

# Zeeman Effect

Maitrey Sharma\*

*School of Physical Sciences, National Institute of Science Education and Research, HBNI, Jatni-752050, India*

(Dated: November 23, 2021)

In this experiment, we study about the phenomena of electron spin resonance, which is often used as a spectroscopic technique to identify paramagnetic materials. We discuss the Larmor precession, the  $g$ -factor, specially Landé  $g$ -factor and justify its appearance in our discussions. We review the classical and quantum mechanical picture of the magnetic moment of electron in paramagnetic materials. We explore the ESR phenomena in macroscopic systems like solids. We formulate the theory for the experiment and workings of the ESR spectrometer. We finally determine the Landé  $g$ -factor for the electron and finally discuss the results so obtained like the appearance of four peaks on the oscilloscope and how the obtained Lissajous figure provides the insight on the effects of external magnetic field when a paramagnetic substance is kept under its influence. In the process of this experiment, we establish various useful results relates to magnetism and behaviour of electron in magnetic fields.

## I. INTRODUCTION

A spectral line is a dark or bright line in an otherwise uniform and continuous spectrum, resulting from emission or absorption of light in a narrow frequency range, compared with the nearby frequencies. Spectral lines are the result of interaction between a quantum system (usually atoms, but sometimes molecules or atomic nuclei) and a single photon. When a photon has about the right amount of energy (which is connected to its frequency) to allow a change in the energy state of the system (in the case of an atom this is usually an electron changing orbitals), the photon is absorbed. Then it will be spontaneously re-emitted, either in the same frequency as the original or in a cascade, where the sum of the energies of the photons emitted will be equal to the energy of the one absorbed (assuming the system returns to its original state).

A spectral line may be observed either as an emission line or an absorption line. Which type of line is observed depends on the type of material and its temperature relative to another emission source. An absorption line is produced when photons from a hot, broad spectrum source pass through a cold material. The intensity of light, over a narrow frequency range, is reduced due to absorption by the material and re-emission in random directions. By contrast, a bright emission line is produced when photons from a hot material are detected in the presence of a broad spectrum from a cold source. The intensity of light, over a narrow frequency range, is increased due to emission by the material.

Spectral lines can be influenced in the presence of electric (*Stark effect*) and magnetic fields (*Zeeman effect*).

In 1896, shortly before moving from Leiden to Amsterdam, Dutch physicist Pieter Zeeman discovered that a spectral line is split into several components in the presence of a magnetic field. Fellow physicist Hendrik Lorentz

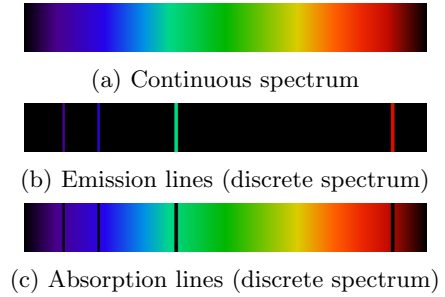


FIG. 1: Visualising various spectra

heard about Zeeman's observations at the meeting of the Royal Netherlands Academy of Arts and Sciences in Amsterdam, where these results were communicated by Kamerlingh Onnes. Lorentz called Zeeman into his office and presented him with an explanation of his observations, based on Lorentz's theory of electromagnetic radiation. Zeeman's discovery had confirmed Lorentz's prediction about the polarization of light emitted in the presence of a magnetic field. Thanks to Zeeman's work it became clear that the oscillating particles that according to Lorentz were the source of light emission were negatively charged, and were a thousandfold lighter than the hydrogen atom. This conclusion was reached well before J. J. Thomson's discovery of the electron. The Zeeman effect thus became an important tool for elucidating the structure of the atom. Zeeman was awarded the 1902 Nobel Prize in Physics for his discovery.

Zeeman effect is analogous to Stark effect and transitions between different components have, in general, different intensities, with some being entirely forbidden (in the dipole approximation), as governed by the selection rules.

Since the distance between the Zeeman sub-levels is a function of magnetic field strength, this effect can be used to measure magnetic field strength, e.g. that of the Sun and other stars or in laboratory plasmas. The Zeeman effect is very important in applications such as nu-

---

\* [maitrey.sharma@niser.ac.in](mailto:maitrey.sharma@niser.ac.in)

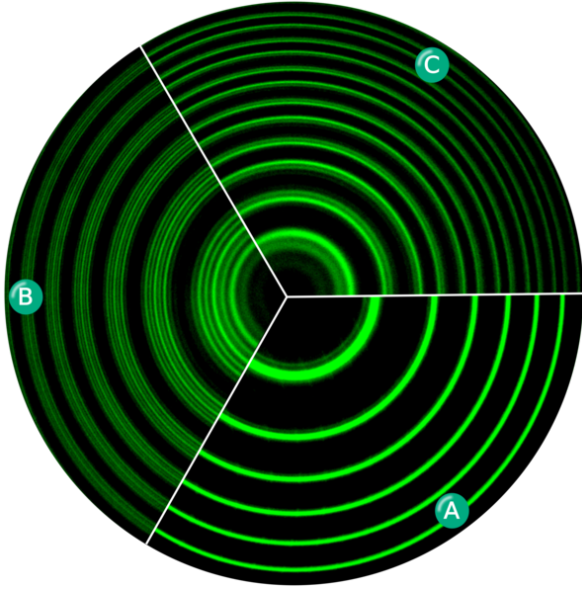


FIG. 2: The spectral lines of mercury vapor lamp at wavelength 546.1 nm, showing anomalous Zeeman effect. (A) Without magnetic field. (B) With magnetic field, spectral lines split as transverse Zeeman effect. (C) With magnetic field, split as longitudinal Zeeman effect. The spectral lines were obtained using a Fabry-Pérot interferometer.

clear magnetic resonance spectroscopy, electron spin resonance spectroscopy, magnetic resonance imaging (MRI) and Mössbauer spectroscopy<sup>1</sup>. It may also be utilized to improve accuracy in atomic absorption spectroscopy. A theory about the magnetic sense of birds assumes that a protein in the retina is changed due to the Zeeman effect.

## II. THEORY

The phenomena discovered by Zeeman in 1896 is now retrospectively called *normal* Zeeman effect. In 1898, Thomas Preston observed quartet pattern of lines in a magnetic field which is now referred to as the *anomalous* Zeeman effect.

Consider an external magnetic field  $\mathbf{B}$  (figure (3)), a magnetic dipole has an amount of potential energy  $U_m$  that depends upon both the magnitude of its magnetic moment and the orientation of this moment with respect to the field. The torque on a magnetic dipole in a magnetic field of flux density  $\mathbf{B}$  is

$$\tau = \mu B \sin \theta \quad (1)$$

<sup>1</sup> a spectroscopy technique based on the resonant and recoil-free emission and absorption of gamma radiation by atomic nuclei bound in a solid.

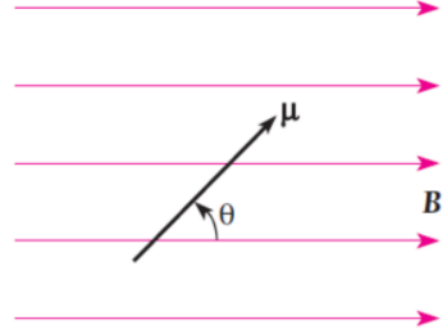


FIG. 3: Torque on a magnetic dipole

If we take  $U_m = 0$  when  $\theta = \pi/2 = 90^\circ$ ,

$$U_m = \int_{\pi/2}^{\theta} \tau d\theta = \mu B \int_{\pi/2}^{\theta} \sin \theta d\theta = -\mu B \cos \theta \quad (2)$$

The magnetic moment of the orbital electron in an atom depends on its angular momentum  $\mathbf{L}$ . By taking the current loop formulation we can write,

$$\mu = -\left(\frac{e}{2m}\right)\mathbf{L} \quad (3)$$

So the magnetic energy can be written as,

$$U_m = \left(\frac{e}{2m}\right)LB \cos \theta \quad (4)$$

As we know the angle  $\theta$  between the  $\mathbf{L}$  and  $z$  direction can only, we have,

$$\cos \theta = \frac{m_l}{\sqrt{l(l+1)}} \quad (5)$$

and

$$L = \sqrt{l(l+1)}\hbar \quad (6)$$

We rewrite magnetic energy as

$$U_m = m_l \mu_B B \quad (7)$$

where  $\mu_B = \frac{e\hbar}{2m}$  is the Bohr magneton which will be later determined in the experiment.

Let's first consider the normal Zeeman effect (figure (4)). since the magnetic quantum number  $m_l$  can have the  $2l+1$  values of  $l$  through 0 to  $l$ , a state of given orbital quantum number  $l$  splits into  $2l+1$  sub-states that differ in energy by  $\mu_B B$  when the atom is in a magnetic field. The selection rule only allow changes  $\Delta m = 0, \pm 1$ . So we expect a spectral line from a transition between two states of different  $l$  to be split into only three components:

$$\begin{aligned} \nu_1 &= \nu_0 - \mu_B \frac{B}{h} = \nu_0 - \frac{e}{4\pi m} B \\ \nu_2 &= \nu_0 \\ \nu_3 &= \nu_0 + \mu_B \frac{B}{h} = \nu_0 + \frac{e}{4\pi m} B \end{aligned} \quad (8)$$

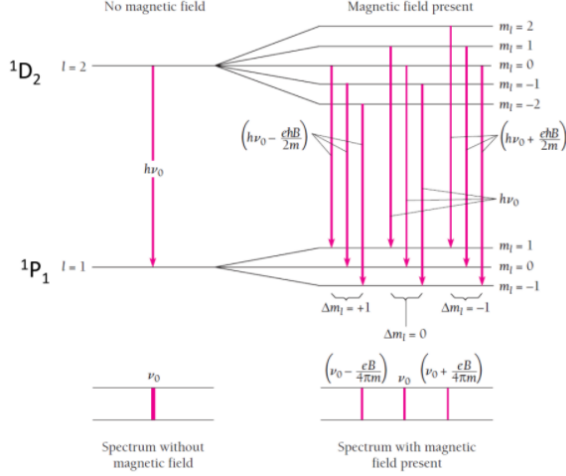


FIG. 4: Normal Zeeman effect

So we will get a single spectral line splits into there in presence of magnetic field. The amount of splitting depends on the strength of magnetic field, which we will verify in this experiment.

The anomalous Zeeman effect is the more general case where the electron spins do not cancel each other and the energy of an atomic state in a magnetic field depends on both the magnetic moments of electron orbit and electron spin. The spin moment can be written as,

$$\vec{\mu}_S = -\left(\frac{e}{2m_e}\right)\vec{S} = -g_S\mu_B\frac{\vec{S}}{\hbar} \quad (9)$$

As  $|\vec{S}| = \hbar\sqrt{s(s+1)}$ , we have

$$\mu_S = \left| -g_S\mu_B\sqrt{s(s+1)} \right| \quad (10)$$

Since the spin and orbital angular momentum are coupled ( $L-S$  coupling), the total angular momentum can be written as

$$|\vec{J}| = |\vec{L} + \vec{S}| = \hbar\sqrt{J(J+1)} \quad (11)$$

The gyromagnetic ratio, which is 1 for pure orbital and 2 for pure spin case, becomes

$$g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \quad (12)$$

The magnetic quantum number  $m_J$  will change from  $J, J-1, \dots, -J$ . The magnetic energy becomes

$$U_{mJ} = -m_J g_J \mu_B B_0 \quad (13)$$

For the anomalous Zeeman effect the used transition in Cd is from  $^3S_1$  to  $^3P_2$ . The superscript (here 3) represents multiplicity, i.e.  $2s+1$ , the subscript represents  $J = l + s$ . Thus, The energy difference between the initial sub-states in magnetic field ( $^3S_1$ ) is  $\Delta E = -2\mu_B B_0$

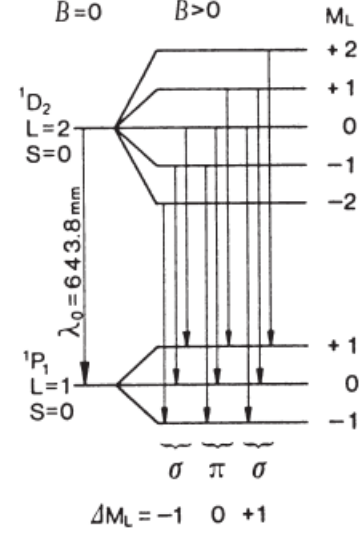


FIG. 5: Splitting up of the components in the magnetic field and permitted transitions.

and The energy difference between the final sub-states in magnetic field ( $^3P_2$ ) is  $\Delta E = -1.5\mu_B B_0$ . The selection rules remain the same.

The splitting of the Cd-spectral line  $\lambda = 643.8 \text{ nm}$  into three lines, the so-called Lorentz triplets, occurs since the Cd-atom represents a singlet system of total spin  $S = 0$ . In the absence of a magnetic field there is only one possible  $D \rightarrow P$  transition of  $643.8 \text{ nm}$ , as indicated by figure (5). In the presence of a magnetic field the associated energy levels split into  $2l + 1$  components. Radiating transitions between these components are possible, provided that the selection rules mentioned earlier are taken into account. In this case, therefore, there are a total of nine permitted transitions. These nine transitions can be grouped into three groups of three transitions each, where all transitions in a group have the same energy and hence the same wavelength. Therefore, only three lines will be visible.

The first group where  $\Delta m_l = -1$  gives a  $\sigma$ -line the light of which is polarized vertically to the magnetic field. The middle group  $\Delta m_l = 0$  gives a  $\pi$ -line. This light is polarized parallel to the direction of the field. The last group where  $\Delta m_l = +1$  gives a  $\sigma$ -line the light of which is again polarized vertically to the magnetic field.

In the absence of the analyser all three lines can be seen simultaneously. Each ring which was observed in the absence of a magnetic field is split into three rings when a magnetic field is applied. Inserting the analyser the two  $\sigma$ -lines can be observed exclusively if the analyser is in the vertical position, while only the  $\pi$ -line appears if the analyser is turned into its horizontal position (*transverse Zeeman effect*). Turning the electromagnet by  $90^\circ$  the light coming from the spectral lamp parallel to the direction of the field can also be studied since the pole-shoes have been drilled. It can be shown that this



From here we obtain

$$r_{p+1}^2 - r_p^2 = \frac{2f^2}{n_0} \quad (23)$$

Now, if there are two components of a spectral line (splitting of one central line into two components) with wavelengths  $\lambda_a$  and  $\lambda_b$ , which are very close to one another, they will have fractional orders at the center  $\varepsilon_a$  and  $\varepsilon_b$ :

$$\begin{aligned} \varepsilon_a &= \frac{2\mu t}{\lambda_a} - n_{1,a} = 2\mu t \bar{\nu}_a - n_{1,a} \\ \varepsilon_b &= \frac{2\mu t}{\lambda_b} - n_{1,b} = 2\mu t \bar{\nu}_b - n_{1,b} \end{aligned} \quad (24)$$

where  $n_{1,a}$ ,  $n_{1,b}$  is the interference order of the first ring. The difference in the wave numbers thus is

$$\Delta \bar{\nu} = \bar{\nu}_a - \bar{\nu}_b = \frac{\varepsilon_a - \varepsilon_b}{2\mu t} \quad (25)$$

The final equation for  $\Delta \bar{\nu}$  we get is

$$\Delta \bar{\nu} = \frac{1}{2\mu t} \left( \frac{r_{p+1,a}^2}{r_{p+1,a}^2 - r_{p,a}^2} - \frac{r_{p+1,b}^2}{r_{p+1,b}^2 - r_{p,b}^2} \right) \quad (26)$$

Rewriting this after taking  $\delta_{a,b}^{p+1,p} = r_{p+1,a}^2 - r_{p,a}^2$  and  $\Delta_a^{p+1,p} = \Delta_b^{p+1,p}$  we finally get our working formula as

$$\Delta \bar{\nu} = \frac{1}{2\mu t} \frac{\delta}{\Delta} \quad (27)$$

The difference in wave numbers of one of the  $\sigma$ -lines with respect to the central lines is  $\Delta \bar{\nu}/2$ . For the radiating electrons this means, for instance, a change in energy of

$$\Delta E = hc \frac{\Delta \bar{\nu}}{2} \quad (28)$$

On the other hand the change in energy  $\Delta E$  is proportional to the magnetic flux density  $B$ . The factor of proportionality between  $\Delta E$  and  $B$  is the Bohr magneton. Thus

$$\mu_B = hc \frac{\Delta \bar{\nu}}{2B} \quad (29)$$

### III. EXPERIMENTAL SET-UP AND PROCEDURE

The experimental set-up of the experiment is presented in figure (9). The electromagnet is put on the rotating table for heavy loads and mounted with the two pole-shoes with holes in such a way that a gap large enough for the Cd-lamp (9-11 mm) remains for the Cd-lamp. The pole-shoes have to be well tightened in such a way that they cannot move later on when the magnetic flux is established. The Cd-lamp is inserted into the gap without touching the pole-shoes and connected to the power



FIG. 9: Experimental setup with CMOS-camera

supply for spectral lamps. The coils of the electromagnet are connected in parallel and via an ammeter connected to the variable power supply of up to 20 VDC, 12 A. A capacitor of 22 000  $\mu$ F is in parallel to the power output to smoothen the DC-voltage.

The optical bench for investigation of the line splitting carries the following elements (their approximate position in cm is given in brackets) and is illustrated in figure (10):

- (80) CMOS Camera
- (73)  $L_3 = 50$  mm
- (68) Screen with scale
- (45) Analyser
- (39)  $L_3 = 300$  mm
- (33) Fabry-Perot Etalon
- (25)  $L_3 = 50$  mm
- (20) Iris diaphragm
- (20) Drilled pole-shoes
- Cd-spectral lamp on rotating table

The iris diaphragm is eliminated for initial adjustment and for the observation of the longitudinal Zeeman effect. During observation of the transverse Zeeman effect the iris diaphragm is illuminated by the Cd-lamp and such it acts as the light source. The lens  $L_1$  and a lens of  $f = 100$  mm, incorporated in the étalon, create a nearly parallel light beam which the Fabry-Perot étalon needs for a proper interference pattern.

The etalon contains a removable colour filter that lets the red cadmium line at 643.8 nm pass. The lens  $L_2$  produces an interference pattern of rings which can be observed through  $L_3$ . The ring diameters can be measured using the CMOS-camera and the software supplied with it. In the classical version the interference pattern is produced within the plane of the screen with a scale mounted on a slide mount which can laterally be displaced with a precision of 1/100th of a millimeter. The measurement here can be done for instance, by systematic displacement of the slash representing the '0' of the scale.



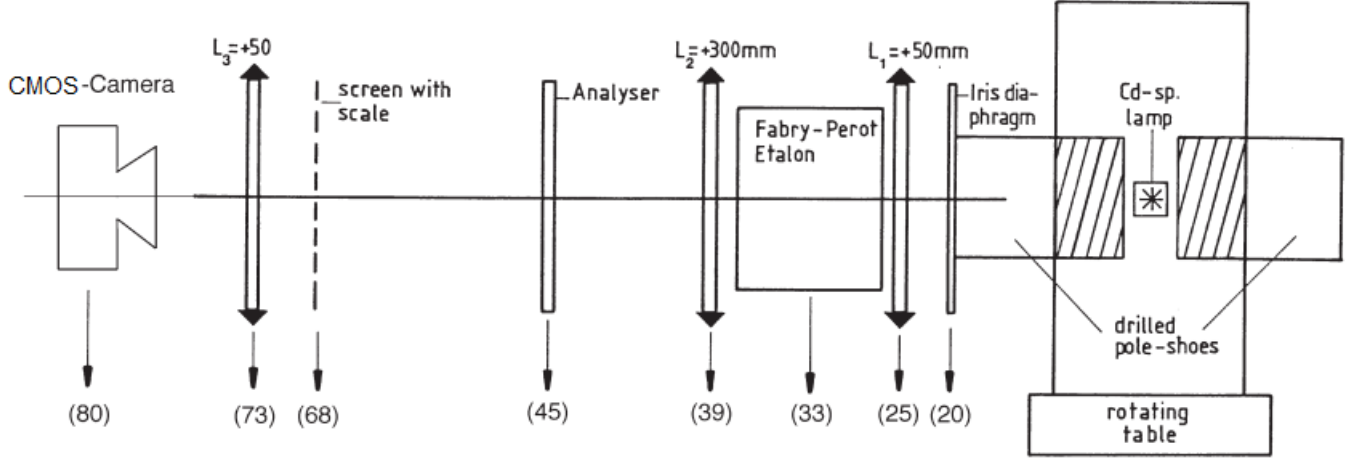


FIG. 10: Arrangement of the optical components.

The initial adjustment is done in the following way: The rotating table with electromagnet, pole-shoes and Cd-lamp already mounted is adjusted so that the center of the holes in the pole-shoes lies about 28 cm above the table. The optical bench with all elements (except iris diaphragm and CMOS-camera) mounted, is then moved closer to the electromagnet in such a way that one of the outlet holes of the pole-shoes coincides with the previous position of the iris diaphragm.  $L_1$  is then adjusted so that the outlet hole is within the focal plane of it. All other optical elements of figure (10). are subsequently readjusted with respect to their height correspondingly. The current of the coils is set for some time to 8 A (increase in light intensity of the Cd-lamp) and the ring interference pattern in axial direction is observed through  $L_3$  by the eye. The pattern must be centered and sharp which is eventually achieved by a last, slight movement of the étalon (to the right or to the left) and by displacement of  $L_2$  (vertically and horizontally).

Finally the CMOS-camera with the 8 mm lens attached is mounted to the optical bench and adjusted in horizontal and vertical position as well as in tilt and focus until a clear picture of the ring pattern is visible on the computer screen. The electromagnet is now turned by  $90^\circ$ , the iris diaphragm is inserted and the analyzer turned until the  $\pi$ -line disappears completely and the two  $\sigma$ -lines appear clearly visible.

#### IV. OBSERVATIONS

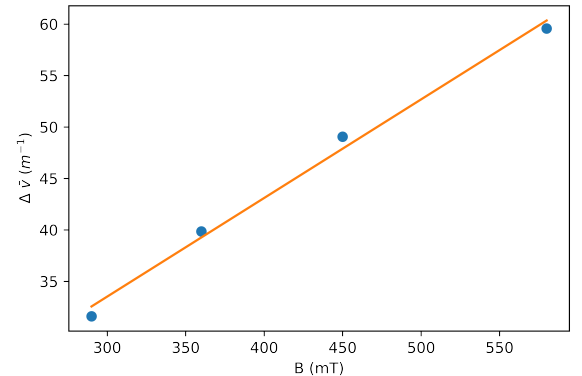
The make of the instrument is PHYWE.

1. The refractive index at  $\lambda = 643.8 \text{ nm}$ ,  $\mu = 1.456$
2. The etalon spacing,  $t = 3 \text{ mm}$

The graph between  $B$  and  $\Delta\bar{\nu}$  from data in table (I) is plotted in figure (11).

TABLE I: Readings for the experiment

Distance (mm)	Order of fringe $p$	$r_a$ ( $\mu \text{ m}$ )	$r_b$ ( $\mu \text{ m}$ )	$B$ (mT)	$\Delta\bar{\nu}$ ( $\text{m}^{-1}$ )
40	1	18.97	66.955		
40	2	89.26	110.47	580	59.56687
40	3	127.09	141.49		
42	1	28.865	62.715		
42	2	92.45	108.1	450	49.0555
42	3	126.51	140.41		
44	1	33.395	60.045		
44	2	94.46	106.96	360	39.84824
44	3	128.12	139.89		
46	1	36.31	57.39		
46	2	93.91	105.16	290	31.60394

FIG. 11: The  $B \sim \Delta\bar{\nu}$  plot

## V. CALCULATIONS AND RESULTS

Now the Bohr magneton is given from (29) as

$$\mu_B = \frac{hc}{2} m_{slope} \quad (30)$$

where  $m_{slope}$  is the slope of the line plotted in figure (11). The five summations of for the data set plotted in figure (11) are as follows:

$$S_x = \sum x_i = 1680 \text{ mT}, \quad S_y = \sum y_i = 180.074 \text{ m}^{-1},$$

$$S_{xx} = \sum x_i^2 = 752\,600 \text{ mT}^2,$$

$$S_{yy} = \sum y_i^2 = 8541.344\,518 \text{ m}^{-2},$$

$$S_{xy} = \sum x_i y_i = 80\,134.264\,88 \text{ mT m}^{-1}.$$

Now the slope is given by,

$$m_{\nu_1} = \frac{SS_{xy} - S_x S_y}{SS_{xx} - S_x^2} = 95.8076 \text{ m}^{-1} \text{ T}^{-1} \quad (31)$$

Now, putting in the values in (30), we get

$$\boxed{\mu_B = 9.522 \times 10^{-24} \text{ J T}^{-1}} \quad (32)$$

## VI. ERROR ANALYSIS

Error in  $\mu_B$  is equal to the error in slope as rest terms are assumed exact. Now error in slope is given by

$$\sigma_m = \sigma_y \times \sqrt{\frac{S}{\Delta}} \quad (33)$$

$\sigma_y$  is the uncertainty in the measurement of  $\Delta\nu$ . We need uncertainties in  $\delta$  and  $\Delta$  to calculate

that. Using propagational error formulae, we obtain  $dm_{slope} = 5.241 \text{ m}^{-1} \text{ T}^{-1}$  (considering the maximum error). Putting in the values in equations (33) and solving, we get

$$\boxed{d\mu_B = 1.041 \times 10^{-24} \text{ J T}^{-1}} \quad (34)$$

## VII. DISCUSSIONS

1. the effect is demonstrated with the light of a Cadmium lamp and the help of a Fabry-Perot interferometer for resolving a small part of the spectrum pre-selected by a color filter or an interference filter so only the light of a single atomic transition line is observed.
2. Without field the magnetic sub-levels have the same energy but with field the degeneration of the levels with different is cancelled and the line is split.
3. Cadmium has a similar outer electron structure to that of Helium but also of Mercury. In a completed shell in it's ground state the electron spins always compensate each other – they are anti-parallel. If the total electron spin is zero, also the magnetic moment connected to electron spin is zero. Atomic states with zero total spin are called singlet states. So in transitions between different singlet states the magnetic moment of spin does not play a role, as is the case with the normal Zeeman effect.
4. The  $B \sim \Delta\nu$  plot was obtained to be a straight line, that is, with increase in magnetic field, the difference in wave numbers increased.

## VIII. CONCLUSIONS

1. The value of  $\mu_B$  was found to be  $\mu_B = (9 \pm 1) \times 10^{-24} \text{ J T}^{-1}$  after taking care of the significant figures.
2. The results so obtained were in close to the literature value. The experiment was thus a success.