UNIVERSITY OF TARTU
FACULTY OF SCIENCE AND TECHNOLOGY
INSTITUTE OF MATHEMATICS AND STATISTICS

AUTHOR

TITLE OF THE THESIS

BACHELOR / MASTER THESIS

in Mathematical Statistics

(30 ECTS)

Supervisor: MSc/PhD/prof. Firstname Lastname

TARTU 2020

TITLE OF THE THESIS
Bachelor (Master) thesis
Author

Abstract
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CERCS research specialisation: P160 Statistics, operations research, programming, financial and actuarial mathematics.

Key Words: Random, sample, probability.

LÕPUTÖÖ PEALKIRI
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Lühikokkuvõte
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CERCS teaduseriala: P160 Statistika, operatsioonianalüüs, programmeerimine, finants- ja kindlustusmatemaatika.

Märksõnad: Juhuslik, valim, tõenäosus.

Contents

Introduction

The main text of a thesis always starts with an "Introduction". You can leave writing it to the final phase of writing.

It is a good idea to start the Introduction with the main thesis statement or research question of the thesis. After that, it is a good idea to clarify things by defining any necessary terms.[1] This section is also a good place to discuss why your thesis statement is scientifically or practically relevant and interesting. Introduction is the place where you explain what is your contribution – knowledge that you have investigated or produced personally. At the end of the section, it is customary to briefly explain the structure of the thesis – what each chapter is about.

Please note that the instructions given in this sample are by no means official. Always follow your supervisor's instructions even if they conflict with what this sample says.

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[1]Definitions after the thesis statement! Also, don't babble in the introduction.

1 The structure of the thesis

There should be 5–8 numbered chapters in a thesis, including Introduction and Conclusion. If necessary, you can use sections and subsections to give the thesis a more fine-grained structure.

The chapters that lie between Introduction and Conclusion are collectively called the *body* of the thesis. It is often said to start with a *background*, which is then followed either by a *main theorem*, a *constructive part* or an *empirical part*.

Instead of simply listing headings of different levels it is recommended to let every heading be followed by at least a short passage of text.

Headings should reflect the content of the coming part (chapter, section).

1.1 Background

The goal of the background part of a thesis is to develop the theoretical background required in the thesis. The idea is that a reader of the thesis should be able to understand all the special concepts and methods used in the thesis. A good thesis also gives well-argued reasons for why exactly these concepts and methods are in use in the thesis (with the main alternatives given in the literature mentioned).

The best way to present and use of the theoretical background depends on what the thesis is about. The background part of a fully mathematical work differs considerably from the background part of a quantitative or qualitative empirical study.

Some suggestions:

- In a fully mathematical (theoretical) thesis this chapter usually introduces the mathematical background, general theory that is the setting of your contribution.

- In an empirical thesis it is usually chapter where you describe background of the problem, the methodology or explain the method(s) you will use to address the main problem of your thesis.

Reading other thesis of the same type will give you a good impression of what is required of your own thesis.

1.2 Main contribution

The background part is followed by your contribution:

- In a fully mathematical (theoretical) thesis it is usually a sequence of definitions and lemmas of your own devising, which then culminate in the proof of your main theorem.

- In an empirical thesis it is a set of empirical results obtained by applying a empirical research method.

1.3   Experimental part

In this part your results (theorems or new methods) are validated. This often means simulation, where your results are compared with other previous results.

You should present your contribution with precision, giving reasons for the choices you have made. You should follow the best practices of the research tradition you are using.

2 Formatting

A thesis is written with a single-column layout on one- or two-sided A4 sheets. The font type of the body text is usually Times New Roman and the font size is 12 pt. The text is fully justified and hyphenated. You do not have to indent the paragraphs.

2.1 Mathematical notations

Numbers are generally written using numerals for the sake of clarity, for example "6 stages" rather than "six stages". You should also use a thousand separators[2], i.e. instead of 55700125 write 55 700 125. Never omit the leading zero in decimals. For example, it is correct to write "0.5" and wrong to write ".5". A comma is used as a decimal separator in the Estonian language and a period in the English language.

Like numbers, it is advisable to abbreviate units of measurement. There is a space between the number and the unit, but you should keep them on the same line. It is better to compile a table or graph than include a great deal of numerical values in the body text. Use precise language and put numbers on a scale.

Newton's Second Law can be presented in the following way:

$$F = ma, \tag{1}$$

where m denotes the mass of an object, a means acceleration, and F means force. Please note that all the variables must be defined at the point of their first appearance. All sentences end with a punctuation mark, and the main elements of a sentence are separated by a comma in accordance with the rules of English grammar. Consider formulas as part of a sentence, therefore use commas or period accordingly after formulas (e.g. see equation (1)).

Formulas can be numbered or not numbered. Numbered formulas are used if there is need to refer to these formulas later on. For example, let us have a formula

$$c^2 = a^2 + b^2. \tag{2}$$

Then, since (2) is numbered, we can always refer to it in later text. For example, specifying $a = 3$ and $b = 4$ and applying (2) implies $c = 5$.

On the other hand, if we do not refer to some formula or equation at all, it should not have a number, for example:

$$S_n = \sum_{i=1}^{n} X_n.$$

---

[2]Use tilde ~ in LaTeX and a special character *non-breaking space* in MS Word

Occasionally mathematical notations are preceded by an identifier, such as 'Definition 1' or 'Theorem 1'. Simple formulas may be displayed within the body of the text without numbering.

Please note that usually different styles applied to same letter/symbol refer to different object, for example, x, x, X, $\boldsymbol{X}$, $\mathbf{X}$, ..., are all different. Also, a general rule is that variables are in italic and function names are in regular font, for example, s, u and p are some variables, and supremum function sup is in regular font so we do not confuse it with the product $sup = s \cdot u \cdot p$.

Please try to use unified notation (i.e. same letter/symbol should mean the same variable or function) throughout the whole document (as much as possible).

LaTeX is the best editor for writing also the more complex equations, such as

$$G^+(t,t') = \int G^+(E) \exp[-iE(t-t')/\hbar]dE. \tag{3}$$

2.2 Theorems and definitions

Mathematical thesis often include elements that require special formatting and numbering such as theorems, definitions, propositions, remarks, corollaries, lemmas and so on. Such formatting can be achieved by defining special environment with the `\newtheorem` command in the preamble of the document. For example:

Theorem 1. *Let f be a function whose derivative exists in every point, then f is a continuous function.*

Proof. If a derivative of function f exists in point x_0 (meaning that f is differentiable at point x_0), then f is continuous at point x_0 (differentiability implies continuity - not proven here).

Assume that function f is differentiable in every point, then that implies that f is continuous at every point and f is a continuous function. □

For the next theorem we need a definition:

Definition. *A right triangle is a triangle in which one angle is a right angle (that is, a 90-degree angle).*

Notice that the definition does not have a number. This can be achieved by defining the definition environment with the `\newtheorem*{}` command.

Theorem 2 (Pythagorean theorem)**.** *This is a theorem about right triangles and can be summarised in the next equation*

$$x^2 + y^2 = z^2$$

And a consequence of Theorem 2 is the statement in the next corollary (you can reference theorems when a label is assigned).

Corollary 2.1. *There's no right rectangle whose sides measure 3cm, 4cm, and 6cm.*

Notice that the Corollary has different numbering, this is because the environment is defined with `\newtheorem{corollary}{Corollary}[theorem]` , where the square brackets determines the environment that will be used for numbering (also refered to as counter). The same logic can be applied to formulas and theorems, if there are many of such elements. Often the counters are determined by section, for example "Theorem 2.3" refers to the 3rd theorem in the 2nd section of a document.

By default, each theorem uses its own counter. However it is common for similar types of theorems (e.g. Theorems, Lemmas and Corollaries) to share a counter. In this case, define subsequent theorems as:

`\newtheorem{<name>}[<counter>]{<Printed output>}`

where counter is the name of the counter to be used.

2.3 Figures

You must refer to all the figures in the body text. The reference should preferably appear on the same page as the actual figure or before it (e.g. Figure 1). Figures and tables must be numbered consistently thesis and primarily placed at the top of the page, but you are free to decide where they fit best. Never start a chapter with a figure, table or list.

Figures and the caption are consistently centered and the caption is placed under the figure and always on the same page as the figure. All figures must be explained in the body text, so that readers know what they are supposed to notice.

The figures should be in the same language as other text. The recommended font size is the same as that of the body text but no smaller than 10 pt. The figures must be readable, even if your thesis is printed in grey-scale!

2.4 Tables

Tables have numbered captions, see Table1 for example. The caption is placed on the same page above the table. You must refer to all the tables in the body text. In addition, you must discuss the content of any tables in the body text to ensure that readers understand their relevance.
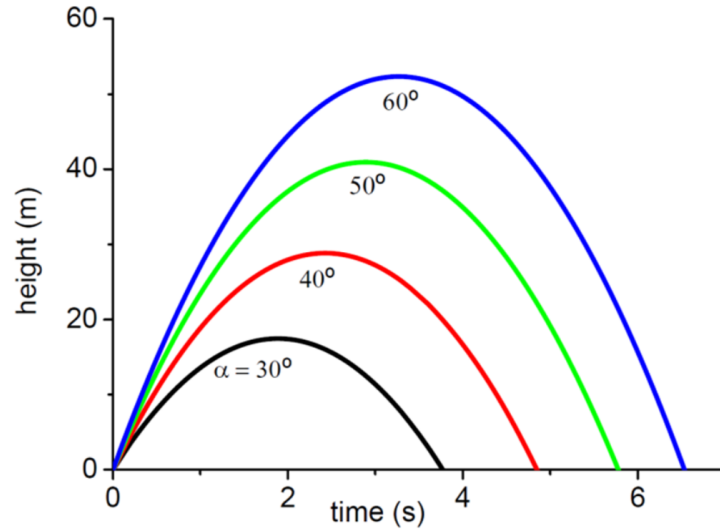
Figure 1: Diagram of achieved height and time of flight with different angles of release.

Table 1: Example of evaporation conditions in a thin film structure.

Substance	Thickness (nm)	Correction coefficient	Pressure (mbar)	Current (mA)	Speed (nm/s)
SiO_2	181.0	1.10	$3.0 \cdot 10^5$	20-23	0.2
TiO_2	122.1	1.55	$1.5 \cdot 10^4$	100-93	0.1

Often it is better to create the table in, e.g. MS Excel, and import it as .eps or .pdf file, for example, when you calculate some of the values automatically.

You can use boldface to highlight the header row. Do not surround all the cells with a border, as it may make your table harder to read. Put a line on top and bottom of the table. You can add a horizontal line grouped into categories.

The numbers are right aligned (optimally lined up at the decimal point) for easy comparison. You should preferably use SI units, established prefixes and rewrite large numbers so that the power of ten should be placed in the title of the column instead of each row, if possible.

2.5   Programs and algorithms

Codes and algorithms are written using mono-spaced font, such as Courier New, Consolas or their variations. If the length of the code or algorithm is less than 10

lines and you do not refer to it later on in the text, you can present it similarly to formulas. Here's an example showing a snippet (install package `listings` for coloring your code according to type):

```
\begin{lstlisting}[style=console, % title={Template files} ]
all: ${TARGET}.tex
pdflatex ${TARGET}.tex
bibtex ${TARGET}
pdflatex ${TARGET}.tex
\end{lstlisting}
```

If the code is longer but shorter than a page, you present like a figure (Program 4.1) titled "Program" or "Algorithm". You should add some comments to the code and indent it consistently. The actions performed by the code must be outlined in broad terms in the body text. Line numbers make it much easier to refer to the code in the text.

LaTeX has a packages that enable automatic code formatting like highlight reserved words, bring out string input or color comments differently.

3 Using references

This section will give an overview on how to use references. But first, a little more about structure. Pay notice that there should always be an introductory part to the whole chapter. If a chapter headline is immediately followed by a subsection, then this is considered bad practice. Having some text between headlines also makes your thesis more pleasant to read. See Background section for more.

3.1 References and citation

The list of references is the last section of the document. References can be ordered in alphabetic order (preferred) or in order of citation. The references section of this document covers three most common types of resources and also has some examples. The style of formatting of references provided in this document is recommended, but not mandatory. The key points to remember are that

- all the relevant information about a resource should be included;

- references of same type should be formatted using same style (e.g., all books have their style, articles their style, etc.).

The most common citation style is via the last name(s) of the author(s) and the year of publication (also known as Chicago style referencing). For example, if this sentence is taken from the first reference in the list (see the last section), it can be referred to as (Articleauthor, year). If a whole paragraph needs reference to the same resource, it can simply be added to the end of the whole paragraph, as is done with the second reference. (Bookauthor et al., year)

Furthermore, if a whole section or subsection needs reference(s) then this can be specified in the beginning of this section or subsection. For example, one may say something like "*We refer to Bookauthor et al. (year) and Bookauthor2 & Bookauthor (year) for the results of this section.*".

For a complete guide on Chicago style referencing see: Chicago 17th B (Author-Date) referencing guide

3.2 Using literature

The theoretical part is almost always based solely on the literature. When discussing your contribution, you may also need to cite some known results in literature.

Remember to avoid plagiarism! If you copy, either verbatim or with slight changes (or, example, in your own translation) text from some source, make it clear to the reader. Mark your quotes (using quotation marks or some other clear manner) and

give a precise citation. If you do not quote verbatim, mark any changes you have made. In most situations, however, it is better to use your own words, based on more than one source. Even then, give clear citations.

This guidelines document uses the BibLATEX system [Leh+12] and Chicago style [Fus12]. You can switch off this automation by using the \documentclass-option manualbib, but that means you have to take care of the bibliography yourself, and the techniques discussed below may not be available. See Appendix 2 for information on manual bibliography. Please note that the Institute recommends using a Chicago style for your bibliography.

3.3 Citations

You can cite sources in two ways. First, you can use the citation as a noun: **aho-compilers** briefly discuss the use of graph coloring in the register allocation phase of a compiler. In this case, use the \textcite command. Second, you can use a citation as a parenthetical, which is not read aloud: Graph coloring is one possibile way to allocate registers [**aho-compilers**]. Use the \parencite command for this.

Both commands (\textcite and \parencite) take three parameters, two of which are optional. The first (optional) parameter is a pre-note, the second (optional) parameter is a post-note, and the third (mandatory) parameter is the citation key [see Leh+12, Section 3.7]. The citation in the preceding sentence was made using the following command:

```
\parencite[see][Section~3.7]{biblatex-manual}
```

If you give these commands just one optional argument (that is, one enclosed in square brackets), it will be interpreted as a post-note. If you want to give only a pre-note, leave the post-note empty [see Leh+12]:

```
\parencite[see][]{biblatex-manual}
```

It is also possible to cite multiple sources in the same citation [see Leh+12, Section 3.7; regarding citations in general, see also Fus12, Section 5.3.2]. Use the command \parencites for this. For each citation, give it the same parameters as you would give a single \parencite command. It is good practice (but often not necessary) to end the command in a \relax, so that no surprises ensue.

```
\parencites%
  [see][Section~3.7]{biblatex-manual}%
  [regarding citations in general, see also][Section~5.3.2]%
    {biblatex-chicago-manual}%
\relax.
```

If you break the command into multiple lines, use the comment sign to end each line, to prevent spurious spaces.

Conclusions

The last chapter of a thesis is the Conclusions (some authors use Conculsion, instead). Keep it short, and discuss what one can conclude about the thesis statement or research question given in the Introduction, in light of all that has been written in the thesis. The Conclusion is also the place to discuss any limitations and weaknesses of the thesis (especially those that cast doubt on the reliability of the results given in the thesis), if they have not been already discussed, for example in a Discussion chapter. It is also customary to state, what further research might be beneficial in light of this thesis.

If the Conclusions threatens to become too long, it is a good idea to split the interpretation of the results into its own chapter, often called Discussion, making Conclusions short and sweet.

After Conclusions, there is the bibliography.

References

[Fus12]   David Fussner. *The biblatex-chicago package. Style files for biblatex*. Version 0.9.9a. July 30, 2012. URL: http://mirror.ctan.org/macros/latex/contrib/biblatex/doc/biblatex.pdf (visited on 01/29/2013).

[Leh+12]   Philipp Lehman et al. *The biblatex Package. Programmable Bibliographies and Citations*. Version 2.2. Aug. 2012. URL: http://mirror.ctan.org/macros/latex/contrib/biblatex/doc/biblatex.pdf (visited on 01/29/2013).

Appendix 1 – Manual bibliography

Appendices are purely optional. All appendices must be referred to in the body text.

You can append to your thesis, for example, lengthy mathematical derivations,an important algorithm in a programming language, input and output listings, an extract of a standard relating to your thesis, a user manual, empirical knowledge produced while preparing the thesis, the results of a survey, lists, pictures, drawings, maps, complex charts (conceptual schema, circuit diagrams, structure charts) etc.

List of references

1. Articleauthor, A. (year) Article title in regular font. *Journal title in italic.* **Volume in bold**, pages.

2. Bookauthor, B., Anotherauthor, A. & Yetanotherauthor, Y. (year) *Book title in italic.* Publishing House.

3. Bookauthor2, B., & Bookauthor, B. (year) *Another book (in italic).* Another Publishing House.

4. Internetauthor, I. Title in regular font. *Available:* http://... (*last visited:* date)

5. Hogg, R. & Klugman, S. (1984) *Loss distributions.* Wiley, New York.

6. Pigeon, M.& Denuit., M. (2010) Composite lognormal-Pareto model with random threshold. *Scandinavian Actuarial Journal*, **10**, 49–64.

7. R Core Team. Documentation for package 'stats'. *Available:* http://stat.ethz.ch/R-manual/R-patched/library/stats/html/00Index.html (*last visited:* 12.11.2014)

Appendix 2 – Some helpful tips

Brief basics of writing style are:

- Always think of your reader when you are writing and proceed logically from general to specific. "Write your thesis so that it would be understandable by your non-mathematician friend", Jüri Lember and "There is no such thing as too much explanation", Jüri Lember

- Highlight your key points, for example, by discussing them in separate chapters or presenting them in a table or figure. Use *italics* or **boldface** for emphasis, but don't overdo it!

- Avoid long sentences and complicated statements. A full stop is the best way to end a sentence.

- Use active verbs to make a dynamic impression but avoid the first person pro-noun "I", except in your preface.

- Avoid jargon and wordiness. Use established terminology and neutral language.

- The minimum length of chapters and sub-chapters is two paragraphs, and you need to consider the balance of chapters. Paragraphs must always consist of more than one sentence.

- Do not use more than three levels of headings, such as 4.4.2.

- Do not use too many abbreviations. Use capital and small letters consistently.

- Use LaTeX for your thesis, because of reasons! Compile your document as often as possible. Debugging a LaTeX document can be a nightmare, but if you at least know the last 4-5 steps that you took, then you can at least roll-back to latest working version.

- Use a good LaTeX Editor! It will save you a lot of time and spare some nerve cells.

- In case of many formulas in the thesis and frequent cross-referencing between sections, then consider using equations numbering that includes the section number in front (instead of equation label (3) you will have (1.3) where 1 refers to the section where this formula is found). For this use the following command before `\begin{document}` statement:

  `\numberwithin{equation}{section}`

- Be responsible for every word you write! This is Your thesis.

Sometimes ending a section with list is considered as bad style. Therefore, it is better to have some text after it.

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