

IE2111 ISE Principles & Practice II
Solutions to Tutorial #4

Question 1.

(a)

	Pessimistic	Most likely	Optimistic
Capital Investment	\$ 120,000	\$ 100,000	\$ 90,000
Useful Life	6 years	10 years	12 years
Market Value at EoL	\$ 0	\$ 20,000	\$ 30,000
Net annual cash flow	\$ 20,000	\$ 30,000	\$ 35,000

$MARR = 10\%$

i. When all the factors are at their most likely values:

$$AW(10\%) = -100,000 [A/P, 10\%, 10] + 30,000 + 20,000 [A/F, 10\%, 10] = \$ 14,980.37 > 0$$

ii. When all the factors are at their optimistic values:

$$AW(10\%) = -90,000 [A/P, 10\%, 12] + 35,000 + 30,000 [A/F, 10\%, 12] = \$ 23,194.20 > 0$$

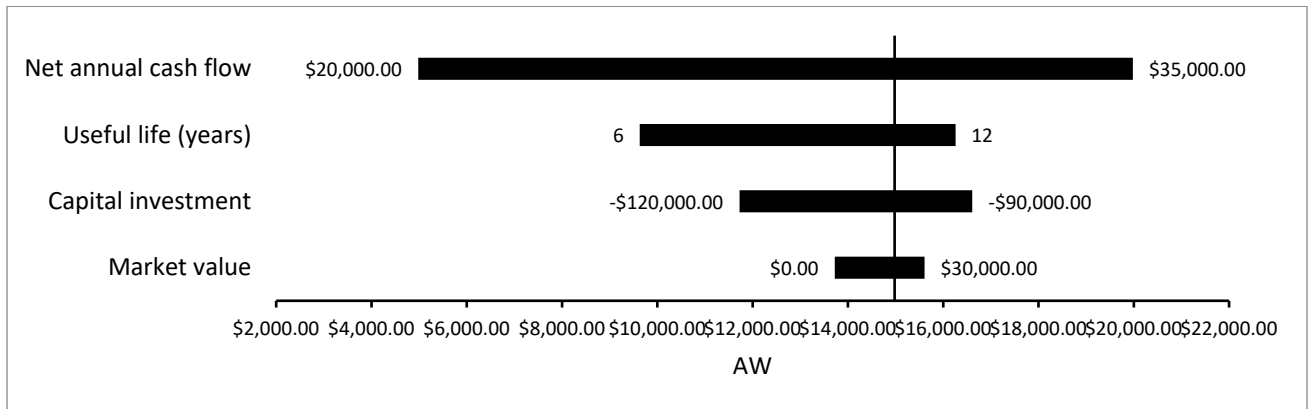
iii. When all the factors are at their pessimistic values:

$$AW(10\%) = -120,000 [A/P, 10\%, 6] + 20,000 + 0 = -\$ 7,552.89 < 0$$

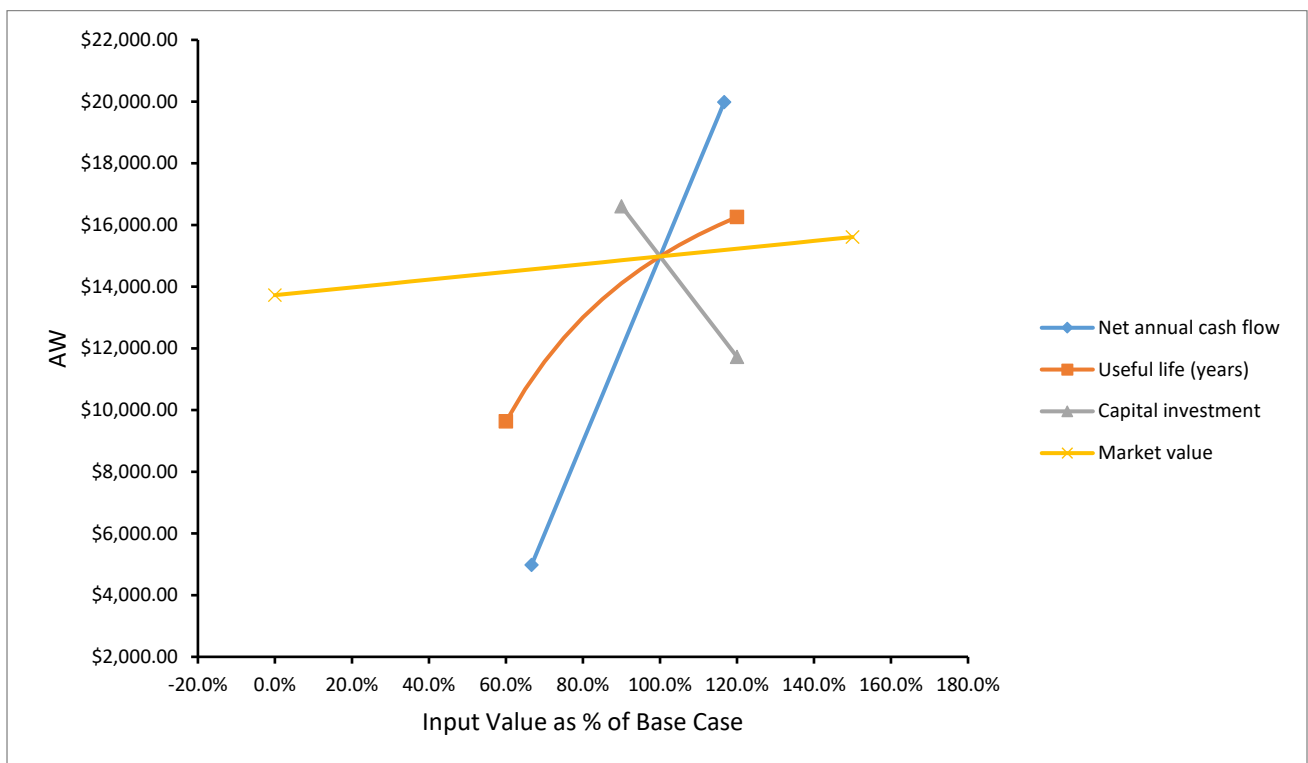
(b) One-Way Range Sensitivity Table:

				AW		
Variable	Pessimistic	Most-likely	Optimistic	Low	High	Swing
Net annual cash flow	\$ 20,000	\$ 30,000	\$ 35,000	\$4,980.37	\$19,980.37	\$15,000.00
Useful life (years)	6	10	12	\$9,631.41	\$16,258.93	\$6,627.52
Capital investment	-\$120,000	-\$100,000	-\$90,000	\$11,725.46	\$16,607.82	\$4,882.36
Market value	\$ 0	\$ 20,000	\$ 30,000	\$13,725.46	\$15,607.82	\$1,882.36

Tornado Diagram for AW



Spider Diagram for AW



(c) Identification of Sensitive and Non-sensitivity factors:

- The project AW is most sensitive to Net Annual Cash Flow, followed by Useful Life, and Capital Investment.
- Market Value at EoL is not very sensitive.

Question 2.

(a)

	Winshear	Blowby	Air-vantage
Capital Investment	\$1,000	\$400	\$1,200
Drag reduction	20%	10%	25%
Maintenance per year	\$10	\$5	\$5
Useful life	10 years	10 years	5 years

Study period = 10 years. Assume repeatability.

Let X = the number of miles driven per year by a tractor.

Reductions in fuel consumption for each alternative:

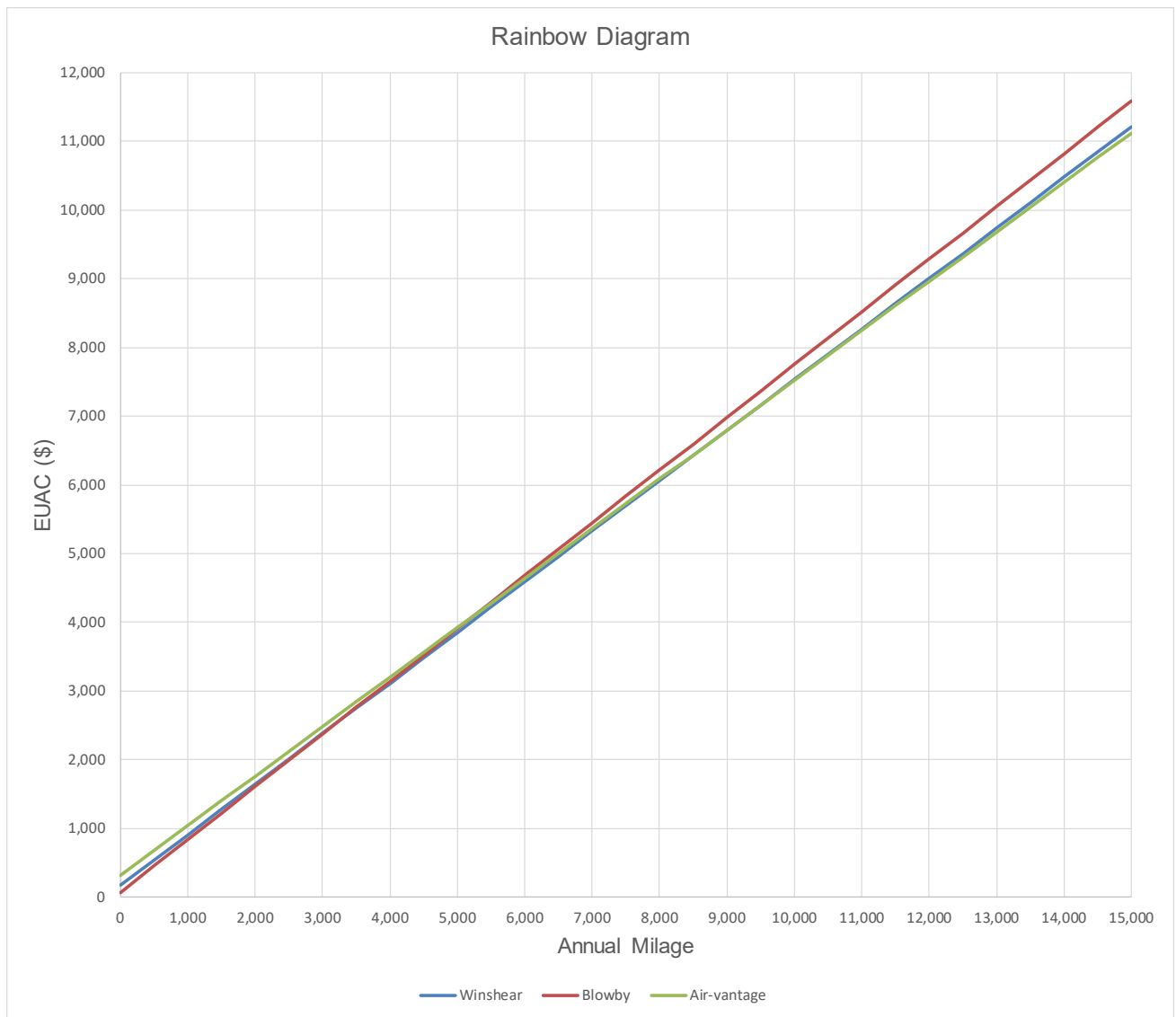
- Windshear: $(20\% / 5\%) (2\% \text{ per miles}) = 8\% \text{ per mile}$
- Blowby: $(10\% / 5\%) (2\% \text{ per miles}) = 4\% \text{ per mile}$
- Air-vantage: $(25\% / 5\%) (2\% \text{ per miles}) = 10\% \text{ per mile}$

Annual Fuel Costs for each alternative:

- Windshear: $0.92 X \text{ (mile/year)} \times 1/5 \text{ (gallon/mile)} \times \$4.00 \text{ (per gallon)} = \$0.736X$
- Blowby: $0.96 X \text{ (mile/year)} \times 1/5 \text{ (gallon/mile)} \times \$4.00 \text{ (per gallon)} = \$0.768X$
- Air-vantage: $0.90 X \text{ (mile/year)} \times 1/5 \text{ (gallon/mile)} \times \$4.00 \text{ (per gallon)} = \$0.720X$

Equivalent Uniform Annual Cost (EUAC) for each alternative:

- $EUAC(\text{Windshear}) = 1,000 [A/P, 10\%, 10] + 10 + 0.736 X$
 $= 1,000 (0.162745395) + 10 + 0.736 X$
 $= 172.74539 + 0.736 X$
- $EUAC(\text{Blowby}) = 400 [A/P, 10\%, 10] + 5 + 0.768 X$
 $= 400 (0.162745395) + 5 + 0.768 X$
 $= 70.09816 + 0.768 X$
- $EUAC(\text{Air-vantage}) = 1,200 [A/P, 10\%, 5] + 5 + 0.720 X$
 $= 1,200 (0.263797481) + 5 + 0.720 X$
 $= 321.55698 + 0.720 X$



(b)

To determine the breakpoint value between Blowby and Windshear, we solve

$$\begin{aligned}
 EUAC(\text{Blowby}) &= EUAC(\text{Windshear}) \\
 70.09816 + 0.768 X &= 172.74539 + 0.736 X \\
 X &= 3,207.73
 \end{aligned}$$

To determine the breakpoint value between Windshear and Air-vantage, we solve

$$\begin{aligned}
 EUAC(\text{Windshear}) &= EUAC(\text{Air-vantage}) \\
 172.74539 + 0.736 X &= 321.55698 + 0.720 X \\
 X &= 9,300.73
 \end{aligned}$$

Optimal Decision Rule

Miles driven per year	Optimal Choice
$0 < X \leq 3,207.73$	Blowby
$3,207.73 \leq X \leq 9,300.73$	Windshear
$9,300.73 \leq X$	Air-vantage

Question 3.

	Alternative 1	Alternative 2
Capital Investment	\$ 4,500	\$ 6,000
Annual revenues	\$ 1,600	\$ 1,850
Annual expenses	\$ 400	\$ 500
Estimated market value	\$ 800	\$ 1,200
Useful life	8 years	10 years

$MARR = 15\%$.

(a) Study period = 40 years. Assume repeatability.

$$\begin{aligned} AW(15\%) \text{ of Alternative 1 over the 40-year study period} \\ &= AW(15\%) \text{ of Alternative 1 over the first 8 years} \\ &= -4,500 [A/P, 15\%, 8] + 1,600 - 400 + 800 [A/F, 15\%, 8] \\ &= \$ \underline{\underline{255.45}} \end{aligned}$$

$$\begin{aligned} AW(15\%) \text{ of Alternative 2 over 40-year study period} \\ &= AW(15\%) \text{ of Alternative 2 over the first 10 years} \\ &= -6,000 [A/P, 15\%, 10] + 1,850 - 500 + 1,200 [A/F, 15\%, 10] \\ &= \$ \underline{\underline{213.59}} \end{aligned}$$

Select **Alternative 1** with higher AW over the study period of 40 years.

(b)

Let I_2 = Capital cost of Alternative 2.

$$AW_2(15\%, I_2) = -I_2 [A/P, 15\%, 10] + 1,850 - 500 + 1,200 [A/F, 15\%, 10]$$

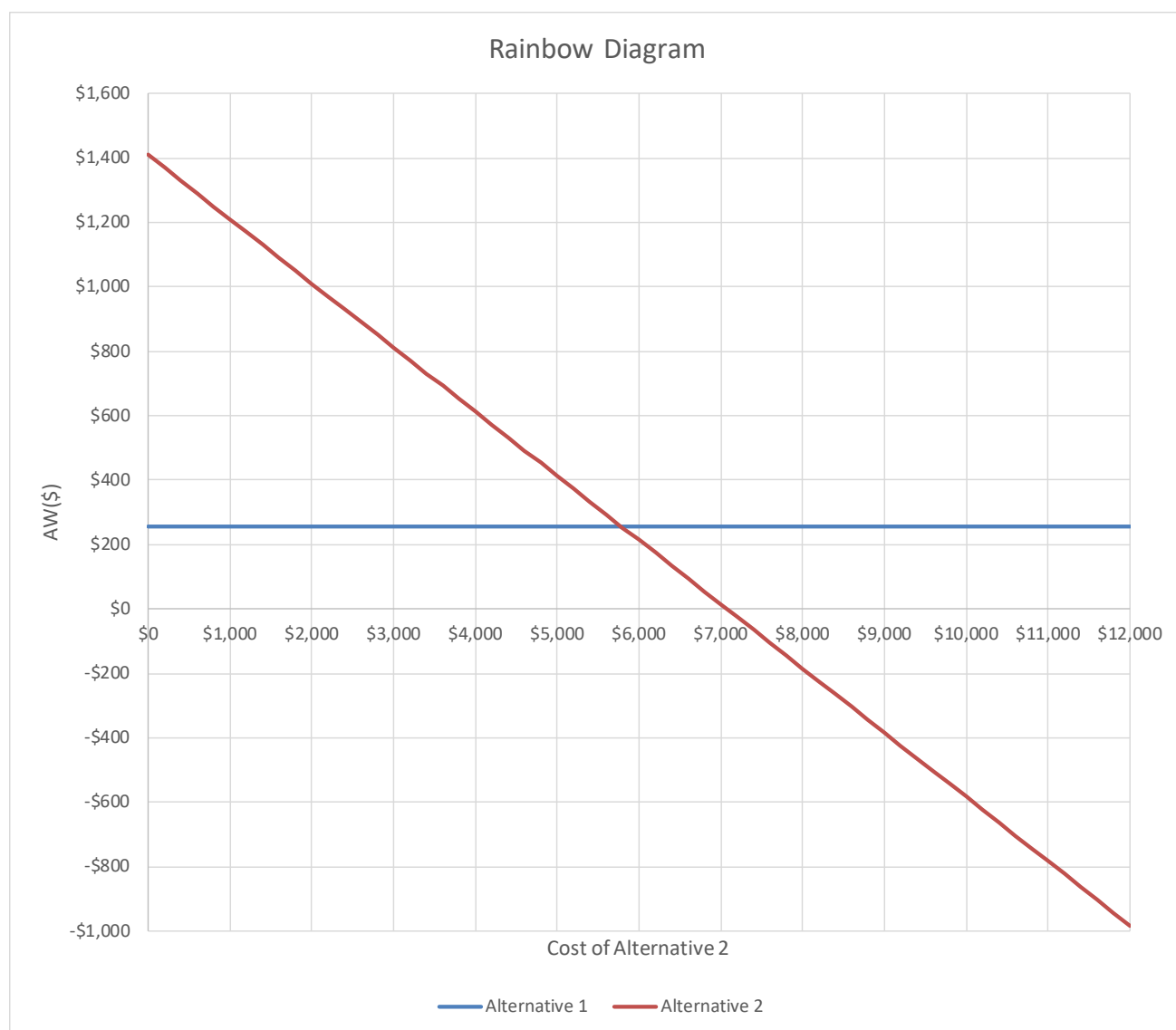
We want to find the value of I_2 such that $AW_2(15\%, I_2) > AW_1(15\%)$:

$$-I_2 [A/P, 15\%, 10] + 1,850 - 500 + 1,200 [A/F, 15\%, 10] > 255.45$$

$$\Rightarrow I_2 < \$ 5,789.89$$

Minimum change in capital cost of Alternative 2 required for decision reversal = $5,789.89 - 6,000$
= **-\$ 210.11**

%-change required = $-210.11 / 6,000 = -\underline{\underline{3.502\%}}$



(c)

Let life of Alternative 1.

$$AW_1(15\%, N_1) = -4,500 [A/P, 15\%, N_1] + 1,600 - 400 + 800 [A/F, 15\%, N_1]$$

We want to find the value of N_1 such that $AW_1(15\%, N_1) < AW_2(15\%)$:

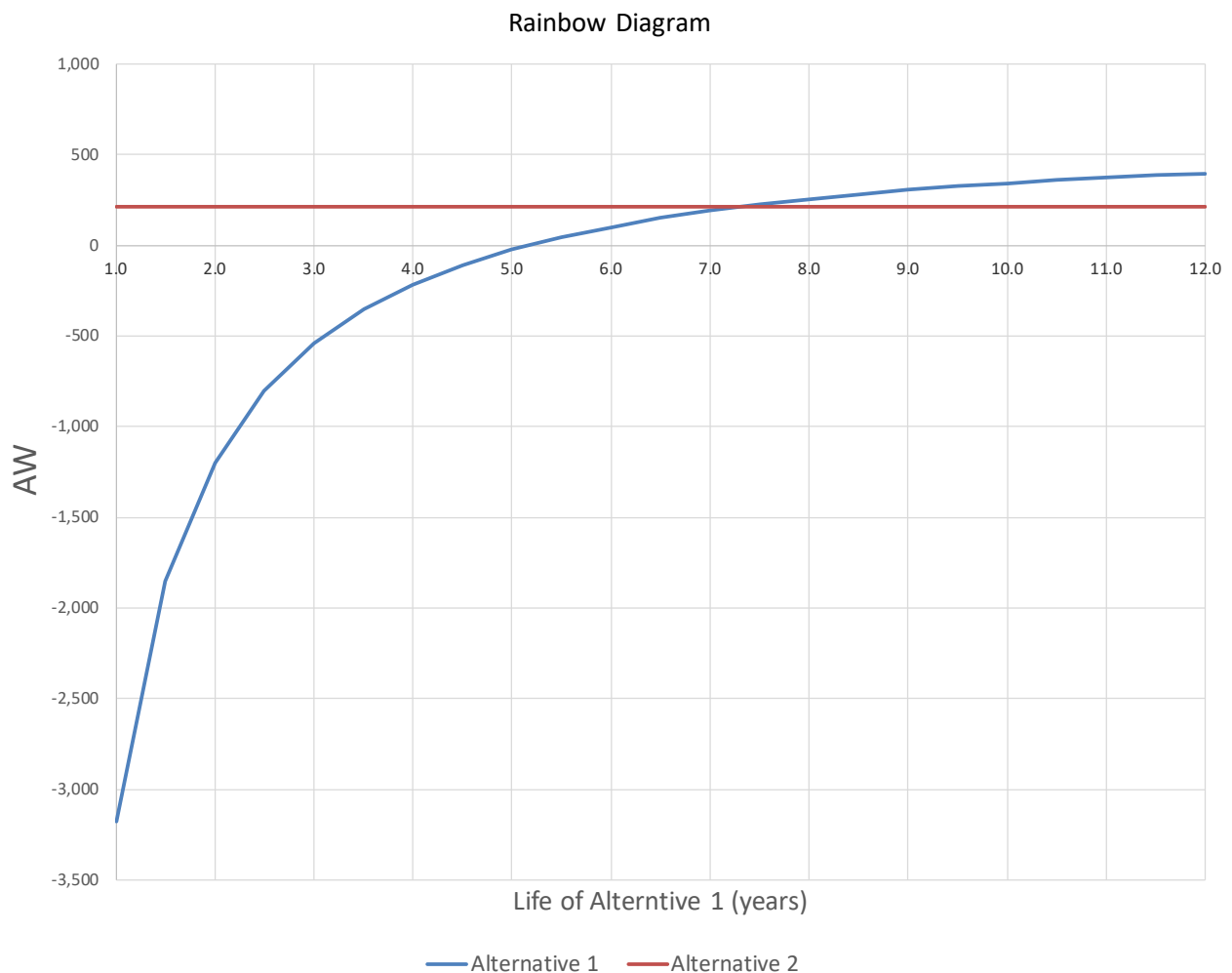
$$\begin{aligned} -4,500 [A/P, 15\%, N_1] + 1,600 - 400 + 800 [A/F, 15\%, N_1] &= 213.59 \\ -4,500 [A/P, 15\%, N_1] + 986.41 + 800 [A/F, 15\%, N_1] &= 0 \end{aligned}$$

Using Excel NPER function

$$N_1 < \text{NPER}(0.15, 986.41, -4500, 800, 0) = 7.321 \text{ years}$$

Minimum change in useful life of Alternative 1 required for decision reversal = $7.321 - 8 = -0.679$ years

$$\% \text{-change required} = -0.679 / 8 = -8.48\%$$



Question 4.

(a) $PW(N) = -418,000 + 148,000 [P/A, 15\%, N]$

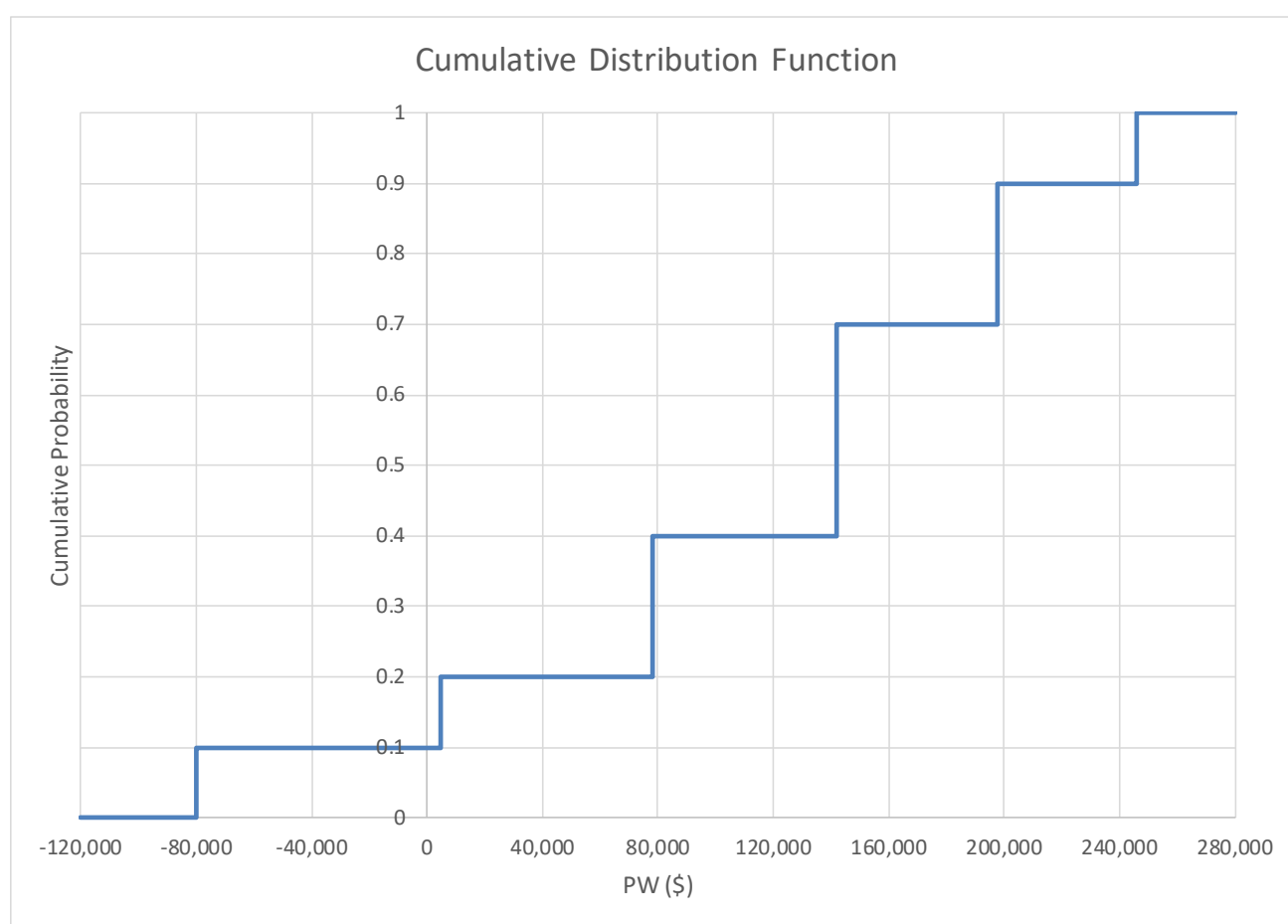
N	p	PW	$p \times PW$	$PW - E[PW]$	$p \times (PW - E[PW])^2$
3	0.1	-80,082.68	-8,008.27	-194,943.70	3,800,304,582.45
4	0.1	4,536.80	453.68	-110,324.22	1,217,143,324.65
5	0.2	78,118.95	15,623.79	-36,742.06	269,995,823.16
6	0.3	142,103.44	42,631.03	27,242.42	222,644,871.05
7	0.2	197,742.12	39,548.42	82,881.10	1,373,855,485.38
8	0.1	246,123.58	24,612.36	131,262.57	1,722,986,141.40
			114,861.02		8,606,930,228.10

$$E[PW] = \$ 114,861.02$$

$$Var[PW] = \$\$ 8,606,930,228.10$$

$$\sigma[PW] = \$ 92,773.54$$

(b) The CDF for the PW is plotted below:



(c)

i. Downside Risk of Project = $\text{Prob}\{PW < 0\} = 0.1$

ii. Chance of achieving an upside potential of $PW \geq \$150,000 = 1 - 0.7 = 0.3$

iii. Present Equivalent Value-at-Risk (95% confidence) = $-(-80,082.68) = \$ 80,082.68$

Question 5.

$MARR = 15\%$.

	Alternative A		Alternative B	
EoY	Expected Cash Flow (\$)	Std Dev. of Cash Flow (\$)	Expected Cash Flow (\$)	Std Dev. of Cash Flow (\$)
0	-8,000	0	-12,000	500
1	4,000	600	4,500	300
2	6,000	600	4,500	300
3	4,000	800	4,500	300
4	6,000	800	4,500	300

(a) When the cash flows are mutually independent:

For Investment A:

$$E[PW(A)] = -8,000 + \frac{4,000}{(1+0.15)} + \frac{6,000}{(1+0.15)^2} + \frac{4,000}{(1+0.15)^3} + \frac{6,000}{(1+0.15)^4}$$

$$= \$6,075.71$$

$$Var[PW(A)] = 0 + \frac{600^2}{(1+0.15)^2} + \frac{600^2}{(1+0.15)^4} + \frac{800^2}{(1+0.15)^6} + \frac{800^2}{(1+0.15)^8}$$

$$= \$\$ 963,949.69$$

$$\sigma[PW(A)] = \sqrt{963,949.69}$$

$$= \$ 981.81$$

For Investment B:

$$E[PW(B)] = -12,000 + \frac{4,500}{(1+0.15)} + \frac{4,500}{(1+0.15)^2} + \frac{4,500}{(1+0.15)^3} + \frac{4,500}{(1+0.15)^4}$$

$$= \$847.40$$

$$Var[PW(B)] = 500^2 + \frac{300^2}{(1+0.15)^2} + \frac{300^2}{(1+0.15)^4} + \frac{300^2}{(1+0.15)^6} + \frac{300^2}{(1+0.15)^8}$$

$$= \$\$ 437,841.37$$

$$\sigma[PW(B)] = \sqrt{437,841.37}$$

$$= \$ 661.70$$

Investment A has a higher expected PW than Investment B, but the standard deviation of Investment A is larger than that of Investment B. Hence the mean-variance criterion is non conclusive.

- (b) When the cash flows are mutually independent;

$$\begin{aligned} E[PW(A - B)] &= E[PW(A)] - E[PW(B)] \\ &= 6,075.71 - 847.40 \\ &= \$ 5,228.30 \end{aligned}$$

$$\begin{aligned} Var[PW(A - B)] &= Var[PW(A)] + Var[PW(B)] \\ &= 963,949.69 + 437,841.37 \\ &= \$\$ 1,401,791.05 \end{aligned}$$

$$\begin{aligned} \sigma[PW(A - B)] &= \sqrt{1,401,791.05} \\ &= \$ 1,183.97 \end{aligned}$$

- (c) When not all the cash flows for investment A are mutually independent and coefficients of correlations are $\rho_{12} = 0.1$, $\rho_{23} = 0.2$, $\rho_{34} = 0.3$:

$$E[PW(A)] = \$ 6,075.71$$

$$\begin{aligned} Var[PW(A)] &= 0 + \frac{600^2}{(1+0.15)^2} + \frac{600^2}{(1+0.15)^4} + \frac{800^2}{(1+0.15)^6} + \frac{800^2}{(1+0.15)^8} \\ &\quad + \frac{2(0.1)(600)(600)}{(1+0.15)^3} + \frac{2(0.2)(600)(800)}{(1+0.15)^5} + \frac{2(0.3)(800)(800)}{(1+0.15)^7} \\ &= \$\$ 1,251,108.61 \end{aligned}$$

$$\begin{aligned} \sigma[PW(A)] &= \sqrt{1,251,108.61} \\ &= \$ 1,118.53 \end{aligned}$$