# Math test 2020, part 1

```
md"# Math test 2020, part 1"
```

```
using PlutoUI
```

## Table of Contents



# Exercise 1a

```
md"## Exercise 1a"
```

$$7.2dm = 72cm = 720mm > 72mm$$

```
md"$7.2 dm = 72 cm = 720 mm \boldsymbol{>} 72 mm$"
```

$$720\ 000cm = 7\ 200m = 7.2km = 7.2km$$

```
md"$720\ 000 cm = 7\ 200 m = 7.2 km \boldsymbol{=} 7.2 km$"
```

# Exercise 1b

```
md"## Exercise 1b"
```

$$612\ 300mm^2 = 6123cm^2 = 61.23dm^2 > 6.123dm^2$$

```
md"$612\ 300 mm^2 = 6 123 cm^2 = 61.23 dm^2 \boldsymbol{>} 6.123 dm^2$"
```

$$0.6123m^2 = 61.23dm^2 = 6\ 123cm^2 = 612\ 300mm^2 < 6\ 123\ 000mm^2$$

```
md"$0.6123 m^2 = 61.23 dm^2 = 6\ 123 cm^2 = 612\ 300 mm^2 \boldsymbol{<} 6\
123\ 000 mm^2$"
```

# Exercise 1c

```
md"## Exercise 1c"
```

$$20.58dm^3 = \{1m^3 = 1000dm^3\} = 0.02058m^3 < 0.2058m^3$$

```
md"$20.58 dm^3 = \{1 m^3 = 1000 dm^3\} = 0.02058 m^3 \boldsymbol{<} 0.2058 m^3$"
```

$$2\ 058\ 000mm^3 = 2058cm^3 = 2.058dm^3 < 2.58dm^3$$

```
md"$2\ 058\ 000 mm^3 = 2058 cm^3 = 2.058 dm^3 \boldsymbol{<} 2.58 dm^3$"
```

# Exercise 2

```
md"## Exercise 2"
```

$$\begin{aligned}(66.75 - 39.75) \cdot (0.43 + 0.27) = \\ 27 \cdot 0.7 = \underline{18.9}\end{aligned}$$

```
md"$\begin{align}
    (66.75-39.75) \cdot (0.43 + 0.27) &= \\
    27 \cdot 0.7 &= \underline{18.9}
\end{align}$"
```

# Exercise 3a

```
md"## Exercise 3a"
```

$$73\ 284 - 8\ 097 - 24\ 702 = 40485$$

```
md"$73\ 284 - 8\ 097 - 24\ 702 = 40485$"
```

# Exercise 3b

```
md"## Exercise 3b"
```

$$\begin{aligned}134\frac{2}{5}hl : 14 = \\ 134.4hl : 14 = \\ 13440l : 14 = \\ \frac{13440}{14}l = \\ \{\frac{13440}{14} = 960\} = \\ = 960l\end{aligned}$$

```
md"$\begin{align}
    134 \frac{2}{5} hl : 14 &= \\
    134.4 hl :14 &= \\
    13440 l :14 &= \\
    \frac{13440}{14}l &= \\
    \{ \frac{13440}{14} = 960 \} &= \\
    &= 960 l
\end{align}$"
```

# Exercise 5

```
md"## Exercise 5"
```

$$\begin{aligned}
\text{grey area} &= \\
18m \cdot 12m \cdot 2 + \frac{12m \cdot 12m}{2} \cdot 2 + 36m \cdot 12m &= \\
18 \cdot 12 \cdot 2m^2 + 12^2 m^2 + 36 \cdot 12m^2 &= \\
72m^2 + 144m^2 + 432m^2 &= \\
(72 + 144 + 432)m^2 &= \underline{648m^2}
\end{aligned}$$

```
md"$\begin{align}
    \text{grey area} &= \\
    18m \cdot 12 m \cdot 2 + \frac{12m \cdot 12m}{2} \cdot 2 + 36m \cdot 12m &=
    \\
    18 \cdot 12 \cdot 2 m^2 + 12^2 m^2 + 36 \cdot 12 m^2 &= \\
    72 m^2 + 144 m^2 + 432 m^2 &= \\
    (72 + 144 + 432) m^2 &= \underline{648 m^2}
\end{align}$"
```

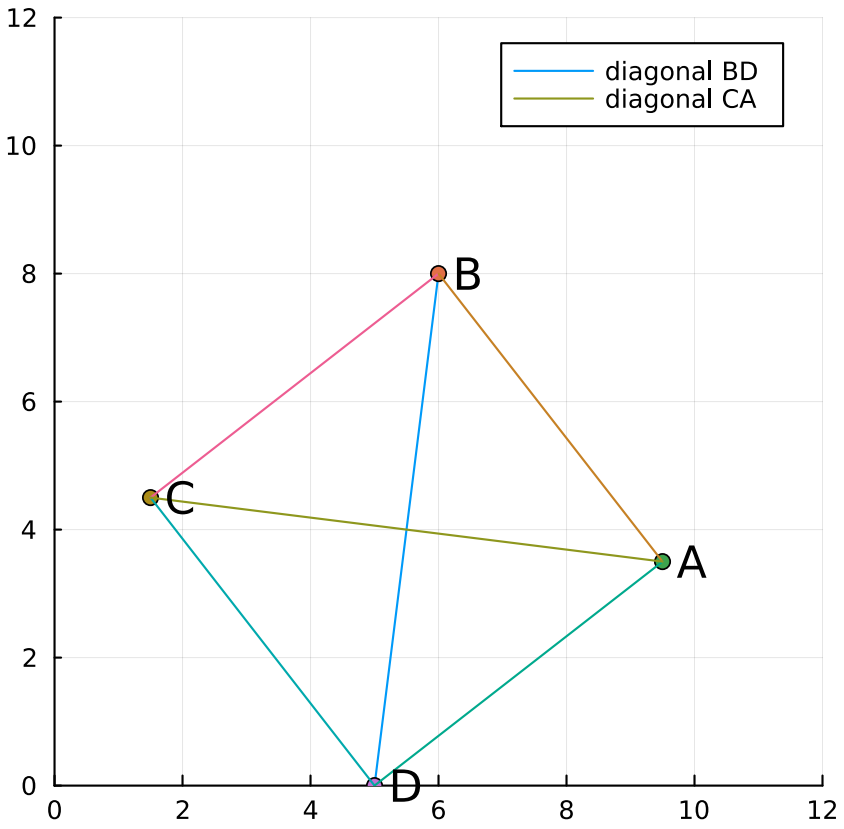
# Exercise 7

```
md"## Exercise 7"
```

```
using Plots
```

```
4×2 Matrix{Float64}:
 6.0  8.0
 5.0  0.0
 1.5  4.5
 9.5  3.5
```

```
begin
    B_point = [6 8]
    D_point = [5 0]
    C_point = [1.5 4.5]
    A_point = [9.5 3.5]
    points = [B_point; D_point; C_point; A_point]
end
```




```
begin
    # Plot the first diagonal, from B to D
    p5 = plot(
        [B_point[1], D_point[1]], [B_point[2], D_point[2]];
        aspect_ratio=:equal,
        label = "diagonal BD",
        xlims = (0,12),
        ylims = (0,12),
    )
    # Add point B
    scatter!(
        p5,
        [B_point[1]], [B_point[2]],
        annotations = (
            B_point[1], B_point[2],
            Plots.text(" B", :left)
        ),
        label="",
    )
    # Add point A
    scatter!(
        p5,
        [A_point[1]], [A_point[2]],
        annotations = (
            A_point[1], A_point[2],
            Plots.text(" A", :left)
        ),
        label="",
    )
    # Add point D
    scatter!(
        p5,
```

```
        [D_point[1]], [D_point[2]],
        annotations = (
            D_point[1], D_point[2],
            Plots.text(" D", :left)
        ),
        label="",
    )
    # Add point C
    scatter!(
        p5,
        [C_point[1]], [C_point[2]],
        annotations = (
            C_point[1], C_point[2],
            Plots.text(" C", :left)
        ),
        label="",
    )

    # Plot line from C to D
    plot!(p5, [C_point[1], D_point[1]], [C_point[2], D_point[2]], label="")
    # Plot line from C to B
    plot!(p5, [C_point[1], B_point[1]], [C_point[2], B_point[2]], label="")
    # Plot line from B to A
    plot!(p5, [B_point[1], A_point[1]], [B_point[2], A_point[2]], label="")
    # Plot line from D to A
    plot!(p5, [D_point[1], A_point[1]], [D_point[2], A_point[2]], label="")

    # Plot the second diagonal, from C to A
    plot!(p5, [C_point[1], A_point[1]], [C_point[2], A_point[2]],
    label="diagonal CA")
end
```

# Exercise 8a

```
md"## Exercise 8a"
```

$$
\begin{aligned}
\text{speed km/h} &= \frac{\text{kilometers}}{\text{hours}} \\
&= \frac{519km}{45min} = \frac{519km}{\frac{3}{4}h} = \frac{519 \cdot 4}{3} km/h \\
&= 173 \cdot 4km/h \\
&= 692km/h \\
&\text{Hence, in one hour at this speed, the plane can fly 692 km}
\end{aligned}
$$

```
md"$\begin{align}
    \text{speed km/h} &= \frac{\text{kilometers}}{\text{hours}} \\
    &= \frac{519 km}{45 min} = \frac{519 km}{\frac{3}{4} h} = \frac{519 \cdot 4}{3} km/h \\
    &= 173 \cdot 4 km/h\\
    &= 692 km/h\\
    & \text{Hence, in one hour at this speed, the plane can fly 692 km} \\
\end{align}$"
```

# Exercise 8b

```
md"## Exercise 8b"
```

$$
\begin{aligned}
\text{speed km/h} &= \frac{\text{kilometers}}{\text{hours}} \\
\text{kilometers} &= \text{speed km/h} \cdot \text{hours} \\
\text{hours} &= \frac{\text{kilometers}}{\text{speed km/h}} \\
&= \frac{48km}{120km/h} \\
&= \frac{48}{120}h \\
&= \frac{48}{120} \cdot 60min \\
&= \frac{48}{2}min \\
&= 24min \\
&\text{So in 24 minutes at the reduced speed 110 km/h, the car drives:} \\
\text{kilometers} &= \text{speed km/h} \cdot \text{hours} \\
\text{kilometers} &= 110km/h \cdot 24min \\
&= 110km/\cancel{h} \cdot \frac{24}{60}\cancel{h} \\
&= \frac{110 \cdot 24}{60}km \\
&= \frac{110 \cdot 4}{10}km \\
&= 11 \cdot 4km \\
&= \underline{44km}
\end{aligned}
$$

```
md"$\begin{align}
    \text{speed km/h} &= \frac{\text{kilometers}}{\text{hours}} \\
    \text{kilometers} &= \text{speed km/h} \cdot \text{hours} \\
    \text{hours} &= \frac{\text{kilometers}}{\text{speed km/h}} \\
    & = \frac{48 km}{120 km/h} \\
    & = \frac{48}{120}h \\
    & = \frac{48}{120} \cdot 60 min \\
    & = \frac{48}{2} min \\
    & = 24 min \\
    & \text{So in 24 minutes at the reduced speed 110 km/h, the car drives:} \\
    \text{kilometers} &= \text{speed km/h} \cdot \text{hours} \\
    \text{kilometers} &= 110 km/h \cdot 24 min \\
    &= 110 km/\cancel{h} \cdot \frac{24}{60} \cancel{h} \\
    &= \frac{110 \cdot 24}{60} km \\
    &= \frac{110 \cdot 4}{10} km \\
    &= 11 \cdot 4 km \\
    &= \underline{44 km} \\
\end{align}$"
```

# Exercise 9a

```
md"## Exercise 9a"
```

$$\begin{aligned}
\text{pages of book} &= \text{pages per day} \cdot \text{number of days} \\
&= 8 \cdot 36 pages \\
&= 288 pages \\
&\text{So at reading speed 12 pages per day,} \\
&\text{it would take this many days to read the book:} \\
\text{number of days} &= \frac{\text{pages of book}}{\text{pages per day}} \\
&= \frac{288 \cancel{pages}}{12 \cancel{pages}/day} \\
&= \frac{288}{12} days \\
&= \frac{144}{6} days \\
&= \frac{72}{3} days \\
&= \underline{24 days} \\
&\text{So with 50\% higher reading speed, the book completion time} \\
&\text{gets reduced to 2/3 of the original time}
\end{aligned}$$

```
md"$\begin{align}
    \text{pages of book} &= \text{pages per day} \cdot \text{number of days} \\
    &= 8 \cdot 36 pages\\
    &= 288 pages \\
    & \text{So at reading speed 12 pages per day,} \\
    & \text{it would take this many days to read the book:} \\
    \text{number of days} &= \frac{\text{pages of book}}{\text{pages per day}}
    \\
    &= \frac{288 \cancel{pages}}{12 \cancel{pages}/day} \\
    &= \frac{288}{12}days \\
    &= \frac{144}{6}days \\
    &= \frac{72}{3}days \\
    &= \underline{24days} \\
    & \text{So with 50\% higher reading speed, the book completion time} \\
    & \text{gets reduced to 2/3 of the original time} \\
\end{align}$"
```

# Exercise 9b

```
md"## Exercise 9b"
```

$$
\begin{aligned}
\text{number of days} &= \frac{\text{pages of book}}{\text{pages per day}} \\
&= \frac{288 + 96\ \cancel{pages}}{12\ \cancel{pages}/day} \\
&= \frac{384}{12} days \\
&= \frac{192}{6} days \\
&= \frac{96}{3} days \\
&= 32 days \\
&= 24 days \cdot x \\
&\text{i.e.} \\
x &= \frac{32}{24} \\
&= \frac{4}{3} \\
&\text{So with } 1/3 \text{ more pages, the completion time} \\
&\text{increases to } 4/3 \text{ of the original time}
\end{aligned}
$$

```
md"$\begin{align}
    \text{number of days} &= \frac{\text{pages of book}}{\text{pages per day}}
    \\
    &= \frac{288+96 \cancel{pages}}{12 \cancel{pages}/day} \\
    &= \frac{384}{12}days \\
    &= \frac{192}{6}days \\
    &= \frac{96}{3}days \\
    &= 32 days \\
    &= 24 days \cdot x \\
    & \text{i.e.} \\
    x &= \frac{32}{24} \\
    &= \frac{4}{3} \\
    & \text{So with 1/3 more pages, the completion time }\\
    & \text{increases to 4/3 of the original time} \\
\end{align}$"
```

# Exercise 10a

```
md"## Exercise 10a"
```

$$\begin{aligned}
&\text{degrees per minute} = \frac{360^\circ}{12 \cdot 60min} = \frac{1}{2}^\circ/min \\
&\text{hence, in 2 minutes the angle will increase from } 137^\circ \text{ to } 138^\circ
\end{aligned}$$

```
md"$\begin{align}
    \text{degrees per minute} &= \frac{360°}{12 \cdot 60min}  = \frac{1}{2}
    °/min \\
    & \text{hence, in 2 minutes the angle will increase from 137° to 138°}
\end{align}$"
```

# Exercise 10b

```
md"## Exercise 10b"
```

$$\begin{aligned}
\frac{137\not{^\circ}}{\frac{1}{2}\not{^\circ}/min} &= \\
137 \cdot 2min &= \\
274min &= \\
\frac{274}{60}h &= 4h34min \\
&\text{The time is 34 minutes past 4}
\end{aligned}$$

```
md"$\begin{align}
    \frac{137\cancel{°}}{\frac{1}{2}\cancel{°}/min} &= \\
    137 \cdot 2 min &= \\
    274 min &= \\
    \frac{274}{60} h &= 4h34min \\
    &\text{The time is 34 minutes past 4}
\end{align}$"
```

# Exercise 11

```
md"## Exercise 11"
```

```
using DataFrames
```

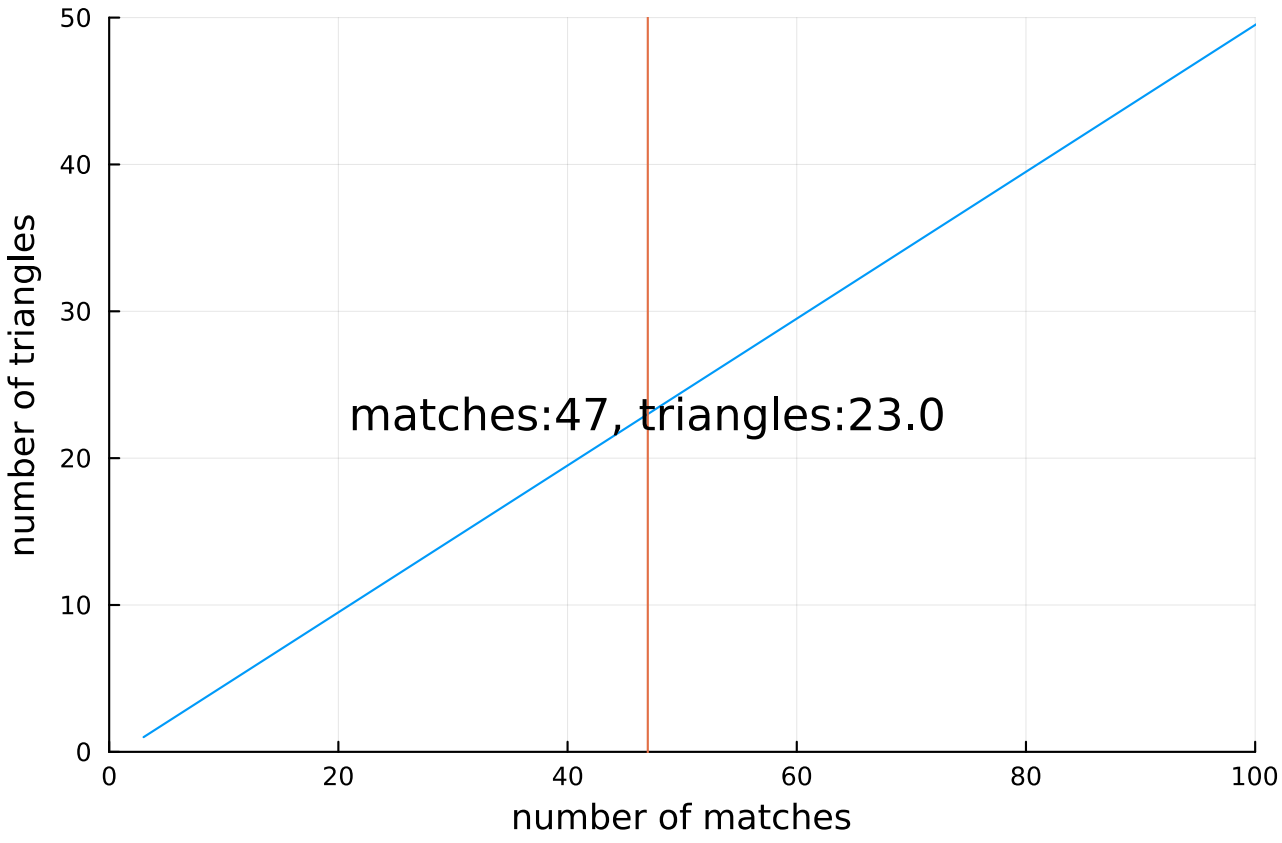
matches_df =

| | num_triangles | num_matches |
|---|---|---|
| 1 | 1 | 3 |
| 2 | 2 | 5 |
| 3 | 3 | 7 |
| 4 | 4 | 9 |
| 5 | 5 | 11 |
| 6 | 10 | 21 |
| 7 | 20 | 41 |
| 8 | 1000 | 2001 |

```
matches_df = DataFrame(
    num_triangles = [1, 2, 3, 4,  5, 10, 20,1000],
    num_matches =   [3, 5, 7, 9, 11, 21, 41, 2001]
)
```

47

```
@bind matches Slider(1:1:100, 11, true)
```



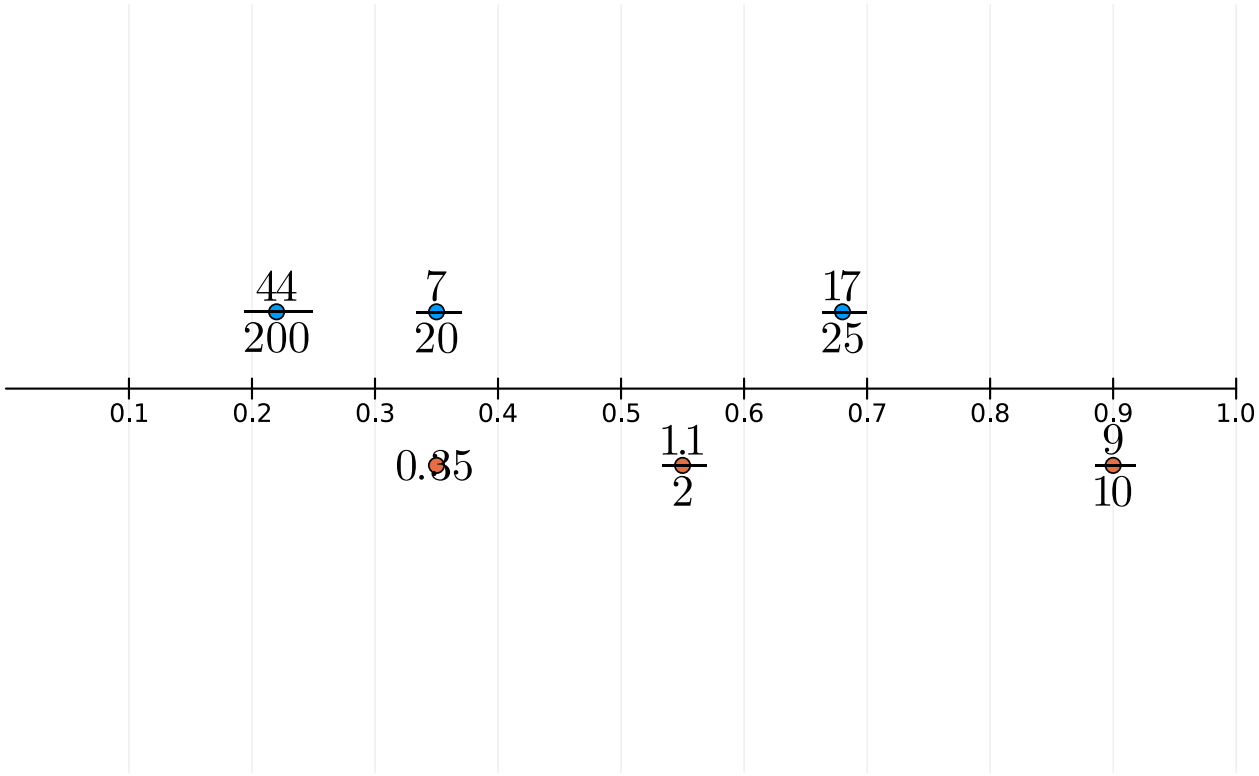

```
begin
    match_plot = plot(
        matches_df.num_matches,
        matches_df.num_triangles,
        xlabel = "number of matches",
        ylabel = "number of triangles",
        legend = false,
        ylims = (0, 50),
        xlims = (0, 100),
        annotations = (
            matches, triangles(matches),
            Plots.text("matches:$matches, triangles:$(triangles(matches))",
            :under)
        )
    )
    vline!([matches], label="")
end
```

```
triangles (generic function with 1 method)
```

```
function triangles(matches)
    return (matches - 1)/2
end
```

# Exercise 12

```
md"## Exercise 12"
```


$\frac{44}{200}$
$\frac{7}{20}$
$\frac{17}{25}$
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
0.35
$\frac{1.1}{2}$
$\frac{9}{10}$

```
begin
    #plotly()
    gr()
    rationals_str = ["7/20", "44/200", "17/25"]
    rationals = [7//20, 44//200, 17//25]
    decimals_str = ["0.35", "9/10", "1.1/2"]
    decimals = [0.35, 9/10, 1.1/2]
    data_df = DataFrame(rationals = rationals, decimals = decimals)
    single_axis_plot = scatter(
        data_df.rationals,
        2 .* ones(length(rationals)),
        annotations = (
            rationals, 2 .* ones(length(rationals)),
            ([Plots.text(latexify(rationals_str[i]), :over) for i in
            1:length(rationals)])
        ),
        legend = false,
        xticks = 0:0.1:1,
        framestyle = :origin,
        yaxis=([], false),
        ylims = (-10, 10),
        xlims = (0,1)
    )
    scatter!(single_axis_plot,
        data_df.decimals,
        -2 .* ones(length(decimals)),
        annotations = (
            decimals, -2 .* ones(length(decimals)),
            ([Plots.text(latexify(decimals_str[i]), :under) for i in
            1:length(decimals)])
        ),
        legend = false,
        yaxis = false,
    )
end
```

```
using Latexify   , LaTeXStrings
```

# Exercise 13a

```
md"## Exercise 13a"
```

| | five_digit_number | digit_sum |
|---|---|---|
| 1 | 10001 | 2 |
| 2 | 10010 | 2 |
| 3 | 10100 | 2 |
| 4 | 11000 | 2 |
| 5 | 20000 | 2 |

```
begin
    five_digit_numbers = 10000:1:99999
    five_d_df = DataFrame(five_digit_number = five_digit_numbers)
    select!(five_d_df,
        :,
        :five_digit_number => ByRow(x -> sum(digits(x))) => :digit_sum
    )
    subset!(five_d_df, :digit_sum => ByRow(x -> x == 2))
end
```

# Exercise 13b

```
md"## Exercise 13b"
```

```
using Primes
```

| | n |
|---|---|
| 1 | 101 |
| 2 | 103 |
| 3 | 107 |
| 4 | 109 |
| 5 | 113 |
| 6 | 127 |

```
begin
    primes_df = DataFrame(n = 100:1:130)
    subset!(primes_df, :n => ByRow(Primes.isprime))
end
```

# Exercise 13c

```
md"## Exercise 13c"
```

| | n | n_div_by_3 | n_div_by_6 | n_div_by_9 |
|---|---|---|---|---|
| 1 | 216 | 72 | 36 | 24 |
| 2 | 234 | 78 | 39 | 26 |
| 3 | 252 | 84 | 42 | 28 |
| 4 | 270 | 90 | 45 | 30 |
| 5 | 288 | 96 | 48 | 32 |

```
begin
    divisible_df = DataFrame(n = 200:1:300)
    subset!(divisible_df,
        :n => ByRow(n -> mod(n,3) == 0),
        :n => ByRow(n -> mod(n,6) == 0),
        :n => ByRow(n -> mod(n,9) == 0)
    )
    select(divisible_df,
        :,
        :n => ByRow(n -> Int(n/3)) => :n_div_by_3,
        :n => ByRow(n -> Int(n/6)) => :n_div_by_6,
        :n => ByRow(n -> Int(n/9)) => :n_div_by_9,
    )
end
```

# Exercise 14

```
md"## Exercise 14"
```

$$\begin{aligned}
\frac{\text{remaining candies after 2nd child}}{2} = 3 &\implies \text{remaining candies after 2nd child} = 6 \\
\frac{\text{remaining candies after 1st child}}{2} = 3+6 = 9 &\implies \text{remaining candies after 1st child} = 18 \\
\frac{\text{bought candies}}{2} = 3+18 = 21 &\implies \text{bought candies} = 42
\end{aligned}$$

```
md"$\begin{align}
    \frac{\text{remaining candies after 2nd child}}{2} = 3 &\implies
    \text{remaining candies after 2nd child} = 6 \\
    \frac{\text{remaining candies after 1st child}}{2} = 3+6 = 9 &\implies
    \text{remaining candies after 1st child} = 18 \\
    \frac{\text{bought candies}}{2} = 3+18 = 21 &\implies
    \text{bought candies} = 42
\end{align}$"
```