TIE4203 Decision Analysis in Industrial & Operations Management
Solutions to Tutorial #9

Question 1 (P9.1)

Computing the main criteria weights w.r.t. Goal:

Goal	Cost	User-friendliness	Software availability	***w*** (exact)	***w*** (RGM)
Cost	1	1/4	1/5	0.09739	0.09739
User-friendliness	4	1	1/2	0.33307	0.33307
Software availability	5	2	1	0.56954	0.56954

$\lambda_{max} = 3.0246$ CR = 0.0212 < 0.10

Computing the Alternative local weights w.r.t. each main criterion:

Cost	Computer 1	Computer 2	Computer 3	***w*** (exact)	***w*** (RGM)
Computer 1	1	3	5	0.64833	0.64833
Computer 2	1/3	1	2	0.22965	0.22965
Computer 3	1/5	1/2	1	0.12202	0.12202

$\lambda_{max} = 3.0037$ CR = 0.00318 < 0.10

User-friendliness	Computer 1	Computer 2	Computer 3	***w*** (exact)	***w*** (RGM)
Computer 1	1	1/3	1/2	0.14662	0.14662
Computer 2	3	1	5	0.65707	0.65707
Computer 3	2	1/5	1	0.19631	0.19631

$\lambda_{max} = 3.16323$ CR = **0.14072** > 0.10

Software availability	Computer 1	Computer 2	Computer 3	***w*** (exact)	***w*** (RGM)
Computer 1	1	1/3	1/7	0.08096	0.08096
Computer 2	3	1	1/5	0.18839	0.18839
Computer 3	7	5	1	0.73064	0.73064

$\lambda_{max} = 3.06489$ CR = 0.05594 < 0.10

Global Weights:

Alternative	***w*** (exact)	***w*** (RGM)
Computer 1	0.15809	0.15809
Computer 2	0.34851	0.34851
Computer 3	**0.49340**	0.49340

Note that the RGM approximation method give very accurate results because the matrixes are all of size 3. This will not be so when the matrix sizes are large.

(*a*) The company should choose Computer 3 which has the highest global weight.

(*b*) Only the pair-wise comparison matrix for "user friendliness" has CR > 10%.

(*c*) Transitivity relation is satisfied for all matrices.

Question 2 (P9.2)

(*a*) Computing the main criteria weights:

	Dependability	Qualification	Experience	Quality	***w***
Dependability	1	2	3	4	0.46730
Qualification	1/2	1	2	3	0.27718
Experience	1/3	1/2	1	2	0.16009
Quality	1/4	1/3	1/2	1	0.09543

$\lambda_{max} = 4.03098$, CR=0.011475 < 0.10

The weights for the four criteria are:

- Dependability = 0.46730
- Qualification = 0.27718
- Experience = 0.16009
- Quality = 0.09543

Weights for Ratings under Dependability:

	Outstanding	Average	Unsatisfactory	***w***	Idealized
Outstanding	1	3	7	0.66942	1
Average	1/3	1	3	0.24264	0.36246
Unsatisfactory	1/7	1/3	1	0.08795	0.13138

$\lambda_{max} = 3.00702$, CR =0.006053 < 0.1

Weights for Ratings under Qualification:

	Postgraduate	Graduate	Non-graduate	***w***	Idealized
Postgraduate	1	3	5	0.63699	1
Graduate	1/3	1	3	0.25828	0.40548
Non-graduate	1/5	1/3	1	0.10473	0.16441

$\lambda_{max} = 3.03851$, CR=0.033199 < 0.1

Weights for Ratings under Experience:

	Exceptional	Average	Little	***w***	Idealized
Exceptional	1	5	9	0.75140	1
Average	1/5	1	3	0.17818	0.23713
Little	1/9	1/3	1	0.07042	0.09371

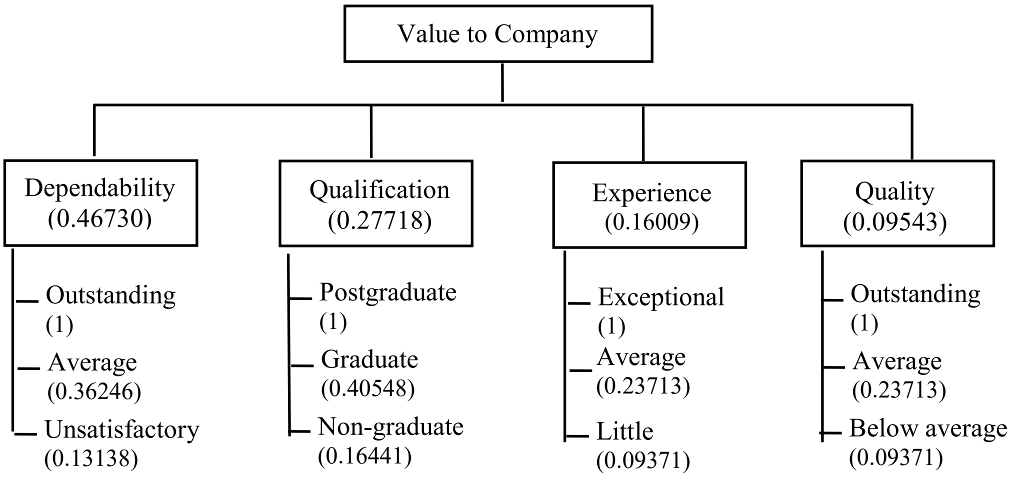
$\lambda_{max} = 3.02906$, CR=0.025055 < 0.1

Weights for Ratings under Quality:

	Outstanding	Average	Below average	***w***	Idealized
Outstanding	1	5	9	0.75140	1
Average	1/5	1	3	0.17818	0.23713
Below average	1/9	1/3	1	0.07042	0.09371

$\lambda_{max} = 3.02906$, CR=0.025055 < 0.1

The Rating System:



(b) Applying the rating systems to John and Bill:

Candidate	**Ratings under Criterion**				**Overall Rating**
	Dependability (0.46730)	**Qualification** (0.27718)	**Experience** (0.16009)	**Quality** (0.09543)	
John	Average (0.36246)	Graduate (0.40548)	Average (0.23713)	Outstanding (1)	0.4145
Bill	Outstanding (1)	Non-graduate (0.16441)	Exceptional (1)	Average (0.23713)	0.6956

- John's overall rating
  = 0.46730 (0.36246) + 0.27718 (0.40548) + 0.16009 (0.23713) + 0.09543 (1)
  = **0. 41516**

- Bill's overall rating
  = 0.46730 (1) + 0.27718 (0.16441) + 0.16009 (1) + 0.09543 (0.23713)
  = **0.69559**

- Hence Bill should get a higher pay increase than John.

Question 3 (P9.3)

(*a*) With only two alternatives (Investments 1 and 2)

Goal	Expected Return	Degree of Risk	Weight
Expected Return	1	1	0.5
Degree of Risk	1	1	0.5

$\lambda_{max} = 2$, CR=0 < 0.1

Expected Return	Investment 1	Investment 2	Weight
Investment 1	1	1/2	0.3333
Investment 2	2	1	0.6667

$\lambda_{max} = 2$, CR=0 < 0.1

Degree of Risk	Investment 1	Investment 2	Weight
Investment 1	1	3	0.7500
Investment 2	1/3	1	0.2500

$\lambda_{max} = 2$, CR=0 < 0.1

Alternative	**Global Weight**
Investment 1	**0.54167**
Investment 2	0.45833

Conclusion: Investment 1 is preferred to Investment 2.

(*b*) With 3 alternatives (Investments 1, 2, & 3)

Expected Return	Investment 1	Investment 2	Investment 3	Weight
Investment 1	1	1/2	4	0.307692
Investment 2	2	1	8	0.615385
Investment 3	1/4	1/8	1	0.076923

$\lambda_{max} = 3$, CR=0 < 0.1

Degree of Risk	Investment 1	Investment 2	Investment 3	Weight
Investment 1	1	3	1/2	0.3
Investment 2	1/3	1	1/6	0.1
Investment 3	2	6	1	0.6

$\lambda_{max} = 3$, CR=0 < 0.1

Alternative	**Global Weight**
Investment 1	0.303846
Investment 2	**0.357692**
Investment 3	0.338462

- Conclusion: Investment 2 is now preferred to Investment 1.

(*c*) Rank Reversal between Investment 1 and Investment 2 has occurred with the introduction of Investment 3 although the pairwise comparison sub-matrices for the first two alternatives have not changed.

(*d*) Using the Ideal Mode

Goal	Expected Return	Degree of Risk	Weight
Expected Return	1	1	0.5
Degree of Risk	1	1	0.5

$\lambda_{max} = 2$, CR=0 < 0.1

With only two alternatives (Investments 1 and 2)

Expected Return	Investment 1	Investment 2	Distributive	Ideal
Investment 1	1	1/2	0.3333	0.5000
Investment 2	2	1	0.6667	1.0000

Degree of Risk	Investment 1	Investment 2	Distributive	Ideal
Investment 1	1	3	0.7500	1.0000
Investment 2	1/3	1	0.2500	0.3333

- Global weights under Ideal Mode before normalization

Alternative	**Global Weight**
Investment 1	**0.7500**
Investment 2	0.6667

- Conclusion: Investment 1 is preferred to Investment 2 under Ideal Mode.

With 3 alternatives (Investments 1, 2, & 3)

Expected Return	Investment 1	Investment 2	Investment 3	Distributive	Ideal
Investment 1	1	1/2	4	0.307692	0.5
Investment 2	2	1	8	0.615385	1
Investment 3	1/4	1/8	1	0.076923	0.125

Degree of Risk	Investment 1	Investment 2	Investment 3	Distributive	Ideal
Investment 1	1	3	1/2	0.3	0.5
Investment 2	1/3	1	1/6	0.1	0.16667
Investment 3	2	6	1	0.6	1

- Global weights under Ideal Mode before normalization

Alternative	**Global Weight**
Investment 1	0.5000
Investment 2	**0.5833**
Investment 3	0.5625

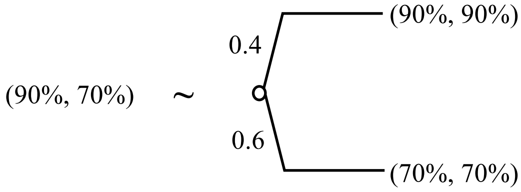
- Investment 2 is now preferred to Investment 1.
- Ideal Mode did not prevent rank reversal for this problem.
- Hence Ideal model does not guarantee rank reversal.

Question 4 (P10.1)

(*a*) Given $u(x_1,x_2) = k_1\, u_1(x_1) + (1 - k_1)\, u_2(x_2)$

$70\% \le x_1 \le 90\%$ and $70\% \le x_1 \le 90\%$.

Let $u(70\%, 70\%) = u_1(70\%) = u_2(70\%) = 0$
$u(90\%, 90\%) = u_1(90\%) = u_2(90\%) = 1$



$\Rightarrow \quad 0.4\, u(90\%, 90\%) + 0.6\, u(70\%, 70\%) = u(90\%, 70\%)$

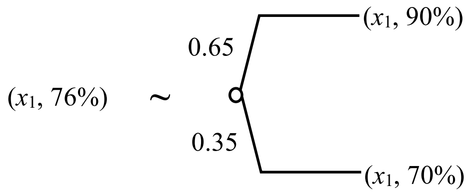
$0.4\, u(90\%, 90\%) + 0.6\, u(70\%, 70\%) = k_1\, u_1(90\%) + (1 - k_1)\, u_2(70\%)$

$0.4\,(1) + 0.6\,(0) = k_1\,(1) + (1 - k_1)\,(0)$

$\Rightarrow \quad k_1 = 0.4$

- Hence the Ministry two-attribute additive utility function is $u(x_1,x_2) = 0.4\, u_1(x_1) + 0.6\, u_2(x_2)$

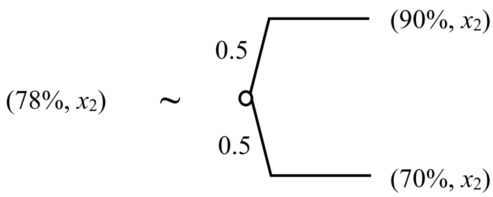
(*b*)



$\Rightarrow 0.65\,[k_1\, u_1(x_1) + k_2\, u_2(90\%)\,] + 0.35\,[k_1\, u_1(x_1) + k_2\, u_2(70\%)\,] = k_1\, u_1(x_1) + k_2\, u_2(76\%)$

$\underline{k}_1\, u_1(x_1) + 0.65\, k_2\, u_1(90\%) + 0.35\, k_2\, u_2(70\%) = \underline{k}_1\, u_1(x_1) + k_2\, u_2(76\%)$

Hence $\underline{u}_2(76\%) = 0.65$.

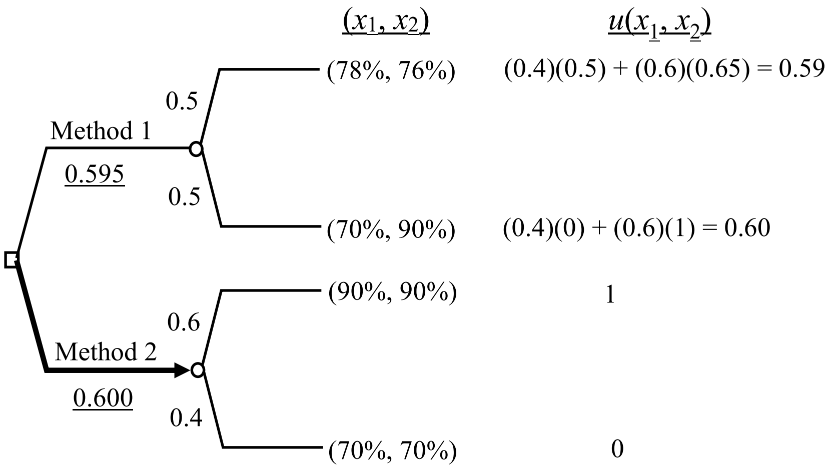


$\Rightarrow 0.5\,[\, k_1\, u_1(90\%) + k_2\, u_2(x_2)\,] + 0.5\,[k_1\, u_1(70\%) + k_2\, u_2(x_2)\,] = k_1\, u_1(78\%) + k_2\, u_2(x_2)$

$0.5\, k_1\, u_1(90\%) + k_2\, u_2(x_2) + 0.5\, k_1\, u_1(70\%) = k_1\, u_1(78\%) + k_2\, u_2(x_2)$

Hence $u_1(78\%) = 0.5$.

- The decision tree for the two teaching techniques is as follows:



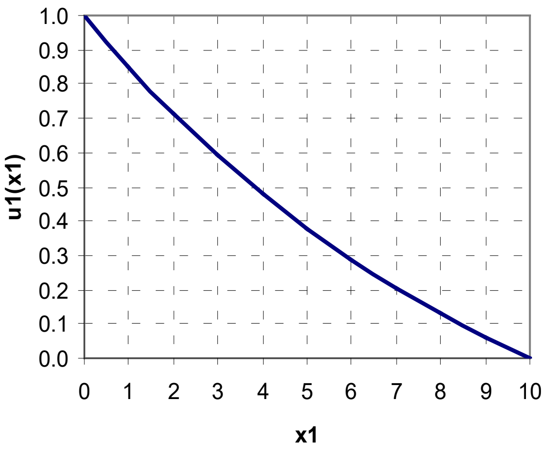
Conclusion: <u>Method 2 is preferred to Method 1.</u>

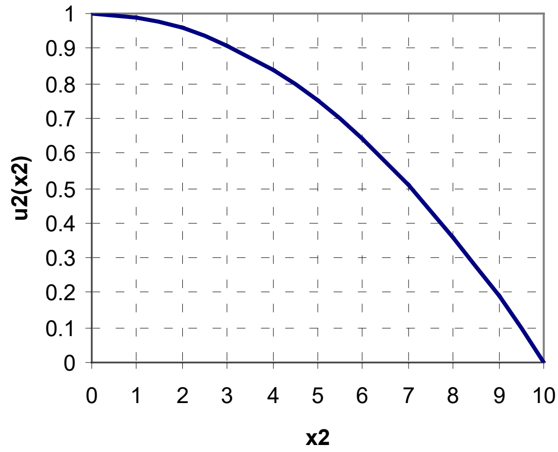
Question 5 (P10.2)

x_1 = Weekly blood shortage ($0 \le x_1 \le 10$)
x_2 = Weekly blood outdated ($0 \le x_2 \le 10$)

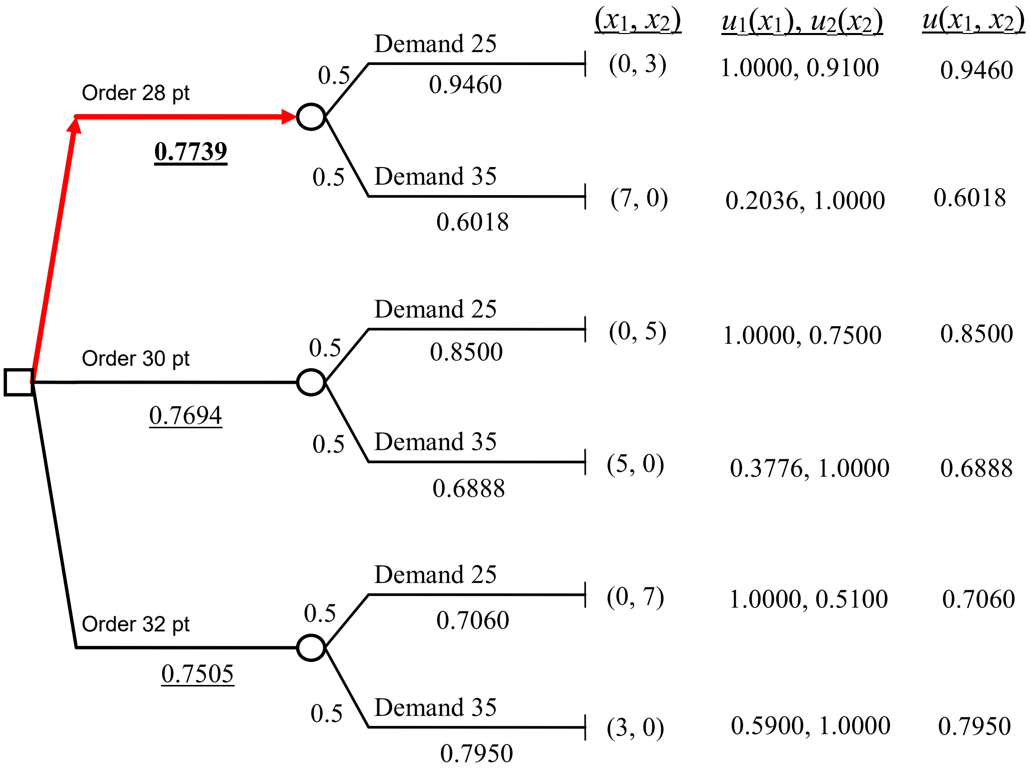
$$u(x_1, x_2) = 0.4\, u_1(x_1) + 0.5\, u_2(x_2) + 0.1\, u_1(x_1)\, u_2(x_2)$$

where $u_1(x_1) = 0.582\left[\exp(1 - \frac{x_1}{10}) - 1\right]$ and $u_2(x_2) = 1 - \frac{x_2^2}{100}$.





The decision tree is



Conclusion: The hospital should order 28 pints of blood weekly.

Question 6 (P10.4)

(*a*) The weights for the main criteria with respect to Goal are computed:

	Human prod	Economics	Design	Operations	Exact w	RGM
Human Productivity	1	3	3	7	0.513052	0.5159
Economics	1/3	1	2	5	0.246592	0.2474
Design	1/3	1/2	1	7	0.193575	0.1903
Operations	1/7	1/5	1/7	1	0.046781	0.0463

λ_{max} = 4.212088, CR= 0.078551 < 10%

The local weights for the alternatives with respect to each criterion are computed:

Human Productivity:

	System A	System B	System C	Exact w	RGM
System A	1	3	5	0.648329	0.6483
System B	1/3	1	2	0.229651	0.2297
System C	1/5	1/2	1	0.122020	0.1220

λ_{max} = 3.003695, CR= 0.003185 < 10%

Economics:

	System A	System B	System C		RGM
System A	1	1/3	1/2	0.157056	0.1571
System B	3	1	3	0.593634	0.5936
System C	2	1/3	1	0.249311	0.2493

λ_{max} = 3.053622 , CR= 0.046225 < 10%

Design:

	System A	System B	System C	Exact w	RGM
System A	1	1/2	1/7	0.093813	0.0938
System B	2	1	1/5	0.166593	0.1666
System C	7	5	1	0.739594	0.7396

λ_{max} = 3.014152, CR= 0.012200 < 10%

Operations:

	System A	System B	System C	Exact w	RGM
System A	1	3	1/5	0.178178	0.1782
System B	1/3	1	1/9	0.070418	0.0704
System C	5	9	1	0.751405	0.7514

λ_{max} = 3.029064, CR= 0.025055 < 10%

The composite weights for the three alternative systems are:

- System A: (0.513052)(0.648329) + (0.246592)(0.157056) + (0.093813)(0.0938) + (0.046781)(0.178178)
  = 0.397850

- System B: (0.513052)(0.229651) + (0.246592)(0.593634) + (0.193575)(0.166593) + (0.046781)(0.070418)
  = 0.299750

- System C: (0.513052)(0.122020) + (0.246592)(0.249311) + (0.193575)(0.739594) + (0.046781)(0.751405)
  = 0.302399

<u>Hence System A should be chosen as it has the highest global weight.</u>

(b)

Alternative	Effectiveness	EUAC ($)
System B	0.397850	80,000
System A	0.299750	100,000
System C	0.302399	110,000

- Efficient Cost-Effective Alternatives are B and A.
- The efficient frontier is shown below: