Regressão Multinomial

31/08/2020

$Y \in \{A, B, C\}$ $\quad Y \sim \text{Multinomial}(\pi_A, \pi_B, \pi_C)$

onde $\pi_A = P(Y=A)$; $\pi_B = P(Y=B)$ e $\pi_C = P(Y=C)$

$\pi_A + \pi_B + \pi_C = 1$.

Por comparação:

$Y \sim \text{Bernoulli}(p)$

$$P(Y=1) = \frac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}}$$

$$\frac{P(Y=1)}{P(Y=0)} = e^{\beta_0+\beta_1 x}$$

↑ referência

Se $X \in \{A, B, C\}$

X	X_A	X_B	X_C
A	1	0	0
B	0	1	0
C	0	0	1

escolhe-se um para referência (β_0)

Se $Y \in \{A, B, C\}$

Y	Y_A	Y_B	Y_C
A	1	0	0
B	0	1	0
C	0	0	1

↑ referência

No modelo Multinomial:

$$\frac{P(Y=B)}{P(Y=A)} = e^{\beta_0^B + \beta_1^B x}$$

$$\frac{P(Y=C)}{P(Y=A)} = e^{\beta_0^C + \beta_1^C x}$$

onde $P(Y=A) + P(Y=B) + P(Y=C) = 1$

$$P(Y=B) = \frac{e^{\beta_0^B + \beta_1^B x}}{1 + e^{\beta_0^B + \beta_1^B x} + e^{\beta_0^C + \beta_1^C x}}$$

$$P(Y=C) = \frac{e^{\beta_0^C + \beta_1^C x}}{1 + e^{\beta_0^B + \beta_1^B x} + e^{\beta_0^C + \beta_1^C x}}$$

$$P(Y=A) = \frac{1}{1 + e^{\beta_0^B + \beta_1^B x} + e^{\beta_0^C + \beta_1^C x}}$$