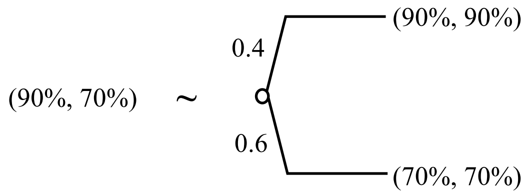
IE5203 Decision Modeling & Risk Analysis
Solutions to Chapter 10 Exercises

P10.1

(*a*) Given $u(x_1,x_2) = k_1\, u_1(x_1) + (1 - k_1)\, u_2(x_2)$

$70\% \le x_1 \le 90\%$ and $70\% \le x_1 \le 90\%$.

Let $u(70\%, 70\%) = u_1(70\%) = u_2(70\%) = 0$
$u(90\%, 90\%) = u_1(90\%) = u_2(90\%) = 1$



$\Rightarrow \quad 0.4\, u(90\%, 90\%) + 0.6\, u(70\%, 70\%) = u(90\%, 70\%)$

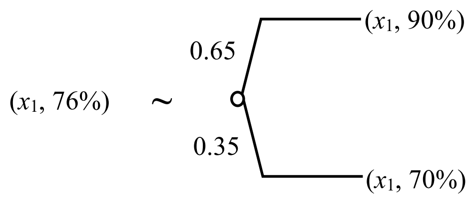
$0.4\, u(90\%, 90\%) + 0.6\, u(70\%, 70\%) = k_1\, u_1(90\%) + (1 - k_1)\, u_2(70\%)$

$0.4\,(1) + 0.6\,(0) = k_1\,(1) + (1 - k_1)\,(0)$

$\Rightarrow \quad k_1 = 0.4$

- Hence the Ministry two-attribute additive utility function is $u(x_1,x_2) = 0.4\, u_1(x_1) + 0.6\, u_2(x_2)$

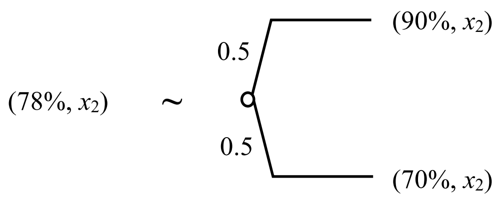
(*b*)



$\Rightarrow 0.65\,[k_1\, u_1(x_1) + k_2\, u_2(90\%)\,] + 0.35\,[k_1\, u_1(x_1) + k_2\, u_2(70\%)\,] = k_1\, u_1(x_1) + k_2\, u_2(76\%)$

$\underline{k}_1\, u_1(x_1) + 0.65\, k_2\, u_1(90\%) + 0.35\, k_2\, u_2(70\%) = \underline{k}_1\, u_1(x_1) + k_2\, u_2(76\%)$

Hence $\underline{u}_2(76\%) = 0.65$.

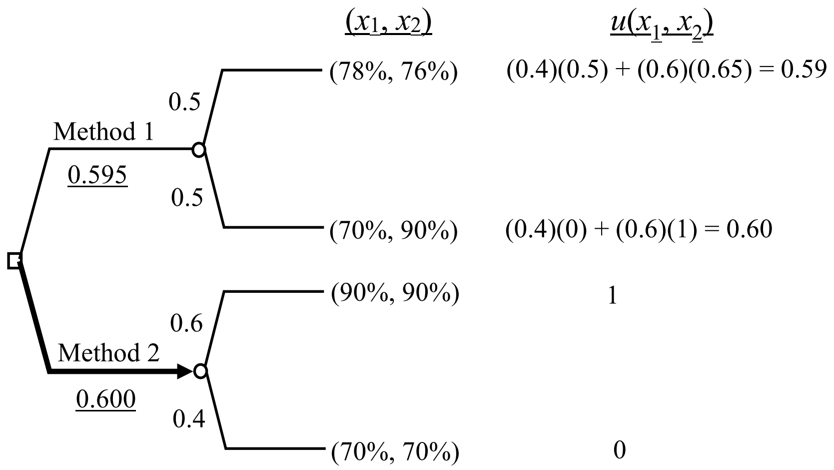


$\Rightarrow 0.5\,[\,k_1\, u_1(90\%) + k_2\, u_2(x_2)\,] + 0.5\,[k_1\, u_1(70\%) + k_2\, u_2(x_2)\,] = k_1\, u_1(78\%) + k_2\, u_2(x_2)$

$0.5\, k_1\, u_1(90\%) + k_2\, u_2(x_2) + 0.5\, k_1\, u_1(70\%) = k_1\, u_1(78\%) + k_2\, u_2(x_2)$

Hence $u_1(78\%) = 0.5$.

- The decision tree for the two teaching techniques is as follows:



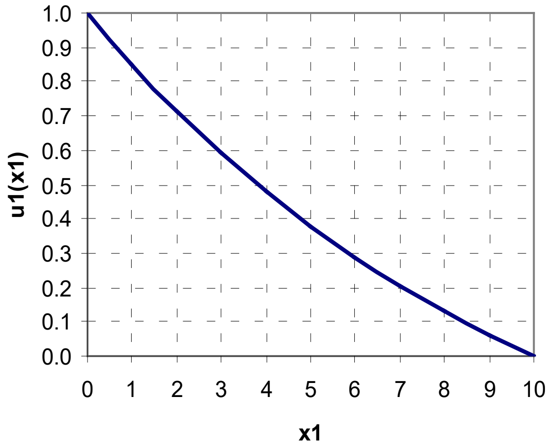
Conclusion: <u>Method 2 is preferred to Method 1.</u>

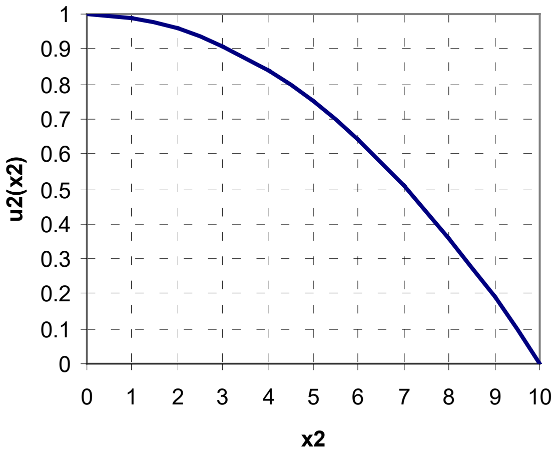
P10.2

x_1 = Weekly blood shortage ($0 \le x_1 \le 10$)
x_2 = Weekly blood outdated ($0 \le x_2 \le 10$)

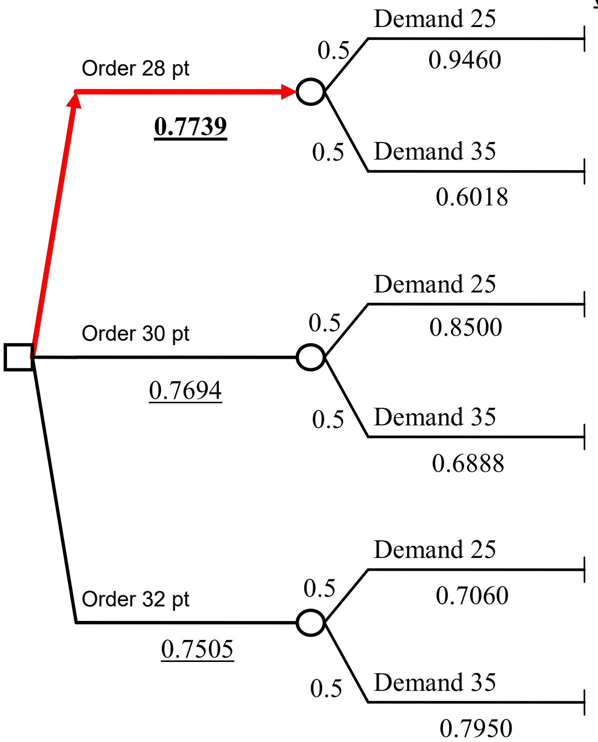
$$u(x_1, x_2) = 0.4\,u_1(x_1) + 0.5\,u_2(x_2) + 0.1\,u_1(x_1)\,u_2(x_2)$$

where $u_1(x_1) = 0.582\left[\exp(1 - \frac{x_1}{10}) - 1\right]$ and $u_2(x_2) = 1 - \frac{x_2^2}{100}$.





The decision tree is

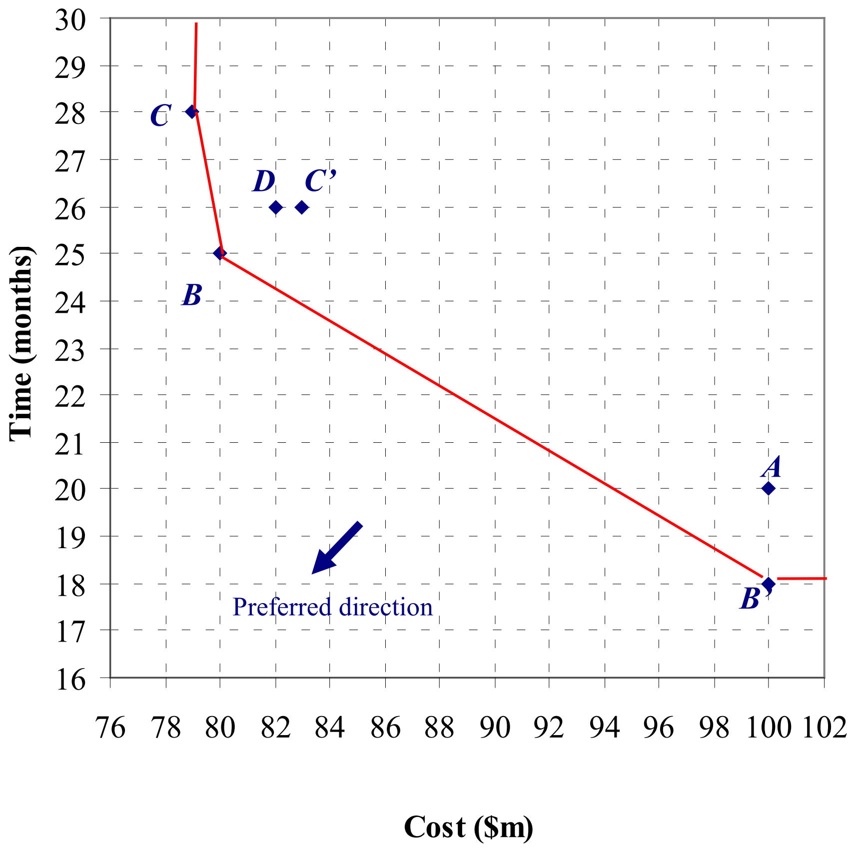


(x_1, x_2)	$u_1(x_1)$, $u_2(x_2)$	$u(x_1, x_2)$
(0, 3)	1.0000, 0.9100	0.9460
(7, 0)	0.2036, 1.0000	0.6018
(0, 5)	1.0000, 0.7500	0.8500
(5, 0)	0.3776, 1.0000	0.6888
(0, 7)	1.0000, 0.5100	0.7060
(3, 0)	0.5900, 1.0000	0.7950

Conclusion: <u>The hospital should order 28 pints of blood weekly.</u>

P10.3

Contractor	Cost (\$$m$)	Time (months)
A	100	20
B	80	25
B'	100	18
C	79	28
C'	83	26
D	82	26



Given $B' \sim B$ and $C' \sim C$.

By dominance analysis, we have:

$B \succ D$, $D \succ C'$ and $B' \succ A$

Hence $B' \sim B \succ D \succ C' \sim C$
and $B' \succ A$.

Answer: Choose either B or B'.

Note that although C is an efficient alternative, i.e., on the efficient frontier, it is not optimal to the decision maker because it has the same utility as C' which is non-efficient.

Hence non-dominance is necessary but not sufficient for optimality with respect to utility.

P10.4

(*a*) The weights for the main criteria with respect to the Goal are computed:

	Human prod	Economics	Design	Operations	Exact w	RGM
Human Productivity	1	3	3	7	0.513052	0.5159
Economics	1/3	1	2	5	0.246592	0.2474
Design	1/3	1/2	1	7	0.193575	0.1903
Operations	1/7	1/5	1/7	1	0.046781	0.0463

λ_{max} = 4.212088, CR= 0.078551 < 10%

The local weights for the alternatives with respect to each criterion are computed:

Human Productivity:

	System A	System B	System C	Exact w	RGM
System A	1	3	5	0.648329	0.6483
System B	1/3	1	2	0.229651	0.2297
System C	1/5	1/2	1	0.122020	0.1220

λ_{max} = 3.003695, CR= 0.003185 < 10%

Economics:

	System A	System B	System C		RGM
System A	1	1/3	1/2	0.157056	0.1571
System B	3	1	3	0.593634	0.5936
System C	2	1/3	1	0.249311	0.2493

λ_{max} = 3.053622 , CR= 0.046225 < 10%

Design:

	System A	System B	System C	Exact w	RGM
System A	1	1/2	1/7	0.093813	0.0938
System B	2	1	1/5	0.166593	0.1666
System C	7	5	1	0.739594	0.7396

λ_{max} = 3.014152, CR= 0.012200 < 10%

Operations:

	System A	System B	System C	Exact w	RGM
System A	1	3	1/5	0.178178	0.1782
System B	1/3	1	1/9	0.070418	0.0704
System C	5	9	1	0.751405	0.7514

λ_{max} = 3.029064, CR= 0.025055 < 10%

The composite weights for the three alternative systems are:

- System A: (0.513052)(0.648329) + (0.246592)(0.157056) + (0.093813)(0.0938) + (0.046781)(0.178178)
  = 0.397850

- System B: (0.513052)(0.229651) + (0.246592)(0.593634) + (0.193575)(0.166593) + (0.046781)(0.070418)
  = 0.299750

- System C: (0.513052)(0.122020) + (0.246592)(0.249311) + (0.193575)(0.739594) + (0.046781)(0.751405)
  = 0.302399

<u>Hence System A should be chosen as it has the highest global weight.</u>

(b)

Alternative	Effectiveness	EUAC ($)
System B	0.299750	80,000
System A	0.397850	100,000
System C	0.302399	110,000

- Efficient Cost-Effective Alternatives are B and A.
- The efficient frontier is shown below: