Suppose we can find N dependent constants α_L and α_U such that
$\alpha_L <= |\alpha_1(s)| <= \alpha_U$

We can lower bound the summand denominators as
$Re(s + (t/2)\alpha_1(s)) <= \sigma + (t/2)\alpha_L$

Among the numerators, we can upper bound different terms as
$|2\alpha_1(s) - log(n)| < |2\alpha_1(s)| <= 2\alpha_U$
since $|(a + bi) - c| < |a + bi|$ if $a, c > 0$ and $a > c$

$|\alpha_1^2(s) - \alpha_1^2(1 - s)| <= |\alpha_1(s) + \alpha_1(1 - s)||\alpha_1(s) - \alpha_1(1 - s)|$

Now $|\alpha_1(s) + \alpha_1(1 - s)| <= 2\alpha_U$

and $\alpha_{diff} = |\alpha_1(s) - \alpha_1(1 - s)|$
$<= |\frac{3}{2}(\frac{1}{s} + \frac{1}{s-1})| + \frac{1}{2}|\log\frac{s}{1-s}|$
$<= \frac{3}{x_N} + \frac{1}{2}\log|\frac{1-y+ix}{1+y-ix}| + \frac{1}{2}|i(tan^{-1}(\frac{x}{1-y}) + tan^{-1}(\frac{x}{1+y}))|$
$<= \frac{3}{x_N} + \frac{1}{4}\log\frac{(1-y)^2+x_{N+1}^2}{(1+y)^2+x_N^2} + \frac{\pi}{2}$
$<= \frac{\pi}{2} + \frac{3}{x_N} + \frac{1}{2}\log\frac{x_{N+1}}{x_N}$ since $\frac{a+b}{c+d} < \frac{b}{d}$ if $a < c$ and $d < b$

(From the wiki, it is possible $|\alpha_1(s) - \alpha_1(1 - s)| <= \frac{y}{x_{N-6}}$ would also work although it was derived from $|\alpha_1(s) - \overline{\alpha_1(1 - s)}|$)

$|\lambda| <= e^{\delta}/N^y$ (from the wiki)

So,
$|\frac{d}{dt}\frac{A^{eff}+B^{eff}}{B_0^{eff}}|$
$<= \frac{1}{4}\sum_{n=1}^{N}\frac{b_n^t|\log^2 n-2\log(n)\alpha_1(s_B)|}{n^{Re(s_B+(t/2)\alpha_1(s_B))}} + \frac{|\lambda|}{4}\sum_{n=1}^{N}\frac{b_n^t|\log^2 n-2\log(n)\alpha_1(s_A)+\alpha_1^2(s_A)-\alpha_1^2(s_B)|}{n^{Re(s_A+(t/2)\alpha_1(s_A))}}$
$<= \frac{1}{4}\sum_{n=1}^{N}\frac{2\alpha_U b_n^t log(n)}{n^{1-\sigma+(t/2)\alpha_L}} + \frac{e^{\delta}}{N^y}\sum_{n=1}^{N}\frac{2\alpha_U b_n^t(log(n)+2\alpha_U\alpha_{diff})}{n^{\sigma+(t/2)\alpha_L}}$ ...(fixing $\sigma=(1-y)/2$)
$<= \frac{\alpha_U}{2}\sum_{n=1}^{N}(\frac{b_n^t log(n)}{n^{1-\sigma+(t/2)\alpha_L}} + \frac{e^{\delta}}{N^y}\frac{b_n^t(log(n)+\alpha_{diff})}{n^{\sigma+(t/2)\alpha_L}})$
$\alpha_L = log(N), \alpha_U = log(N) + 1$ works but may not be that tight
$\alpha_L = \frac{1}{2}\log\frac{|1-y+ix_N|}{4\pi}, \alpha_U = \frac{\pi}{4} + \frac{3}{2x_N} + \frac{1}{2}\log\frac{|1+y+ix_{N+1}|}{4\pi}$ is much tighter

To improve the bound further, we can treat $|2\alpha_1(s) - log(n)|$
$= |2(\frac{1}{2s} + \frac{1}{s-1}) + log\frac{s}{2\pi} - log(n)|$
$<= |2(\frac{1}{2s} + \frac{1}{s-1}) + log\frac{|1+y+ix_{N+1}|}{4\pi n} + i\frac{\pi}{2}|$
$<= \frac{\pi}{2} + \frac{3}{x_N} + log\frac{|1+y+ix_{N+1}|}{4\pi n}$
$= 2\alpha_U - log(n)$

This gives an improved bound,
$|\frac{d}{dt}\frac{A^{eff}+B^{eff}}{B_0^{eff}}|$
$<= \frac{1}{4}\sum_{n=1}^{N}(\frac{b_n^t log(n)(2\alpha_U-\log(n))}{n^{1-\sigma+(t/2)\alpha_L}} + \frac{e^{\delta}}{N^y}\frac{b_n^t(log(n)(2\alpha_U-\log(n))+2\alpha_U\alpha_{diff})}{n^{\sigma+(t/2)\alpha_L}})$