E102 Midterm Project

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Introduction

Analysis of the Plant

The plant that we are trying to control is the circuit depicted in figure 1. This is a simple single input, single output system.

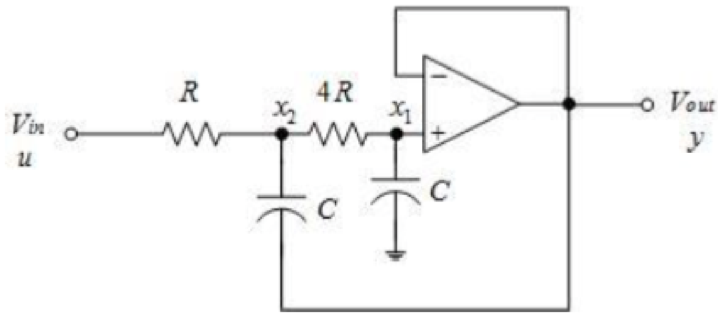

Figure 1: The circuit schematic for our plant. $C = 10\mu F$ and $R = 50k\Omega$

Immediately, we notice that $y = x_1$, because the $+$ and $-$ terminals of the op-amp must be at approximately equal potential. This means that our output equation is:

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{1}$$

We can use Kirchhoff's Current Law at the two nodes x_1 and x_2 to determine the rest of the state space equations for this system.

$$x_1 : \quad \frac{1}{4R}(x_1 - x_2) + C\dot{x}_1 = 0 \tag{2}$$

$$x_2 : \quad \frac{1}{R}(x_2 - u) + \frac{1}{4R}(x_2 - x_1) + C(\dot{x}_2 - \dot{x}_1) = 0 \tag{3}$$

Now if we solve equation 2 for $\dot{x}_1$, we get:

$$\dot{x}_1 = \frac{1}{4RC}(-x_1 + x_2) \tag{4}$$

Solving equation 3 for $\dot{x}_2$ (and plugging in the result above) we get:

$$\dot{x}_2 = \dot{x}_1 + \frac{1}{4RC}(x_1 - x_2) + \frac{1}{RC}(-x_2 + u) = \frac{1}{RC}(-x_2 + u) \tag{5}$$

Putting these equations in matrix form, we obtain:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{4RC} & -\frac{1}{4RC} \\ 0 & \frac{1}{RC} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{RC} \end{bmatrix} u \tag{6}$$

For our system, $\frac{1}{RC} = \frac{1}{50k\Omega * 10\mu F} = 2\text{Hz}$. Using this, our equations become:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u \tag{7}$$

Designing the Controller

Results