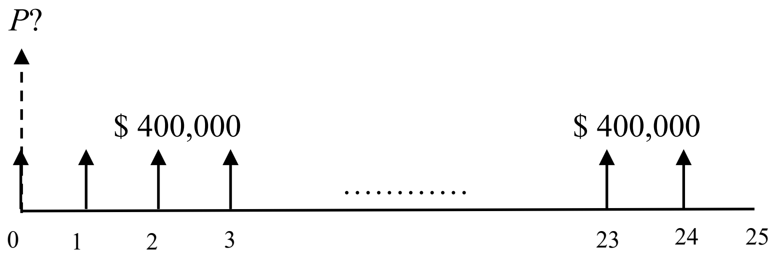
IE2111 ISE Principles & Practice II
Solutions to Tutorial #1

Question 1.

Spivey's cash flows diagram:



Present equivalent value of Spivey's cash flows is

$$
\begin{aligned}
P &= 400{,}000 + 400{,}000\,[P/A, 2\%, 24] \\
&= 400{,}000\,(1 + 18.913925603) \\
&= \mathbf{\$\ \underline{7{,}965{,}570.24}}
\end{aligned}
$$

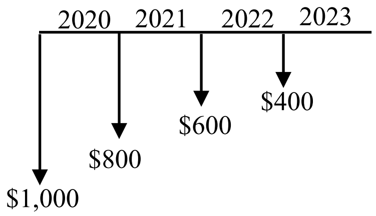
Or

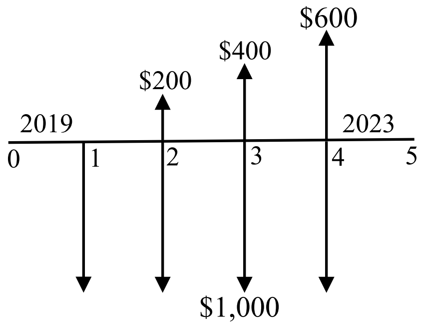
$$
\begin{aligned}
P &= 400{,}000\,[P/A, 2\%, 25]\,[F/P, 2\%, 1] \\
&= 400{,}000\,(19.52345647)\,(1.0200) \\
&= \mathbf{\$\ \underline{7{,}965{,}570.24}}
\end{aligned}
$$

Spivey did not really win $10 million because the payments were received by installments and time value of money means the actual amount he won is actually lower.

Question 2.

The cash flows may be decomposed into 2 parts:





Equivalent value at the end of Year 2022:

F_4 = –1,000 [F/A, 8%, 4] + 200 [F/G, 8%, 4]
= – 1,000 (4.506112) + 200 (6.326400)
= – 3,240.83

Equivalent value at the end of Year 2023:

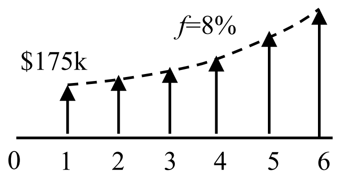
F_5 = F_4 [F/P, 8%, 1]
= – 3,240.83 (1.08)
= – $<u>**3,500.10**</u>

Alternative solution:

F_5 = (–1,000 [P/A, 8%, 4] + 200 [P/G, 8%, 4]) [F/P, 8%, 5] = – $<u>**3,500.10**</u>

Question 3.

Note that the total number of years = 6, i.e., 5 more years in addition to the first.
If a higher quality heat exchanger is purchase, the cash flow diagram for the savings in replacement and downtime cost is as follows:



Given

- $i = 0.18$
- $A_1 = 175{,}000$
- $f = 0.08$
- N=6

The present equivalent value (for the case of $f \neq i$) of the savings =

$$\begin{aligned} P &= \frac{A_1[1-(1+i)^{-N}(1+f)^N]}{(i-f)} \\ &= \frac{175{,}000[1-(1+0.18)^{-6}(1+0.08)^6]}{(0.18-0.08)} \\ &= \$\ 721{,}300.48 \end{aligned}$$

Hence, you can afford to spend as much as **$ <u>721,300.48</u>** now for a higher quality heat exchanger.

Question 4.

Amount borrowed = $5,000

(*a*)

Number of monthly payments = 48.

Interest rate = 6% per year compounded monthly which is equivalent to 6/12 = 0.5% per month compounded monthly

Monthly payment = 5,000 [*A*/*P*, 0.5%, 48]
= 5,000 (0.0234850)
= **$ <u>117.43</u>**

(*b*)

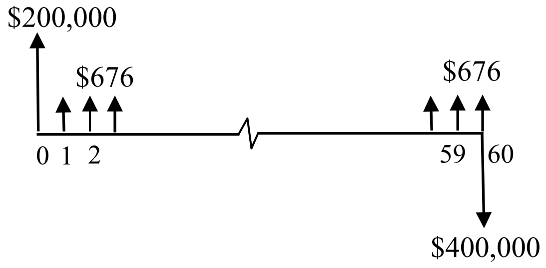
Number of monthly payments = 60

Interest rate = 9% per year compounded monthly which is equivalent to 9/12 = 0.75% per month compounded monthly

Monthly payment = 5,000 [*A*/*P*, 0.75%, 60]
= 5,000 (0.0207584)
= **$ <u>103.79</u>**

Question 5.

There are 60 months from 1 January 2020 to 1 January 2025. The cash flow diagram is as follows:



Let i = interest rate per month.

We require that

$$400{,}000 = 200{,}000\ [F/P,\ i\%,\ 60] + 676\ [F/A,\ i\%,\ 60]$$

By trial and error and linear interpolation:

Try $i = 0.75\%$ 400,000 > 364,126.69 $\Rightarrow i > 0.75\%$
Try $i = 1.00\%$ 400,000 < 418,548.72 $\Rightarrow i < 1.00\%$

$$\frac{i - 0.75}{400{,}000 - 364{,}126.69} = \frac{1.00 - 0.75}{418{,}548.72 - 364{,}126.69}$$

$$i = 0.75 + \frac{(400{,}000.00 - 364{,}126.69)}{(418{,}548.72 - 364{,}126.69)}(1.00 - 0.75)$$

= 0.009148 or 0.9148% per month

Using Excel function: rate(60, 676, 200000, -400000, 0.1) = 0.00918742

Using Python: numpy_financial.rate(60, 676, 200_000, -400_000) = 0.00918742

The *nominal* interest rate = 12 (0.918742%)
= **11.0 % per year compounded monthly.**

The *effective* interest rate = $(1 + 0.00918742)^{12} - 1$
= 0.116 or **11.6% per year**.

Question 6.

Amount borrowed = \$20,000.00
Repayment period =10 years
Payment frequency = monthly
Interest rate = 9% per year compounded continuously which is equivalent to 9/12 = 0.75% per month compounded continuously

Number of monthly payments =120.

Monthly payment =

$$
\begin{aligned}
A &= 20{,}000\ [A/P, \underline{0.75}\%,\ 120] \\
&= 20{,}000\left(\frac{e^{0.0075(120)}(e^{0.0075}-1)}{e^{0.0075(120)}-1}\right) \\
&= 20{,}000(0.012685896) \\
&= \$253.72
\end{aligned}
$$

Hence the amount for each payment is **<u>\$ 253.72</u>**