Decision Analysis
Solutions to Assignment #2

(***a***)

$$u(w)=\begin{cases}\frac{w^2}{100{,}000} & \text{for } w\ge 0\\ \frac{-w^2}{100{,}000} & \text{for } w<0\end{cases}$$

$$\Rightarrow \qquad u'(w)=\begin{cases}\frac{2w}{100{,}000} & \text{for } w\ge 0\\ \frac{-2w}{100{,}000} & \text{for } w<0\end{cases}$$

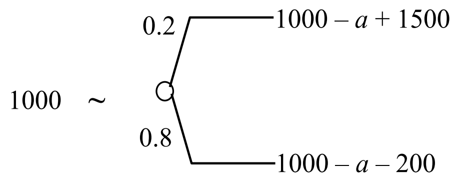
$$\Rightarrow \qquad u''(w)=\begin{cases}\frac{2}{100{,}000} & \text{for } w\ge 0\\ \frac{-2}{100{,}000} & \text{for } w<0\end{cases}$$

Risk tolerance $\rho(w)=\dfrac{-u'(w)}{u''(w)}=-w$ for all w

Alice's current risk tolerance = - \$1,000.

(***b***) Hence, Alice is risk-seeking as her current risk tolerance is negative.

(***c***) Let Alice's personal indifferent buying price of Investment A = a



Equating the utility of not buying with the expected utility of buying Investment A

$u(1000) = 0.2\,u(1000 - a + 1500) + 0.8\,u(1000 - a - 200)$
$u(1000) = 0.2\,u(2500 - a) + 0.8\,u(800 - a)$
$1000^2 = 0.2\,(2500 - a)^2 + 0.8\,(800 - a)^2$

Solving: a = \$406.79

Hence Alice's personal indifferent buying price of Investment A = **\$ <u>406.79</u>**

We can also find the answer by plotting a rainbow diagram.

(*d*) Investment A costs \$350. Investment B costs \$600

Alternative 1: Alice purchases only A at a cost of \$350

Expected wealth utility

$= 0.2\ u(1000 - 350 + 1500) + 0.8\ u(1000 - 350 - 200)$

$= 0.2\ u(2150) + 0.8\ u(450)$

$= (\ 0.2\ (2150)^2 + 0.8\ (450)^2)/100{,}000$

$= \underline{\mathbf{10.8650}}$

Alternative 2: Alice purchases only B at a cost of \$600

Expected wealth utility

$= 0.9\ u(1000 - 600 + 750) + 0.1\ u(1000 - 600 - 500)$

$= 0.9\ u(1150) + 0.1\ u(-100)$

$= (\ 0.9\ (1150)^2 - 0.1\ (100)^2)\ /100{,}000$

$= \mathbf{11.8925}$

Alternative 3: Alice purchases A and B at a total cost of = \$950:

Expected wealth utility

$= 0.2\ [\ 0.9\ u(1000 - 950 + 1500 + 750) + 0.1\ u(1000 - 950 + 1500 - 500)\ ] +$
$0.8\ [\ 0.9\ u(1000 - 950 - 200 + 750) + 0.1\ u(1000 - 950 - 200 - 500)\ ]$

$= 0.2\ [\ 0.9\ u(2300) + 0.1\ u(1050)\ ] + 0.8\ [\ 0.9\ u(600) + 0.1\ u(-650)\ ]$

$= \{\ 0.2\ [\ 0.9\ (2300)^2 + 0.1\ (1050)^2\ ] + 0.8\ [\ 0.9\ (600)^2 - 0.1\ (650)^2\ ]\ \}\ /\ 100{,}000$

$= \underline{\mathbf{11.9965}}$

Alternative 4: Alice purchases none

Expected wealth utility

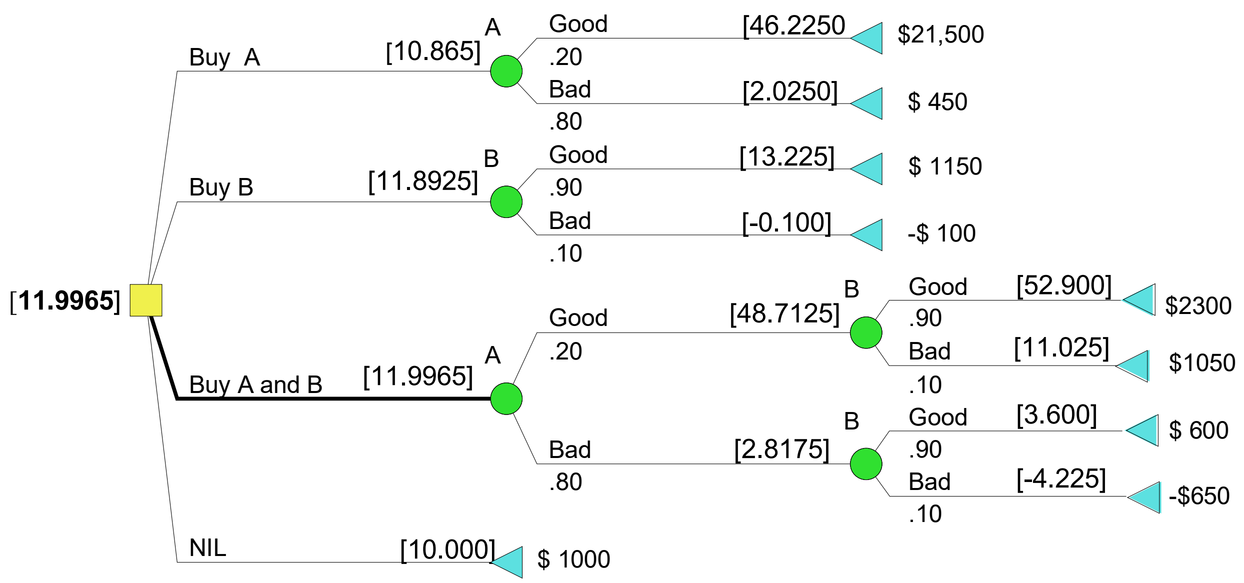
$= u(1000)$

$= (1000)^2\ /\ 100{,}000$

$= \underline{\mathbf{10.0000}}$

Hence Alice should purchase both Investments A and B.

The decision tree is shown below. The rolled backed values are the expected utilities.



(*e*)

Upon purchasing Investments, A and B, Alice's wealth = 1000 – 950 = \$50.

Let Alice's personal indifferent selling price for Investment A = s

If Alice does not sell Investment A she faces the outcome of both A and B.
Expected wealth utility = 11.9965 // from part (d)

If Alice sells Investment A and keeps B:
Expected wealth utility
$= 0.9\, u(50 + s + 750) + 0.1\, u(50 + s - 500)$
$= 0.9\, u(800 + s) + 0.1\, u(-450 + s)$

Solving: $0.9\, u(800 + s) + 0.1\, u(-450 + s) = 11.9965$

s = \$ **354.97**

Hence Alice personal indifferent selling price for Investment A = \$354.97.

We can also find the answer buy plotting a rainbow diagram.

(*f*)

We would expect the two answers to be different, as Alice's utility function is neither linear nor exponential. Hence, her personal indifferent selling price for Investment A is not equal to her personal indifferent buying price. Moreover, these two quantities also depend on her wealth, which is also different in the two situations.