IE2111 ISE Principles & Practice II
Solutions to Assignment #4

	Project A	Project B
Initial Investment Cost	$300,000	$180,000
Annual Benefits	$100,000	$60,000
Salvage Value	$12,000	$6,000
Useful Life	4 years	4 years

MARR = 8%. Study period = 4 years.

(*a*) $PW(8\%)$ of Project A $= -300{,}000 + 100{,}000\,[P/A, 8\%, 4] + 12{,}000\,[P/F, 8\%, 4]$
$= -300{,}000 + 100{,}000\,(3.3121268) + 12{,}000\,(0.7350299)$
$= \$\ \underline{\mathbf{40{,}033.04}}$

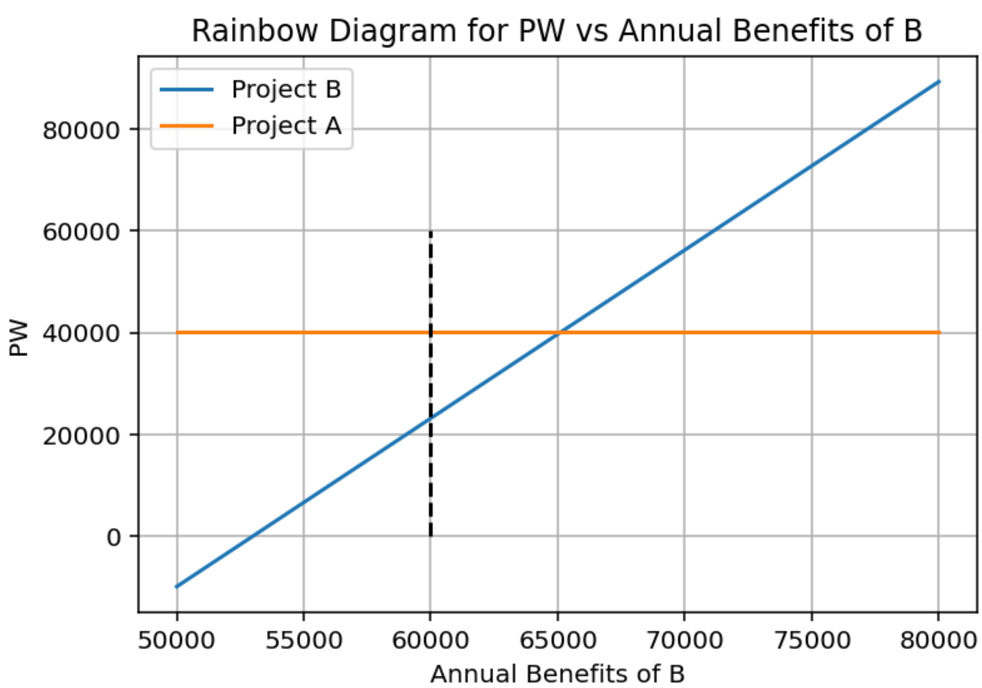
$PW(8\%)$ of Project B $= -180{,}000 + 60{,}000\,[P/A, 8\%, 4] + 6{,}000\,[P/F, 8\%, 4]$
$= -180{,}000 + 60{,}000\,(3.3121268) + 6{,}000\,(0.7350299)$
$= \$\ \underline{\mathbf{23{,}137.79}}$

Hence choose Project A which has a higher $PW(8\%)$.

(*b*) Let A_B = Annual benefits of Project B.

For decision reversal, $PW(8\%)$ of B must be increased by 40,033.04 – 23,137.79
= $16,895.25

This can be achieved by increasing the annual benefits of B by 16,895.25 $[A/P, 8\%, 4]$
= 16,895.25 (0.3019208)
= **$ 5,101.03**



(*c*)

Annual benefits of A ~ Normal($100000, $22000)

i.

$$
\begin{aligned}
E[PW_A(8\%) &= \sum_{k=0}^{N} \frac{EV[F_k]}{(1+0.08)^k} \\
&= -300{,}000 + \frac{100{,}000}{(1+0.08)} + \frac{100{,}000}{(1+0.08)^2} + \frac{100{,}000}{(1+0.08)^3} + \frac{100{,}000+12{,}000}{(1+0.08)^4} \\
&= \$\ \underline{\mathbf{40{,}033.04}}
\end{aligned}
$$

ii. When all the annual cash flows are mutually independent:

$$
\begin{aligned}
Var[PW_A(8\%)] &= \sum_{k=0}^{N} \frac{Var[F_k]}{(1+0.08)^{2k}} \\
&= 0 + \frac{22{,}000^2}{(1+0.08)^2} + \frac{22{,}000^2}{(1+0.08)^4} + \frac{22{,}000^2}{(1+0.08)^6} + \frac{22{,}000^2+0}{(1+0.08)^8} \\
&= \$\$1{,}337{,}198{,}677.29
\end{aligned}
$$

$SD[PW_A(8\%)]$ = **$ <u>36,567.73</u>**

(*d*) Let I_B = Initial Investment cash flow for Project B

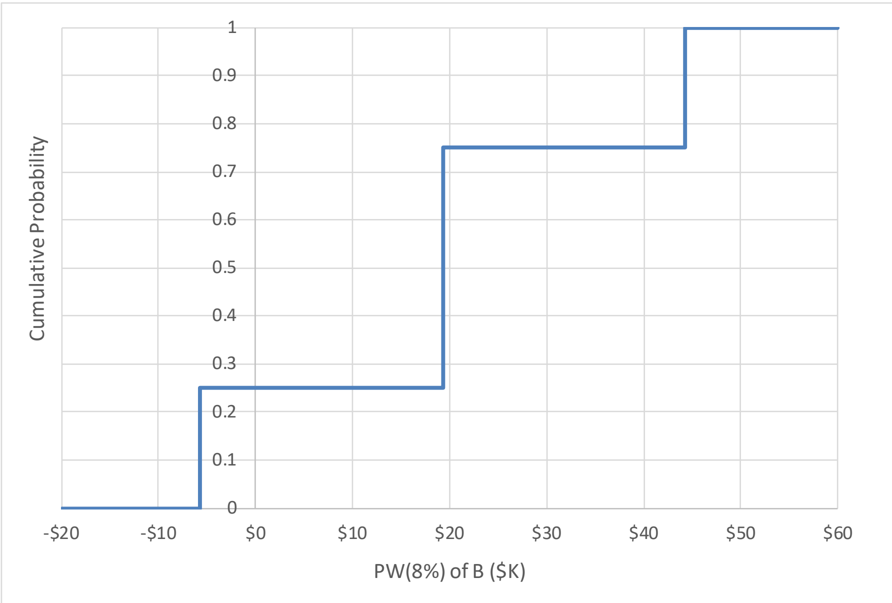
$$PW(8\%) \text{ of B} = -I_B + 60{,}000\,[P/A, 8\%, 4] + 6{,}000\,[P/F, 8\%, 4]$$
$$= -I_B + 60{,}000\,(3.3121268) + 6{,}000\,(0.7350299)$$

$PW(8\%)$ of B has the following distribution:

Initial Cost of B	*PW* of B	Probability
155,000	48,137.79	0.25
180,000	23,137.79	0.50
205,000	-1,862.21	0.25

***i*.** Risk profile for Project B's *PW*:

PW of B	Probability	*PW* - E[*PW*]	$(PW - E[PW])^2$	Prob x $(PW-E[PW])^2$
48,137.79	0.25	-25,000	625,000,000	156,250,000
23,137.79	0.50	0	0	0
-1,862.21	0.25	25,000	625,000,000	156,250,000
			Sum =	312,500,000



ii. $E[PW(8\%)]$ of Project *B*
$= 0.25\,(-1{,}862.21) + 0.5\,(23{,}137.79) + 0.25\,(48{,}137.79)$
= **$ <u>23,137.79</u>**

iii. $Var[PW(8\%)]$ of Project B = $$ 312,500,000 // Note that Var[*PW* of B] = Var[I_B]

$SD[PW(8\%)]$ of Project B = **$ <u>17,677.67</u>**

iv. Downside risk for B = Prob$\{PW(8\%) \text{ of B} \le 0\}$ = 0.25

v. Prob$\{PW(8\%) \text{ of B} \ge \$40{,}000\}$ = 1 – 0.75 = 0.25

(e)

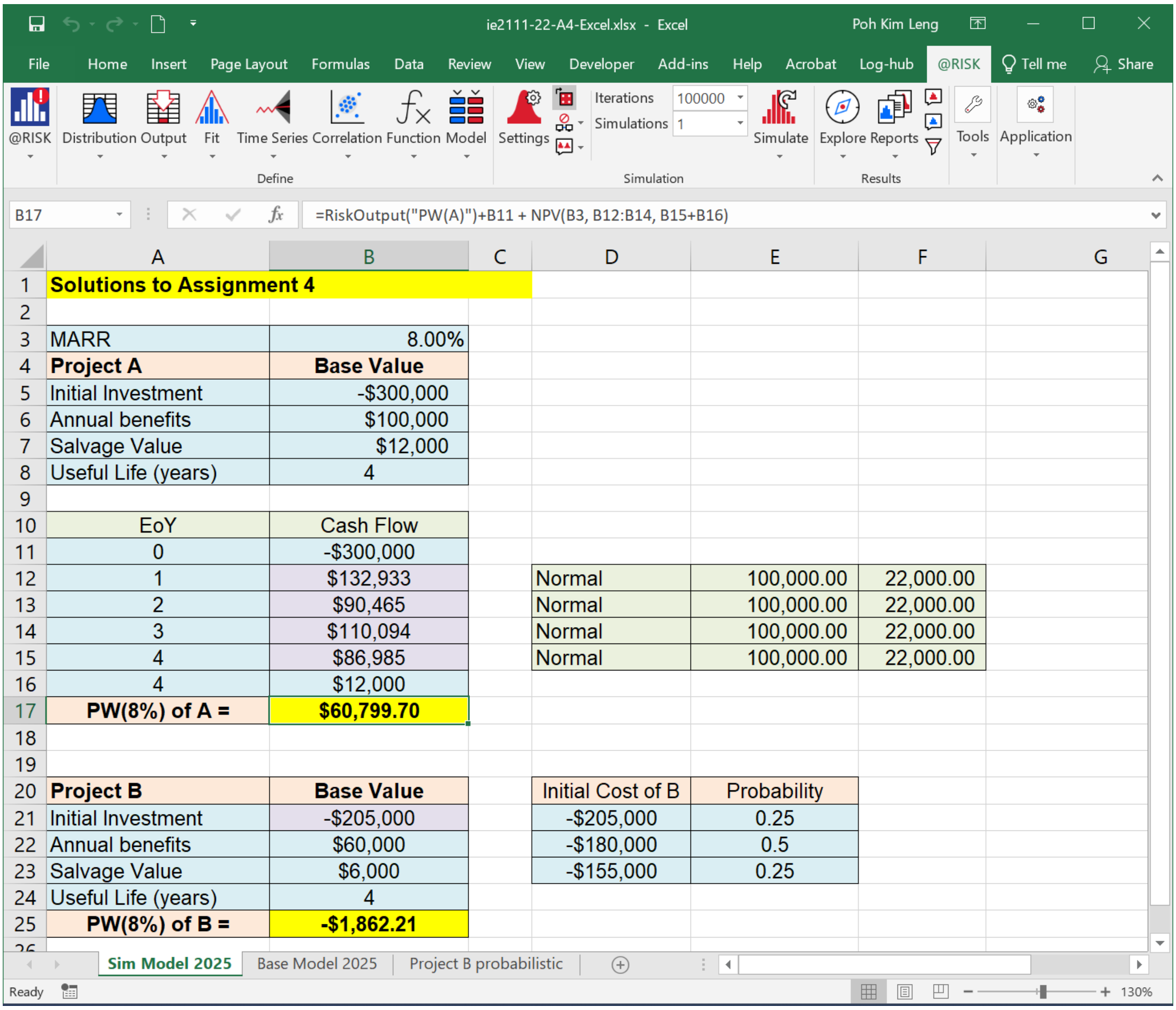
i.

Alternative	$E[PW(8\%)]$	$SD[PW(8\%)]$
Project A	\$ 40,033.04	\$ 36,567.73
Project B	\$ 23,137.79	\$ 17,677.67

$E[PW(8\%)]$ of A $> E[PW(8\%)$ of B but $SD[PW(8\%)$ of A] $> SD[PW(8\%)$ of B]

Hence, there is no mean-variance dominance between the two alternatives.

(f) Monte Carlo Simulation using @Risk:

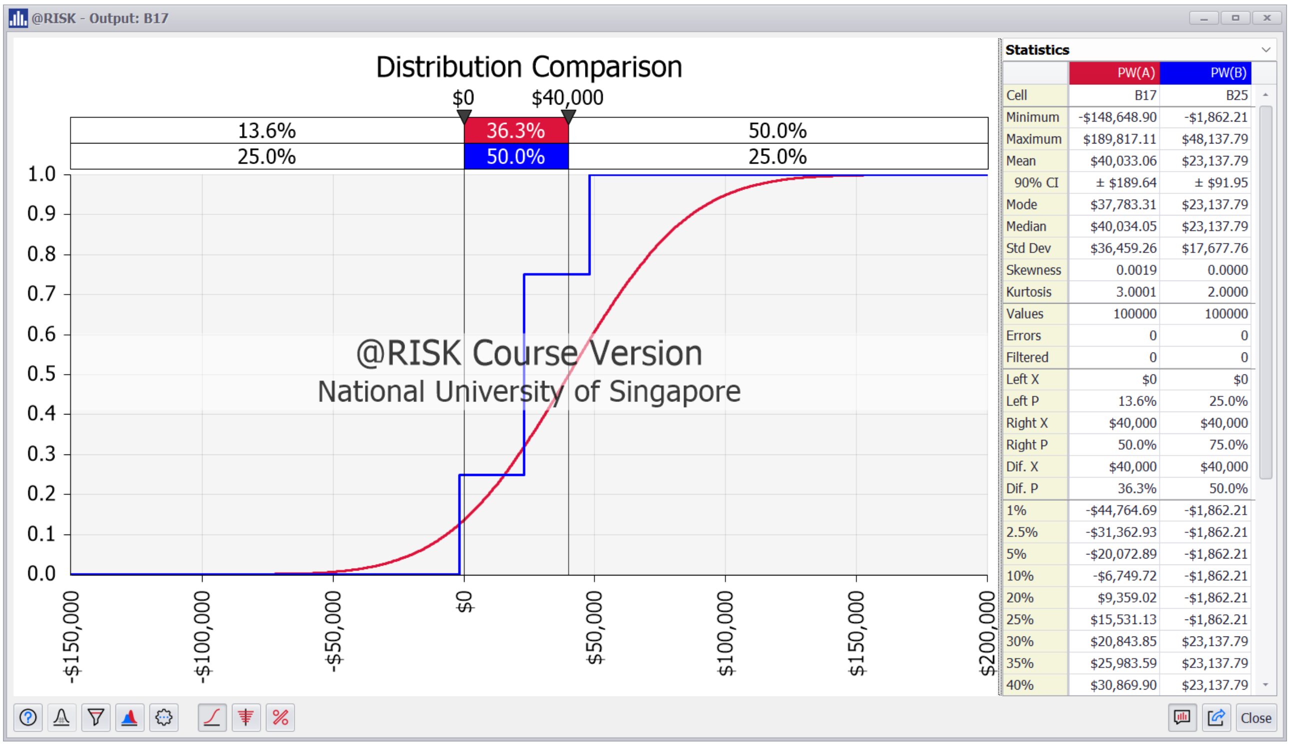


B17 =RiskOutput("PW(A)")+B11 + NPV(B3, B12:B14, B15+B16)

	A	B	C	D	E	F	G
1	**Solutions to Assignment 4**						
2							
3	MARR	8.00%					
4	**Project A**	**Base Value**					
5	Initial Investment	-\$300,000					
6	Annual benefits	\$100,000					
7	Salvage Value	\$12,000					
8	Useful Life (years)	4					
9							
10	EoY	Cash Flow					
11	0	-\$300,000					
12	1	\$132,933		Normal	100,000.00	22,000.00	
13	2	\$90,465		Normal	100,000.00	22,000.00	
14	3	\$110,094		Normal	100,000.00	22,000.00	
15	4	\$86,985		Normal	100,000.00	22,000.00	
16	4	\$12,000					
17	**PW(8%) of A =**	**\$60,799.70**					
18							
19							
20	**Project B**	**Base Value**		Initial Cost of B	Probability		
21	Initial Investment	-\$205,000		-\$205,000	0.25		
22	Annual benefits	\$60,000		-\$180,000	0.5		
23	Salvage Value	\$6,000		-\$155,000	0.25		
24	Useful Life (years)	4					
25	**PW(8%) of B =**	**-\$1,862.21**					

Sim Model 2025 | Base Model 2025 | Project B probabilistic

Results from 100,000 trials:



There is no first-order stochastic dominance between the two risk profiles as the crosses each other.