**TIE4203 Decision Analysis in Industrial & Operations Management**
**Solutions to Tutorial #6**

**Question 1 (P6.1)**

**(*a*)** $u(w) = w - \beta w^2$

$u'(w) = 1 - 2\beta w$

$u''(w) = -2\beta$

Risk tolerance $\rho(w) = \dfrac{-u'(w)}{u''(w)} = \dfrac{1-2\beta w}{2\beta}$

Degree of absolute risk aversion $r(w) = \dfrac{2\beta}{1-2\beta w}$

Note that for $u(w)$ to be increasing and concave (i.e., risk averse), it is sufficient for the coefficient $\beta$ to be strictly positive.

**(*b*)** $u(w) = ln\ w$

$u'(w) = 1/w$

$u''(w) = -1/w^2$

Risk tolerance $\rho(w) = \dfrac{-u'(w)}{u''(w)} = w$

Degree of absolute risk aversion = $r(w) = \dfrac{1}{w}$

**(*c*)** $u(w) = sgn(\beta)\ w^{\beta}$

$u'(w) = sgn(\beta)\ \beta\ w^{\beta-1}$

$u''(w) = sgn(\beta)\ \beta\ (\beta - 1)\ w^{\beta-2}$

Risk tolerance $\rho(w) = \dfrac{-u'(w)}{u''(w)} = \dfrac{w}{1-\beta}$

Degree of absolute risk aversion = $r(w) = \dfrac{1-\beta}{w}$

## Question 2 (P6.2)

Given that John has the utility function $u(x) = 1 - 3^{-x/50}$ over the range of $x$ = -\$50 to \$5,000.

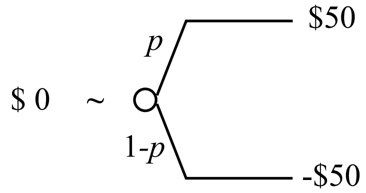
The utility function may be rewritten as $u(x) = 1 - 3^{-x/50} = 1 - e^{-x \ln 3/50}$

**(*a*)** The utility function is increasing and concave. Hence John is risk averse.

**(*b*)** John's risk tolerance = \$ 50 / *ln* 3.

Hence John's degree of risk aversion = 1 / risk tolerance = *ln* 3 / 50 = 0.0219722 $\$^{-1}$

**(*c*)** We want to find $p$ such that the certainty equivalent of the deal is zero.

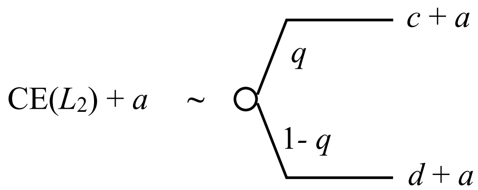



$$
\begin{aligned}
u(0) &= p\, u(50) + (1-p)\, u(-50) \\
1 - 3^0 &= p\,(1 - 3^{-1}) + (1 - p)\,(1 - 3^1) \\
0 &= p\,(2/3) + (1-p)(-2) \\
p &= 3/4
\end{aligned}
$$

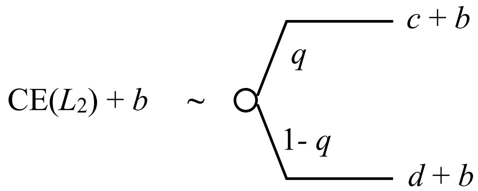
## Question 3 (P6.3)

Consider $L_2$:

By the delta property, if we add the amount $a$ to all its outcomes of $L_2$, we get

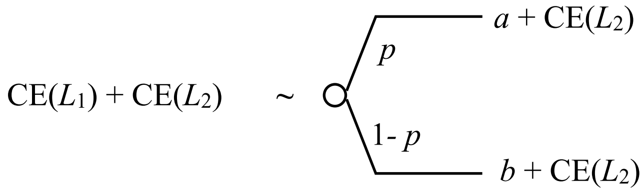



Similarly, by the delta property, if we add the amount $b$ to all the outcomes of $L_2$, , we get



Consider $L_1$:

By the delta property, if we add the amount CE($L_2$) to all its outcomes, we get



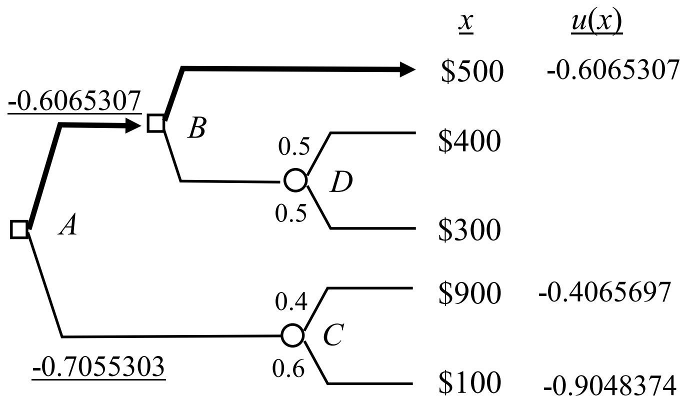

Finally, by the subsitution rule, the above deal is equivalent to $L_3$.

Hence CE($L_3$) = CE($L_1$) + CE($L_2$).

**Question 4 (P6.4)**

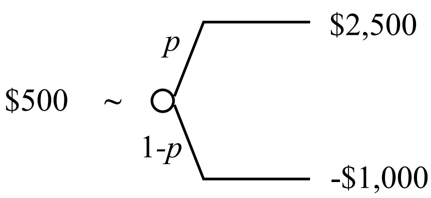
Since George has a constant risk tolerance of $1,000, it follows that he has constant absolute risk aversion or the delta property. Hence his utility function is exponential in form.

We let $u(x) = -\exp(-x/1{,}000)$.




Maximum Expected Utility at $A$ = -0.6065307
Certainty Equivalent at $A = u^{-1}(-0.60653)$ = $500

The required preference probability is $p$, such that




Hence $\quad u(500) = p\, u(2{,}500) + (1-p)\, u(-1{,}000)$

$$\Rightarrow \quad -e^{\frac{-500}{1{,}000}} = (p)(-e^{\frac{-2{,}500}{1{,}000}}) + (1-p)(-e^{\frac{1{,}000}{1{,}000}})$$

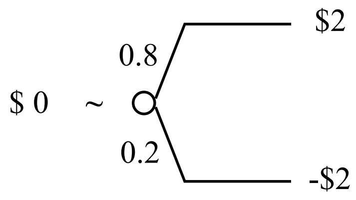
$$\Rightarrow \quad p = 0.80106$$

Note that we would have got the same answer if we had used the utility function $u(x) = 1 - exp(-x/1000)$, or if we had assumed $u(x) = a - b\, exp(-x/1000)$ and fitted the constants $a$ and $b$ to the boundary conditions $u(-\$1{,}000) = 0$ and $u(\$2{,}500) = 1$. Make sure you understand why this is so.

**Question 5 (P6.5)**

**(*a*)** Susan satisfies delta property $\Rightarrow$ utility function is of the form $u(x) = a - b\, e^{-x/\rho}$ where $\rho$ is the risk tolerance.

Given




$u(0) = 0.8\, u(2) + 0.2\, u(-2)$

$a - b = 0.8\, (a - b\, e^{-2/\rho}) + 0.2\, (a - b\, e^{2/\rho})$

$1 = 0.8\, e^{-2/\rho} + 0.2\, e^{2/\rho}$

Let $x = e^{2/\rho}$

$x^2 - 5x + 4 = 0 \;\Rightarrow\; (x - 1)(x - 4) = 0 \;\Rightarrow\; x = 1$ or $x = 4.$

Hence $= e^{2/\rho} = 4 \;\Rightarrow\; \rho = \dfrac{1}{\ln 2} = \$1.44$

Susan risk tolerance = \$1.44

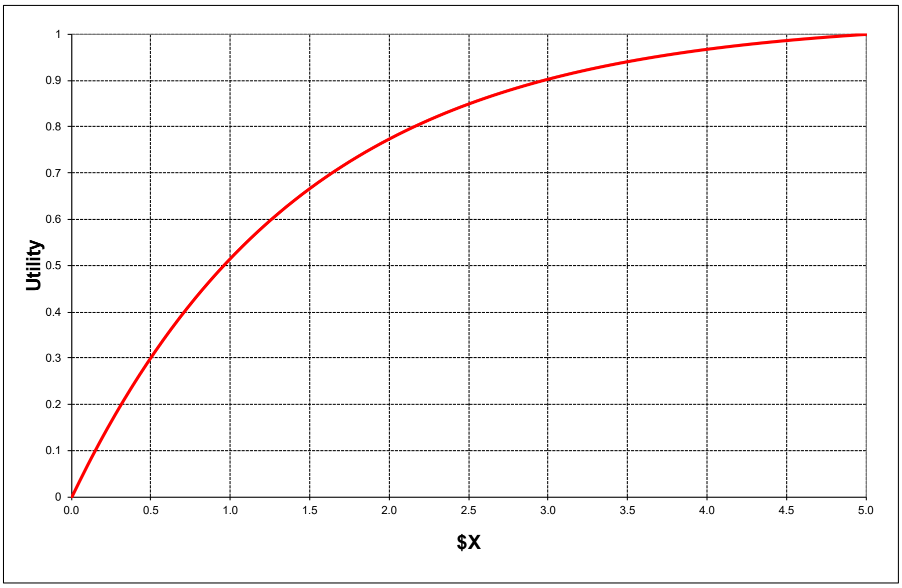
**(*b*)** Susan's risk attitude is risk adverse since her risk tolerance is positive.

**(*c*)** Susan's utility function such that $u(L = 0) = 0$ and $u(H = 5) = 1$ is

$$u(x) = \frac{1 - e^{-(x-L)/\rho}}{1 - e^{-(H-L)/\rho}}$$

$$= 1.0322581\ (1 - e^{-x\ln 2})$$

$$= 1.0322581\ (1 - 2^{-x})$$

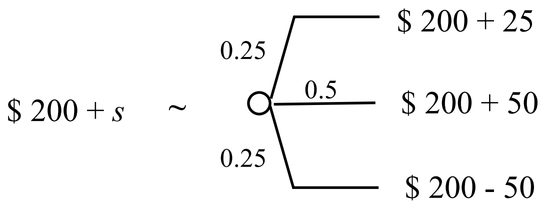


**Question 6 (P6.6)**

Current wealth = \$200.

Wealth Utility function $u(w) = \dfrac{w^2}{2000}$, $w \geq 0$.

**(*a*)** Let $s$ = personal indifferent selling price.




$u(200 + s) = 0.25\, u(200+25) + 0.5\, u(200 + 50\,) + 0.25\, u(200 – 50)$

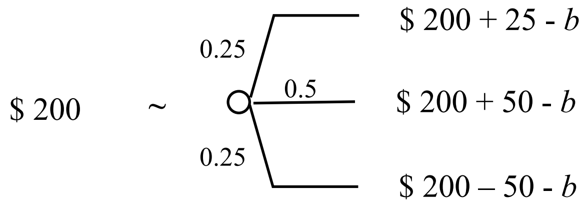
$$\frac{(200+s)^2}{2000} = 0.25\left(\frac{225^2}{2000}\right) + 0.5\left(\frac{250^2}{2000}\right) + 0.25\left(\frac{150^2}{2000}\right)$$

$$(200+s)^2 = 0.25\,(225)^2 + 0.5\,(250)^2 + 0.25\,(150)^2$$

$$s = \$\,22.5562$$

Hence Susan's PISP = <u>**\$22.56**</u>

**(*b*)** Let $b$ = personal indifferent buying price.




$u(200) = 0.25\, u(200+25 – b) + 0.5\, u(200 + 50 – b) + 0.25\, u(200 – 50 – b)$

$$\frac{200^2}{2000} = 0.25\left(\frac{(225-b)^2}{2000}\right) + 0.5\left(\frac{(250-b)^2}{2000}\right) + 0.25\left(\frac{(150-b)^2}{2000}\right)$$

$$4(200)^2 = (225-b)^2 + 2(250-b)^2 + (150-b)^2$$

$$4b^2 - 1750b + 38125 = 0$$

Solving: $b$ = \$22.99425 (okay) or \$414.5057 (rejected)

Hence Susan's PIBP = <u>**\$22.99**</u>