TIE2140 Engineering Economy
Solutions to Tutorial #5

Question 1.

(***a***)

	Pessimistic	Most likely	Optimistic
Capital Investment	\$ 120,000	\$ 100,000	\$ 90,000
Useful Life	6 years	10 years	12 years
Market Value at EoL	\$ 0	\$ 20,000	\$ 30,000
Net annual cash flow	\$ 20,000	\$ 30,000	\$ 35,000

MARR = 10%

i. When all the factors are at their most likely values:

$AW(10\%) = -100{,}000\,[A/P,10\%,10] + 30{,}000 + 20{,}000\,[A/F,10\%,10] = \$\,14{,}980.37 > 0$

ii. When all the factors are at their optimistic values:

$AW(10\%) = -90{,}000\,[A/P,10\%,12] + 35{,}000 + 30{,}000\,[A/F,11\%,12] = \$\,23{,}194.20 > 0$

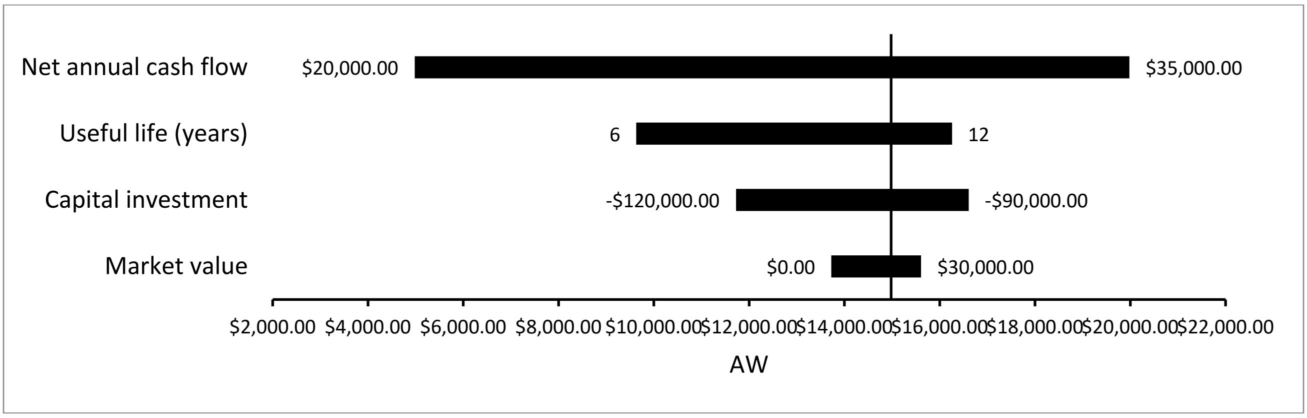
iii. When all the factors are at their pessimistic values:

$AW(10\%) = -120{,}000\,[A/P, 10\%,6] + 20{,}000 + 0 = -\$\,7{,}552.89 < 0$

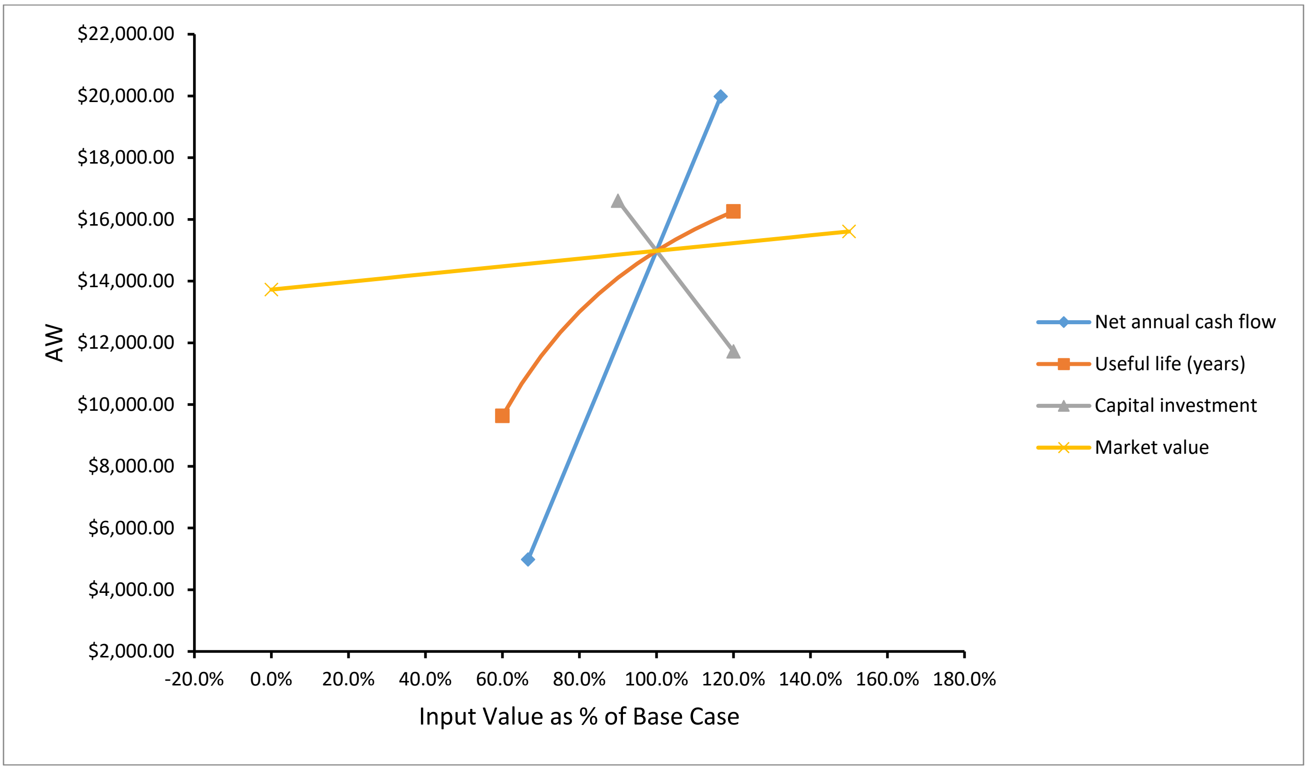
(***b***) **One-Way Range Sensitivity Table**:

				AW		
Variable	**Pessimistic**	**Most-likely**	**Optimistic**	**Low**	**High**	**Swing**
Net annual cash flow	\$ 20,000	\$ 30,000	\$ 35,000	\$4,980.37	\$19,980.37	\$15,000.00
Useful life (years)	6	10	12	\$9,631.41	\$16,258.93	\$6,627.52
Capital investment	-\$120,000	-\$100,000	-\$90,000	\$11,725.46	\$16,607.82	\$4,882.36
Market value	\$ 0	\$ 20,000	\$ 30,000	\$13,725.46	\$15,607.82	\$1,882.36

Tornado Diagram for *AW*



Spider Diagram for *AW*



(*c*) Identification of Sensitive and Non-sensitivity factors:

- The project *AW* is most sensitive to Net Annual Cash Flow, followed by Useful Life, and Capital Investment.
- Market Value at EoL is not very sensitive.

Question 2.

(*a*)

	Winshear	Blowby	Air-vantage
Capital Investment	\$1,000	\$400	\$1,200
Drag reduction	20%	10%	25%
Maintenance per year	\$10	\$5	\$5
Useful life	10 years	10 years	5 years

Study period = 10 years. Assume repeatability.

Let X = the number of miles driven per year by a tractor.

Reductions in fuel consumption for each alternative:

- Windshear: (20 % / 5 %) (2 % per miles) = 8% per mile
- Blowby: (10 % / 5 %) (2 % per miles) = 4% per mile
- Air-vantage: (25 % / 5 %) (2 % per miles) = 10% per mile

Annual Fuel Costs for each alternative:

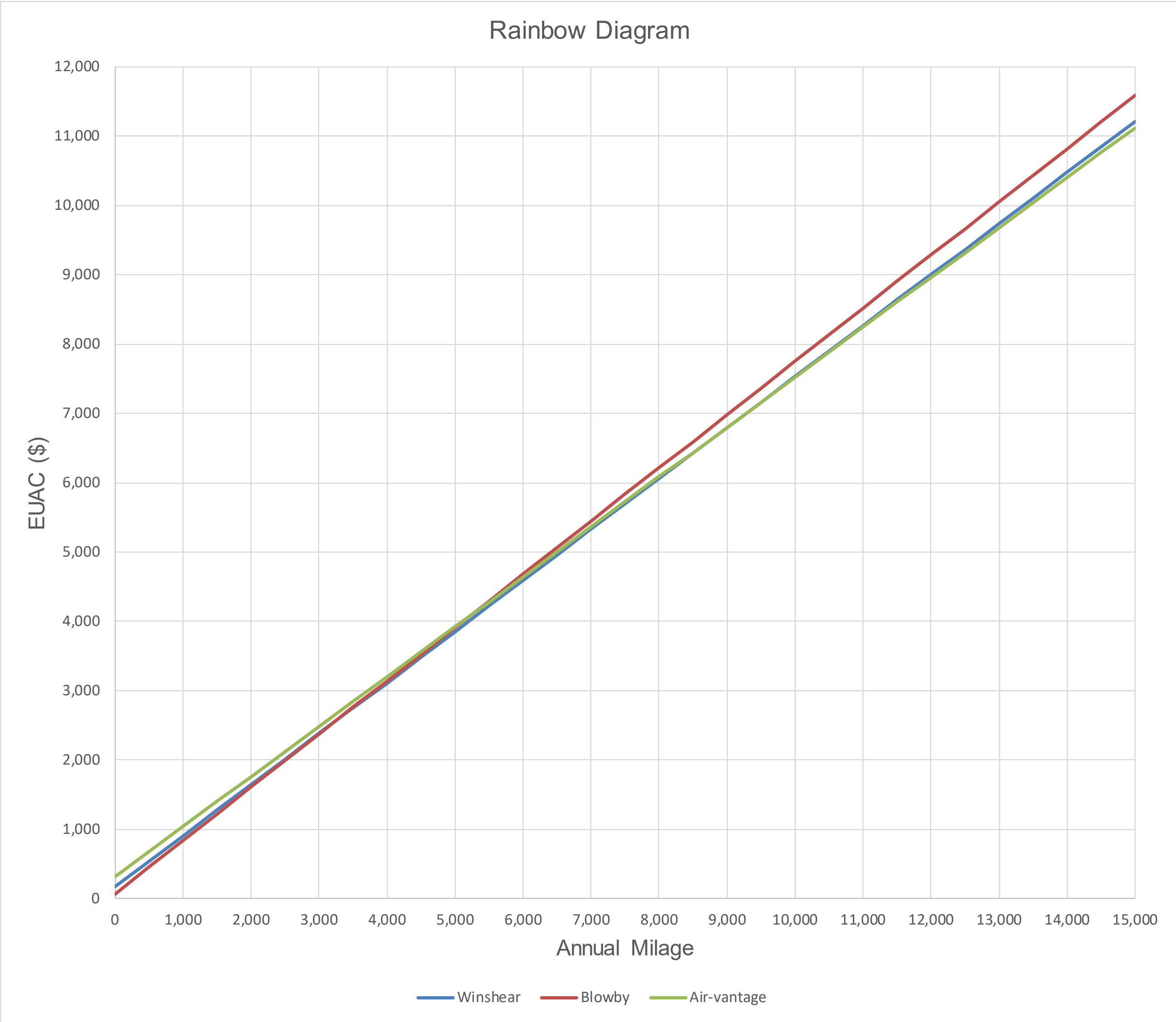
- Windshear: 0.92 X (mile/year) × 1/5 (gallon/mile) × \$4.00 (per gallon) = \$0.736X
- Blowby: 0.96 X (mile/year) × 1/5 (gallon/mile) × \$4.00 (per gallon) = \$0.768X
- Air-vantage: 0.90 X (mile/year) × 1/5 (gallon/mile) × \$4.00 (per gallon) = \$0.720X

Equivalent Uniform Annual Cost (EUAC) for each alternative:

- $EUAC$(Windshear) = 1,000 [A/P, 10%, 10] + 10 + 0.736 X
  = 1,000 (0.162745395) + 10 + 0.736 X
  = 172.74539 + 0.736 X

- $EUAC$(Blowby) = 400 [A/P, 10%, 10] + 5 + 0.768 X
  = 400 (0.162745395) + 5 + 0.768 X
  = 70.09816 + 0.768 X

- $EUAC$(Air-vantage)= 1,200 [A/P, 10%, 5] + 5 + 0.720 X
  = 1,200 (0.263797481) + 5 + 0.720 X
  = 321.55698 + 0.720 X



(*b*)

To determine the breakpoint value between Blowby and Windshear, we solve

$$
\begin{aligned}
EUAC(\text{Blowby}) &= EUAC(\text{Windshear}) \\
70.09816 + 0.768\,X &= 172.74539 + 0.736\,X \\
X &= 3{,}207.73
\end{aligned}
$$

To determine the breakpoint value between Windshear and Air-vantage, we solve

$$
\begin{aligned}
EUAC(\text{Windshear}) &= EUAC(\text{Air-vantage}) \\
172.74539 + 0.736\,X &= 321.55698 + 0.720\,X \\
X &= 9{,}300.73
\end{aligned}
$$

Optimal Decision Rule

Miles driven per year	Optimal Choice
$0 < X \le 3{,}207.73$	Blowby
$3207.73 \le X \le 9{,}300.73$	Windshear
$9{,}300.73 \le X$	Air-vantage

Question 3.

	Alternative 1	**Alternative 2**
Capital Investment	\$ 4,500	\$ 6,000
Annual revenues	\$ 1,600	\$ 1,850
Annual expenses	\$ 400	\$ 500
Estimated market value	\$ 800	\$ 1,200
Useful life	8 years	10 years

MARR = 15%.

(*a*) Study period = 40 years. Assume repeatability.

AW(15%) of Alternative 1 over 40-year study period
= *AW*(15%) of Alternative 1 over first 8 years
= – 4,500 [*A*/*P*, 15%, 8] + 1,600 – 400 + 800 [*A*/*F*, 15%,8]
= \$ <u>255.45</u>

AW(15%) of Alternative 2 over 40-year study period
= *AW*(15%) of Alternative 2 over 40-year study period
= – 6,000 [*A*/*P*, 15%, 10] + 1,850 – 500 + 1,200 [*A*/*F*, 15%, 10]
= \$ <u>213.59</u>

Select **<u>Alternative 1</u>** with higher *AW* over study period 40 years.

(*b*)

Let I_2 = Capital cost of Alternative 2.

$$AW_2(15\%, I_2) = -I_2\,[A/P, 15\%, 10] + 1{,}850 - 500 + 1{,}200\,[A/F, 15\%, 10]$$

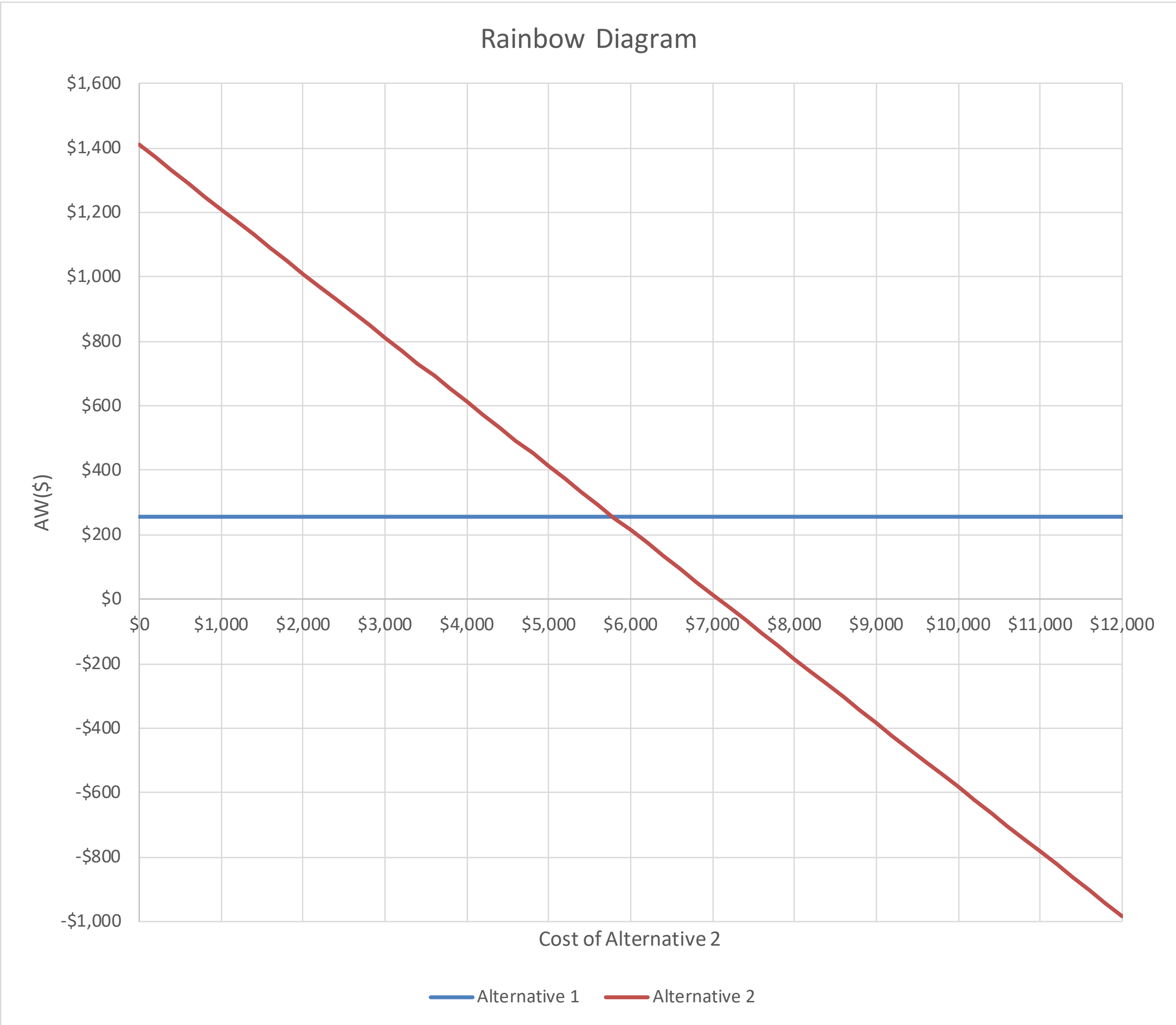
We want to find the value of I_2 such that $AW_2(15\%, , I_2) = AW_1(15\%)$:

$$-\ I_2\,[A/P, 15\%, 10] + 1{,}850 - 500 + 1{,}200\,[A/F, 15\%, 10] = 255.45$$

$$\Rightarrow \quad I_2 = \$\ 5{,}789.89$$

Change in capital cost of Alternative 2 required for decision reversal = 5,789.89 – 6,000
= – $ **<u>210.11</u>**

%-change required = – 210.11 / 6,000 = **– <u>3.502</u>%**



(*c*)

Let life of Alternative 1.

$AW_1(15\%, N_1) = -4{,}500\ [A/P, 15\%, N_1] + 1{,}600 - 400 + 800\ [A/F, 15\%, N_1]$

We want to find the value of N_1 such that $AW_1(15\%, N_1) = AW_2(15\%)$:

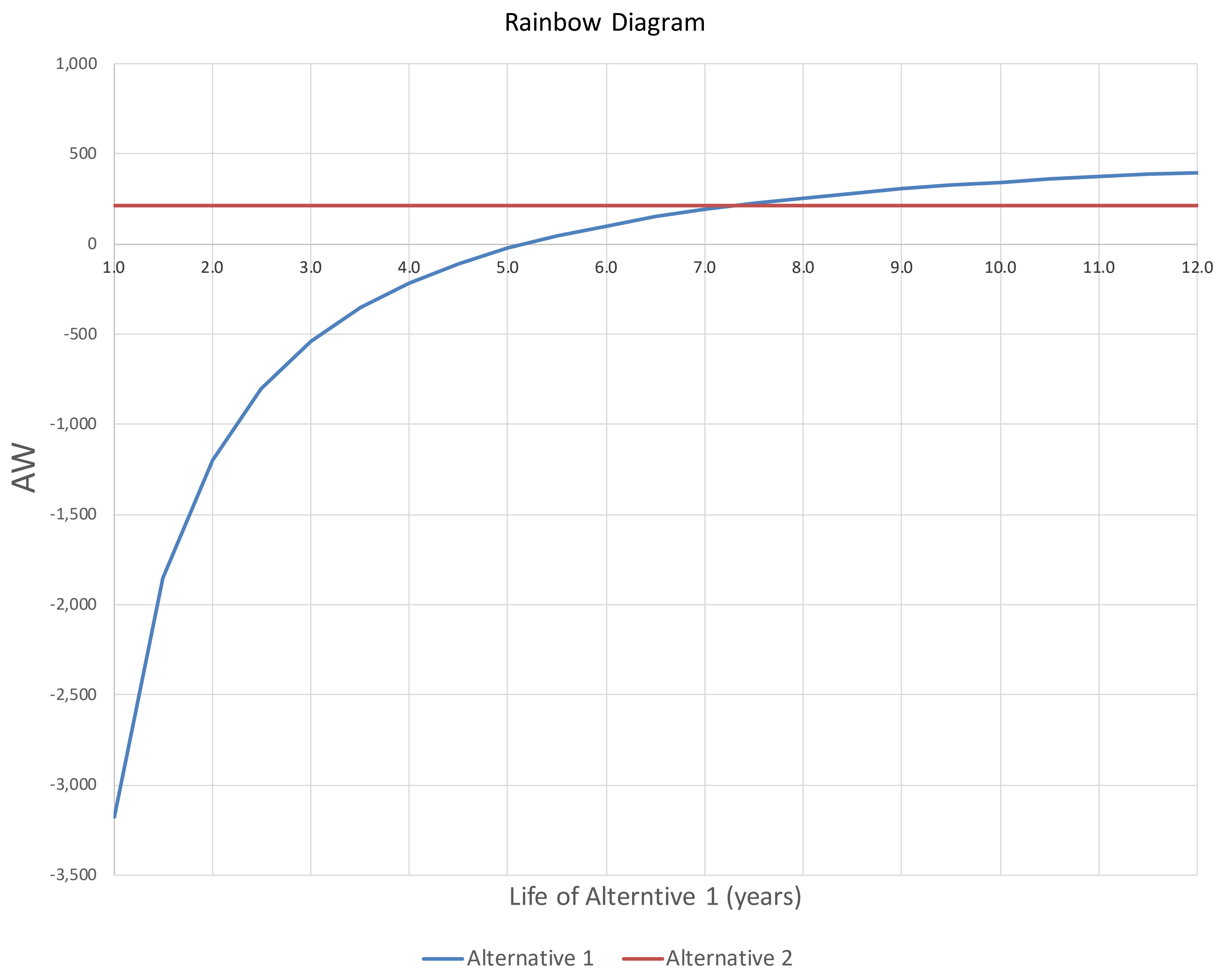
$$-4{,}500\ [A/P, 15\%, N_1] + 1{,}600 - 400 + 800\ [A/F, 15\%, N_1] = 213.59$$
$$-4{,}500\ [A/P, 15\%, N_1] + 986.41 \quad + 800\ [A/F, 15\%, N_1] = 0$$

Using Excel NPER function

N_1 =NPER(0.15, 986.41, -4500, 800, 0) = 7.321 years

Change in useful life of Alternative 1 required for decision reversal = 7.321 – 8 = – 0.679 years

%-change required = – 0.679 / 8 = **– 8.48%**



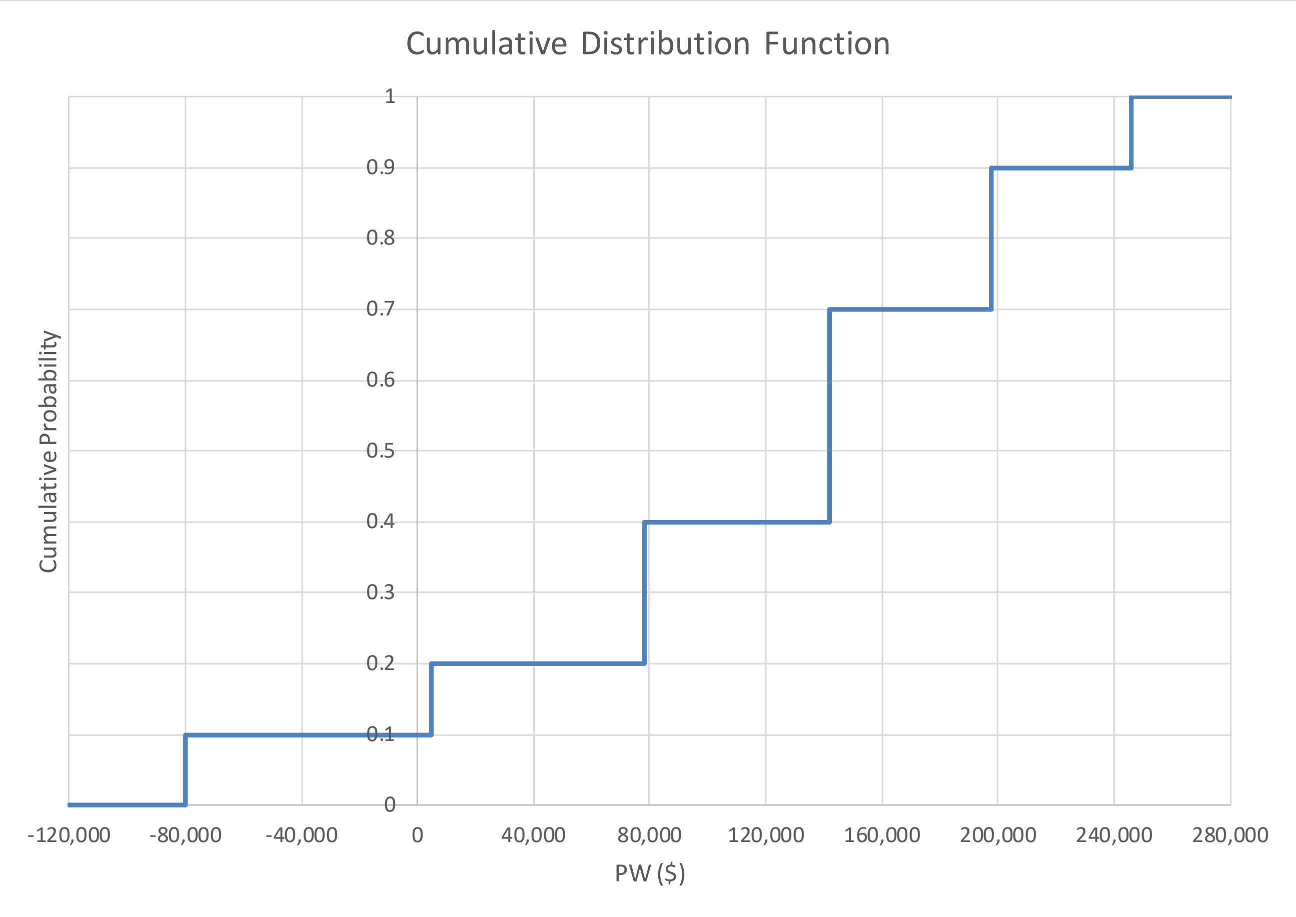
Question 4.

(*a*) $PW(N) = -500{,}000 + 200{,}000\,[P/A,\ 12\%,\ N]$

N	p	PW	$p \times PW$	$PW - E[PW]$	$(PW - E[PW])^2$
3	0.1	-80,082.68	-8,008.27	-194,943.70	3,800,304,582.45
4	0.1	4,536.80	453.68	-110,324.22	1,217,143,324.65
5	0.2	78,118.95	15,623.79	-36,742.06	269,995,823.16
6	0.3	142,103.44	42,631.03	27,242.42	222,644,871.05
7	0.2	197,742.12	39,548.42	82,881.10	1,373,855,485.38
8	0.1	246,123.58	24,612.36	131,262.57	1,722,986,141.40
			114,861.02		8,606,930,228.10

$E[PW]$ = \$ 114,861.02
$Var[PW]$ = \$\$ 8,606,930,228.10
$\sigma[PW]$ = \$ 92,773.54

(*b*) The CDF for the PW is plotted below:



(*c*)

i. Downside Risk of Project = Prob$\{PW < 0\}$ = **0.1**

ii. Chance of achieving an upside potential of $PW \geq \$150{,}000$ = 1 – 0.7 = **0.3**

iii. Present Equivalent Value-at-Risk (95% confidence) = – (–80,082.68) = **\$ 80,082.68**

Question 5.

$MARR$ = 15%.

	Alternative A		Alternative B	
EoY	Expected Cash Flow ($)	Std Dev. of Cash Flow ($)	Expected Cash Flow ($)	Std Dev. of Cash Flow ($)
0	-8,000	0	-12,000	500
1	4,000	600	4,500	300
2	6,000	600	4,500	300
3	4,000	800	4,500	300
4	6,000	800	4,500	300

(*a*) When the cash flows are mutually independent:

For Investment A:

$$E[PW(A)] = -8{,}000 + \frac{4{,}000}{(1+0.15)} + \frac{6{,}000}{(1+0.15)^2} + \frac{4{,}000}{(1+0.15)^3} + \frac{6{,}000}{(1+0.15)^4}$$
$$= \$6{,}075.71$$

$$Var[PW(A)] = 0 + \frac{600^2}{(1+0.15)^2} + \frac{600^2}{(1+0.15)^4} + \frac{800^2}{(1+0.15)^6} + \frac{800^2}{(1+0.15)^8}$$
$$= \$\$\ 963{,}949.69$$

$$\sigma[PW(A)] = \sqrt{963{,}949.69}$$
$$= \$\,981.81$$

For Investment B:

$$E[PW(B)] = -12{,}000 + \frac{4{,}500}{(1+0.15)} + \frac{4{,}500}{(1+0.15)^2} + \frac{4{,}500}{(1+0.15)^3} + \frac{4{,}500}{(1+0.15)^4}$$
$$= \$847.40$$

$$Var[PW(B)] = 500^2 + \frac{300^2}{(1+0.15)^2} + \frac{300^2}{(1+0.15)^4} + \frac{300^2}{(1+0.15)^6} + \frac{300^2}{(1+0.15)^8}$$
$$= \$\$\ 437{,}841.37$$

$$\sigma[PW(B)] = \sqrt{437{,}841.37}$$
$$= \$\,661.70$$

Investment A has a higher expected PW than Investment B, but the standard deviation of Invstment A is larger than that of Investment B. Hence the mean-variance criterion is non conclusive.

(*b*) When the cash flows are mutually independent;

$$E[PW(A-B)] = E[PW(A)] - E[PW(B)]$$
$$= 6{,}075.71 - 847.40$$
$$= \$\ 5{,}228.30$$

$$Var[PW(A-B)] = Var[PW(A)] + Var[PW(B)]$$
$$= 963{,}949.69 + 437{,}841.37$$
$$= \$\$\ 1{,}401{,}791.05$$

$$\sigma[PW(A-B)] = \sqrt{1{,}401{,}791.05}$$
$$= \$1{,}183.97$$

(*c*) When not all the cash flows for investment *A* are mutually independent and coefficients of correlations are $\rho_{12} = 0.1$, $\rho_{23} = 0.2$, $\rho_{34} = 0.3$:

$$E[PW(A)] = \$\ 6{,}075.71$$

$$Var[PW(A)] = 0 + \frac{600^2}{(1+0.15)^2} + \frac{600^2}{(1+0.15)^4} + \frac{800^2}{(1+0.15)^6} + \frac{800^2}{(1+0.15)^8}$$
$$+ \frac{2(0.1)(600)(600)}{(1+0.15)^3} + \frac{2(0.2)(600)(800)}{(1+0.15)^5} + \frac{2(0.3)(800)(800)}{(1+0.15)^7}$$
$$= \$\$1{,}251{,}108.61$$

$$\sigma[PW(A)] = \sqrt{1{,}251{,}108.61}$$
$$= \$1{,}118.53$$