

IE2111 ISE Principles and Practice II

Solutions to Assignment 6

$MARR = 12\%$

- (a) The challenger's total marginal costs and *EUAC* for each year of service are computed as follows:

	(A)	(B)	(C)	(D)	(E)=(B)+(C)+(D)	(F)
EoY	MV(k)	Loss of MV during year k	Cost of capital = $i \cdot MV(k-1)$	Annual expenses E(k)	Total Marginal Cost TC(k)	EUAC
0	10,000.00					
1	6,000.00	4,000.00	1,200.00	2,200.00	7,400.00	7,400.00
2	5,100.00	900.00	720.00	2,640.00	4,260.00	5,918.87
3	4,335.00	765.00	612.00	3,168.00	4,545.00	5,511.72
4	3,684.75	650.25	520.20	3,801.60	4,972.05	5,398.81
5	3,132.04	552.71	442.17	4,561.92	5,556.80	5,423.68
6	2,662.23	469.81	375.84	5,474.30	6,319.95	5,534.12

The economic service life of the challenger = 4 years

Optimal *EUAC* of **\$5,398.81**

This means that if the challenger is purchased for use and its service is required for a very long time, it should be replaced with an identical one every 4 years under the repeatability assumption.

(b) Study period = Infinity.

Based on the opportunity cost approach, the year-by-year total marginal costs of the defender are computed as follows:

	(A)	(B)	(C)	(D)	(E)=(B)+(C)+(D)
EoY	MV(k)	Loss of MV during year k	Cost of capital = $i \cdot MV(k-1)$	Annual expenses $E(k)$	Total Marginal Cost $TC(k)$
0	4,000.00				
1	3,000.00	1,000.00	480.00	2,000.00	3,480.00
2	2,250.00	750.00	360.00	3,500.00	4,610.00
3	1,687.50	562.50	270.00	5,000.00	5,832.50
4	1,265.63	421.88	202.50	6,500.00	7,124.38

We observed that the Defender's TC_k values are monotonically non-decreasing, and that

$$TC_2 \text{ of defender} = \$4,610.00 < EUAC^* \text{ of Challenger} = \$5,398.81 < \$5,832.50$$

Optimal replacement plan:

Keep the Defender for two more years.

Replace it with the Challenger at EoY 2.

The Challenger is then repeated every 4 years.

EPC (opportunity cost) of optimal replacement plan

$$\begin{aligned}
 &= \frac{3,480}{(1 + 0.12)} + \frac{4,610 + 5,398.81/0.12}{(1 + 0.12)^2} \\
 &= \$42,647.99
 \end{aligned}$$

$EUAC$ (cash flow) of the optimal replacement plan

$$= (\$42,647.99 - \$4,000) (0.12) = \$ 4,637.76$$

(c) Study period = 1 year

Let (k_1, k_2) denote the plan for using the defender for k_1 more years before replacing it with the Challenger, using it for k_2 years. There are 2 feasible alternative replacement plans:

No	Plan	Year 1	EPC
1	(0, 1)	7,400.00	\$6,607.14
4	(1, 0)	3,480.00	\$3,107.14

The optimal replacement plan is (1, 0), i.e., keep the defender for 1 more year.

EPC (opportunity cost) of optimal plan= **\$3,107.14**

EUAC (cash flows) of the optimal replacement plan
= $(3,107.14 - 4,000.00) [A/P, 12\%, 1] = -\$892.86 (1.12) = \mathbf{-\$1,000}$

The *EUAC* (cash flow) is negative because the defender's *MV* at EoY 1 (\$3,000) is greater than the Expense (\$2,000) in Year 1.

(d) Study period = 4 years with max 2 replacements

Using the same notations as above, there are 11 feasible alternative replacement plans if only up to two replacements are allowed:

No	Plan	Year 1	Year 2	Year 3	Year 4	EPC
1	(0, 1, 3)	7,400	7,400	4,260	4,545	\$18,426.99
2	(0, 2, 2)	7,400	4,260	7,400	4,260	\$17,977.67
3	(0, 3, 1)	7,400	4,260	4,545	7,400	\$17,941.06
4	(0, 4, 0)	7,400	4,260	4,545	4,972	\$16,398.06
5	(1, 1, 2)	3,480	7,400	7,400	4,260	\$16,980.86
6	(1, 2, 1)	3,480	7,400	4,260	7,400	\$16,741.40
7	(1, 3, 0)	3,480	7,400	4,260	4,545	\$14,926.99
8	(2, 1, 1)	3,480	4,610	7,400	7,400	\$16,752.21
9	(2, 2, 0)	3,480	4,610	7,400	4,260	\$14,756.69
10	(3, 1, 0)	3,480	4,610	5,833	7,400	\$15,636.50
11	(4, 0, 0)	3,480	4,610	5,833	7,124	\$15,461.33

The optimal replacement plan is (2,020), i.e., keep the defender for 2 more years before replacing it with the challenger and using it for 2 years.

EPC (opportunity cost) of optimal plan = \$14,756.69

EUAC (cash flow) of the optimal replacement plan
= $(\$14,756.69 - \$4,000) [A/P, 12\%, 4] = \$10,756.69 (0.3292344) = \mathbf{\$3,541.47}$