Int II + Pre-Adv Math Circle Sessions	**Instructor: Zvezda; Time: 9:10-10am**
Date Distributed	**Day 4, June 16 2022**
Read	Problem 1 in BMC book, vol. I, Session 2, Combinatorics, pages 25-26, 35-39, 41 Handout "Counting Regions in the Plane and More": pages 2-3 • Binomial coefficients (pp. 35-41, BMC Book I): ○ What they mean; the factorial formula for $\binom{n}{k}$; the shortcut formula for $\binom{n}{2}$; ○ Pascal's triangle; Exercise 18 (p. 41, BMC Book I).
Think/Review	Your class notes.
Instructions	When writing solutions to problems, follow the Co-Co-Clear rubric (correct-complete-clear). Do NOT blindly rewrite from the book (where you will see some solutions, partial solutions, and ideas). After you read, think, and solve, put aside everything, and write the solutions in your own words.
Written Exercises	**1.** In **Problem 1 (handout, Regions in a circle)** with n points on a circle (no 3 diagonals concurrent), we conjectured that the formula $1 + \binom{n}{2} + \binom{n}{4}$ counts the number of regions into which the circle is split. ○ Verify that this formula gives the correct answers for $n = 1, 2, 3, 4$ points. ○ Calculate what the formula gives for $n = 8, 9, 10$ points on the circle. **2.** Verify the formula $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$ for $n = 1, 2, \ldots, 7$; that is, plug into both sides and simplify until you get to the same result on both sides; for example, for $n = 2$, this equality would read $1 + 2 = \frac{2(2+1)}{2}$ are the sides equal? **3.** In **Problem 8 (handout, Lines in generic position)** with n lines in general position in the plane, consider the formula $1 + \binom{n+1}{2}$. Plug $n = 8, 9, 10$ lines to find out the number of regions into which the plane is split. **4.** Solve **Problem 9 (handout, Games in tournament)** for $n = 8, 9, 10$ teams. Is the formula $\binom{n}{2}$ helpful here? Why? Explain.
Bonus/Challenge Expected for Pre-Advanced Suggested for Intermediate II	**5. Problem 15(a) (handout, Summing up):** Prove the formula ○ $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$ for any natural number $n = 1, 2, 3, \ldots$ (There are many ways to prove this formula. Choose one way and prove it.) **6. Problem 10 (handout, Counting diagonals)** ○ Do this problem for $n = 5, 6, 7, 8$. ○ Think of a formula that works for any n-gon. (No proof necessary yet.)
Project for the die-hards: do NOT turn in with HW! Suggested for Pre-Advanced	**7.** In **Problem 8 (handout, Lines in generic position)** with n lines in general position in the plane, show that the two formulas below are equal and that each correctly counts the number of regions: $\binom{n+1}{0} + \binom{n+1}{2} = \binom{n}{0} + \binom{n}{1} + \binom{n}{2}$. **8.** Prove that $\binom{n}{2} = \frac{n!}{2!(n-2)!} = \frac{n(n-1)}{2}$ and $\binom{n}{4} = \frac{n!}{4!(n-4)!} = \frac{n(n-1)(n-2)(n-3)}{24}$ and that these formulas count the number of pairs (resp. 4-tuples) among n objects.