Assignment 4

Data Analysis and Modeling Techniques

10.31

$$\pi(\theta|x) = \frac{f(x|\theta)\cdot\pi(\theta)}{m(x)}$$

Given,

$$\pi(\theta) = \text{Gamma}(5, 1)$$

$$f(x|\theta) = \text{Poisson}(\theta)$$

$$\pi(\theta|x) = \text{Gamma}(\alpha + n\bar{x}, \ \lambda + n)$$

$n = 1, \quad \bar{x} = 4, \quad \alpha = 5, \quad \lambda = 1$

$$= \text{Gamma}(5 + (1)(4), \ 1 + 1)$$

$$= \text{Gamma}(9, 2)$$

$$\hat{\theta}_B = \frac{9}{2} = 4.5$$

$$\text{Posterior-risk} = \frac{9}{4} = 2.25$$

10.37

$$\pi(p|x) = \frac{f(x|p)\ \pi(p)}{m(x)}$$

$$\pi(p) = 1$$

$f(x/p) = f(x=10/p) = p^{10}$ (Coin is tossed for 10 times)

$$m(x) = \int_0^1 f(x=10/p) \cdot \pi(p) \cdot dp$$

$$= \int_0^1 p^{10} \cdot (1)\, dp = \left(\frac{p^{10+1}}{10+1}\right)_0^1$$

$$= \frac{1}{11}$$

$$\pi(p|x) = \frac{p^{10}(1)}{1/11} = 11\, p^{10}$$

The coin is highly biased as it is concentrated near the $p = 0.99$.

5.1

$$f(x) = 1.5\sqrt{x}$$

C.D.F is given as,

$$F(x) = P(X \le x)$$

$$= \int_0^x 1.5\sqrt{x}$$

$$= (1.5)\left(\frac{x^{3/2}}{3/2}\right)_0^x$$

$$= x^{3/2}$$

$F(x) = u$ (u is an random number generated by uniform standard distribution)

$x^{3/2} = u$

so $x = u^{2/3}$

If $u = 0.001$, then $x = (0.001)^{2/3}$

$x = 0.01$

5.6

For first Mechanic,

$X \rightarrow$ Service time

$\lambda = 5$

$E(X) = \frac{1}{5} \times 60 = 12$ min

For second Mechanic,

$\lambda = 20$

$E(X) = \frac{1}{20} \times 60 = 3$ min

Probability of being served by second machanic $= 4/5$

Probability of being served by first mechanic $= 1/5$

Random variable X takes 12 min and 3 min with probabilities $1/5$ and $4/5$.