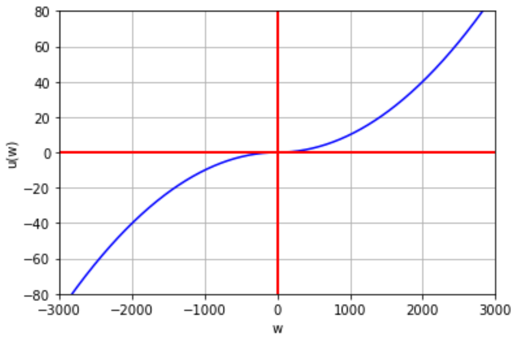
IE5203 Decision Analysis
Solutions to Assignment #2

(*a*) Anna's wealth utility function:

$$u(w) = \begin{cases} \frac{w^2}{100{,}000} & w \geq 0 \\ \frac{-w^2}{100{,}000} & w < 0 \end{cases}$$



$$u'(w) = \begin{cases} \frac{2w}{100{,}000} & w \geq 0 \\ \frac{-2w}{100{,}000} & w < 0 \end{cases}$$

$$u''(w) = \begin{cases} \frac{2}{100{,}000} & w \geq 0 \\ \frac{-2}{100{,}000} & w < 0 \end{cases}$$

Risk tolerance $\rho(w) = \dfrac{-u'(w)}{u''(w)} = -w$ for all w

At current wealth of \$2,200, Anna's risk tolerance = – **<u>\$2,200</u>**.

(*b*) Anna is currently **<u>Risk-Seeking</u>** in attitude as her risk tolerance is negative.

(*c*) Let Anna's PIBP for Deal $A = b_1$.

<u>Don't Buy</u> <u>Buy Deal A</u>

2,200 ~ A: with probability 0.7: $2{,}200 - b_1 + 1{,}500$; with probability 0.3: $2{,}200 - b_1 - \$200$

Equating the utility of not buying A with the expected utility of buying A

$$u(2{,}200) = 0.7\, u(2{,}200 - b_1 + 1{,}500) + 0.3\, u(2{,}200 - b_1 - 200)$$
$$u(2{,}200) = 0.7\, u(3{,}700 - b_1) + 0.3\, u(2{,}000 - b_1)$$

Assuming $b_1 \leq 2{,}000$

$$2{,}200^2 = 0.7\,(3{,}700 - b_1)^2 + 0.3\,(2{,}000 - b_1)^2$$

Using an equation solver: $b_1 = \$1{,}132.55$

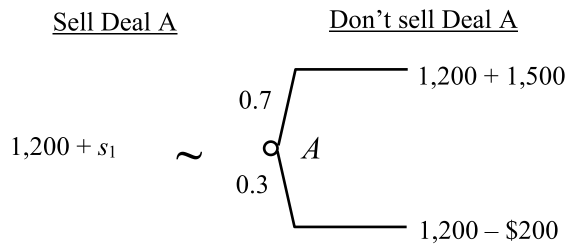
Hence Anna's PIBP for Deal A = **<u>\$ 1,132.55</u>**

(*d*) Anna paid \$1,000 for Deal A.

Her new wealth = \$2,200 – \$1,000 = \$1,200 plus Deal A.

To find Anna's risk tolerance in this situation, we need to find her CE for Deal A.

Let Anna's CE = PISP for Deal $A = s_1$



Equating the utility of selling A to the expected utility of not selling A:

$$\begin{aligned} u(1{,}200 + s_1) &= 0.7\, u(2{,}700) + 0.3\, u(1{,}000) \\ &= 0.7\,(72.900) + 0.3\,(10.000) \\ &= 54.030 \end{aligned}$$

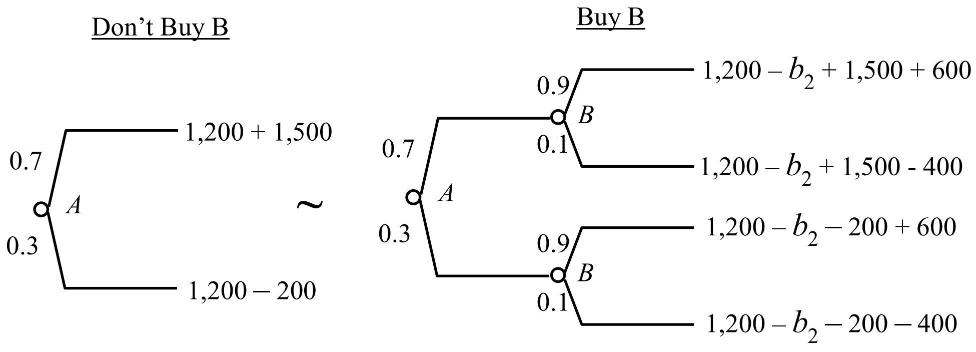
Assume $s_1 \geq -1{,}200$: $(1{,}200 + s_1)^2 = 54.030 \times 100{,}000$

$s_1 = \$1{,}124.44$

Anna's wealth certainty equivalent = 1,200 + 1,124.44 = \$2,324.44

Anna's risk tolerance = **- \$<u>2,324.44</u>**

($\boldsymbol{e}$) Let Anna's PIBP for Deal $B = b_2$



Equating the expected utility of not buying Deal B with the expected utility of buying Deal B:

$$0.7\, u(1{,}200 + 1{,}500) = 0.3\, u(1{,}200 - 200) = \\ 0.7\,(0.9\, u(1{,}200 - b_2 + 1{,}500 + 600) + 0.1\, u(1{,}200 - b_2 + 1500 - 400)) + \\ 0.3\,(0.9\, u(1{,}200 - b_2 - 200 + 600) + 0.1\, u(1{,}200 - b_2 - 200 - 400))$$

$$0.7\, u(2{,}700) + 0.3\, u(1{,}000) = \\ 0.7\,(0.9\, u(3{,}300 - b_2) + 0.1\, u(2{,}300 - b_2)) + 0.3\,(0.9\, u(1{,}600 - b_2) + 0.1\, u(600 - b_2))$$

Assume $b_2 \leq 600$:

$$0.7\,(2{,}700)^2 + 0.3\,(1{,}000)^2 = \\ 0.7\,(0.9\,(3{,}300 - b_2)^2 + 0.1\,(2{,}300 - b_2)^2) + 0.3\,(0.9\,(1{,}600 - b_2)^2 + 0.1\,(600 - b_2)^2)$$

Using an equation solver: $b_2 =$ \$ 520.65

Anna's PIBP for Deal B = **\$ <u>520.65</u>**

(f)

Charlie is risk-neutral.

Charlie's PISP for Deal $B = EV(B) = 0.9\,(600) + 0.1\,(-400) = \$500.00$ which is less than Anna's PIBP of \$520.65.

Hence a transaction on the sale of Deal B between Anna and Charlie at a price between \$500.00 and \$520.65 is possible.