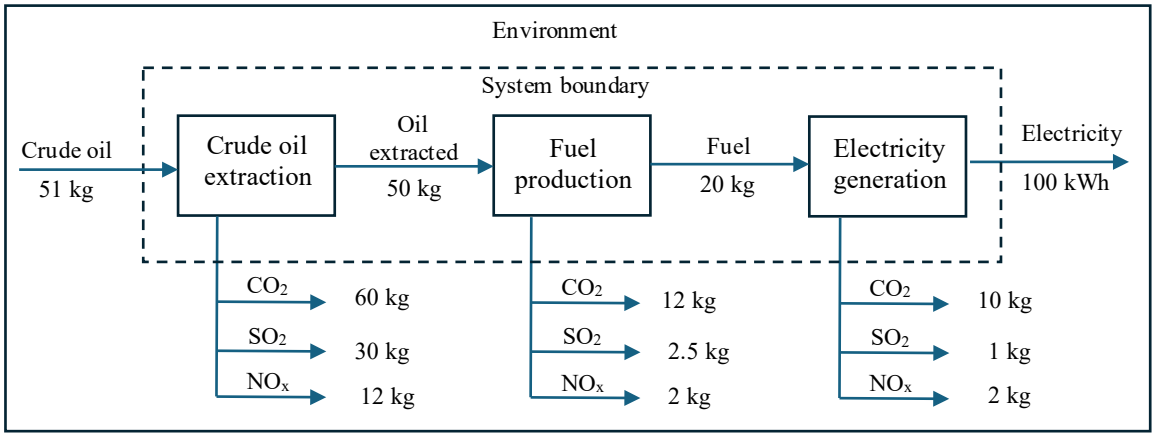
DSS5202 Sustainable Systems Analysis
Solutions to Assignment 1

1. Product System Diagram



2. Scaling factors computations

The scaling factors required to produce 100 kWh of electricity can be determined by working backward from the final product flows:

Electricity generation:

- Electricity output required = 100 kWh
- Scaling factor =100 kWh/10 kWh = **10**.

	Input		Quantity	Ref flow
1	Fuel	kg	2	20
	Output			
1	Electricity	kWh	10	**100**
2	CO_2	kg	1	10
3	SO_2	kg	0.1	1
4	NO_x	kg	0.2	2

Fuel production:

- Fuel output required = 20 kg.
- Scaling factor = 20 kg/20 kg = **1**

	Input		Quantity	Ref flow
1	Oil extracted	kg	50	50
	Output			
1	Fuel	kg	20	**20**
2	CO_2	kg	12	12
3	SO_2	kg	2.5	2.5
4	NO_x	kg	2	2

Crude oil extraction:

- Oil extracted required = 50 kg
- Scaling factor = 50 kg / 100 kg = **0.5**

	Input		Quantity	Ref flow
1	Crude oil	kg	102	51
	Output			
1	Oil extracted	kg	100	50
2	CO_2	kg	120	60
3	SO_2	kg	60	30
4	NO_x	kg	24	12

Scaling factors are:

- Crude oil extraction 0.5
- Fuel production 1
- Electricity generation 10

3. Life Cycle Inventory Analysis

Product Flows

1. Oil extracted 50 kg
2. Fuel oil 20 kg
3. Electricity 100 kWh // functional unit

Elementary Flows

	Elementary flow		Crude oil extraction	Fuel production	Electricity generation	Total Inventory
1	CO_2	kg	60	12	10	82
2	SO_2	kg	30	2.5	1	33.5
3	NO_x	kg	12	2	2	16
4	Crude Oil	kg	51	0	0	51

4. Life Cycle Midpoints Impact Analysis

Let the midpoints impact categories characterization factors matrix be denoted by

$$CF_{\text{midpoint}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0.29 & 0.11 & 0 \\ 0 & 1 & 0.36 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The scores for each midpoint impact category for each production process may be computed by matrix multiplication:

$$\begin{bmatrix} \text{GWP100} \\ \text{HOFP} \\ \text{PMFP} \\ \text{AP} \\ \text{EOFP} \\ \text{FFP} \end{bmatrix} = CF_{\text{midpoint}} \begin{bmatrix} 60 & 12 & 10 & 82 \\ 30 & 2.5 & 1 & 33.5 \\ 12 & 2 & 2 & 16 \\ 51 & 0 & 0 & 51 \end{bmatrix}$$

The results are as shown in the table below:

	Midpoint category		Crude oil extraction	Fuel production	Electricity generation	Total midpoint impact score
1	GWP100	kg CO_2-eq	60	12	10	82
2	HOFP	kg NOx-eq	12	2	2	16
3	PMFP	kg PM2.5 eq	10.02	0.945	0.51	11.475
4	AP	kg SO_2-eq;	34.32	3.22	1.72	39.26
5	EOFP	kg NOx-eq	12	2	2	16
6	FFP	kg oil-eq	51	0	0	51

We observe that Crude oil extraction has the most significant environmental impacts compared to fuel production and electricity generation.

5. Life Cycle Endpoints Impact Analysis

Let the midpoints to endpoint areas of protection characterization factors matrix be

$$CF_{end} = \begin{bmatrix} 9.28E-07 & 9.10E-07 & 6.29E-04 & 0 & 0 & 0 \\ 2.80E-09 & 0 & 0 & 2.12E-07 & 1.29E-07 & 0 \\ 7.65E-14 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.4566 \end{bmatrix}$$

The scores for the end impact category for each production process may be computed by matrix multiplication:

$$\begin{bmatrix} \text{Human Health} \\ \text{Terestrial ecosystems} \\ \text{Freshwater ecosystems} \\ \text{Resource} \end{bmatrix} = CF_{\text{end}} \begin{bmatrix} 60 & 12 & 10 & 82 \\ 12 & 2 & 2 & 16 \\ 10.02 & 0.945 & 0.51 & 11.475 \\ 34.32 & 3.22 & 1.72 & 39.26 \\ 12 & 2 & 2 & 16 \\ 51 & 0 & 0 & 51 \end{bmatrix}$$

The endpoint impact scores are:

	Area of protection	Unit	Crude oil extraction	Fuel production	Electricity generation	Total endpoint score
1	Human health	DALY	6.37E-03	6.07E-04	3.32E-04	7.31E-03
2a	Terrestrial ecosystems	Species.year	8.99E-06	9.74E-07	6.51E-07	1.06E-05
2b	Freshwater ecosystems	Species.year	4.59E-12	9.18E-13	7.65E-13	6.273E-12
3	Resource	USD2013	2.33E+01	0	0	2.33E+01

Again, we observe that Crude oil extraction has the most significant environmental impacts compared to fuel production and electricity generation.

Consolidated Endpoint Areas of Protection Impact Scores

	Area of protection	Unit	Crude oil extraction	Fuel production	Electricity generation	Total endpoint score
1	Human health	DALY	6.37E-03	6.07E-04	3.32E-04	7.31E-03
2	Ecosystem protection	Species.year	8.99E-06	9.74E-07	6.51E-07	1.06E-05
3	Resource	USD2013	2.33E+01	0.00E+00	0.00E+00	2.33E+01

6. Crude oil extraction with electricity input:

	Input		Quantity
1	Crude oil	kg	102
2	Electricity	kWh	2
	Output		
1	Oil extracted	kg	100
2	CO2	kg	120
3	SO2	kg	60
4	NOx	kg	24

Let s_1 = scaling factor for Crude oil extraction
s_2 = scaling factor for Fuel production
s_3 = scaling factor for Electricity generation

Based on product flow balances between processes to achieve the functional unit:

$$\begin{aligned} 100\,s_1 - 50\,s_2 &= 0 && \text{// oil extracted} \\ 20\,s_2 - 2\,s_3 &= 0 && \text{// fuel} \\ -2\,s_1 + 10\,s_3 &= 100 && \text{// energy for crude oil extraction} \end{aligned}$$

In matrix notations:

$$\begin{bmatrix} 100 & -50 & 0 \\ 0 & 20 & -2 \\ -2 & 0 & 10 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 100 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 100 & -50 & 0 \\ 0 & 20 & -2 \\ -2 & 0 & 10 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 100 \end{bmatrix} = \begin{bmatrix} 0.5051 \\ 1.0101 \\ 10.1010 \end{bmatrix}$$

The new scaling factors are:

- Crude oil extraction 0.5051
- Fuel production 1.0101
- Electricity generation 10.1010