Designing an MINLP model for optimal scheduling of Supply Chain Network

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1. ABSTRACT

The optimal design of supply chain networks can reduce the high costs and long wait times for industrial organizations. Indeed, the optimal design can contribute to refine logistical objects as well as logistical strategies, improve on the architecture logistics network, and above all, support decision making; however, optimization of integrated network design is still a challenge. With respect to the number of different types of decision variables, most current studies just addressed the network design problem considering three or fewer layers and deterministic demands. They are not practical for situations with uncertainty. Since this industrial engineering optimization problem is an NP-hard (Non-Linear-hard) problem even for a small-size network. Traditional optimization methods tried to calculate every potential plan one by one, which is very time consuming and impractical. Thus the traditional methods are unadaptable for realistic industrial logistics networks planning.

To better address this problem, we proposed to model a way to aid the decision makers with the task of planning single-horizon logistics in supply chain networks with consideration of different types of decision variables that perplex the decision makers of this supply chain network problem. This model can be applied to supply chain designs with many items, layers, different decision variables and stochastic demands. It will make it more realistic and will save both cost and time for industrial planning.

Our main results prove that our model concerns not only facility locations, production, and transportation, but also inventory levels in warehouses based on the stochastic demand of customers, for a more realistic perspective. Our model can be perfectly used in multi-layer supply chain problem, which can also solve the problem caused by the uncertainty in the actual logistics and stochastic demands.

2. INTRODUCTION AND PROBLEM STATEMENT

2.1. Motivation

There has been substantial research towards optimal design of supply chain networks recently due to high costs and wait times involved. However, optimization of integrated network design is still a challenge, especially if there exist several layers and logistical components. The motivation for this project is to model a way to aid the decision makers with the task of planning single-horizon logistics in supply chain networks. Several decision variables and stochastic demands will be considered in this model to obtain a minimum cost solution for managing the supply chain network.

2.2. Problem statement

The mathematical formulation for the logistics network can have more than three layers. The model will integrate decisions on the location of plants and warehouses, levels of production and inventory, and on transportation modes (Cordeau et al., 2006). The Optimization is solved using GAMS (GAMS Development Corporation) software. The optimization problem is formulated as an MINLP for which BARON solver is used. Other solvers like ANTIGONE and DICOPT can also be used to compare the results for the same problem.

We will be deriving our model from an example problem that minimized cost for a supply chain network with stochastic demand. With consideration of uncertainties, we proposed a more realistic mathematical formulation. The uncertainties of customer demand lead to a bi-criterion for the optimal design of responsive process supply chains with inventories. The data used in the model is inspired from the example problem (Cordeau et al., 2006) and is generated randomly.

3. LITERATURE SURVEY

A supply chain is a complex network that involves resources, people, technologies, activities, and information in order to convert raw materials into finished goods and distribute them to final customers (Lambiase, A. et al., 2013). There are usually different supplier options for purchasing raw materials, different production options for the assembly of final products and different distribution options to carry final products to market. Generally, the design and management of the supply chain are to obtain the best performance of the integrated network (Aslam, T., and Amos, H.C.N., 2010). It offers decision-makers who decide at different levels of the supply chain robust tools to evaluate the impact of alternative strategies on industrial organizations' performance, prior to making them in the real environment.

The main purpose of supply chain modeling is to minimize or maximize an objective function under constraints of satisfying the demands of the nodes that follow. These equations can be improved by the identification of decisions and trade-off solutions. Therefore, generating a mathematical model for the optimization problem is pertinent for designing supply chains (Ramzi, H. et al., 2008).

To build a practical mathematical model, several items, layers, logistics components, and different types of decision variables and stochastic demands should be considered. However, just a few current studies have considered three or more layers and deterministic demands with four different types of decision variables with mixed integer linear programming models(MILP) (Melo, M. et al., 2006; and Wilhelm, W., et al., 2005). Some studies have achieved the design optimization of a single-period planning horizon logistics network with three or more layers involving single decision function of location using MILP model (Salema, M.I.G. et al., 2007; and Santoso, T., Ahmed, S., 2005).

From a realistic point of view, the uncertainty of customer demands is also an important criterion to be considered in the model in order to determine the optimal network design, transportation and inventory levels of the supply chain. Studies towards designing an MINLP model for the optimal design of responsive process supply chains with inventories, considering economic and responsiveness objectives can also be applied (You, F., and Grossmann, I. E., 2011).

Based on the previous studies, the aim of this report is to propose a model that formulates a four-type decision-variable logistics network design problem considering three layers and multi-products with stochastic demands, as an MINLP. The formulation presented here is flexible and can easily be adapted to handle multiple production and distribution stages as well as multiple capacity alternatives at any given location. With respect to uncertainty in the customer demands, we incorporate the probability of the customer demands as a normal distribution.

Further research could concentrate on extending the model and solution method to handle the cases of dynamic(time-varying) and costs associated with stocks in transit, backlogging, variability in lead time per product.

4. MATHEMATICAL MODELS AND METHODS

We created a model of a three-layer network to determine optimal solutions for a supply chain network. Our network consists of multiple suppliers, process plants, storage warehouses, and customers. To start developing our model we created an objective function in order to minimize the cost of the total network. The cost of this network is from the capital cost of the origins of the network, marginal cost of producing a unit of each commodity, cost of transportation and storage costs in warehouses. Our goal is to reduce the total cost of these four segments. In the model we developed, we analyzed a network of three potential suppliers, two potential plants, four potential warehouses with five customers. The model had five generic variables to modify to minimize the cost. They are explained below:

X_{wc}^{fm}: $the\ amount\ of\ product\ f\ provided\ by\ warehouse\ w\ to\ customer\ c\ using\ transportation\ mode\ m$

X_{od}^{km}: $the\ amount\ of\ commodity\ k\ provided\ by\ origin\ o\ to\ destination\ d\ using\ transportation\ mode\ m$

U_o: $indicate\ if\ origin\ o\ is\ selected$

$V_o{}^k$: $indicate\ if\ commodity\ k\ is\ assigned\ to\ origin\ o$

Y_{wc}^f: $indicate\ if\ warehouse\ w\ provides\ product\ f\ to\ customer\ c$

Y_{od}^k: $indicate\ if\ origin\ o\ provides\ commodity\ k\ to\ destination\ d$

Z_{wc}^m: $indicate\ if\ transportation\ mode\ m\ is\ selected\ to\ serve\ from\ warehouse\ w\ to\ customer\ c$

Z_{od}^m: $indicate\ if\ transportation\ mode\ m\ is\ selected\ to\ serve\ from\ origin\ o\ to\ destination\ d$

4.1 Material balance

In order for the model to accurately represent a realistic supply chain network constraints were made to limit the model. The first of these constants is a material balance of each plant in the network. This constraint equates the quantity of raw material being shipped into the amount of the raw material used in the products coming out of the plant as seen below.

<u>Constraint equation (1)</u>

$$\sum_{s \in S^r} \sum_{m \in M_{sp}^r} X_{sp}^{rm} - \sum_{f \in F^r} \sum_{w \in W^f} \sum_{m \in M_{pw}^f} b^{rf} X_{pw}^{fm} = 0, \quad r \in R, p \in P$$

In the equation, b^{rf} represent the amount of raw material r used in the creation of one unit of product p. The second constraint was also material balance equality of the product movement in and out of the warehouses. This constraint is created to make sure that the amount of product that enters the warehouse eventually leaves. This equation is shown below.

Constraint equation (2)

$$\sum_{p \in P^f} \sum_{m \in M_{pw}^f} X_{pw}^{fm} - \sum_{x \in C} \sum_{m \in M_{wc}^f} X_{wc}^{fm} = 0, \quad f \in F, w \in W^f,$$

The last of the equality constraints is to ensure that customer demand is met. Customers will only be given what their demand no more or less and this constraint mathematically describes this. This constraint is shown below where a_c^f is the demand of product f by customer c.

Constraint equation (3)

$$\sum_{w \in W^f} \sum_{m \in M_{wc}^f} X_{wc}^{fm} = a_c^f, f \in F, \quad c \in C^f,$$

4.2 Capacity balance

Additional inequality constraints were used to limit the model into a feasible space. The feasible space was set by the limits of the supply chain network with the maximum amount of plants, warehouses, and capacities. The first of these inequality constraints s depicted below where u_o is the capacity of the node and u_o^k is the amount of capacity used by a unit of commodity k.

Constraint equation (4)

$$\sum_{k \in K} \sum_{d \in D^k} \sum_{m \in M_{od}^k} u_o^k X_{od}^{km} - u_o U_o \leq 0, \quad o \in O$$

This equation limits the model to allow only the amount of commodities moving through any origin o to be less than or equal to the capacity of the origin within the network. The next constraint is similarly restricting the amount of product moving due to capacity limits. This equation restricts the amount of a commodity k that can be shipped from origin o. This equation is shown below with q_o^k being the maximum amount of commodity k that can be shipped out of origin o.

Constraint equation (5)

$$\sum_{d \in D^k} \sum_{m \in M_{od}^k} X_{od}^{km} - q_o^k V_o^k \leq 0, \qquad k \in K, \ o \in O^k$$

This makes our model more realistic making sure that a single location does not utilize beyond what is capable of shipping for each commodity. In order to guarantee that the transportation of commodity k to destination d is executed when origin o is selected, the following constraint is applied.

Constraint equation (6)

$$\sum_{m \in M_{od}^{km}} X_{od}^{km} - q_{od}^k Y_{od}^k \ \leq 0, \ \ k \in K, o \in O^k, d \in D^k$$

The next inequality constraint limits the amount of commodity shipped by a mode of transportation from an origin to a destination. Each mode of transportation has a capacity for moving commodities from one origin to another destination this capacity is described by g_{od}^m. In this inequality there is a second parameter, g^{km} which describes the number of cubic meters of capacity utilized by one ton of commodity for particular transportation mode. The inequality is shown below where g_{od}^m is the capacity in cubic meters of transportation mode m from origin o to destination d. Also in the equation represents the amount of volume in cubic meters taken up by one kilogram of commodity k.

Constraint equation (7)

$$\sum_{k \in K} g^{km} X_{od}^{km} - \ g_{od}^m Z_{wc}^m \ \leq 0, \ \ o \in O, d \in D, m \in M_{od}$$

This inequality above restricts the model to make sure that the resources used for transportation have space for our data set. The next parameter is another capacity constraint to limit the model to within the upper feasible limits. Just like before this is last capacity constraint creates a feasible space that fits real-world network capabilities. This equation is specific to the connection between the warehouse and the customers. The equation is shown below where f is the final product, w is a warehouse and c is the customer.

Constraint equation (8)

$$\sum_{f \in F} u_w^f g^{fm} X_{wc}^{fm} - \ g_{wc}^m Z_{wc}^m \ \leq 0, \ \ w \in W, c \in C, m \in M_{wc}$$

Following the capacity constraints above, two additional constraints were made to limit the maximum number of plant and warehouse locations that can be selected by our supply chain network.

Constraint equation (9) and (10) respectively

$$\sum_{w \in W} U_w \leq N_1$$

$$\sum_{p \in P} U_p \leq N_2$$

The first equation of the two above represents the limitation of the plant and the second of the warehouses. Both inequalities further reduce the feasible space for the model and make the constraints tighter.

The next constraint determines that each customer only receives a particular product from one warehouse. This constraint is put in to emulate that customers would only reach out to one location to receive their product. The constraint is shown in equation form below.

Constraint equation (11)

$$\sum_{w \in W} Y_{wc}^{f} = 1, \qquad f \in F, c \in C$$

4.3 Objective function

The rest of the constraints to the model are definitions of the variable types. All of the following variables are binary, limiting the answers to 0 or 1: $U_o, V_o{}^k, Y_{od}^k, Z_{wc}^m$. After the addition of these constraints the final feasible space has been determined. With all the variables there is a large number of degrees of freedom for decisions to be made to optimize the objective function which is to minimize the cost. As described before there are four segments that comprise the costs which are storage, transportation, production and capital. The equation is shown below.

Objective equation (12)

$$
\begin{aligned}
min \sum_{o\in O}(c_o U_o + \sum\nolimits_{d\in D} \sum_{m\in M_{od}} c_{od}^m Z_{od}^m) \\
+ \sum_{k\in K}\sum_{o\in O^k}\left[c_o^k V_o^k + \sum_{d\in D^k}\left(c_{od}^k Y_{od}^k + \sum_{m\in M_{od}^k} c_{od}^{km} X_{od}^{km}\right)\right] \\
+ \frac{TH}{n}\sum_{f\in F}\sum_{w\in W}\left(CPI_w^f \sum_{c\in C}\sum_{m\in M} X_{wc}^{fm}\right) \\
+ TH\sum_{f\in F}\sum_{w\in W}[\sqrt{\frac{2CP_w^f IC_w^f}{n}}\left(\sum_{c\in C}\sum_{m\in M} X_{wc}^{fm}\right)^{\frac{1}{2}} \\
+ IC_w^f\sqrt{LT_w^f Z_{w_{1-\alpha}}^m}(\sum_{c\in C} v_c^f\ Y_{wc}^f)^{1/2}]
\end{aligned}
$$

The first term of the objective function accounts for the capital and production costs. In this section c_O accounts for the capital costs of building an origin o; c_{od}^m is the capital cost associated with using transportation mode m from origin o and destination d; c_o^k is the capital cost of the equipment to transport commodity k out of origin o; c_{od}^k is the capital cost of obtaining the infrastructure to ship commodity k from origin o to destination d. The second part of the first term is the production cost with c_{od}^{km} representing the cost of moving and producing commodity k via mode m from origin o to destination d. The last two terms of the objective function represent the capital cost and storage costs associated with the warehouse portion of the network. In the equation TH represents the monetary factor; n is the time horizons; CPI_w^f is the handling cost of product f in warehouse w; IC_w^f is the cost of storing product f in warehouse w; LT_w^f is the lead time of product f to warehouse w. The model uses Barron to solve the MINLP on GAMS to find the minimized solution within the feasible space set by the constraints.

To solve our model, we minimized the objective function (12) using constraint equations (1) – (11). The values of the data were generated randomly but they were based on the supply chain health commodity shipment data (Doby, 2015). In order to make the problem manageable the network was set to only having 3 suppliers, 2 plants, 4 warehouses, 5 customers, 4 raw materials, 2 products and 2 transportation modes (via airplane and trucks). As the numbers of each of these categories grew the number of variables increased exponentially so in order to keep the runtimes low and the amount of data manageable, we used a small network. Based on the averages and the standard of deviation of the data set we randomly produced data for the parameters. We finalized our model by combing through the data set to determine if the randomly generated data made physical sense in the comparison to what would be expected. This could be seen by showing that the cost of transporting commodities by plane is more expensive than by truck. After all the constraints, objective function and data sets were entered into a games code, the model was run. The model was able to obtain a feasible solution within the bounded feasible space.

4.4. Degrees of freedom in the model

Our model consists of 3113 variables and 935 equations, so the degrees of freedom is 2178 (degrees of freedom = number of variables - number of equations). We also define following decision variables to construct and improve our objective function.

4.5. Critical inequality constraints

The critical inequality constraints in our model are equations (4), (5), (7) and (8). These inequalities limit the relationship between the products produced and the number of products shipped to different stores, and the relationship between the factory, warehouse and transportation capacity in the supply chain, which make the model feasible and more realistic.

4.6. Solver used

The Branch-And-Reduce Optimization Navigator (BARON) is a GAMS solver for the *global solution* of nonlinear (NLP) and mixed-integer nonlinear programs (MINLP). While traditional NLP and MINLP algorithms are guaranteed to converge only under certain convexity assumptions, BARON implements deterministic global optimization algorithms of the branch-and-bound type that are *guaranteed to provide global optima* under fairly general assumptions.

5. RESULT AND DISCUSSION

We test the proposed model on the data that generated randomly for a certain supply chain network design. the instance was randomly generated to present the following supply chain characteristics:

1. a network with 3 layers or echelons composed by 3 suppliers, 2 plants, 4 warehouses, and 5 costumes is considered.
2. It has a total of 7 distinct raw materials and 3 different finished products.
3. There are 2 transportation modes (air and road) with different charges.

5.1. Cost minimization

For our model, when we try to minimize the total cost of the supply chain network we obtain the following Network.

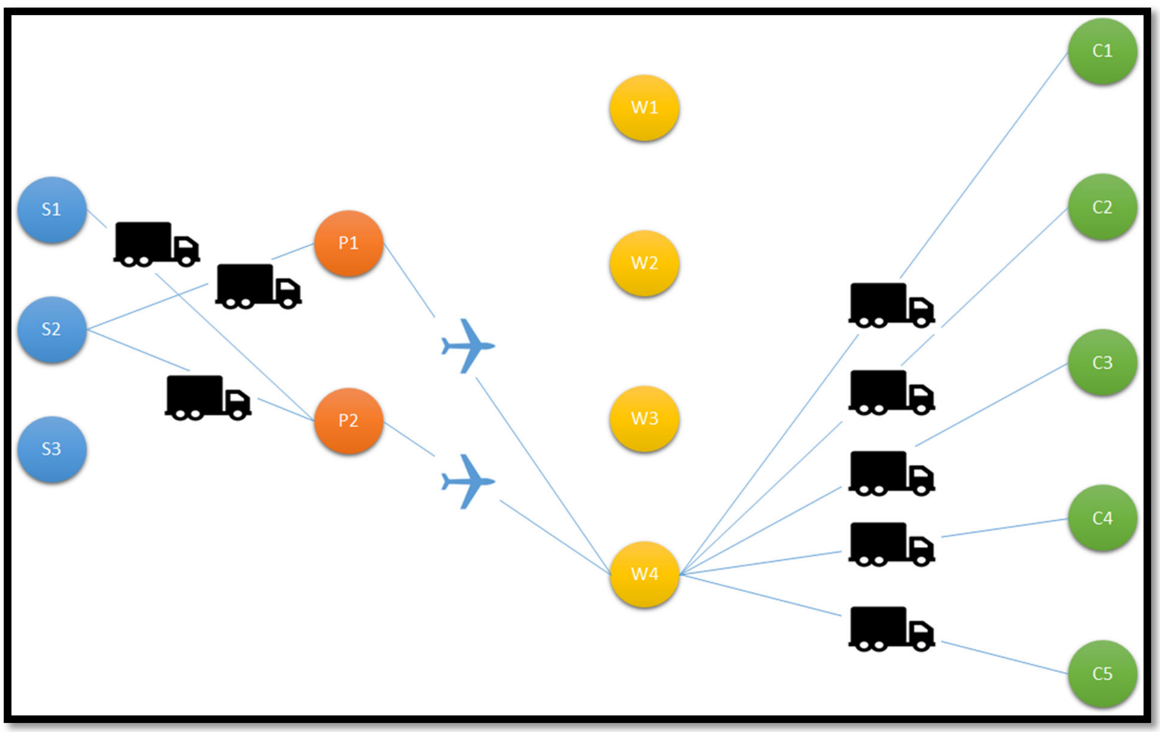

Figure 5.1: Optimal solution to minimize the Supply Chain Cost

It's observed that transportation of the commodity was carried out mostly by trucks instead of using the costlier option of transporting via an airplane. Also, it's observed that the finished product is only shipped to Warehouse W4 because it has the least storage cost. The total optimized cost was found to be $55.795 million, however, the execution takes about 272 days.

5.2. Time-span minimization

Since prompt delivery of the products is also an important aspect of an optimally executed Supply Chain Network, the application of this model can be extended by simultaneously minimizing both the costs and time involved. For this multi-objective optimization problem, the solution to these conflicting objectives can be modeled with Pareto Curves which can give us optimal trade-off in our model.

For this, we first minimize the total time required to transport all the commodities in the entire network. The optimal schematic for this is shown below.

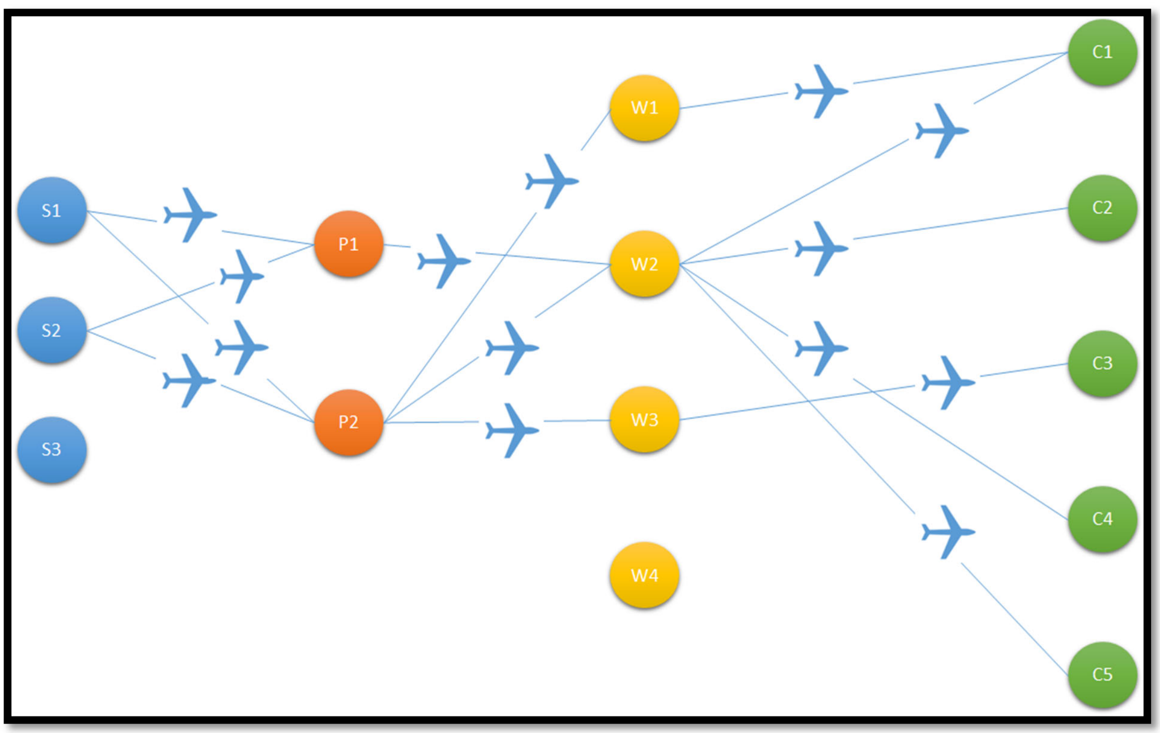

Figure 5.2: Optimal solution to minimize total time-span of the Supply Chain

The above figure is expected since we are minimizing time and hence airplane is used as the transportation mode for all shipping. Also, most of the nodes are used in order to minimize the total time.

5.3. Simultaneous optimization of timespan and cost

Now, in order to obtain the Pareto optimal curve, we optimize the time constraint while limiting the cost objective over a slack variable (ϵ) that varies from 0 to the difference between the maximum and minimum objective value obtained. The above is explained as follows:

$$\textit{min} \text{ Objective of Time}$$

$$\text{s.t. Objective of Cost} \leq \epsilon_i \qquad \text{for } i = 1, 2, \ldots, n$$

$$h(x) = 0,\ g(x) \leq 0 \quad \text{[These equations are the constraints (1) – (11) discussed previously]}$$

Here n is the number of optimal evaluations for the curve. In the present extension, we took n = 7 and the objective values are tabulated below.

Cost (Million $)	Time (days)
55.795	272
61.042	40.645
65.495	37.822
66.739	37.533
70.925	37.353
75.776	37.3
107.526	37.3

Table 5.1 Objective values for Pareto optimal curve

The above values are plotted in the figure below.

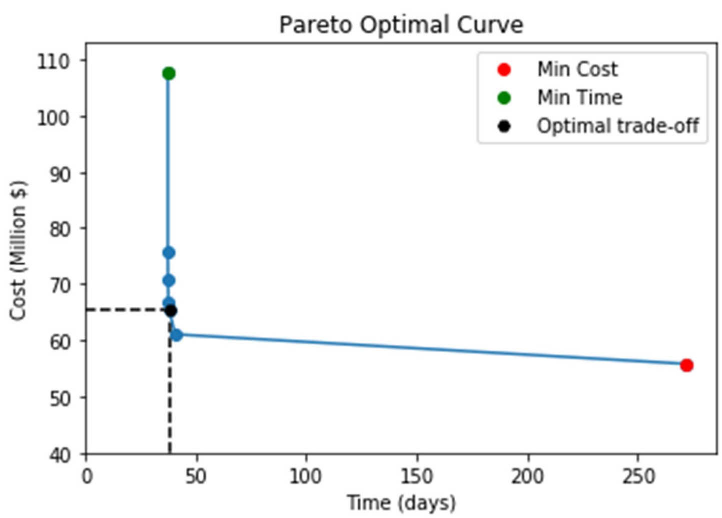

Figure 5.3: Pareto optimal curve using BARON solver

Using the above curve, we selected the point which has a minimum cost of $65.495 million and takes 37.822 ≈ 38 days. The network corresponding to this point is shown below.

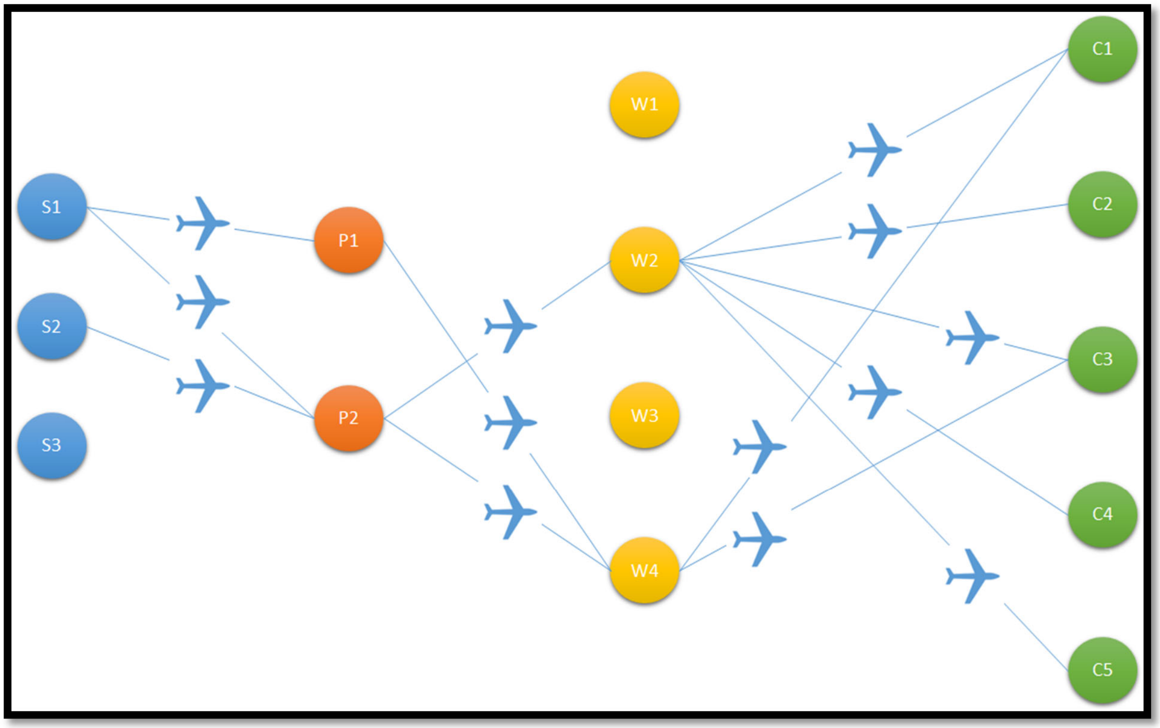

Figure 5.4: Optimal trade-off selected minimizing total cost and time-span of the Supply Chain

In order to compare and verify the results, the analysis was carried out using other MINLP solvers. The results obtained are compared in the table below.

S. N.	Solver	Results Comparison
1	BARON	Gives us the above discussed curve.
2	ANTIGONE	Returns optimal values which aren't as good as those returned by BARON.
3	ALPHAECP	Not suitable for the model since it returns singularity while solving as the variable values decrease.
4	DICOPT	Returns accurate values for individual objectives but returns relaxed solution for multi-objective function. Hence, it can't be used to produce the optimal curve.
5	OQNLP	Cannot solve the model used.
6	SBB	Produces correct solution to minimize time but not the cost. Hence, it produces inaccurate curve.

Table 5.2: Comparison of various MINLP solvers

From the above table, we find out that ANTIGONE and BARON are the only solvers that can return accurate Pareto curve. However, the latter provides more optimal solutions than the former. Since the dataset used was small, the execution time for the model was very less. Hence, we weren't able to classify which solver produced the results in least amount of time.

6. CONCLUSION AND FUTURE WORK

6.1. Conclusion

We have proposed a new integrated and flexible mathematical formulation for the design of a supply chain network that supports decision makers on the planning of diverse fields and markets. This model considers not only facility locations, production, and transportation. but also inventory levels in warehouses based on the stochastic demand of customers, for a more realistic perspective. As the objective function of this model consists of a nonconvex term, we applied MINLP solver BARON to obtain an optimal solution within little computational effort. Experiments show that our model can deal with multi-layer supply chain problems very well. It is worth noting that in the past supply chain optimization problems, people have tried to optimize transportation costs to maximize profits. However, supply time is also a very important optimization variable. When these two objectives are optimized at the same time, we need to use some slack variables to constrain one of them while minimizing the other.

6.2. Future Work

The model has a lot of scope to improve and can be used in real-life settings if following improvements are incorporated. Firstly, the planning has been carried out considering fixed requirement of customers and their stochastic demands in the future. Instead, we can employ Rolling Horizon Method to reorganize the Supply Chain decisions after specific time periods. To execute this, the continuous planning horizon would have to be discretized (say monthly). Due to the stochastic nature of customer demands, after the end of each month, renewed demands of the customer can be used to update the model to provide us with continuous updating of the amounts to be transported. Secondly, for a more realistic perspective, the cost associated with stocks in transit, backlogging, variability in lead time per product can also be considered to modify the proposed model. Presently we consider these elements do not affect the supply chain design.

To provide a realistic representation, we must also employ timing constraints. For instance, time for production can be proportional to the amount used. Additionally, we can modify the model to allow the Supply Chain to have backlogs with respect to the given commodity having certain penalties.

Moreover, in the case of multi-products, we can constrain certain order of production to be allowed (for example, product C cannot be produced following product A).

This model is very flexible, which means one can easily change the complexity of the logistics network in the number of components and of layers to fit the realistic situation. To check the validity of the model, tests can be carried out on real datasets and we can improve the model by comparing the values of the two.

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