Decision Analysis
Solutions to Homework #3

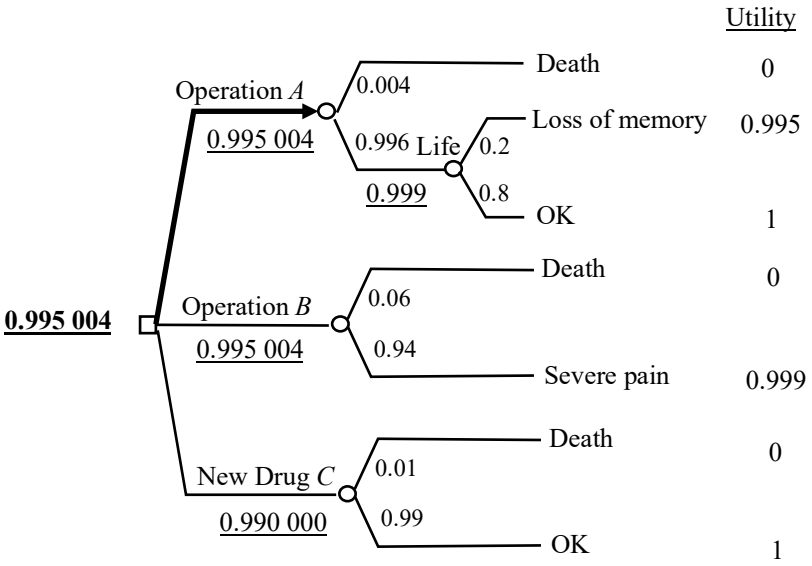
Question 1

(*a*)

- Note that the best outcome is "OK", and the worst outcome is "Death".
- Hence let u(OK) = 1, and u(Death) = 0.
- Based on Dr. Tan's preferences, the utilities for the various outcomes are as follows:

Outcome	Utility
OK	1
Severe pain (SP)	0.999
Loss of memory (LOM)	0.995
Death (D)	0

- Using Dr. Tan's utilities in the decision tree:



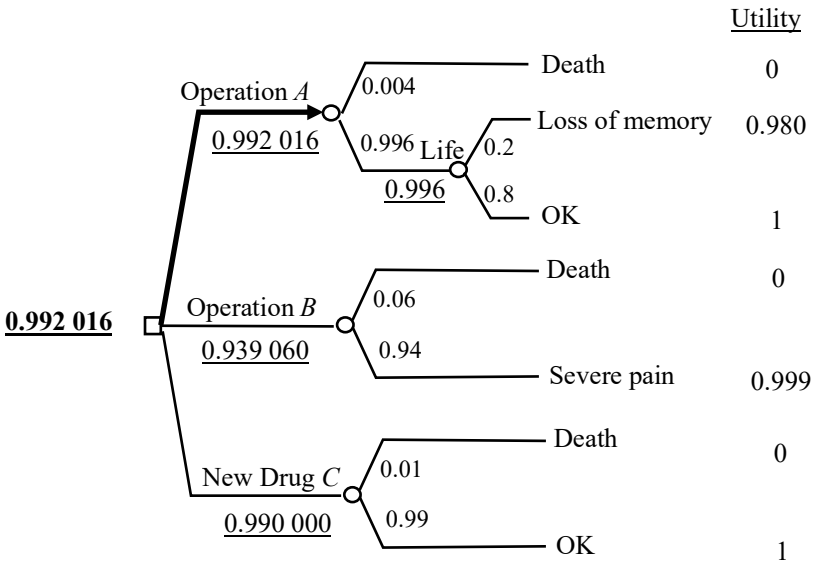
- By rolling back the decision tree using Dr. Tan's preferences, the best choice is Operation A.

(*b*)

- Based on Mr. Goh's preferences, and again assuming that u(OK)=1, and u(Death) = 0, the utilities for the various outcomes are as follows:

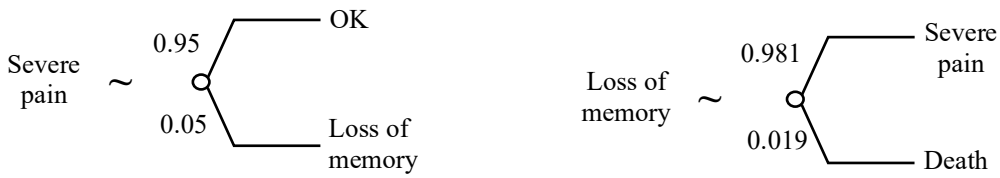
Outcome	Utility
OK	1
Severe pain (SP)	0.999
Loss of memory (LOM)	0.980
Death (D)	0

- Using Mr. Goh's utilities in the decision tree:



- Based on Mr. Goh's preferences, the best choice is Operation A.

(*c*) Given



u(SP) = 0.95 u(OK) + 0.05 u(LOM) = 0.95 + 0.05 u(LOM)

u(LOM) = 0.981 u(SP) + 0.019 u(D) = 0.981u(SP)

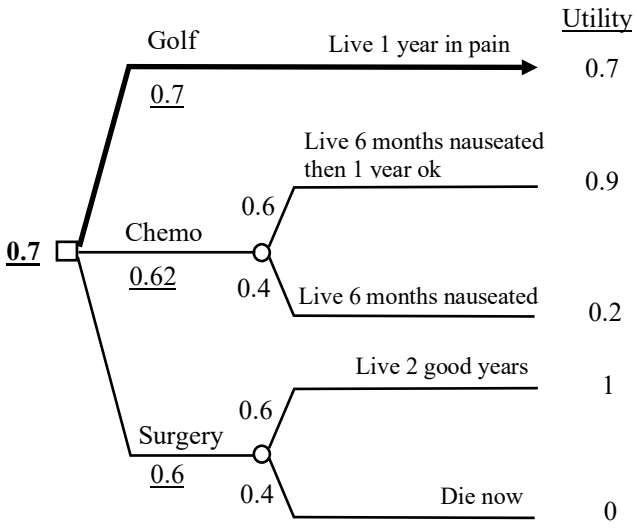
$\Rightarrow$ u(SP) = 0.999 and u(LOM) = 0.980

Therefore, optimal decision remains unchanged for Mr. Goh.

Question 2

- If we let u(Live two good years) = 1, and u(Die now) = 0, then Roy's preference probabilities for the various outcomes are the same as his utilities.

- Roy's decision problem:



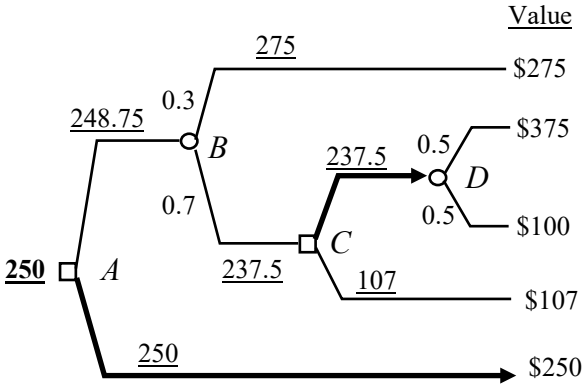
(*a*) Roy should play golf, which has the maximum expected utility of 0.7.

(*b*) We cannot calculate the value of information on anything here because we do not have Roy's equivalent dollar values for the outcomes.

(*c*) Roy might want the expected value of perfect information so that he would know how much he should spend on any form of information collection activities concerning any of the uncertainties he faces.

Question 3

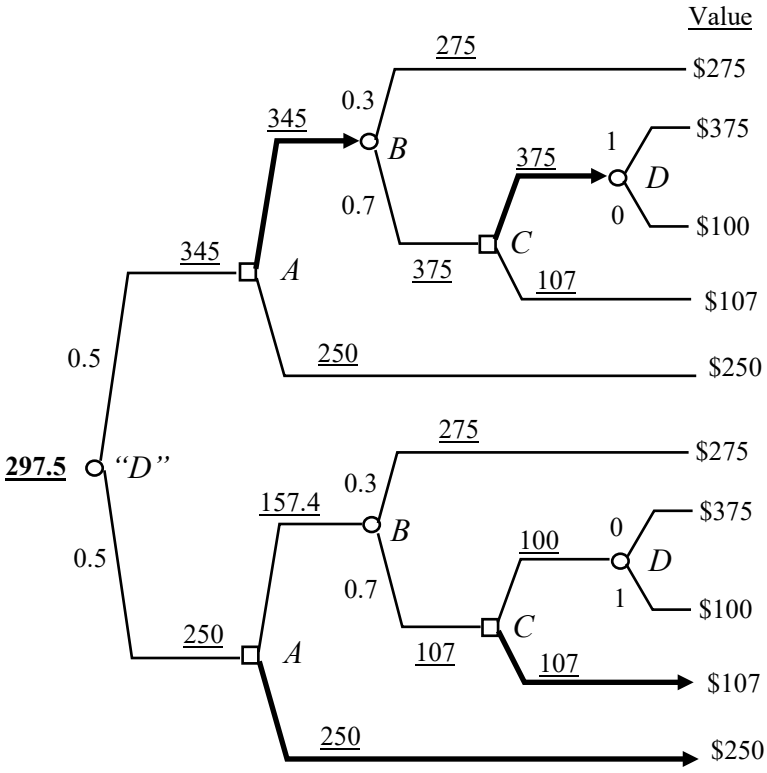
(*a*) Jeanne's decision tree:



- Jeanne is risk-neutral. This means that she makes her decision based on maximum expected dollar values.
- Also, Risk neutral ⇒ certainty equivalent = expected dollar value.
- Jeanne's certainty equivalent for decision A = **$250**.

(*b*) Value of Clairvoyance on D before A.

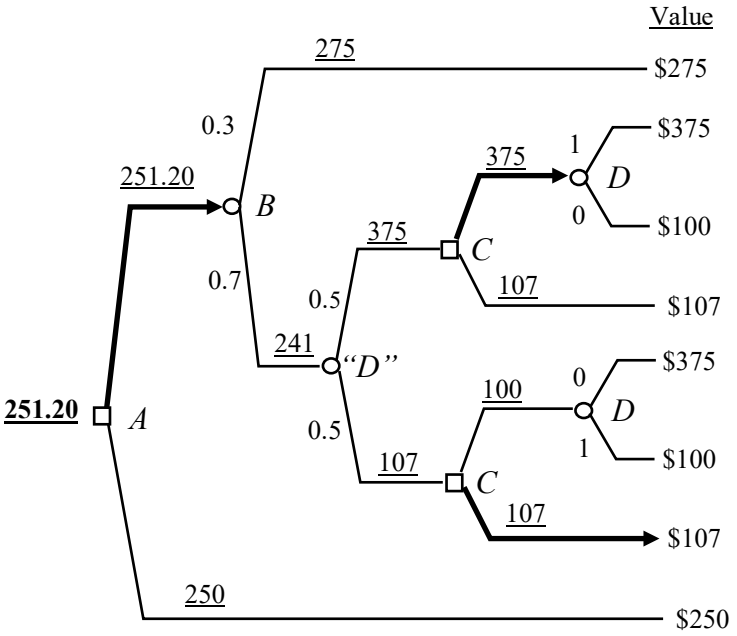
- If Jeanne receives free perfect information on the outcome of D just before she makes decision A, then the decision model with free clairvoyance on D before decision A is:



- Note that we have redrawn the decision tree so that chance node D appears before decision A.
- Certainty equivalent when there is free clairvoyance on event D before decision A = $ 297.50
- Certainty equivalent when there is no information (from part (*a*)) = $ 250.
- Value of Clairvoyance on event D before decision A = $ 297.50 – $250 = $ 47.50
- Hence the most Jeanne should pay for clairvoyance on event D before decision A = **$ 47.50**

(*c*) Value of Clairvoyance on D before making decision C but after making decision A.

- If Jeanne receives free perfect information on the outcome of uncertain event D after making decision A, but before making decision C, then the decision model is:



- Note that we have redrawn the decision tree so that chance node D appears after decision A but before decision C.

- Also, D and B are independent events. Hence, we can also draw information node "D" before node B and obtain the same answer, but the decision tree would be larger in size.

- Certainty equivalent when there is free clairvoyance on event D before making decision C, but after making decision A = $251.20.

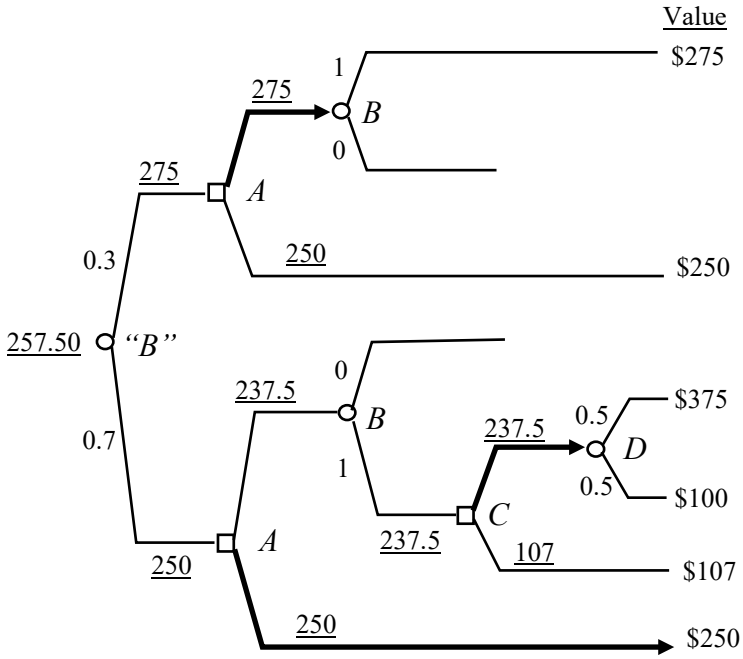
- Certainty equivalent when there is no information (from part (a)) = $250.

- Value of Clairvoyance on event D before making decision C, but after making decision A = $251.20 – $250.00 = $1.20

- Hence the most Jeanne should pay for clairvoyance on event D before making decision C, but after making decision A = **$1.20**

(*d*) Value of Clairvoyance on B before making decision A.

- Decision model with free clairvoyance on uncertain event B before making decision A:



- Certainty equivalent when there is free clairvoyance on event B before making decision A = $257.50

- Certainty equivalent when there is no information (from part (a)) = $250.

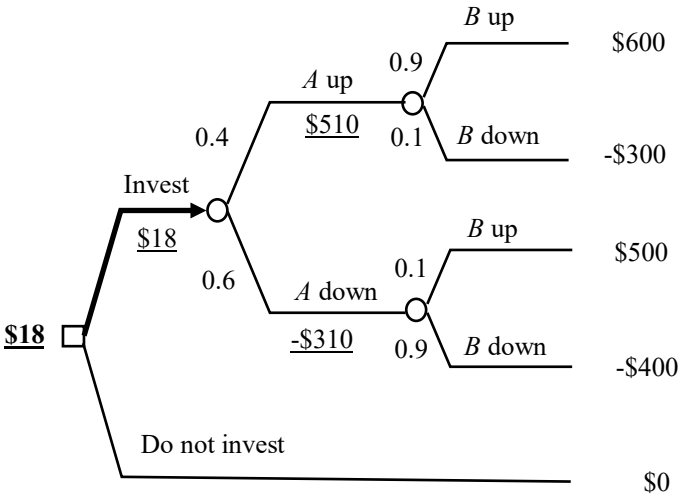
- Value of Clairvoyance on event B before making decision A = $257.50 – 250.00 = $7.50

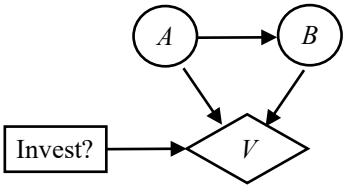
- Hence the most Jeanne should pay for clairvoyance on event B before making decision A = **$7.50**

Question 4

Alice is risk-neutral. Hence, we compute her expected dollar values on the decision tree.

(*a*) Base decision model:

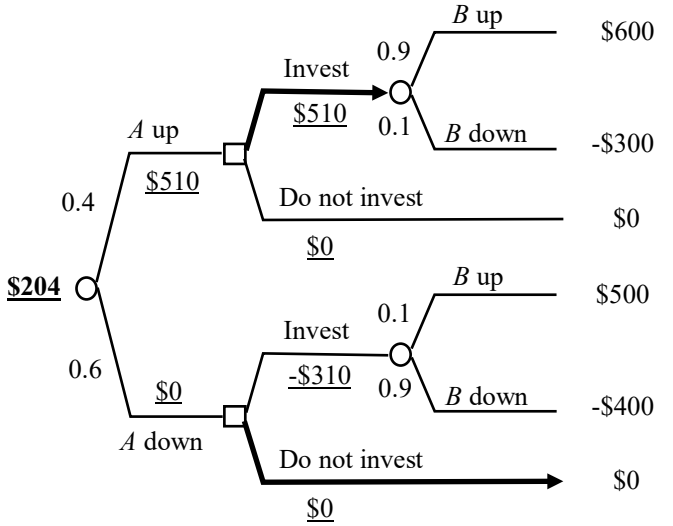


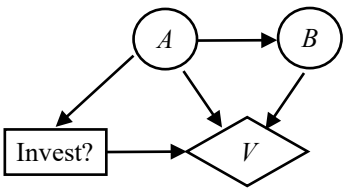


- Alice's best decision is to invest.
- Alice's certainty equivalent for the deal = $18.00

(*b*) Value of Clairvoyance Analysis on A:

- Decision model with free clairvoyance on performance of stock A:





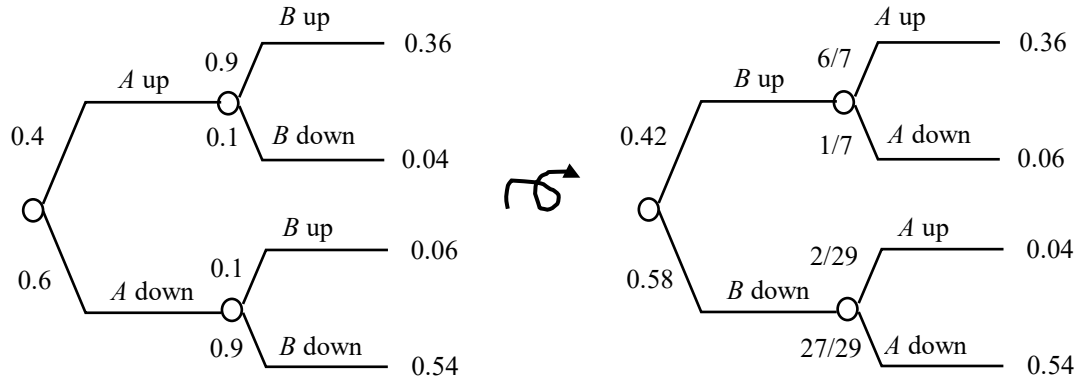
- CE with free clairvoyance on the performance of Stock A = $204.00
- CE with no clairvoyance = $18.00
- Hence Alice's value of clairvoyance on the performance of Stock A
  = $204.00 - $18.00 = **$186.00**

(*c*) Value of Clairvoyance Analysis on B:

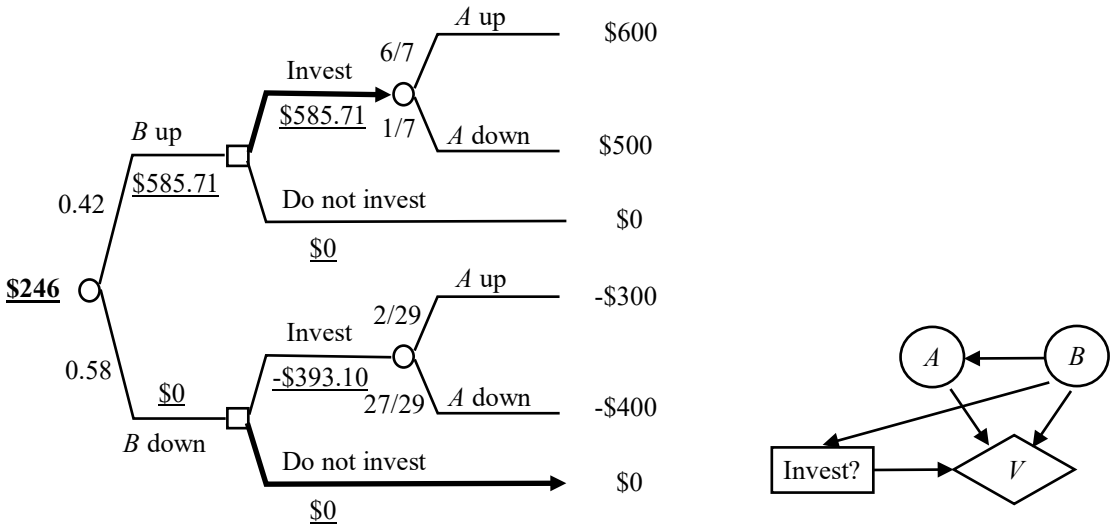
- When there is free clairvoyance on B, we need to draw the decision tree with node B appearing before the investment decision node which is before node A. Hence B will appear before A in the tree.

- As A and B are not independent, we need to flip tree between *A* and *B* and find $p(A \mid B)$ for the decision tree.

- Note that if A and B were independent, then tree flipping is not necessary. See for example Question 3.



- Decision tree with free clairvoyance on B:



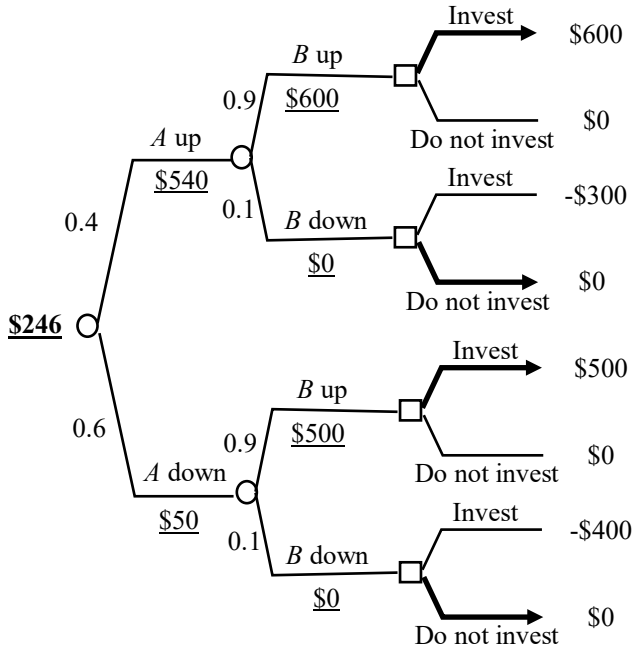
- CE with free clairvoyance on the performance of Stock B = \$245.99.

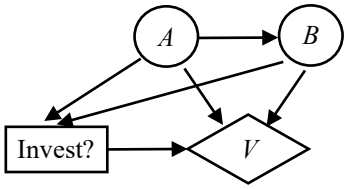
- CE with no clairvoyance = \$18.00

- Hence Alice's value of clairvoyance on the performance of Stock B = \$245.99 - \$18.00 = **\$227.99**

(*d*) Joint Value of Clairvoyance Analysis on *A* and *B*:

- Decision model with free clairvoyance on performance of both stocks A and B:





- CE with free clairvoyance on performance of both stocks A and B = $246.00
- CE with no clairvoyance = $18.00
- Alice's value of clairvoyance on performance of both stocks A and B together = $246.00 – $18.00 = **$228.00**