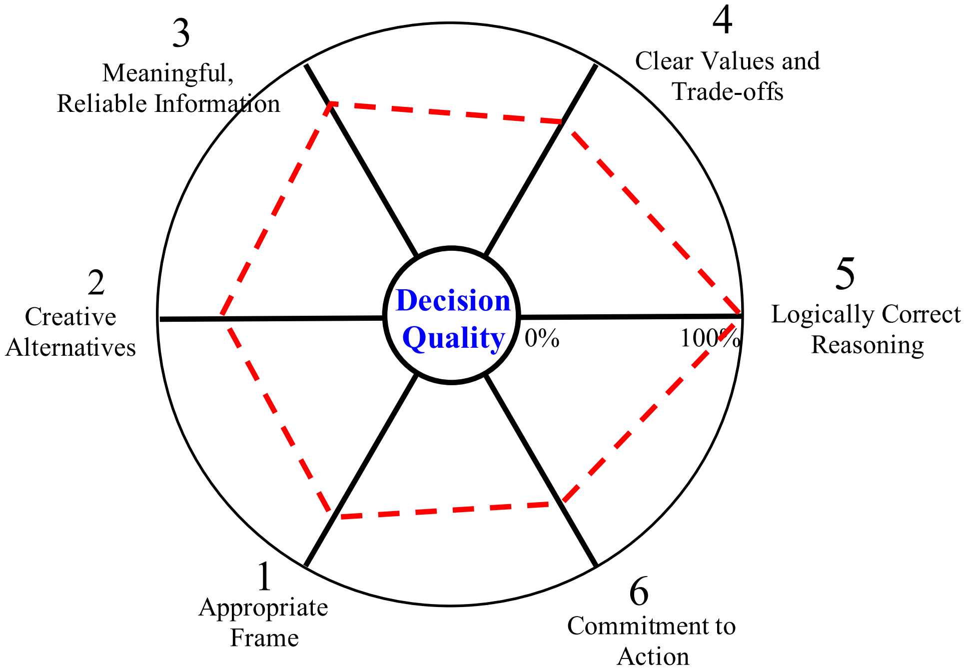
- The quality of any decision can be measured by the quality of six elements: the information, alternatives and preference of the decision maker, collectively called the decision basis, as well as the representative frame, the sound reasoning, and the commitment to follow through on the decision.



- The decision quality chain:



- To assess the overall decision quality, we can use a radar chart to display the score of each of the variables:

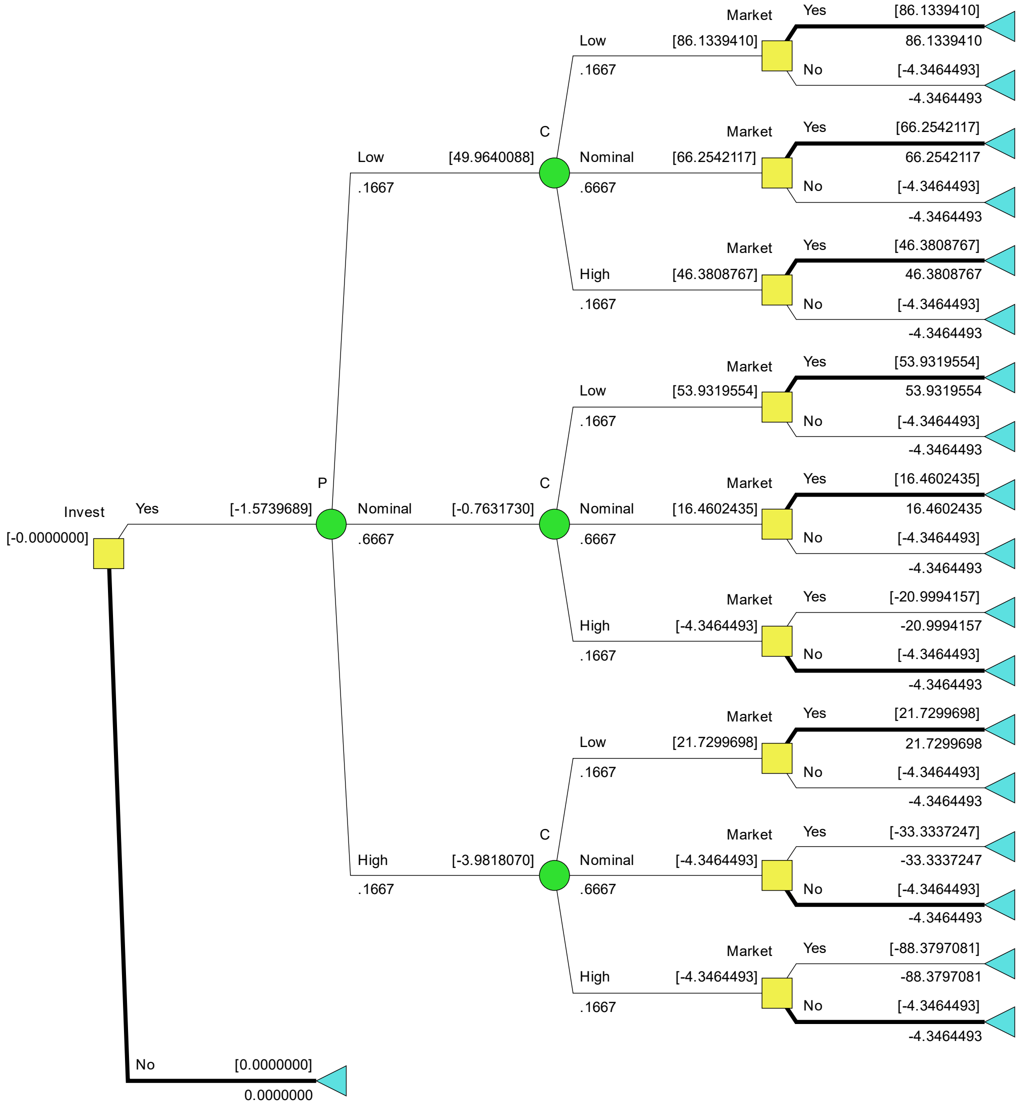


8.2.3 Second Pass: Considering Risk Aversion

- In the first pass through the DA cycle, we assumed the Exxoff is risk neutral, and based on the expected NPV of cash flows, the optimal decision was to “Invest in R&D”.

- If Exxoff is not risk-neutral, the result is inclusive as there is neither first nor second-order stochastic dominance.

- Assuming that Exxoff has an exponential utility function with a risk tolerance of $2 million, the following optimal decision policy (with CE values displayed) is obtained:



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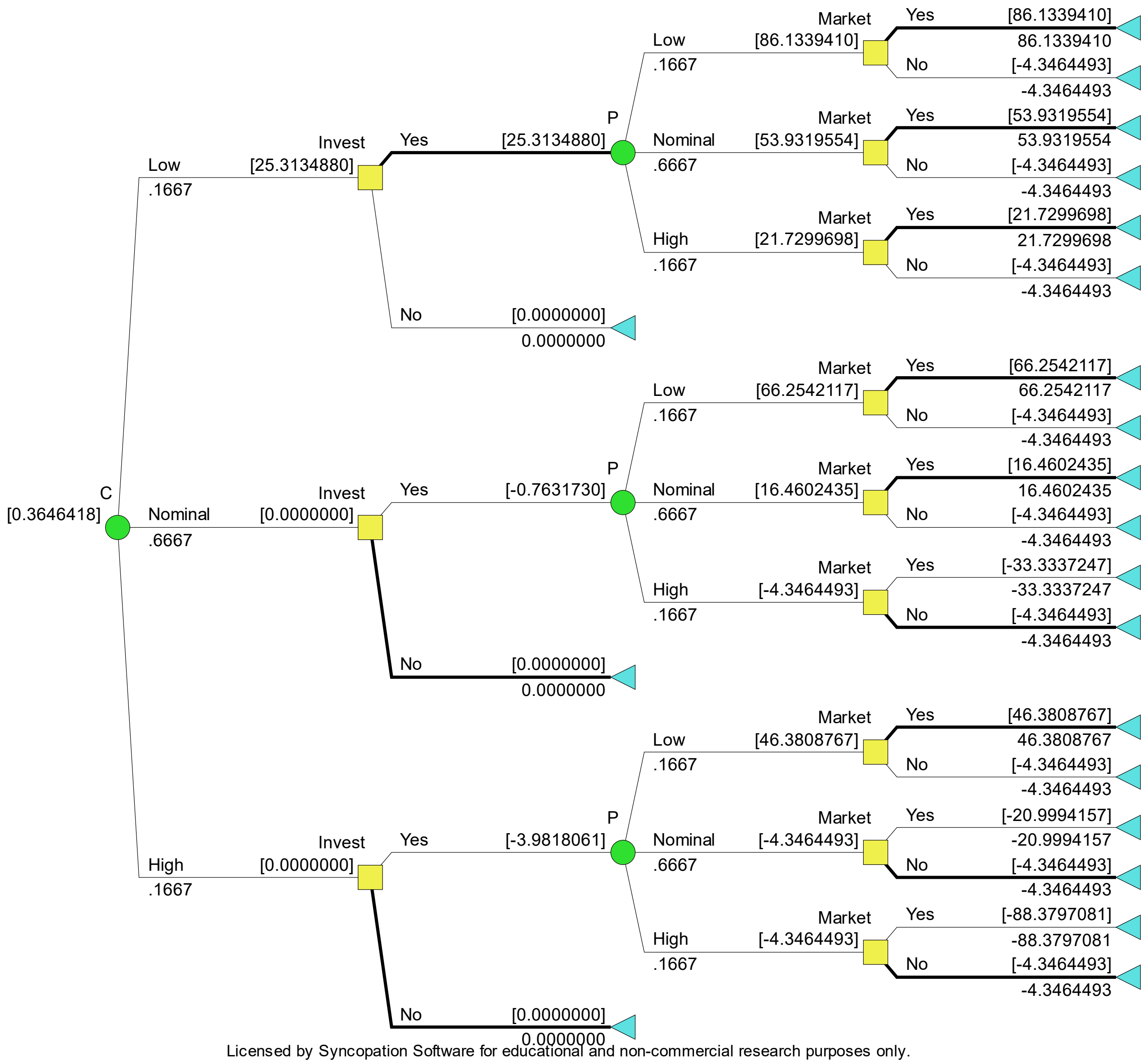
- Hence at a constant risk tolerance of $2 million, Exxoff should not invest in R&D.

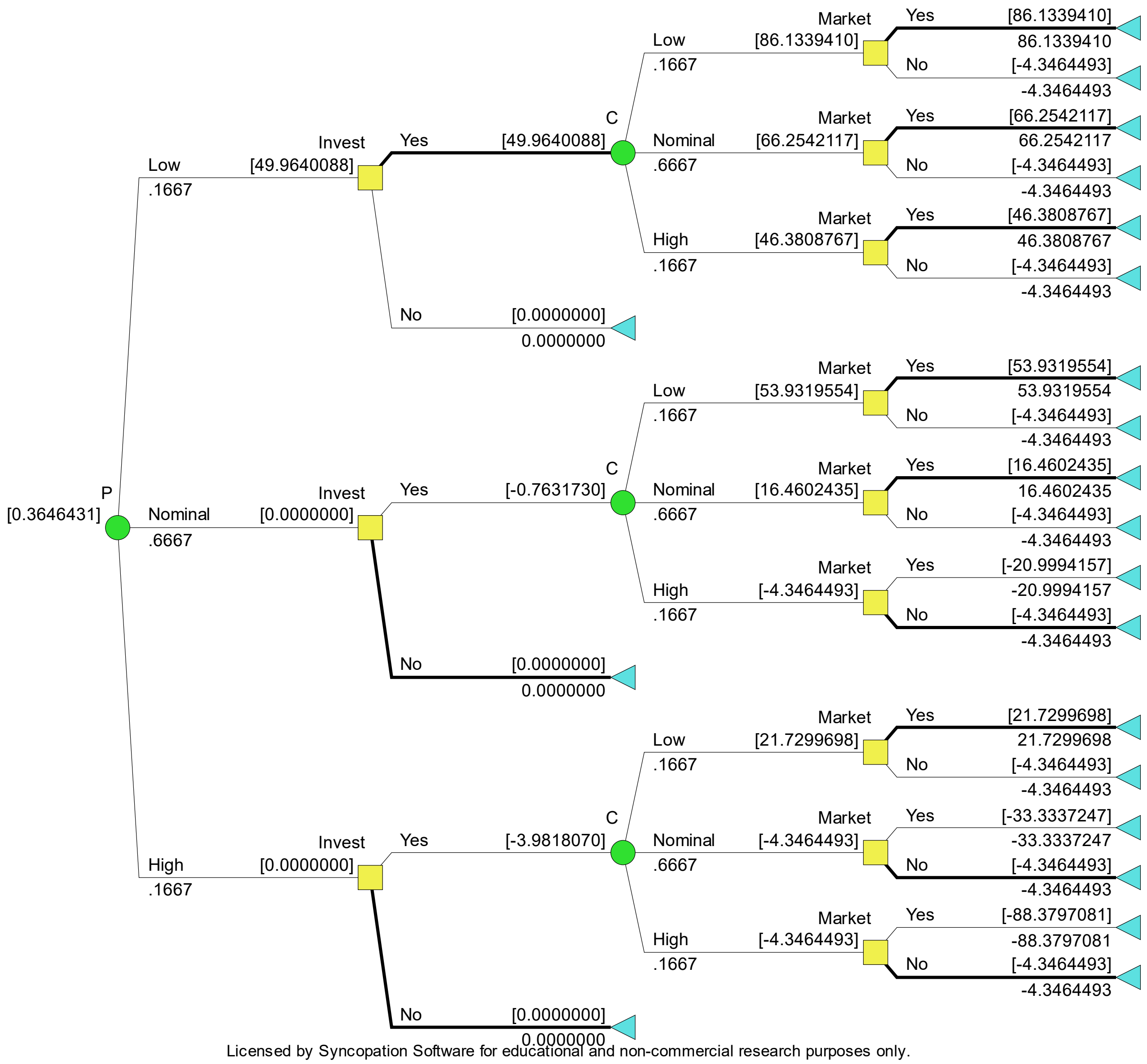
Value of Information Analysis under Risk Aversion (RT=$2 million)

Decision Model with Free Perfect Information on Production Cost



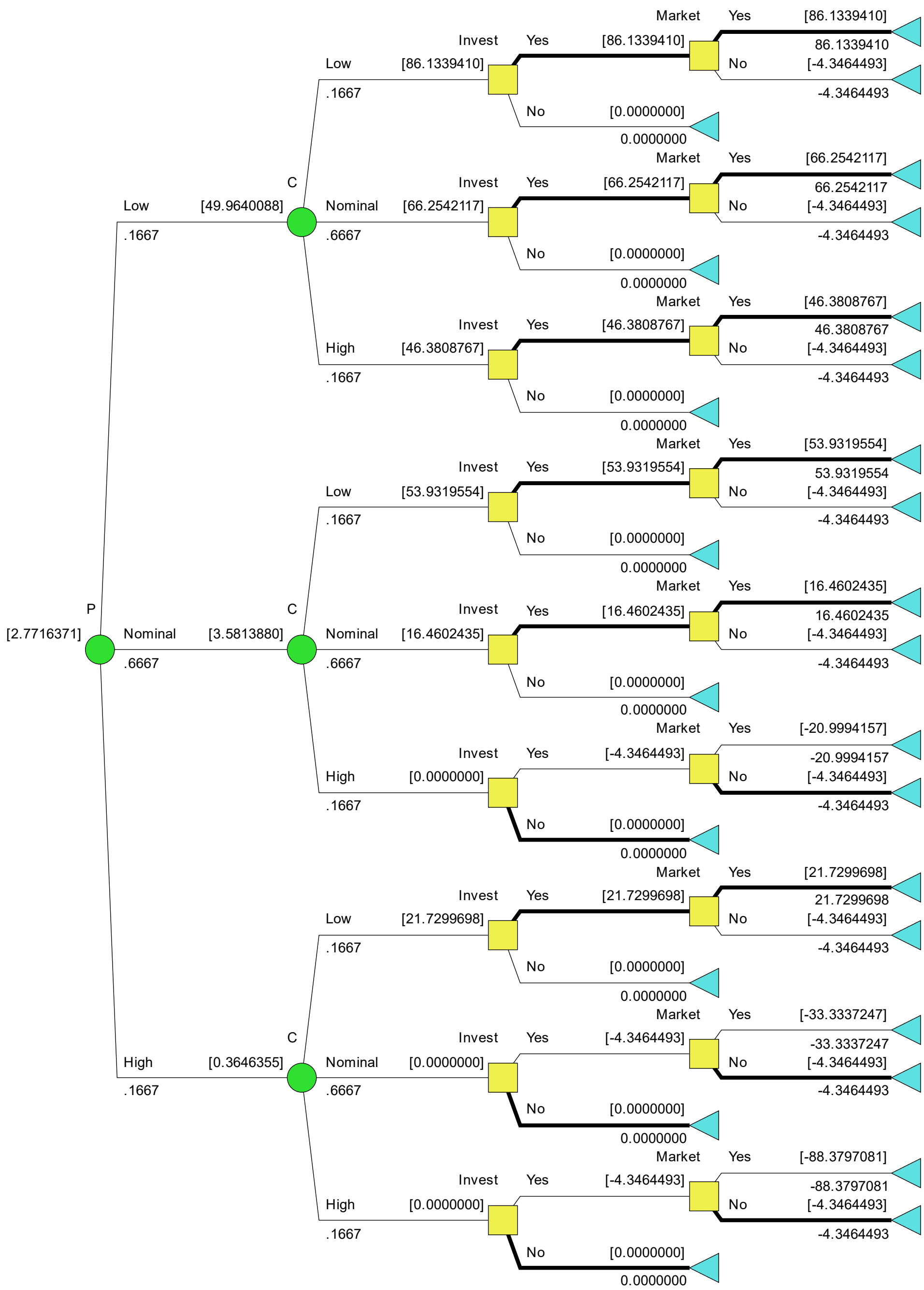
- Certainty equivalent with no information = $0.
- Certainty equivalent with free perfect information on Production Cost = $0.3646418 million
- EVPI on Production Cost at risk tolerance $2m = **$0.3646418 million**

Decision Model with Free Perfect Information on Potency Index



- Certainty equivalent with no information = $0.
- Certainty equivalent with free perfect information on Potency Index = $0.3646431 million
- EVPI on Potency Index at risk tolerance $2m = **<u>$0.3646431 million</u>**

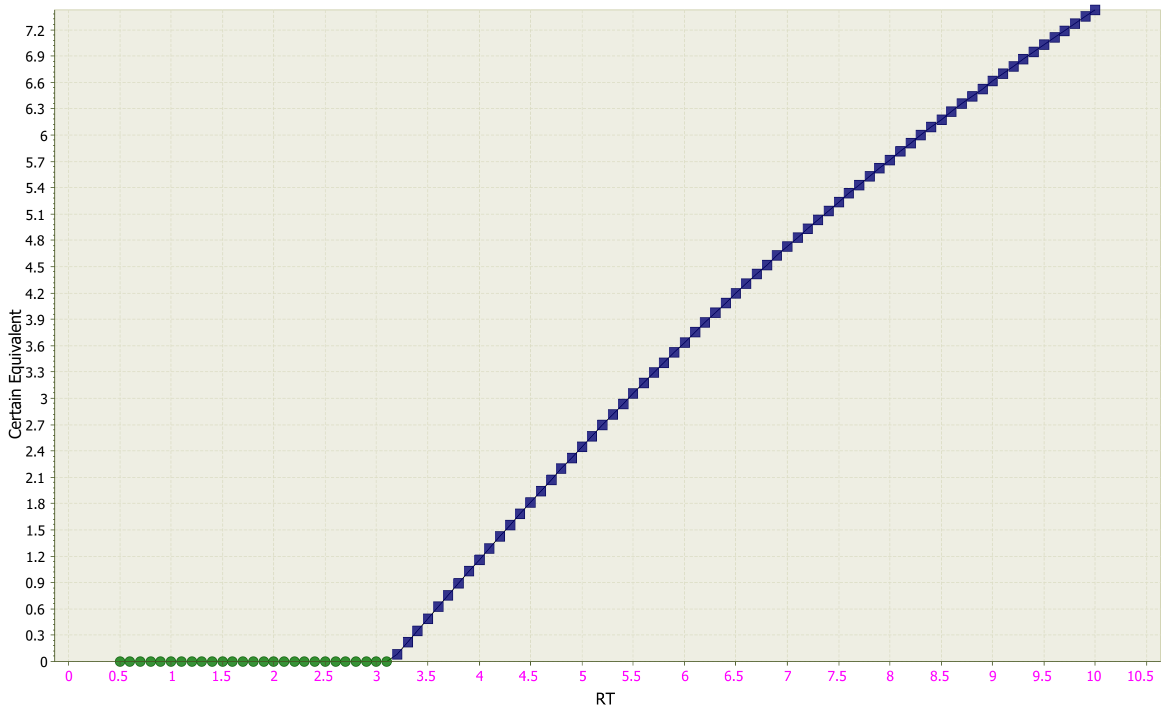
Decision Model with Free Joint Perfect Information on Product Cost and Potency Index



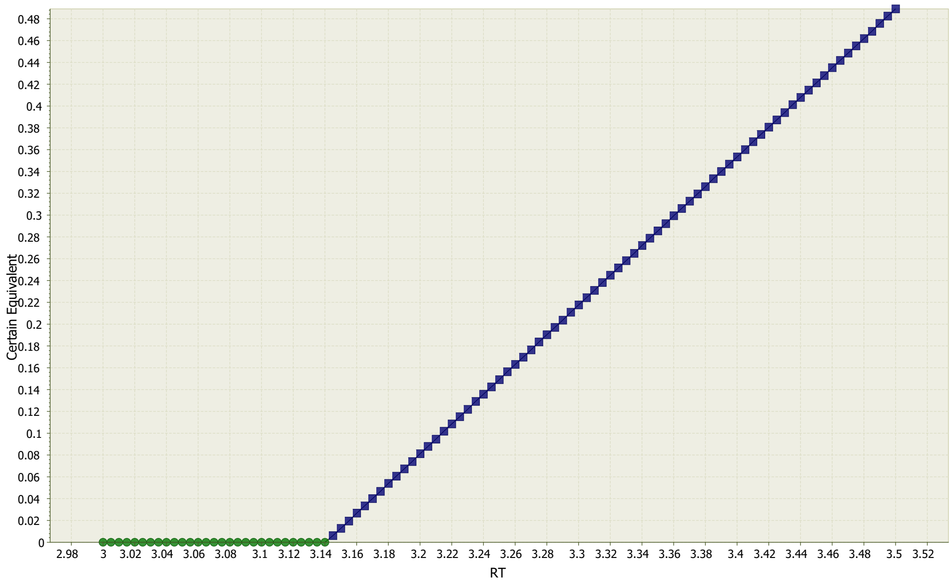
- Certainty equivalent with no information = $0.
- Certainty equivalent with free perfect joint information on Production Cost and Potency Index
  = $ 2.7716371 million
- Joint EVPI on Production Cost and Potency Index at risk tolerance $2m
  = **$ 2.7716371 million**

Sensitivity of Exxoff's Optimal Decision to Risk Tolerance:

- At a risk tolerance of $2 million, Exxoff's decision is not to Invest in R&D. We would like to find out the minimum risk tolerance needed before Exxoff would Invest in R&D.



- To get a more accurate break-even value, we vary the risk tolerance for Exxoff from $ 3 million to 3.5 million in smaller step sizes.



Exxoff's Investment Decision Rule based on Risk Tolerance

If 0 < Risk Tolerance ≤ $3.14 million
    Do Not Invest in the R&D
Else
    Invest in R&D

8.3 Using the DPL Software

- The DPL Software may be easily used in every step of the Decision Analysis Process/Cycle.

8.3.1 A Step-by-Step Guide

1. Deterministic Structuring

(a) Develop the **Value Models** for each alternative or strategy using only value nodes and their base-values. No decision tree is generated.

(b) Generate a **Tornado Diagram** for each alternative or strategy by specifying the extreme values (low and high) for all the uncertain variables. Use Sensitivity → Tornado → Value.

(c) Duplicate the last model and construct a **Deterministic Decision Model** by adding decision nodes. Place the decision strategy value at the end points of the deterministic decision tree generated. The optimal decision under deterministic base variables can be determined.

(d) Generate a **Combined Tornado** for all the initial alternatives using Sensitivity → Tornado → Value and don't forget to select "Initial Decision Alternatives" at the bottom of the dialog box. Identify any **Tornado Dominance** among the initial alternatives.

(e) Identify the **Sensitive and Insensitive Variables** from the Tornado Diagrams generated in Step 1(b).

2. Probabilistic Evaluation

(a) Duplicate the last model and create a **Probabilistic Decision Model** by converting all sensitivity variables from value nodes to discrete chance nodes. Drop any alternative that is tornado-dominated (Step 1(d)). Insensitive variables remain as value nodes for the rest of the analysis.

(b) **Solve** the probabilistic decision model and generate the **Optimal Decision Policy Tree**, **Value of Information/Control Chart**, and Combined **Risk Profiles** for initial alternatives.

(c) Identify any first or second-order **Stochastic Dominance** in the combined risk profiles. The end-point values of the optimal decision tree may be extracted from the End-Point Database and processed off-line.

(d) Perform Joint Value of Information Analysis by:

i. Duplicate the last model and **add information arcs** from the chance nodes to the decision node in the probabilistic decision model. Reorder the nodes in the decision tree built by DPL. Solve the model to determine the certainty equivalent with free perfect joint information.

ii. Compute the difference between the certainty equivalent with free joint information and the certainty equivalent with no information. Note that the Delta property is assumed here.

3. Sensitivity, What-If and Option Value Analyses

(a) Perform **Sensitivity Analysis or What-if Analyses** on parameters of interest by plotting **One-way** or **Two-way Rainbow Diagrams**.

(b) Perform **Option Value Analysis** (if there are down-stream decision nodes) by using Control on the branches of the decision node representing the option. Solve the model without options and compute the difference in the certainty equivalents with the case with option. Note that Delta property is assumed here.

4. Consideration of Risk Preferences

(a) Consider aversion to risk by introducing a utility function in the model using either the built-in exponential function (specify the risk tolerance) or a user-defined utility function. Determine the optimal decision policy and value of information/control under risk aversion.

(b) Perform expected value of joint perfect information analysis by the steps described in 2(d).

(c) Perform sensitivity analysis on the risk preference parameters such as Risk Tolerance by plotting rainbow diagrams and locate their break points. Note that the DPL default exponential utility function requires a positive risk tolerance (i.e. risk-averse only). For risk-seeking cases, use the user-defined utility function with negative risk tolerance.

(d) Repeat Steps 3(a) and 3(b).

8.3.2 Modeling and Solving the Exxoff Problem using DPL

- See instructional video on Canvas for a step-by-step tutorial.

8.4 Case Study 2: Large Instrument Manufacturer Problem

8.4.1 Problem Description

Large Instrument Manufacturer, Inc. (LIM) is the producer of the ROE-1, a sophisticated research instrument. The government of UDC is among those that have purchased several ROE-1's. Two years ago, UDC brought the ROE-1's for $77,000 each and installed them throughout the country. Unfortunately, UDC has had difficulties maintaining the proper operation of the ROE-1's.

LIM's management is concerned about the large loss from the sale of ROE-1's to UDC. In reviewing the business plan for the ROE-2 (the follow-up version of the ROE-1), LIM is considering several servicing alternatives. The first alternative is to continue with the present servicing arrangement. This means that LIM itself must send trained service personnel to UDC. The next alternative is to train UDC engineers so that they can repair the machines on their own. However, since not all breakdowns can be repaired by the user, LIM would still have to send its service personnel to UDC periodically. LIM can also sign a contract with IPX, an independent service firm. The contract would cover any type of breakdown that might occur.

LIM's warranty period is limited to five years. Any breakdowns that occur beyond the warranty period must be repaired at the expense of the purchaser. LIM is interested in maximizing the net present value (NPV) of profits. LIM's CEO has instructed you to use a 3% discount rate and to assume risk neutrality. The sales price of the ROE-2 is $100,000. The cost of manufacturing, marketing, and installing an ROE-2 is $70,000.

LIM's experts have thought carefully about a few uncertain variables: the number of machines LIM can sell to UDC during the next year; the average service cost per machine per year during the five-year warranty period which depends on the servicing alternative selected; the training cost per machine sold.

All machine failures are one of two types: alpha or beta. The percentage of machines per year that will experience an alpha or beta failure is uncertain. The average costs for LIM to repair alpha and beta failures are two more uncertainties.

If the "train for self-maintenance" servicing alternative is selected, LIM will only be liable for alpha failures. However, it will have to include a supply of parts and servicing equipment with each unit sold and trains the users to fix beta failures. The parts and servicing equipment per machine sold will cost $10,000. If the service contract is signed, LIM will pay IPX $750,000 or 23% of the sales price for every ROE-2 sold in UDC during the next year, whichever is greater. Of course, LIM could also not sell ROE-2's to UDC. Since this would adversely impact the sales of LIM's other products, LIM considers the loss associated with this alternative to be $25,000.

Assume a study period of 5 years.

The company's weighted average cost of capital (*WACC*) or *MARR* is 3%.

8.4.2 First Pass through the DA Cycle

LIM's Decision Basis

Decision Variables:
- Marketing Decision
  - Sell in UDC
  - Not sell in UDC

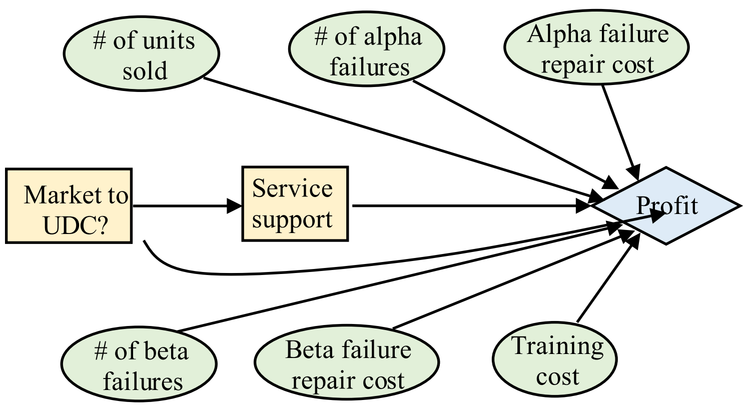
- Service Support
  - Present arrangement
  - Train users for some self-maintenance
  - Contract IPX

State or System Variables:
- number of units sold: n
- average cost to repair alpha mode failure: c_α
- average fraction of alpha failures/machine/year: α
- average cost to repair beta mode failure: c_β
- average fraction of beta failures/machine/year: β
- training cost per machine sold: τ.

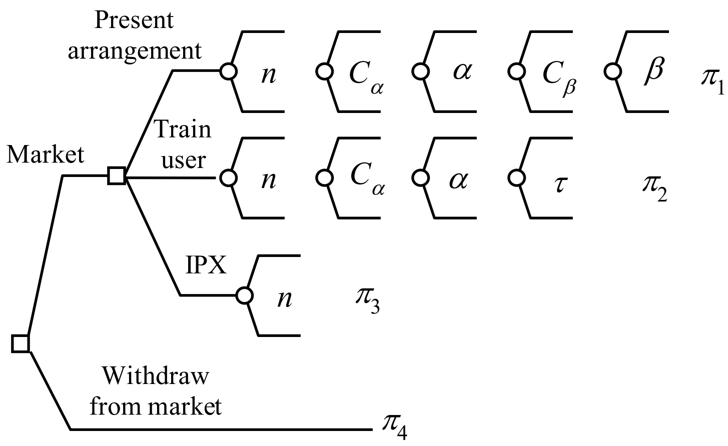
Preference and Value:
- NPV of profits at a 3% discount rate.
- Risk neutral for preliminary analysis.

Influence Diagram

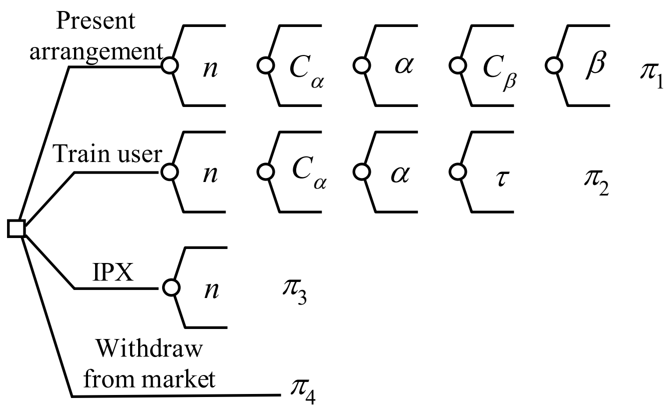
- The Influence Diagram for the problem:



Generic Decision Tree



Equivalent Decision Tree with four initial decision alternatives

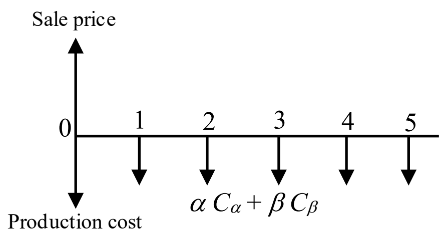


Value (Business) Model

- Net present value factor $[P/A, 3\%, 5] = \sum_{i=1}^{5} \frac{1}{(1+0.03)^i} = 4.579072$

  Alternative I: Present Arrangement

  Cash flow diagram for every unit sold:



Value Model:

$$\pi_1 = n\,[\,100 - 70 - 4.579072\,(\alpha C_\alpha + \beta C_\beta)\,] \times 10^3$$
$$= n\,[\,30 - 4.579072\,(\alpha C_\alpha + \beta C_\beta)\,] \times 10^3$$

Engineering Economy notation:

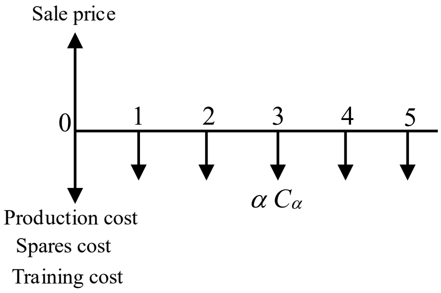
$$\pi_1 = n\,[\,100 - 70 - (\alpha C_\alpha + \beta C_\beta)\,[P/A, 3\%, 5\,]\,] \times 10^3$$

DPL value expression:

$$\pi_1 = (n * (100 - 70 - @\mathrm{pv}(\alpha C_\alpha + \beta C_\beta, 0.03, 5)))*1000$$

Alternative II: Train users for some self-maintenance

Cash flow diagram for every unit sold:



Value Model:

$$\pi_2 = n\,[\,100 - 70 - 10 - \tau\, - 4.579072\ \alpha\, C_\alpha] \times 10^3$$
$$= n\,[\,20 - \tau\, - 4.579072\ \alpha\, C_\alpha\,] \times 10^3$$

Engineering Economy notation:

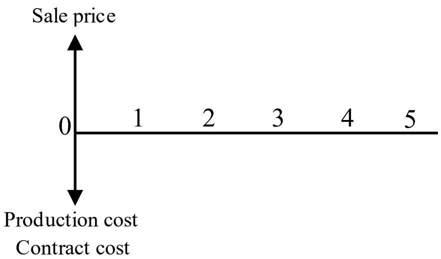
$$\pi_2 = \; n\,[\,100 - 70 - 10 - \tau\; - \;\alpha\, C_\alpha\,[P/A, 3\%, 5\,]\,] \times 10^3$$

DPL value expression:

$$\pi_2 = \; (n\ {*}\,(100 - 70 - 10\; - \tau\, - @\text{pv}(\alpha\, C_\alpha, 0.03, 5))){*}1000$$

Alternative III: Contract IPX

Cash flow diagram:



Value Model:

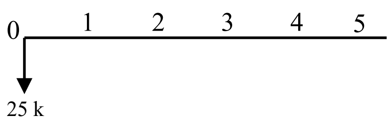
$$\pi_3 = [(100 - 70)\, n - \max[750, (0.23)(100)n]\,] \times 10^3$$
$$= [30n - \max[750, 23\, n]\,] \times 10^3$$

DPL value expression:

$$\pi_3\; = (30{*}n - @\max(750, 23\, {*}n)){*}1000$$

Alternative IV: Withdraw from UDC market

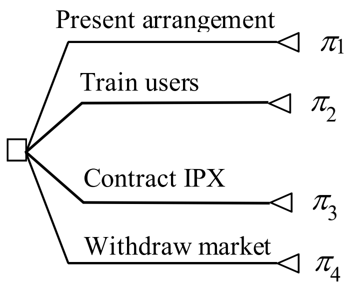
Cash flow diagram:



Value Model:

$$\pi_4\; = \; -25 \times 10^3.$$

Deterministic Decision Model



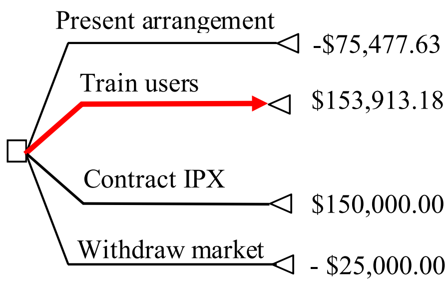
- This model is useful for generating combined tornados diagram in DPL later.

Deterministic Base-Value Solutions

- The base or nominal values for each of the variables are assessed by the company's experts:

Variable	Base value
Average repair cost, alpha failure ($K)	10
Average repair cost, beta failure ($K)	8.0
Alpha failures/year (fraction of machines)	0.15
Beta failures/year (fraction of machines)	0.70
Number of machines sold	30
Training cost per machine ($K)	8

- The base-value NPV for the alternatives are:



	Alternative	Base case NPV ($)
I	Present arrangement	-75,477.63
II	Train user	**153,913.18**
III	Contract IPX	150,000.00
IV	Withdraw from market	-25,000.00

- The optimal base-value solution is Alternative II "Train User", but Alternative III "Contract IPX" is close.

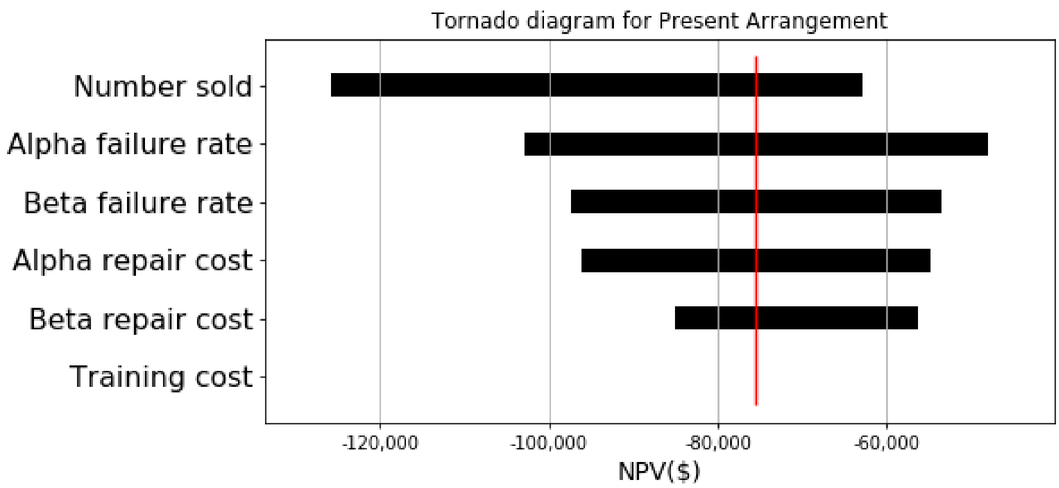
Deterministic Sensitivity Analysis

- The low, base and high values for all the variables are assessed by the company experts:

Variable	Low	Base	High
Average repair cost, alpha failure ($K)	9	10	11
Average repair cost, beta failure ($K)	7.8	8.0	8.1
Alpha failures/year (fraction of machines)	0.13	0.15	0.17
Beta failures/year (fraction of machines)	0.68	0.70	0.72
Number of machines sold	25	30	50
Training cost per machine ($K)	4	8	10

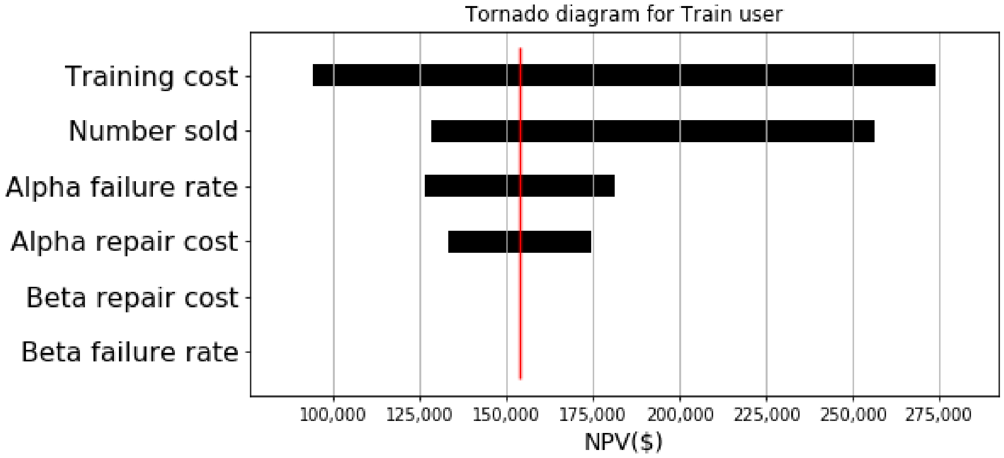
The One-Way Range Sensitivity Analysis on "Present Arrangement"

Variable	Variable value			π_1 ($)		
	Low	Base	High	One extreme	Other extreme	Swing
n	25	30	50	-62,898.03	-125,796.05	62,898.03
C_α	9	10	11	-54,868.95	-96,086.31	41,217.36
α	0.13	0.15	0.17	-47,999.39	-102,955.87	54,956.49
C_β	7.8	8.0	8.1	-56,242.86	-85,095.02	28,852.16
β	0.68	0.70	0.72	-53,495.04	-97,460.23	43,965.19
τ	4	8	10	-75,477.63		0



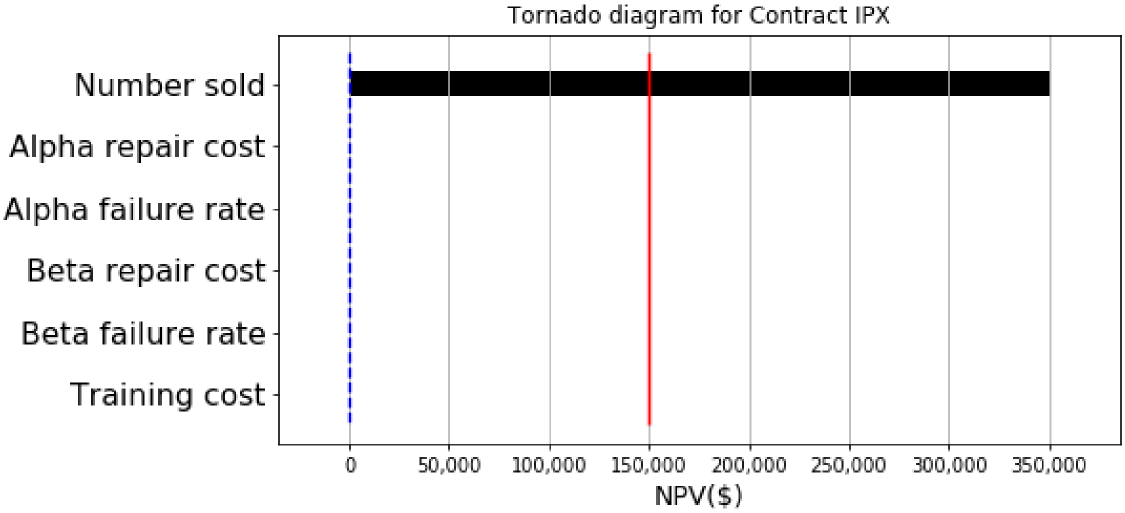
The One-Way Range Sensitivity Analysis on "Train Users"

Variable	Variable value			π_2 ($)		
	Low	Base	High	One extreme	Other extreme	Swing
n	25	30	50	128,260.98	256,521.96	128,260.98
C_α	9	10	11	174,521.86	133,304.49	41,217.36
α	0.13	0.15	0.17	181,391.42	126,434.93	54,956.49
C_β	7.8	8.0	8.1	153,913.18		0
β	0.68	0.70	0.72	153,913.18		0
τ	4	8	10	273,913.18	93,913.18	180,000.00



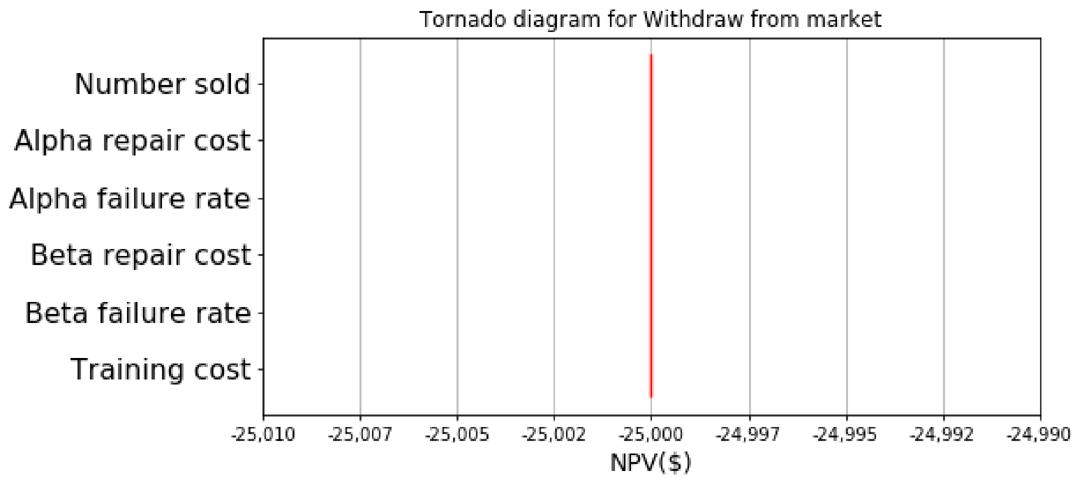
The One-Way Range Sensitivity Analysis on "Contract IPX"

Variable	Variable value			π_3 ($)		
	Low	Base	High	One extreme	Other extreme	Swing
n	25	30	50	0	350,000.00	350,000.00
C_α	9	10	11	150,000.00		0
α	0.13	0.15	0.17	150,000.00		0
C_β	7.8	8.0	8.1	150,000.00		0
β	0.68	0.70	0.72	150,000.00		0
τ	4	8	10	150,000.00		0

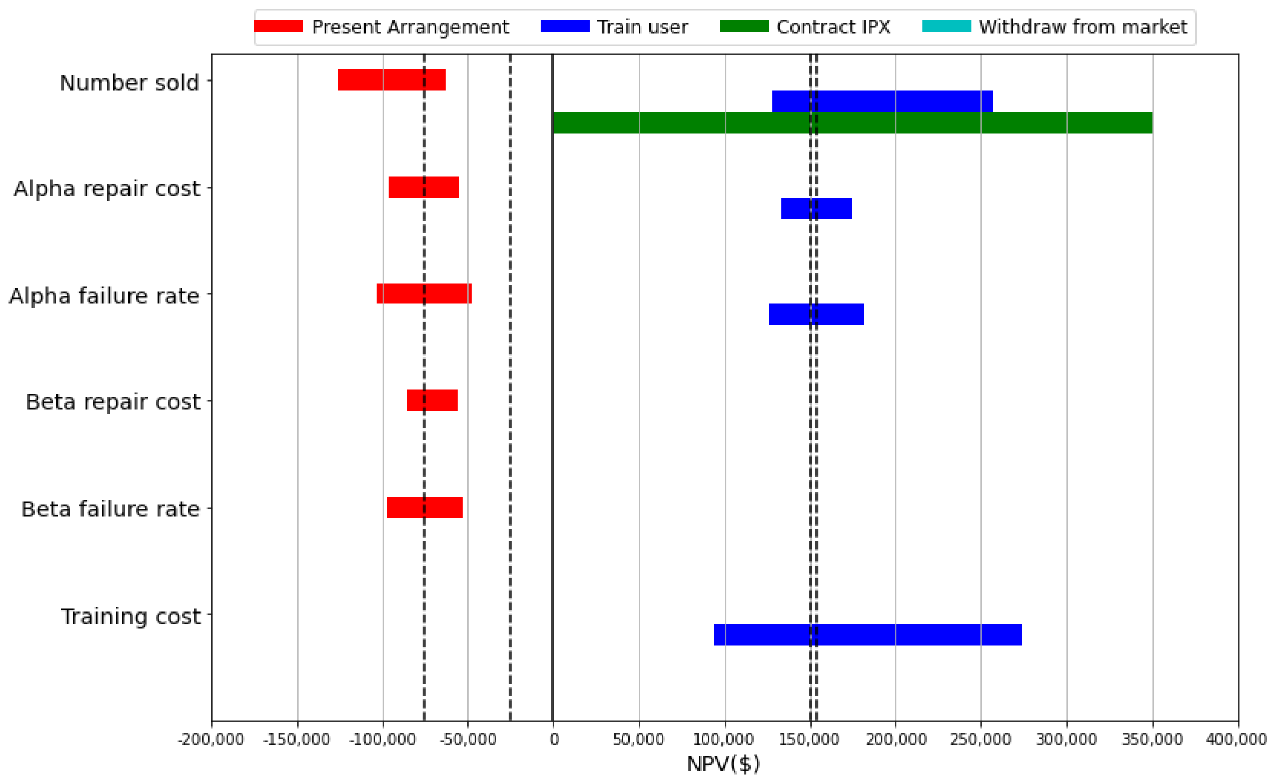


The One-Way Range Sensitivity Analysis on "Withdraw from the Market"

Variable	Variable value			π_4 ($)		
	Low	Base	High	One extreme	Other extreme	Swing
n	25	30	50	-25,000.00		0
C_α	9	10	11	-25,000.00		0
α	0.13	0.15	0.17	-25,000.00		0
C_β	7.8	8.0	8.1	-25,000.00		0
β	0.68	0.70	0.72	-25,000.00		0
τ	4	8	10	-25,000.00		0



Combined Tornado Diagrams



Identification of Sensitive Variables

- **Sensitive Variables**: n (number of units sold) and τ (Training cost).

- Hence only n and τ will be given full probabilistic assessments in the next phase.

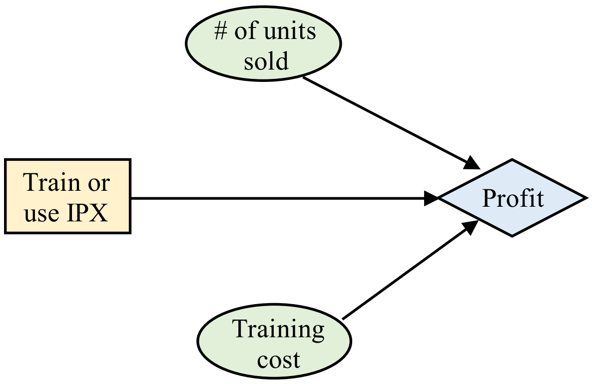
- The rest of the variables will be fixed at their nominal values.

Tornado Dominance Analysis

- From the combined tornado diagram:
  - II (Train user) dominates I (Present arrangement)
  - II (Train user) dominates IV (Withdraw from the market)
  - III (Contract IPX) dominates I (Present arrangement)
  - III (Contract IPX) dominates IV (Withdraw from the market)
  - IV (Withdraw from the market) dominates I (Present arrangement)

- Conclusion: We can drop alternatives I and IV from further consideration.

The Simplified Influence Diagram

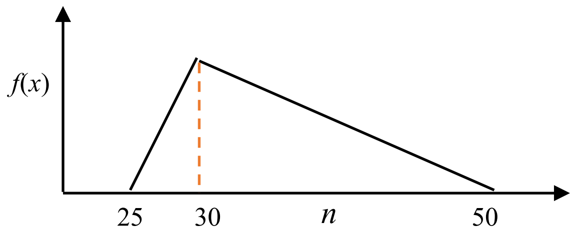


Probability Assessment for Sensitive Variables

- The experts at LIM assessed the probability distributions for the Number of units sold and the Training cost per machine as follows:

Variable	Distribution	Parameters	Unit
n	Triangular	Min=25, Mode=30, Max =50	Pieces
τ	Triangular	Min=4, Mode=8, Max =10	$k

Discretization of distribution for *n*



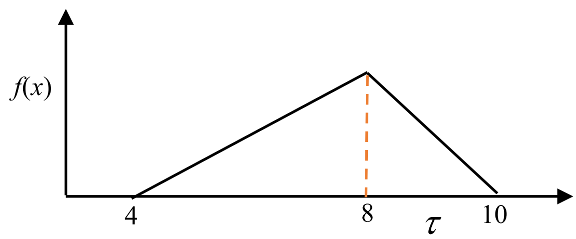
- Shape parameter = (30 – 25)/(50 – 25) = 0.2.
- From Chapter 7, the 3-branch discrete approximation for Triangular(0, 1, 0.2) is

Branch	Value	Probability
1	0.15441205	0.34663662
2	0.44738893	0.50021458
3	0.80108143	0.15314880

The 3-branch discrete approximation for Triangular(25, 30, 50) is

Branch	Value	Probability
1	25 + 0.15441205 (50 – 25) = 28.860301	0.34663662
2	25 + 0.44738893 (50 – 25) = 36.184723	0.50021458
3	25 + 0.80108143 (50 – 25) = 45.027036	0.15314880

Discretization of distribution for τ



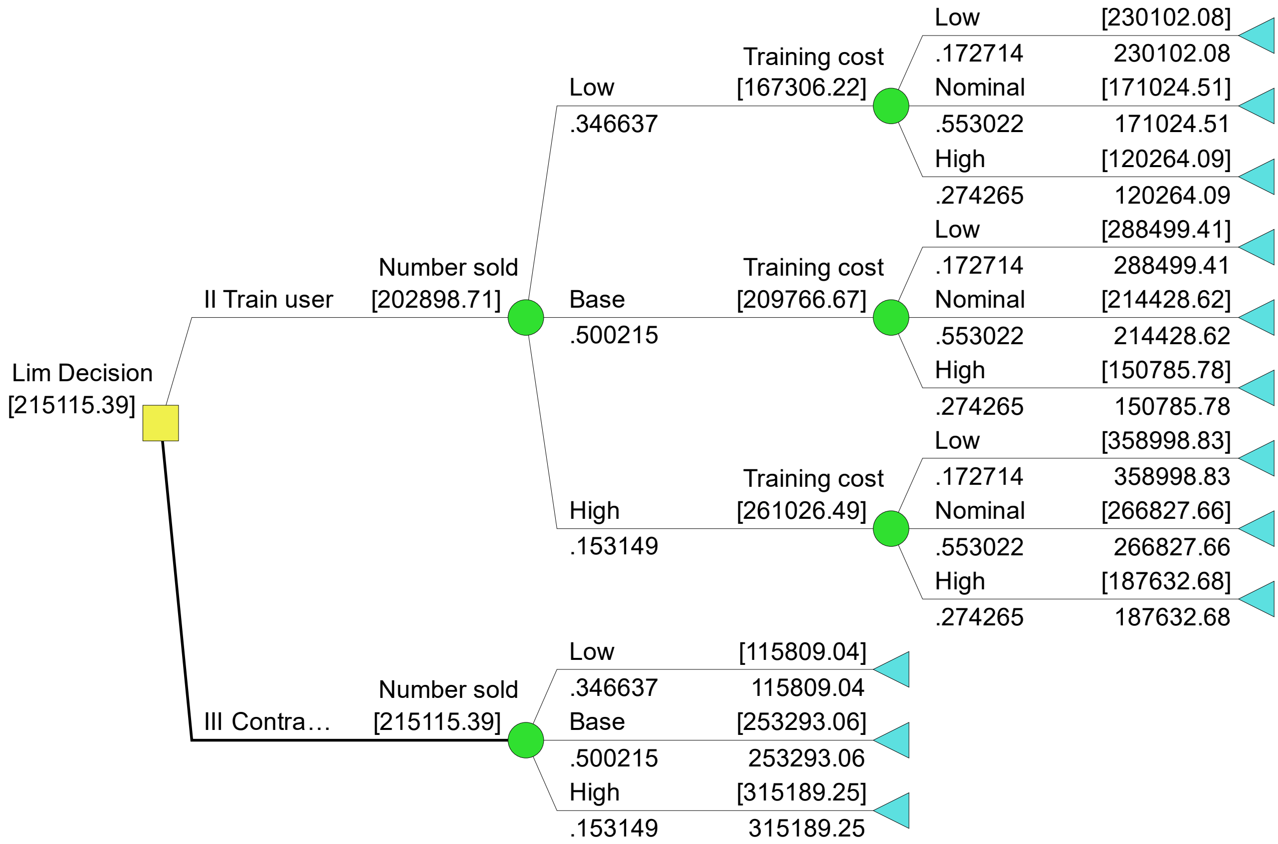
- Shape parameter = (8 – 4)/(10 – 4) = 2/3
- From Chapter 7, the 3-branch discrete approximation for Triangular(0, 1, 2/3) is

Branch	Value	Probability
1	0.1929129	0.17271373
2	0.5340827	0.55302152
3	0.8272213	0.27426476

The 3-branch discrete approximation for Triangular(4, 10, 8) is

Branch	Value	Probability
1	4 + 0.1929129 (10 – 4) = 5.157478	0.17271373
2	4 + 0.5340827 (10 – 4) = 7.204496	0.55302152
3	4 + 0.8272213 (10 – 4) = 8.963328	0.27426476

The Decision Tree



- The **Optimal Decision** is Alternative III, i.e., Contract IPX.
- Certainty Equivalent = $215,115.39

Plotting the Risk Profiles

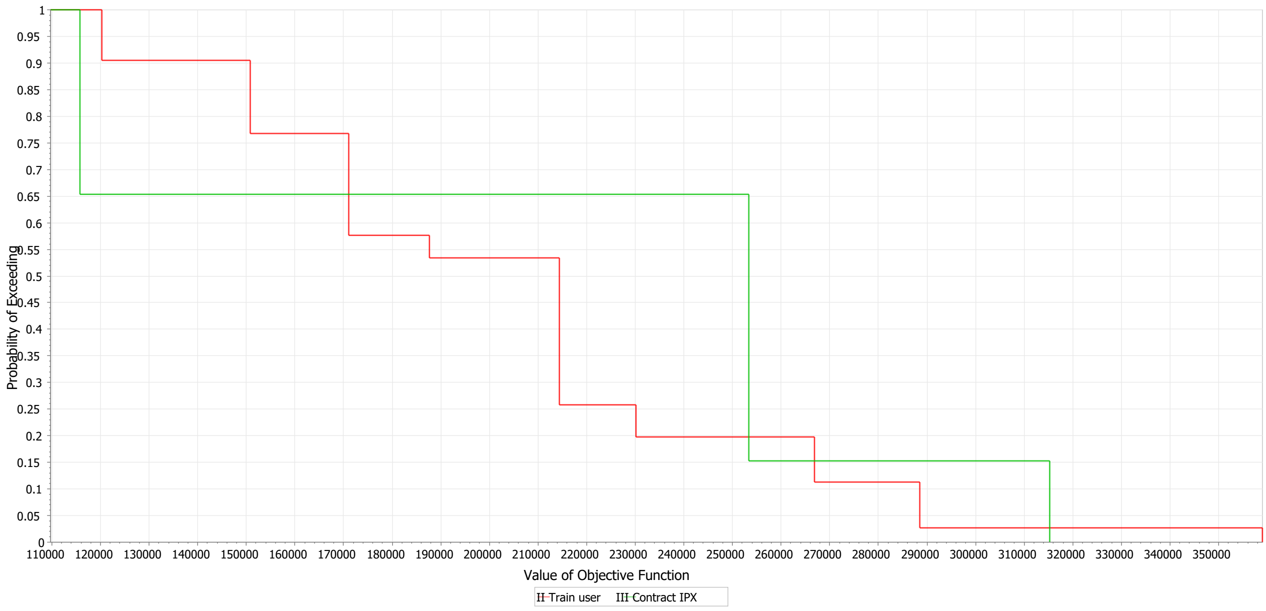
Alternative II (Train user)

	NPV ($)	Probability	Cumulative Probability	Excess Probability
1	120,264.09	0.095070	0.095070	0.904930
2	150,785.78	0.137191	0.232261	0.767739
3	171,024.51	0.191698	0.423959	0.576041
4	187,632.68	0.042003	0.465962	0.534038
5	214,428.62	0.276629	0.742591	0.257409
6	230,102.08	0.059869	0.802460	0.197540
7	266,827.66	0.084695	0.887155	0.112845
8	288,499.41	0.086394	0.973549	0.026451
9	358,998.83	0.026451	1	0

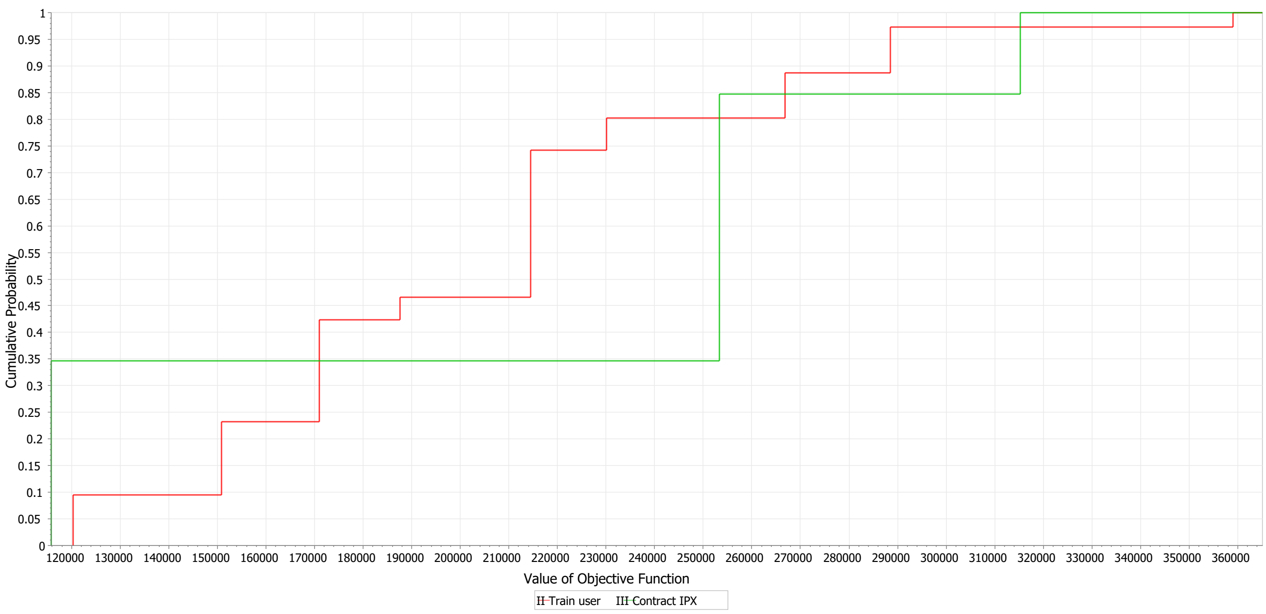
Alternative III (Contract IPX)

	NPV ($)	Probability	Cumulative Probability	Excess Probability
1	115,809.04	0.346637	0.346637	0.653363
2	253,293.06	0.500215	0.846852	0.153148
3	315,189.25	0.153149	1	0

Risk Profiles in Excess Probability



Risk Profiles in Cumulative Probability

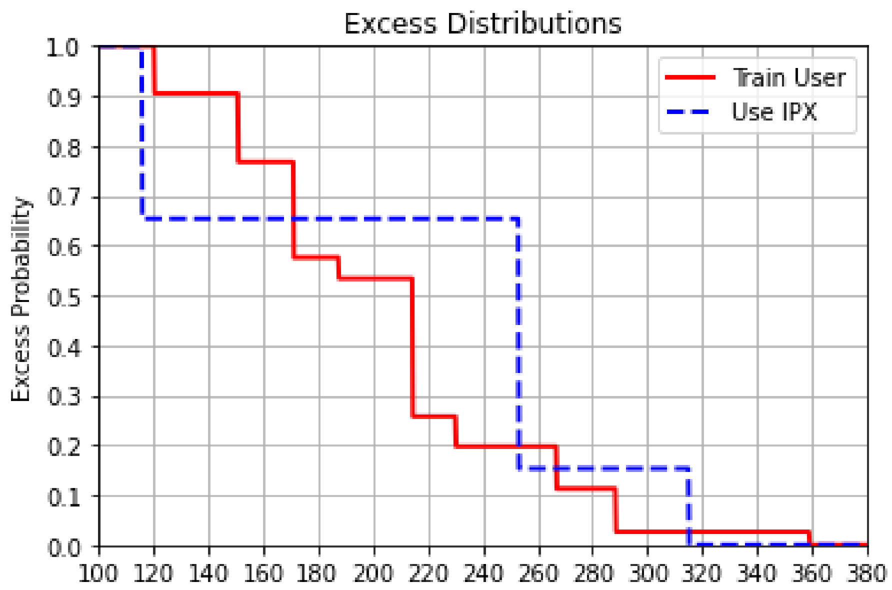


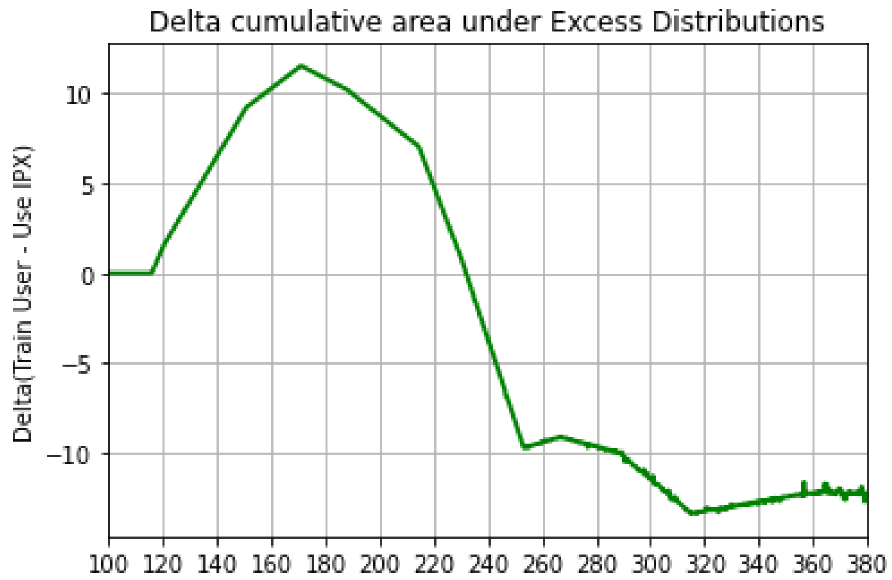
First-Order Stochastic Dominance Analysis

- There is no first-order stochastic dominance as the two risk profiles crossed each other.

Second-Order Stochastic Dominance Analysis

- From the risk profile, we note that "Train User" may 2SD "Contract IPX", but not vice versa.
- The difference in cumulative areas under the excess distributions for "Train user" and "Contract IPX" for all outcomes generated by the Python program is as follows:

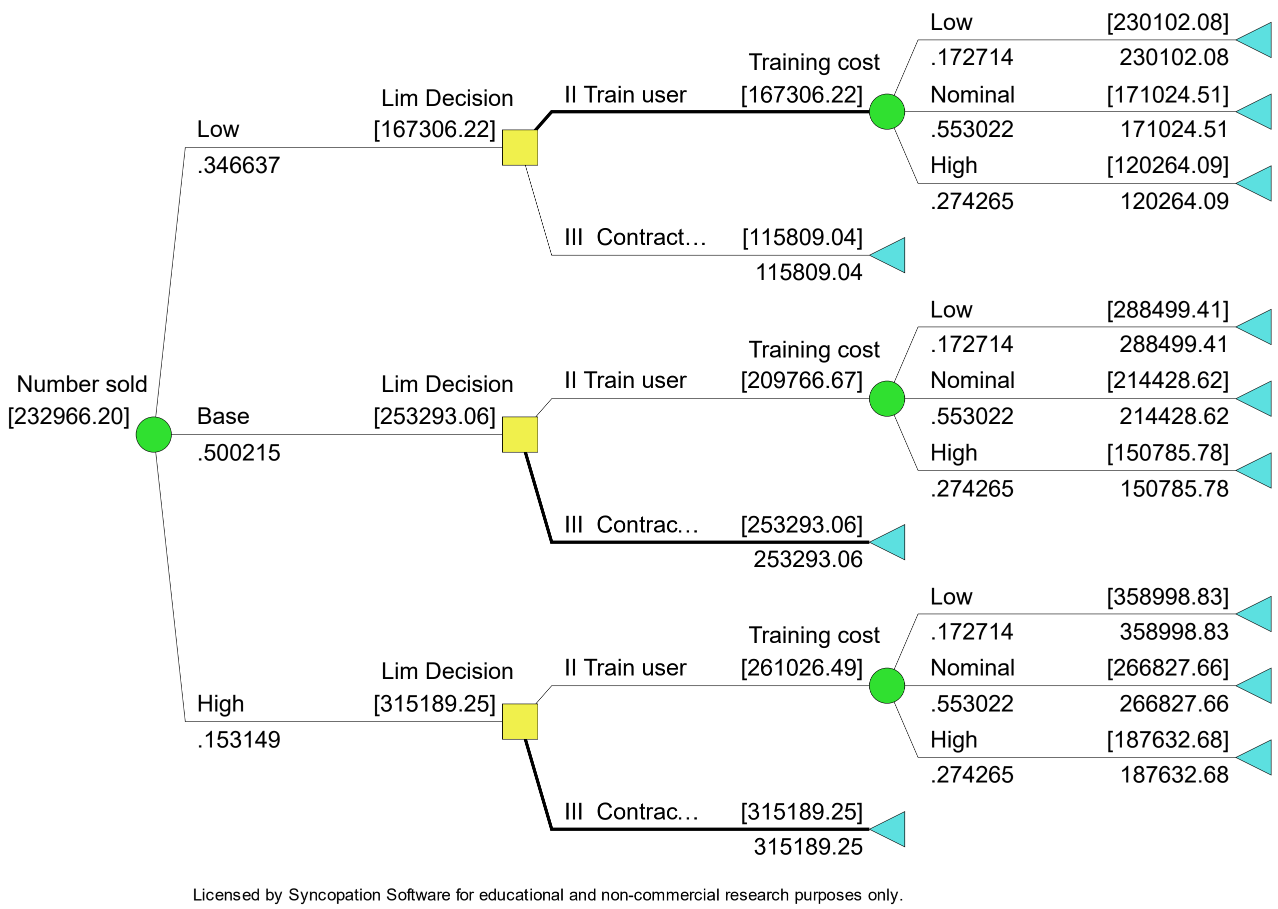




- We conclude that there is no second-order stochastic dominance.

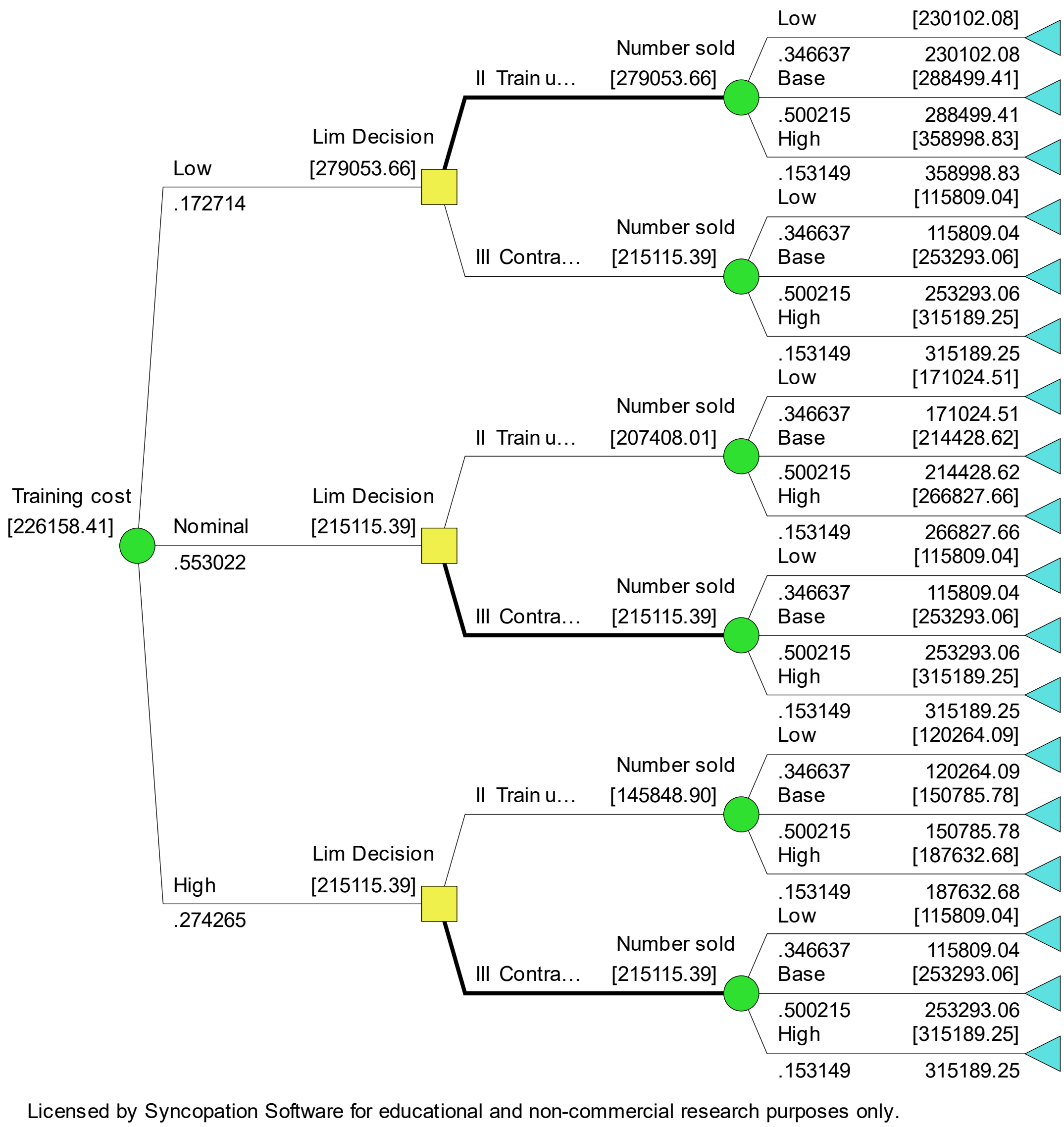
Value of Information Analyses

- **Decision model with free perfect information on Number Sold**



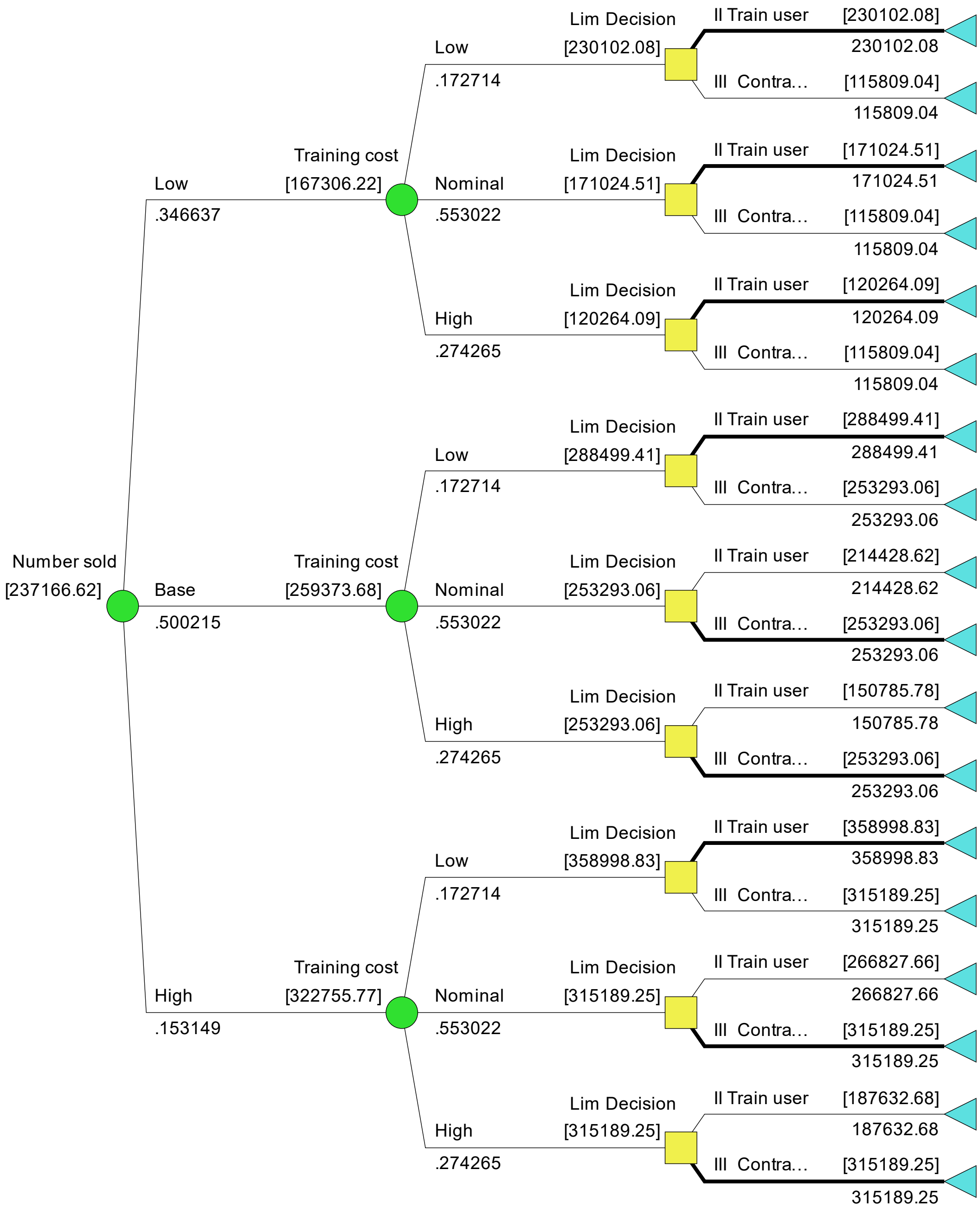
- Expected NPV with free perfect information on Number Sold = $232,966.20
- Expected NPV with no information = $215,115.39
- EVPI for Number Sold = $232,966.20 – $215,115.39 = **$17,580.81**

- **Decision model with free perfect information on Training Cost**



- Expected NPV with free perfect information on Training Cost = $226,158.41
- Expected NPV with no information = $215,115.39
- EVPI for Training Cost = $226,158.41 – $215,115.39 = $11,043.02

- **Decision model with joint perfect info on Training Cost and Number Sold:**



- Expected NPV with free joint perfect information on Training Cost and Number Sold
  = \$237,166.62

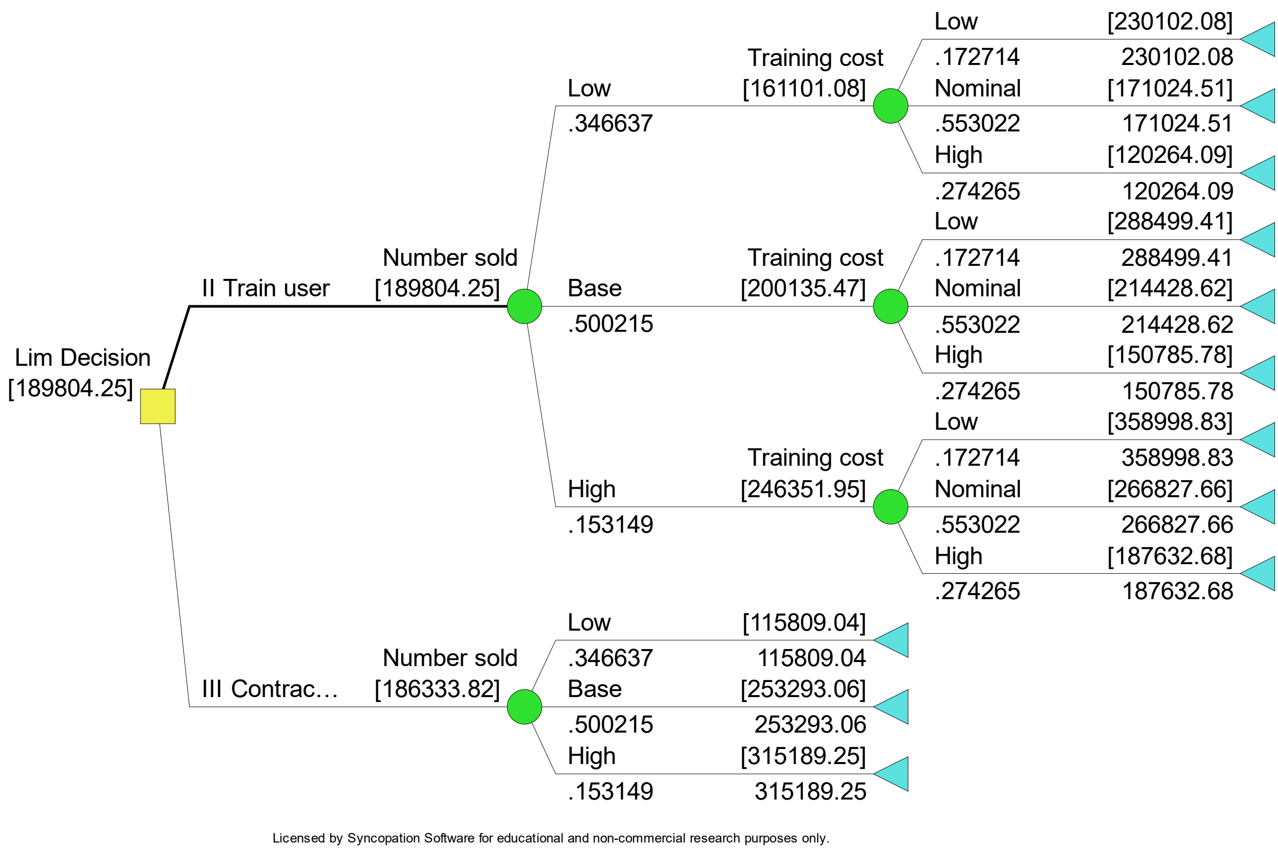
- Expected NPV with no information = \$215,115.39

- Joint EVPI for Train Cost and Number Sold = \$237,166.62 – \$215,115.39 = <u>**\$22,051.23**</u>

8.4.3 Second Pass: Consideration of Risk Aversion

- Assume that the company's risk tolerance is $100,000.

Optimal Decision Policy Tree at RT = $100,000.

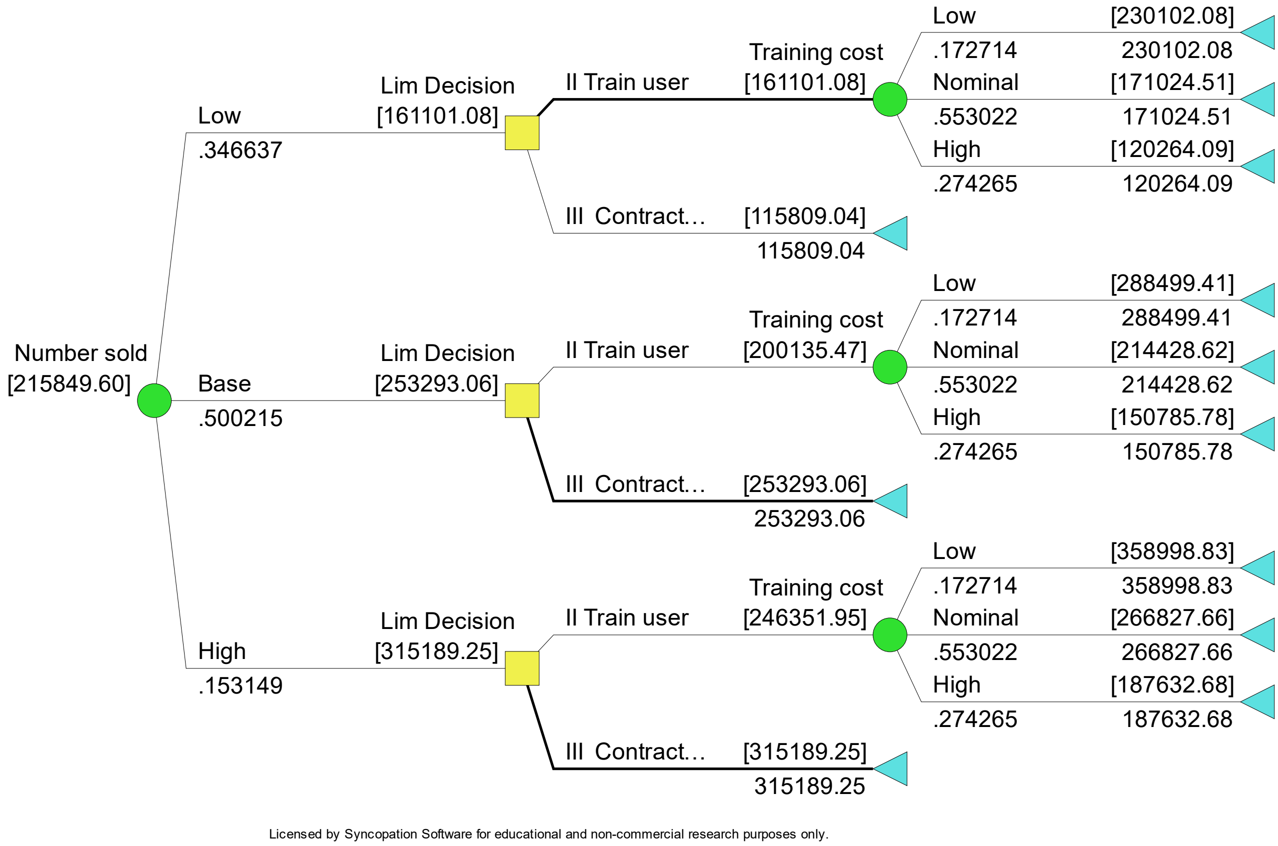


- At a risk tolerance of $100,000, LIM should train the user for self-maintenance.

- Certainty Equivalent = $189,804.25

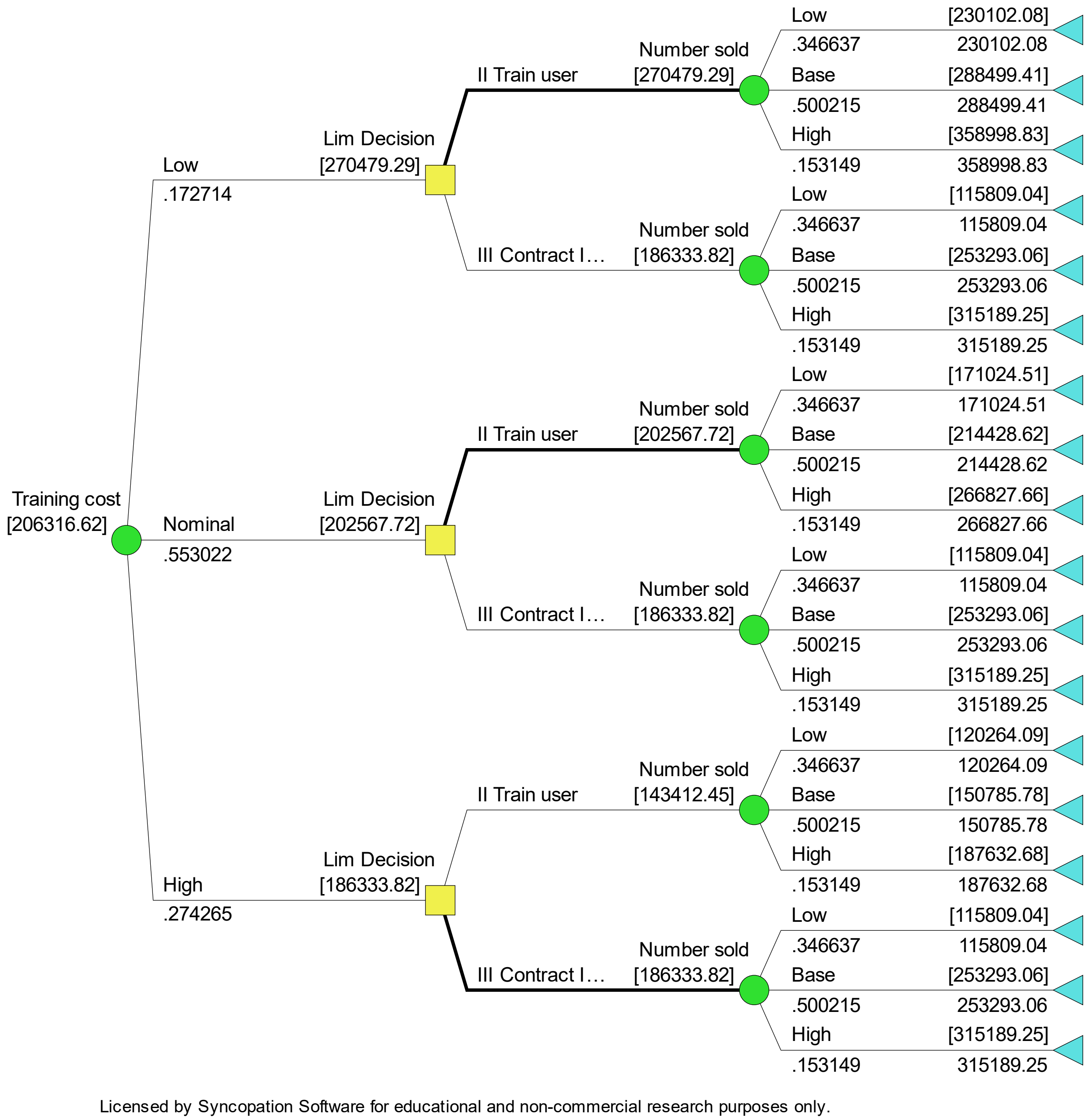
Value of Information Analyses under Risk Aversion

- **Decision model with perfect information on Number Sold (RT=$100,000)**



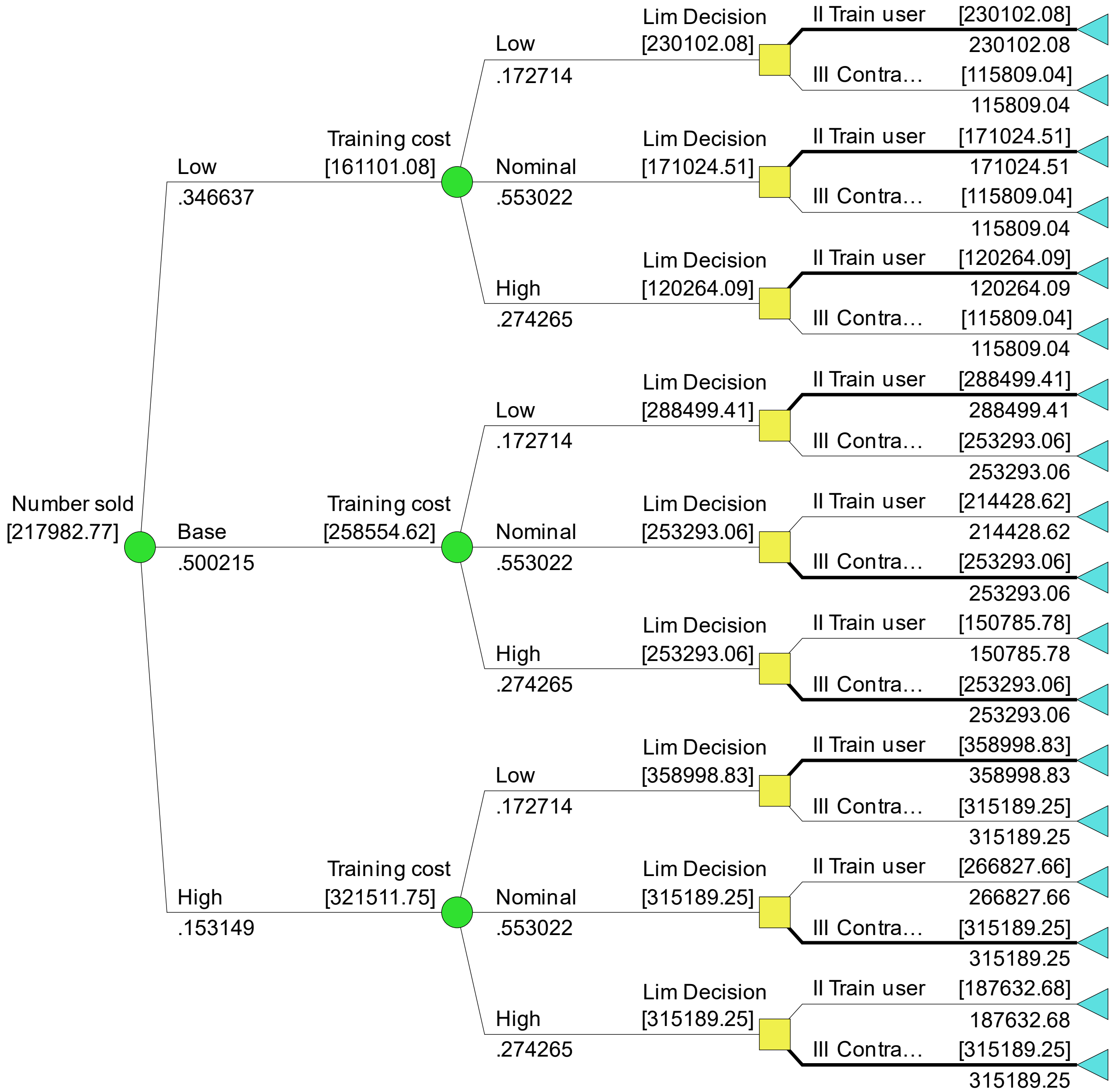
- Certainty Equivalent with free perfect information on Number Sold = $215,849.60
- Certainty Equivalent with no information = $189,804.25
- EVPI for Number Sold = $215,849.60 – $189,804.25 = $26,045.35

- **Decision model with perfect information on Training Cost (RT=$100,000)**



- Certainty Equivalent with free perfect information on Training Cost = $206,316.62
- Certainty Equivalent with no information = $189,804.25
- EVPI for Number Sold = $206,316.62 – $189,804.25 = $16,512.37

- **Decision model with joint perfect information on Number Sold and Training Cost (RT=$100,000)**



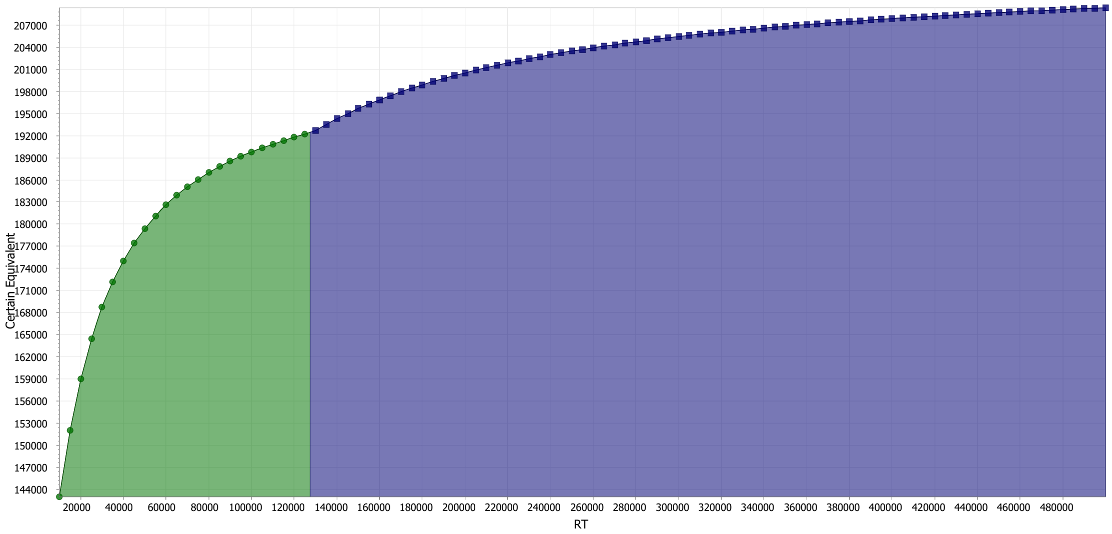
- Certainty Equivalent with free perfect joint information on Training Cost and Number Sold
  = $217,982.77

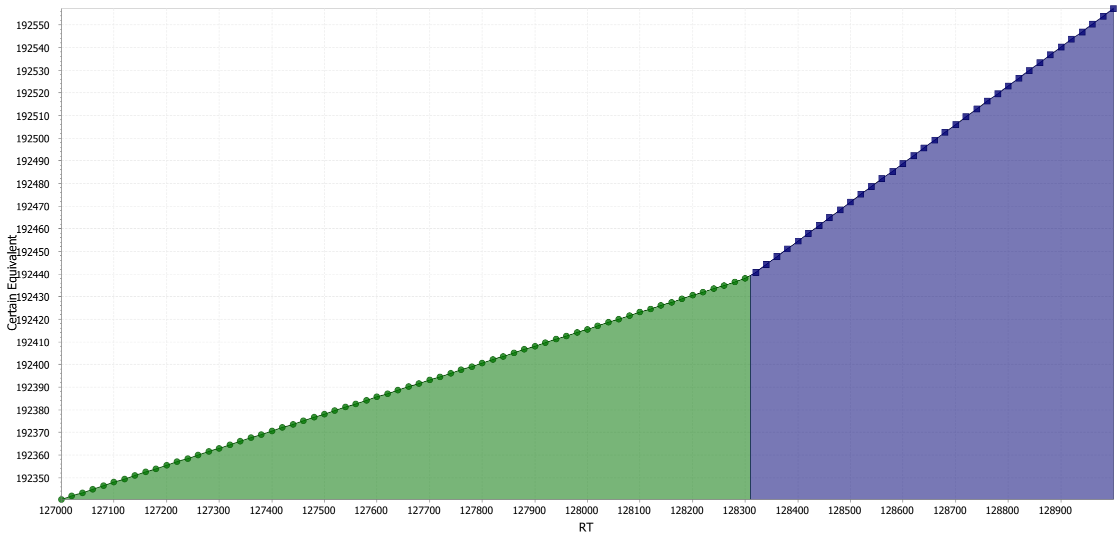
- Certainty Equivalent with no information = $189,804.25

- Joint EVPI for Number Sold and Training Cost = $217,982.77 – $189,804.25
  = $28,178.52

Sensitivity to Risk Tolerance Analysis

Rainbow Diagram by Varying Risk Tolerance





LIM's Decision Rule based on Risk Tolerance

If 0 < Risk Tolerance ≤ $128,310
  Train Users
Else
  Contract IPX

8.4.4 Modeling and Solving the LIM Problem using DPL

- See instructional video on Canvas for a step-by-step tutorial.

Exercise Case Study: Bioteckno Problem

Background

Biotekno is a small biotechnology company. Their long-term research efforts involve human therapeutic drugs, but due to the long development times and uncertain regulatory process for drugs administered to humans, Biotekno has not yet brought any significant drugs to market. They have, however, introduced several diagnostic tests in the past few years, and revenue from these products has funded the overall research program.

Research scientists have recently developed a new diagnostic test for detecting a common viral infection in cattle. While the final work is being completed in the lab, management is currently considering how best to proceed with the production and marketing of the new product. They have considered strategic alliances with several large pharmaceutical firms, and have singled out a potential candidate, National Diagnostics. National Diagnostics has made two offers, which are outlined below:

Biotekno management is interested in an alliance because they are concerned that their existing production facilities might not be sufficient if demand is high. Yet, on the other hand, the investment in a new facility might be a mistake if demand is low. In addition, National Diagnostics has an established line of diagnostic products and a large sales staff and may be able to sell the product more effectively than Biotekno.

The Decisions

You have been called in as a decision analyst to assist Biotekno management with their decision. There are four alternative business strategies: whether or not to accept either type of partnership offered by National Diagnostics, and whether to build a new plant at Biotekno to produce the product. Such a plant could be completed by the time the test is ready to market.

Because the innovation cycle in this field is so rapid (and to simplify the analysis), a two-year study period has been selected for the analysis. The animal diagnostics field is very competitive and Biotekno is a price-taker. Management wants you to consider a fixed price of $20 for the new product.

The Alternatives

I and **II.** No Partnership: Biotekno may choose not to form a partnership with National Diagnostic, and choose to produce and market the product themselves. Then they have to choose whether or not to build a new facility. The uncertainty around the sales volume for the first year using the Biotekno sale staff is a major concern. Costs for such a plant are also uncertain, and this is one reason management has agreed to try the decision analysis approach. The plant is prefabricated, so it is able to begin production instantly if Biotekno chooses to build it. The reason for considering a new plant is that the existing facility could produce a maximum of 4 million units/year of the new product. Let alternative I be "No Partnership/Build Plant" and alternative II be "No Partnership/Don't Build Plant".

III. Joint Venture Partnership/Build Plant: National Diagnostic has offered to pay the entire cost of the new production facility in return for a 50% share in the net profits (sales revenue minus production costs) for the new product. The production would take place at the new Biotekno facility, while the marketing would be done by National Diagnostic. Sales volume for the first year using the National Diagnostic sales staff is uncertain.

Note that for alternatives I, II, and III, the production would take place at Biotekno. The process would be the same whether production takes place in Biotekno's existing facility or the proposed new facility. Regardless of which facility is used, management is uncertain about the production cost per unit.

IV. Royalty Partnership/Don't build Plant. National Diagnostic has also offered to undertake both the production and market of the product all alone. Under this arrangement, Biotekno would receive a royalty payment of 2% of sales revenue. Note that because the royalty payment is based on sales revenue rather than net profits, the production costs at the National Diagnostic facility are irrelevant to Biotekno's decision. Also, since there would be no production at Biotekno, there is no need for Biotekno to consider building a new production facility if the royalty offer is accepted.

Additional Information

Second-year sale is to be modeled as a function of first-year sales. The uncertainty management feels most comfortable considering is the growth rate for second-year sales, which is a multiplier of first-year sales. This growth rate depends on market conditions and should be used regardless of which company's sales force markets the product.

If a new facility is constructed, it must be paid for in full at the time of completion. Biotekno management does not want you to consider the value of a new facility for uses beyond the two-year time frame. You do not need to consider marketing costs in this analysis. There is no need to discount cash flows, and you can assume that Biotekno is risk neutral for the initial analysis.

Conduct one full pass of the DA Cycle.

For the deterministic sensitivity analysis, use the following values which are based on the 5, 50 and 95 percentiles:

Variables	Low	Base	High
Plant construction cost (million $)	10	15	20
Sales growth rate (%)	-10	20	50
Sales, 1st year, Biotekno (million)	1	3	5
Sales, 1st year, National Diagnostics (million)	1	4	7
Unit production cost ($)	7	9	11

For probabilistic analysis, you may assume all the sensitive uncertain variables may be represented by normal distributions, and use 3-branch discrete approximations in your decision tree.

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