IE2111 ISE Principles & Practice II
Solutions to Tutorial #3

Question 1.

	A	*B*	*C*
Investment cost	$30,000	$60,000	$50,000
Estimated units to be sold/year	15,000	20,000	18,000
Unit selling price, $/unit	$3.50	$4.40	$4.10
Variable costs, $/unit	$1.00	$1.40	$1.15
Annual expenses (fixed)	$15,000	$30,000	$26,000
Market value	0	$20,000	$15,000
Useful life	10 years	10 years	10 years

$MARR = 20\%$
Study period = 10 years

Assume all units are produced and sold each year.

The AW for the alternatives are:

$$\begin{aligned} AW_A(20\%) &= -30{,}000\,[A/P,20\%,10] + 15{,}000\,(3.50-1.00) - 15{,}000 \\ &= -30{,}000\,(0.238523) \quad + 15{,}000\,(3.50-1.00) - 15{,}000 \\ &= \$15{,}344.32 \end{aligned}$$

$$\begin{aligned} AW_B(20\%) &= -60{,}000\,[A/P,20\%,10] + 20{,}000\,(4.40-1.40) - 30{,}000 + 20{,}000\,[A/F,20\%,10] \\ &= -60{,}000\,(0.238523) \quad + 20{,}000\,(4.40-1.40) - 30{,}000 + 20{,}000\,(0.038523) \\ &= \$16{,}459.09 \end{aligned}$$

$$\begin{aligned} AW_C(20\%) &= -50{,}000\,[A/P,20\%,10] + 18{,}000\,(4.10-1.15) - 26{,}000 + 15{,}000\,[A/F,20\%,10] \\ &= -50{,}000\,(0.238523) \quad + 18{,}000\,(4.10-1.15) - 26{,}000 + 15{,}000\,(0.038523) \\ &= \$15{,}751.70 \end{aligned}$$

Select Design *B* which has the highest AW.

Question 2.

	A	*B*
Capital investment	\$ 272,000	\$346,000
Annual expenses (Y1 to Y9)	\$ 28,800	\$19,300
Useful life	6 years	9 years
Market Value at end of useful life	\$ 25,000	\$40,000
Annual leasing cost for year 7 to 9	\$ 66,000	

$MARR = 15\%$

(*a*) *PW* Method:

$$PW_A(15\%) = -272{,}000 - 28{,}800\,[P/A, 15\%, 9] + 25{,}000\,[P/F, 15\%, 6]$$
$$\quad - 66{,}000\,[P/A, 15\%, 3]\,[P/F, 15\%, 6]$$
$$= -\$\,463{,}762.11$$

$$PW_B(15\%) = -346{,}000 - 19{,}300\,[P/A, 15\%, 9] + 40{,}000\,[P/F, 15\%, 9]$$
$$= -\$\,\underline{\mathbf{426{,}721.07}}$$

Select Alternative *B* which has the highest *PW*.

(*b*) *IRR* Method:

EoY	*A*	*B*	*B - A*
0	-272,000	-346,000	-74,000.
1	-28,800	-19,300	9,500
2	-28,800	-19,300	9,500
3	-28,800	-19,300	9,500
4	-28,800	-19,300	9,500
5	-28,800	-19,300	9,500
6	-28,800 + 25,000 = -3,800	-19,300	-15,500
7	-28,800 – 66,000 = -94,800	-19,300	75,500
8	-28,800 – 66,000 = -94,800	-19,300	75,500
9	-28,800 - 66,000 =-94,800	-19,300 + 40,000 = 20,700	115,500

Incremental *IRR* Analysis:

1. Alternative sorted in increasing capital investment List = [*A, B*]
   We are considering cost projects.
   Base alternative = *A*
   Next alternative = *B*.
   List = [ ]

2. Consider the increment ($B - A$):

   To fine the *IRR* of B-A, we solve:

   $$-74{,}000 + 9{,}500\,[P/A, i, 5] - 15{,}500\,[P/F, i, 6] + 75{,}500\,[P/F, i, 7] + 75{,}500\,[P/F, i, 8] + 115{,}500\,[P/F, i, 9] = 0$$

   Using any solver: $IRR\,(B - A) = 22.51\% > MARR$.

   The incremental investment ($B - A$) is feasible.

3. Base alternative = B.
   Choose Alternative B.

(*c*) If Crane A is leased for 9 years:

$$\begin{aligned} PW_{\text{Lease}}(15\%) &= -\,(66{,}000 + 28{,}800)\,[P/A, 15\%, 9] \\ &= -\,94{,}800\,[P/A, 15\%, 9] \\ &= -\,\$\,452{,}346.16 \end{aligned}$$

Since $PW_A < PW_{\text{Lease}} < PW_B$, leasing crane A is not preferred to the selected Alternatives B, but would be preferred to the purchase of crane A.

Question 3.

	Boiler A	**Boiler B**
Capital investment	\$50,000	\$100,000
Useful life, N	20 years	40 years
Market value at EOY N	\$10,000	\$20,000
Annual operating costs	\$9,000	\$3,000, increasing \$300 per year after first year

$MARR$ = 10%
Study period = 40 years
Assume that Boiler A is repeated at the end of year 20.

AW(10%) of A over 40 years = AW(10%) of A over first 20 years

$$= -50{,}000\,[A/P,10\%,20] + 10{,}000\,[A/F,10\%,20] - \$9{,}000$$
$$= -50{,}000\,(0.117460) + 10{,}000\,(0.017460) - \$9{,}000$$
$$= -\$14{,}698.38$$

AW(10%) of B over 40 years

$$= -100{,}000\,[A/P,10\%,40] + 20{,}000\,[A/F,10\%,40] - 3{,}000 - 300\,[A/G,10\%,40]$$
$$= -100{,}000\,(0.102259) + 20{,}000\,(0.002259) - 3{,}000 - 300\,(9.096234)$$
$$= -\$15{,}909.59$$

Select Boiler A which has a higher AW over the study period.

Note that if PW with repeatability assumption is used instead, then

- $PW_A(10\%)$ = −\$143,736.25 over 2 life cycles or 40 years
- $PW_B(10\%)$ = −\$155,581.01 over 1 life cycle or 40 years

You must get the same decision as in the AW method with repeatability assumption.

Question 4.

	Alternative *A*	Alternative *B*
Capital investment	\$20,000	\$38,000
Annual expenses	5,500	4,000
Market value at end of useful life	1,000	4,200
Useful life	5 years	10 years

$MARR$ = 20%.

(*a*) If the service is needed indefinitely.

Study period = infinity and assume Repeatability.

AW(20%) of A over study period infinity = AW(20%) of A over its first 5 years
= –20,000 [A/P, 20%, 5] – 5,500 + 1,000 [A/F, 20%, 5]
= – 20,000 (0.334380) – 5,500 + 1,000 (0.134380)
= – \$ 12,053.21

AW(20%) of B over study period infinity = AW(20%) of B over its first 10 years
= –38,000 [A/P,20%, 10] – 4,000 + 4,200 [A/F,20%,10]
= –38,000 (0.238523) – 4,000 + 4,200 (0.038523)
= – \$ 12,902.07

Select Alternative A which has a higher AW over study period under repeatability assumption.

(*b*) If the service is required for only 5 years:

Study period = 5 years, and assume co-termination at EoY 5.

AW(20%) of A over study period 5 years = – \$ 12,053.21 as in Part (*a*)

Given market value (MV_5) for B at EoY 5 = \$15,000

AW(20%) of B over study period 5 years
= – 38,000 [A/P, 20%, 5] – 4,000 + 15,000 [A/F, 20%, 5]
= – 38,000 (0.334380) – 4,000 + 15,000 (0.134380)
= – \$14,690.73

Select Alternative A which has higher AW over study period 5 years under co-termination assumption.

Question 5.

Route	Construction costs	Annual maintenance cost	Annual Savings in fire damage	Annual recreational benefit	Annual Time access benefit
A	\$185,000	\$2,000	\$5,000	\$3,000	\$500
B	\$220,000	\$3,000	\$7,000	\$6.500	\$1,500
C	\$290,000	\$4,000	\$12,000	\$6,000	\$2,800

$MARR$ = 8%
Study period = 50 years
Assume zero salvage value for all alternatives.

The initial investment costs, total annual costs, and total annual benefits are:

Route	Initial Cost	Total Annual Cost	Total Annual Benefits
A	\$185,000	\$2,000	\$8,500
B	\$220,000	\$3,000	\$15,000
C	\$290,000	\$4,000	\$20,800

The PW of costs for each alternative:

- PW(8%) of Costs for route A = 185,000 + 2,000 [P/A, 8%, 50] = \$ 209,466.97
- PW(8%) of Costs for route B = 220,000 + 3,000 [P/A, 8%, 50] = \$ 256,700.45
- PW(8%) of Costs for route C = 290,000 + 4,000 [P/A, 8%, 50] = \$ 338,933.94

The PW of benefits for each alternative:

- PW(8%) of Benefits for route A = 8,500 [P/A, 8%, 50] = \$103,984.62
- PW(8%) of Benefits for route B = 15,000 [P/A, 8%, 50] = \$ 183,502.27
- PW(8%) of Benefits for route C = 20,800 [P/A, 8%, 50] = \$ 254,456.48

(*a*)

If this is considered as a cost (service) project and there is no need to consider the "do-nothing" alterative.

Perform incremental $\Delta B/\Delta C$ ratio analysis to find the best alternative.

Alternatives sorted increasing PW of Costs = [A, B, C]

Base alternative = A
Next alternative = B.
List = [C]

$\Delta B/\Delta C$ ($B - A$) = (183,502.27 – 103,984.62) / (256,700.45 – 209,466.97)
= 1.6835 > 1
The incremental investment (B–A) is feasible.

Base alternative = B
Next alternative = C
List = [ ]

$\Delta B/\Delta C$ (C–B) = (254,456.48 – 183,502.27) / (338,933.94 – 256,700.45)
= 0.8628 < 1
The incremental investment (C–B) is infeasible.

Base alternative remains as B.
Choose Route B.

(*b*)

If this problem is considered as an investment project, then economic feasibility is also a requirement.

None of the routes are feasible as the individual B/C ratios are all less than one.

Route	B/C ratio
A	0.4964
B	0.7148
C	0.7508

If the "do-nothing" is included in the analysis, it will be the first base alternative, and each of the incremental investment will be rejected. Do-nothing will then emerge as the best alternative at the end.

Also take note that in Part (a), the best alternative B does not have the maximum B/C ratio.