Millikan's Oil Drop Experiment

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In this experiment, we study the study the oil drop experiment which was performed by Richard A. Millikan and Harvey Fletcher in 1909. The main intention was to measure the elementary charge after the discrete nature of charge had already been propounded by Faraday in mid-nineteenth century. The experiment is structure to touch upon important aspects of modern physics like quantization of charge, existence of charge particles with integer and fractional charges, and how the latter cannot be isolated. The experiment involves concepts from diverse fields of Physics like fluid mechanics, gravitation, and electromagnetism. We employ the Newton's law of motion to oil droplets where the forces of gravity, viscosity and electromagnetism balance out each other, and the droplets reach the terminal velocity. Analysing the speeds of droplets under and without electric field provides us crucial information about the radius and charge on droplets. And that charge is further observed to have only obtained in integer multiples of the elementary charge, e. We reperform this experiment with slightly different and modified apparatus and match our observational value to the literature value of charge of an electron.

I. INTRODUCTION

The human fascination with electric phenomena has been there from time immemorial. The efforts to discern the true physical nature of the electricity and thus electric charge over the period of hundreds years finally culminated when Michael Faraday concluded the electric charge possessed a discrete nature, or in other words, it was *quantized*.

The elementary charge, e, is the electric charge carried by a single proton or, equivalently, the magnitude of the negative electric charge carried by a single electron. It is a fundamental physical constant. Charge quantization is the principle that the charge of any object is an integer multiple of the elementary charge. There are some entities which are known to possess fractional charge (quarks and quasiparticles) but they are never found in isolation but in some configuration that the total charge is some multiple of e.

The oil drop experiment was performed by Robert A. Millikan and Harvey Fletcher in 1909 to measure the elementary electric charge (the charge of the electron). The experiment entailed observing tiny electrically charged droplets of oil located between two parallel metal surfaces, forming the plates of a capacitor. The plates were oriented horizontally, with one plate above the other. A mist of atomized oil drops was introduced through a small hole in the top plate and was ionized by an x-ray, making them negatively charged. First, with zero applied electric field, the velocity of a falling droplet was measured. At terminal velocity, the drag force equals the gravitational force. As both forces depend on the radius in different ways, the radius of the droplet, and therefore the mass and gravitational force, could be determined (using the known density of the oil). Next, a voltage inducing an electric field was applied between the plates and adjusted until the drops were suspended in mechanical equilibrium, indicating that the electrical force and the gravitational force were in balance. Using the known electric field, Millikan and Fletcher could determine the charge on the oil droplet. By repeating the experiment for many droplets, they confirmed that the charges were all small integer multiples of a certain base value, which was found to be 1.5924×10^{-19} C, about 0.6% difference from the currently accepted value of $1.602\,176\,634 \times 10^{-19}$ C. They proposed that this was the magnitude of the negative charge of a single electron.

In this experiment we will subject charged oil droplets to an electric field and to gravity between the plates of a capacitor. Charged oil droplets subjected to an electric field and to gravity between the plates of a capacitor are accelerated by application of a voltage. The elementary charge is determined from the velocities in the direction of gravity and in the opposite direction.

II. EXPERIMENTAL SETUP

Millikan's and Fletcher's apparatus incorporated a parallel pair of horizontal metal plates. By applying a potential difference across the plates, a uniform electric field was created in the space between them. A ring of insulating material was used to hold the plates apart. Four holes were cut into the ring, three for illumination by a bright light, and another to allow viewing through a microscope (see figure (1)).

A fine mist of oil droplets was sprayed into a chamber above the plates. The oil was of a type usually used in vacuum apparatus and was chosen because it had an extremely low vapour pressure. Ordinary oil would evaporate under the heat of the light source causing the mass of the oil drop to change over the course of the experiment. Some oil drops became electrically charged through friction with the nozzle as they were sprayed. Alternatively,

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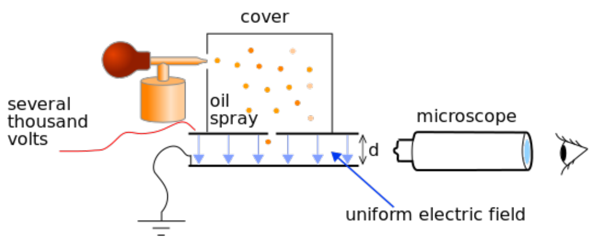

FIG. 1: Simplified scheme of Millikan's oil drop experiment

charging could be brought about by including an ionising radiation source (such as an X-ray tube). The droplets entered the space between the plates and, because they were charged, could be made to rise and fall by changing the voltage across the plates.

The experimental set up is as shown in Fig. 1. The power unit supplies the necessary voltages for the Millikan apparatus. The lighting system is connected to the 6.3 V a.c. sockets.

The first task is to calibrate the eyepiece micrometer with a stage micrometer. This is done by connecting the fixed (300 V d.c.) and the variable (0 V to 300 V d.c.) outputs in series, and through this, a voltage supply of more than 300 V d.c. can be obtained. The commutator switch is used to invert the polarity of the capacitor.

The setup used in our experiment is shown in figure (2).

III. THEORY

The main principle of this experiment derives from using the fundamental forces of nature to accomplish our task of find the elementary charge.

Charged oil droplets subjected to an electric field and to gravity between the plates of a capacitor are accelerated by application of a voltage. The elementary charge is determined from the velocities in the direction of gravity and in the opposite direction. In the following discussion, we will revisit and highlight the fundamental aspects that will come into play in the experiment.

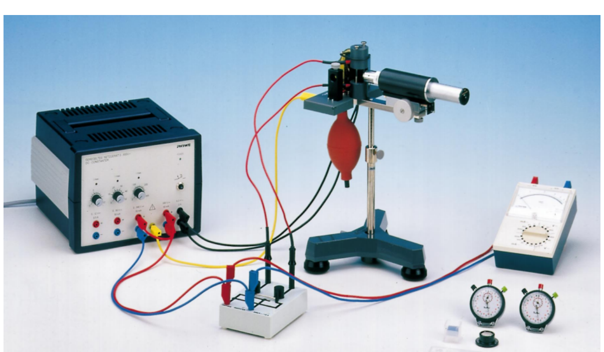

FIG. 2: Experimental set up for determining the elementary charge with the Millikan apparatus

An electric field is the physical field that surrounds electrically-charged particles and exerts force on all other charged particles in the field, either attracting or repelling them. It is a manifestation of the electromagnetic force, one of the most fundamental forces (or interactions) in nature.

The viscosity of a fluid is a measure of its resistance to deformation at a given rate. Viscosity can be conceptualized as quantifying the internal frictional force that arises between adjacent layers of fluid that are in relative motion.

In 1851, George Gabriel Stokes derived an expression, now known as Stokes law, for the frictional force – also called drag force – exerted on spherical objects with very small Reynolds numbers in a viscous fluid. The Reynolds number (**Re**) helps predict flow patterns in different fluid flow situations. At low Reynolds numbers, flows tend to be dominated by laminar (sheet-like) flow, while at high Reynolds numbers flows tend to be turbulent. Stokes' law is derived by solving the Stokes flow limit for small Reynolds numbers of the Navier–Stokes equations.

Terminal velocity is the maximum velocity attainable by an object as it falls through a fluid. It occurs when the sum of the drag force (F_d) and the buoyancy is equal to the downward force of gravity (F_G) acting on the object. Since the net force on the object is zero, the object has zero acceleration, that is, constant velocity.

Coming back to the experiment, we observe the rising and falling movement of the charged oil droplets in the electric field of the capacitor. Initially the oil drops are allowed to fall between the plates with the electric field turned off. They very quickly reach a terminal velocity because of friction with the air in the chamber. The field is then turned on and, if it is large enough, some of the drops (the charged ones) will start to rise. (This is because the upwards electric force FE is greater for them than the downwards gravitational force Fg, in the same way bits of paper can be picked by a charged rubber rod). A likely looking drop is selected and kept in the middle of the field of view by alternately switching off the voltage until all the other drops have fallen. The experiment is then continued with this one drop.

From Stokes' law, the force F experienced by a sphere of radius r and velocity v in a viscous fluid of viscosity η is:

$$F = 6\pi\eta r v \tag{1}$$

As this spherical droplet of mass m, volume V and density ρ_1 is also under the influence of earth's gravitational field.

$$F = m \cdot g = \rho_1 \cdot V \cdot g \tag{2}$$

Now force of buoyancy is given by

$$F = \rho_2 \cdot V \cdot g \tag{3}$$

where ρ_2 is the density of air.
The force of electric field is given by

$$F = Q \cdot E = Q \cdot \frac{U}{d} \tag{4}$$

where U is the applied voltage across the capacitor with distance between the plates equal to d. From the sum of the forces affecting a charged particle, the fall and rise velocities of the droplets are obtained.

$$v_1 = \frac{1}{6\pi\eta r}\left(Q \cdot E + \frac{4}{3}\pi r^3 g(\rho_1 - \rho_2)\right) \tag{5}$$

$$v_2 = \frac{1}{6\pi\eta r}\left(Q \cdot E - \frac{4}{3}\pi r^3 g(\rho_1 - \rho_2)\right) \tag{6}$$

Subtraction or addition of these equations gives the radius and the charge of the droplet.
With

$$\begin{aligned} Q &= C_1 \cdot \frac{v_1 + v_2}{U}\sqrt{v_1 - v_2} \\ C_1 &= \frac{9}{2}\pi d \cdot \sqrt{\frac{\eta^3}{g(\rho_1 - \rho_2)}} \\ C_1 &= 2.73 \times 10^{-11}\,\mathrm{kg} \cdot \mathrm{m}^{1/2} \cdot \mathrm{s}^{-1/2} \end{aligned} \tag{7}$$

with

$$\begin{aligned} r &= C_2 \cdot \sqrt{v_1 - v_2} \\ C_2 &= \frac{3}{2}\sqrt{\frac{\eta}{g(\rho_1 - \rho_2)}} \\ C_2 &= 6.37 \times 10^{-5}\,(\mathrm{m} \cdot \mathrm{s})^{1/2} \end{aligned} \tag{8}$$

IV. EXPERIMENTAL PROCEDURE

1. Set the capacitor voltage to a value between 300 V and 500 V.
2. Blow in the oil droplets.
3. Select an oil droplet and by operating the commutator switch move the droplet between the highest and lowest graduations on the eyepiece micrometer.
4. Correct the focusing of the microscope if necessary.
5. Note the following criteria when selecting the droplet:
   (a) The droplet must not move too fast, then it has a small charge (it should need around one to three seconds for the way of 30 divisions).
   (b) The droplet must not move too slowly and should not exhibit any swaying movements. Increase the capacitor voltage if required.
6. Sum together some rise times using the first stopwatch.
7. Sum together some fall times using the second stopwatch.
8. The added times should be greater than 5 s in both cases.

V. OBSERVATIONS

1. Capacitor inter-electrode distance, $d = (2.50 \pm 0.01)\,\mathrm{mm}$.
2. Density of silicone oil, $\rho_1 = 1.03 \times 10^3\,\mathrm{kg\,m^{-3}}$.
3. Viscosity of air, $\eta = 1.82 \times 10^{-5}\,\mathrm{kg} \cdot (\mathrm{m} \cdot \mathrm{s})^{-1}$.
4. Gravitational acceleration, $g = 9.81\,\mathrm{m\,s^{-2}}$.
5. Density of air, $\rho_2 = 1.293\,\mathrm{kg} \cdot \mathrm{m}^{-3}$.

VI. RESULTS AND DISCUSSIONS

The following is the plot of radius of droplets vs the charge of droplets. It is clearly evident from the plot that distinct steps are seen in Q, which implies the droplets acquire only integral multiples of the elementary charge e. On approximately estimating the levels, i.e. multiples (n), we come to see that the data almost matches our assumed model. From table (I) the average value of e obtained is $\boxed{1.66 \times 10^{-19}\,\mathrm{C}}$.

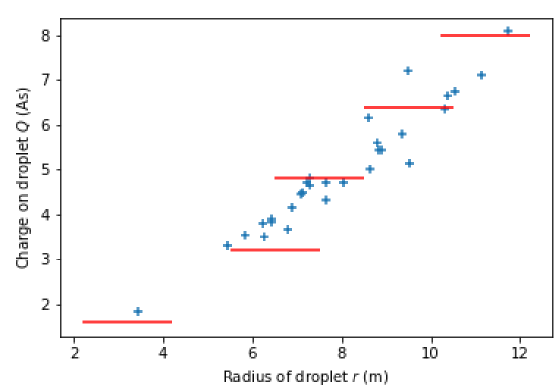

FIG. 3: Measurements on various droplets for determining the elementary charge by the Millikan method.

TABLE I: Measurements on various droplets for determining the elementary charge by the Millikan method. t_1 and t_2 are the fall and rise times of the droplets

$\mathbf{U}$ (V)	$\mathbf{t_1}$ (s)	$\mathbf{s_1}$ (div.)	$\mathbf{t_2}$ (s)	$\mathbf{s_2}$ (div.)	$\mathbf{s_1}$ (mm)	$\mathbf{s_2}$ (mm)	$\mathbf{v_1}$ (m/s)	$\mathbf{v_2}$ (m/s)	$\mathbf{v_1 - v_2}$ (m/s)	$\mathbf{r}$ (m)	$\mathbf{Q}$ (As)	n	$\mathbf{e}$ (As)
360	5.72	60	8.42	60	1.96	1.78	3.43E-04	2.11E-04	1.31E-04	7.30E-07	4.81E-19	3	1.60E-19
360	6.61	60	8.39	60	1.96	1.78	2.97E-04	2.12E-04	8.44E-05	5.85E-07	3.54E-19	2	1.77E-19
360	6.29	60	9.17	60	1.96	1.78	3.12E-04	1.94E-04	1.17E-04	6.90E-07	4.16E-19	3	1.39E-19
360	5.78	60	8.43	60	1.96	1.78	3.39E-04	2.11E-04	1.28E-04	7.21E-07	4.72E-19	3	1.57E-19
360	6.03	60	8.84	60	1.96	1.78	3.25E-04	2.01E-04	1.24E-04	7.08E-07	4.44E-19	3	1.48E-19
360	6.83	90	8.56	60	2.94	1.78	4.30E-04	2.08E-04	2.23E-04	9.50E-07	7.22E-19	5	1.44E-19
360	5.93	60	9.56	60	1.96	1.78	3.31E-04	1.86E-04	1.44E-04	7.65E-07	4.71E-19	3	1.57E-19
360	6.50	60	8.93	60	1.96	1.78	3.02E-04	1.99E-04	1.02E-04	6.44E-07	3.84E-19	2	1.92E-19
360	6.43	60	8.52	60	1.96	1.78	3.05E-04	2.09E-04	9.59E-05	6.24E-07	3.82E-19	2	1.91E-19
360	5.88	60	8.78	60	1.96	1.78	3.33E-04	2.03E-04	1.31E-04	7.28E-07	4.65E-19	3	1.55E-19
460	7.33	90	6.45	60	2.94	1.78	4.01E-04	2.76E-04	1.25E-04	7.13E-07	4.49E-19	3	1.50E-19
460	8.23	120	8.45	60	3.92	1.78	4.76E-04	2.11E-04	2.66E-04	1.04E-06	6.64E-19	4	1.66E-19
460	7.83	90	7.68	60	2.94	1.78	3.75E-04	2.32E-04	1.44E-04	7.64E-07	4.32E-19	3	1.44E-19
460	7.91	120	9.34	60	3.92	1.78	4.96E-04	1.91E-04	3.05E-04	1.11E-06	7.11E-19	4	1.78E-19
460	7.27	120	8.84	60	3.92	1.78	5.39E-04	2.01E-04	3.38E-04	1.17E-06	8.08E-19	5	1.62E-19
460	8.15	120	8.59	60	3.92	1.78	4.81E-04	2.07E-04	2.74E-04	1.05E-06	6.76E-19	4	1.69E-19
460	8.06	90	9.14	90	2.94	2.67	3.65E-04	2.92E-04	7.26E-05	5.43E-07	3.32E-19	2	1.66E-19
460	7.78	90	9.66	90	2.94	2.67	3.78E-04	2.76E-04	1.01E-04	6.42E-07	3.91E-19	2	1.96E-19
460	8.25	120	9.13	90	3.92	2.67	4.75E-04	2.92E-04	1.83E-04	8.61E-07	6.16E-19	4	1.54E-19
460	7.45	90	7.55	60	2.94	1.78	3.95E-04	2.36E-04	1.59E-04	8.03E-07	4.72E-19	3	1.57E-19
560	6.32	90	7.36	60	2.94	1.78	4.65E-04	2.42E-04	2.23E-04	9.52E-07	5.15E-19	3	1.72E-19
560	7.34	120	9.78	90	3.92	2.67	5.34E-04	2.73E-04	2.61E-04	1.03E-06	6.36E-19	4	1.59E-19
560	7.65	120	9.01	90	3.92	2.67	5.12E-04	2.96E-04	2.16E-04	9.36E-07	5.80E-19	4	1.45E-19
560	7.12	90	8.45	90	2.94	2.67	4.13E-04	3.16E-04	9.69E-05	6.27E-07	3.50E-19	2	1.75E-19
560	7.86	120	8.70	90	3.92	2.67	4.99E-04	3.07E-04	1.92E-04	8.82E-07	5.44E-19	3	1.81E-19
560	7.91	120	8.90	90	3.92	2.67	4.96E-04	3.00E-04	1.96E-04	8.91E-07	5.42E-19	3	1.81E-19
560	8.09	90	7.99	90	2.94	2.67	3.63E-04	3.34E-04	2.92E-05	3.44E-07	1.84E-19	1	1.84E-19
560	7.67	120	8.32	90	3.92	2.67	5.11E-04	3.21E-04	1.90E-04	8.78E-07	5.59E-19	3	1.86E-19
560	7.17	90	9.01	90	2.94	2.67	4.10E-04	2.96E-04	1.14E-04	6.79E-07	3.67E-19	2	1.84E-19
560	8.34	120	9.34	90	3.92	2.67	4.70E-04	2.86E-04	1.84E-04	8.64E-07	5.00E-19	3	1.67E-19

VII. ERROR ANALYSIS

We would be calculating the standard deviation of the values of e obtained. The formula for standard deviation is

$$\sigma = \sqrt{\frac{\Sigma(x_i - \mu)^2}{N-1}} \tag{9}$$

We have average, $\mu = 1.66 \times 10^{-19}\,\mathrm{C}$ and number of values, $N = 30$. Putting in the values we get, $\sigma = 0.16 \times 10^{-19}\,\mathrm{C}$.

VIII. CONCLUSIONS

1. The net charge on an oil droplet is an integral multiple of the elementary charge or charge of an electron (e).
2. The value of this elementary charge is found to be (after rounding off to most significant digit) $\mathbf{1.7 \pm 0.2 \times 10^{-19} C}$.
3. Although the literature value of e is $1.602 \times 10^{-19} C$, the observed value is quite close to this and falls within 1 % of the literature value.
4. The error seen in the data can be accredited to the experimental error, since no systematic error was observed in the instruments used. Some of the error may also be accredited to human error, which occurs when we record the falling and rising times of the droplets using stopwatch.