

**IE2111 ISE Principles & Practice II**  
**Solutions to Tutorial #3**

**Question 1.**

	<i>A</i>	<i>B</i>	<i>C</i>
Investment cost	\$30,000	\$60,000	\$50,000
Estimated units to be sold/year	15,000	20,000	18,000
Unit selling price, \$/unit	\$3.50	\$4.40	\$4.10
Variable costs, \$/unit	\$1.00	\$1.40	\$1.15
Annual expenses (fixed)	\$15,000	\$30,000	\$26,000
Market value	0	\$20,000	\$15,000
Useful life	10 years	10 years	10 years

$MARR = 20\%$

Study period = 10 years

Assume all units are produced and sold each year.

The  $AW$  for the alternatives are:

$$\begin{aligned}
 AW_A(20\%) &= -30,000 [A/P, 20\%, 10] + 15,000 (3.50 - 1.00) - 15,000 \\
 &= -30,000 (0.238523) + 15,000 (3.50 - 1.00) - 15,000 \\
 &= \$15,344.32
 \end{aligned}$$

$$\begin{aligned}
 AW_B(20\%) &= -60,000 [A/P, 20\%, 10] + 20,000 (4.40 - 1.40) - 30,000 + 20,000 [A/F, 20\%, 10] \\
 &= -60,000 (0.238523) + 20,000 (4.40 - 1.40) - 30,000 + 20,000 (0.038523) \\
 &= \$16,459.09
 \end{aligned}$$

$$\begin{aligned}
 AW_C(20\%) &= -50,000 [A/P, 20\%, 10] + 18,000 (4.10 - 1.15) - 26,000 + 15,000 [A/F, 20\%, 10] \\
 &= -50,000 (0.238523) + 18,000 (4.10 - 1.15) - 26,000 + 15,000 (0.038523) \\
 &= \$15,751.70
 \end{aligned}$$

Select Design *B* which has the highest  $AW$ .

## Question 2.

	<i>A</i>	<i>B</i>
Capital investment	\$ 272,000	\$346,000
Annual expenses (Y1 to Y9)	\$ 28,800	\$19,300
Useful life	6 years	9 years
Market Value at end of useful life	\$ 25,000	\$40,000
Annual leasing cost for year 7 to 9	\$ 66,000	

$MARR = 15\%$

(a) *PW* Method:

$$\begin{aligned}
 PW_A(15\%) &= -272,000 - 28,800 [P/A, 15\%, 9] + 25,000 [P/F, 15\%, 6] \\
 &\quad - 66,000 [P/A, 15\%, 3] [P/F, 15\%, 6] \\
 &= -\$ 463,762.11
 \end{aligned}$$

$$\begin{aligned}
 PW_B(15\%) &= -346,000 - 19,300 [P/A, 15\%, 9] + 40,000 [P/F, 15\%, 9] \\
 &= -\$ \underline{\underline{426,721.07}}
 \end{aligned}$$

Select Alternative *B* which has the highest *PW*.

(b) *IRR* Method:

EOY	<i>A</i>	<i>B</i>	<i>B - A</i>
0	-272,000	-346,000	-74,000.
1	-28,800	-19,300	9,500
2	-28,800	-19,300	9,500
3	-28,800	-19,300	9,500
4	-28,800	-19,300	9,500
5	-28,800	-19,300	9,500
6	-28,800 + 25,000 = -3,800	-19,300	-15,500
7	-28,800 - 66,000 = -94,800	-19,300	75,500
8	-28,800 - 66,000 = -94,800	-19,300	75,500
9	-28,800 - 66,000 = -94,800	-19,300 + 40,000 = 20,700	115,500

### Incremental *IRR* Analysis:

1. Alternative sorted in increasing capital investment List = [ *A*, *B* ]  
We are considering cost projects.  
Base alternative = *A*  
Next alternative = *B*.  
List = [ ]

2. Consider the increment (*B* – *A*):

To find the *IRR* of *B*-*A*, we solve:

$$\begin{aligned} -74,000 + 9,500 [P/A, i, 5] - 15,500 [P/F, i, 6] + 75,500 [P/F, i, 7] \\ + 75,500 [P/F, i, 8] + 115,500 [P/F, i, 9] = 0 \end{aligned}$$

Using any solver: *IRR* (*B* – *A*) = 22.51% > *MARR*.

The incremental investment (*B* – *A*) is feasible.

3. Base alternative = *B*.  
Choose Alternative *B*.

- (c) If Crane *A* is leased for 9 years:

$$\begin{aligned} PW_{\text{Lease}}(15\%) &= - (66,000 + 28,800) [P/A, 15\%, 9] \\ &= - 94,800 [P/A, 15\%, 9] \\ &= - \$ 452,346.16 \end{aligned}$$

Since  $PW_A < PW_{\text{Lease}} < PW_B$ , leasing crane *A* is not preferred to the selected Alternatives *B*, but would be preferred to the purchase of crane *A*.

### Question 3.

	Boiler A	Boiler B
Capital investment	\$50,000	\$100,000
Useful life, $N$	20 years	40 years
Market value at EOY $N$	\$10,000	\$20,000
Annual operating costs	\$9,000	\$3,000, increasing \$300 per year after first year

$MARR = 10\%$

Study period = 40 years

Assume that Boiler A is repeated at the end of year 20.

$$\begin{aligned} AW(10\%) \text{ of } A \text{ over 40 years} &= AW(10\%) \text{ of } A \text{ over first 20 years} \\ &= -50,000 [A/P, 10\%, 20] + 10,000 [A/F, 10\%, 20] - \$9,000 \\ &= -50,000 (0.117460) + 10,000 (0.017460) - \$9,000 \\ &= -\$14,698.38 \end{aligned}$$

$$\begin{aligned} AW(10\%) \text{ of } B \text{ over 40 years} &= -100,000 [A/P, 10\%, 40] + 20,000 [A/F, 10\%, 40] - 3,000 - 300 [A/G, 10\%, 40] \\ &= -100,000 (0.102259) + 20,000 (0.002259) - 3,000 - 300 (9.096234) \\ &= -\$15,909.59 \end{aligned}$$

Select Boiler  $A$  which has a higher  $AW$  over the study period.

Note that if  $PW$  with repeatability assumption is used instead, then

- $PW_A(10\%) = -\$143,736.25$  over 2 life cycles or 40 years
- $PW_B(10\%) = -\$155,581.01$  over 1 life cycle or 40 years

You must get the same decision as in the  $AW$  method with repeatability assumption.

**Question 4.**

	Alternative <i>A</i>	Alternative <i>B</i>
Capital investment	\$20,000	\$38,000
Annual expenses	5,500	4,000
Market value at end of useful life	1,000	4,200
Useful life	5 years	10 years

$MARR = 20\%$ .

**(a)** If the service is needed indefinitely.

Study period = infinity and assume Repeatability.

$$\begin{aligned}
 AW(20\%) \text{ of } A \text{ over study period infinity} &= AW(20\%) \text{ of } A \text{ over its first 5 years} \\
 &= -20,000 [A/P, 20\%, 5] - 5,500 + 1,000 [A/F, 20\%, 5] \\
 &= -20,000 (0.334380) - 5,500 + 1,000 (0.134380) \\
 &= -\$12,053.21
 \end{aligned}$$

$$\begin{aligned}
 AW(20\%) \text{ of } B \text{ over study period infinity} &= AW(20\%) \text{ of } B \text{ over its first 10 years} \\
 &= -38,000 [A/P, 20\%, 10] - 4,000 + 4,200 [A/F, 20\%, 10] \\
 &= -38,000 (0.238523) - 4,000 + 4,200 (0.038523) \\
 &= -\$12,902.07
 \end{aligned}$$

Select Alternative *A* which has a higher  $AW$  over study period under repeatability assumption.

**(b)** If the service is required for only 5 years:

Study period = 5 years, and assume co-termination at EoY 5.

$$AW(20\%) \text{ of } A \text{ over study period 5 years} = -\$12,053.21 \text{ as in Part (a)}$$

Given market value ( $MV_5$ ) for *B* at EoY 5 = \$15,000

$$\begin{aligned}
 AW(20\%) \text{ of } B \text{ over study period 5 years} \\
 &= -38,000 [A/P, 20\%, 5] - 4,000 + 15,000 [A/F, 20\%, 5] \\
 &= -38,000 (0.334380) - 4,000 + 15,000 (0.134380) \\
 &= -\$14,690.73
 \end{aligned}$$

Select Alternative *A* which has higher  $AW$  over study period 5 years under co-termination assumption.

**Question 5.**

Route	Construction costs	Annual maintenance cost	Annual Savings in fire damage	Annual recreational benefit	Annual Time access benefit
<i>A</i>	\$185,000	\$2,000	\$5,000	\$3,000	\$500
<i>B</i>	\$220,000	\$3,000	\$7,000	\$6,500	\$1,500
<i>C</i>	\$290,000	\$4,000	\$12,000	\$6,000	\$2,800

$MARR = 8\%$

Study period = 50 years

Assume zero salvage value for all alternatives.

The initial investment costs, total annual costs, and total annual benefits are:

Route	Initial Cost	Total Annual Cost	Total Annual Benefits
<i>A</i>	\$185,000	\$2,000	\$8,500
<i>B</i>	\$220,000	\$3,000	\$15,000
<i>C</i>	\$290,000	\$4,000	\$20,800

The  $PW$  of costs for each alternative:

- $PW(8\%)$  of Costs for route *A* =  $185,000 + 2,000 [P/A, 8\%, 50] = \$ 209,466.97$
- $PW(8\%)$  of Costs for route *B* =  $220,000 + 3,000 [P/A, 8\%, 50] = \$ 256,700.45$
- $PW(8\%)$  of Costs for route *C* =  $290,000 + 4,000 [P/A, 8\%, 50] = \$ 338,933.94$

The  $PW$  of benefits for each alternative:

- $PW(8\%)$  of Benefits for route *A* =  $8,500 [P/A, 8\%, 50] = \$103,984.62$
- $PW(8\%)$  of Benefits for route *B* =  $15,000 [P/A, 8\%, 50] = \$ 183,502.27$
- $PW(8\%)$  of Benefits for route *C* =  $20,800 [P/A, 8\%, 50] = \$ 254,456.48$

**(a)**

If this is considered as a cost (service) project and there is no need to consider the “do-nothing” alternative.

Perform incremental  $\Delta B/\Delta C$  ratio analysis to find the best alternative.

Alternatives sorted increasing  $PW$  of Costs = [  $A$ ,  $B$ ,  $C$  ]

Base alternative =  $A$

Next alternative =  $B$ .

List = [  $C$  ]

$$\Delta B/\Delta C (B - A) = (183,502.27 - 103,984.62) / (256,700.45 - 209,466.97) \\ = 1.6835 > 1$$

The incremental investment ( $B-A$ ) is feasible.

Base alternative =  $B$

Next alternative =  $C$

List = [ ]

$$\Delta B/\Delta C (C-B) = (254,456.48 - 183,502.27) / (338,933.94 - 256,700.45) \\ = 0.8628 < 1$$

The incremental investment ( $C-B$ ) is infeasible.

Base alternative remains as  $B$ .

Choose Route  $B$ .

**(b)**

If this problem is considered as an investment project, then economic feasibility is also a requirement.

None of the routes are feasible as the individual  $B/C$  ratios are all less than one.

Route	$B/C$ ratio
$A$	0.4964
$B$	0.7148
$C$	0.7508

If the “do-nothing” is included in the analysis, it will be the first base alternative, and each of the incremental investment will be rejected. Do-nothing will then emerge as the best alternative at the end.

Also take note that in Part (a), the best alternative  $B$  does not have the maximum  $B/C$  ratio.