IE2111 ISE Principles & Practice II
Solutions to Tutorial # 6

Question 1.

Study period = 8 years
Before-tax *MARR* = 10%.

Remaining useful life of old crane = 10 years is not relevant to our analysis here.

Keep the Old Crane (Defender):

The investment value of the defender is its current market value plus the cost of the overhaul required to keep it in service.

Capital investment = \$8,000 + \$5,000 = \$13,000
Annual O&M costs = \$3,000
Market value at EoY 8 = \$0

$$\begin{aligned} AW(10\%) \text{ of Defender} &= -13{,}000\,[A/P, 10\%, 8] - 3{,}000 \\ &= -13{,}000\,(0.1874440) - 3{,}000 \\ &= \mathbf{-\$\ \underline{5{,}436.77}} \end{aligned}$$

Buy New Crane (Challenger):

Capital investment = \$20,000
Annual O&M costs = \$1,000
Market value EOY 8 = \$4,000

$$\begin{aligned} AW(10\%) \text{ of challenger} &= -20{,}000\,[A/P, 10\%, 8] - 1{,}000 + 4{,}000\,[A/F, 10\%, 8] \\ &= -20{,}000\,(0.1874440) - 1{,}000 + 4{,}000\,(0.0874440) \\ &= \mathbf{-\$\ \underline{4{,}399.10}} \end{aligned}$$

Decision: Replace the old crane with the new crane as $AW_{\text{Challenger}} > AW_{\text{Defender}}$.

Question 2.

Cost of new ESP = $80,000

MARR = 15%

Using the Total Marginal Cost Approach, the *EUAC* of using the ESP for k = 1 to 5 years are computed:

	(A)	(B)	(C)	(D)	(E)=(B)+(C)+(D)	(F)
EoY	$MV(k)$	Loss of MV during year k	Cost of capital $i*MV(k\text{-}1)$	Annual expenses $E(k)$	Total Marginal Cost $TC(k)$	EUAC(k)
0	80,000					
1	60,000	20,000	12,000	30,000	62,000	62,000.00
2	50,000	10,000	9,000	30,000	49,000	55,953.49
3	40,000	10,000	7,500	35,000	52,500	**54,958.96**
4	25,000	15,000	6,000	40,000	61,000	56,168.77
5	12,500	12,500	3,750	45,000	61,250	56,922.40

$$EUAC_1 = \left(\frac{62{,}000}{(1+0.15)}\right)[A/P,15\%,1] = 62{,}000.00$$

$$EUAC_2 = \left(\frac{62{,}000}{(1+0.15)} + \frac{49{,}000}{(1+0.15)^2}\right)[A/P,15\%,2] = 55{,}953.49$$

$$EUAC_3 = \left(\frac{62{,}000}{(1+0.15)} + \frac{49{,}000}{(1+0.15)^2} + \frac{52{,}500}{(1+0.15)^3}\right)[A/P,15\%,3] = 54{,}958.96$$

$$EUAC_4 = \left(\frac{62{,}000}{(1+0.15)} + \frac{49{,}000}{(1+0.15)^2} + \frac{52{,}500}{(1+0.15)^3} + \frac{61{,}000}{(1+0.15)^4}\right)[A/P,15\%,4] = 56{,}168.77$$

$$EUAC_5 = \left(\frac{62{,}000}{(1+0.15)} + \frac{49{,}000}{(1+0.15)^2} + \frac{52{,}500}{(1+0.15)^3} + \frac{61{,}000}{(1+0.15)^4} + \frac{61{,}250}{(1+0.15)^5}\right)[A/P,15\%,5] = 56{,}922.40$$

Minimum *EUAC* occurs at EoY 3 = **$ 54,958.96**

Economic Service life of ESP = 3 years.

Question 3.

$MARR = 10\%$

(*a*) Economic Service Life of Challenger

Challenger:

The *EUAC* of using the challenger for k years (k = 1 to 4) are as follows:

	(A)	(B)	(C)	(D)	(E)=(B)+(C)+(D)	(F)
EoY k	$MV(k)$	Loss of MV during year k	Cost of capital $i*MV(k\text{-}1)$	Annual expenses $E(k)$	Total Marginal Cost $TC(k)$	EUAC(k)
0	50,000					
1	40,000	10,000	5,000	13,000	28,000	28,000.00
2	32,000	8,000	4,000	15,500	27,500	**27,761.90**
3	24,000	8,000	3,200	18,000	29,200	28,196.37
4	16,000	8,000	2,400	20,500	30,900	28,778.93

$$EUAC_1 = \left(\frac{28{,}000}{(1+0.1)}\right)[A/P,10\%,1] = 28{,}000.00$$

$$EUAC_2 = \left(\frac{28{,}000}{(1+0.1)} + \frac{27{,}500}{(1+0.1)^2}\right)[A/P,10\%,2] = 27{,}761.90$$

$$EUAC_3 = \left(\frac{28{,}000}{(1+0.1)} + \frac{27{,}500}{(1+0.1)^2} + \frac{29{,}200}{(1+0.1)^3}\right)[A/P,10\%,3] = 28{,}196.37$$

$$EUAC_4 = \left(\frac{28{,}000}{(1+0.1)} + \frac{27{,}500}{(1+0.1)^2} + \frac{29{,}200}{(1+0.1)^3} + \frac{30{,}900}{(1+0.1)}\right)[A/P,10\%,4] = 28{,}778{,}.93$$

Minimum *EUAC* for Challenger = **$27,761.90** occurs at k = 2.

Economic Service Life of Challenger = **2 years**

(*b*) Optimal replacement time for Defender.

Defender:

The Total Marginal cost of keeping the defender for k more years (k = 0 to 4 years) are given in column (E) below:

	(A)	(B)	(C)	(D)	(E)=(B)+(C)+(D)	(F)
EoY	$MV(k)$	Loss of MV during year k	Cost of capital $i*MV(k\text{-}1)$	Annual expenses $E(k)$	Total Marginal Cost $TC(k)$	$EPC(k)$
0	35000					$277,619.05
1	25,000	10,000	3,500	18,500	32,000	$281,471.86
2	21,000	4,000	2,500	21,000	27,500	$281,255.41
3	17,000	4,000	2,100	23,500	29,600	$282,636.40
4	13,000	4,000	1,700	26,000	31,700	$285,326.17

We note that the defender's $TC(k)$ values are not monotonically non-decreasing in k.
Hence a year-by-year computation of EPC_k if the defender is replaced at EoY k, is done to determine the optimal replacement time.

$$EPC_0 = \frac{27{,}761.90}{0.1} = 277{,}619.05$$

$$EPC_1 = \frac{32{,}000}{(1+0.1)} + \frac{27{,}761.90/0.1}{(1+0.1)} = 281{,}471.86$$

$$EPC_2 = \frac{32{,}000}{(1+0.1)} + \frac{27{,}500}{(1+0.1)^2} + \frac{27{,}761.90/0.1}{(1+0.1)^2} = 281{,}255.41$$

$$EPC_3 = \frac{32{,}000}{(1+0.1)} + \frac{27{,}500}{(1+0.1)^2} + \frac{29{,}600}{(1+0.1)^3} + \frac{27{,}761.90/0.1}{(1+0.1)^3} = 282{,}636.40$$

$$EPC_4 = \frac{32{,}000}{(1+0.1)} + \frac{27{,}500}{(1+0.1)^2} + \frac{29{,}600}{(1+0.1)^3} + \frac{31{,}700}{(1+0.1)^4} \frac{27{,}761.90/0.1}{(1+0.1)^4} = 285{,}326.17$$

Results are summarized in Column (F).

Minimum EPC_k (Opportunity Cost) = \$277,619.05 occurs at $k = 0$

Hence the Defender should be replaced **immediately** by the Challenger.
The Challenger is then repeated every 2 years under the repeatability assumption.

EUAC (Cash flow) over study period infinity = (277,619.05 – 35,000) 0.1 = \$24,261.90

Question 4.

MARR = 10%

(*a*) Economic Service Life of Challenger

The *EUAC* of using the challenger for *k* years (*k*=1 to 4) are as follows:

	(A)	(B)	(C)	(D)	(E)=(B)+(C)+(D)	(F)
EoY k	MV(k)	Loss of MV during year k	Cost of capital = i*MV(k-1)	Annual expenses E(k)	Total Marginal Cost TC(k)	EUAC(k)
0	70,000					
1	56,000	14,000	7,000	5,500	26,500	26,500.00
2	44,000	12,000	5,600	6,800	24,400	25,500.00
3	34,000	10,000	4,400	7,400	21,800	24,382.18
4	22,000	12,000	3,400	9,700	25,100	24,536.85

$$EUAC_1 = \left(\frac{26{,}500}{(1+0.1)}\right)[A/P,10\%,1] = 26{,}500.00$$

$$EUAC_2 = \left(\frac{26{,}500}{(1+0.1)} + \frac{24{,}400}{(1+0.1)^2}\right)[A/P,10\%,2] = 25{,}500.00$$

$$EUAC_3 = \left(\frac{26{,}500}{(1+0.1)} + \frac{24{,}400}{(1+0.1)^2} + \frac{21{,}800}{(1+0.1)^3}\right)[A/P,10\%,3] = 24{,}382.18$$

$$EUAC_4 = \left(\frac{26{,}500}{(1+0.1)} + \frac{24{,}400}{(1+0.1)^2} + \frac{21{,}800}{(1+0.1)^3} + \frac{25{,}100}{(1+0.1)^4}\right)[A/P,10\%,4] = 24{,}536.85$$

Minimum *EUAC* for Challenger = $EUAC^*$ = **$24,382.18** occurs at $k = 3$.
Economic Service Life of Challenger = **3 years**

(*b*) Optimal replacement time for Defender under infinite planning horizon.

The Total Marginal cost of keeping the defender for *k* more years ($k = 0$ to 4 years) are given in column (E) below:

	(A)	(B)	(C)	(D)	(E)=(B)+(C)+(D)
EoY	*MV*(*k*)	Loss of MV during year *k*	Cost of capital = *i***MV*(*k*-1)	Annual expenses *E*(*k*)	Total Marginal Cost *TC*(*k*)
0	40,000				
1	30,000	10,000	4,000	8,500	22,500
2	20,000	10,000	3,000	10,500	23,500
3	12,000	8,000	2,000	14,000	24,000
4	4,000	8,000	1,200	16,000	25,200

The defender's TC_k values are monotonically non-decreasing in *k* and

$$TC_3 \text{ of defender } (24{,}000) < EUAC^* \text{ of challenger } (24{,}382.18) < TC_4 \text{ of defender } (25{,}200)$$

Hence the Defender should be replaced by the Challenger at EoY 3.
The Challenger is then repeated every 3 years.

(*c*) Optimal replacement plan under finite planning horizon.

MARR = 10%
Study period = 4 years.
Challenger can be repeated only once within the next 4 years.

Current *MV* of defender = \$40,000.

	TC_1	TC_2	TC_3	TC_4
Defender	22,500	23,500	24,000	25,200
Challenger	26,500	24,400	21,800	25,100

Let a replacement plan be represented by the tuple (k_1, k_2, k_3) where

k_1 = number of years the defender is kept in use before being replaced by a challenger.
k_2 = number of years the first challenger is used before being replaced by another identical challenger
k_3 = number of years the second challenger is used (if at all).

The 11 possible replacement plans and the EPC and EUAC of each plan are given below:

	Def	**C1**	C2	Marginal Cost for the year					
Plan	k1	k2	k3	1	2	3	4	***EPC***	***EUAC***
1	0	1	3	26,500	26,500	24,400	21,800	79,213.51	24,989.55
2	0	2	2	26,500	24,400	26,500	24,400	80,831.57	25,500.00
3	0	3	1	26,500	24,400	21,800	26,500	78,734.72	24,838.50
4	0	4	0	26,500	24,400	21,800	25,100	77,778.50	24,536.85
5	1	1	2	22,500	26,500	26,500	24,400	78,930.74	24,900.34
6	1	2	1	22,500	26,500	24,400	26,500	78,787.31	24,855.10
7	1	3	0	22,500	26,500	24,400	21,800	75,577.15	23,842.38
8	2	1	1	22,500	23,500	26,500	26,500	77,885.73	24,570.67
9	2	2	0	22,500	23,500	26,500	24,400	76,451.40	24,118.19
10	3	1	0	22,500	23,500	24,000	26,500	76,007.44	23,978.13
11	4	0	0	22,500	23,500	24,000	25,200	75,119.53	23,698.02

- The optimal plan is (4, 0, 0) which is to keep the defender for 4 more years.
- *EPC*(Opportunity Cost) of optimal plan = \$75,119.53
- *EUAC*(Opportunity Cost) of optimal plan = 75,119.53[*A*/*P*, 10%, 4] = \$ 23,698.02
- *EUAC*(Cash Flows) of optimal plan = (75,119.53 – 40,000) [*A*/*P*, 10%, 4] = \$11,079.19